**Birla Institute of Technology & Science, Pilani**

**Second Semester 2022-2023**

**Online B.Sc. Computer Science Programme**

**Comprehensive Make-up Exam**

Course No. : BCS ZC219

Course Title : Discrete Mathematics

Nature of Exam : Open Book

Weightage : 50%, 150 marks

No. of Pages = 2

# No. of Questions = 8

Duration : 2 Hours 30 Minutes

Date of Exam : 03 Mar 23

Note to Students:

1. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
2. Assumptions made if any, should be stated clearly at the beginning of your answer.
3. Suppose n=m2 , so that n and m are integers and n is an even number. Prove that m is also an even number.

[20 Marks]

1. Provide a counterexample to the following statement:

If f∘g is one-to-one, then f is one-to-one.

[20 Marks]

1. Give an example of two uncountably infinite sets A and B, such that A - B is
2. Finite
3. Countably infinite
4. Uncountably infinite

[15 Marks]

1. Consider the vertex set V={v1, … , vn}.

How many distinct directed graphs exist that have V as their vertex set?

[15Marks]

1. Given an undirected graph, its degree sequence is the monotonic nonincreasing sequence of the degrees of the vertices of the graph. For example, the cycle C4 has the degree sequence 2, 2, 2, 2 and the wheel W4 has the degree sequence 4, 3, 3, 3, 3.
2. What is the degree sequence of the complete bipartite graph Km,n , where m and n are positive integers. Briefly justify. [5 Marks]
3. Provide a counterexample to the following statement:  
   If two simple undirected graphs have the same degree sequence, then they are isomorphic. [20 Marks]
4. Show that all vertices visited in a directed path connecting two vertices in the same strongly connected component of a directed graph are also in that strongly connected component

[15 Marks]

Q.7. For the following weighted undirected graph, briefly trace the steps taken by the following algorithms in determining the minimum spanning tree: (a) Prim’s Algorithm (b) Kruskal’s Algorithm.

[20 Marks]

Diagram, shape

Description automatically generated

Q.8. Prove that the union of two subgroups of a group need not necessarily be a subgroup.

[20 Marks]

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