3. Regression.

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Last Minute Notes.

- **Introduction:**

Q- What is Linear regression ? & application of LR.

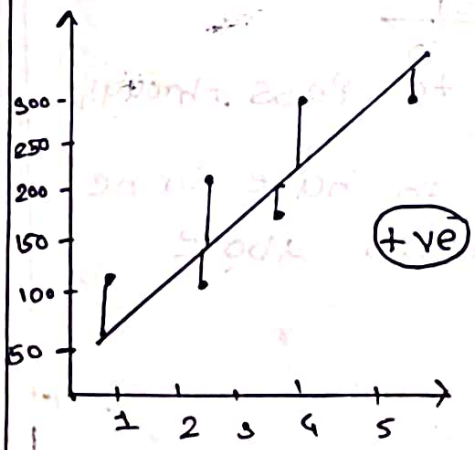
→ Liniear regression illustrate the relationship between two variables or factors.

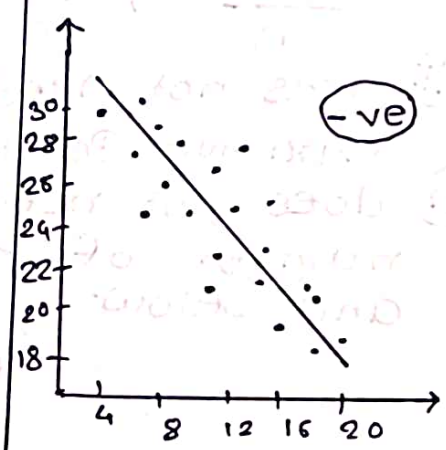
Y → dependent Variable

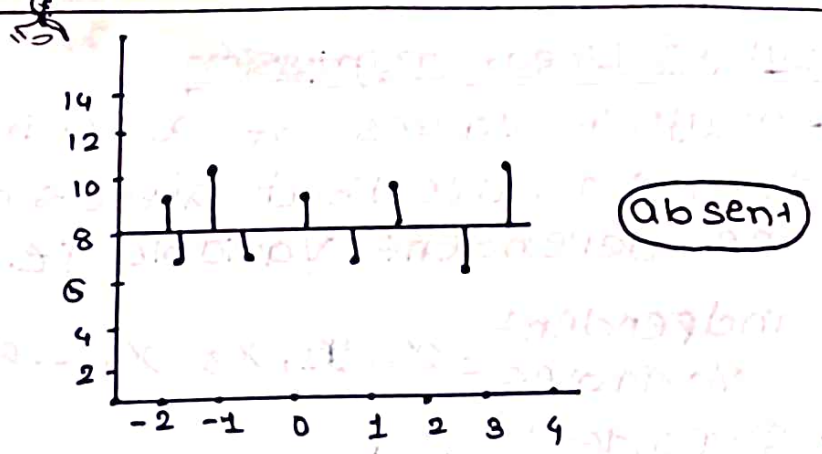
x → Independent Variable

$y = b_0 + b_1 * x \Rightarrow y = c + mx$

- Positive relationship
- negative relation ship.
- absent relationship







- Models – (i) Univariate Linear regression.
  (ii) Multiple Linear regression.

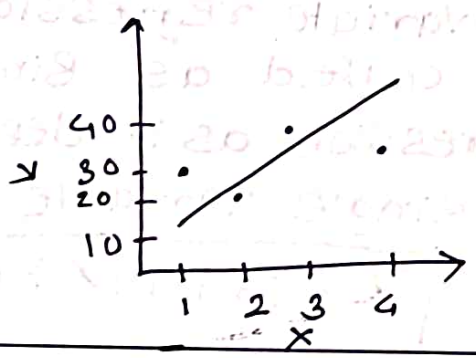
- **univariate Linear regression:**
  - only one dependent & indepedent Variable
  - Linear regression where only one (uni) input variable i.e. (x) known as univariate Linear regression

∴ $y = \theta_1 + \theta_2 x$

θ_1 - intercept

θ_2 = coefficient of x (i.e. slope).

x	y
1	15
2	30
3	40
4	30



• Multiple Linear regression

- Multiple values of x i.e. independent variable that affects only one dependent variable i.e. (y).
- independent variables $= x_1, x_2, x_3, x_4 \ldots x_n$
- Dependent variables $= y$.

$\therefore$ $\boxed{y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \ldots \beta x_n}$

Examples:

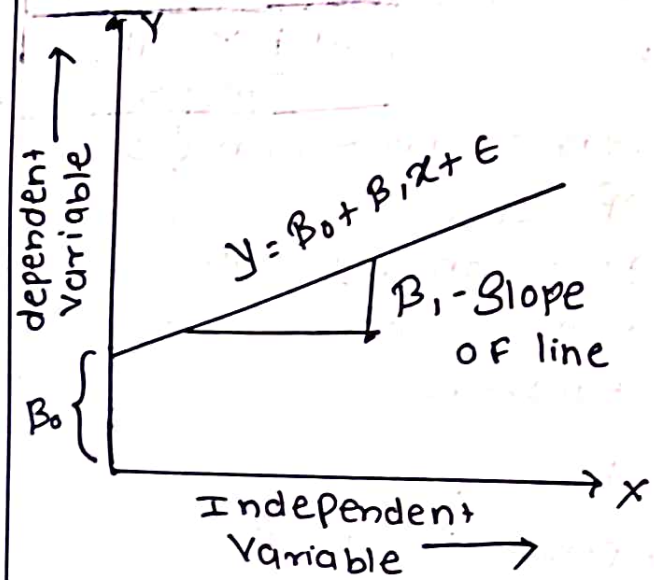
1. Selling price of house depends upon multiple variables like location, no. of bedrooms, year of buiding
2. Height of child depend on harmons, nutrition, enviromental factors.

• Least-square method:

- Univariate regression can be also called as simple linear regression: as it depends only of single variable

eg:- $\boxed{Y = \beta_0 + \beta_1 x + \epsilon}$

ϵ - random error



• Best fit regression

(i) Regression through iminimum sum of error.

(ii) it must pass through centroid of data. $(\bar{X}, \bar{Y})$

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}, \quad \bar{Y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

(iii) does not need to pass through maximum point

(iv) does not need to have same number of points above and below.

$$y = \beta_0 + \beta_1 x + \varepsilon$$

$$\beta_0 = \bar{Y} - \beta_1 \bar{X}$$

$$\beta_1 = \frac{\sum_{i=1}^{n} (y_i - \bar{Y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

The least square method is the process of finding the best-fitted curve or line of best fit for a set of data points by reducing the sum of the squares of offsets of the points from the curve.

least square ⇒ Sum of all square of error is minimum

Cost function:

MSE (Mean Squared error)

avarage of sum of squared of diffrence between actual value and predicted value known as mean squared error.

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2}{n}$$

$$MSE = \frac{SSE}{N}$$

ME Mean error

Avarage of diffrance of actual values and predicted values

$$ME = \frac{\sum_{i=1}^{n} (y_i - \bar{y}_i)}{n}$$

MAE

The Avarage Absolute diffrance bet^n actual & predicted values.

$$MAE = \frac{\sum_{i=1}^{n} |y_i - \bar{y}_i|}{n}$$

R-Squared (R2-Score)

Statastical measure that represent the goodness of fit of regression model

$$R^2 = 1 - \frac{SSE}{Var(y)}$$

NMSE (Normalized mean squared error)

$$NMSE = \frac{SSE}{Var(y)} = \frac{1}{Var(y) \times} \times \sum_{i=1}^{n} (y_i - \bar{y}_i)^2$$

RMSE (Root mean squared error)

$$RMSE = \sqrt{MSE}$$

$$SSE = \sum_{i=1}^{n} (y_i - \bar{y}_i)^2$$

- Problems

• Performance evaluation :

(i) Various ML algorithm are evaluated using performance metrics.

(ii) After training Performance is done on Testing dataset.

(iii) confusion matrix used to evaluate model.

★ Confusion Matrix

Predicted \ Actual	1	0
1	TP	FP
0	FN	TN

(i) Accuracy :-

$$A = \frac{TP+TN}{Total}$$

A		
TP	FP	
FN	TN	

(ii) precision

$$P = \frac{TP}{TP+FP}$$

TP	FP
FN	TN

(iii) Recall

$$R = \frac{TP}{TP+FN}$$

TP	FP
FN	TN

(iv) F1-Score

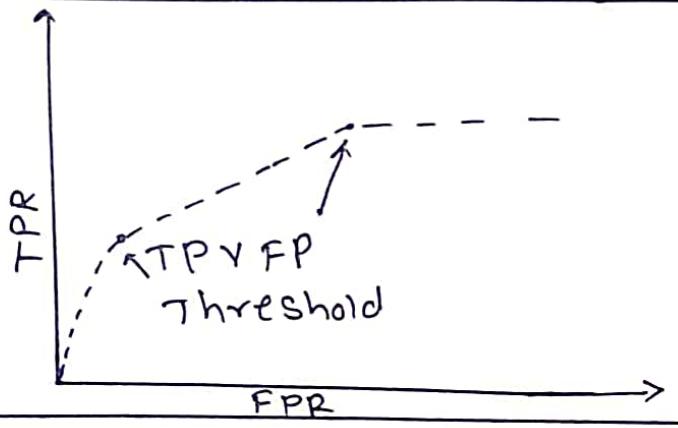
$$F1 = \frac{2 \times Precision \times recall}{(Precision + Recall)}$$

• Roc curve

Reciver oprating charecterstic curve is a graphical Plot that explain the diagnostic ability of a binary classifier system.

$$TPR \text{ (True +ve rate)} = \frac{TP}{TP+FN}$$

$$FPR \text{ (false +ve rate)} = \frac{FP}{FP+TN}$$



• Problems

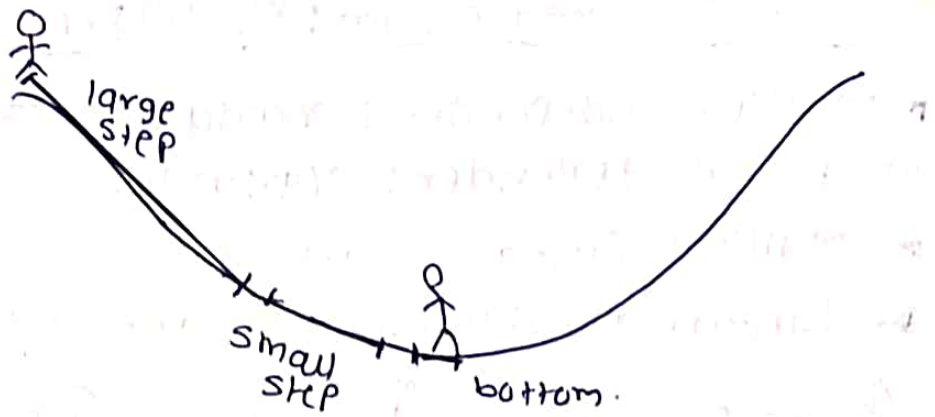
Optimization of Simple linear regression with Gradient descent - Example

- **Gradient Descent Algorithm**
  it is an iterative optimization algorithm to find minimum of fun^{n}. Here that function is Loss fuction

- By using gradient descent we perform derivative of Variable to get the best-fit

- Model makes prediction → error is calculater → model updated → error is reduced → next prediction → (back to prediction)

- Example:- Person wan'ts to reach bottom of velly with no sense of direction. when the slope (steep) is inclide he takes bigger steps & when the value of D (slope) is low he takes the smaller steps.



Steps

① m=0 C=0 L=0.0001

if L is high the low accurcy (Person will keep oscilating)

② $Dm = \frac{1}{n}\sum_{i=0}^{n} 2(y_i - (mx_i + c))(-x_i)$

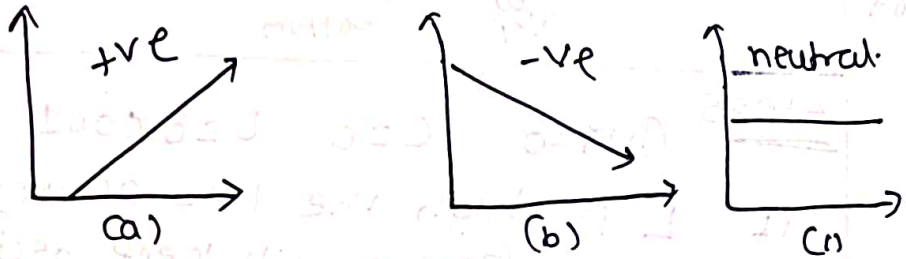
$Dm = \frac{1}{n}\sum_{i=0}^{n} 2(y_i - \bar{y})(-x_i)$

$Dc = \frac{-2}{n}\sum_{i=0}^{n} (y_i - \bar{y})$

③ Repeat function till Lis very low → 0

Multivaridale regression: model Representation

- Multiple independent variable for only one dependent variable.
- Multiple linear regression
- Linear relationship maybe +ve or -ve



- representation

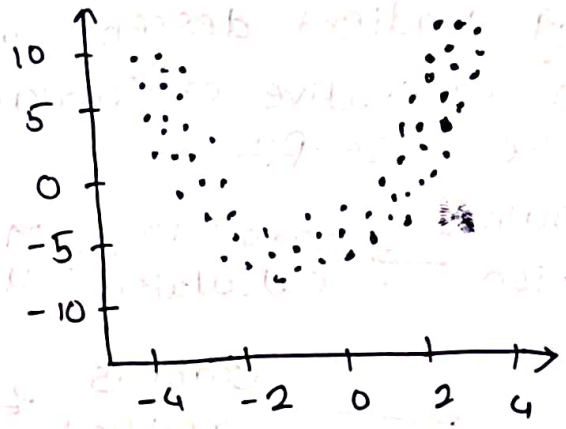
$$Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 \dots b_nx_n$$

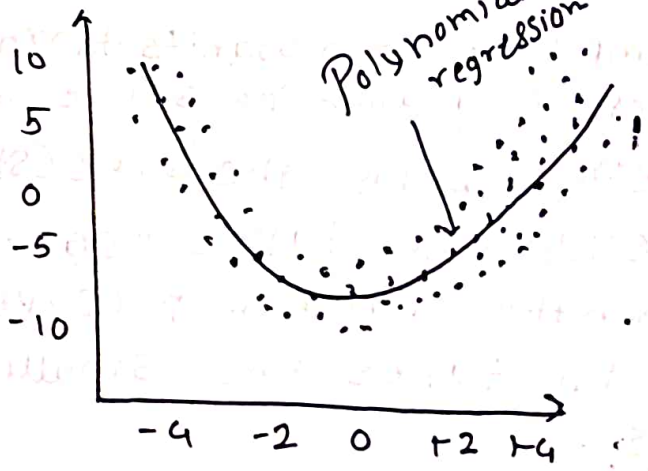
- Multiple Linear regression attempt to model the relationship between two or more features and d_espole by fitting a linear equation to bobserved data.

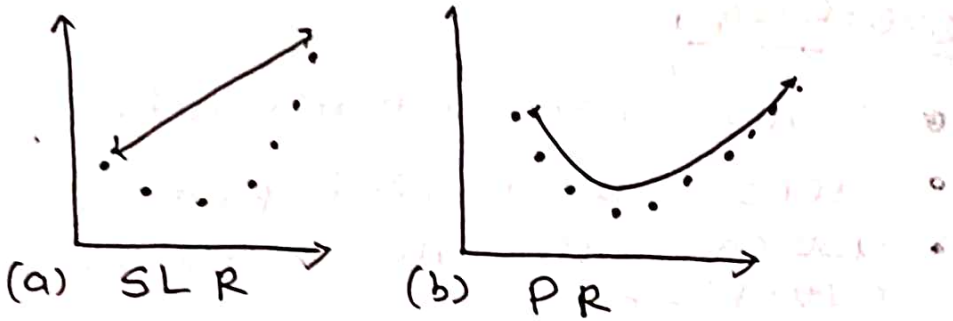
Predictive ability	Interpredictive.
Test error is combibinat of two or more variable	coeficient is assigned to every element/ variable.
accuracy can be ↑ by losing some amount of bias	

Polynomial Regression-

- relation is kind of regression analysis in which the relationship between indpedent variable x & dependent y is in nth degree of polynomial.







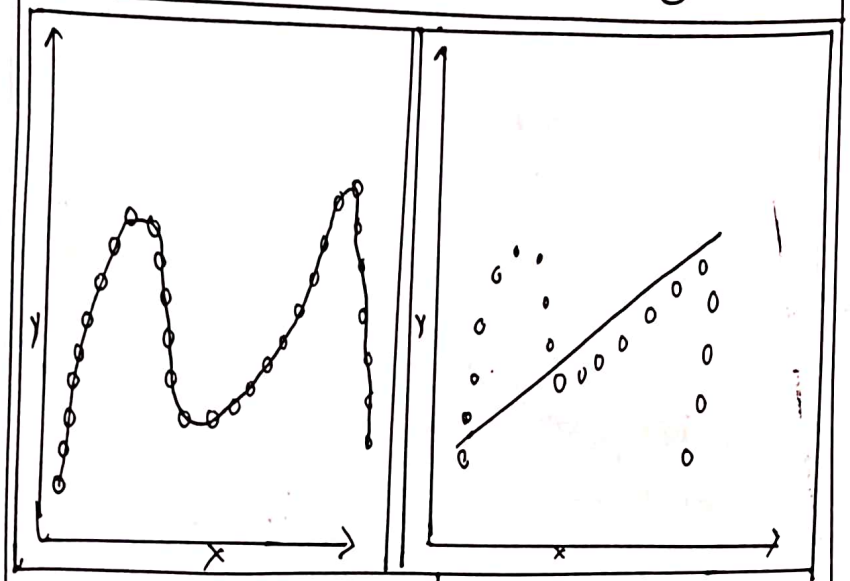
- High Accuracy
- For non-linear dataset
- low loss fuction
- Less error
- Special case of mutiple Linear regression.

$$Y = b_0 + b_1 x^2 + b_2 x^3 + b_3 x^4$$

Advantages	Disadvantages
• Best approximatin • Widerange of fun^n • wide range of curve	• Even one or two outliner can affect resuit • very sensitive • fewer validation tools.

Q. write Short Note on Sigmoid

• Underfitting & overfitting



overfitting | underfitting

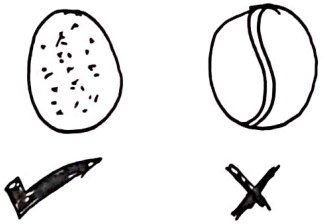
overfitting

- Trained with so many data
- Cannot categorized property
- use of linear algo to reduce overfitting.

underfitting

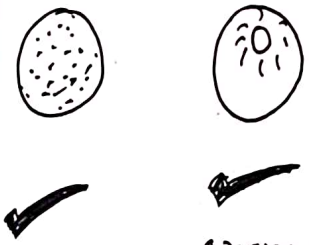
- Trained with very low data
- does not fit well
- more feature selection to overcome underfitting

① Ball
→ Sphare
→ raidus - 5cm
→ color - red
→ matrial - Plastic

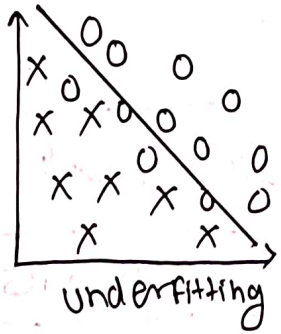


Plastic ball | tennis ball

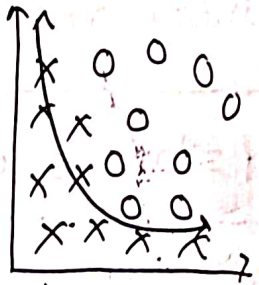
② Ball
→ Sphare



plastic ball | orange



underfitting



best-fit

overfit

Bias V/S Variance

Let consider we have two values

Predicted ⟷ actual

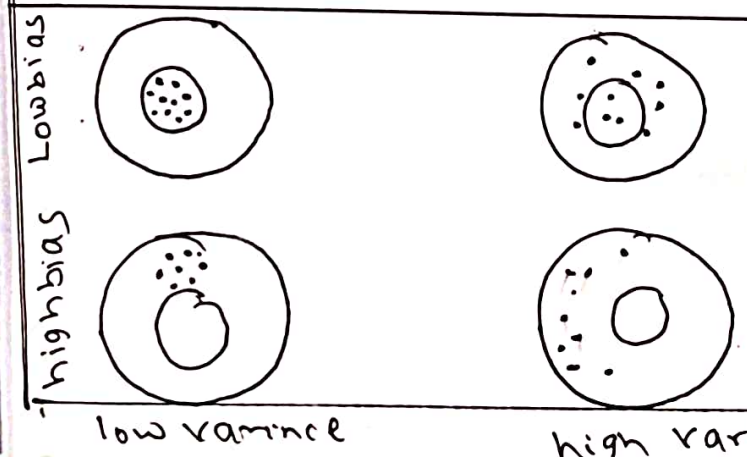
Bias = gap between two values

high Bias → Large gap	Low Bias → smaller gap

- Variance refers to the spread of expected values in relation to one another.

low variance → All predicted values we will see in a group

High variance → Predicted values will be scattred.



Bias	Variance
① Diffn between Prediction & actual Values	spread of our data is called as Varince of model
② Model with high bias pay little attension to training dataset	High variance model pay close attension to training data.
③ It leads to high error on training & test data.	perform very well on traing dataset