

INTRODUCTION TO CSP:

A CSP is a problem composed of a finite set of variables, each of which has a finite domain of values and a set of constraints. Each constraint is defined over some subset of the original set of variables and restricts the values these variables can simultaneously take. The task is to find an assignment of a value for each variable such that the assignments satisfy all the constraints. In some problems the goal is to find all such assignments. Constraint satisfaction problems on finite domains are typically solved using a form of search. The most used techniques are variants of backtracking, constraint propagation, and local search.

GRAPH COLORING:

Given a graph $G = (V, E)$ and an integer k , a k -coloring of G is a one-one mapping of vertices to colors, such that adjacent vertices are assigned to different colors. The Minimum Graph Coloring Problem (Min-GCP) consists in finding the minimum k such that a k -coloring exists. Such minimum k is known as the chromatic number of G and is denoted by $\chi(G)$, or simply by χ . Min-GCP is NP-hard. The chromatic number is bounded from below by the size of the maximum clique of G , known as the clique number $\omega(G)$ which is equal to $\chi(G)$ when G is a perfect graph. Chromatic number is the minimum number of colors required to satisfy the constraint that adjacent vertices do not have the same color.

MAP COLORING PROBLEM STATEMENT:

The Map coloring problem is similar to the graph coloring problem. In map coloring the constraint is that states which are adjacent to each other ie share a border should not have the same color. In the case of map coloring of Australia the chromatic number is 3 meaning if number of colors used is less than 3 then we would have some states which cannot be assigned colors as their domain size would reduce to 0. In the case of USA map coloring chromatic number is 4.

MAP COLORING SOLUTION APPROACHES:

There are 3 approaches we have used in this project DFS, DFS with forward checking, DFS with forward checking and propagation through singleton domain.

DEPTH FIRST SEARCH (BACKTRACKING):

The concept in backtracking applied to map coloring is that once any variable's domain reaches 0 then the algorithm goes back to previous states and see if using other options in the domain would yield a color for the one with the empty set in its domain. Every time the algorithm checks the next state only after reaching there, there are no pre checks done.

DFS WITH FORWARD CHECKING:

The concept here is exactly the same as backtracking only that next check is pre-checked making the algorithm smarter than Just backtracking. Number of backtracks are significantly reduced.

DFS WITH FORWARD CHECKING AND PROPAGATION THROUGH SINGLETON DOMAINS:

Here the algorithm checks among all possibilities of next states and chooses the one with domain value equal to 1 and propagates to the next unassigned variables from the one with domain =1. Number of backtracks are further reduced and the algorithm is relatively faster.

USING HEURISTICS:

There may be a number of options or nodes to choose from the next states. Any one of the states may be chosen at random and we could progress the map coloring algorithm. With using heuristics we get an order of choosing the variables depending on some factors. Some of the most commonly used heuristics are as follows below.

1. MINIMUM REMAINING VALUES:

In this heuristic propagation follows in the order of those nodes with least number of values in its domain . With respect to map coloring problem If one state has 2 permissible values in its domain and another state has 3 then the state with 2 values would be chosen first , here permissible refers to reducing domain size because of constraints imposed that adjacent states cannot have the same color.

2. DEGREE HEURISTIC:

The idea here is to assign a value to the variable that is involved in the largest number of constraints on other unassigned variables. It is often used as a means to reduce the number of same next possibilities ie as a tie breaker with Minimum remaining values heuristic to choose the best next when all next nodes have the same number of domain values after a variable assignment is done using Mrv.

3. LEAST CONSTRAINING VALUE:

Here the chosen heuristic rules out the fewest values in the remaining variables.

Variables used:-

Color List:- a list used to store list of colors

Backtracks:-to store the total number of backtracks

Start_time and end_time

time to calculate total run duration Numcolors:-

chromatic number Statedict:- list of states in usa

or aus root: stores color of root node.

Mystatedict:-color assigned as program

proceeds.

SIMULATION RESULTS

DFS for US Map:

No of Runs	No of Backtrack	Time(sec)
1	1012451	13.110958814620972
2	1012451	13.13487696647644
3	1012451	14.415603876113892
4	1012451	13.344930648803711

Search Path:

(1, {'Maine': 'red', 'Minnesota': 'red', 'South Dakota': 'blue', 'Illinois': 'red', 'Utah': 'red', 'Wyoming': 'green', 'Texas': 'blue', 'Idaho': 'blue', 'Wisconsin': 'blue', 'Connecticut': 'red', 'Pennsylvania': 'red', 'Kansas': 'green', 'West Virginia': 'blue', 'North Carolina': 'green', 'Colorado': 'blue', 'California': 'red', 'Florida': 'red', 'Vermont': 'red', 'Virginia': 'red', 'North Dakota': 'green', 'Michigan': 'green', 'New Jersey': 'blue', 'Nevada': 'green', 'Arkansas': 'green', 'Mississippi': 'red', 'Iowa': 'green', 'Kentucky': 'green', 'Maryland': 'green', 'Louisiana': 'black', 'Alabama': 'green', 'Oklahoma': 'red', 'New Mexico': 'green', 'Rhode Island': 'blue', 'Massachusetts': 'green', 'South Carolina': 'red', 'Indiana': 'blue', 'Delaware': 'black', 'Tennessee': 'blue', 'Georgia': 'black', 'Arizona': 'black', 'Nebraska': 'red', 'Missouri': 'black', 'New Hampshire': 'blue', 'Ohio': 'black', 'Oregon': 'black', 'Washington': 'red', 'Montana': 'red', 'New York': 'black'})

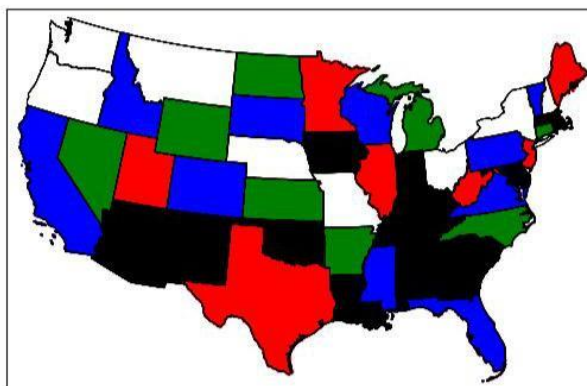


Figure: Image is captured at the time of generation so all the adjacent states may not have different colour.

DFS with Forward Checking for US Map:

No of Runs	No of Backtrack	Time(sec)
1	10238	71.71806001663208
2	10238	71.86888551712036
3	10238	72.59776878356934
4	10238	72.32587456211966

Search Path:

(1, {'New Hampshire': 'red', 'Oklahoma': 'red', 'Tennessee': 'red', 'Illinois': 'red', 'New Mexico': 'blue', 'Kentucky': 'blue', 'West Virginia': 'green', 'Maryland': 'red', 'Maine': 'blue', 'Wisconsin': 'blue', 'Missouri': 'green', 'Minnesota': 'red', 'Montana': 'red', 'Massachusetts': 'blue', 'South Carolina': 'red', 'North Dakota': 'blue', 'Pennsylvania': 'blue', 'Arizona': 'green', 'South Dakota': 'green', 'Ohio': 'red', 'Oregon': 'red', 'Alabama': 'blue', 'Indiana': 'green', 'Rhode Island': 'red', 'Virginia': 'black', 'Idaho': 'green', 'Nevada': 'blue', 'Nebraska': 'red', 'New York': 'green', 'Utah': 'red', 'Michigan': 'black', 'Kansas': 'blue', 'Florida': 'red', 'Connecticut': 'black', 'Iowa': 'black', 'Wyoming': 'blue', 'Louisiana': 'red', 'California': 'black', 'Vermont': 'black', 'Texas': 'green', 'Georgia': 'green', 'New Jersey': 'red', 'North Carolina': 'blue', 'Washington': 'blue', 'Delaware': 'green', 'Colorado': 'black', 'Mississippi': 'green', 'Arkansas': 'blue'})

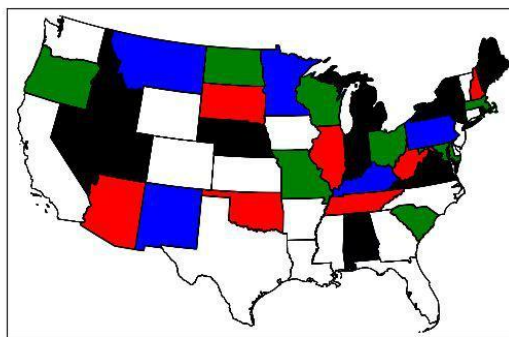


Figure: Image is captured at the time of generation so all the adjacent states may not have different colour.

DFS WITH SINGLETON AND FORWARD CHECK FOR US MAP:

No of Runs	No of Backtrack	Time(sec)
1	9691	0.30711936950683594
2	9691	0.2932472229003906
3	9691	0.3011901378631592
4	9691	0.30134129524230957

Search path:

['Ohio', 'Hawaii', 'Vermont', 'Maine', 'Tennessee', 'Oklahoma', 'Colorado', 'Alabama', 'Oregon', 'Minnesota', 'New Mexico', 'Mississippi', 'Kansas', 'New Hampshire', 'Louisiana', 'Rhode Island', 'Montana', 'Wisconsin', 'Michigan', 'Arkansas', 'Maryland', 'Missouri', 'Massachusetts', 'North Dakota', 'Nevada', 'South Dakota', 'Illinois', 'Washington', 'Virginia', 'Indiana', 'Alaska', 'Connecticut', 'North Carolina', 'New York', 'New Jersey', 'Iowa', 'Kentucky', 'South Carolina', 'West Virginia', 'Idaho', 'Florida', 'Delaware', 'Nebraska', 'Arizona', 'Wyoming', 'California', 'Utah', 'Texas', 'Pennsylvania', 'Georgia']

HEURISTICS FOR DFS:

No of Runs	No of Backtrack	Time(sec)
1	701288	48.62122273445129
2	701288	47.30119013786315
3	701288	49.56984423256123
4	701288	48.59866231002135

EXECUTION RESULTS WITH HEURISTICS FORWARD CHECK WITH DFS USA:-

No of Runs	No of Backtrack	Time(sec)
1	1714	0.07276415824890137
2	1714	0.0827786922454834
3	1714	0.07480001449584961
4	1714	0.07632985566584961

Search Path:

(1, { 'Illinois': 'red', 'Oklahoma': 'red', 'California': 'red', 'Utah': 'red', 'Wyoming': 'blue', 'Missouri': 'blue', 'Michigan': 'green', 'Texas': 'blue', 'Iowa': 'green', 'Delaware': 'red', 'Tennessee': 'red', 'Maryland': 'blue', 'Kentucky': 'green', 'Montana': 'red', 'Minnesota': 'red', 'Connecticut': 'red', 'Louisiana': 'red', 'West Virginia': 'red', 'Pennsylvania': 'green', 'Nebraska': 'red', 'Kansas': 'green', 'Indiana': 'blue', 'Rhode Island': 'blue', 'Arizona': 'blue', 'Florida': 'red', 'Massachusetts': 'green', 'South Dakota': 'black', 'Nevada': 'green', 'South Carolina': 'red', 'Ohio': 'black', 'New Hampshire': 'red', 'Idaho': 'black', 'Washington': 'red', 'Colorado': 'black', 'Oregon': 'blue', 'New Jersey': 'blue', 'Mississippi': 'blue', 'Arkansas': 'green', 'Vermont': 'blue', 'Wisconsin': 'blue', 'Alabama': 'green', 'Georgia': 'blue', 'Maine': 'blue', 'New Mexico': 'green', 'North Carolina': 'green', 'New York': 'black', 'Virginia': 'black', 'North Dakota': 'blue'})

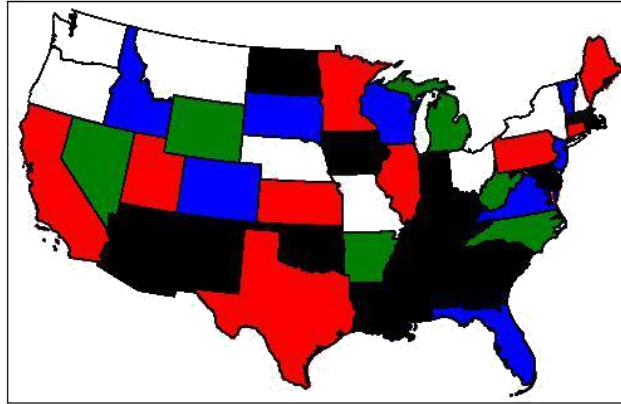


Figure: Image is captured at the time of generation so all the adjacent states may not have different colour.

Singleton With Heuristic for USA:

No of Runs	No of Backtrack	Time(sec)
1	10286	0.13962650299072266
2	10286	0.1356356143951416
3	10286	0.20747780799865723
4	10286	0.14262890815734863

Search Path:

(1, {'New Hampshire': 'red', 'Oklahoma': 'red', 'Tennessee': 'red', 'Illinois': 'red', 'New Mexico': 'blue', 'Kentucky': 'blue', 'West Virginia': 'green', 'Maryland': 'red', 'Maine': 'blue', 'Wisconsin': 'blue', 'Missouri': 'green', 'Minnesota': 'red', 'Montana': 'red', 'Massachusetts': 'blue', 'South Carolina': 'red', 'North Dakota': 'blue', 'Pennsylvania': 'blue', 'Arizona': 'green', 'South Dakota': 'green', 'Ohio': 'red', 'Oregon': 'red', 'Alabama': 'blue', 'Indiana': 'green', 'Rhode Island': 'red', 'Virginia': 'black', 'Idaho': 'green', 'Nevada': 'blue', 'Nebraska': 'red', 'New York': 'green', 'Utah': 'red', 'Michigan': 'black', 'Kansas': 'blue', 'Florida': 'red', 'Connecticut': 'black', 'Iowa': 'black', 'Wyoming': 'blue', 'Louisiana': 'red', 'California': 'black', 'Vermont': 'black', 'Texas': 'green',

'Georgia': 'green', 'New Jersey': 'red', 'North Carolina': 'blue', 'Washington': 'blue', 'Delaware': 'green', 'Colorado': 'black', 'Mississippi': 'green', 'Arkansas': 'blue'})

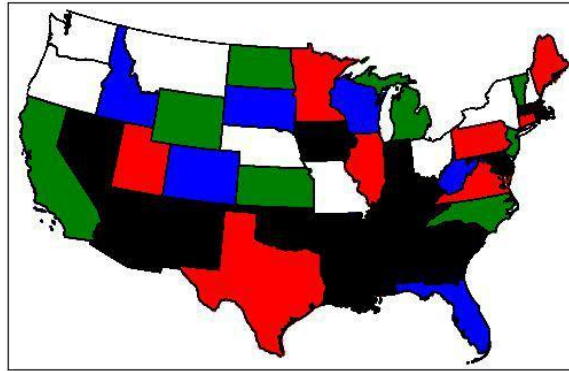


Figure: Image is captured at the time of generation so all the adjacent states may not have different colour.

TABULATION FOR AUSTRALIA:

DFS:

No of Runs	No of Backtrack	Time(sec)
1	0	0.017955541610717773
2	0	0.01798391342163086
3	0	0.019946813583374023
4	0	0.016956806182861328

Search Path:

(1, {'wa': 'red', 'nt': 'blue', 'q': 'red', 'nsw': 'blue', 'v': 'red', 'sa': 'green'})

WITH HEURISTIC:

No of Runs	No of Backtrack	Time(sec)
1	0	0.0013962659072266
2	0	0.0001356356143416
3	0	0.0002074778079986
4	0	0.00014262890815734

Search Path:

(1, {'wa': 'red', 'nt': 'blue', 'q': 'red', 'nsw': 'blue', 'v': 'red', 'sa': 'green'})

DFS WITH FORWARD CHECK:

No of Runs	No of Backtrack	Time(sec)
1	0	0.017955541610717773
2	0	0.01798391342163086
3	0	0.019946813583374023
4	0	0.016956806182861328

Search path:

(1, {'wa': 'red', 'nt': 'blue', 'q': 'red', 'nsw': 'blue', 'v': 'red', 'sa': 'green'})

WITH HEURISTIC:

No of Runs	No of Backtrack	Time(sec)
1	0	0.000997304916381836
2	0	0.000145326895231456
3	0	0.000123472350002238
4	0	0.000236598462134599

Search path:

(1, {'wa': 'red', 'nt': 'blue', 'q': 'red', 'nsw': 'blue', 'v': 'red', 'sa': 'green'})

DFS WITH SINGLETON AND FORWARD CHECK:-

No of Runs	No of Backtrack	Time(sec)
1	0	0.000997781753540039
2	0	0.000235000997781753
3	0	0.000123472350002238
4	0	0.00002568153235000

Search path:

{'wa': 'blue', 'nt': 'green', 'q': 'blue', 'nsw': 'green', 'v': 'blue', 'sa': 'red'}

WITH HEURISTIC:

No of Runs	No of Backtrack	Time(sec)
1	0	0.000997304916381836
2	0	0.000145326895231456
3	0	0.000123472350002238
4	0	0.000236598462134599

Search path:

{'wa': 'blue', 'nt': 'green', 'q': 'blue', 'nsw': 'green', 'v': 'blue', 'sa': 'red'}