Monte Carlo Simulation

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Problem Statement

Given the set of coupled ODEs:

$$\frac{dx}{dt} = 10(y - x) \tag{1}$$

$$\frac{dy}{dt} = x(28 - z) - y \tag{2}$$

$$\frac{dz}{dt} = xy - \frac{8}{3}z \tag{3}$$

$$x(0) = y(0) = z(0) = 1 \tag{4}$$

a) Write an explicit RK-4 scheme to solve the coupled system.
b) Now consider that the initial condition is not exactly known – it is only known that the initial value of (x,y,z) is a joint Gaussian with mean (1,1,1) and covariance of identity matrix. Sample 10,000 elements from this joint PDF and simulate the above coupled ODEs with each of those initial conditions. Plot the pairwise joint PDF of (x,y,z) at a few timesteps and explain your findings.

Solution

RK-4 scheme Let $X = [x, y, z]$ then $\frac{dX}{dT} = f(X)$ and $T_{i+1} - T_i = h$ and i is the simulation time step.
For RK 4:

$$k1 = f(X) \tag{5}$$

$$k2 = f(X + 0.5 * k1 * h) \tag{6}$$

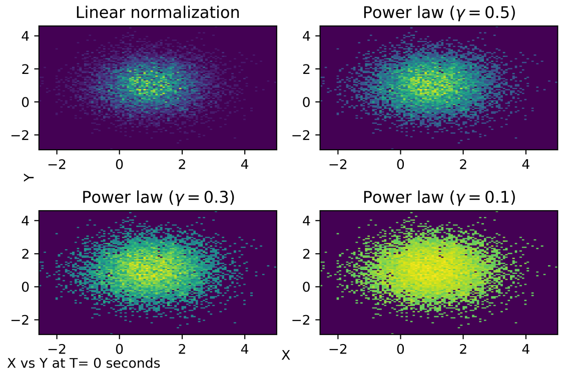
$$k3 = f(X + 0.5 * k2 * h) \tag{7}$$

$$k4 = f(X + k3 * h) \tag{8}$$

$$X_{i+i} = X_i + \frac{1}{6}(k1 + 2 * k2 + 2 * k3 + k4) * h \tag{9}$$

Setting $h = 0.05$ and simulating till $t = 15$ gives the values $X = [x, y, z] = [-9.9, -13.95, 23.23]$ when the initial conditions are $X = [1, 1, 1]$.
Simulation results
Link to code and output images: https://github.com/krutikapatil0109/MonteCarloSimulation
Simulation parameters: Setting $h = 0.05$ and simulating till $t = 15$

Discussion on results: At $T = 0$ the pairwise joint densities of x, y, z suggest that the variables x, y and z are pairwise independent.



Powerlaw normalization (available in matplotlib) is used for the density plots. It helps in clear visualization of the evolution of the dependence between any two variables.