Assignment 1

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Download all latex-tikz codes from

```
https://github.com/Krutikmehta/C−and−DS/tree/
    master/assignment1
```

1 Problem

(CS-2020/CS/Q.NO:46) Consider the following C functions.

```
int f1(int n){
    static int i=0;
    if(n>0){
        i++;
        f1(n−1);
    }
    return i;
}
```

```
int f2(int n){
    static int i=0;
    if(n>0){
        i += f1(n);
        f2(n−1);
    }
    return i;
}
```

Return the value of $f_2(5)$ -

2 Solution

Both the functions have static local variable. Static local variables have two characteristics -

1) The scope of such variable is just within the function.
2) The variable is not destroyed after function execution(it retains value). So, when function modifies the static local variable during the first function call, then this modified value will be available for the next function call also.

Explanation $f_1(n)$ returns the value of i(which has scope limited to f_1) after incrementing it by n.
$f_2(n)$ calls $f_1(i)$ from i=n to i=1 and adds the value returned by f_1 to i(which has scope limited to f_2).
when the first time f_1 is called for n=5, i is initialzed to 0.
The return value of each call is tabulated below.

$f_1(5)$	i=i+5;	i = 5.
$f_1(4)$	i=i+4;	i = 9.
$f_1(3)$	i=i+3;	i = 12.
$f_1(2)$	i=i+2;	i = 14.
$f_1(1)$	i=i+1;	i = 15.

All values returned by f_1 are added to static local variable i in f_2 -
therefore ,

$$i = 5 + 9 + 12 + 14 + 15 = 55.$$

Hence, ans = 55.

3 Mathematical Formulation

let n be the argument passed to f_2. From the table we can derive f_1 as -
After the first call to f_1, n is added to i and returned by f_1.

$$f_1(n) = n \tag{3.0.1}$$

After the second call to f_1, n-1 is added to i, which can be written as-

$$f_1(n-1) = (n-1) + n \tag{3.0.2}$$

$$f_1(n-1) = (n-1) + f_1(n) \tag{3.0.3}$$

Therefore,

$$f_1(n) = f_1(n-1) - (n-1) \tag{3.0.4}$$

similarly for next calls, n-2 is added to i,

$$f_1(n-2) = (n-2) + f_1(n-1) \tag{3.0.5}$$

$$f_1(n-1) = f_1(n-2) - (n-2) \tag{3.0.6}$$

Therefore, we can formulate f_1 as -

$$f_1(x) = \sum_{i=0}^{n-x} (n-i) \tag{3.0.7}$$

$$f_1(x) = \frac{n(n+1) - x(x-1)}{2} \tag{3.0.8}$$

and since $f_2(n)$ is the summation of $f_1(x)$ from x=1 to n.

$$f_2(n) = \sum_{x=1}^{n} f_1(x) \tag{3.0.9}$$

$$f_2(n) = \sum_{x=1}^{n} \frac{n(n+1) - x(x-1)}{2} \tag{3.0.10}$$

$$f_2(n) = \frac{1}{2}(\sum_{x=1}^{n}(n^2+n) + \sum_{x=1}^{n}(x-x^2)) \tag{3.0.11}$$

$$f_2(n) = \frac{1}{6}(2n^3 + 3n^2 + n) \tag{3.0.12}$$

4 Verification

For n = 5, the output of the c code was 55. Using the mathematical formula -

$$f_2(n) = \frac{1}{6}(2n^3 + 3n^2 + n) \tag{4.0.1}$$

$$f_2(5) = \frac{1}{6}(2 * 5^3 + 3 * 5^2 + 5) \tag{4.0.2}$$

$$f_2(5) = \frac{1}{6}(250 + 75 + 5) \tag{4.0.3}$$

$$f_2(5) = \frac{1}{6}330 = 55. \tag{4.0.4}$$

Hence verified.