

CONTENTS

1	Stability	1
1.1	Second order System	1
2	Routh Hurwitz Criterion	1
3	Compensators	1
4	Nyquist Plot	1

Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

1 STABILITY

1.1 Second order System

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT

4.1. Which of the following is **incorrect** ?

- (A) Lead compensator is used to reduce the settling time
- (B) Lag compensator is used to reduce the steady state error
- (C) Lead compensator may increase the order of a system
- (D) Lag compensator always stabilizes an unstable system

Solution:

Statement - A

Lead compensator is used to reduce settling time.

$$\text{system} - G(s) = \frac{1}{s(3s + 1)} \quad (4.1.1)$$

$$\text{lead compensator} - D(s) = \frac{3(s + \frac{1}{3})}{(s + 1)} \quad (4.1.2)$$

$$\text{hence, new system} - G_1(s) = \frac{1}{s(s + 1)} \quad (4.1.3)$$

unit impulse response

without lead compensator -

$$G(s) = \frac{1}{(s)(3s + 1)} \quad (4.1.4)$$

$$Y(s) = G(s).1 \quad (4.1.5)$$

$$Y(s) = \frac{1}{(s)(3s + 1)} \quad (4.1.6)$$

splitting into partial fractions

$$Y(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{3}} \quad (4.1.7)$$

taking inverse laplace transform,

$$y(t) = [1 - e^{-\frac{t}{3}}]u(t) \quad (4.1.8)$$

with lead compensator -

$$G(s) = \frac{1}{(s)(3s + 1)} \quad (4.1.9)$$

$$D(s) = \frac{3(s + \frac{1}{3})}{(s + 1)} \quad (4.1.10)$$

$$G_1(s) = \frac{1}{s(s + 1)} \quad (4.1.11)$$

$$Y_1(s) = G_1(s).1 \quad (4.1.12)$$

$$Y_1(s) = \frac{1}{s(s + 1)} \quad (4.1.13)$$

splitting into partial fractions

$$Y_1(s) = \frac{1}{s} - \frac{1}{s + 1} \quad (4.1.14)$$

taking inverse laplace transform,

$$y(t) = [1 - e^{-t}]u(t) \quad (4.1.15)$$

unit step response without lead compensator -

$$G(s) = \frac{1}{(s)(3s + 1)} \quad (4.1.16)$$

$$Y(s) = G(s).\frac{1}{s} \quad (4.1.17)$$

$$Y(s) = \frac{1}{(s^2)(3s + 1)} \quad (4.1.18)$$

splitting into partial fractions

$$Y(s) = \frac{1}{s^2} + \frac{3}{s + \frac{1}{3}} + \frac{-3}{s} \quad (4.1.19)$$

taking inverse laplace transform,

$$y(t) = [t + 3e^{\frac{-t}{3}} - 3]u(t) \quad (4.1.20)$$

with lead compensator -

$$G(s) = \frac{1}{(s)(3s + 1)} \quad (4.1.21)$$

$$D(s) = \frac{3(s + \frac{1}{3})}{(s + 1)} \quad (4.1.22)$$

$$G_1(s) = \frac{1}{s(s + 1)} \quad (4.1.23)$$

$$Y_1(s) = G_1(s) \cdot \frac{1}{s} \quad (4.1.24)$$

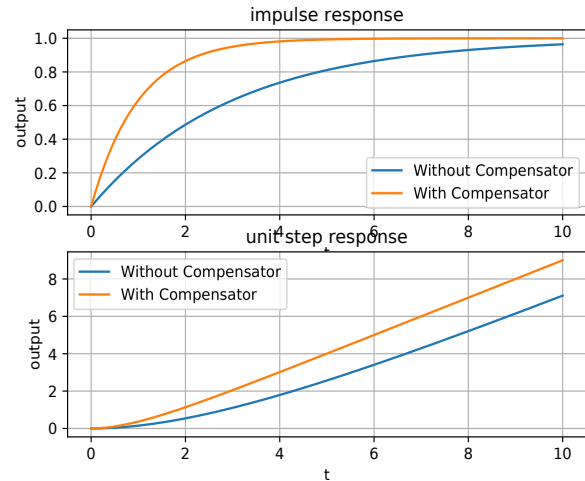
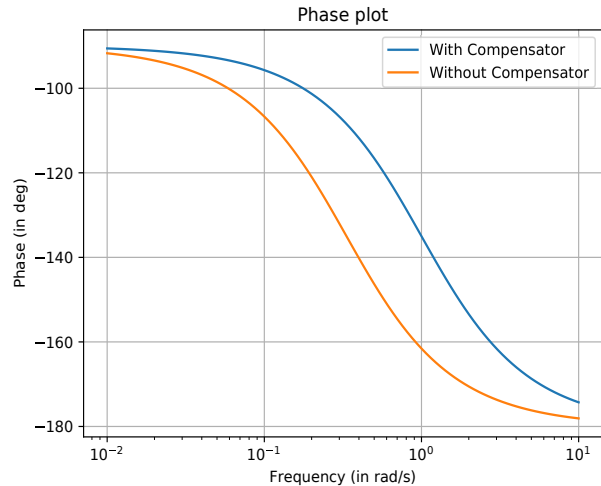
$$Y_1(s) = \frac{1}{(s^2)(s + 1)} \quad (4.1.25)$$

splitting into partial fractions

$$Y_1(s) = \frac{1}{s^2} + \frac{1}{s + 1} - \frac{1}{s} \quad (4.1.26)$$

taking inverse laplace transform,

$$y(t) = [t + e^{-t} - 1]u(t) \quad (4.1.27)$$



Since lead compensator adds + phase for any value of frequency, the bode plot for phase vs frequency is above the one which is without lead compensator.

phase margin $\phi_m = 180 + \phi_{gain=0}$

as lead compensator adds additional phase at all frequencies,

$\phi_{gain=0}$ gets increased, and hence phase margin
Relation between phase margin and damping ratio
Now, $\zeta = 0.01 \times \phi_m$

from this we get that damping factor also increases.

\Rightarrow damping is increased

\Rightarrow settling time decreased

Relation between phase margin and damping ratio.
Consider a second order system,

$$G(s) = \omega_n^2 \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4.1.28)$$

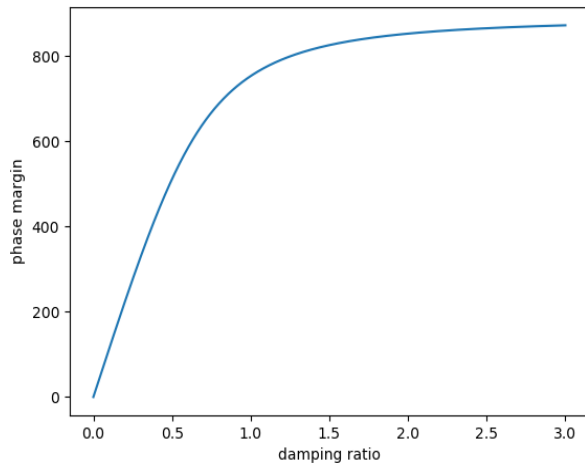
using value of ϕ_m to solve for ζ .

set $20 \log |G(s)| = -3\text{dB}$ to solve for ω_n

using this equations we get,

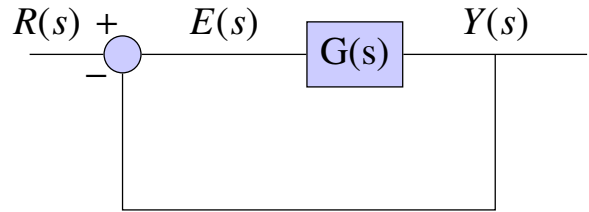
$$\phi_m = \tan^{-1} \frac{2\zeta}{\sqrt{1 + 4\zeta^4} - 2\zeta^2} \quad (4.1.29)$$

a handy relation is - $\zeta = 0.01\phi_m$



compensator has an high gain for low frequencies.
since steady state error is given by -

$$e(\infty) = \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)} \quad (4.1.33)$$



for low frequencies - the gain is high
 $\Rightarrow G(s) \rightarrow \text{large number}$

$\Rightarrow e(\infty) \rightarrow \text{smaller value} \Rightarrow \text{steady state error decrease}$

Statement - C

*Lead compensator may increase the order of a system.
consider -*

Statement - B

Lag compensator is used to reduce the steady state error.

Verification - Let the system transfer function be -

$$c(s) = \frac{1}{s+2} \quad (4.1.30)$$

and the lag compensator be-

$$t(s) = \frac{s+3}{s+1} \quad (4.1.31)$$

therefore resulting system transfer function-

$$G(s) = \frac{s+3}{(s+2)(s+1)} \quad (4.1.32)$$

Hence steady state error for unit step input,

$$\text{Without lag compensator} = \frac{1}{1+c(s)}$$

$$\text{as } s \rightarrow 0 \Rightarrow E_{ss} = \frac{1}{1+0.5} \Rightarrow E_{ss} = 0.66$$

$$\text{With lag compensator} = \frac{1}{1+G(s)}$$

$$\text{as } s \rightarrow 0 \Rightarrow E_{ss} = \frac{1}{1+1.5} \Rightarrow E_{ss} = 0.4$$

hence, the steady state error reduces.

From the bode plot it is clear that the lag

$$G(s) = \frac{1}{s+2} \quad (4.1.34)$$

$$D(s) = \frac{s+1}{s+3} \quad (4.1.35)$$

$$G(s).D(s) = \frac{s+1}{(s+2)(s+3)} \quad (4.1.36)$$

$$G(s).D(s) = \frac{s+1}{s^2+5s+6} \quad (4.1.37)$$

Maximum power in denominator = 2

Hence order increased to 2 from 1

Lead compensator may increase the order of a system since the transfer function adds a pole and a zero therefore it may increase the order of a system.

Statement - D

Lag compensator always stabilizes an unstable system.

This statement is wrong. Consider,

$$G(s) = \frac{1}{s-2} \quad (4.1.38)$$

$$D(s) = \frac{s+3}{s+1} \quad (4.1.39)$$

$$G(s).D(s) = \frac{(s+3)}{(s-2)(s+1)} \quad (4.1.40)$$

$$(4.1.41)$$

$$\text{output} = 1.66e^{(2t)} + 0.66e^{(-t)} \quad (4.1.42)$$

If a system has a pole on right side of s plane, lag compensator cannot stabilize those systems. This is because the resulting system also has an pole on the right side of s plane.

The transfer function of this lead compensator is -

$$\frac{V_o(s)}{V_i(s)} = \beta \frac{s\tau + 1}{s\beta\tau + 1} \quad (4.1.44)$$

$$\text{where} \quad (4.1.45)$$

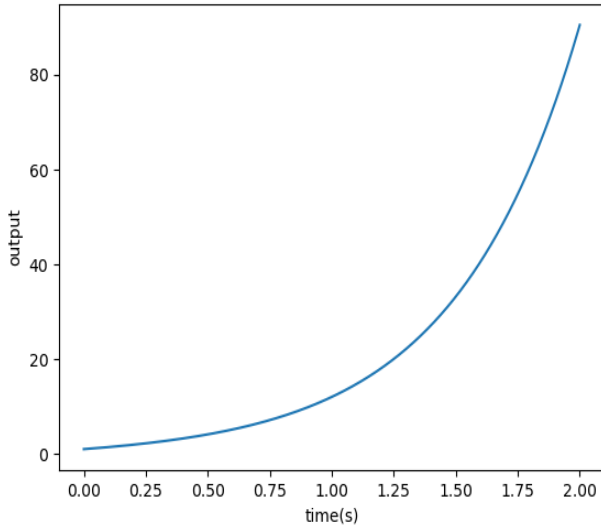
$$\tau = R_1 C \beta = \frac{R_2}{R_1 + R_2} \quad (4.1.46)$$

$$\text{substituting } s=j\omega, \quad \frac{V_o(j\omega)}{V_i(j\omega)} = \beta \frac{j\omega\tau + 1}{j\omega\beta\tau + 1}$$

$$\text{phase angle } \phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\beta\tau) \quad (4.1.47)$$

$$\text{since } 0 < \beta < 1 \quad (4.1.48)$$

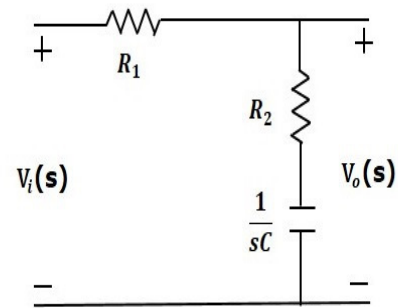
$$\phi > 0 \quad (4.1.49)$$



Lag compensator - The Lag Compensator is an electrical network which produces a output having the **phase lag** when a input is applied.

$$H(s) = \frac{s + z}{s + p} \quad 0 < p < z \quad (4.1.50)$$

The lag compensator circuit in the 's' domain is shown in the following figure.



0.5

The transfer function of this lag compensator is -

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha} \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}} \quad (4.1.51)$$

$$\text{where} \quad (4.1.52)$$

$$\tau = R_2 C \alpha = \frac{R_1 + R_2}{R_2} \quad (4.1.53)$$

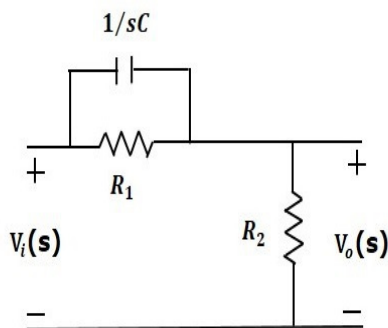
$$\text{substituting } s=j\omega, \quad \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha} \frac{j\omega + \frac{1}{\tau}}{j\omega + \frac{1}{\alpha\tau}}$$

Lead and Lag compensators -

Lead compensator - The lead compensator is an electrical network which produces a output having **phase lead** when a input is applied.

$$H(s) = \frac{s + z}{s + p} \quad 0 < z < p \quad (4.1.43)$$

The lead compensator circuit in the 's' domain is shown in the following figure.



0.5

phaseangle $\phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\alpha\tau)$
(4.1.54)

since $\alpha > 1$
(4.1.55)

$\phi < 0$
(4.1.56)

BODE PLOT

