Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot

4.1. Which of the following is **incorrect** ?
(A) Lead compensator is used to reduce the settling time
(B) Lag compensator is used to reduce the steady state error
(C) Lead compensator may increase the order of a system
(D) Lag compensator always stabilzes an unstable system

Solution: Lead and Lag compensators -

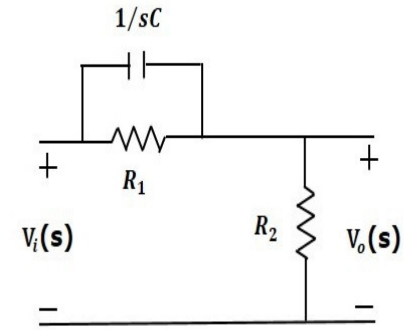
Lead compensator - The lead compensator is an electrical network which produces a output having **phase lead** when a input is applied.

$$H(s) = \frac{s+z}{s+p} 0<z<p \tag{4.1.1}$$

Lag compensator - The Lag Compensator is an electrical network which produces a output having the **phase lag** when a input is applied.

$$H(s) = \frac{s+z}{s+p} \qquad 0<p<z \tag{4.1.2}$$

Lead compensators - The lead compensator circuit in the 's' domain is shown in the following figure.



0.5

Lead compensator - The transfer function of this lead compensator is -

$$\frac{V_o(s)}{V_i(s)} = \beta \frac{s\tau + 1}{s\beta\tau + 1} \tag{4.1.3}$$

$$where \tag{4.1.4}$$

$$\tau = R_1 C\beta = \frac{R_2}{R_1 + R_2} \tag{4.1.5}$$

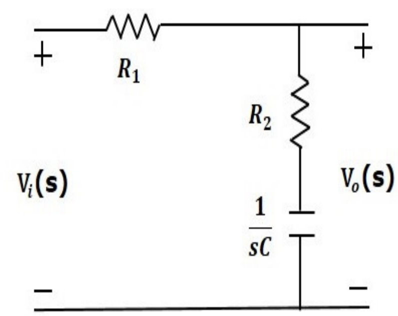
substituting s=jω
$\frac{V_o(j\omega)}{V_i(j\omega)} = \beta \frac{j\omega\tau+1}{j\omega\beta\tau+1}$(4.1.6)

$$phaseangle\phi = \tan^{-1}(\omega\tau) - \tan -1(\omega\beta\tau) \tag{1.7}$$

$$since 0< \beta < 1 \tag{1.8}$$

$$\phi > 0 \tag{1.9}$$

Lag compensators - The lag compensator circuit in the 's' domain is shown in the following figure.



0.5

The transfer function of this lag compensator is -

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha} \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}} \tag{1.10}$$

$$where \tag{1.11}$$

$$\tau = R_2 C\alpha = \frac{R_1 + R_2}{R_2} \tag{1.12}$$

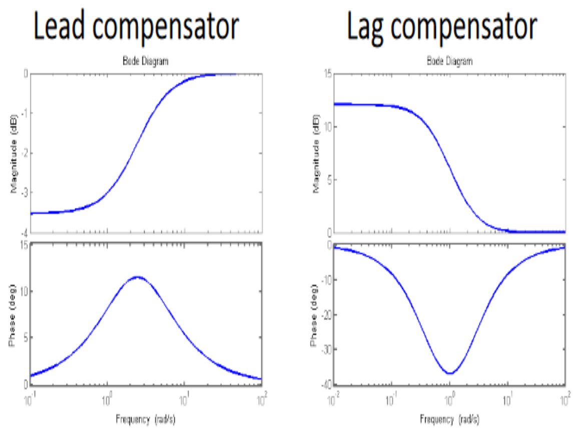
substituting s=jω
$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha}\frac{j\omega+\frac{1}{\tau}}{j\omega+\frac{1}{\alpha\tau}}$(1.13)

$$phaseangle \qquad \phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\alpha\tau) \qquad (1.14)$$

$$since\alpha > 1 \qquad (1.15)$$

$$\phi < 0 \qquad (1.16)$$

BODE PLOTS -



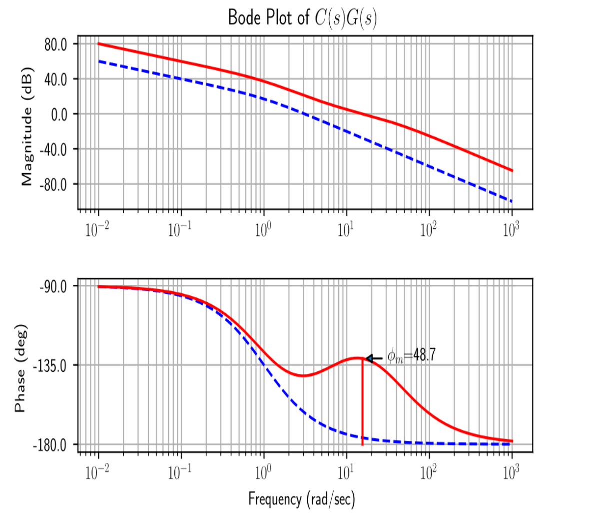
Statement - A
Lead compensator is used to reduce settling time. Since lead compensator adds + phase for any value of frequency, the bode plot for phase vs frequency is above the one which is without lead compensator.

phase margin $\phi_m = 180 + \phi_{gain=0}$
$asleadcompensatoraddsadditionalphaseatallfrequen$
gets increased, hence phase gain
Bode plot - increased phase margin -
Relation between phase margin and damping ratio Now, $\zeta = 0.01\times\ \phi_m$

from this we get that damping factor also increases.

$\implies$ *damping is increased*

$\implies$ *settling time decreased*



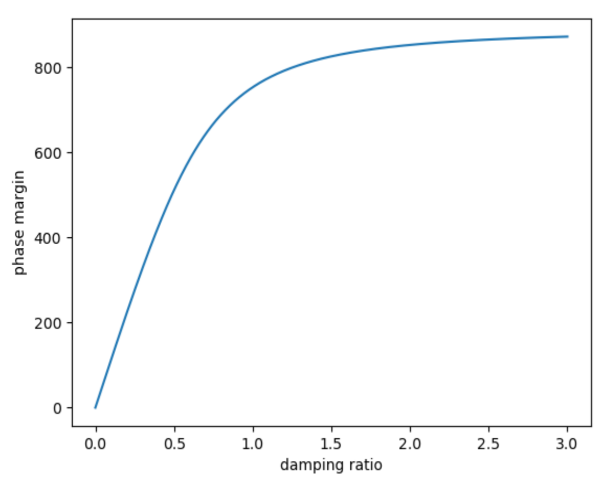
Relation between phase margin and damping ratio Consider a second order system,

G(s) = $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
using value of ϕ_m to solve for ζ.

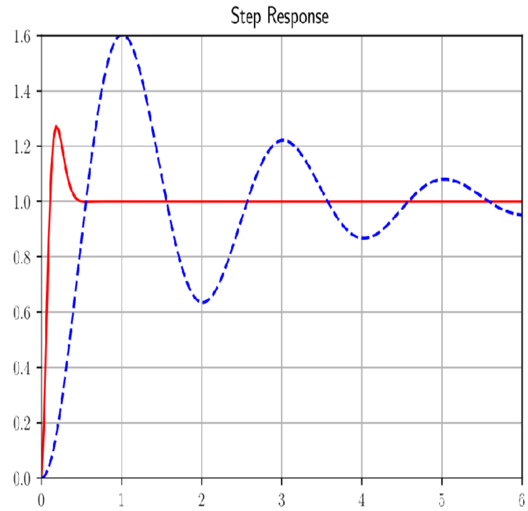
set 20 log$|G(s)|$ = -3dB to solve for ω_n

using this equations we get,

$\phi_m = \tan^{-1}\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4}-2\zeta^2}}$
$ahandyrelationis - \zeta = 0.01\phi_m$



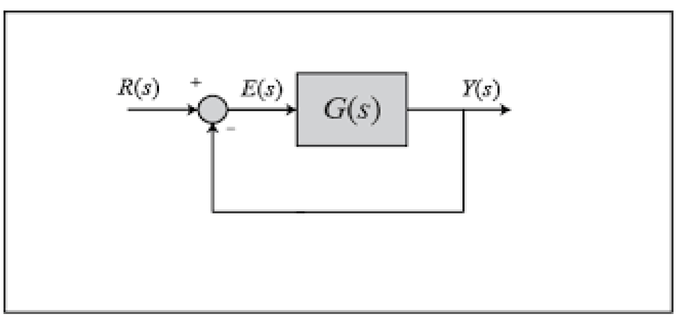
Time response - reduced settling time -



Statement - B

Lag compensator is used to reduce the steady state error From the bode plot it is clear that the lag compensator has an high gain for low frequencies. since steady state error is given by -

$$e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)} \tag{1.17}$$



for low frequencies - the gain is high $\implies G(s) \to number$
$\implies e(\infty) \to smallervalue$
$\implies steady\ state\ error\ decreases$

Verification - Let the system transfer function be -

$$c(s) = \frac{1}{s + 2} \tag{1.18}$$

and the lag compensator be-

$$t(s) = \frac{s + 3}{s + 1} \tag{1.19}$$

therefore resulting system transfer function-

$$G(s) = \frac{s + 3}{(s + 2)(s + 1)} \tag{1.20}$$

Hence steady state error for unit step input,

Without lag compensator = $\frac{1}{1+c(s)}$

as s $\to 0 \implies$ E$_{ss}$ = $\frac{1}{1+0.5} \implies E_{ss} = 0.66$

With lag compensator = $\frac{1}{1+G(s)}$

as s $\to 0 \implies$ E$_{ss}$ = $\frac{1}{1+1.5} \implies E_{ss} = 0.4$

Statement - C

Lead compensator may increase the order of a system since the transfer function adds a pole and a zero therefore it may increase the order of a system. For example -

$$\mathrm{R(s)} = \frac{1}{s+2}$$
$$\mathrm{G(s)} = \frac{s+1}{s+3}$$
$$\mathrm{R(s).G(s)} = \frac{s+1}{(s+2)(s+3)}$$
$$\mathrm{R(s).G(s)} = \frac{s+1}{s^2+5s+6}$$

Maximum power in denominator = 2
Hence order increased to 2 from 1

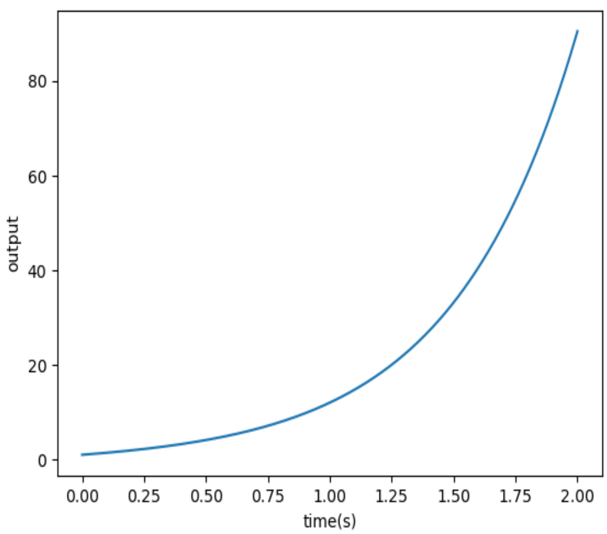
Statement - D

Lag compensator always stabilizes an unstable system

This statement is wrong.
If a system has a pole on right side of s plane, lag compensator cannot stabilize those systems. This is because the resulting system also has an pole on the right side of s plane.

$$R(s) = \frac{1}{s-2}$$
$$G(s) = \frac{s+3}{s+1}$$
$$R(s).G(s) = \frac{(s+3)}{(s-2)(s+1)}$$

$$output = 1.66e^{(2t)} + 0.66e^{(-t)} \tag{1.21}$$



output
time(s)
0
20
40
60
80
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00