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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

1 STABILITY

1.1 Second order System

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT

4.1. Which of the following is **incorrect** ?

- (A) Lead compensator is used to reduce the settling time
- (B) Lag compensator is used to reduce the steady state error
- (C) Lead compensator may increase the order of a system
- (D) Lag compensator always stabilizes an unstable system

Solution: Lead and Lag compensators -

Lead compensator - The lead compensator is an electrical network which produces a output having **phase lead** when a input is applied.

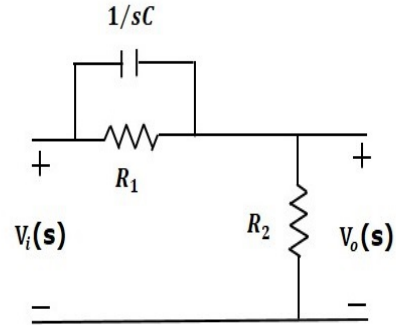
$$H(s) = \frac{s + z}{s + p} \quad 0 < z < p \quad (4.1.1)$$

Lag compensator - The Lag Compensator is an electrical network which produces a output having the **phase lag** when a input is applied.

$$H(s) = \frac{s + z}{s + p} \quad 0 < p < z \quad (4.1.2)$$

Lead compensators - The lead compensator circuit in the 's' domain is shown in the following figure.

0.5



Lead compensator - The transfer function of this lead compensator is -

$$\frac{V_o(s)}{V_i(s)} = \beta \frac{s\tau + 1}{s\beta\tau + 1} \quad (4.1.3)$$

$$\text{where} \quad (4.1.4)$$

$$\tau = R_1 C \quad \beta = \frac{R_2}{R_1 + R_2} \quad (4.1.5)$$

substituting $s = j\omega$

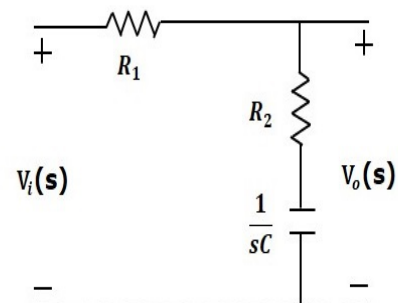
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \beta \frac{j\omega\tau + 1}{j\omega\beta\tau + 1} \quad (4.1.6)$$

$$\text{phase angle } \phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\beta\tau) \quad (1.7)$$

$$\text{since } 0 < \beta < 1 \quad (1.8)$$

$$\phi > 0 \quad (1.9)$$

Lag compensators - The lag compensator circuit in the 's' domain is shown in the following figure.



0.5

The transfer function of this lag compensator is -

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha} \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}} \quad (1.10)$$

where
(1.11)

$$\tau = R_2 C \quad \alpha = \frac{R_1 + R_2}{R_2} \quad (1.12)$$

substituting $s=j\omega$

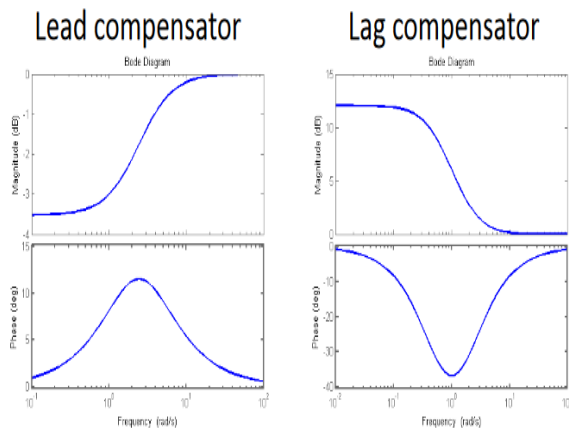
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha} \frac{j\omega + \frac{1}{\tau}}{j\omega + \frac{1}{\alpha\tau}} \quad (1.13)$$

phaseangle $\phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\alpha\tau)$
(1.14)

since $\alpha > 1$
(1.15)

$\phi < 0$
(1.16)

BODE PLOTS -



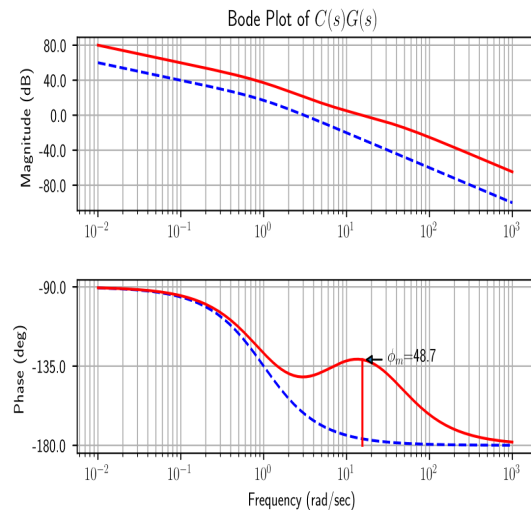
Statement - A

Lead compensator is used to reduce settling time. Since lead compensator adds + phase for any value of frequency, the bode plot for phase vs frequency is above the one which is without lead compensator.

phase margin $\phi_m = 180 + \phi_{\text{gain}=0}$

as lead compensator adds additional phase at all frequencies, $\phi_{\text{gain}=0}$ gets increased, hence phase gain

Bode plot - increased phase margin -



Relation between phase margin and damping ratio
 Now, $\zeta = 0.01 \times \phi_m$
 from this we get that damping factor also increases.

\Rightarrow damping is increased

\Rightarrow settling time decreased

Relation between phase margin and damping ratio
 Consider a second order system,

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

using value of ϕ_m to solve for ζ .

set $20 \log |G(s)| = -3\text{dB}$ to solve for ω_n

using these equations we get,

$$\phi_m = \tan^{-1} \frac{2\zeta}{\sqrt{1+4\zeta^4-2\zeta^2}}$$

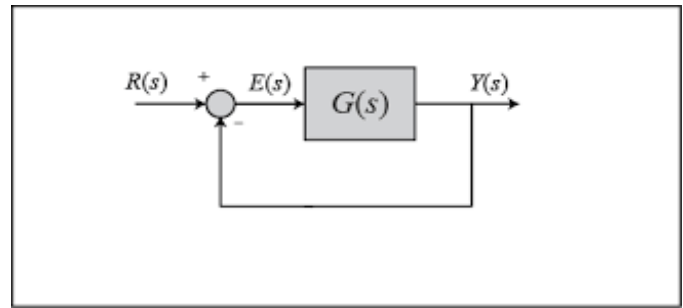
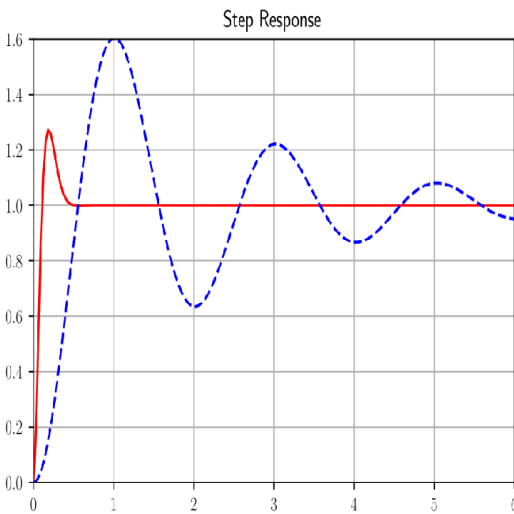
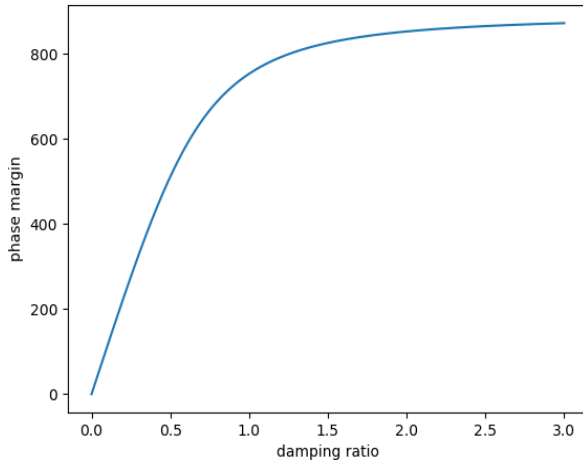
a handy relation is $\zeta = 0.01\phi_m$

Time response - reduced settling time -

Statement - B

Lag compensator is used to reduce the steady state error. From the bode plot it is clear that the lag compensator has a high gain for low frequencies. since steady state error is given by -

$$e(\infty) = \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)} \quad (1.17)$$



Hence steady state error for unit step input,
Without lag compensator = $\frac{1}{1+c(s)}$

$$\text{as } s \rightarrow 0 \Rightarrow E_{ss} = \frac{1}{1+0.5} \Rightarrow E_{ss} = 0.66$$

$$\text{With lag compensator} = \frac{1}{1+G(s)}$$

$$\text{as } s \rightarrow 0 \Rightarrow E_{ss} = \frac{1}{1+1.5} \Rightarrow E_{ss} = 0.4$$

Statement - C

Lead compensator may increase the order of a system since the transfer function adds a pole and a zero therefore it may increase the order of a system.
For example -

$$R(s) = \frac{1}{s+2}$$

$$G(s) = \frac{s+1}{s+3}$$

$$R(s).G(s) = \frac{s+1}{(s+2)(s+3)}$$

$$R(s).G(s) = \frac{s+1}{s^2+5s+6}$$

for low frequencies - the gain is high
 $\Rightarrow G(s) \rightarrow \text{number}$
 $\Rightarrow e(\infty) \rightarrow \text{small value}$
 $\Rightarrow \text{steady state error decreases}$

Verification - Let the system transfer function be

$$c(s) = \frac{1}{s+2} \quad (1.18)$$

and the lag compensator be-

$$t(s) = \frac{s+3}{s+1} \quad (1.19)$$

therefore resulting system transfer function-

$$G(s) = \frac{s+3}{(s+2)(s+1)} \quad (1.20)$$

Maximum power in denominator = 2
Hence order increased to 2 from 1

Statement - D

Lag compensator always stabilizes an unstable system

This statement is wrong.

If a system has a pole on right side of s plane, lag compensator cannot stabilize those systems. This is because the resulting system also has an pole on the right side of s plane.

$$R(s) = \frac{1}{s-2}$$

$$G(s) = \frac{s+3}{s+1}$$

$$R(s).G(s) = \frac{(s+3)}{(s-2)(s+1)}$$

$$output = 1.66e^{(2t)} + 0.66e^{(-t)} \quad (1.21)$$

