Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot

4.1. Which of the following is **incorrect** ?
(A) Lead compensator is used to reduce the settling time
(B) Lag compensator is used to reduce the steady state error
(C) Lead compensator may increase the order of a system
(D) Lag compensator always stabilzes an unstable system

Solution: Lead and Lag compensators -

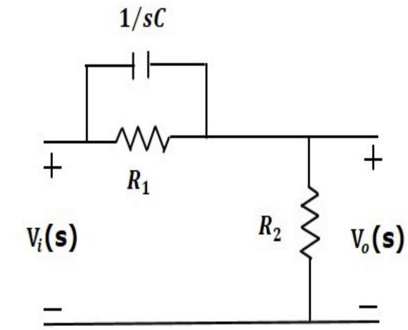
Lead compensator - The lead compensator is an electrical network which produces a output having **phase lead** when a input is applied.

$$H(s) = \frac{s+z}{s+p} 0<z<p \tag{4.1.1}$$

Lag compensator - The Lag Compensator is an electrical network which produces a output having the **phase lag** when a input is applied.

$$H(s) = \frac{s+z}{s+p} \qquad 0<p<z \tag{4.1.2}$$

Lead compensators - The lead compensator circuit in the ‘s’ domain is shown in the following figure.



0.5

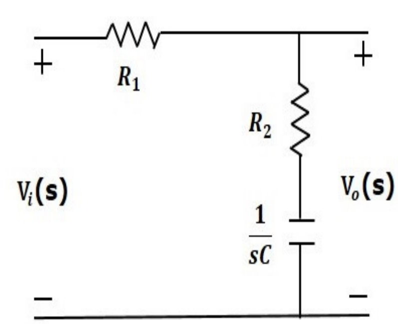
Lead compensator - The transfer function of this lead compensator is -

$$\frac{V_o(s)}{V_i(s)} = \beta \frac{s\tau+1}{s\beta\tau+1} \tag{4.1.3}$$
$$where \tag{4.1.4}$$
$$\tau = R_1 C \beta = \frac{R_2}{R_1+R_2} \tag{4.1.5}$$

substituting s=jω, $\frac{V_o(j\omega)}{V_i(j\omega)} = \beta \frac{j\omega\tau+1}{j\omega\beta\tau+1}$

$$phaseangle\phi = \tan^{-1}(\omega\tau) - \tan -1(\omega\beta\tau) \tag{4.1.6}$$
$$since 0< \beta < 1 \tag{4.1.7}$$
$$\phi > 0 \tag{4.1.8}$$

Lag compensators - The lag compensator circuit in the ‘s’ domain is shown in the following figure.



0.5

The transfer function of this lag compensator is -

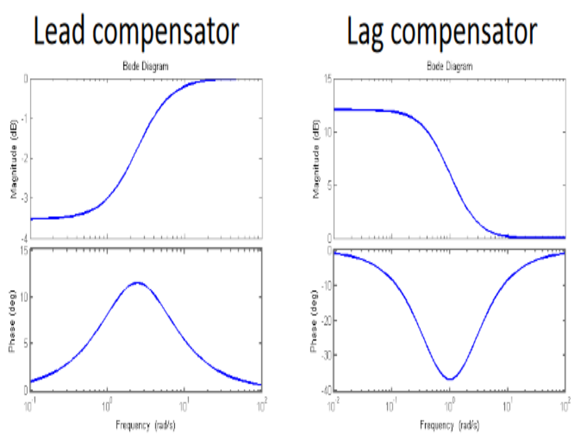
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha} \frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}} \tag{4.1.9}$$
$$where \tag{4.1.10}$$
$$\tau = R_2 C \alpha = \frac{R_1+R_2}{R_2} \tag{4.1.11}$$

substituting s=jω, $\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha}\frac{j\omega+\frac{1}{\tau}}{j\omega+\frac{1}{\alpha\tau}}$

$$phaseangle \qquad \phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\alpha\tau) \tag{4.1.12}$$

$$since\alpha > 1 \tag{4.1.13}$$

$$\phi < 0 \tag{4.1.14}$$



Statement - A
Lead compensator is used to reduce settling time.

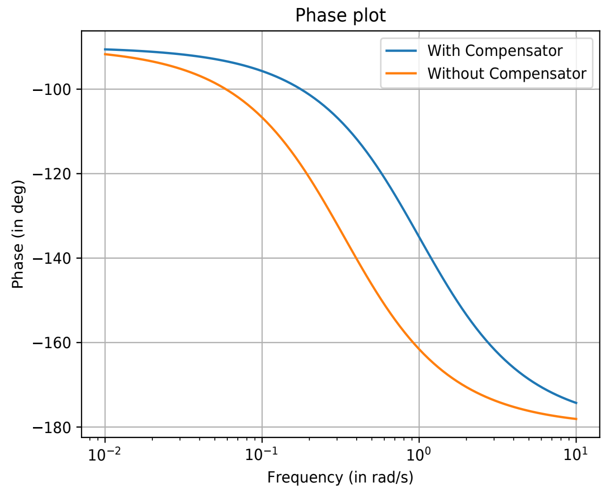
$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.15}$$

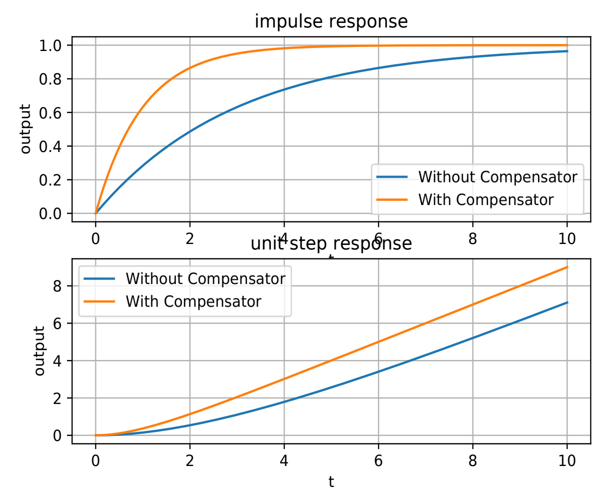
$$D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.16}$$

$$G_1(s) = \frac{1}{s(s+1)} \tag{4.1.17}$$

Since lead compensator adds + phase for any value of frequency, the bode plot for phase vs frequency is above the one which is without lead compensator.

phase margin $\phi_m = 180 + \phi_{gain=0}$
as lead compensator addsadditional phase at all frequencies,
$\phi_{gain=0}$ gets increased, and hence phase margin
Relation between phase margin and damping ratio
Now, $\zeta = 0.01\times\ \phi_m$





from this we get that damping factor also increases.

$\implies$ *damping is increased*

$\implies$ *settling time decreased*

Relation between phase margin and damping ratio. Consider a second order system,

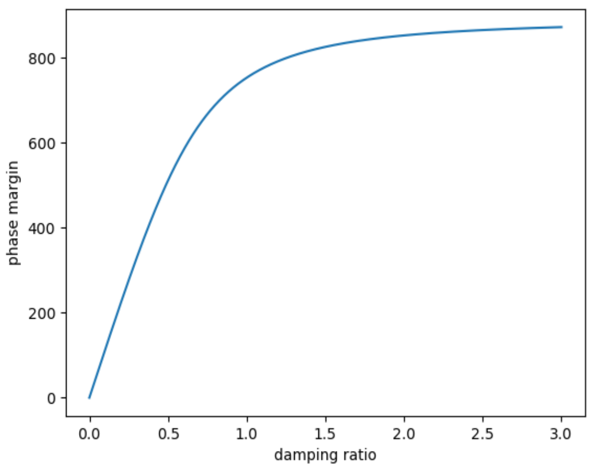
G(s) = $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
using value of ϕ_m to solve for ζ.

set 20 log $|G(s)|$ = -3dB to solve for ω_n

using this equations we get,

$\phi_m = \tan^{-1} \frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4}-2\zeta^2}}$

$ahandyrelationis - \zeta = 0.01\phi_m$



Statement - B
Lag compensator is used to reduce the steady state error.
Verification - Let the system transfer function be -

$$c(s) = \frac{1}{s+2} \tag{4.1.18}$$

and the lag compensator be-

$$t(s) = \frac{s+3}{s+1} \tag{4.1.19}$$

therefore resulting system transfer function-

$$G(s) = \frac{s+3}{(s+2)(s+1)} \tag{4.1.20}$$

Hence steady state error for unit step input,

Without lag compensator $= \frac{1}{1+c(s)}$

as s $\rightarrow 0 \implies$ E$_{ss}$ $= \frac{1}{1+0.5} \implies E_{ss} = 0.66$

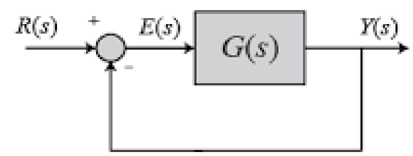
With lag compensator $= \frac{1}{1+G(s)}$

as s $\rightarrow 0 \implies$ E$_{ss}$ $= \frac{1}{1+1.5} \implies E_{ss} = 0.4$

hence, the steady state error reduces.

From the bode plot it is clear that the lag compensator has an high gain for low frequencies. since steady state error is given by -

$$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} \tag{4.1.21}$$



for low frequencies - the gain is high $\implies G(s) \rightarrow number$
$\implies e(\infty) \rightarrow smallervalue$
$\implies$ *steady state error decreases*

$Statement - C$
$consider -$

$$\text{G(s)} = \frac{1}{s+2}$$
$$\text{D(s)} = \frac{s+1}{s+3}$$
$$\text{G(s).D(s)} = \frac{s+1}{(s+2)(s+3)}$$
$$\text{G(s).D(s)} = \frac{s+1}{s^2+5s+6}$$

Maximum power in denominator = 2
Hence order increased to 2 from 1
Lead compensator may increase the order of a system since the transfer function adds a pole and a zero therefore it may increase the order of a system.

Statement - D
Lag compensator always stabilizes an unstable system.
This statement is wrong.Consider,

$$G(s) = \frac{1}{s-2} \tag{4.1.22}$$
$$D(s) = \frac{s+3}{s+1} \tag{4.1.23}$$
$$G(s).D(s) = \frac{(s+3)}{(s-2)(s+1)} \tag{4.1.24}$$
$$\tag{4.1.25}$$
$$output = 1.66e^{(2t)} + 0.66e^{(-t)} \tag{4.1.26}$$

If a system has a pole on right side of s plane, lag compensator cannot stabilize those systems. This is because the resulting system also has an pole on the right side of s plane.