Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

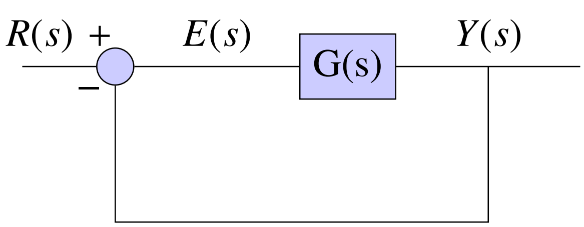
3 Compensators

4 Nyquist Plot

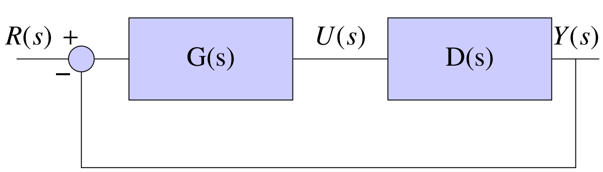
4.1. Lead compensator is used to reduce settlling time. Explain.

Solution:

unity feedback system



compensated system



Settling time -
It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%.
Lead compensator is used to reduce settling time -

let,

$$system - G(s) = \frac{1}{s(3s+1)} \tag{4.1.1}$$

$$lead\ compensator - D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.2}$$

$$hence, new\ system - G_1(s) = G(s)D(s)$$
$$\frac{1}{s(s+1)} \tag{4.1.3}$$

unit impulse response
a). without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.4}$$

$$Y(s) = \frac{G(s)}{1+G(s).1}.R(s) \tag{4.1.5}$$

$$Y(s) = \frac{G(s)}{1+G(s).1}.1 \tag{4.1.6}$$

$$Y(s) = \frac{1}{3s^2+s+1} \tag{4.1.7}$$

$$Y(s) = \frac{1}{3}\frac{1}{(s+\frac{1}{6})^2+\frac{11}{36}} \tag{4.1.8}$$

taking inverse laplace transform,

$$y(t) = [\frac{2}{\sqrt{11}}e^{\frac{-t}{6}}sin(\frac{\sqrt{11}t}{6}))]u(t) \tag{4.1.9}$$

b). with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.10}$$

$$D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.11}$$

$$G_1(s) = \frac{1}{s(s+1)} \tag{4.1.12}$$

$$Y_1(s) = \frac{G_1(s)}{1+G_1(s)}.R(s) \tag{4.1.13}$$

$$Y_1(s) = \frac{G_1(s)}{1+G_1(s)}.1 \tag{4.1.14}$$

$$Y_1(s) = \frac{1}{s^2+s+1} \tag{4.1.15}$$

$$Y_1(s) = \frac{1}{(s+\frac{1}{2})^2+\frac{3}{4}} \tag{4.1.16}$$

taking inverse laplace transform,

$$y(t) = [\frac{2}{\sqrt{3}} e^{\frac{-t}{2}} sin(\frac{\sqrt{3}t)}{2})]u(t) \tag{4.1.17}$$

unit step response
a). without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.18}$$

$$Y(s) = \frac{G(s)}{1+G(s).1}.R(s) \tag{4.1.19}$$

$$Y(s) = \frac{G(s)}{1+G(s)}.\frac{1}{s} \tag{4.1.20}$$

$$Y(S) = \frac{1}{(s)(3s^2+s+1)} \tag{4.1.21}$$

$$Y(s) = \frac{-3s-1}{3s^2+s+1} + \frac{1}{s} \tag{4.1.22}$$

$$\begin{aligned} Y(s) = \frac{1}{s} - \frac{s+\frac{1}{6}}{(s+\frac{1}{6})^2+\frac{11}{36}} \\ -\frac{1}{6}\frac{1}{(s+\frac{1}{6})^2\frac{11}{36}}+ \end{aligned} \tag{4.1.23}$$

taking inverse laplace transform,

$$\begin{aligned} y(t) = [1 - e^{\frac{-t}{6}} cos(\frac{\sqrt{11}t}{6}) \\ -\frac{1}{\sqrt{11}} e^{\frac{-t}{6}} sin(\frac{\sqrt{11}t}{6})]u(t) \end{aligned} \tag{4.1.24}$$

b). with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.25}$$

$$D(s) = \frac{3(s+\frac{1}{3})}{s+1} \tag{4.1.26}$$

$$G_1(s) = \frac{1}{s(s+1)} \tag{4.1.27}$$

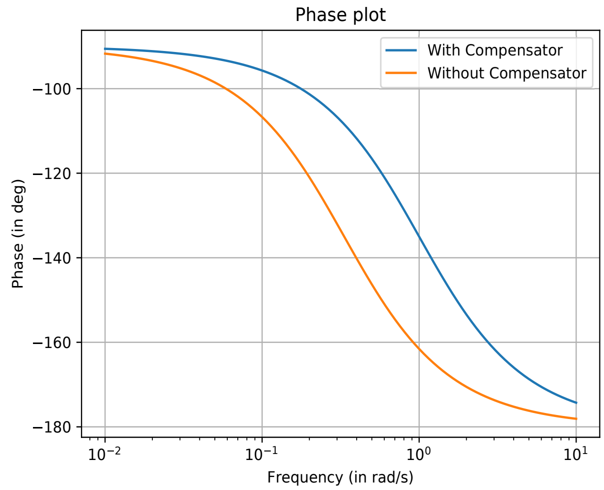
$$Y_1(s) = \frac{G_1(s)}{1+G_1(s)}.R(s) \tag{4.1.28}$$

$$Y_1(s) = \frac{G_1(s)}{1+G_1(s)}.\frac{1}{s} \tag{4.1.29}$$

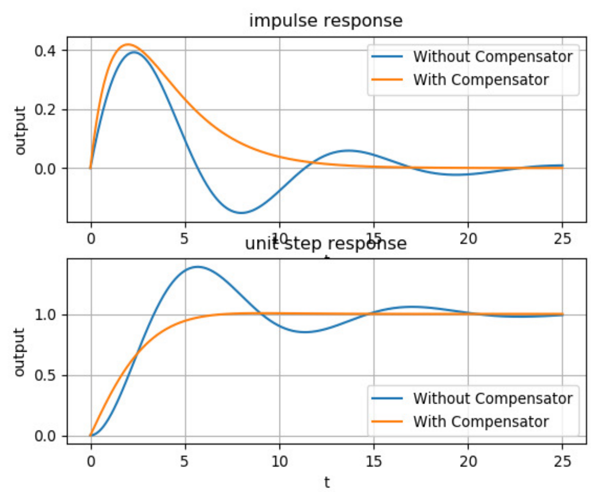
$$Y_1(s) = \frac{1}{s(s^2+s+1)} \tag{4.1.30}$$

$$Y_1(s) = \frac{-s-1}{s^2+s+1} + \frac{1}{s} \tag{4.1.31}$$

phase plot



reduced settling time



$$\begin{aligned} Y_1(s) = \frac{1}{s} - \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} \\ -\frac{1}{2}\frac{1}{(s+\frac{1}{2})^2+\frac{3}{4}}+ \end{aligned} \tag{4.1.32}$$

taking inverse laplace transform,

$$\begin{aligned} y(t) = [1 - e^{\frac{-t}{2}} cos(\frac{\sqrt{3}t}{2}) \\ -\frac{1}{\sqrt{3}} e^{\frac{-t}{2}} sin(\frac{\sqrt{3}t}{2})]u(t) \end{aligned} \tag{4.1.33}$$

Hence, from both examples we can see that settling time is reduced by using a lead compensator.

Theoritically - Since lead compensator adds + phase for any value of frequency, the bode plot for phase vs frequency is above the one which is without lead compensator.

phase margin $\phi_m = 180 + \phi_{gain=0}$
$as\ lead\ compensator\ adds\ additional\ phase\ at\ all\ frequencies,$
$\phi_{gain=0}$ gets increased, and hence phase margin.

Relation between phase margin and damping ratio.

$\zeta = 0.01\times\ \phi_m$

From this we get that damping factor also increases with phase margin.

$\implies$ *damping is increased*

$\implies$ *settling time decreased*

Deriving a relation between phase margin and damping ratio. Consider a second order system,

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{4.1.34}$$

using value of ϕ_m to solve for ζ.

set 20 log $|G(s)|$ = -3dB to solve for ω_n

using this equations we get,

$$\phi_m = \tan^{-1}\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}} \tag{4.1.35}$$

a handy relation is - $\zeta = 0.01\phi_m$

Hence,the settling time is reduced by using a lead compensator.
Similarly if we use a lag compensator the settling time increases as the phase margin decreases.

phase margin vs damping ratio