Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

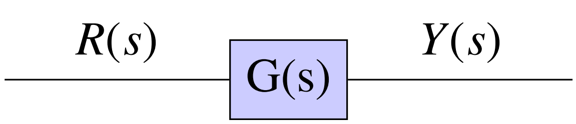
3 Compensators

4 Nyquist Plot

4.1. Lead compensator is used to reduce settlling time. Explain.

Solution:

system



compensated system

Settling time -
It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%.
Lead compensator is used to reduce settling time -

let,

$$system - G(s) = \frac{1}{s(3s+1)} \tag{4.1.1}$$

$$lead\ compensator - D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.2}$$

$$hence, new\ system - G_1(s) = \frac{1}{s(s+1)} \tag{4.1.3}$$

unit impulse response
a). without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.4}$$

$$Y(s) = G(s).1 \tag{4.1.5}$$

$$Y(s) = \frac{1}{(s)(3s+1)} \tag{4.1.6}$$

splitting into partial fractions

$$Y(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{3}} \tag{4.1.7}$$

taking inverse laplace transform,

$$y(t) = [1 - e^{\frac{-t}{3}}]u(t) \tag{4.1.8}$$

b). with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.9}$$

$$D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.10}$$

$$G_1(s) = \frac{1}{s(s+1)} \tag{4.1.11}$$

$$Y_1(s) = G_1(s).1 \tag{4.1.12}$$

$$Y_1(s) = \frac{1}{s(s+1)} \tag{4.1.13}$$

splitting into partial fractions

$$Y_1(s) = \frac{1}{s} - \frac{1}{s+1} \tag{4.1.14}$$

taking inverse laplace transform,

$$y(t) = [1 - e^{-t}]u(t) \tag{4.1.15}$$

unit step response

a). without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.16}$$

$$Y(s) = G(s).\frac{1}{s} \tag{4.1.17}$$

$$Y(S) = \frac{1}{(s^2)(3s+1)} \tag{4.1.18}$$

splitting into partial fractions

$$Y(s) = \frac{1}{s^2} + \frac{3}{s+\frac{1}{3}} + \frac{-3}{s} \tag{4.1.19}$$

taking inverse laplace transform,

$$y(t) = [t + 3e^{\frac{-t}{3}} - 3]u(t) \tag{4.1.20}$$

b). with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.21}$$

$$D(s) = \frac{3(s+\frac{1}{3})}{s+1} \tag{4.1.22}$$

$$G_1(s) = \frac{1}{s(s+1)} \tag{4.1.23}$$

$$Y_1(s) = G_1(s).\frac{1}{s} \tag{4.1.24}$$

$$Y_1(s) = \frac{1}{(s^2)(s+1)} \tag{4.1.25}$$

splitting into partial fractions

$$Y_1(s) = \frac{1}{s^2} + \frac{1}{s+1} - \frac{1}{s} \tag{4.1.26}$$

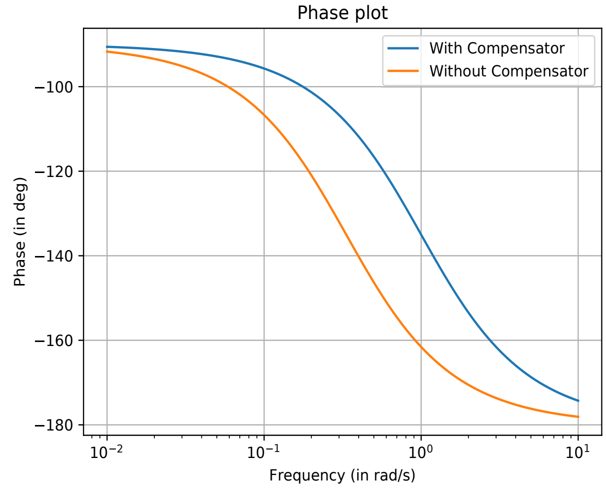
taking inverse laplace transform,

$$y(t) = [t + e^{-t} - 1]u(t) \tag{4.1.27}$$

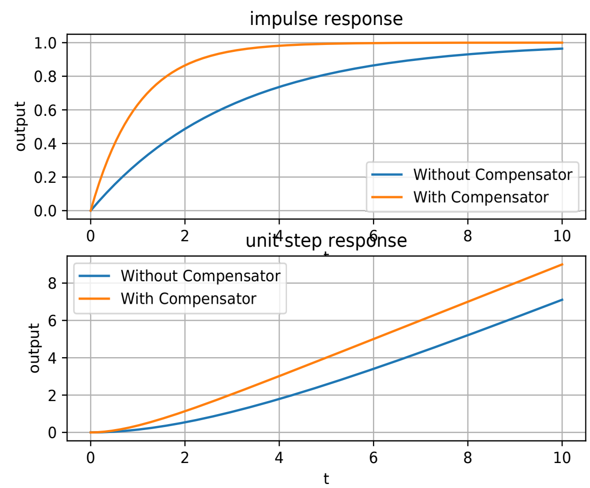
Hence, from both examples we can see that settling time is reduced by using a lead compensator.

Theoritically - Since lead compensator adds + phase for any value of frequency, the bode plot for phase vs frequency is above the one

phase plot



reduced settling time



which is without lead compensator.

phase margin $\phi_m = 180 + \phi_{gain=0}$
as lead compensator adds additional phase at all frequencies,
$\phi_{gain=0}$ gets increased, and hence phase margin.

Relation between phase margin and damping ratio.

$\zeta = 0.01\times\ \phi_m$

From this we get that damping factor also increases with phase margin.

$\implies$ *damping is increased*

$\implies$ *settling time decreased*

Deriving a relation between phase margin and damping ratio. Consider a second order system,

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{4.1.28}$$

using value of ϕ_m to solve for ζ.

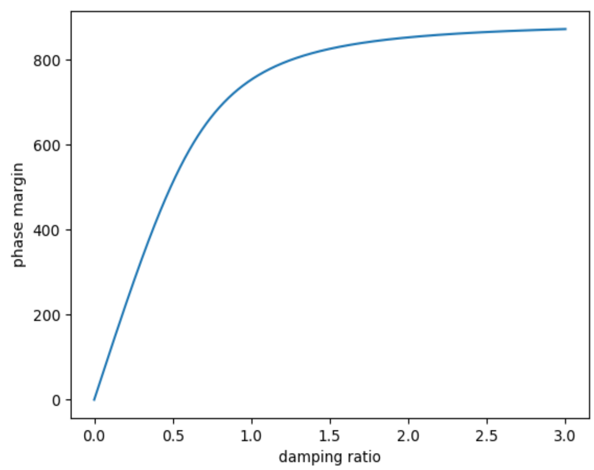
set 20 log $|G(s)|$ = -3dB to solve for ω_n

using this equations we get,

$$\phi_m = \tan^{-1}\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}} \tag{4.1.29}$$

a handy relation is - $\zeta = 0.01\phi_m$

phase margin vs damping ratio



Hence,the settling time is reduced by using a lead compensator.
Similarly if we use a lag compensator the settling time increases as the phase margin decreases.