Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot

4.1. Which of the following is **incorrect** ?
(A) Lead compensator is used to reduce the settling time
(B) Lag compensator is used to reduce the steady state error
(C) Lead compensator may increase the order of a system
(D) Lag compensator always stabilzes an unstable system

Solution:
Statement - A
Lead compensator is used to reduce settling time.

$$system - G(s) = \frac{1}{s(3s+1)} \tag{4.1.1}$$
$$lead\ compensator - D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.2}$$
$$hence, new\ system - G_1(s) = \frac{1}{s(s+1)} \tag{4.1.3}$$

unit impulse response without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.4}$$
$$Y(s) = G(s).1 \tag{4.1.5}$$
$$Y(s) = \frac{1}{(s)(3s+1)} \tag{4.1.6}$$

splitting into partial fractions

$$Y(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{3}} \tag{4.1.7}$$

taking inverse laplace transform,

$$y(t) = [1 - e^{\frac{-t}{3}}]u(t) \tag{4.1.8}$$

with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.9}$$
$$D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.10}$$
$$G_1(s) = \frac{1}{s(s+1)} \tag{4.1.11}$$
$$Y_1(s) = G_1(s).1 \tag{4.1.12}$$
$$Y_1(s) = \frac{1}{s(s+1)} \tag{4.1.13}$$

splitting into partial fractions

$$Y_1(s) = \frac{1}{s} - \frac{1}{s+1} \tag{4.1.14}$$

taking inverse laplace transform,

$$y(t) = [1 - e^{-t}]u(t) \tag{4.1.15}$$

unit step response without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.16}$$
$$Y(s) = G(s).\frac{1}{s} \tag{4.1.17}$$
$$Y(S) = \frac{1}{(s^2)(3s+1)} \tag{4.1.18}$$

splitting into partial fractions

$$Y(s) = \frac{1}{s^2} + \frac{3}{s+\frac{1}{3})} + \frac{-3}{s} \tag{4.1.19}$$

taking inverse laplace transform,

$$y(t) = [t + 3e^{\frac{-t}{3}} - 3]u(t) \tag{4.1.20}$$

with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.21}$$

$$D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.22}$$

$$G_1(s) = \frac{1}{s(s+1)} \tag{4.1.23}$$

$$Y_1(s) = G_1(s).\frac{1}{s} \tag{4.1.24}$$

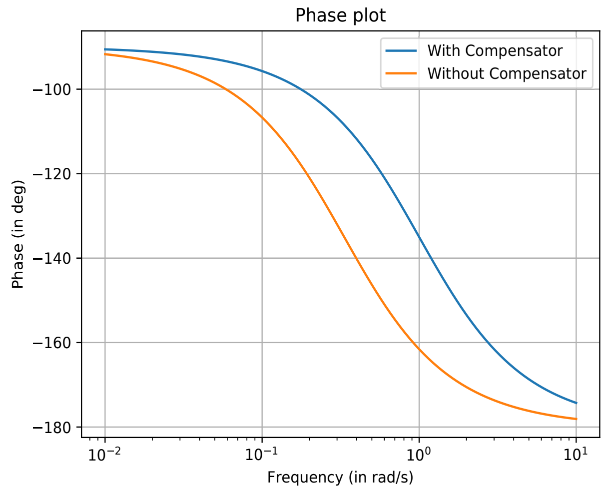
$$Y_1(s) = \frac{1}{(s^2)(s+1)} \tag{4.1.25}$$

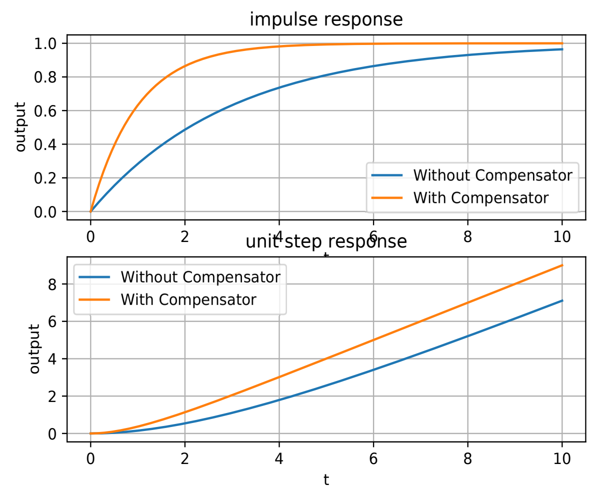
splitting into partial fractions

$$Y_1(s) = \frac{1}{s^2} + \frac{1}{s+1} - \frac{1}{s} \tag{4.1.26}$$

taking inverse laplace transform,

$$y(t) = [t + e^{-t} - 1]u(t) \tag{4.1.27}$$





Since lead compensator adds + phase for any value of frequency, the bode plot for phase vs frequency is above the one which is without lead compensator.

phase margin $\phi_m = 180 + \phi_{gain=0}$
as lead compensator adds additional phase at all frequencies,
$\phi_{gain=0}$ gets increased, and hence phase margin Relation between phase margin and damping ratio Now, $\zeta = 0.01\times$ ϕ_m

from this we get that damping factor also increases.

$\implies$ *damping is increased*

$\implies$ *settling time decreased*

Relation between phase margin and damping ratio. Consider a second order system,

$$G(s) = \omega_n^2 \frac{}{s^2+2\zeta\omega_n s+\omega_n^2} \tag{4.1.28}$$

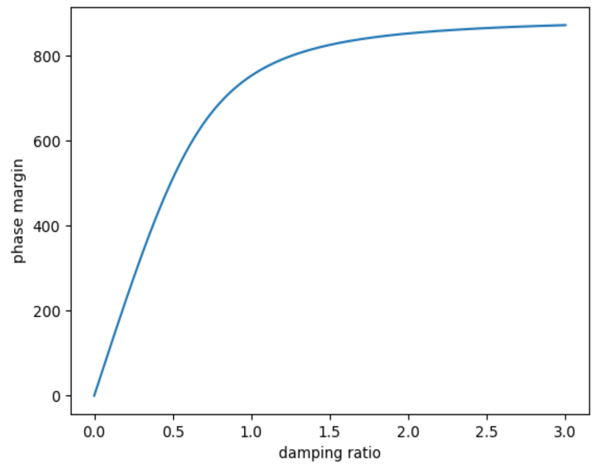
using value of ϕ_m to solve for ζ.

set 20 log $|G(s)|$ = -3dB to solve for ω_n

using this equations we get,

$$\phi_m = \tan^{-1}\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}} \tag{4.1.29}$$

a handy relation is - $\zeta = 0.01\phi_m$



Statement - B
Lag compensator is used to reduce the steady state error.
Verification - Let the system transfer function be -

$$c(s) = \frac{1}{s+2} \tag{4.1.30}$$

and the lag compensator be-

$$t(s) = \frac{s+3}{s+1} \tag{4.1.31}$$

therefore resulting system transfer function-

$$G(s) = \frac{s+3}{(s+2)(s+1)} \tag{4.1.32}$$

Hence steady state error for unit step input,

Without lag compensator = $\frac{1}{1+c(s)}$

as s $\to 0 \implies$ E$_{ss}$ = $\frac{1}{1+0.5} \implies E_{ss} = 0.66$

With lag compensator = $\frac{1}{1+G(s)}$

as s $\to 0 \implies$ E$_{ss}$ = $\frac{1}{1+1.5} \implies E_{ss} = 0.4$

hence, the steady state error reduces.

From the bode plot it is clear that the lag compensator has an high gain for low frequencies. since steady state error is given by -

$$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} \tag{4.1.33}$$

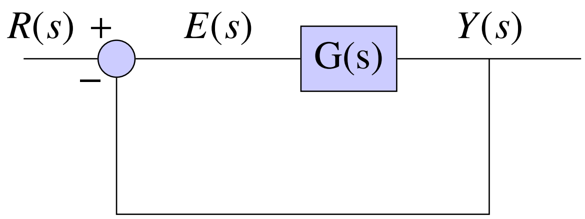


Fig. 4.1

for low frequencies - the gain is high $\implies G(s) \to largenumber$
$\implies e(\infty) \to smallervalue \implies$ *steady state error decreases*

Statement – C
Lead compensator may increase the order of a system.
consider –

$$G(s) = \frac{1}{s+2} \tag{4.1.34}$$

$$D(s) = \frac{s+1}{s+3} \tag{4.1.35}$$

$$G(s).D(s) = \frac{s+1}{(s+2)(s+3)} \tag{4.1.36}$$

$$G(s).D(s) = \frac{s+1}{s^2+5s+6} \tag{4.1.37}$$

Maximum power in denominator = 2
Hence order increased to 2 from 1
Lead compensator may increase the order of a system since the transfer function adds a pole and a zero therefore it may increase the order of a system.

Statement - D
Lag compensator always stabilizes an unstable system.

This statement is wrong.Consider,

$$G(s) = \frac{1}{s-2} \tag{4.1.38}$$

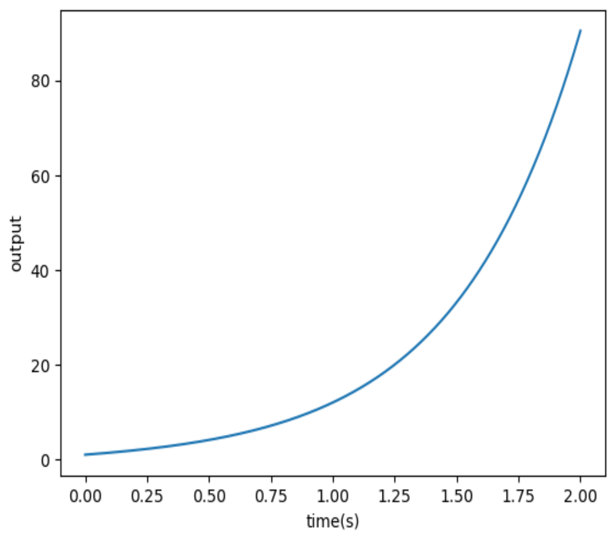
$$D(s) = \frac{s+3}{s+1} \tag{4.1.39}$$

$$G(s).D(s) = \frac{(s+3)}{(s-2)(s+1)} \tag{4.1.40}$$

$$\tag{4.1.41}$$

$$output = 1.66e^{(2t)} + 0.66e^{(-t)} \tag{4.1.42}$$

If a system has a pole on right side of s plane, lag compensator cannot stabilize those systems. This is because the resulting system also has an pole on the right side of s plane.



Lead and Lag compensators -
Lead compensator - The lead compensator is an electrical network which produces a output having **phase lead** when a input is applied.

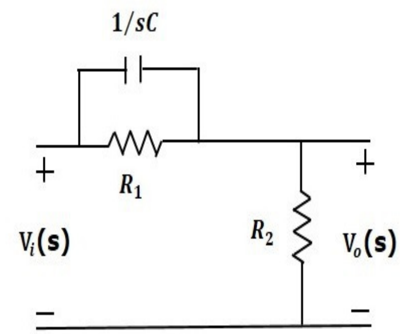
$$H(s) = \frac{s+z}{s+p} 0<z<p \tag{4.1.43}$$

The lead compensator circuit in the 's' domain is shown in the following figure.
The transfer function of this lead compensator is -

$$\frac{V_o(s)}{V_i(s)} = \beta \frac{s\tau + 1}{s\beta\tau + 1} \tag{4.1.44}$$

$$where \tag{4.1.45}$$

$$\tau = R_1 C \beta = \frac{R_2}{R_1 + R_2} \tag{4.1.46}$$



0.5

substituting s=jω, $\frac{V_o(j\omega)}{V_i(j\omega)} = \beta \frac{j\omega\tau+1}{j\omega\beta\tau+1}$

$$phaseangle\phi = \tan^{-1}(\omega\tau) - \tan -1(\omega\beta\tau) \tag{4.1.47}$$

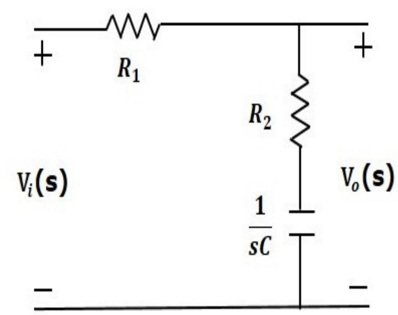
$$since 0 < \beta < 1 \tag{4.1.48}$$

$$\phi > 0 \tag{4.1.49}$$

Lag compensator - The Lag Compensator is an electrical network which produces a output having the **phase lag** when a input is applied.

$$H(s) = \frac{s+z}{s+p} \qquad 0<p<z \tag{4.1.50}$$

The lag compensator circuit in the 's' domain is shown in the following figure.



0.5

The transfer function of this lag compensator is -

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha} \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}} \tag{4.1.51}$$

$$where \tag{4.1.52}$$

$$\tau = R_2 C \alpha = \frac{R_1 + R_2}{R_2} \tag{4.1.53}$$

substituting s=jω, $\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha} \frac{j\omega + \frac{1}{\tau}}{j\omega + \frac{1}{\alpha\tau}}$

$$phase angle \qquad \phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\alpha\tau) \tag{4.1.54}$$

$$since \alpha > 1 \tag{4.1.55}$$

$$\phi < 0 \tag{4.1.56}$$

BODE PLOT