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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

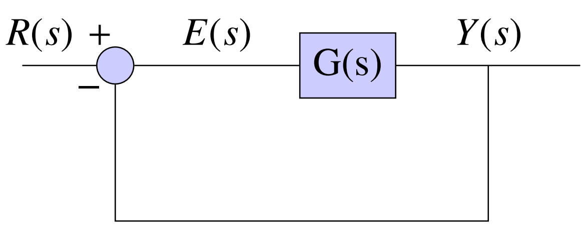
3 Compensators

4 Nyquist Plot

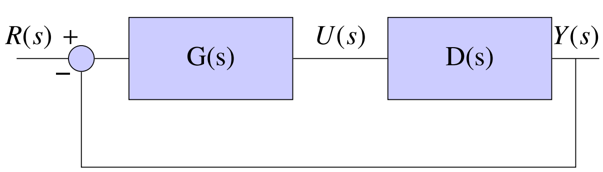
4.1. Show by plotting the output that a lead compensator reduces settling time of a control system.

Solution: considering the above discussed control system (**??**) and the lead compensator.

Unity feedback system -



Compensated system -



Settling time -

It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%.

let,

$$system - G(s) = \frac{1}{s(3s+1)} \tag{4.1.1}$$

$$lead\ compensator - D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.2}$$

$$\begin{aligned} hence, new\ system - G_1(s) &= G(s)D(s) \\ &= \frac{1}{s(s+1)} \end{aligned} \tag{4.1.3}$$

1). Unit impulse response -

a). without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.4}$$

$$Y(s) = \frac{G(s)}{1+G(s).1}.R(s) \tag{4.1.5}$$

$$Y(s) = \frac{G(s)}{1+G(s).1}.1 \tag{4.1.6}$$

$$Y(s) = \frac{1}{3s^2+s+1} \tag{4.1.7}$$

$$Y(s) = \frac{1}{3}\frac{1}{(s+\frac{1}{6})^2+\frac{11}{36}} \tag{4.1.8}$$

taking inverse laplace transform,

$$y(t) = [\frac{2}{\sqrt{11}}e^{\frac{-t}{6}}sin(\frac{\sqrt{11}t}{6}))]u(t) \tag{4.1.9}$$

b). with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.10}$$

$$D(s) = \frac{3(s+\frac{1}{3})}{(s+1)} \tag{4.1.11}$$

$$G_1(s) = \frac{1}{s(s+1)} \tag{4.1.12}$$

$$Y_1(s) = \frac{G_1(s)}{1+G_1(s)}.R(s) \tag{4.1.13}$$

$$Y_1(s) = \frac{G_1(s)}{1+G_1(s)}.1 \tag{4.1.14}$$

$$Y_1(s) = \frac{1}{s^2+s+1} \tag{4.1.15}$$

$$Y_1(s) = \frac{1}{(s+\frac{1}{2})^2+\frac{3}{4}} \tag{4.1.16}$$

taking inverse laplace transform,

$$y(t) = [\frac{2}{\sqrt{3}}e^{\frac{-t}{2}}sin(\frac{\sqrt{3}t)}{2})]u(t) \tag{4.1.17}$$

2). unit step response -
a). without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.18}$$
$$Y(s) = \frac{G(s)}{1+G(s).1}.R(s) \tag{4.1.19}$$
$$Y(s) = \frac{G(s)}{1+G(s)}.\frac{1}{s} \tag{4.1.20}$$
$$Y(S) = \frac{1}{(s)(3s^2+s+1)} \tag{4.1.21}$$
$$Y(s) = \frac{-3s-1}{3s^2+s+1} + \frac{1}{s} \tag{4.1.22}$$

$$\begin{aligned} Y(s) = \frac{1}{s} - \frac{s+\frac{1}{6}}{(s+\frac{1}{6})^2+\frac{11}{36}} \\ -\frac{1}{6}\frac{1}{(s+\frac{1}{6})^2\frac{11}{36}} + \end{aligned} \tag{4.1.23}$$

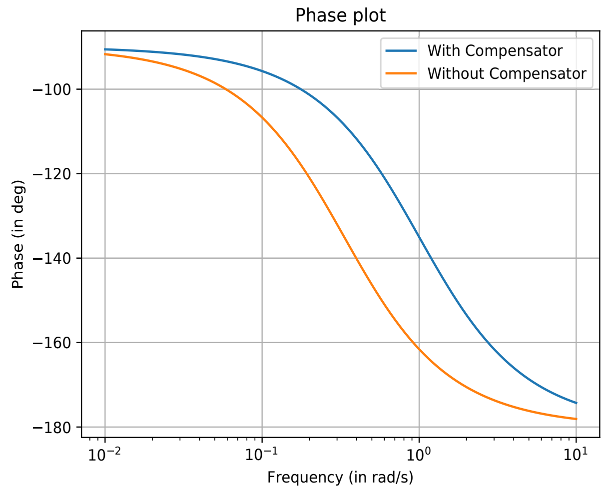
taking inverse laplace transform,

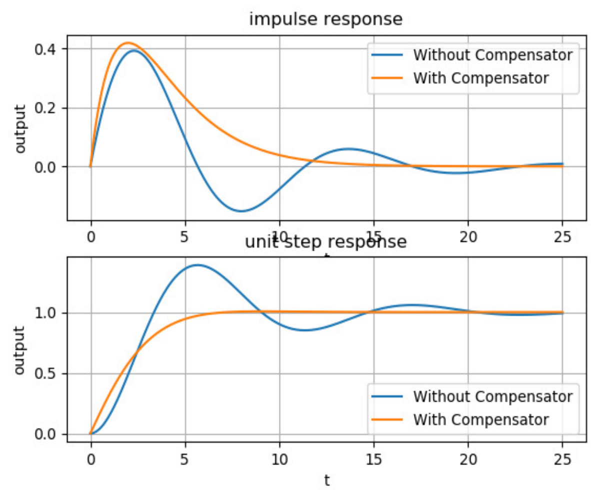
$$\begin{aligned} y(t) = [1 - e^{\frac{-t}{6}}cos(\frac{\sqrt{11}t}{6}) \\ -\frac{1}{\sqrt{11}}e^{\frac{-t}{6}}sin(\frac{\sqrt{11}t}{6})]u(t) \end{aligned} \tag{4.1.24}$$

b). with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \tag{4.1.25}$$
$$D(s) = \frac{3(s+\frac{1}{3})}{s+1} \tag{4.1.26}$$
$$G_1(s) = \frac{1}{s(s+1)} \tag{4.1.27}$$
$$Y_1(s) = \frac{G_1(s)}{1+G_1(s)}.R(s) \tag{4.1.28}$$
$$Y_1(s) = \frac{G_1(s)}{1+G_1(s)}.\frac{1}{s} \tag{4.1.29}$$
$$Y_1(s) = \frac{1}{s(s^2+s+1)} \tag{4.1.30}$$
$$Y_1(s) = \frac{-s-1}{s^2+s+1} + \frac{1}{s} \tag{4.1.31}$$

$$\begin{aligned} Y_1(s) = \frac{1}{s} - \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} \\ -\frac{1}{2}\frac{1}{(s+\frac{1}{2})^2+\frac{3}{4}} + \end{aligned} \tag{4.1.32}$$





taking inverse laplace transform,

$$\begin{aligned} y(t) = [1 - e^{\frac{-t}{2}}cos(\frac{\sqrt{3}t}{2}) \\ -\frac{1}{\sqrt{3}}e^{\frac{-t}{2}}sin(\frac{\sqrt{3}t}{2})]u(t) \end{aligned} \tag{4.1.33}$$

Hence, from both examples we can see that settling time is reduced by using a lead compensator.

4.2. Theoritically state how the lead compensator reduces settling time.

Solution: Since lead compensator adds +ve phase for any value of frequency, the phase bode plot of compensated system is above the one which is without lead compensator.

$$\phi_m = 180 + \phi_{gain=0} \tag{4.2.1}$$

as lead compensator adds additional phase at all frequencies, $\phi_{gain=0}$ gets increased, and hence phase margin.
Relation between phase margin and damping ratio. $\zeta = 0.01 \times \phi_m$
From this we get that damping factor also increases with phase margin.

$$\implies \textit{damping is increased} \tag{4.2.2}$$

$$\implies \textit{settling time decreased} \tag{4.2.3}$$

4.3. Derive a relation between phase margin and damping ratio.
Solution: Consider a second order system,

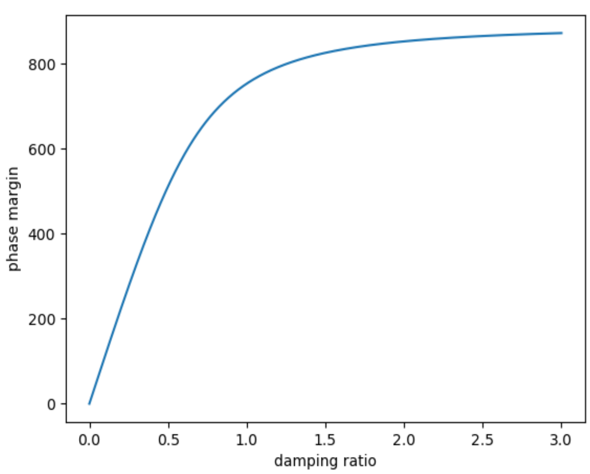
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{4.3.1}$$

using value of ϕ_m to solve for ζ. set $20 \log |G(s)| =$ -3dB to solve for ω_n
using this equations we get,

$$\phi_m = \tan^{-1} \frac{2\zeta}{\sqrt{\sqrt{1 + 4\zeta^4} - 2\zeta^2}} \tag{4.3.2}$$

a handy relation is - $\zeta = 0.01\phi_m$

phase margin vs damping ratio



Hence,for a lead compensator phase margin increases, therfore damping increases, resulting in reduced settling time. Similarly if we use a lag compensator the settling time increases beacuse the phase margin decreases.