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*Abstract*—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

## 1 STABILITY

## 1.1 Second order System

## 2 ROUTH HURWITZ CRITERION

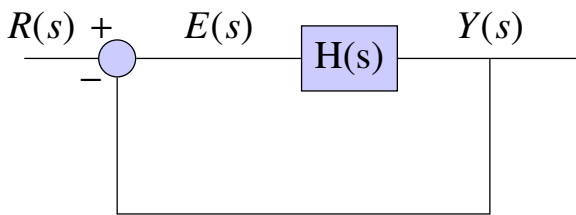
## 3 COMPENSATORS

## 4 NYQUIST PLOT

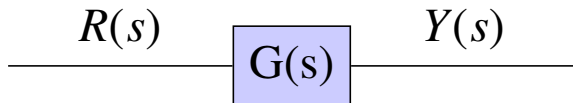
4.1. Lead compensator is used to reduce settling time. Explain.

**Solution:**

**unity feedback system**



**system(closed loop transfer function)**



**compensated system**



**Settling time -**

It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%. Lead compensator is used to reduce settling time - let,

$$\text{system} - G(s) = \frac{1}{s(3s+1)} \quad (4.1.1)$$

$$\text{lead compensator} - D(s) = \frac{3(s + \frac{1}{3})}{(s+1)} \quad (4.1.2)$$

$$\text{hence, new system} - G_1(s) = \frac{1}{s(s+1)} \quad (4.1.3)$$

**unit impulse response**

a). without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \quad (4.1.4)$$

$$Y(s) = G(s).1 \quad (4.1.5)$$

$$Y(s) = \frac{1}{(s)(3s+1)} \quad (4.1.6)$$

splitting into partial fractions

$$Y(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{3}} \quad (4.1.7)$$

taking inverse laplace transform,

$$y(t) = [1 - e^{-\frac{t}{3}}]u(t) \quad (4.1.8)$$

b). with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \quad (4.1.9)$$

$$D(s) = \frac{3(s + \frac{1}{3})}{(s+1)} \quad (4.1.10)$$

$$G_1(s) = \frac{1}{s(s+1)} \quad (4.1.11)$$

$$Y_1(s) = G_1(s).1 \quad (4.1.12)$$

$$Y_1(s) = \frac{1}{s(s+1)} \quad (4.1.13)$$

splitting into partial fractions

$$Y_1(s) = \frac{1}{s} - \frac{1}{s+1} \quad (4.1.14)$$

taking inverse laplace transform,

$$y(t) = [1 - e^{-t}]u(t) \quad (4.1.15)$$

### unit step response

a). without lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \quad (4.1.16)$$

$$Y(s) = G(s) \cdot \frac{1}{s} \quad (4.1.17)$$

$$Y(s) = \frac{1}{(s^2)(3s+1)} \quad (4.1.18)$$

splitting into partial fractions

$$Y(s) = \frac{1}{s^2} + \frac{3}{s + \frac{1}{3}} + \frac{-3}{s} \quad (4.1.19)$$

taking inverse laplace transform,

$$y(t) = [t + 3e^{\frac{-t}{3}} - 3]u(t) \quad (4.1.20)$$

b). with lead compensator -

$$G(s) = \frac{1}{(s)(3s+1)} \quad (4.1.21)$$

$$D(s) = \frac{3(s + \frac{1}{3})}{s+1} \quad (4.1.22)$$

$$G_1(s) = \frac{1}{s(s+1)} \quad (4.1.23)$$

$$Y_1(s) = G_1(s) \cdot \frac{1}{s} \quad (4.1.24)$$

$$Y_1(s) = \frac{1}{(s^2)(s+1)} \quad (4.1.25)$$

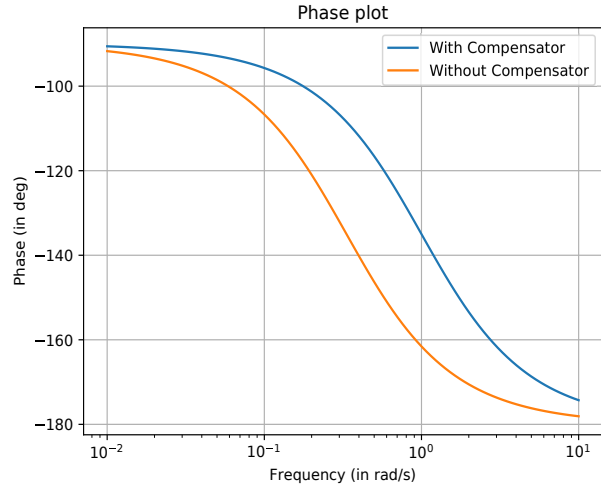
splitting into partial fractions

$$Y_1(s) = \frac{1}{s^2} + \frac{1}{s+1} - \frac{1}{s} \quad (4.1.26)$$

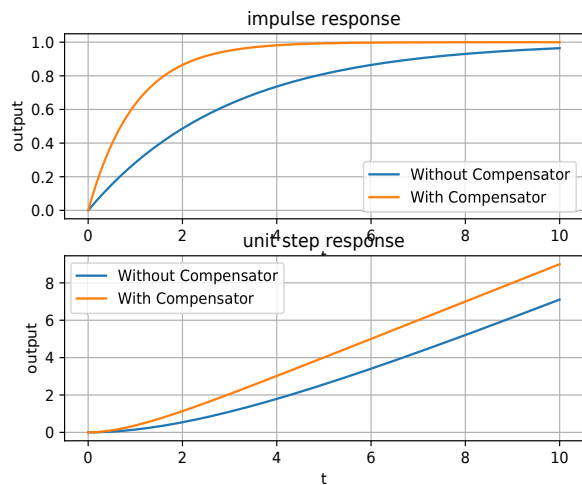
taking inverse laplace transform,

$$y(t) = [t + e^{-t} - 1]u(t) \quad (4.1.27)$$

### phase plot



### reduced settling time



Hence, from both examples we can see that settling time is reduced by using a lead compensator.

**Theoretically** - Since lead compensator adds + phase for any value of frequency, the bode plot for phase vs frequency is above the one which is without lead compensator.

phase margin  $\phi_m = 180 + \phi_{gain=0}$   
as lead compensator adds additional phase at all frequencies,  
 $\phi_{gain=0}$  gets increased, and hence phase margin.

Relation between phase margin and damping ratio.

$$\zeta = 0.01 \times \phi_m$$

From this we get that damping factor also increases with phase margin.

$\Rightarrow$  *damping is increased*

$\Rightarrow$  *settling time decreased*

Deriving a relation between phase margin and damping ratio. Consider a second order system,

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4.1.28)$$

using value of  $\phi_m$  to solve for  $\zeta$ .

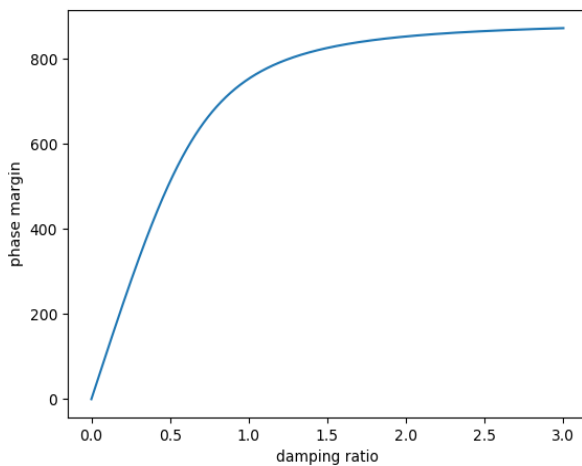
set  $20 \log |G(s)| = -3\text{dB}$  to solve for  $\omega_n$

using this equations we get,

$$\phi_m = \tan^{-1} \frac{2\zeta}{\sqrt{\sqrt{1 + 4\zeta^4} - 2\zeta^2}} \quad (4.1.29)$$

a handy relation is -  $\zeta = 0.01\phi_m$

**phase margin vs damping ratio**



Hence, the settling time is reduced by using a lead compensator.

Similarly if we use a lag compensator the settling time increases as the phase margin decreases.