EE2227 Control Systems Presentation-1

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GATE EC 2017 Question-20

Which of the following is **incorrect** ?

(A) Lead compensator is used to reduce the settling time
(B) Lag compensator is used to reduce the steady state error
(C) Lead compensator may increase the order of a system
(D) Lag compensator always stabilzes an unstable system

Lead and Lag compensators

Lead compensator - The lead compensator is an electrical network which produces a output having **phase lead** when a input is applied.

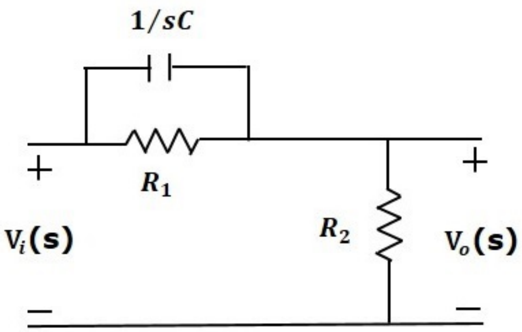
$$H(s) = \frac{s+z}{s+p} \qquad 0<z<p \tag{1}$$

Lag compensator - The Lag Compensator is an electrical network which produces a output having the **phase lag** when a input is applied.

$$H(s) = \frac{s+z}{s+p} \qquad 0<p<z \tag{2}$$

Lead compensators

The lead compensator circuit in the ‘s’ domain is shown in the following figure.



0.5

Lead compensator

The transfer function of this lead compensator is -

$$\frac{V_o(s)}{V_i(s)} = \beta \frac{s\tau + 1}{s\beta\tau + 1}$$

where

$$\tau = R_1 C \qquad \beta = \frac{R_2}{R_1 + R_2} (3)$$

substituting s=jω

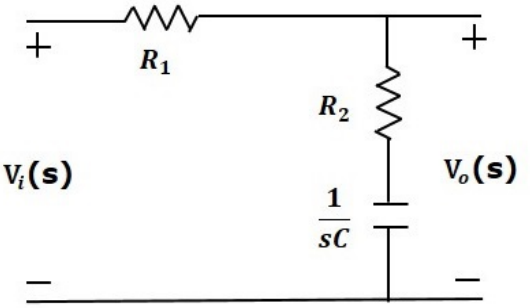
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \beta \frac{j\omega\tau + 1}{j\omega\beta\tau + 1} \tag{4}$$

phase angle $\phi = \tan^{-1}(\omega\tau) - \tan -1(\omega\beta\tau)$

$$\text{since } 0 < \beta < 1$$
$$\phi > 0$$

Lag compensators

The lag compensator circuit in the 's' domain is shown in the following figure.



0.5

Lag compensator

The transfer function of this lag compensator is -

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha}\frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}}$$

where

$$\tau = R_2 C \qquad \alpha = \frac{R_1+R_2}{R_2}(5)$$

substituting s=jω

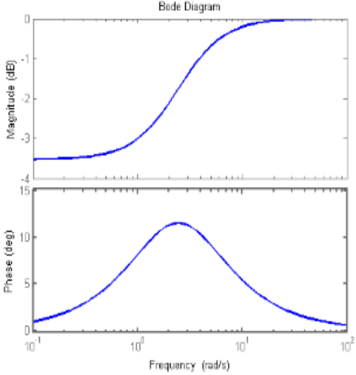
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha}\frac{j\omega+\frac{1}{\tau}}{j\omega+\frac{1}{\alpha\tau}} \tag{6}$$

phase angle $\qquad \phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\alpha\tau)$

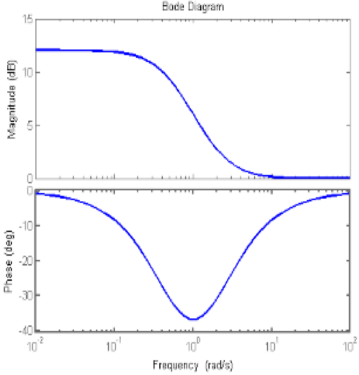
$$\text{since } \alpha > 1$$
$$\phi < 0$$

BODE PLOTS

Lead compensator



Lag compensator

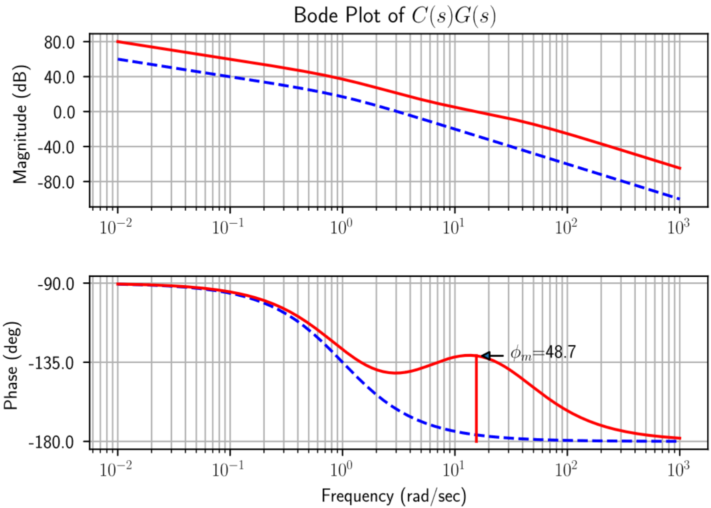


Statement - A
Lead compensator is used to reduce settling time.

Since lead compensator adds $+$ phase for any value of frequency, the bode plot for phase vs frequency is above the one which is without lead compensator.

phase margin $\phi_m = 180 + \phi_{gain=0}$
as lead compensator adds additional phase at all frequencies, $\phi_{gain=0}$ gets increased, hence phase gain

Bode plot - increased phase margin



Relation between phase margin and damping ratio

Now, $\zeta = 0.01\times\ \phi_m$

from this we get that damping factor also increases.

$\implies$ *damping is increased*

$\implies$ *settling time decreased*

Relation between phase margin and damping ratio

Consider a second order system,

$$G(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

using value of ϕ_m to solve for ζ.

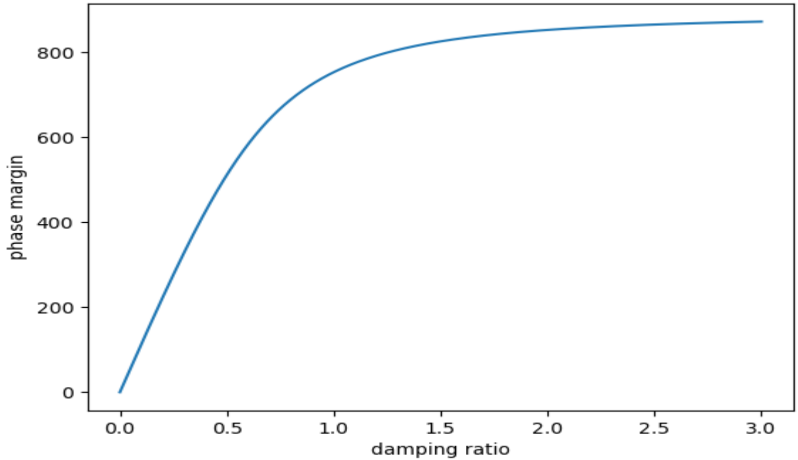
set 20 log $|G(s)|$ = -3dB to solve for ω_n

using this equations we get,

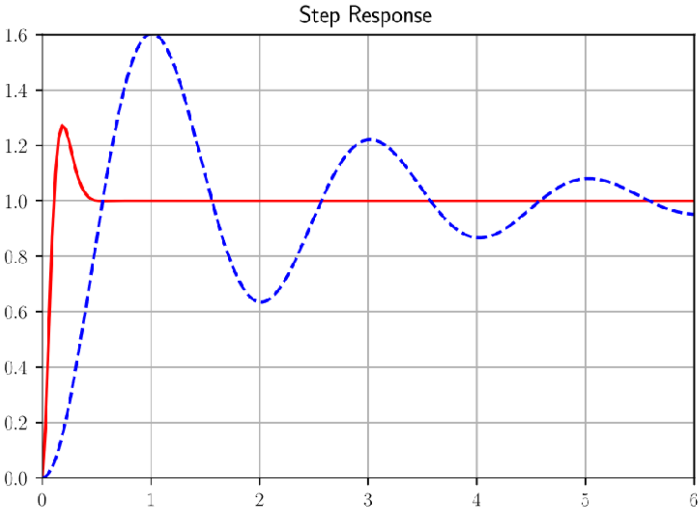
$$\phi_m = \tan^{-1}\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4}-2\zeta^2}}$$

a handy relation is - $\zeta = 0.01\phi_m$

Relation between phase margin and damping ratio



Time response - reduced settling time

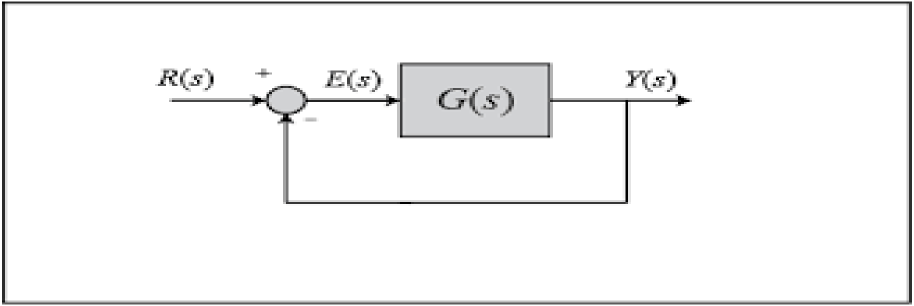


Statement - B

Lag compensator is used to reduce the steady state error

From the bode plot it is clear that the lag compensator has an high gain for low frequencies. since steady state error is given by -

$$e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)} \tag{7}$$



for low frequencies - the gain is high $\implies G(s) \rightarrow number$
$\implies e(\infty) \rightarrow smallervalue$
$\implies steady\ \ state\ \ error\ \ decreases$

Input signal	Steady state error e_{ss}	Error constant
unit step signal	$\frac{1}{1+k_p}$	$K_p = \lim_{s\to 0} G(s)$
unit ramp signal	$\frac{1}{K_v}$	$K_v = \lim_{s\to 0} sG(s)$
unit parabolic signal	$\frac{1}{K_a}$	$K_a = \lim_{s\to 0} s^2G(s)$

Verification

Let the system transfer function be -

$$c(s) = \frac{1}{s+2} \tag{8}$$

and the lag compensator be-

$$t(s) = \frac{s+3}{s+1} \tag{9}$$

therefore resulting system transfer function-

$$G(s) = \frac{s+3}{(s+2)(s+1)} \tag{10}$$

Verification

Hence steady state error for unit step input,

Without lag compensator $= \frac{1}{1+c(s)}$

as s $\rightarrow 0 \implies \mathrm{E}_{ss} = \frac{1}{1+0.5} \implies E_{ss} = 0.66$

With lag compensator $= \frac{1}{1+G(s)}$

as s $\rightarrow 0 \implies \mathrm{E}_{ss} = \frac{1}{1+1.5} \implies E_{ss} = 0.4$

Statement - C
Lead compensator may increase the order of a system

since the transfer function adds a pole and a zero therefore it may increase the order of a system.
For example -

$$R(s) = \frac{1}{s+2}$$

$$G(s) = \frac{s+1}{s+3}$$

$$R(s).G(s) = \frac{s+1}{(s+2)(s+3)}$$

$$R(s).G(s) = \frac{s+1}{s^2+5s+6}$$

Maximum power in denominator = 2
Hence order increased to 2 from 1

Statement - D
Lag compensator always stabilizes an unstable system

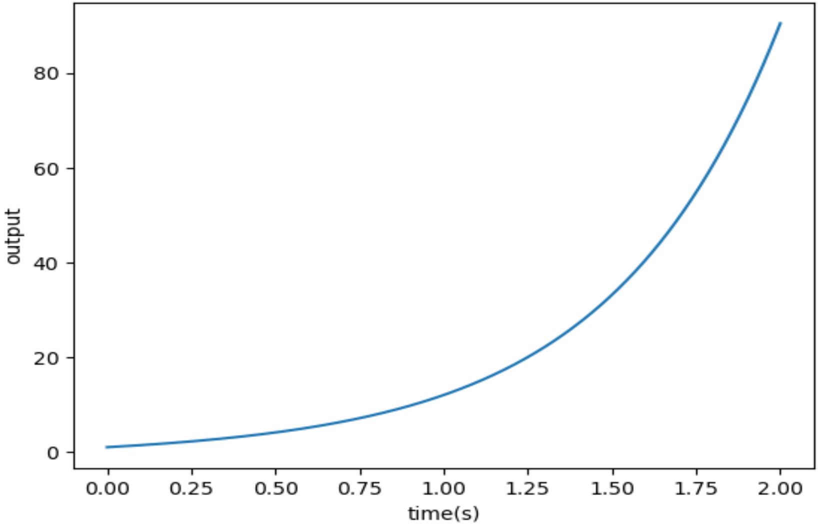
This statement is wrong.
If a system has a pole on right side of s plane, lag compensator cannot stabilize those systems. This is because the resulting system also has an pole on the right side of s plane.

$$R(s) = \frac{1}{s-2}$$

$$G(s) = \frac{s+3}{s+1}$$

$$R(s).G(s) = \frac{(s+3)}{(s-2)(s+1)}$$

$$output = 1.66e^{(2t)} + 0.66e^{(-t)} \tag{11}$$



output
0
20
40
60
80
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
time(s)