Sphere Decoding

Application to a MC-CSK system

Sphere Decoding

- <u>Mathematical approach</u>

  Minimization of a cost function $f(x_1, \ldots, x_K)$ with respect to its K arguments taking value in a discrete set of cardinality L

- <u>Digital communications approach</u>

  Minimization of the distance between the received symbol and the possible transmitted symbols in order to recover the information in a multi-user system

Sphere Decoding

- Computation of the distance based on the optimum Maximum Likelihood operator
  $\longrightarrow$ absolute minimum

- Smart hypotheses leading to a smaller complexity than classic ML algorithm

  Main idea: searching over the noiseless possible received signals that lie within a hypersphere of radius R around the actual received signal

- Multi-user detection

Principle of Sphere Decoding

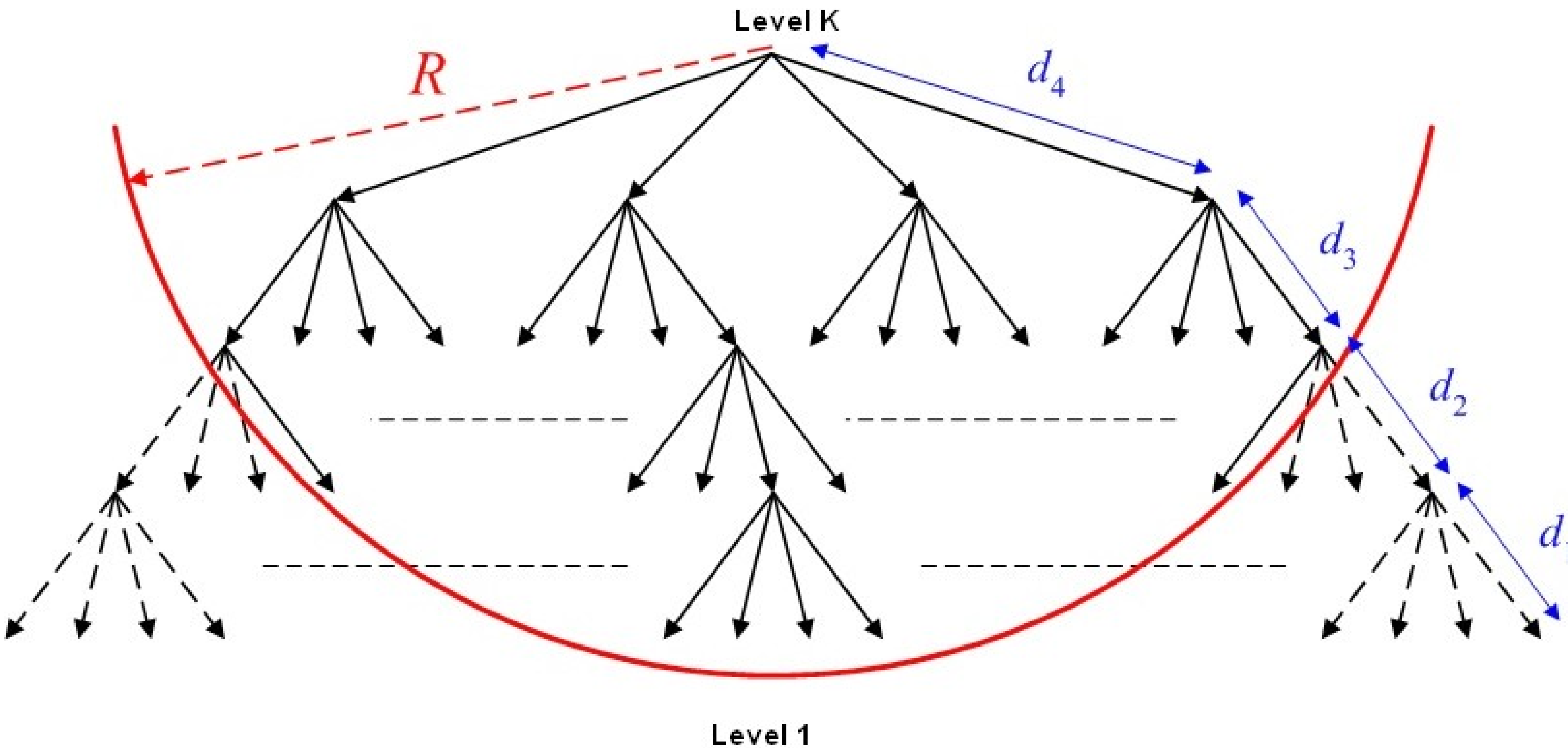
- The distance is seen as a sum of nonnegative functions with an increasing number of arguments

  $d = f(x_1, \ldots, x_K) = h(x_K) + h(x_K, x_{K-1}) + \ldots + h(x_K, \ldots, x_1)$

- Graphically, the process is a (K)-level tree graph with one uppermost node (level K) and L^{K-1} leaves (level 1)

- Each branch corresponds to an intermediate distance

Graphical approach



Algorithm

- 2 stages
  - Initialization
  - Pruning

Algorithm

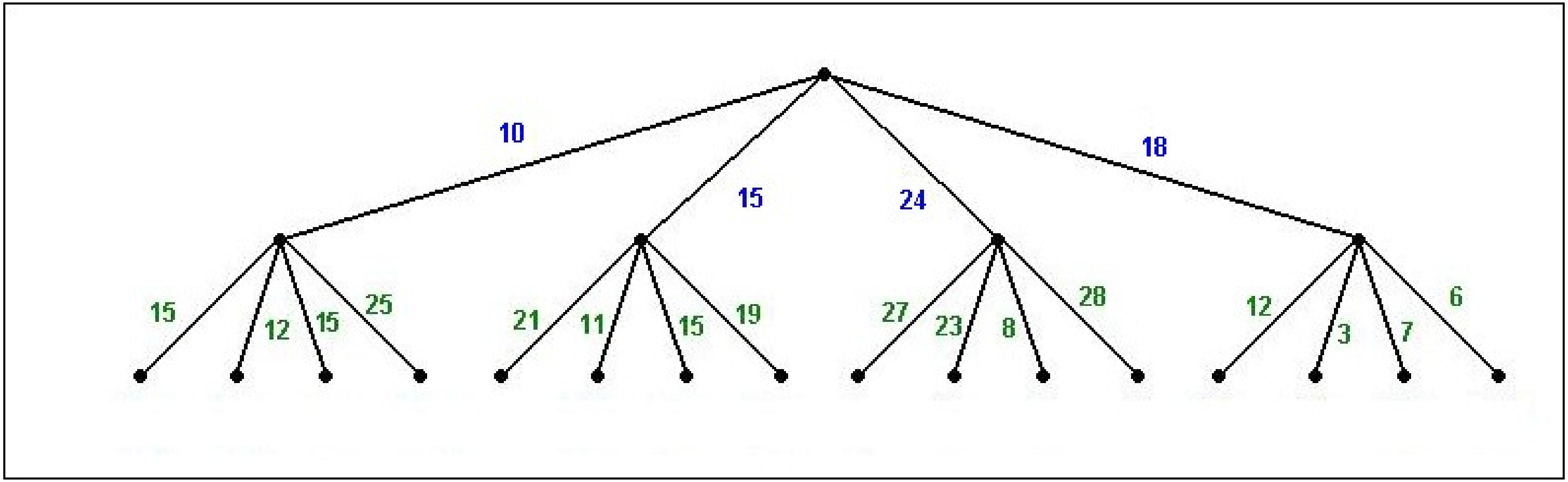
- 2 stages
  - Initialization
    - At level K $\longrightarrow$ smallest intermediate value leads to L next nodes at level K-1.
    - At level K-1 $\longrightarrow$ smallest value among the L nodes and so on until the level 1.
    - Sum of the K values gives a first minimum of the function, the starting radius **μo**
    - Other methods

Algorithm

- 2 stages
  - Pruning
    - Exploration of the other branches
    - Each branch which will certainly give a higher **μo** is pruning out
    - If a leaf is reached with a smaller sum than **μo**, **μo** is updated with that new value
    - The process continues until all branches have been explored or pruned out

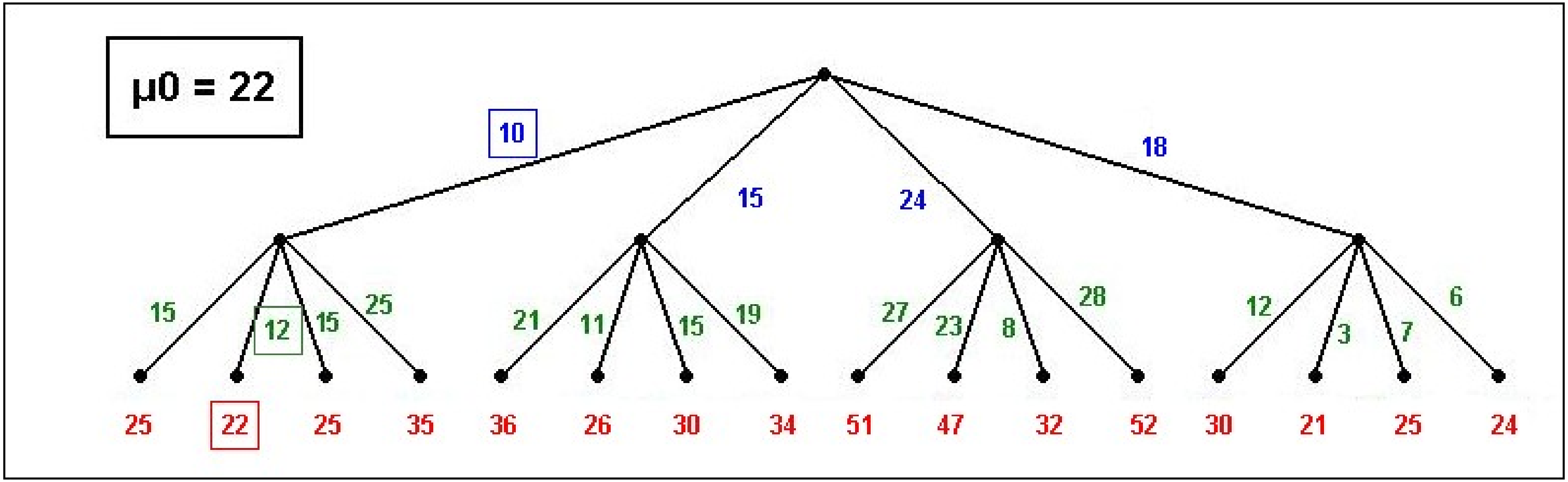
Exemple: K=2, L=4

- Initialization



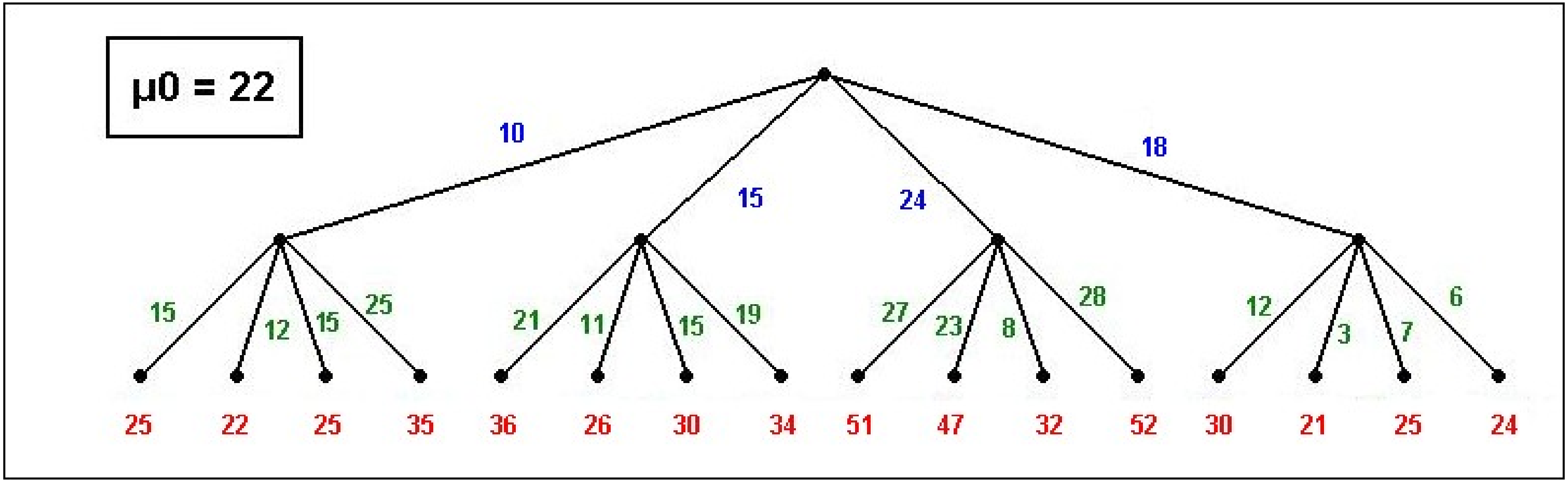
Exemple: K=2, L=4

- Initialization



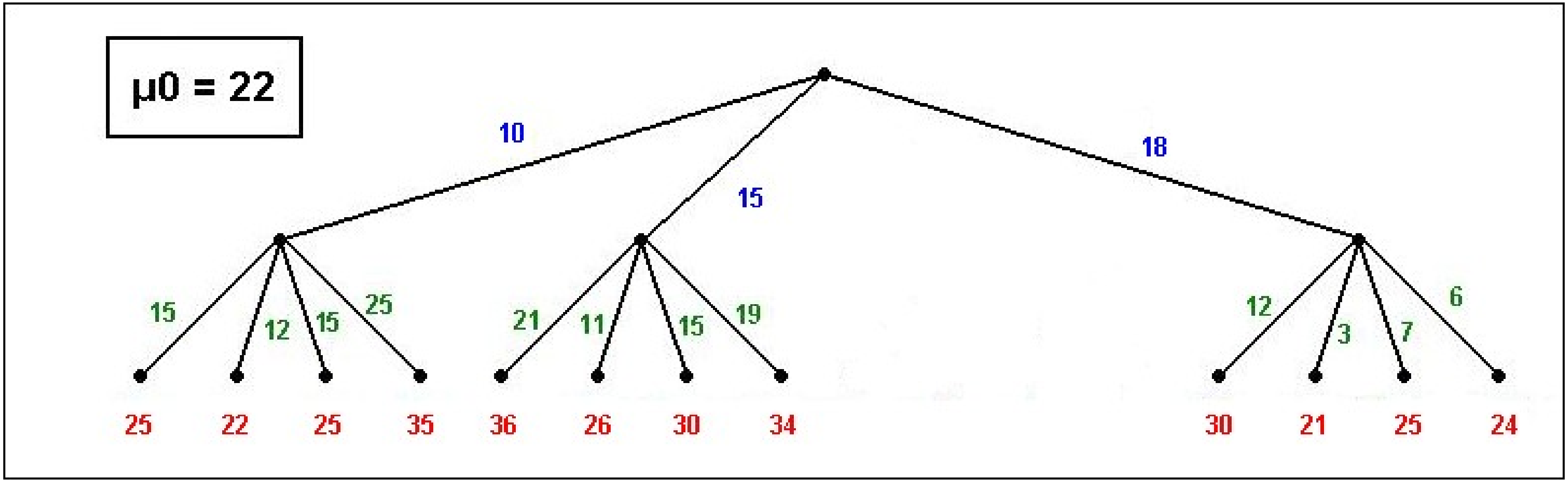
Exemple: K=2, L=4

- Pruning



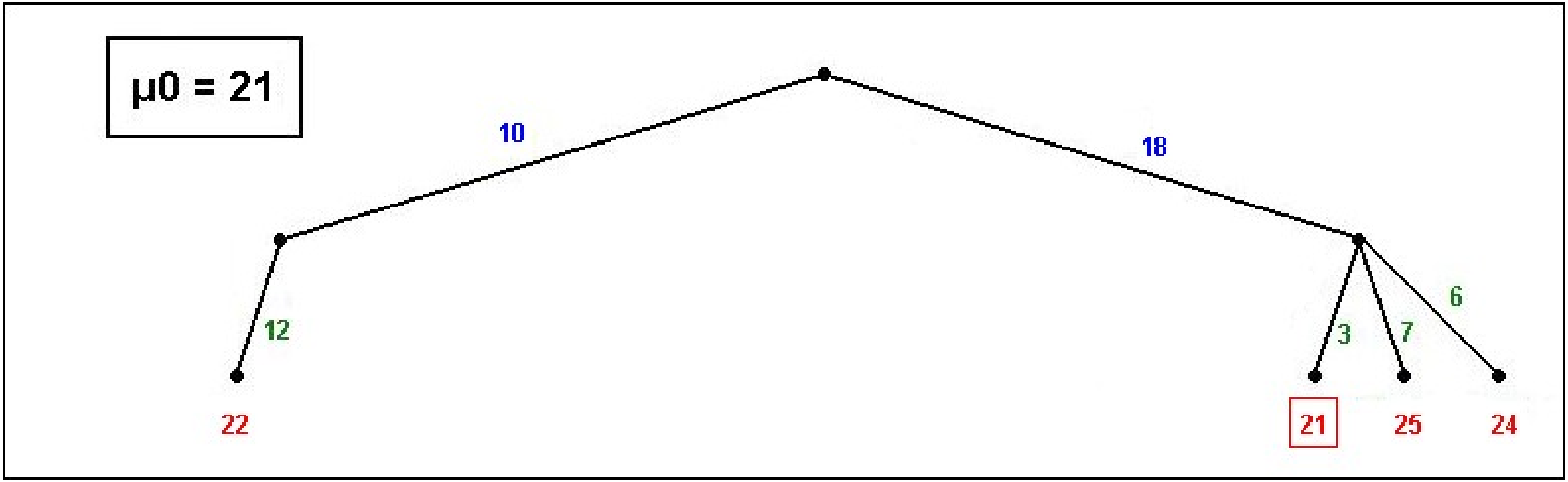
Exemple: K=2, L=4

- Pruning (first level)



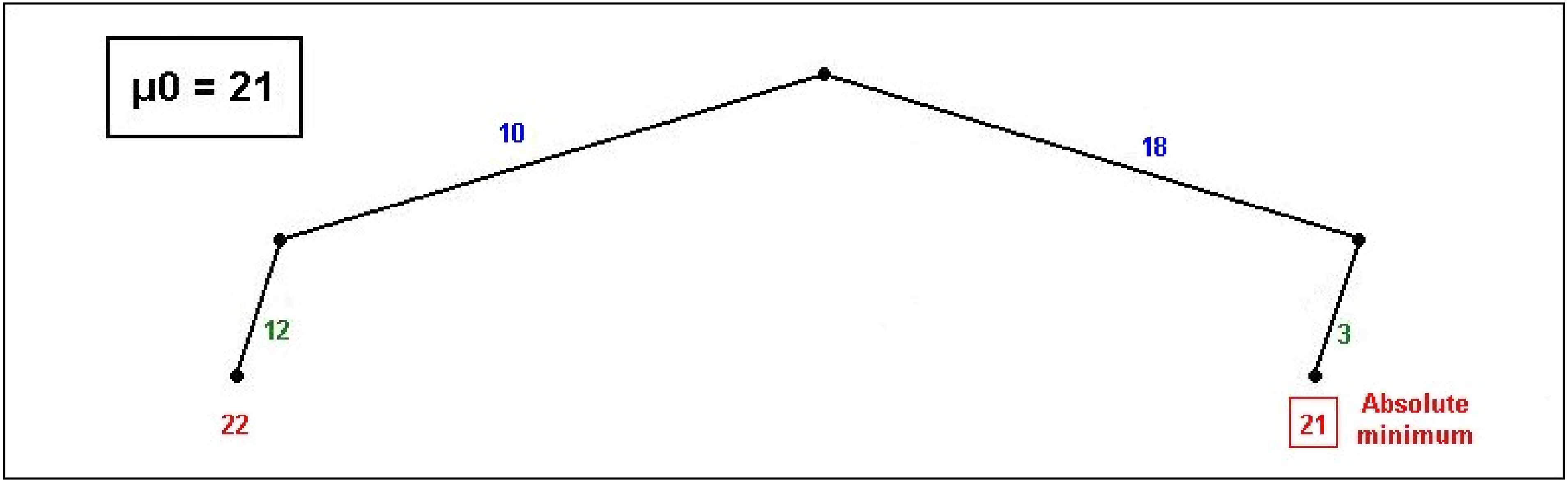
Exemple: K=2, L=4

- Pruning (second level)



Exemple: K=2, L=4

- Pruning (second level)



Conclusion

- The algorithm gives the absolute minimum of the cost function
- The complexity is smaller than a classic ML algorithm when the SNR grows as the pruning is more and more efficient
- The information of all the users is recovered in one time

Application to a MC-CSK system

Multiple Carrier Code Shift Keying

Reminder

- Spectral efficiency

$$\eta_s = R_b/B \quad (\text{bit.Hz}^{-1}.\text{s}^{-1})$$

- Power efficiency

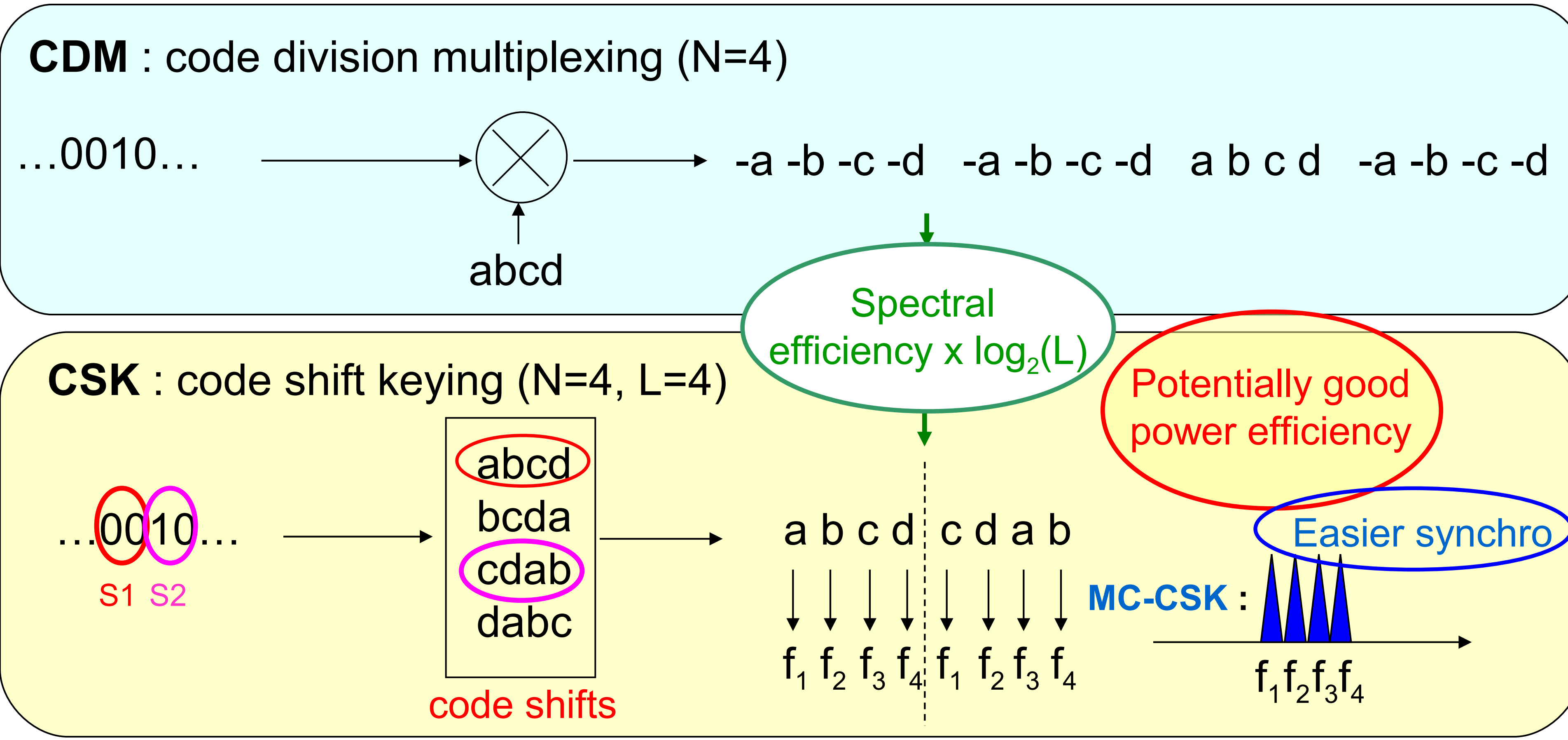
$$\eta_p = d^2/E_b$$

- Trade-off between η_s and η_p

MC-CSK system

- Broadcast synchronous system
- K flows assimilated to K “users”
- Each user
  - L spreading sequences of length N (Gold sequences)
  - Coding log2(L) bits
- Multi-user detection, not interference limited

MC-CSK system



MC-CSK system

$\underline{\mathbf{C}}_1$ = matrix containing all the sequences of user 1

$\underline{\mathbf{C}} = [\underline{\mathbf{C}}_1\ \underline{\mathbf{C}}_2\ \ldots\ \underline{\mathbf{C}}_K]$ = generation matrix

$\mathbf{g}_1$ vector of length L containing one '1' and L-1 '0'. The '1' points out the transmitted sequence for user 1

$\mathbf{g} = [\mathbf{g}_1\ \mathbf{g}_2\ \ldots\ \mathbf{g}_K]^T$

Received signal: $\mathbf{z} = \underline{\mathbf{C}}.\mathbf{g} + \mathbf{w}_\mathbf{n}$

MC-CSK system and Sphere Decoding

- In reception
  - Estimation of $\mathbf{g} = [\mathbf{g}_1\ \mathbf{g}_2\ \ldots\ \mathbf{g}_K]^T$
  - $\hat{\mathbf{g}} = \arg\min_{\mathbf{g}_1,\ldots,\mathbf{g}_K} ||\mathbf{z} - \underline{\mathbf{C}}\mathbf{g}||_2^2$ (cost function)
  - QR decomposition of $\underline{\mathbf{C}}$
    - $\underline{\mathbf{C}} = \underline{\mathbf{Q}} * \underline{\mathbf{R}}$
    - $\underline{\mathbf{Q}}$ unitary, $\underline{\mathbf{R}}$ upper triangular
  - $\hat{\mathbf{g}} = \arg\min_{\mathbf{g}_1,\ldots,\mathbf{g}_K} ||\mathbf{y} - \underline{\mathbf{R}}\mathbf{g}||_2^2$, $\mathbf{y} = \underline{\mathbf{Q}}^{T*}.\mathbf{z}$

MC-CSK system and Sphere Decoding

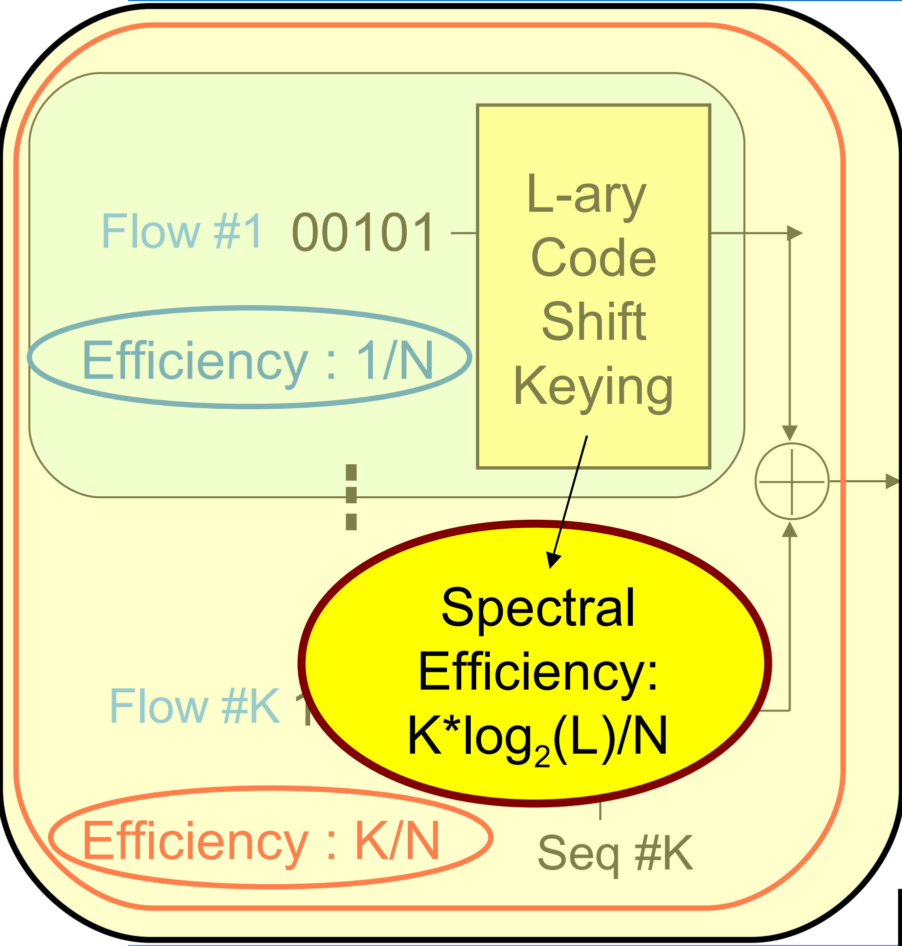
$$\underline{\mathbf{R}} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & \dots & r_{1,LK} \\ 0 & r_{22} & r_{23} & r_{24} & \dots & r_{2,LK} \\ 0 & 0 & r_{33} & r_{34} & \dots & r_{3,LK} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & r_{LK-1,LK-1} & r_{LK-1,LK} \\ 0 & 0 & 0 & \dots & 0 & r_{LK,LK} \\ 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

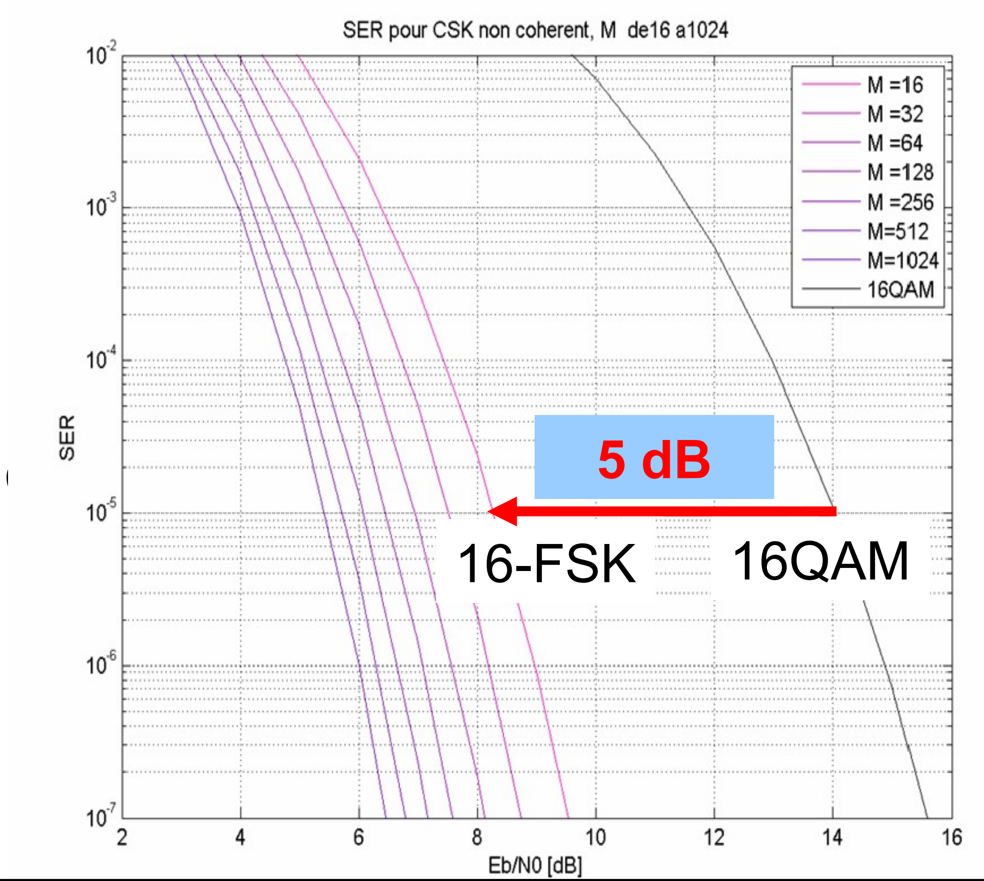
$$d = ||\mathbf{y} - \underline{\mathbf{R}}\mathbf{g}||_2^2 = h(\mathbf{g}_K) + h(\mathbf{g}_K,\mathbf{g}_{K-1}) + \dots + h(\mathbf{g}_K,\dots,\mathbf{g}_1)$$

MC-CSK system and Sphere Decoding

- Initialization
  - $\hat{\mathbf{g}}_K = \arg\min_{\mathbf{g}_K} h(\mathbf{g}_K)$
  - $\hat{\mathbf{g}}_{K-1} = \arg\min_{\mathbf{g}_{K-1}} h(\hat{\mathbf{g}}_K, \mathbf{g}_{K-1})$
  - $\hat{\mathbf{g}}_1 = \arg\min_{\mathbf{g}_1} h(\hat{\mathbf{g}}_K, \ldots, \mathbf{g}_1)$
  - $\mu_0 = h(\hat{\mathbf{g}}_K) + h(\hat{\mathbf{g}}_K, \hat{\mathbf{g}}_{K-1}) + \ldots + h(\hat{\mathbf{g}}_K, \hat{\mathbf{g}}_{K-1}, \ldots, \hat{\mathbf{g}}_1)$
- Pruning

Why using MC-CSK and Sphere Decoding?



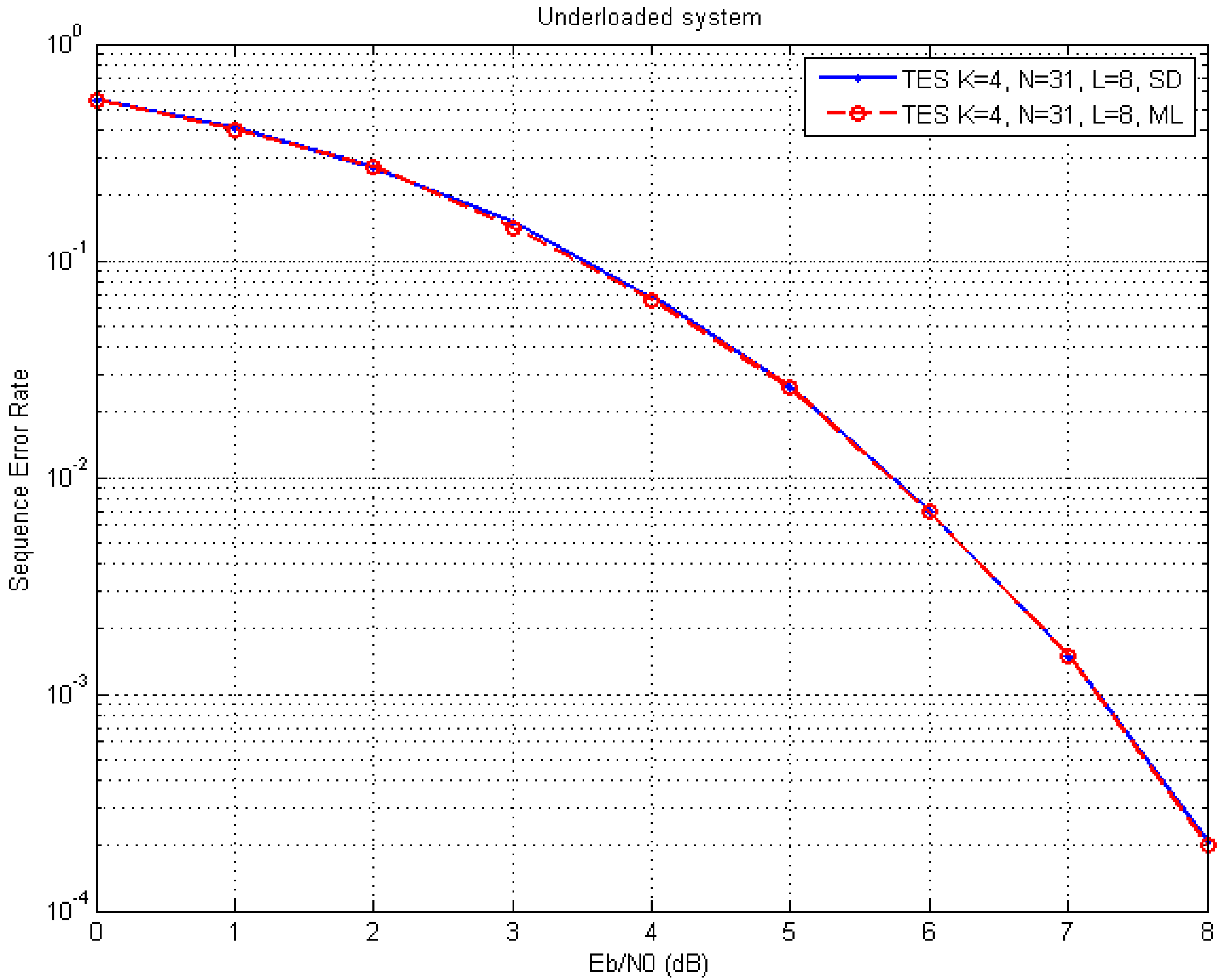


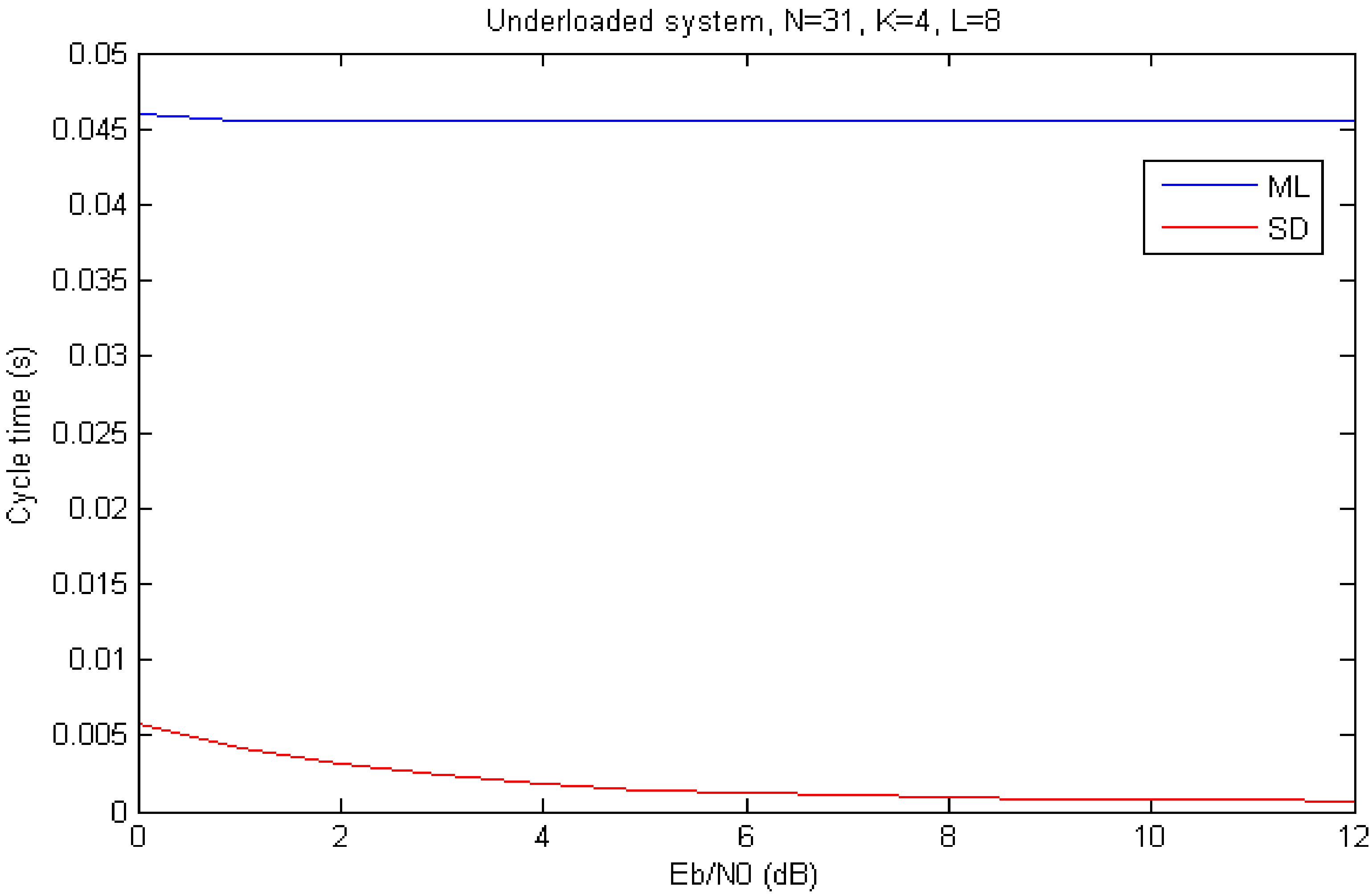
IF orthogonality of sequences & permutations
→ MC-CSK power efficiency
= FSK power efficiency !

Why using MC-CSK and Sphere Decoding?

- System not interference limited
- Improving in the same time spectral and power efficiency
- Optimal decoding with lower complexity
- Scalable to (small) overloaded systems

Results


Underloaded system
TES K=4, N=31, L=8, SD
TES K=4, N=31, L=8, ML
Sequence Error Rate
Eb/N0 (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
1
2
3
4
5
6
7
8


Underloaded system, N=31, K=4, L=8
Cycle time (s)
Eb/N0 (dB)
ML
SD
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
2
4
6
8
10
12

Limitations

- For higher loads (>15%), the complexity and time simulation are too important
- Need of another decoder to reach interesting spectral efficiency
- LASSO

Brief presentation of the LASSO

- Least Absolute Shrinkage and Selection Operator – (Tibshirani, 1994)

- Working on the hypothesis of the sparsity of vector **g**

- Using a penalty on the L1 norm and the residual sum of squares

$$\min \|\mathbf{y} - \mathbf{Cg}\|_2^2 \text{ s.t. } \|\mathbf{g}\|_1^2 <= \mathbf{t}$$

Thank you for your attention

Questions?