微分方程(组)求解

求解析解

```
% 若不指定自变量, 则默认为 t
dsolve('y-Dy=2*exp(x)','y(0)=1','x'); % Matlab 官方已不推荐使用
```

警告: Support of character vectors and strings will be removed in a future release. Use sym objects to define differential equations instead.

```
syms y(x) a % 该形式定义会指定自变量为 x
eqn = (y-diff(y,x,1) == 2*a*x);
dsolve(eqn,y(0)==1)
```

ans = $2a + 2ax - e^x (2a - 1)$

方程组

```
syms y(t) x(t) z(t)
eqn = [diff(x,t)==2*x-3*y+3*z+t, diff(y,t)==4*x-5*y+3*z+t, diff(z,t)==4*x-4*y+2*z+t];
eqn
```

eqn(t) =

$$\left(\frac{\partial}{\partial t}\, x(t) = t + 2\,x(t) - 3\,y(t) + 3\,z(t) \quad \frac{\partial}{\partial t}\, y(t) = t + 4\,x(t) - 5\,y(t) + 3\,z(t) \quad \frac{\partial}{\partial t}\, z(t) = t + 4\,x(t) - 4\,y(t) \cdot\right.$$

```
Dy = diff(y,t); % 由于不支持连续括号语法, 需先定义 Dy
cond = [y(0)==0, Dy(1)==15];
solution = dsolve(eqn,cond)
```

solution = *包含以下字段的 struct:*

```
    y: [1×1 sym]
    x: [1×1 sym]
    z: [1×1 sym]
```

```
solution.x = simplify(solution.x); % 可以尝试先化简
latex_reslut = latex(solution.x); % 转化为 Latex 格式代码输出
solution.x, solution.y, solution.z
```

ans =

$$z\,e^{2t} - \frac{t}{2} + \frac{e^{-t}\,(4z - 31\,e^2 + 4z\,e^4 - 1)}{2\,(e-2)} - \frac{1}{4}$$

ans =

$$e^{2t}\left(z - \frac{e^{-2t}\,(2t+1)}{4}\right) + \frac{e^{-t}\,(4z - 31\,e^2 + 4z\,e^4 - 1)}{2\,(e-2)} - \frac{e^{-2t}\,e\,(4z - 62\,e + 8z\,e^3 - 1)}{4\,(e-2)}$$

ans =

$$e^{2t}\left(z - \frac{e^{-2t}\,(2t+1)}{4}\right) - \frac{e^{-2t}\,e\,(4z - 62\,e + 8z\,e^3 - 1)}{4\,(e-2)}$$

求数值解

求解析解失败的例子

```
v = 1;
syms P(t) Q(t) x(t) y(t)
P = 20 + sqrt(5)/2*v*t;
Q = sqrt(5)/2*v*t;
Dx = diff(x,t);
Dy = diff(y,t);
com = 3*v/sqrt((P-x)^2 + (Q-y)^2);
eqn = [Dx==com*(P-x), Dy==com*(Q-y)];
cond = [x(0)==0, y(0)==0];
solution = dsolve(eqn, cond); % 求解析解失败
```

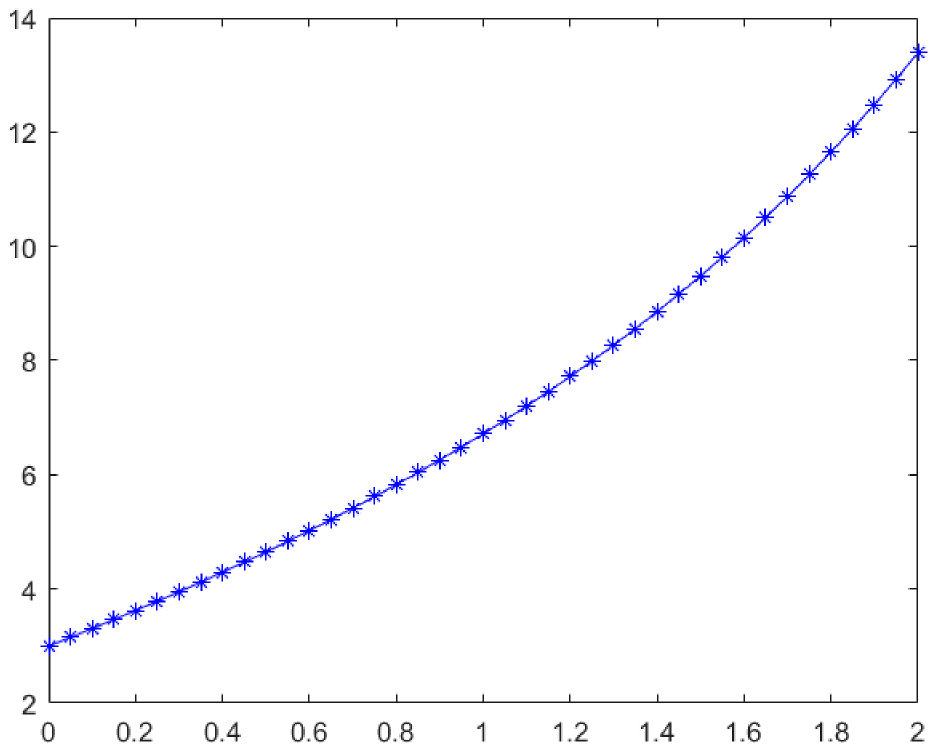
```
警告: Unable to find symbolic solution.
```

非刚性

```
tspan = [0, 2]; % 自变量范围
y0 = 3; % 初始值
[x, y] = ode45(@df1, tspan, y0); % 龙格-库塔法(利用计算机仿真)
x, y
```

```
x = 41×1
         0
    0.0500
    0.1000
    0.1500
    0.2000
    0.2500
    0.3000
    0.3500
    0.4000
    0.4500
      ⋮
y = 41×1
    3.0000
    3.1513
    3.3052
    3.4618
    3.6214
    3.7840
    3.9499
    4.1191
    4.2918
    4.4683
      ⋮
```

```
figure(1);
plot(x, y, 'b*-');
```

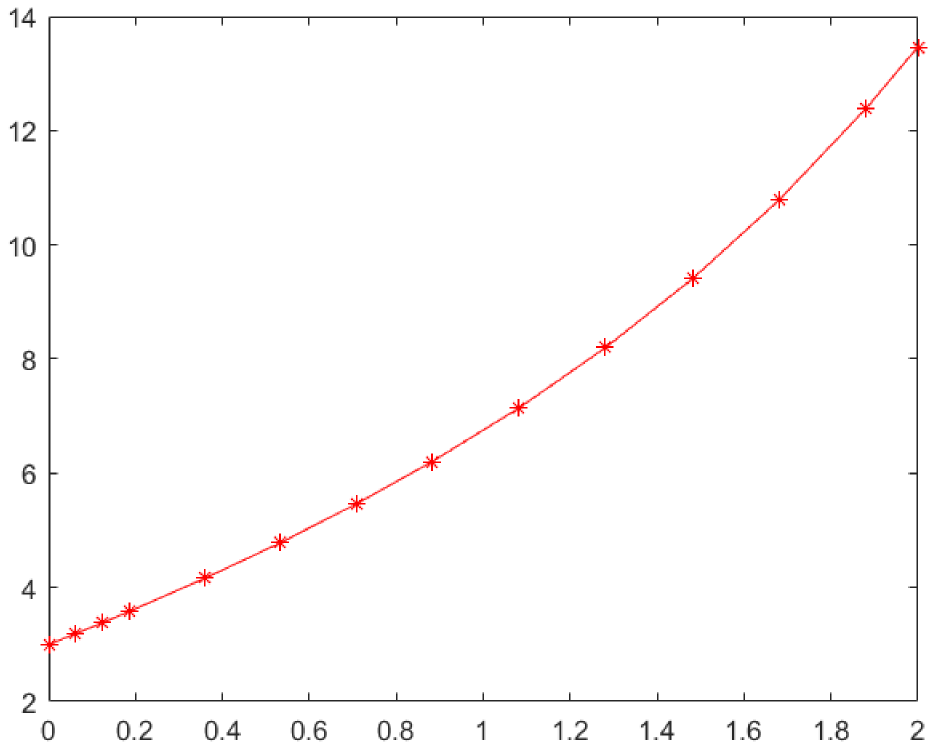


刚性

```
[x, y] = ode15s(@df1,tspan,y0);
x, y
```

```
x = 14×1
         0
    0.0620
    0.1239
    0.1859
    0.3598
    0.5336
    0.7075
    0.8814
    1.0814
    1.2814
      ⋮

y = 14×1
    3.0000
    3.1893
    3.3828
    3.5808
    4.1611
    4.7846
    5.4603
    6.1986
    7.1395
    8.1994
      ⋮
```

```
figure(2);
plot(x, y, 'r*-');
```



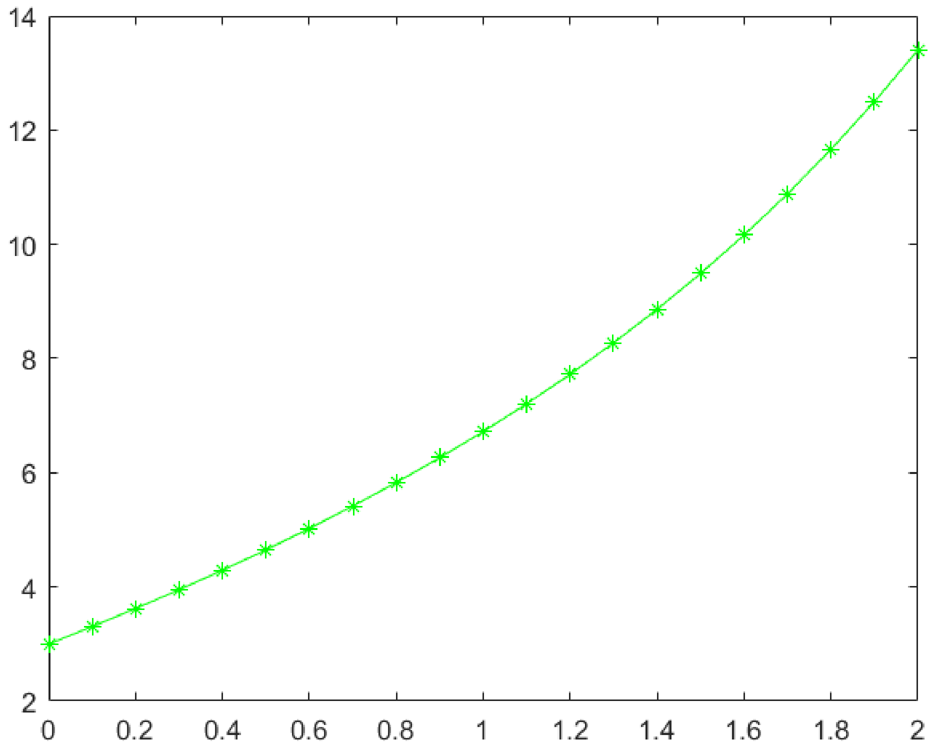
手动指定求解区间步长，指定相对误差与绝对误差

```
tspan = 0:1e-1:2;
option = odeset("RelTol",1e-4, "AbsTol",1e-8);
[x, y] = ode15s(@df1,tspan,y0, option);
x, y
```

```
x = 21×1
         0
    0.1000
    0.2000
    0.3000
    0.4000
    0.5000
    0.6000
    0.7000
    0.8000
    0.9000
      ⋮

y = 21×1
    3.0000
    3.3058
    3.6223
    3.9509
    4.2931
    4.6503
    5.0240
```

```
    5.4160
    5.8282
    6.2627
      ⋮
```

```
figure(3);
plot(x, y, 'g*-');
```



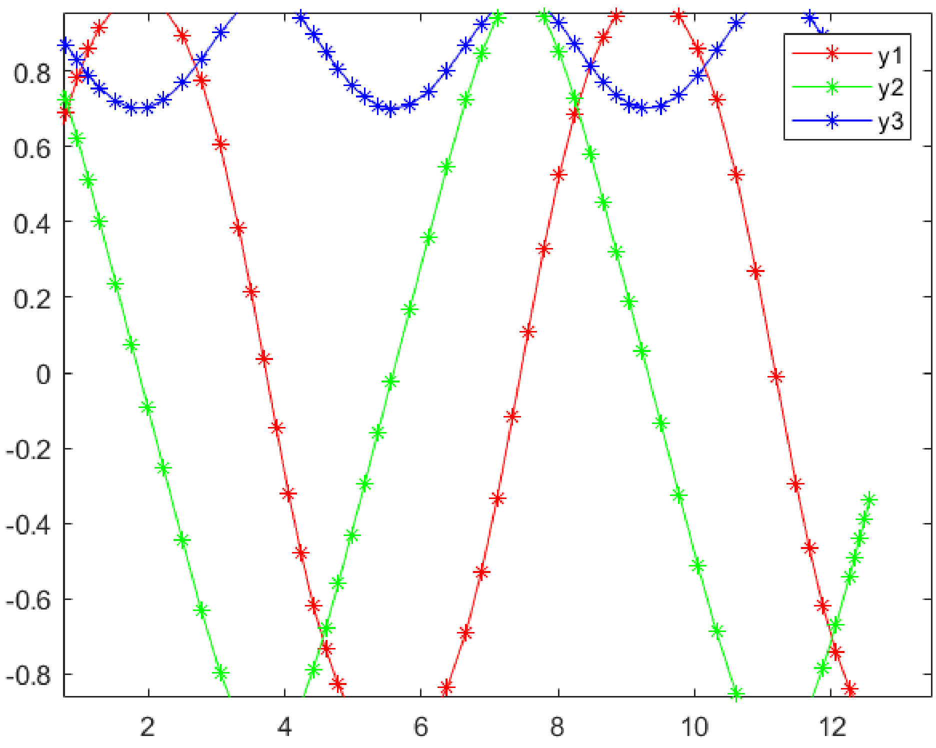
方程组

```
tspan = [0, 4*pi];
y0 = [0 1 1]; % 即 x = 0 时 y 的取值
[x, y] = ode45(@df2, tspan,y0);
x, y
```

```
x = 81×1
         0
    0.0001
    0.0001
    0.0002
    0.0002
    0.0005
    0.0007
    0.0010
    0.0012
    0.0025
      ⋮

y = 81×3
         0    1.0000    1.0000
```

```
    0.0001    1.0000    1.0000
    0.0001    1.0000    1.0000
    0.0002    1.0000    1.0000
    0.0002    1.0000    1.0000
    0.0005    1.0000    1.0000
    0.0007    1.0000    1.0000
    0.0010    1.0000    1.0000
    0.0012    1.0000    1.0000
    0.0025    1.0000    1.0000
     ⋮
```

```
figure(4);
plot(x, y(:,1), 'r*-');
hold on;
plot(x, y(:,2), 'g*-');
hold on;
plot(x, y(:,3), 'b*-');
legend('y1','y2','y3');
```



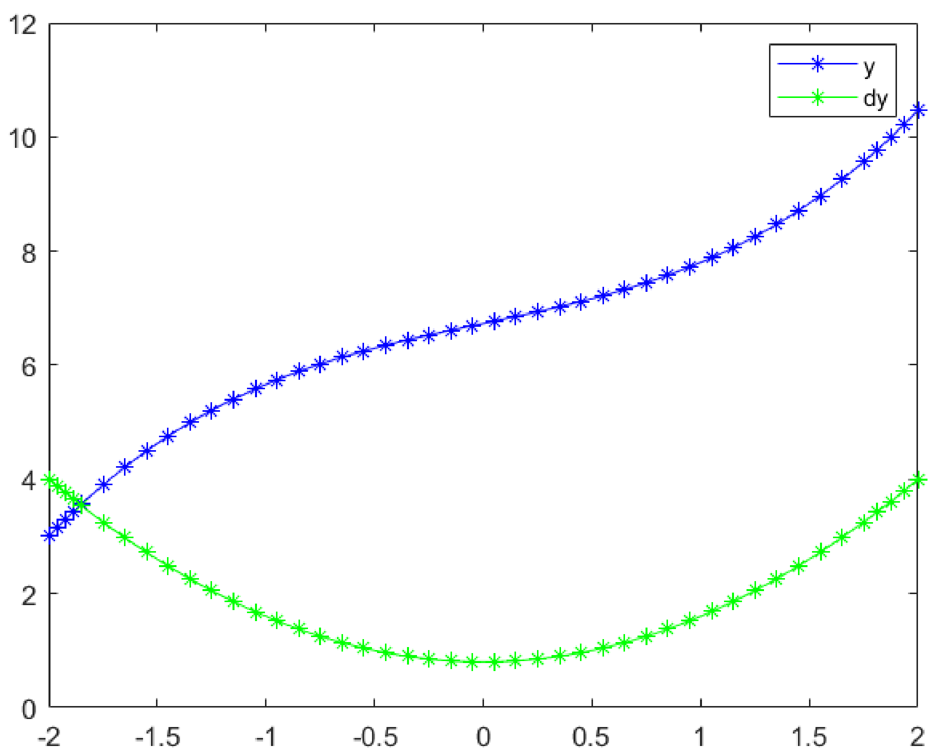
高阶微分方程

```
% 将高阶导数重命名为新变量(如 y1=y,y2=Dy,y3=D2y), 重命名后要将原来存在的求导关系转换为方程
% (如 y2=Dy1)加入 dy_n

% (每一个 dy(k)都是对 y(k)的一阶求导, 这是严格对应的)方程组(这是重命名可行性的核心原理)
% 标准形式为 dy_n = f(...) 其中 n=1,2,...
tspan = [-2, 2];
y0 = [3 4]; % y0 是基于 tspan 的起始值(此处为-2,即 y(-2)=3,dy(-2)=4)
```

```
[x, y] = ode45(@df3, tspan, y0); % 对 n 阶微分方程, solver 返回 0 ~ n-1 阶的结果
x, y
```

```
x = 45×1
   -2.0000
   -1.9623
   -1.9246
   -1.8870
   -1.8493
   -1.7493
   -1.6493
   -1.5493
   -1.4493
   -1.3493
      ⋮

y = 45×2
    3.0000    4.0000
    3.1485    3.8806
    3.2925    3.7634
    3.4321    3.6485
    3.5674    3.5359
    3.9065    3.2480
    4.2176    2.9761
    4.5022    2.7202
    4.7621    2.4803
    4.9988    2.2565
      ⋮
```

```
plot(x, y(:,1),'b*-', x, y(:,2),'g*-');
legend('y','dy');
```



函数定义

```
function dy = df1(x, y)
    dy = y-2*x;
end

function dy = df2(~, y)
    dy = zeros(3, 1);
    dy(1) = y(2)*y(3);
    dy(2) = -y(1)*y(3);
    dy(3) = -0.51*y(1)*y(2);
end

function dy = df3(x, y)
    dy = zeros(2, 1);
    % 对应 y0=[3,4]
    dy(1) = y(2);
    dy(2) = 2*x*y(2)/(1+x^2);
    % 对应 y0=[4,3] 两者结果相同(但在 y 中的列顺序不同)
%     dy(1) = 2*x*y(1)/(1+x^2);
%     dy(2) = y(1);
end
```