Mongkolsery Lin (1) Time to complete: 00:40 Points: 50/50

Final Examination Subject-1

## 1. Question 1

5 / 5 pts

*Auto-graded*

Solve the differential equation

$$(3e^t y + t)\,dt + e^t dy = 0$$

- $2t - y^4 = Cy^2$
- $e^{3t}y + \frac{1}{2}te^{2t} - t = c$
- $e^{3t}y + \frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} = c$ ✓
- $t^3 y - \frac{1}{2}t^2 y^2 = c$
- $\frac{1}{2}t^2 + ty + \ln|\sin y| = c$

## 2.

5 / 5 pts

*Auto-graded*

Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' - 2y' + y = te^t + 1$$

- $y_p(t) = (a_2t^2 + a_3t^3)e^t + b_0 + b_1t + b_2t^2$
- $y_p(t) = (a_2t^2 + a_3t^3)e^t + b_0 + b_1t$
- $y_p(t) = (a_2t^2 + a_3t^3)e^t + b_0$ ✓
- $y_p(t) = (a_1t + a_2t^2 + a_3t^3)e^{3t} + (b_0 + b_1t)e^t$
- $y_p(t) = a_0e^t + b_1t\cos t + c_1t\sin t$

3. Find the general solution of the differential equation

$$2\sqrt{t}y' = y^2 + 1$$

5 / 5 pts
*Auto-graded*

- $y(t) = \tan(\sqrt{t} + C) + 2t$
- $y(t) = \sin(\sqrt{t} + C)$
- **$y(t) = \tan(\sqrt{t} + C)$** ✓
- $y(t) = \sqrt{t} + \tan t + C$
- other

4. Solve the differential equation

$$(5t + 4y)dt + (4t - 8y^3)\,dy = 0$$

5 / 5 pts
*Auto-graded*

- $t^2 + 4ty - 2y^4 = C$
- **$\frac{5}{2}t^2 + 4ty - 2y^4 = C$** ✓
- $3t^4 + 4y^3 + 12y\ln t = C$
- $\frac{3}{2}t^2y^2 - ty^3 = C$
- $t + e^{ty} + y^2 = C$

5. Find the general solution of the differential equation

$$y'' - 3y' = te^{3t} + 1$$

5 / 5 pts
*Auto-graded*

- $y(t) = a_0 + b_0e^{3t} - \frac{1}{3}t + \frac{1}{6}t^2e^{3t}$
- $y(t) = a_0e^{3t} + b_0e^{4t} - 2te^{3t} - t^2e^{3t} + \frac{5}{144} - \frac{t}{12}$
- $y(t) = a_0\cos t + b_0\sin t + \frac{1}{4}t\sin t - \frac{1}{4}t^2\cos t - \frac{1}{3}\cos(2t)$
- **$y(t) = a_0 + b_0e^{3t} - \frac{1}{3}t - \frac{1}{9}te^{3t} + \frac{1}{6}t^2e^{3t}$** ✓
- $y(t) = (a_0 + a_1t)e^{3t} + \frac{1}{2}t^2e^{3t} + \frac{1}{9}$

6. Solve the differential equation

$$ty' = \frac{1}{\sin\left(\frac{y}{t}\right)} + y$$

5 / 5 pts

*Auto-graded*

- $y^2 = t^2(\ln|t| + C)$
- $1 + \left(\frac{y}{t}\right)^2 = Ct$
- $\cos\left(\frac{y}{t}\right) = \frac{1}{t} + C$ ✓
- $\sin\left(\frac{y}{t}\right) = \frac{1}{t} + C$
- $\frac{y+1}{t} = -\frac{1}{\ln(|t|)+C}$

7. Question

Solve the differential equation

$$y' + y\cot t = y^4 \sin t$$

5 / 5 pts

*Auto-graded*

- $\frac{1}{y^3} = (3\cot t + C)\sin^3 t$ ✓
- $\frac{1}{y^3} = (\cos t + C)\sin^3 t$
- $y^3 = (3\cot t + C)\sin^3 t$
- $t\sqrt{y} = \frac{t^3}{3} + C$
- $y^2 = \frac{t^2}{3} + C\sqrt{t}$

## 8. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & -2 & -3 \\ 2 & 3 & 4 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) = C_1 e^t \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} + C_2 e^t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + C_3 e^t \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$

(B) $X(t) = C_1 e^t \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} + C_2 e^t \left[ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} \right]$

(C) $X(t) = C_1 e^t \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} + C_2 e^t \left[ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} \right] + C_3 e^t \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$

- ○ **(A)**
- ○ **(B)**
- ◉ **(C)** ✓

## 9. Question

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 3x(t) - 4y(t) + 1 \\ y'(t) = x(t) - y(t) + t \end{cases}$$

(A) $X(t) = C_1 e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -7 - 4t \\ -4 - 3t \end{pmatrix}$

(B) $X(t) = C_1 e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^t \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right] + \begin{pmatrix} -7 - 4t \\ -4 - 3t \end{pmatrix}$

(C) $X(t) = C_1 e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{2t} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right] + \begin{pmatrix} 1 \\ t \end{pmatrix}$

(D) $X(t) = Ce^t \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right] + \begin{pmatrix} -7 - 4t \\ -4 - 3t \end{pmatrix}$

- ○ **(A)**
- ◉ **(B)** ✓
- ○ **(C)**
- ○ **(D)**

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Time to complete: 01:17 Points: 50/50

Final Examination Subject-2

1. Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' - 5y' + 6y = te^{2t} + 3e^t$$

5 / 5 pts
*Auto-graded*

- (selected) $y_p(t) = (a_1t + a_2t^2)e^{2t} + b_0e^t$ ✓
- $y_p(t) = (a_0 + a_1t)e^{2t} + b_0e^t$
- $y_p(t) = (a_2t^2 + a_3t^3)e^t + b_0$
- $y_p(t) = (a_1t + a_2t^2 + a_3t^3)e^{3t} + (b_0 + b_1t)e^t$
- $y_p(t) = a_0e^t + b_1t\cos t + c_1t\sin t$

2. Find the general solution of the differential equation

$$y'' + y' = 2 + t + te^{-t}$$

5 / 5 pts
*Auto-graded*

- (selected) $y(t) = a_0 + b_0e^{-t} + t + \frac{1}{2}t^2 - te^{-t} - \frac{1}{2}t^2e^{-t}$ ✓
- $y(t) = a_0 + b_0e^{-t} + \frac{1}{2}t^2 - te^{-t} - \frac{1}{2}t^2e^{-t}$
- $y(t) = a_0\cos t + b_0\sin t + \frac{1}{4}t\sin t - \frac{1}{4}t^2\cos t - \frac{1}{3}\cos(2t)$
- $y(t) = t + \frac{1}{2}t^2 - te^{-t} - \frac{1}{2}t^2e^{-t}$
- $y(t) = (a_0 + a_1t)e^{3t} + \frac{1}{2}t^2e^{3t} + \frac{1}{9}$

**3.** Find the general solution of the differential equation

$$y' = \frac{t}{y + t^2 y}$$

5 / 5 pts
*Auto-graded*

- $y = \ln(1 + t^2) + C$
- $y^2 = \ln(1 + t^2) + C$ ✓
- $y(t) = \tan(\sqrt{t} + C)$
- $y^2 = \ln(1 + t) + C$
- $y' = 6t(y - 1)^{\frac{2}{3}}$

**4.** Solve the differential equation

$$(3t^2 + 2y^2)\,dt + (4ty + 6y^2)\,dy = 0$$

5 / 5 pts
*Auto-graded*

- $t^3 + 2y^2t + 2y^3 = C$ ✓
- $\frac{5}{2}t^2 + 4ty - 2y^4 = C$
- $3t^4 + 4y^3 + 12y\ln t = C$
- $t^3 + y^2t + y^3 = C$
- $t + e^{ty} + y^2 = C$

**5.** Solve the differential equation

$$(t + y)\sin y\,dt + (t\sin y + \cos y)dy = 0$$

5 / 5 pts
*Auto-graded*

- $\frac{1}{2}t^2 + ty + \sin y = c$
- $\frac{1}{2}t^2 + ty + \ln|y| = c$
- $e^{3t}y + \frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} = c$
- $t^3y - \frac{1}{2}t^2y^2 = c$
- $\frac{1}{2}t^2 + ty + \ln|\sin y| = c$ ✓

**6.** Solve the differential equation

$$y' = \frac{1}{1+\cos\left(\frac{y}{t}\right)} + \frac{y}{t}$$

5 / 5 pts

*Auto-graded*

- $\frac{y}{t} + \sin\left(\frac{y}{t}\right) = t + C$
- $1 + \left(\frac{y}{t}\right)^2 = Ct$
- $\cos\left(\frac{y}{t}\right) = \frac{1}{t} + C$
- $\frac{y}{t} + \sin\left(\frac{y}{t}\right) = \ln|t| + C$ ✓
- $\frac{y+1}{t} = -\frac{1}{\ln(|t|)+C}$

**7. Question**

5 / 5 pts

*Auto-graded*

Solve the differential equation

$$4tyy' = y^2 + t^2$$

- $\frac{1}{y^3} = (3\cot t + C)\sin^3 t$
- $y = \frac{1}{Ct - t^2}$
- $\frac{1}{y^3} = \frac{1-t}{2} + Ce^{-t}$
- $t\sqrt{y} = \frac{t^2}{3} + C$
- $y^2 = \frac{t^2}{3} + C\sqrt{t}$ ✓

## 8. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} 3 & -1 & -3 \\ 1 & 1 & -3 \\ 0 & 0 & 2 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) = C_1 e^{2t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + C_2 e^{2t} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + C_3 e^{2t} \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$

(B) $X(t) = C_1 e^{2t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \left[ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right] + C_3 t^2 e^{2t} \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$

(C) $X(t) = C_1 e^{2t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \left[ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right] + C_3 e^{2t} \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$

- (A)
- (B)
- (C) ✓

## 9. Question

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 3x(t) - 4y(t) - t \\ y'(t) = x(t) - y(t) + 2t \end{cases}$$

(A) $X(t) = C_1 e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^t \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right] - \begin{pmatrix} -t \\ 2t \end{pmatrix}$

(B) $X(t) = C_1 e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 19 + 12t \\ 4 + 7t \end{pmatrix}$

(C) $X(t) = C e^t \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right] - \begin{pmatrix} 19 + 12t \\ 4 + 7t \end{pmatrix}$

(D) $X(t) = C_1 e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^t \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right] - \begin{pmatrix} 19 + 12t \\ 4 + 7t \end{pmatrix}$

- (A)
- (B)
- (C)
- (D) ✓

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Time to complete: 00:55 Points: 50/50

Final Examination Subject-3

1. 5 / 5 pts

*Auto-graded*

Solve the differential equation

$$y' + 2ty^2 = 0$$

- [x] $y = \frac{1}{t^2+C}$
- [ ] $y = t^2 + C$
- [ ] $y(t) = \tan(\sqrt{t} + C)$
- [ ] $y(t) = \sqrt{t} + \tan t + C$
- [ ] $y^2 = \frac{1}{t^2+C}$

2. 5 / 5 pts

*Auto-graded*

Find the general solution of the differential equation

$$y'' - 7y' + 12y = 2te^{3t} + 1 - t$$

- [ ] $y(t) = a_0e^{3t} + b_0e^{4t} - 2te^{3t} - t^2e^{3t} - \frac{t}{12}$
- [x] $y(t) = a_0e^{3t} + b_0e^{4t} - 2te^{3t} - t^2e^{3t} + \frac{5}{144} - \frac{t}{12}$
- [ ] $y(t) = a_0\cos t + b_0\sin t + \frac{1}{4}t\sin t - \frac{1}{4}t^2\cos t - \frac{1}{3}\cos(2t)$
- [ ] $y(t) = a_0 + b_0e^{3t} - \frac{1}{3}t - \frac{1}{9}te^{3t} + \frac{1}{6}t^2e^{3t}$
- [ ] $y(t) = (a_0 + a_1t)e^{3t} + \frac{1}{2}t^2e^{3t} + \frac{1}{9}$

3. Solve the differential equation

$$(3ty^2 - y^3)\,dt + (3t^2y - 3ty^2)\,dy = 0$$

5 / 5 pts

*Auto-graded*

- $t^2 + 4ty - 2y^4 = C$
- $\frac{5}{2}t^2 + 4ty - 2y^4 = C$
- $\frac{3}{2}t^2y - ty^2 = C$
- $\frac{3}{2}t^2y^2 - ty^3 = C$ ✓
- $t + e^{ty} + y^2 = C$

4. Solve the differential equation

$$(3ty - y^2)\,dt + t(t - y)dy = 0$$

5 / 5 pts

*Auto-graded*

- $2t - y^4 = Cy^2$
- $e^{3t}y + \frac{1}{2}te^{2t} - t = c$
- $e^{3t}y + \frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} = c$
- $t^3y - \frac{1}{2}t^2y^2 = c$ ✓
- $\frac{1}{2}t^2 + ty + \ln|\sin y| = c$

5. Solve the differential equation

$$t^2y' = t(y + 1) + (y + 1)^2$$

5 / 5 pts

*Auto-graded*

- $y^2 = t^2(\ln|t| + C)$
- $1 + \left(\frac{y}{t}\right)^2 = Ct$
- $\cos\left(\frac{y}{t}\right) = \frac{1}{t} + C$
- $\sin\left(\frac{y}{t}\right) = \frac{1}{t} + C$
- $\frac{y+1}{t} = -\frac{1}{\ln(|t|)+C}$ ✓

6. Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' - 2y' + y = (2t+1)e^t + 3t$$

5 / 5 pts

*Auto-graded*

- $y_p(t) = (a_2t^2 + a_3t^3)e^t + b_0 + b_1t + b_2t^2$
- $y_p(t) = (a_2t^2 + a_3t^3)e^t + b_0 + b_1t$ ✓
- $y_p(t) = (a_2t^2 + a_3t^3)e^t + b_0$
- $y_p(t) = (a_1t + a_2t^2 + a_3t^3)e^{3t} + (b_0 + b_1t)e^t$
- $y_p(t) = a_0e^t + b_1t\cos t + c_1t\sin t$

7. Question

5 / 5 pts

*Auto-graded*

Solve the differential equation

$$y' + \frac{2y}{t} = 2t\sqrt{y}$$

- $\frac{1}{y^3} = (3\cot t + C)\sin^3 t$
- $\frac{1}{y^3} = (\cos t + C)\sin^3 t$
- $y^3 = (3\cot t + C)\sin^3 t$
- $t\sqrt{y} = \frac{t^3}{3} + C$ ✓
- $y^2 = \frac{t^2}{3} + C\sqrt{t}$

## 8. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} 3 & 1 & -1 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) = C_1 e^{2t} \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} + C_2 e^{2t} \left[ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \right] +$
$C_3 e^{2t} \left[ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \frac{t^2}{2} \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \right]$

(B) $X(t) =$
$C_1 e^{2t} \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} + C_2 e^{2t} \left[ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \right] + C_3 e^{2t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

- (A) ✓
- (B)

## 9. Question

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 2x(t) - 13y(t) - t \\ y'(t) = x(t) - 4y(t) + 2t \end{cases}$$

(A) $X(t) = C_1 e^{-t} \begin{pmatrix} 3\cos(2t) - 2\sin(2t) \\ \cos(2t) \end{pmatrix} +$
$C_2 e^{-t} \begin{pmatrix} 2\cos(2t) + 3\sin(2t) \\ \sin(2t) \end{pmatrix} + \begin{pmatrix} \frac{11}{5} - 6t \\ \frac{4}{5} - t \end{pmatrix}$

(B) $X(t) = C_1 e^{-t} \begin{pmatrix} 3\cos(2t) - 2\sin(2t) \\ \cos(2t) \end{pmatrix} +$
$C_2 e^{-t} \begin{pmatrix} 2\cos(2t) + 3\sin(2t) \\ \sin(2t) \end{pmatrix} + \begin{pmatrix} -t \\ 2t \end{pmatrix}$

(C) $X(t) = C_1 e^{-t} \begin{pmatrix} 3\cos(2t) \\ \cos(2t) \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} +3\sin(2t) \\ \sin(2t) \end{pmatrix} + \begin{pmatrix} \frac{11}{5} - 6t \\ \frac{4}{5} - t \end{pmatrix}$

- (A) ✓
- (B)
- (C)

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Time to complete: 00:52 Points: 50/50

Final Examination Subject-4

1. 5 / 5 pts

*Auto-graded*

Solve the differential equation

$$y' = 3\sqrt{ty}$$

- $y = t^{\frac{3}{2}} + C$
- $y = \left(t^{\frac{1}{2}} + C\right)^2$
- $y = \left(t^{\frac{3}{2}} + C\right)^2$ ✓
- $y(t) = \sqrt{t} + \tan t + C$
- other

2. 5 / 5 pts

*Auto-graded*

Solve the differential equation

$$\left(1 + ye^{ty}\right) dt + \left(2y + te^{ty}\right) dy = 0$$

- $t^2 + 4ty - 2y^4 = C$
- $\frac{5}{2}t^2 + 4ty - 2y^4 = C$
- $3t^4 + 4y^3 + 12y \ln t = C$
- $\frac{3}{2}t^2y^2 - ty^3 = C$
- $t + e^{ty} + y^2 = C$ ✓

**3.** Find the general solution of the differential equation

$$y'' + y = \cos(2t) + t\sin t$$

5 / 5 pts

*Auto-graded*

- $y(t) = a_0 + b_0 e^{3t} - \frac{1}{3}t + \frac{1}{6}t^2 e^{3t}$
- $y(t) = a_0 \cos t + b_0 \sin t + \frac{1}{4}t^2 \cos t - \frac{1}{3}\cos(2t)$
- $y(t) = a_0 \cos t + b_0 \sin t + \frac{1}{4}t\sin t - \frac{1}{4}t^2 \cos t - \frac{1}{3}\cos(2t)$ ✓
- $y(t) = a_0 + b_0 e^{3t} - \frac{1}{3}t - \frac{1}{9}te^{3t} + \frac{1}{6}t^2 e^{3t}$
- $y(t) = (a_0 + a_1 t)e^{3t} + \frac{1}{2}t^2 e^{3t} + \frac{1}{9}$

**4.** Solve the differential equation

$$(ty - 1)dt + t^2 dy = 0$$

5 / 5 pts

*Auto-graded*

- $y = \ln|t| + C$
- $ty - \ln|t| = C$ ✓
- $e^{3t}y + \frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} = c$
- $t^3 y - \frac{1}{2}t^2 y^2 = c$
- $\frac{1}{2}t^2 + ty + \ln|\sin y| = c$

**5.** Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' - 5y' + 6y = te^t + 3t^2 e^{3t}$$

5 / 5 pts

*Auto-graded*

- $y_p(t) = (a_2 t^2 + a_3 t^3)e^t + b_0 + b_1 t + b_2 t^2$
- $y_p(t) = (a_1 t + a_2 t^2 + a_3 t^3)e^{3t} + b_0 e^t$
- $y_p(t) = (a_1 t + a_2 t^2)e^{3t} + (b_0 + b_1 t)e^t$
- $y_p(t) = (a_1 t + a_2 t^2 + a_3 t^3)e^{3t} + (b_0 + b_1 t)e^t$ ✓
- $y_p(t) = a_0 e^t + b_1 t\cos t + c_1 t\sin t$

**6.** Solve the differential equation

$$y' = \frac{t^2 + 3y^2}{2ty}$$

5 / 5 pts

*Auto-graded*

- $y^2 = t^2(\ln|t| + C)$
- $1 + \left(\frac{y}{t}\right)^2 = Ct$ ✓
- $\cos\left(\frac{y}{t}\right) = \frac{1}{t} + C$
- $\sin\left(\frac{y}{t}\right) = \frac{1}{t} + C$
- $\frac{y+1}{t} = -\frac{1}{\ln(|t|)+C}$

**7.** Question

Solve the differential equation

$$6y' - 2y = ty^4$$

5 / 5 pts

*Auto-graded*

- $\frac{1}{y^3} = (3\cot t + C)\sin^3 t$
- $y^3 = \frac{1-t}{2} + Ce^{-t}$
- $\frac{1}{y^3} = \frac{1-t}{2} + Ce^{-t}$ ✓
- $t\sqrt{y} = \frac{t^3}{3} + C$
- $y^2 = \frac{t^2}{3} + C\sqrt{t}$

## 8. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} 3 & 0 & 4 \\ 1 & 1 & 1 \\ -1 & 0 & -1 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) = C_1 e^t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + C_2 e^t \left[ \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] + C_3 e^t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

(B) $X(t) = C_1 e^t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + C_2 e^t \left[ \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] +$
$C_3 e^t \left[ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \frac{t^2}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right]$

○ **(A)**

◉ **(B)** ✓

## 9. Question

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 2x(t) - 13y(t) - 5t \\ y'(t) = x(t) - 4y(t) \end{cases}$$

(A) $X(t) = C_1 e^{-t} \begin{pmatrix} 3\cos(2t) \\ \cos(2t) \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 3\sin(2t) \\ \sin(2t) \end{pmatrix} + \begin{pmatrix} \frac{3}{5} - 4t \\ \frac{2}{5} - t \end{pmatrix}$

(B) $X(t) = C_1 e^{-t} \begin{pmatrix} 3\cos(2t) - 2\sin(2t) \\ \cos(2t) \end{pmatrix} +$
$C_2 e^{-t} \begin{pmatrix} 2\cos(2t) + 3\sin(2t) \\ \sin(2t) \end{pmatrix} + \begin{pmatrix} -5t \\ 0 \end{pmatrix}$

(C) $X(t) = C_1 e^{-t} \begin{pmatrix} 3\cos(2t) - 2\sin(2t) \\ \cos(2t) \end{pmatrix} +$
$C_2 e^{-t} \begin{pmatrix} 2\cos(2t) + 3\sin(2t) \\ \sin(2t) \end{pmatrix} + \begin{pmatrix} \frac{3}{5} - 4t \\ \frac{2}{5} - t \end{pmatrix}$

○ **(A)**

○ **(B)**

◉ **(C)** ✓

Mongkolsery Lin

Time to complete: 00:51 Points: 50/50

Final Examination Subject-5

1. Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' + y = e^t + \sin t$$

5 / 5 pts
*Auto-graded*

- $y_p(t) = (a_2t^2 + a_3t^3)e^t + b_0 + b_1t + b_2t^2$
- $y_p(t) = b_1t\cos t + c_1t\sin t$
- $y_p(t) = a_0e^t + c_1t\sin t$
- $y_p(t) = a_0e^t + b_1t\cos t + c_1t\sin t + c_2t^2\sin t$
- **$y_p(t) = a_0e^t + b_1t\cos t + c_1t\sin t$** ✓

2. Solve the differential equation

$$y' = 6t(y-1)^{\frac{2}{3}}$$

5 / 5 pts
*Auto-graded*

- $y = 1 + t^2 + C$
- **$y = 1 + (t^2 + C)^3$** ✓
- $y(t) = \sqrt{t} + C$
- $y(t) = \sqrt{t} + \tan t + C$
- other

3. Solve the differential equation

$$\left(t^3 + \frac{y}{t}\right) dt + \left(y^2 + \ln t\right) dy = 0$$

5 / 5 pts
*Auto-graded*

- $t^2 + 4ty - 2y^4 = C$
- $\frac{5}{2}t^2 + 4ty - 2y^4 = C$
- $3t^4 + 4y^3 + 12y \ln t = C$ ✓
- $\frac{3}{2}t^2y^2 - ty^3 = C$
- $t + e^{ty} + y^2 = C$

4. Solve the differential equation

$$ydt - \left(2t + y^4\right) dy = 0$$

5 / 5 pts
*Auto-graded*

- $2t - y^4 = Cy^2$ ✓
- $e^{3t}y + \frac{1}{2}te^{2t} - t = c$
- $e^{3t}y + \frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} = c$
- $t^3y - \frac{1}{2}t^2y^2 = c$
- $\frac{1}{2}t^2 + ty + \ln|\sin y| = c$

5. Solve the differential equation

$$y' = \frac{t^2 + 2y^2}{2ty}$$

5 / 5 pts
Auto-graded

- (selected) $y^2 = t^2(\ln|t| + C)$ ✓
- $1 + \left(\frac{y}{t}\right)^2 = Ct$
- $\cos\left(\frac{y}{t}\right) = \frac{1}{t} + C$
- $\sin\left(\frac{y}{t}\right) = \frac{1}{t} + C$
- $\frac{y+1}{t} = -\frac{1}{\ln(|t|)+C}$

6. Find the general solution of the differential equation

$$y'' - 6y' + 9y = e^{3t} + 1$$

5 / 5 pts
Auto-graded

- $y(t) = a_0 + b_0e^{3t} - \frac{1}{3}t + \frac{1}{6}t^2e^{3t}$
- $y(t) = a_0e^{3t} + b_0e^{4t} - 2te^{3t} - t^2e^{3t} - \frac{7}{144} - \frac{t}{12}$
- $y(t) = (a_0 + a_1t)e^{3t} + \frac{1}{2}t^2e^{3t} + 9$
- $y(t) = a_0 + b_0e^{3t} - \frac{1}{3}t - \frac{1}{9}te^{3t} + \frac{1}{6}t^2e^{3t}$
- (selected) $y(t) = (a_0 + a_1t)e^{3t} + \frac{1}{2}t^2e^{3t} + \frac{1}{9}$ ✓

## 7. Question

5 / 5 pts

*Auto-graded*

Solve the differential equation

$$ty' + y = t^2y^2$$

- $\frac{1}{y^3} = (3\cot t + C)\sin^3 t$
- $\frac{1}{y^3} = (\cos t + C)\sin^3 t$
- $y = \frac{1}{Ct - t^2}$ ✓
- $t\sqrt{y} = \frac{t^3}{3} + C$
- $y^2 = \frac{t^2}{3} + C\sqrt{t}$

## 8. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & -4 \\ 0 & 1 & -3 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) = C_1e^{-t}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + C_2e^{-t}\left[\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] +$
$C_3e^{-t}\left[\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \frac{t^2}{2}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]$

(B) $X(t) = C_1e^{-t}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + C_2e^{-t}\left[\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] + C_3e^{-t}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

- **(A)** ✓
- **(B)**

## 9. Question

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 2x(t) - 5y(t) - 5t \\ y'(t) = x(t) - 2y(t) \end{cases}$$

(A) $X(t) = C_1 \begin{pmatrix} 2\cos(t) - \sin(t) \\ \cos(t) \end{pmatrix} + C_2 \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} -5t \\ -5t \end{pmatrix}$

(B) $X(t) = C_1 \begin{pmatrix} 2\cos(t) - \sin(t) \\ \cos(t) \end{pmatrix} + C_2 \begin{pmatrix} \cos(t) + 2\sin(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} t \\ 2 \end{pmatrix}$

(C) $X(t) = C_1 \begin{pmatrix} 2\cos(t) - \sin(t) \\ \cos(t) \end{pmatrix} + C_2 \begin{pmatrix} \cos(t) + 2\sin(t) \\ \sin(t) \end{pmatrix} - \begin{pmatrix} 5 + 10t \\ 5t \end{pmatrix}$

- ○ **(A)**
- ○ **(B)**
- ◉ **(C)** ✓

Mongkolsery Lin

Time to complete: 00:49 Points: 50/50

Final Examination Subject-6

1. Find the general solution of the differential equation

$$y'' + 4y = \cos(2t) + t$$

5 / 5 pts
*Auto-graded*

- $y(t) = a_0 + b_0 e^{3t} - \frac{1}{3}t + \frac{1}{6}t^2 e^{3t}$
- $y(t) = a_0 \cos(2t) + b_0 \sin(2t) + \frac{1}{8}t\sin(2t) - \frac{1}{4}t^2\cos(2t) + \frac{1}{4}$
- $y(t) = a_0 \cos t + b_0 \sin t + \frac{1}{2}t\sin t - \frac{1}{2}t^2 \cos t + 1$
- $y(t) = a_0 e^t \cos(2t) + b_0 e^t \sin(2t) + \frac{1}{16}te^t \sin(2t) - \frac{1}{8}t^2 e^t \cos($
- $y(t) = a_0 \cos(2t) + b_0 \sin(2t) + \frac{1}{4}t\sin(2t) + \frac{t}{4}$ ✓

2. Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' + y = 1 + 2t\cos t$$

5 / 5 pts
*Auto-graded*

- $y_p(t) = a_0 \cos t + b_0 \sin t + c_1 te^{3t}$
- $y_p(t) = a_0 + a_1 t + a_2 t^2 + (b_1 t + b_2 t^2)e^{4t}$
- $y_p(t) = a_0 \cos t + b_0 \sin t + (c_1 t + c_2 t^2)e^{3t}$
- $y_p(t) = a_0 + (b_1 t + b_2 t^2)\cos t + (c_1 t + c_2 t^2)\sin t$ ✓
- $y_p(t) = a_1 t\cos(2t) + b_1 t\sin(2t) + c_0 \cos t + d_0 \sin t$

3. Solve the differential equation

$$y' = y \sin t$$

5 / 5 pts
*Auto-graded*

- $y^2 + 1 = Ce^{t^2}$
- $y = Ce^{-\cos t}$ ✓
- $\ln|1+y| = t + \frac{1}{2}t^2 + C$
- $-\frac{1}{2y^2} = \sqrt{1+t^2} + c$
- $e^{-y}(\sin(y) - \cos(y)) = -2e^{-t}\left(t^2 + 2t + 3\right) + C$

4. Solve the differential equation

$$t^2 y dt + y\left(t^3 + e^{-3y}\sin y\right) dy = 0$$

5 / 5 pts
*Auto-graded*

- $\frac{t^3 e^{3y}}{3} - \cos y = C$ ✓
- $\left(1+t^2\right)y - \arctan t = C$
- $2t^{1/2}y - \frac{2t^{5/2}}{5} = C$
- $t^3 y - t^2 y^{-1} = C$
- $t^2 \ln y + t^2 \ln t = c$

5. Solve the differential equation

$$ty' = y + 2\sqrt{ty}$$

5 / 5 pts
*Auto-graded*

- $y = -t\ln(C - \ln|t|)$
- $y = t(C + \ln|t|)^2$ ✓
- $\ln|ty| = ty^{-1} + C$
- $y = t\sin(C + \ln|t|)$
- $y = \frac{t}{C - \ln|t|}$

6. Solve the differential equation

$$(\cos t + \ln y)dt + \left(\frac{t}{y} + e^y\right)dy = 0$$

5 / 5 pts

*Auto-graded*

- $t^2 + 4ty - 2y^4 = C$
- $ty + \ln|\frac{t}{y}| = C$
- $\sin t + t\ln y + e^y = C$ ✓
- $\tan^{-1}(\frac{y}{t}) + \ln|t| = C$
- $6e^{2t} + 3t^2y - 3ty^2 + y^3 = C$

7. Question

Solve the differential equation

$$t^2y' + 2ty = 5y^4$$

5 / 5 pts

*Auto-graded*

- $y^{-1} = t^2\cos t + Ct^2$
- $y^3 = \frac{7t}{15+Ct^7}$ ✓
- $y^{-8} = \frac{8}{9}t + Ct^{-8}$
- $\ln(y^{-6} - 1) = t^6 + C$
- $y = \frac{1}{Ct - t^2}$

## 8. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} -1 & -1 & 0 \\ 4 & 3 & 0 \\ -1 & -3 & 2 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) =$

$$C_1 e^t \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} + C_2 e^{2t} \left[ \begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \right] + C_3 e^{2t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

(B) $X(t) = C_1 e^t \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} + C_2 e^t \left[ \begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \right] + C_3 e^{2t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(C) $X(t) = C_1 e^t \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} + C_2 e^t \begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix} + C_3 e^{2t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

○ **(A)**

◉ **(B)** ✓

○ **(C)**

## 9. Question

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 2x(t) - 5y(t) - 5t \\ y'(t) = x(t) - 2y(t) + 10 \end{cases}$$

(A) $X(t) =$

$$C_1 \begin{pmatrix} 2\cos(t) - \sin(t) \\ \cos(t) \end{pmatrix} + C_2 \begin{pmatrix} \cos(t) + 2\sin(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} -5t \\ 10 \end{pmatrix}$$

(B) $X(t) =$

$$C_1 \begin{pmatrix} 2\cos(t) - \sin(t) \\ \cos(t) \end{pmatrix} + C_2 \begin{pmatrix} \cos(t) + 2\sin(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} -55 - 10t \\ -20 - 5t \end{pmatrix}$$

(C) $X(t) = C_1 \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} + C_2 \begin{pmatrix} \cos(t) + 2\sin(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} -55 \\ -5t \end{pmatrix}$

○ **(A)**

◉ **(B)** ✓

○ **(C)**

Mongkolsery Lin

Time to complete: 00:46 Points: 50/50

Final Examination Subject-7

1. Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' - 7y' + 12y = \sin t + 3e^{3t}$$

5 / 5 pts
*Auto-graded*

- $y_p(t) = a_0 \cos t + b_0 \sin t + c_1 t e^{3t}$ ✓
- $y_p(t) = a_0 + a_1 t + a_2 t^2 + (b_1 t + b_2 t^2) e^{4t}$
- $y_p(t) = a_0 \cos t + b_0 \sin t + (c_1 t + c_2 t^2) e^{3t}$
- $y_p(t) = a_0 + (b_1 t + b_2 t^2) \cos t + (c_1 t + c_2 t^2) \sin t$
- $y_p(t) = a_1 t \cos(2t) + b_1 t \sin(2t) + c_0 \cos t + d_0 \sin t$

2. Solve the differential equation

$$yy' = t\,(y^2 + 1)$$

5 / 5 pts
*Auto-graded*

- $y^2 + 1 = Ce^{t^2}$ ✓
- $y = Ce^{-\cos t}$
- $\ln|1 + y| = t + \frac{1}{2}t^2 + C$
- $-\frac{1}{2y^2} = \sqrt{1 + t^2} + c$
- $e^{-y}(\sin(y) - \cos(y)) = -2e^{-t}\left(t^2 + 2t + 3\right) + C$

**3.** Solve the differential equation

$$(y^2 + \cos t)\,dt + (2ty + \sin y)dy = 0$$

5 / 5 pts

*Auto-graded*

- $t^2 + 4ty - 2y^4 = C$
- $ty^2 + \sin t - \cos y = C$ ✓
- $\sin t + t\ln y + e^y = C$
- $\tan^{-1}(\frac{y}{t}) + \ln|t| = C$
- $6e^{2t} + 3t^2y - 3ty^2 + y^3 = C$

**4.** Find the general solution of the differential equation

$$y'' - 2y' + 5y = te^t \sin(2t)$$

5 / 5 pts

*Auto-graded*

- $y(t) = a_0 + b_0e^{3t} - \frac{1}{3}t + \frac{1}{6}t^2e^{3t}$
- $y(t) = a_0\cos(2t) + b_0\sin(2t) + \frac{1}{8}t\sin(2t) - \frac{1}{4}t^2\cos(2t) + \frac{1}{4}$
- $y(t) = a_0\cos t + b_0\sin t + \frac{1}{2}t\sin t - \frac{1}{2}t^2\cos t + 1$
- $y(t) = a_0e^t\cos(2t) + b_0e^t\sin(2t) + \frac{1}{16}te^t\sin(2t) - \frac{1}{8}t^2e^t\cos($ ✓
- $y(t) = a_0\cos(2t) + b_0\sin(2t) + \frac{1}{4}t\sin(2t) + \frac{t}{4}$

**5.** Solve the differential equation

$$t^2y' = ty + y^2$$

5 / 5 pts

*Auto-graded*

- $y = -t\ln(C - \ln|t|)$
- $y = t(C + \ln|t|)^2$
- $\ln|ty| = ty^{-1} + C$
- $y = t\sin(C + \ln|t|)$
- $y = \frac{t}{C - \ln|t|}$ ✓

## 6. Question

5 / 5 pts

*Auto-graded*

Solve the differential equation

$$ty' = y(t^2y - 1)$$

- $y^{-1} = t^2 \cos t + Ct^2$
- $y^3 = \frac{7t}{15+Ct^7}$
- $y^{-8} = \frac{8}{9}t + Ct^{-8}$
- $\ln(y^{-6} - 1) = t^6 + C$
- $y = \frac{1}{Ct-t^2}$ ✓

## 7. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & 0 \\ -1 & -3 & 1 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) = C_1e^{2t}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + C_2e^{2t}\begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix} + C_3e^{t}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(B) $X(t) = C_1e^{2t}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + C_2e^{2t}\left[\begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}\right] + C_3e^{t}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(C) $X(t) = C_1e^{2t}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + C_2e^{t}\left[\begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}\right] + C_3e^{t}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

- (A)
- (B) ✓
- (C)

**8.** Solve the differential equation

$$(y - t^2)\,dt + 2t\,dy = 0$$

5 / 5 pts

*Auto-graded*

- $\frac{t^3 e^{3y}}{3} - \cos y = C$
- $(1 + t^2)y - \arctan t = C$
- $2t^{1/2}y - \frac{2t^{5/2}}{5} = C$ ✓
- $t^3 y - t^2 y^{-1} = C$
- $t^2 \ln y + t^2 \ln t = c$

**9. Question**

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 2x(t) - y(t) - 5t \\ y'(t) = x(t) + 2y(t) \end{cases}$$

(A) $X(t) = C_1 e^{2t} \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} \frac{3}{5} + 2t \\ -\frac{4}{5} - t \end{pmatrix}$

(B) $X(t) = C_1 e^{2t} \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} 5 + 2t \\ -4 - t \end{pmatrix}$

(C) $X(t) = C_1 e^{2t} \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} -5t \\ 0 \end{pmatrix}$

- **(A)** ✓
- **(B)**
- **(C)**

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Time to complete: 00:30 Points: 50/50

Final Examination Subject-8

1. Find the general solution of the differential equation

$$y'' + 4y = 2t\sin(2t) + 1$$

5 / 5 pts

*Auto-graded*

- $y(t) = a_0 + b_0 e^{3t} - \frac{1}{3}t + \frac{1}{6}t^2 e^{3t}$
- $y(t) = a_0\cos(2t) + b_0\sin(2t) + \frac{1}{8}t\sin(2t) - \frac{1}{4}t^2\cos(2t) + \frac{1}{4}$
- $y(t) = a_0\cos t + b_0\sin t + \frac{1}{2}t\sin t - \frac{1}{2}t^2\cos t + 1$
- $y(t) = a_0 e^t\cos(2t) + b_0 e^t\sin(2t) + \frac{1}{16}te^t\sin(2t) - \frac{1}{8}t^2 e^t\cos(2$
- $y(t) = a_0\cos(2t) + b_0\sin(2t) + \frac{1}{4}t\sin(2t) + \frac{t}{4}$ ✓

2. Solve the differential equation

$$y' = 1 + t + y + ty$$

5 / 5 pts

*Auto-graded*

- $y^2 + 1 = Ce^{t^2}$
- $y = Ce^{-\cos t}$
- $\ln|1+y| = t + \frac{1}{2}t^2 + C$ ✓
- $-\frac{1}{2y^2} = \sqrt{1+t^2} + c$
- $e^{-y}(\sin(y) - \cos(y)) = -2e^{-t}\left(t^2 + 2t + 3\right) + C$

3. Solve the differential equation

$$\frac{ty-1}{t}dt + \frac{ty+1}{y}dy = 0$$

5 / 5 pts
Auto-graded

- (●) $ty + \ln|\frac{y}{t}| = C$ ✓
- ( ) $ty^2 + \sin t - \cos y = C$
- ( ) $\sin t + t\ln y + e^y = C$
- ( ) $\tan^{-1}(\frac{y}{t}) + \ln|t| = C$
- ( ) $6e^{2t} + 3t^2y - 3ty^2 + y^3 = C$

4. Solve the differential equation

$$ty[2\ln(ty) + 1]dt + t^2dy = 0$$

5 / 5 pts
Auto-graded

- ( ) $\frac{t^3e^{3y}}{3} - \cos y = C$
- ( ) $(1+t^2)y - \arctan t = C$
- ( ) $2t^{1/2}y - \frac{2t^{5/2}}{5} = C$
- ( ) $t^3y - t^2y^{-1} = C$
- (●) $t^2\ln y + t^2\ln t = c$ ✓

5. Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' - 7y' + 12y = t^2 + 4te^{4t}$$

5 / 5 pts
Auto-graded

- ( ) $y_p(t) = a_0 + a_1t + a_2t^2 + b_1te^{4t}$
- (●) $y_p(t) = a_0 + a_1t + a_2t^2 + (b_1t + b_2t^2)e^{4t}$ ✓
- ( ) $y_p(t) = a_0\cos t + b_0\sin t + (c_1t + c_2t^2)e^{3t}$
- ( ) $y_p(t) = a_0 + (b_1t + b_2t^2)\cos t + (c_1t + c_2t^2)\sin t$
- ( ) $y_p(t) = a_1t\cos(2t) + b_1t\sin(2t) + c_0\cos t + d_0\sin t$

## 6. Question

5 / 5 pts

*Auto-graded*

Solve the differential equation

$$y' + t^5 y = t^5 y^7$$

- $y^{-1} = t^2 \cos t + Ct^2$
- $y^3 = \frac{7t}{15+Ct^7}$
- $y^{-8} = \frac{8}{9}t + Ct^{-8}$
- $\ln(y^{-6} - 1) = t^6 + C$ ✓
- $y = \frac{1}{Ct-t^2}$

## 7.

5 / 5 pts

*Auto-graded*

Solve the differential equation

$$ty' = y + \sqrt{t^2 - y^2}$$

- $y = -t \ln(C - \ln|t|)$
- $y = t(C + \ln|t|)^2$
- $\ln|ty| = ty^{-1} + C$
- $y = t \sin(C + \ln|t|)$ ✓
- $y = \frac{t}{C-\ln|t|}$

## 8. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3 & 0 \\ -1 & -3 & 2 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) =$

$$C_1e^{2t}\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + C_2e^{3t}\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\right] + C_3e^{3t}\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

(B) $X(t) =$

$$C_1e^{2t}\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + C_2e^{2t}\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\right] + C_3e^{3t}\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

(C) $X(t) = C_1e^{2t}\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + C_2e^{2t}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + C_3e^{3t}\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

○ **(A)**

◉ **(B)** ✓

○ **(C)**

## 9. Question

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 2x(t) - y(t) + t \\ y'(t) = x(t) + 2y(t) - 1 \end{cases}$$

(A) $X(t) = C_1e^{2t}\begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} + C_2e^{2t}\begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} -t \\ 1 \end{pmatrix}$

(B) $X(t) = C_1e^{2t}\begin{pmatrix} 0 \\ \cos(t) \end{pmatrix} + C_2e^{2t}\begin{pmatrix} \cos(t) \\ 1 \end{pmatrix} + \begin{pmatrix} 2 - \frac{2}{5}t \\ \frac{14}{25} + 5t \end{pmatrix}$

(C) $X(t) = C_1e^{2t}\begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} + C_2e^{2t}\begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} + \begin{pmatrix} \frac{2}{25} - \frac{2}{5}t \\ \frac{14}{25} + \frac{1}{5}t \end{pmatrix}$

○ **(A)**

○ **(B)**

◉ **(C)** ✓

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Time to complete: 00:31 Points: 50/50

Final Examination Subject-9

1. Find the general solution of the differential equation

$$y'' - 4y' + 4y = 2te^{2t} + e^t$$

5 / 5 pts

*Auto-graded*

- $y(t) = a_0 + b_0e^{3t} - \frac{1}{3}t + \frac{1}{6}t^2e^{3t}$
- $y(t) = (a_0 + a_1t)e^{2t} + \frac{1}{3}t^3e^{2t} + e^t$ ✓
- $y(t) = a_0\cos t + b_0\sin t + \frac{1}{2}t\sin t - \frac{1}{2}t^2\cos t + 1$
- $y(t) = a_0e^{2t} + \frac{1}{3}t^3e^{2t} + e^t$
- $y(t) = a_0\cos(2t) + b_0\sin(2t) + \frac{1}{4}t\sin(2t) + \frac{t}{4}$

2. Solve the differential equation

$$\frac{dy}{dt} = \frac{e^y e^{-t}}{\cos(y)}(1 + t^2)$$

5 / 5 pts

*Auto-graded*

- $y^2 + 1 = Ce^{t^2}$
- $y = Ce^{-\cos t}$
- $\ln|1 + y| = t + \frac{1}{2}t^2 + C$
- $-\frac{1}{2y^2} = \sqrt{1 + t^2} + c$
- $e^{-y}(\sin(y) - \cos(y)) = -2e^{-t}\left(t^2 + 2t + 3\right) + C$ ✓

**3.** Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' - 6y' + 9y = \sin t + 3e^{3t}$$

5 / 5 pts
*Auto-graded*

- $y_p(t) = a_0 \cos t + b_0 \sin t + c_1 t e^{3t}$
- $y_p(t) = a_0 + a_1 t + a_2 t^2 + (b_1 t + b_2 t^2) e^{4t}$
- $y_p(t) = a_0 \cos t + b_0 \sin t + c_2 t^2 e^{3t}$ ✓
- $y_p(t) = a_0 + (b_1 t + b_2 t^2) \cos t + (c_1 t + c_2 t^2) \sin t$
- $y_p(t) = a_1 t \cos(2t) + b_1 t \sin(2t) + c_0 \cos t + d_0 \sin t$

**4.** Solve the differential equation

$$(2ty + 2t^3 y - 1)\, dt + (1 + t^2)^2\, dy = 0$$

5 / 5 pts
*Auto-graded*

- $\frac{t^3 e^{3y}}{3} - \cos y = C$
- $(1 + t^2)\, y - \arctan t = C$ ✓
- $2t^{1/2} y - \frac{2t^{5/2}}{5} = C$
- $t^3 y - t^2 y^{-1} = C$
- $t^2 \ln y + t^2 \ln t = c$

**5.** Solve the differential equation

$$t(t + y)y' = y(t - y)$$

5 / 5 pts
*Auto-graded*

- $y = -t \ln(C - \ln|t|)$
- $y = t(C + \ln|t|)^2$
- $\ln|ty| = ty^{-1} + C$ ✓
- $y = t \sin(C + \ln|t|)$
- $y = \frac{t}{C - \ln|t|}$

6. Solve the differential equation

$$(4e^{2t} + 2ty - y^2)\,dt + (t - y)^2 dy = 0$$

5 / 5 pts

*Auto-graded*

- $t^2 + 4ty - 2y^4 = C$
- $ty + \ln|\frac{t}{y}| = C$
- $\sin t + t \ln y + e^y = C$
- $\tan^{-1}(\frac{y}{t}) + \ln|t| = C$
- $6e^{2t} + 3t^2y - 3ty^2 + y^3 = C$ ✓

7. Question

Solve the differential equation

$$y' - \frac{1}{t}y = y^9$$

5 / 5 pts

*Auto-graded*

- $y^{-1} = t^2 \cos t + Ct^2$
- $y^3 = \frac{7t}{15+Ct^7}$
- $y^{-8} = -\frac{8}{9}t + Ct^{-8}$ ✓
- $\ln(y^{-6} - 1) = t^6 + C$
- $y = \frac{1}{Ct - t^2}$

## 8. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} -1 & 0 & 3 \\ 1 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) = C_1 e^{3t} \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} + C_3 e^{-t} \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$

(B) $X(t) = C_1 e^{3t} \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + C_2 e^{3t} \left[ \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} + t \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} \right] + C_3 e^{-t} \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$

(C) $X(t) =$

$$C_1 e^{-t} \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + C_2 e^{-t} \left[ \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} + t \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} \right] + C_3 e^{3t} \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$$

○ **(A)**

◉ **(B)** ✓

○ **(C)**

## 9. Question

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 3x(t) - 2y(t) + t \\ y'(t) = 2x(t) - 2y(t) - 1 \end{cases}$$

(A) $X(t) = C_1 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -1-t \\ -1-t \end{pmatrix}$

(B) $X(t) = C_1 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1-t \end{pmatrix}$

(C) $X(t) = C_1 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3t \\ -1 \end{pmatrix}$

◉ **(A)** ✓

○ **(B)**

○ **(C)**

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Time to complete: 00:46 Points: 50/50

Final Examination Subject-10

1. Determine the form of the particular solution $y_p(t)$ of the differential equation

$$y'' + 4y = \cos(2t) + \sin t$$

5 / 5 pts
*Auto-graded*

- $y_p(t) = a_0 \cos t + b_0 \sin t + c_1 t e^{3t}$
- $y_p(t) = a_0 + a_1 t + a_2 t^2 + (b_1 t + b_2 t^2) e^{4t}$
- $y_p(t) = a_0 \cos t + b_0 \sin t + (c_1 t + c_2 t^2) e^{3t}$
- $y_p(t) = a_0 + (b_1 t + b_2 t^2) \cos t + (c_1 t + c_2 t^2) \sin t$
- $y_p(t) = a_1 t \cos(2t) + b_1 t \sin(2t) + c_0 \cos t + d_0 \sin t$ ✓

2. Find the general solution of the differential equation

$$y'' + y = 1 + 2t \sin t$$

5 / 5 pts
*Auto-graded*

- $y(t) = a_0 + b_0 e^{3t} - \frac{1}{3}t + \frac{1}{6}t^2 e^{3t}$
- $y(t) = (a_0 + a_1 t) e^{2t} + \frac{1}{3}t^3 e^{2t} + e^t$
- $y(t) = a_0 \cos t + b_0 \sin t + \frac{1}{2}t \sin t - \frac{1}{2}t^2 \cos t + 1$ ✓
- $y(t) = a_0 e^{2t} + \frac{1}{3}t^3 e^{2t} + e^t$
- $y(t) = a_0 \cos(2t) + b_0 \sin(2t) + \frac{1}{4}t \sin(2t) + \frac{t}{4}$

3. Solve the differential equation

$$y' = \frac{ty^3}{\sqrt{1+t^2}}$$

5 / 5 pts

*Auto-graded*

- $y^2 + 1 = Ce^{t^2}$
- $y = Ce^{-\cos t}$
- $\ln|1+y| = t + \frac{1}{2}t^2 + C$
- $-\frac{1}{2y^2} = \sqrt{1+t^2} + c$ ✓
- $e^{-y}(\sin(y) - \cos(y)) = -2e^{-t}\left(t^2 + 2t + 3\right) + C$

4. Solve the differential equation

$$\left(\frac{1}{t} - \frac{y}{t^2+y^2}\right)dt + \frac{t}{t^2+y^2}dy = 0$$

5 / 5 pts

*Auto-graded*

- $t^2 + 4ty - 2y^4 = C$
- $ty + \ln|\frac{t}{y}| = C$
- $\sin t + t\ln y + e^y = C$
- $\tan^{-1}(\frac{y}{t}) + \ln|t| = C$ ✓
- $6e^{2t} + 3t^2y - 3ty^2 + y^3 = C$

**5.** Solve the differential equation

$$(3ty - 2y^{-1})\,dt + t\,(t + y^{-2})\,dy = 0$$

5 / 5 pts

*Auto-graded*

- $\frac{t^3e^{3y}}{3} - \cos y = C$
- $(1+t^2)y - \arctan t = C$
- $2t^{1/2}y - \frac{2t^{5/2}}{5} = C$
- $t^3y - t^2y^{-1} = C$ ✓
- $t^2 \ln y + t^2 \ln t = c$

**6.** Solve the differential equation

$$t^2y' = ty + t^2e^{\frac{y}{t}}$$

5 / 5 pts

*Auto-graded*

- $y = -t\ln(C - \ln|t|)$ ✓
- $y = t(C + \ln|t|)^2$
- $\ln|ty| = ty^{-1} + C$
- $y = t\sin(C + \ln|t|)$
- $y = \frac{t}{C - \ln|t|}$

## 7. Question

5 / 5 pts

*Auto-graded*

Solve the differential equation

$$y' - \frac{2}{t}y = y^2 t^2 \sin t$$

- ◉ $y^{-1} = t^2 \cos t + Ct^2$ ✓
- ○ $y^3 = \frac{7t}{15+Ct^7}$
- ○ $y^{-8} = \frac{8}{9}t + Ct^{-8}$
- ○ $\ln(y^{-6} - 1) = t^6 + C$
- ○ $y = \frac{1}{Ct - t^2}$

## 8. Question

10 / 10 pts

*Auto-graded*

Solve the system of ordinary differential equation $X'(t) = A.X(t)$ where

$$A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}, \quad \text{and} \quad X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

(A) $X(t) = C_1 e^{3t} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + C_3 e^{-t} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

(B) $X(t) = C_1 e^{3t} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + C_3 e^{-t} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

(C) $X(t) = C_1 e^{3t} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + C_3 e^{5t} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

- ○ *(A)*
- ○ *(B)*
- ◉ *(C)* ✓

## 9. Question

5 / 5 pts

*Auto-graded*

Solve the system of ordinary differential equation

$$\begin{cases} x'(t) = 3x(t) - 2y(t) + e^{-t} \\ y'(t) = 2x(t) - 2y(t) \end{cases}$$

(A) $X(t) = C_1 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{t} \begin{pmatrix} 3 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} t \\ 2+6t \end{pmatrix}$

(B) $X(t) = C_1 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \frac{1}{9} e^{-t} \begin{pmatrix} -4-3t \\ -2-6t \end{pmatrix}$

(C) $X(t) = C_1 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \frac{1}{9} e^{-t} \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

- ( ) **(A)**
- (•) **(B)** ✓
- ( ) **(C)**