I2–TD2
(Discrete R.V and Probability Distribution)

1. A mail-order computer business has six telephone lines. Let X denote the number of lines in use at a specified time. Suppose the pmf of X is as given in the accompanying table.

x	0	1	2	3	4	5	6
$p(x)$	0.10	0.15	0.20	0.25	0.20	0.06	0.04

Calculate the probability of each of the following events.

(a) At most three lines are in use.

(b) Fewer than three lines are in use.

(c) At least three lines are in use.

(d) Between two and five lines, inclusive, are in use.

(e) Between two and four lines, inclusive, are not in use.

(f) At least four lines are not in use.

2. A group of four components is known to contain two defectives. An inspector tests the components one at a time until the two defectives are located. Let X denote the number of the test on which the second defective is found. Find the probability distribution for X.

3. A problem in a test given to small children asks them to match each of three pictures of animals to the word identifying that animal. If a child assigns the three words at random to the three pictures, find the probability distribution for Y , the number of correct matches.

4. In order to verify the accuracy of their financial accounts, companies use auditors on a regular basis to verify accounting entries. The company's employees make erroneous entries 5% of the time. Suppose that an auditor randomly checks three entries.

(a) Find the probability distribution for X , the number of errors detected by the auditor.

(b) Find the probability that the auditor will detect more than one error.

5. A new battery's voltage may be acceptable (A) or unacceptable (U). A certain flashlight requires two batteries, so batteries will be independently selected and tested until two acceptable ones have been found. Suppose that 90% of all batteries have acceptable voltages. Let Y denote the number of batteries that must be tested.

(a) What is $p(2)$, that is, $P(Y = 2)$?

(b) What is $p(3)$? [Hint: There are two different outcomes that result in $Y = 3$.]

(c) To have $Y = 5$, what must be true of the fifth battery selected? List the four outcomes for which $Y = 5$ and then determine $p(5)$.

(d) Use the answers for parts (a)-(c) to obtain a general formula for p(y).

6. An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let X be the number of months between successive payments. The cdf of X is as follows:

$$F(x) = \begin{cases} 0, & x < 1 \\ 0.10, & 1 \le x < 3 \\ 0.40, & 3 \le x < 7 \\ 0.80, & 7 \le x < 12 \\ 1, & 12 \le x. \end{cases}$$

   (a) What is the pmf of X?

   (b) Using just the cdf, compute $P(3 \le X \le 6)$ and $P(6 \le X)$.

7. An Internet survey estimates that, when given a choice between English and French, 60% of the population prefers to study English. Three students are randomly selected and asked which of the two languages they prefer.

   (a) Find the probability distribution for Y, the number of students in the sample who prefer English.

   (b) What is the probability that exactly one of the three students prefer English?

   (c) What are the mean and standard deviation for Y?

   (d) What is the probability that the number of students prefer English falls within 2 standard deviations of the mean?

8. In a gambling game a person draws a single card from an ordinary 52-card playing deck. A person is paid \$15 for drawing a jack or a queen and \$5 for drawing a king or an ace. A person who draws any other card pays \$4. If a person plays this game, what is the expected gain?

9. An expedition is sent to the Himalayas with the objective of catching a pair of wild yaks for breeding. Assume yaks are loners and roam about Himalayas at random. The probability $p \in (0, 1)$ that a given trapped yak is male is independent of prior outcomes. Let N be the number of yaks that must be caught until a pair is obtained.

   (a) Show that the expected value of N is $1 + p/q + q/p$, where $q = 1 - p$.

   (b) Find the variance of N.

10. A complex electronic system is built with a certain number of backup components in its subsystems. One subsystem has four identical components, each with a probability of 0.2 of failing in less than 1000 hours. The subsystem will operate if any two of the four components are operating. Assume that the components operate independently. Find the probability that

   (a) exactly two of the four components last longer than 1000 hours.

   (b) the subsystem operates longer than 1000 hours.

11. Many utility companies promote energy conservation by offering discount rates to consumers who keep their energy usage below certain established subsidy standards. A

recent report notes that 70% of the people live in Phnom Penh have reduced their electricity usage sufficiently to qualify for discounted rates. If five residential subscribers are randomly selected from Phnom Penh, find the probability of each of the following events:

(a) All five qualify for the favorable rates.

(b) At least four qualify for the favorable rates.

(c) At least two do not qualify the favorable rates.

12. Knowing that 80% of the volunteers donating blood in a clinic have type A blood.

    (a) If five volunteers are randomly selected, what is the probability that at least one does not have type A blood?

    (b) If five volunteers are randomly selected, what is the probability that at most four have type A blood?

    (c) What is the smallest number of volunteers who must be selected if we want to be at least 90% certain that we obtain at least five donors with type A blood?

13. An electronics store has received a shipment of 20 table radios that have connections for an iPod or iPhone. Twelve of these have two slots (so they can accommodate both devices), and the other eight have a single slot. Suppose that six of the 20 radios are randomly selected to be stored under a shelf where the radios are displayed, and the remaining ones are placed in a storeroom. Let $X=$ the number among the radios stored under the display shelf that have two slots.

    (a) What kind of a distribution does X have (name and values of all parameters)?

    (b) Compute $P(X = 2), P(X \leq 2)$, and $P(X \geq 2)$.

    (c) Calculate the mean value and standard deviation of X.

14. A manufacturing company uses an acceptance scheme on items from a production line before they are shipped. The plan is a two-stage one. Boxes of 25 items are readied for shipment, and a sample of 3 items is tested for defectives. If any defectives are found, the entire box is sent back for 100% screening. If no defectives are found, the box is shipped.

    (a) What is the probability that a box containing 3 defectives will be shipped?

    (b) What is the probability that a box containing only 1 defective will be sent back for screening?

15. In an assembly-line production of industrial robots, gearbox assemblies can be installed in one minute each if holes have been properly drilled in the boxes and in ten minutes if the holes must be redrilled. Twenty gearboxes are in stock, 2 with improperly drilled holes. Five gearboxes must be selected from the 20 that are available for installation in the next five robots.

    (a) Find the probability that all 5 gearboxes will fit properly.

    (b) Find the mean, variance, and standard deviation of the time it takes to install these 5 gearboxes.

16. Suppose that $p = P(\text{male birth}) = 0.3$. A couple wishes to have exactly two female children in their family. They will have children until this condition is fulfilled.
    (a) What is the probability that the family has x male children?
    (b) What is the probability that the family has four children?
    (c) What is the probability that the family has at most four children?
    (d) How many male children would you expect this family to have? How many children would you expect this family to have?
17. In an NBA (National Basketball Association) championship series, the team that wins four games out of seven is the winner. Suppose that teams A and B face each other in the championship games and that team A has probability 0.55 of winning a game over team B.
    (a) What is the probability that team A will win the series in 5 games?
    (b) What is the probability that team A will win the series?
    (c) What is the probability that team B will win the series in 6 games?
18. Ten percent of the engines manufactured on an assembly line are defective. What is the probability that the third nondefective engine will be found
    (a) on the fifth trial?
    (b) on or before the fifth trial?
    (c) Given that the first two engines tested were defective, what is the probability that at least two more engines must be tested before the first nondefective is found?
    (d) Find the mean and variance of the number of the trial on which a the first non-defective engine is found.
    (e) Find the mean and variance of the number of the trial on which the third nondefective engine is found.
19. A large stockpile of used pumps contains 20% that are in need of repair. A maintenance worker is sent to the stockpile with three repair kits. She selects pumps at random and tests them one at a time. If the pump works, she sets it aside for future use. However, if the pump does not work, she uses one of her repair kits on it. Suppose that it takes 10 minutes to test a pump that is in working condition and 30 minutes to test and repair a pump that does not work. Find the mean and variance of the total time it takes the maintenance worker to use her three repair kits.
20. Suppose small aircraft arrive at a certain airport according to a Poisson process with rate $\alpha = 8$ per hour, so that the number of arrivals during a time period of t hours is a Poisson rv with parameter $\lambda = 8t$.
    (a) What is the probability that exactly 6 small aircraft arrive during a 1-hour period? At least 6? At least 10?
    (b) What are the expected value and standard deviation of the number of small aircraft that arrive during a 90-min period?
    (c) What is the probability that at least 20 small aircraft arrive during a 2.5-hour period? That at most 10 arrive during this period?

21. Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. If it takes approximately ten minutes to serve each customer, find the mean and variance of the total service time for customers arriving during a 1-hour period. (Assume that a sufficient number of servers are available so that no customer must wait for service.) Is it likely that the total service time will exceed 2.5 hours?

22. Suppose that trees are distributed in a forest according to a two-dimensional Poisson process with parameter α, the expected number of trees per acre, equal to 80.

    (a) What is the probability that in a certain quarter-acre plot, there will be at most 16 trees?

    (b) If the forest covers 85,000 acres, what is the expected number of trees in the forest?

    (c) Suppose you select a point in the forest and construct a circle of radius .1 mile. Let $X=$ the number of trees within that circular region. What is the pmf of X? [Hint: 1 sq mile = 640 acres.]

23. In proof testing of circuit boards, the probability that any particular diode will fail is 0.01. Suppose a circuit board contains 200 diodes.

    (a) How many diodes would you expect to fail, and what is the standard deviation of the number that are expected to fail?

    (b) What is the (approximate) probability that at least four diodes will fail on a randomly selected board?

    (c) If five boards are shipped to a particular customer, how likely is it that at least four of them will work prop- erly? (A board works properly only if all its diodes work.)

24. [**Markov inequality**] If $X \geq 0$, i.e. X takes only nonnegative values, then for an $a > 0$, we have

$$P(X \geq a) \leq \frac{E(X)}{a}$$

25. [**Chebyshev inequality**] For any random variable X and any $a > 0$, we have

$$P(|X - E(X)| \geq a) \leq \frac{V(X)}{a^2}$$

26. The number of customers per day at a sales counter, Y , has been observed for a long period of time and found to have mean 20 and standard deviation 2. The probability distribution of Y is not known. What can be said about the probability that, tomorrow, Y will be greater than 16 but less than 24?

27. Let X be a random variable with mean 11 and variance 9. Using Tchebysheff's theorem, find

    (a) a lower bound for $P(6 < X < 16)$.

    (b) the value of C such that $P(|X - 11| \geq C) \leq 0.09$.