I2–TD1
(Probability)

1. Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles.

   (a) List all outcomes in the event A that all three vehicles go in the same direction.
   (b) List all outcomes in the event B that all three vehicles take different directions.i
   (c) List all outcomes in the event C that exactly two of the three vehicles turn right.
   (d) List all outcomes in the event D that exactly two vehicles go in the same direction.
   (e) List outcomes in D', $C \cup D$, and $C \cap D$.

2. An engineering construction firm is currently working on power plants at three different sites. Let A_i denote the event that the plant at site i is completed by the contract date. Use the operations of union, intersection, and complementation to describe each of the following events in terms of A_1, A_2, and A_3, draw a Venn diagram, and shade the region corresponding to each one.

   (a) At least one plant is completed by the contract date.
   (b) All plants are completed by the contract date.
   (c) Only the plant at site 1 is completed by the contract date.
   (d) Exactly one plant is completed by the contract date.
   (e) Either the plant at site 1 or both of the other two plants are completed by the contract date.

3. An academic department has just completed voting by secret ballot for a department head. The ballot box contains four slips with votes for candidate A and three slips with votes for candidate B. Suppose these slips are removed from the box one by one.

   (a) List all possible outcomes.
   (b) Suppose a running tally is kept as slips are removed. For what outcomes does A remain ahead of B throughout the tally?

4. Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a MasterCard. Suppose that $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.25$.

   (a) Compute the probability that the selected individual has at least one of the two types of cards (i.e., the probability of the event $A \cup B$).
   (b) What is the probability that the selected individual has neither type of card?
   (c) Describe, in terms of A and B, the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of this event.

5. Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

   (a) What is the probability that a randomly selected adult regularly consumes both coffee and soda?

    (b) What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?

6. Consider the type of clothes dryer (gas or electric) purchased by each of five different customers at a certain store.

    (a) If the probability that at most one of these purchases an electric dryer is .428, what is the probability that at least two purchase an electric dryer?

    (b) If and P(all five purchase gas) $=0.116$ and P(all five purchase electric) $=0.005$, what is the probability that at least one of each type is purchased?

7. An individual is presented with three different glasses of cola, labeled C, D, and P. He is asked to taste all three and then list them in order of preference. Suppose the same cola has actually been put into all three glasses.

    (a) What are the simple events in this ranking experiment, and what probability would you assign to each one?

    (b) What is the probability that C is ranked first?

    (c) What is the probability that C is ranked first and D is ranked last?

8. Let A denote the event that the next request for assistance from a statistical software consultant relates to the SPSS package, and let B be the event that the next request is for help with SAS. Suppose that $P(A) = .30$ and $P(B) = 0.50$.

    (a) Why is it not the case that $P(A) + P(B) = 1$?

    (b) Calculate $P(A')$.

    (c) Calculate $P(A \cup B)$.

    (d) Calculate $P(A' \cap B')$.

9. The three most popular options on a certain type of new car are a built-in GPS (A), a sunroof (B), and an automatic transmission (C). If 40% of all purchasers request A, 55request B, 70% request C, 63% request A or B, 77% request A or C, 80% request B or C, and 85% request A or B or C, determine the probabilities of the following events. [Hint: "A or B" is the event that at least one of the two options is requested; try drawing a Venn diagram and labeling all regions.]

    (a) The next purchaser will request at least one of the three options.

    (b) The next purchaser will select none of the three options.

    (c) The next purchaser will request only an automatic transmission and not either of the other two options.

    (d) The next purchaser will select exactly one of these three options.

10. An academic department with five faculty members— Anderson, Box, Cox, Cramer, and Fisher—must select two of its members to serve on a personnel review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representative will be selected by putting the names on identical pieces of paper and then randomly selecting two.

(a) What is the probability that both Anderson and Box will be selected? [Hint: List the equally likely outcomes.]

(b) What is the probability that at least one of the two members whose name begins with C is selected?

(c) If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have a total of at least 15 years teaching experience there?

11. Certain system can experience three different types of defects. Let $A_i(i = 1, 2, 3)$ denote the event that the system has a defect of type i. Suppose that $P(A_1) = 0.12, P(A_2) = 0.07, P(A_3) = 0.05, P(A_1 \cup A_2) = 0.13, P(A_1 \cup A_3) = 0.14, P(A_2 \cup A_3) = 0.10, P(A_1 \cap A_2 \cap A_3) = 0.01$.

(a) What is the probability that the system does not have a type 1 defect?

(b) What is the probability that the system has both type 1 and type 2 defects?

(c) What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?

(d) What is the probability that the system has at most two of these defects?

12. Pollution of the rivers in Cambodia has been a problem for many years. Consider the following events: A: the river is polluted; B: a sample of water tested detects pollution; C: fishing is permitted.
Assume $P(A) = 0.3, P(B|A) = 0.75, P(B|A') = 0.20, P(C|A \cap B) = 0.20, P(C|A' \cap B) = 0.15, P(C|A \cap B') = 0.80$, and $P(C|A' \cap B') = 0.90$.

(a) Find $P(A \cap B \cap C)$.

(b) Find $P(B' \cap C)$.

(c) Find $P(C)$.

(d) Find the probability that the river is polluted, given that fishing is permitted and the sample tested did not detect pollution.

13. A production facility employs 20 workers on the day shift, 15 workers on the swing shift, and 10 workers on the graveyard shift. A quality control consultant is to select 6 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 45).

(a) How many selections result in all 6 workers coming from the day shift? What is the probability that all 6 selected workers will be from the day shift?

(b) What is the probability that all 6 selected workers will be from the same shift?

(c) What is the probability that at least two different shifts will be represented among the selected workers?

(d) What is the probability that at least one of the shifts will be unrepresented in the sample of workers?

14. An academic department with five faculty members narrowed its choice for department head to either candidate A or candidate B. Each member then voted on a slip of paper

for one of the candidates. Suppose there are actually three votes for A and two for B. If the slips are selected for tallying in random order, what is the probability that A remains ahead of B throughout the vote count (e.g., this event occurs if the selected ordering is $AABAB$, but not for $ABBAA$)?

15. In five-card poker, a straight consists of five cards with adjacent denominations (e.g., 9 of clubs, 10 of hearts, jack of hearts, queen of spades, and king of clubs). Assuming that aces can be high or low, if you are dealt a five-card hand, what is the probability that it will be a straight with high card 10? What is the probability that it will be a straight? What is the probability that it will be a straight flush (all cards in the same suit)?

16. Suppose an individual is randomly selected from the population of all adult males living in the United States. Let A be the event that the selected individual is over 6 ft in height, and let B be the event that the selected individual is a professional basketball player. Which do you think is larger, $P(A|B)$ or $P(B|A)$? Why?

17. A certain shop repairs both audio and video components. Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that $P(A) = 0.6$ and $P(B) = 0.05$. What is $P(B|A)$?

18. Deer ticks can be carriers of either Lyme disease or human granulocytic ehrlichiosis (HGE). Based on a recent study, suppose that 16% of all ticks in a certain location carry Lyme disease, 10% carry HGE, and 10% of the ticks that carry at least one of these diseases in fact carry both of them. If a randomly selected tick is found to have carried HGE, what is the probability that the selected tick is also a carrier of Lyme disease?

19. A student has to sit for an examination consisting of 3 questions selected randomly from a list of 100 questions. To pass, he needs to answer all three questions. What is the probability that the student will pass the examination if he knows the answers to 90 questions on the list?

20. The Reviews editor for a certain scientific journal decides whether the review for any particular book should be short (1–2 pages), medium (3–4 pages), or long (5–6 pages). Data on recent reviews indicates that 60% of them are short, 30are medium, and the other 10% are long. Reviews are submitted in either Word or LaTeX. For short reviews, 80% are in Word, whereas 50% of medium reviews are in Word and 30% of long reviews are in Word. Suppose a recent review is randomly selected.

    (a) What is the probability that the selected review was submitted in Word format?

    (b) If the selected review was submitted in Word format, what are the posterior probabilities of it being short, medium, or long?

21. We have two dice, a red and a white one. The red die is unfair so the probability of getting 6 is $\frac{1}{3}$, the remaining outcomes being equally likely among themselves. The white die is fair. We can pick one die, leaving the other one to the opponent. The dice are rolled and the player with the highest result wins. In case of equal results, the player with the white die wins. Which die does one choose to have a better chance of winning the game?

22. Suppose that you have three coins, two fair ones but the third biased with probability of heads p and tails $1 - p$. One coin selected at random drops to the floor, landing heads up. How likely is it that it is one of the fair coins?

23. If A and B are independent events, show that A' and B are also independent. [Hint: First establish a relationship between $P(A' \cap B)$, $P(B)$, and $P(A \cap B)$.]

24. An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful and B be the event that the European project is successful. Suppose that A and B are independent events with $P(A) = 0.4$ and $P(B) = 0.7$.

    (a) If the Asian project is not successful, what is the probability that the European project is also not successful? Explain your reasoning.

    (b) What is the probability that at least one of the two projects will be successful?

    (c) Given that at least one of the two projects is successful, what is the probability that only the Asian project is successful?

25. Consider independently rolling two fair dice, one red and the other green. Let A be the event that the red die shows 3 dots, B be the event that the green die shows 4 dots, and C be the event that the total number of dots showing on the two dice is 7. Are these events pairwise independent (i.e., are A and B independent events, are A and C independent, and are B and C independent)? Are the three events mutually independent?

26. A fair coin is tossed three times and the following events are considered:

$$A = \text{toss 1 and toss 2 produce different outcomes,}$$
$$B = \text{toss 2 and toss 3 produce different outcomes,}$$
$$C = \text{toss 3 and toss 1 produce different outcomes.}$$

    Show that $P(A) = P(A|B) = P(A|C)$, but $P(A) \neq P(A|B \cap C)$.

27. Suppose the diagram of an electrical system is as given in Figure. What is the probability that the system works? Assume the components fail independently.