Chapter 1 Modeling Change (TD1)

1.1. Modeling Change with Difference Equations.

1.2. Approximating Change with Difference Equations.

Example. Six years ago your parents purchased a home by financing $\$80,000$ for 20 years, paying monthly payments of $\$880.87$ with a monthly interest of 1%. They have made 72 payments and wish to know how much they owe on the mortgage, which they are considering paying off with an inheritance they received. Or they could be considering refinancing the mortgage with several interest rate options, depending on the length of the payback period. The change in the amount owed each period increases by the amount of interest and decreases by the amount of the payment:

$$\Delta b_n = b_{n+1} - b_n = 0.01 b_n - 880.87$$

Solving for b_{n+1} and incorporating the initial condition gives the dynamical system model

$$\begin{aligned} b_{n+1} &= b_n + 0.01 b_n - 880.87 \\ b_0 &= 80000 \end{aligned}$$

where b_n represents the amount owed after n months. Thus,

$$\begin{aligned} b_1 &= 80000 + 0.01(80000) - 880.87 = 79919.13 \\ b_2 &= 79919.13 + 0.01(79919.13) - 880.87 = 79837.45 \end{aligned}$$

yielding the sequence

$$B = (80000, 79919.13, 79837.45, \ldots)$$

Calculating b_3 from b_2, b_4 from b_3, and so forth in turn, we obtain $b_{72} = \$71,523.11$. This sequenc is graphed in **Figure 1.6**.

Months n	Amount owed b_n
0	80000.00
1	79919.13
2	79837.45
3	79754.96
4	79671.64
5	79587.48
6	79502.49
7	79416.64
8	79329.94
9	79242.37
10	79153.92
11	79064.59
12	78974.37

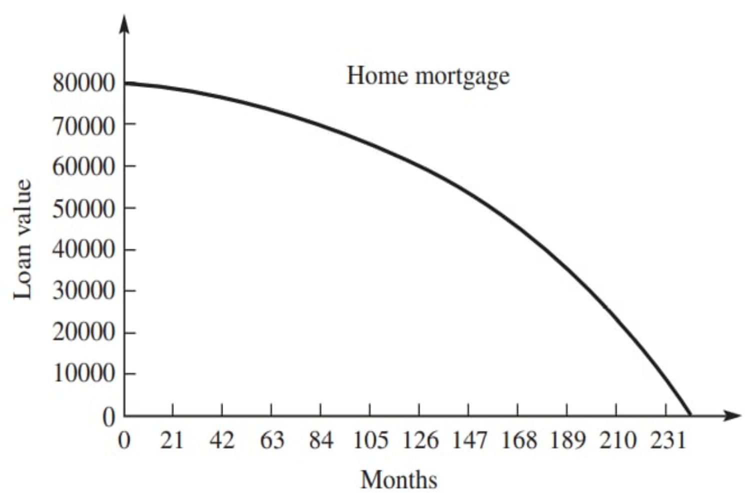

Figure 1.6

Exercise in class.

1). Write out the first five terms $a_0 - a_4$ of the sequences:

a. $a_{n+1} = 3a_n,\ a_0 = 1$ b. $a_{n+1} = 2a_n(a_n + 3),\ a_0 = 4$

2). Find a formula for the nth term of the sequence :

a. $\{1, 4, 16, 64, 256, ...\}$ b. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, ...\right\}$

3). By examining the following sequences, write a difference equation to represent the change during the nth interval as a function of the previous term in the sequence.

a. {2, 4, 16, 256,...} b. {1, 2, 5, 11, 23,...}

4). Write out the first five terms of the sequence satisfying the following difference equations:

a. $\Delta a_n = \frac{1}{2}a_n,\ a_0 = 1$ b. $\Delta p_n = 0.001(500 - p_n),\ p_0 = 10$

5). By substituting $n = 0, 1, 2, 3$ write out the first four algebraic equations represented by the following dynamical systems:

a. $a_{n+1} = 3a_n,\ a_0 = 1$ b. $a_{n+1} = 2a_n(a_n + 3),\ a_0 = 4$

***T*he problem 6 and 7, formulate a dynamical system that models change eactly for the described situation.**

6). You owe \$500 on a credit card that charges 1.5% interest each month. You pay \$50 each month and you make no new charges.

7). Your grandparents have an annuity. The value of the annuity increases each month by an automatic deposit of 1% interest on the previous month's balance. Your grandparents withdraw \$1000 at the beginning of each month for living expenses. Currently, they have \$50,000 in the annuity. Model the annuity with a dynamical system. Will the annuity run out of money? When? $Hint$: What value will an have when the annuity is depleted?

8). The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania.

Year	1814	1824	1834	1844	1854	1864
Population	125	275	830	1200	1750	1650

Plot the data. Is there a trend? Plot the change in population versus years elapsed after 1814. Formulate a discrete dynamical system that reasonably approximates the change you have observed.

9). Assume that we are considering the survival of whales and that if the number of whales falls below a minimum survival level m, the species will become extinct. Assume also that the population is limited by the carrying capacity M of the environment. That is, if the whale population is above M, it will experience a decline because the environment cannot sustain that large a population level. In the following model, a_n represents the whale population after n years. Discuss the model.

$$a_{n+1} - a_n = k(M - a_n)(a_n - m)$$

10). A certain drug is effective in treating a disease if the concentration remains above $100mg/L$. The initial concentration is $640mg/L$. It is known from laboratory experiments that the drug decays at the rate of 20% of the amount present each hour.

a. Formulate a model representing the concentration at each hour.

b. Build a table of values and determine when the concentration reaches $100mg/L$.

Assignmet

Deadline: 20-October, 2022.

1). With the price of gas continuing to rise, you wish to look at cars that get better gas mileage. You narrow down your choices to the following 2012 models: Ford Fiesta, Ford Focus, Chevy Volt, Chevy Cruz, Toyota Camry, Toyota Camry Hybrid, Toyota Prius and Toyota Corolla. Each company has offered you their "best deal" as listed in the following table. You are able to allocate approximately $500 for a car payment each month up to 60 months, although less time would be preferable. Use dynamical systems to determine which new car you can afford.

2012 Model	Best Deal Price	Cash Down	Interest and Duration
Ford Fiesta	$\$14,200$	$500	4.5% APR for 60 months
Ford Focus	$\$20,705$	$750	4.38% APR for 60 months
Chevy Volt	$\$39,312$	$\$1,000$	3.28% APR for 48 months
Chevy Cruz	$\$16,800$	$500	4.4% APR for 60 months
Toyota Camry	$\$22,955$	0	4.8% APR for 60 months
Toyota Camry Hybrid	$\$26,500$	0	3% APR for 48 months
Toyota Corolla	$\$16,500$	$900	4.25% for 60 months
Toyota Prius	$\$19,950$	$\$1,000$	44.3% for 60 months

2). You are considering a 30−year mortgage that charges 0.4% interest each month to pay off a $\$250,000$ mortgage.

a. Determine the monthly payment p that allows the loan to be paid off at 360 months.

b. Now assume that you have been paying the mortgage for 8 years and now have an opportunity to refinance the loan. You have a choice between a 20-year loan at 4% per year with interest charged monthly and a 15-year loan at 3.8% per year with interest charged monthly. Each of the loans charges a closing cost of $2500. Determine the monthly payment p for both the 20-year loan and the 15-year loan. Do you think refinancing is the right thing to do? If so, do you prefer the 20-year or the 15-year option?

3). Consider the spreading of a highly communicable disease on an isolated island with population size N. A portion of the population travels abroad and returns to the island infected with the disease. Formulate a dynamical system to approximate the change in the number of people in the population who have the disease.

4). The data in the accompanying table show the speed n (in increments of 5mph) of an automobile and the associated distance a_n in feet required to stop it once the brakes are applied. For instance, $n = 6$ (representing $6 \times 5 = 30$ mph) requires a stopping distance of $a_6 = 47$ ft.

a. Calculate and plot the change Δa_n an versus n. Does the graph reasonably approximate a linear relationship?

b. Based on your conclusions in part (a), find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against n. Discuss the appropriateness of the model.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
a_n	3	6	11	21	32	47	65	87	112	140	171	204	241	282	325	376