Assignment

Group I: I3-AMS-A

Group 10

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Exercise 01: The following data represent the growth of a population of fruit flies over a 6-week period. Test the following models by plotting an appropriate set of data. Estimate the parameters of the following model.

a.) $P(t) = c_1 t$

b.) $P(t) = ae^{bt}$

t(days)	7	14	21	28	35	42
P(# observed flies)	8	41	133	250	280	297

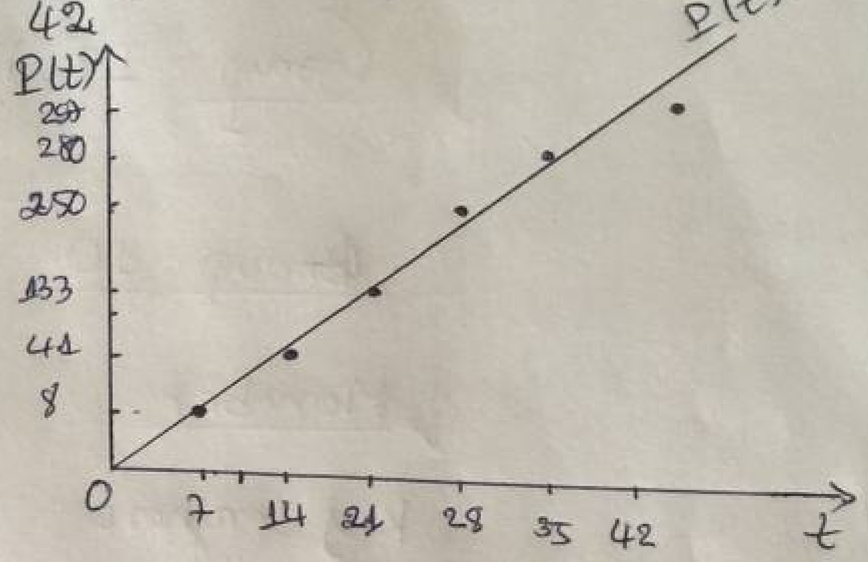
Proof:

a.) $P(t) = c_1 t$

consider following substitution:

- $P(7) = 7c_1 = 8 \Rightarrow c_1 = \frac{8}{7} = 1.142857$
- $P(14) = 14c_1 = 41 \Rightarrow c_1 = \frac{41}{14} = 2.928571$
- $P(21) = 21c_1 = 133 \Rightarrow c_1 = \frac{133}{21} = 6.3333$

- $P(28) = 28C_1 = 250 \Rightarrow C_1 = \frac{250}{28} = 8.9285714$
- $P(35) = 35C_1 = 280 \Rightarrow C_1 = \frac{280}{35} = 8$
- $P(42) = 42C_1 = 297 \Rightarrow C_1 = \frac{297}{42} = 7.0714285$



$\Rightarrow C_1 = \frac{1}{6}\sum_{i=1}^{6} C_i = 5.7341.$

b.) $P(t) = ae^{bt}$

<u>Proof:</u>

- $P(7) = 8 = ae^{7b}$ (1)
- $P(14) = 41 = ae^{14b}$ (2)
- $P(21) = 133 = ae^{21b}$ (3)
- $P(28) = 250 = ae^{28b}$ (4)
- $P(35) = 280 = ae^{35b}$ (5)
- $P(42) = 297 = ae^{42b}$ (6)

$\frac{(2)}{(1)}: \frac{41}{8} = \frac{e^{14b}}{e^{7b}} = e^{7b} \Rightarrow 7b = \ln\left(\frac{41}{8}\right) \Rightarrow b = 0.23344$

$\frac{(3)}{(2)}: \frac{133}{41} = \frac{e^{21b}}{e^{14b}} = e^{7b} \Rightarrow 7b = \ln\left(\frac{133}{41}\right) \Rightarrow b = 0.168111$

$\frac{(4)}{(3)}: \frac{250}{133} = e^{7b} \Rightarrow 7b = \ln\left(\frac{250}{133}\right) \Rightarrow b = 0.09015882$

$\frac{(5)}{(4)}: \frac{280}{250} = e^{7b} \Rightarrow 7b = \ln\left(\frac{28}{25}\right) \Rightarrow b = 0.0161898$

$\frac{(6)}{(5)}: \frac{297}{280} = e^{7b} \Rightarrow 7b = \ln\left(\frac{297}{280}\right) \Rightarrow b = 0.008420362$

(1): $a = \frac{8}{e^{7b}} = \frac{8}{e^{7\times 0.23344}} = \frac{8}{e^{1.63408}} \Rightarrow a = 1.56105$

(2): $a = \frac{41}{e^{14b}} = \frac{41}{e^{14\times 0.168111}} = \frac{41}{e^{2.353554}} \Rightarrow a = 3.896263$

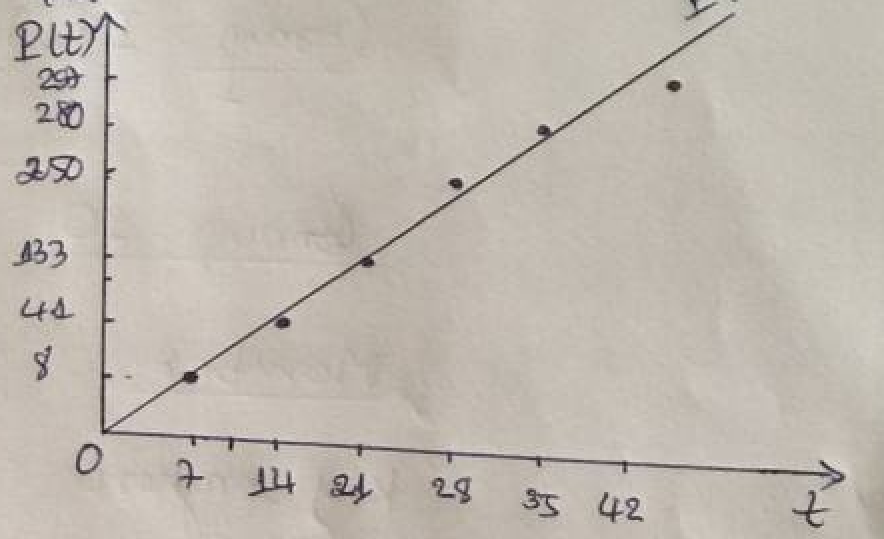
(3): $a = \frac{233}{e^{21b}} = \frac{133}{e^{21\times 0.09015882}} = \frac{133}{e^{1.8933}} = 20.02564$

- $P(28) = 28c_1 = 250 \Rightarrow c_1 = \frac{250}{28} = 8.9285714$

- $P(35) = 35c_1 = 280 \Rightarrow c_1 = \frac{280}{35} = 8$

- $P(42) = 42c_1 = 297 \Rightarrow c_1 = \frac{297}{42} = 7.0714285$

$\Rightarrow c_1 = \frac{1}{6}\sum_{i=1}^{6} c_i = 5.7341.$



b.) $P(t) = ae^{bt}$

Proof:

- $P(7) = 8 = ae^{7b}$ (1)
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$\frac{(2)}{(1)}: \frac{41}{8} = \frac{e^{14b}}{e^{7b}} = e^{7b} \Rightarrow 7b = \ln\left(\frac{41}{8}\right) \Rightarrow b = 0.23344$

$\frac{(3)}{(2)}: \frac{133}{41} = \frac{e^{21b}}{e^{14b}} = e^{7b} \Rightarrow 7b = \ln\left(\frac{133}{41}\right) \Rightarrow b = 0.168111$

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$\frac{(5)}{(4)}: \frac{280}{250} = e^{7b} \Rightarrow 7b = \ln\left(\frac{28}{25}\right) \Rightarrow b = 0.0161898$

$\frac{(6)}{(5)}: \frac{297}{280} = e^{7b} \Rightarrow 7b = \ln\left(\frac{297}{280}\right) \Rightarrow b = 0.00842036$

$(1): a = \frac{8}{e^{7b}} = \frac{8}{e^{7\times 0.23344}} = \frac{8}{e^{1.63408}} \Rightarrow a = 1.56105$

$(2): a = \frac{41}{e^{14b}} = \frac{41}{e^{14\times 0.168111}} = \frac{41}{e^{2.353554}} \Rightarrow a = 3.896263$

$(3): a = \frac{133}{e^{21b}} = \frac{133}{e^{21\times 0.09015882}} = \frac{133}{e^{1.8933}} = 20.02564$

(4) : $a = \dfrac{250}{e^{28b}} = \dfrac{250}{e^{28 \times 0.0161898}} = \dfrac{250}{1.5735188} = 158.8755$

(5) : $a = \dfrac{280}{e^{35b}} = \dfrac{280}{e^{35 \times 0.0161898}} = \dfrac{280}{1.762340} = 158.8795$

(6) : $a = \dfrac{297}{e^{42b}} = \dfrac{297}{e^{42 \times 0.008420362}} = 208.5287$

$\Rightarrow a = \dfrac{1}{5}\sum_{i=1}^{6} a_i = \cancel{91.9611}\ 78.5782$; $b = \dfrac{1}{5}\sum_{i=1}^{6} b_i = \cancel{0.047146}\ 0.103265$

Therefore, we can estimate a, b that $a = 91.9611$ and $b = 0.047146$.

Exercise 2: In 1610 the german astronomer Johannes Kepler became direction of the Prague observatory. Kepler had been helping Ticho Brahe is collecting 13 years old of observation on the relative motion of the planet Mars. By 1609 Kepler had formulated his first two laws:

i.) Each planet moves on an ellipse with the sun at one focus.

ii.) For each point planet, the line from the sun to planet sweeps out equal areas in equal times. Kepler spent many years verifying these laws and formulated a third law, which relates the planets' orbital periods and mean distances from the sun.

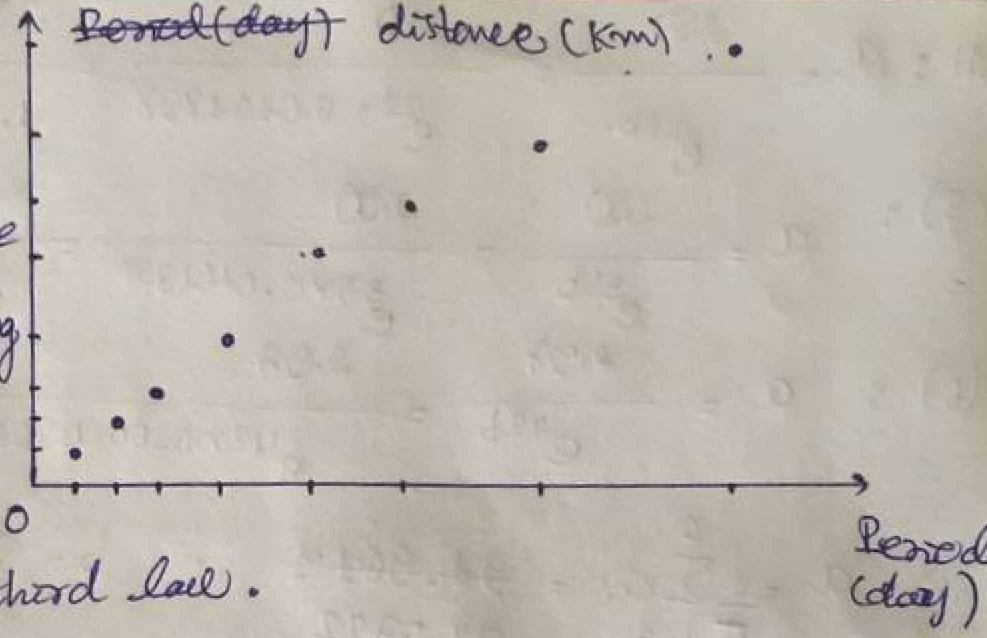
a.) Plot period time T versus the mean distance r using the following updated observational data.

Planet	Period(day)	Mean distance from Sun (Mkm)
Mercury	88	57.9
Venus	225	108.2
Earth	365	149.6
Mars	687	227.9
Jupyter	4329	778.1
Saturn	10753	1428.2
Uranus	30660	2837.9
Neptune	60150	4488.5

b.) Assuming a relationship of the form

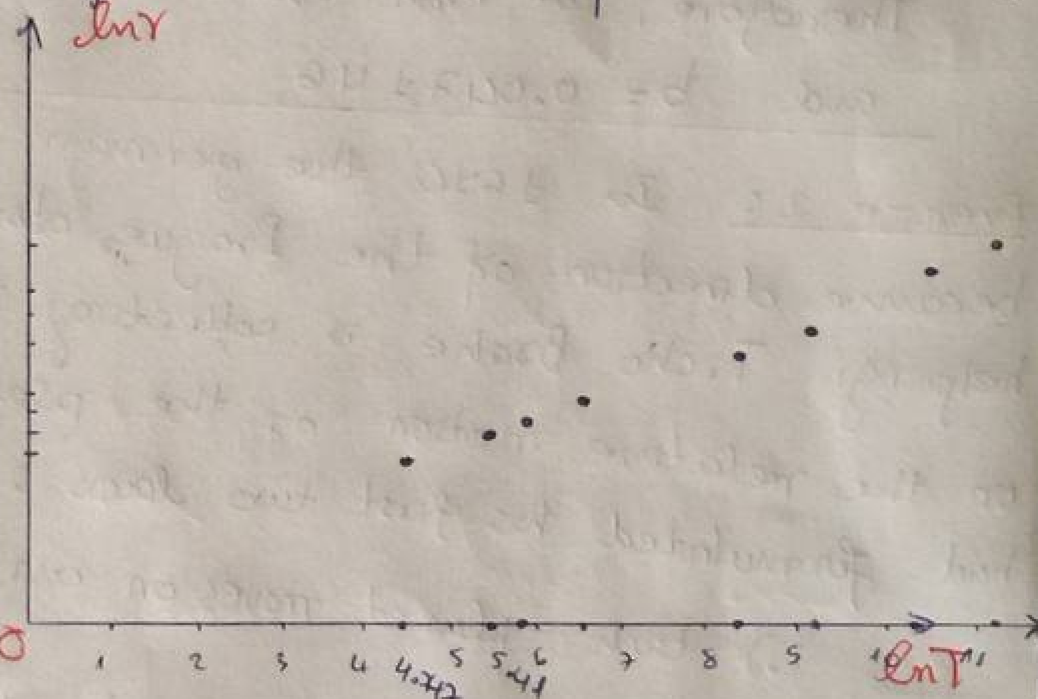
$T = cr^{\alpha}$. Determine the parameters c and α by plotting $\ln T$ versus $\ln r$. Does the model seem reasonable?

Try to formulate Kepler's third law.



Proof: we have the table of $\ln T$ and $\ln r$ represent as below:

Planet	$\ln T$	$\ln r$
Mercury	4.47733	4.058717
Venus	5.4161	4.683981
Earth	5.899897	5.007965
Mars	6.5323	5.4289
Jupiter	8.3730	6.6568
Saturn	9.2829	7.264170
Uranus	10.3307	7.950819
Neptune	11.0045	8.4093



we have $T = cr^{\alpha} \Rightarrow$ ~~$\ln T = \alpha \ln cr = \alpha(\ln c + \ln r)$~~

$\ln T = \ln c + \alpha \ln r \Leftrightarrow \ln c + \alpha \ln r = \ln T$ is the line equation

$\ln c + 4.058717\alpha = 4.47733$ (1)

$\ln c + 4.683981\alpha = 5.4161$ (2)

$\ln c + 5.007965\alpha = 5.899857$ (3)

$\ln c + 5.4289\alpha = 6.5323$ (4)

$\ln c + 6.6568\alpha = 8.3730$ (5)

$\ln c + 7.264170\alpha = 9.2829$ (6)

$\ln c + 7.950819 = 10.3307$ (7)

$\ln c + 8.4093 = 11.0045$ (8)

$\Rightarrow$ (2)−(1): $0.625264\alpha = 0.938764$

$\Rightarrow \alpha = 1.501388$, $C = 0.158618$

Caculate on excel we have:

- ($\alpha = 1.453276$, $C = 0.2063101$)
- ($\alpha = 1.502433$, $C = 0.1570625$)
- ($\alpha = 1.456052$, $C = 0.2027134$)
- ($\alpha = 1.458148$, $C = 0.2015238$)
- ($\alpha = 1.525523$, $C = 0.1650307$)
- ($\alpha = 1.465616$, $C = 0.2582218$)

$\Rightarrow \alpha = 1.4585$; $C = 0.2039$

According from data above therefore, we can estimate
$\alpha = 1.4585$ and $c = 0.2039$.

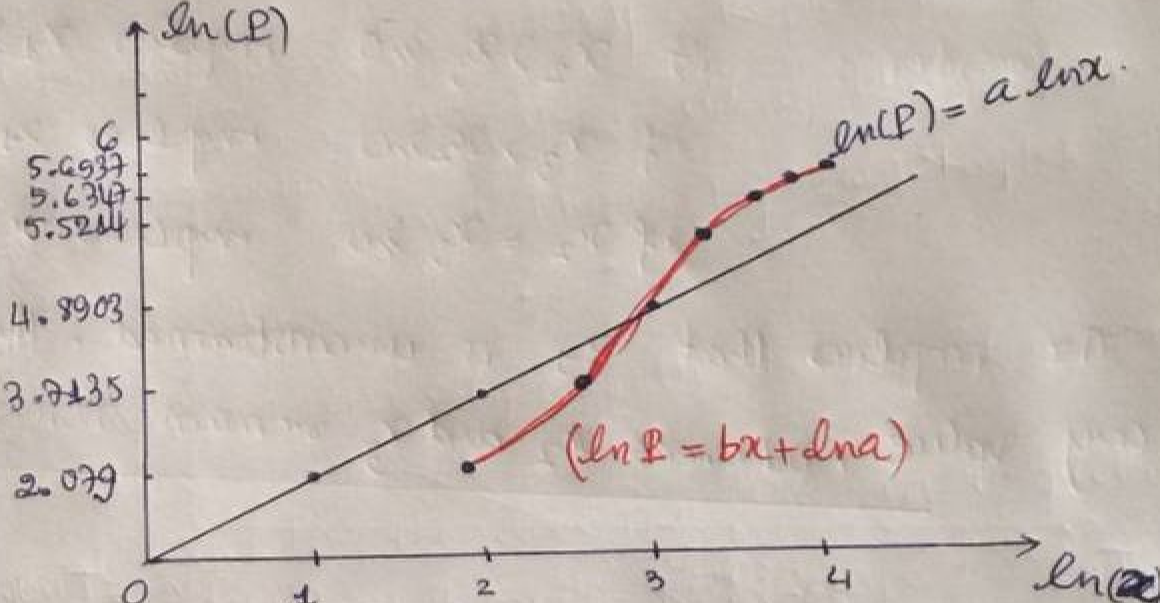
* It is reasonable to make conclusion with data above
* The third of Kepler's law is represented as the form:

$$T = 0.2039 r^{1.4985}$$

3.) For the following data, formulate mathematical model that minimize the largest deviation between data and model $P = ae^{bx}$, if computer is available, solve for the estimate of a and b.

we have $P = ae^{bx} \Rightarrow \ln P = bx + \ln a$.

$\ln P$	$\ln x$
2.079441	1.945910
3.713572	2.639057
4.890345	3.044522
5.521460	3.332204
5.634789	3.555348
5.653732	3.737669



$7b + \ln a = 2.079441$ (1)

$14b + \ln a = 3.713572$ (2)

$21b + \ln a = 4.890345$ (3)

$28b + \ln a = 5.521460$ (4)

$35b + \ln a = 5.634789$ (5)

$42b + \ln a = 5.693722$ (6)

$\Rightarrow$ After calculate on excel we have

a	b
1.5609	0.2334
3.8962	0.1681
20.0256	0.09015
158.5287	0.018420
208.5287	0.00842
78.5782	0.1032

then we can estimate that

$$a = \frac{1}{5}\sum_{i=1}^{5} a_i = 78.5782 \quad ; \quad b = \frac{1}{5}\sum_{i=1}^{5} b_i = 0.1032$$

$\boxed{\text{Therefore, } a \approx 78.5782,\ b \approx 0.1032}$