INSTITUTE OF TECHNOLOGY OF CAMBODIA
Department: AMS

Assignment

MATHEMATICAL MODELING

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Group8

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Assignment

1) As allocation of \$500 is allowable up to 60 mouths for the car,

Check: Both the time needed and total amount needed to pay off the loan in cases of all the cars mentioned in the question

Frame a computation formula for our calculations with the help of dynamical systems,

Let S_n denote the Standing Amount at the end of n mouth

Change per period (in this case a mouth) is denoted by $\Delta S_n = S_{n+1} - S_n = rS_n - P$

Here ΔS_n denotes change (decrement) in the loan amount at the end of each mount, r Denotes the rate of interest in APR (Annual Percentage Rate) calculated monthly and P is the monthly payment made to clear of the loan \$500.

Rearrange the equation:

$\Delta S_n = S_n + rS_n - P = (1+r)S_n - P$

Substitute $n = 0$ to obtain S_1

$$
\begin{aligned}
S_1 &= (1+r)S_0 - P \\
S_2 &= (1+r)S_1 - P \\
&= (1+r)\left[(1+r)S_0 - P\right] - P \\
&= (1+r)^2 S_0 - (1-r)P - P \\
S_3 &= (1+r)S_2 - P \\
&= (1+r)\left[(1+r)^2 S_0 - (1-r)P - P\right] - P \\
&= (1+r)^3 S_0 - (1+r)^2 P - (1+r)P - P
\end{aligned}
$$

Now I can write S_n

Thus $S_n = (1+r)^n S_0 - P\left[(1+r)^{n-1} + (1+r)^{n-2} + \ldots + 1\right]$

$$S_n = (1+r)^n S_0 - P\left\{\frac{(1+r)^{n-1} - 1}{r}\right\}$$

Total allowable time 60 months

Total allowable amount $\boxed{\$(500 \times 60) = \$30\,000}$

Further proceed with our calculation and comparisons of the Best Deals Offered by each car company

Ford Fiesta

Best Deal Price: $14 200

Cash Down: $500

Initial Amount $S_0 = \left(\text{Best Deal Price} - \text{Cash Down}\right) = 14\ 200 - 500 = \$13700$

To calculate the time of completion of the Loan S_n must be zero

$r = \dfrac{4.5}{12} = 0.375\% = 0.00375$ (APR expressed monthly for 60 months)

Use the formula above:

$$0 = \left(1 + 0.00375\right)^n \times 13700 - 500\left\{\frac{\left(1+0.00375\right)^{n-1} - 1}{0.00375}\right\}$$

$$13700\left(1.00375\right)^n = \frac{500}{0.00375}\left\{\left(1.00375\right)^{n-1} - 1\right\}$$

$$\left(1.00375\right)^n = \frac{500}{0.00375 \times 13700}\left\{\left(1.00375\right)^{n-1} - 1\right\}$$

$$\left(1.00375\right)^n = \frac{4000}{411}\left(1.00375\right)^{n-1} - \frac{4000}{411}$$

$$\frac{\left(1.00375\right)^n}{\left(1.00375\right)^{n-1}} = \frac{4000}{411} \times \frac{\left(1.00375\right)^{n-1}}{\left(1.00375\right)^{n-1}} - \frac{4000}{411\left(1.00375\right)^{n-1}}$$

$$1.00375 = \frac{4000}{411} - \frac{4000}{411\left(1.00375\right)^{n-1}}$$

$$\left(1.00375\right)^{n-1} = 1.115$$

$$\left(n-1\right)\ln\left(1.00375\right) = \ln\left(1.115\right)$$

$$\left(n-1\right) = 29.432$$

$$n \approx 31 \text{months}$$

Total amount required in this period $\boxed{\$\left(500 \times 31\right) + \$500 = \$16000}$

Ford Focus:

Best Deal Price: $20 705

Cash Down: $750

Initial Amount $S_0 = (\text{Best Deal Price} - \text{Cash Down}) = 20705 - 750 = \$19\ 995$

To calculate the time of completion of the Loan S_n must be zero

$r = \frac{4.38}{12} = 0.365\% = 0.00365$ (APR expressed monthly for 60 months)

Use the formula above:

$$0 = (1+0.00365)^n \times 19\ 995 - 500\left\{\frac{(1+0.00365)^{n-1} - 1}{0.00365}\right\}$$

$$19\ 995(1.00365)^n = \frac{500}{0.00365}\left\{(1.00365)^{n-1} - 1\right\}$$

$$(1.00365)^n = 6.85\left\{(1.00365)^{n-1} - 1\right\}$$

$$1.00365 = 6.85 - \frac{6.85}{(1.00365)^{n-1}}$$

$$(1.00375)^{n-1} = 1.172$$

$$(n-1)\ln(1.00365) = \ln(1.172)$$

$$(n-1) = 43.56$$

$$n \approx 45 \text{ months}$$

Total amount required in this period $\boxed{\$(500 \times 45) + \$750 = \$23250}$

Chery Volt:

Best Deal Price: \$39 312

Cash Down: \$1 000

Initial Amount $S_0 = 39312 - 1000 = \$38312$

To calculate the time of completion of the Loan S_n must be zero

$r = \frac{3.28}{12} = 0.273\% = 0.00273$ (APR expressed monthly for 48 months)

Use the formula above:

$$0 = (1+0.00273)^n \times 38312 - 500\left\{\frac{(1+0.00273)^{n-1} - 1}{0.00273}\right\}$$

$$n = 87.3$$

$$n \approx 87 \text{ months}$$

Total amount required in this period $\boxed{\$\left(500\times 87\right)+\$1000=\$44500}$

Chery Cruz:

Best Deal Price: $16 800

Cash Down: $500

Initial Amount $S_0=\$16300$

To calculate the time of completion of the Loan S_n must be zero

$r=\frac{4.4}{12}=0.366\%=0.00366$ (APR expressed monthly for 60 months)

Use the formula above:

$$0=\left(1+0.00366\right)^{n}\times 16300-500\left\{\frac{\left(1+0.00366\right)^{n-1}-1}{0.00366}\right\}$$

$n=34.9$

$n\approx 35$ months

Total amount $\boxed{\left(500\times 35\right)+500=\$18\ 000}$

Toyota Camry:

Best Deal Price: $ 22 995

Cash Down: $0

Initial Amount $S_0=\$22995$

To calculate the time of completion of the Loan S_n must be zero

$r=\frac{4.8}{12}=0.4\%=0.004$ (APR expressed monthly for 60 months)

Use the formula above:

$$0=\left(1+0.004\right)^{n}\times 22995-500\left\{\frac{\left(1+0.004\right)^{n-1}-1}{0.004}\right\}$$

$n=52.15$

$n\approx 50$ months

Total amount $\boxed{(500\times 50)+0=\$25\ 000}$

Toyota Camry Hybrid:

Best Deal Price: \$ 26 500

Cash Down: \$0

Initial Amount $S_0=\$26500$

To calculate the time of completion of the Loan S_n must be zero

$r=\dfrac{3}{12}=0.25\%=0.0025$ (APR expressed monthly for 48 months)

Use the formula above:

$$0=(1+0.0025)^n\times 26\ 500-500\left\{\frac{(1+0.0025)^{n-1}-1}{0.0025}\right\}$$

$n=57.6$
$n\approx 58$ months

Total amount $\boxed{(500\times 58)+0=\$29000}$

Toyota Corolla:

Best Deal Price: \$ 16 500

Cash Down: \$900

Initial Amount $S_0=\$15\ 600$

To calculate the time of completion of the Loan S_n must be zero

$r=\dfrac{4.25}{12}=0.354\%=0.00354$ (APR expressed monthly for 60 months)

Use the formula above:

$$0=(1+0.00354)^n\times 16600-500\left\{\frac{(1+0.00354)^{n-1}-1}{0.00354}\right\}$$

$n=36.51$
$n\approx 37$ months

Toyota amount required is $\boxed{\$(500\times 37)+900=\$19400}$

Toyota Prius:

Best Deal Price: $ 19 950

Cash Down: $1 000

Initial Amount $S_0 = \$18\,950$

To calculate the time of completion of the Loan S_n must be zero

$r = \frac{4.3}{12} = 0.358\% = 0.00358$ (APR expressed monthly for 60 months)

Use the formula above:

$$0 = (1+0.00358)^n \times 18\,950 - 500\left\{\frac{(1+0.00358)^{n-1} - 1}{0.00358}\right\}$$

$n = 41.8$

$n \approx 42$ months

Hence value of n is equal to 34 months

Toyota amount required is $\boxed{\$(500 \times 42) + 1000 = \$22\,000}$

From the above observe that **Chevy Volt** and **Toyota Camry Hybrid** are not affordable, the best choice in terms of least number of months required to clear the payment is **Ford Fiesta.**

2) a). Let S_n denote the Standing Capital at the end of n months.

$\Delta S_n = S_{n-1} - S_n = rS_n - P$

We have interest $r = 0.4\% = 0.04$ and initial amount $S_0 = \$250\,000$

P denotes the monthly payment that allows the loan to be paid off

Rearrange the equation: $S_{n+1} = (1-r)S_n - P$

Loan is to be paid off in 360 months

So, the Standing capital at the end of 360 month or S_{360} should be 0

To option a formula for S_n to equate S_{360} to 0

Form Ex1. $S_n = (1+r)^n S_0 - P\left\{\frac{(1+r)^{n-1} - 1}{r}\right\}$

But $S_{360} = 0 \Leftrightarrow (1+0.004)^{360}\, 250\,000 - P\left\{\frac{(1+0.004)^{360} - 1}{0.004}\right\} = 0$

Thus $\boxed{P = \$1\,318{,}5538}$

b). Now assume that you have been paying the mortgage for 8 years and now have an opportunity to refinance the term. You have a choice between a 20-year loan at 4% per year with interest charged monthly and a 15-year loan at 3.8% per year with interest charged monthly. Each of the loans charges a closing cost of $2500*:* Determine the monthly payment p for both the 20-year loan and the 15-year loan. Do you think refinancing is the right thing to do? If so, do you prefer the 20-year or the 15-year option?

Use formula for $S_n = (1+r)^n S_0 - P\left\{\dfrac{(1+r)^{n-1}-1}{r}\right\}$

We have $r = 0.004$, $P = 1318.5538$ and $S_0 = 250\,000$

And $n = 8\times 12 = 96$ months.

$$\Rightarrow S_{96} = (1+0.004)^{96}\, 250000 - 1318.5538\left\{\frac{(1+0.004)^{95}-1}{0.004}\right\}$$

$\Rightarrow S_{96} = \$214\ 733.785$

Thus $\boxed{S_{96} = \$214\ 733.785}$

+ For the first option the following is obtained:

$r = 4\%$ per year with interest charged monthly

$\Rightarrow r = \dfrac{4}{12} = 0.33\%$

$S_0 = \$214\ 733.785$

For $n = 20\times 12 = 240$ months.

That $S_{240} = (1+0.0033)^{240}\, 214733.785 - P\left\{\dfrac{(1+0.0033)^{239}-1}{0.0033}\right\}$

Thus $\boxed{P = \$1\ 304.5751}$

+ a 15-year loan at 3.8% per year with interest charged monthly.

$r = 0.003, n = 180$

$$S_{180} = 0 \Leftrightarrow (1+0.0031)^{180}\, 214733.785 - P_2\left\{\frac{(1+0.0031)^{179}-1}{0.0031}\right\} = 0$$

$$\Rightarrow P_2 = \$1569.908$$

For the first case: $1318.5538\times(360-96) = \$348098.203$

Second: $1304.5751\times 240 + 2500 = \$315\ 598.023$

Third; $1304.5751\times 180 + 2500 = \$237\,323.518$

Therefore, refinancing the loan is the right option and it is recommended to go with the third case (the 15-year period loan) as the total amount required in its case is lowest of the three.

3) Consider the spreading of a highly communicable disease on an isolated island with population size N. A portion of the population travels abroad and returns to the island infected with the disease. Formulate a dynamical system to approximate the change in the number of people in the population who have the disease.

ANSWER

Consider that the population size is N .

Let a_n be the portion of the population that return infected with the disease.

The objective is to formulate a dynamic system to approximate the change in the diseased population.

Then the portion of population free from disease is given by,

$\Rightarrow$ Disease free population = Total Population Size – Diseased Population

$$= N - a_n$$

Consider that the rate of spreading of such a disease is proportional to the product of disease-free and diseased population, $\Rightarrow a_n \left(N - a_n\right)$

So, the dynamic system for the approximate diseased population is given as,

Total diseased population=Existing diseased population + increase in the diseased population

Substitute the values, $\Rightarrow a_{n+1} = a_n + \left(N - a_n\right) a_n$

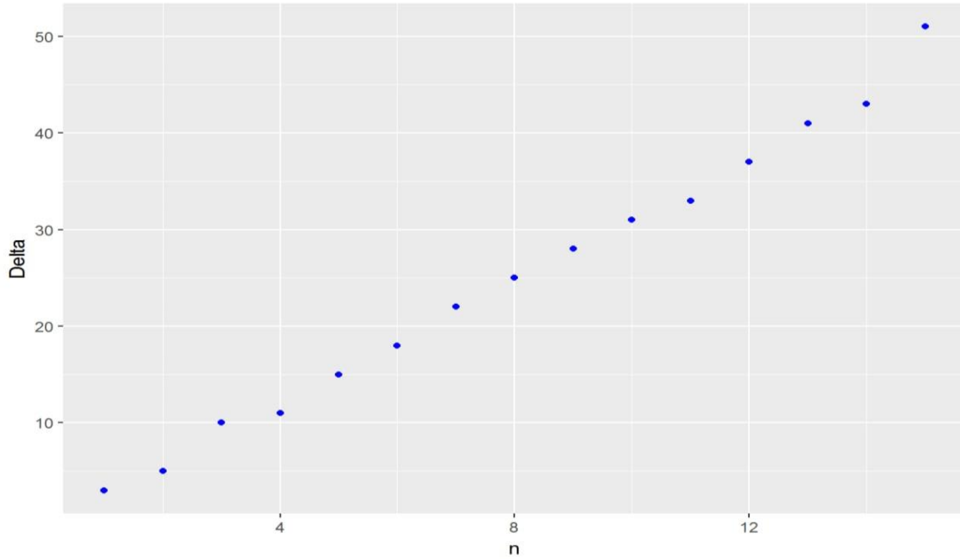
Thus, the required dynamic system is obtained.

4) a). Calculate and plot the change Δa_n an versus *n*. Does the graph reasonably approximate a linear relationship?

We know that $\Delta a_n = a_{n+1} - a_n$

The table

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
a_n	3	6	11	21	32	47	65	87	112	140	171	204	241	284	325	376
Δa_n	3	5	10	11	15	18	22	25	28	31	33	37	41	43	51	



The graph reasonably approximates a linear relation ship

b). Based on your conclusions in part (a), find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against *n*. Discuss the appropriateness of the model.

- Calculate stopping distance a_n

  $a_{n+1} = a_n + \Delta a_n$
- Find a formula for delta in terms of n
- Calculate K by finding the slope that passes in between the points $(n=5,\ \Delta a_5 = 15)$ and $(n=6,\ \Delta a_6 = 18)$.

  $$K = \frac{18-15}{6-5} = 3$$
- Find b by use the point $(n=1,\ \Delta a_1 = 3)$

  $\Delta a_n = Kn + b$

  $3 = 3 \cdot 1 + b$

  $\Rightarrow b = 0$

  $\Rightarrow \Delta a_n = 3n$

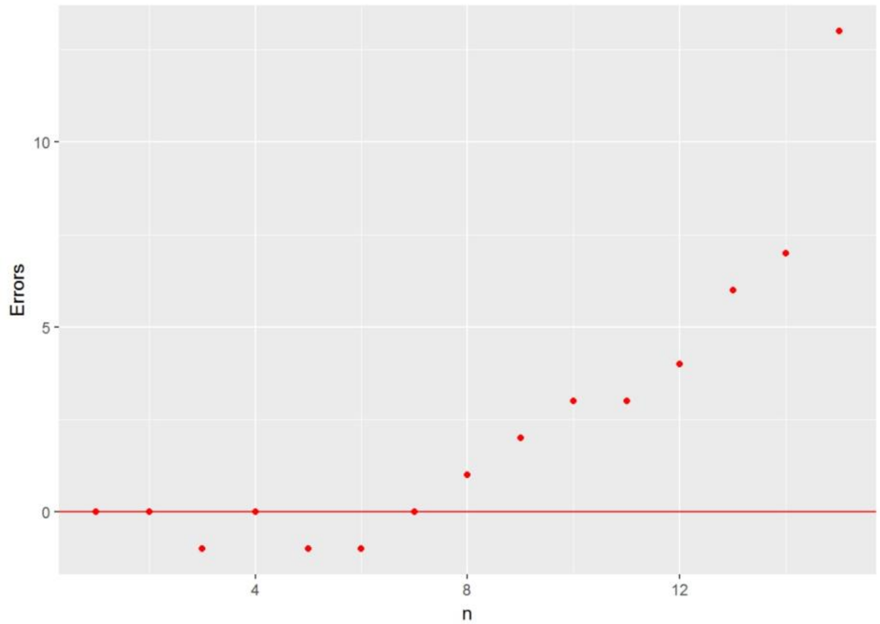
Predicted a_{n+1} = Predicted $a_{n+1} + \Delta a_n$

$\Leftrightarrow$ Predicted a_{n+1} = Predicted $a_{n+1} + 3n$

$\Rightarrow$Predicted $a_0 = 3$

$\Rightarrow$Predicted $a_1 = 3$

*Plotting the error



The line represents no error, it is clear that the breaks drown for Hight values of n as the error is increasing. This would be expected as larger n would make the error more apparent.