

Chapter 2

The Modeling Process, Proportionality, and Geometric Similarity (TD2)

2.1 Mathematical Model

2.2. Modeling Using Proportionality.

2.3 .Modeling Using Geometric Similarity.

Exercise in Class

1. Show graphically the meaning of the proportionality $y \propto u/v$.
2. If a spring is stretched 0.37 in. by a 14-lb force, what stretch will be produced by a 9-lbforce? By a 22-lb force? Assume Hooke's law, which asserts that the distance stretched is proportional to the force applied.
3. Determine whether the following data support a proportionality argument for $y \propto z^{1/2}$. If so, estimate the slope.

y	3.5	5	6	7	8
z	3	6	9	12	15

4. Determine whether the data set supports the stated proportionality model.
 - a. Force \propto Stretch

Force	10	20	30	40	50	60	70	80	90
Stretch	19	57	94	134	173	216	256	297	343

- b. $y \propto x^3$

y	19	25	32	51	57	71	113	141	123	187	192	205	252	259	294
x	17	19	20	22	23	25	28	31	32	33	36	37	38	39	41

5. Consider a 20 – lb pink flamingo that stands 3 ft in height and has legs that are 2 ft inlength. Model the height and leg length of a 100 – lb flamingo. What assumptions are necessary? Are they reasonable assumptions?
6. An object is sliding down a ramp inclined at an angle of θ radians and attains a terminal velocity before reaching the bottom. Assume that the drag force caused by the air is proportional to Sv^2 , where S is the cross-sectional area perpendicular to the direction of motion and v is the speed. Further assume that the sliding friction between the object and the ramp is proportional to the normal weight of the object. Determine the relationship between the terminal velocity and the mass of the object. If two different boxes, weighing 600 and 800lb, are pushed down the ramp, find the relationship between their terminal velocities.

Assignmet

Deadline: 10-November, 2022.

1. Consider an automobile suspension system. Build a model that relates the stretch (or compression) of the spring to the mass it supports. If possible, obtain a car spring and collect data by measuring the change in spring size to the mass supported by the spring. Graphically test your proportionality argument. If it is reasonable, find the constant of proportionality.
2. Research and prepare a 10-minute report on Hooke's law.
3. Consider the models $W \propto l^2 G$ and $W \propto g^3$. Interpret each of these models geometrically. Explain how these two models differ from Models (2.11) and (2.13), respectively. In what circumstances, if any, would the four models coincide? Which model do you think would do the best job of predicting W ? Why? In Chapter 3 you will be asked to compare the four models analytically.
- a. Let $A(x)$ denote a typical cross-sectional area of a bass, $0 \leq x \leq l$, where l denotes the length of the fish. Use the mean value theorem from calculus to show that the volume V of the fish is given by

$$V = l \cdot \bar{A}$$

where A is the average value of $A(x)$.

- b. Assuming that A is proportional to the square of the girth g and that weight density for the bass is constant, establish that

$$W \propto l g^2$$

4. Heart Rate of Mammals—The following data relate the weights of some mammals to their heart rate in beats per minute. Based on the discussion relating blood flow through the heart to body weight, as presented in Project 2, construct a model that relates heart rate to body weight. Discuss the assumptions of your model. Use the data to check your model.

Mammal	Body weight (g)	Pulse rate (beats/min)
Vespergo pipistrellas	4	660
Mouse	25	670
Rat	200	420
Guinea pig	300	300
Rabbit	2000	205
Little dog	5000	120
Big dog	30000	85
Sheep	50000	70
Man	70000	72
Horse	450000	38
Ox	500000	40
Elephant	3000000	48

5. Lumber Cutters—Lumber cutters wish to use readily available measurements to estimate the number of board feet of lumber in a tree. Assume they measure the diameter of the tree in inches at waist height. Develop a model that predicts board feet as a function of diameter in inches. Use the following data for your test:

x	17	19	20	23	25	28	32	38	39	41
y	19	25	32	57	71	113	123	252	259	294

The variable x is the diameter of a ponderosa pine in inches, and y is the number of board feet divided by 10.

a. Consider two separate assumptions, allowing each to lead to a model. Completely analyze each model.

(i.) Assume that all trees are right-circular cylinders and are approximately the same height.

(ii.) Assume that all trees are right-circular cylinders and that the height of the tree is proportional to the diameter.

b. Which model appears to be better? Why? Justify your conclusions.

2.4. Automobile Gasoline Mileage

17-November, 2022.

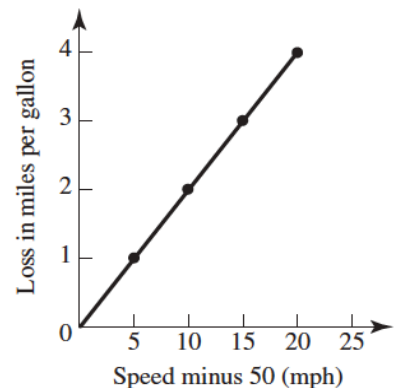
2.5 Body Weight and Height, Strength and Agility

Exercise in class

1. In the automobile gasoline mileage example, support you plot miles per gallon against speed as a graphical representation of the general rule (instead of the graph depicted in **Figure 2.7**). explain why it would be difficult to deduce a proportionality relationship from that graph.

■ **Figure 2.27**

For every 5 mph over 50 mph, there is a loss of 1 mile to the gallon.



2. In the automobile gasoline mileage example, assume the drag forces are proportional to Sv , where S is the cross-sectional area perpendicular to the direction of the moving car and v is its speed. what conclusions can you draw? Discuss the factors that might influence the choice of Sv^2 over Sv for the drag forces submodel. How could you test the submodel?

3. Describe in detail the data you would like to obtain to test the various submodels supporting Model (2.21) (in Text book page 102). How would you go about collection the data?

4. Tests exist to measure the percentage of body fat. Assume that such tests are accurate and that a great many carefully collected data are available. You may specify any other statistics, such as waist size and height, that you would like collected. Explain how the data could be arranged to check the assumptions underlying the submodels in this section. For example, suppose the data for males between ages 17 and 21 with constant body fat and height are examined. Explain how the assumption of constant density of the inner core could be checked.

Assignmet

Deadline: 24-November, 2022.

1. Discuss several factors that were completely ignored in our analysis of the gasoline mileage problem.
2. A popular measure of physical condition and personal appearance is the pinch test. To administer this test, you measure the thickness of the outer core at selected locations on the body by pinching. Where and how should the pinch be made? What thickness of pinch should be allowed? Should the pinch thickness be allowed to vary with height?
3. Consider an endurance test that measures only aerobic fitness. This teat could be a swimming test, running test, or bike test. Assume that we want all competitors to do an equal amount of work. Build a mathematical model that relates work done by the competitor to some measurable characteristic, such as height or weight. Next consider a refinement using kinetic energy in your model. Collect some data for one of these aerobic tests and determine the reasonableness of these models.