Chapter 5 Simulation Modeling (TD5)

5.1 Simulation Deterministic Behavior
5.2 Generating Random Numbers

Exercise in Class

1.

Each ticket in a lottery contains a single "hidden" number according to the following scheme: 55% of the tickets contain a 1, 35% contain a 2, and 10% contain a 3. A participant in the lottery wins a prize by obtaining all three numbers 1, 2, and 3. Describe an experiment that could be used to determine how many tickets you would expect to buy to win a prize.

2.

Using Monte Carlo simulation, write an algorithm to calculate an approximation to π by considering the number of random points selected inside the quarter circle

$$Q : x^2 + y^2 = 1, x \geq 0, y \geq 0$$

where the quarter circle is taken to be inside the square

$$S : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1$$

Use the equation $\pi/4 = \text{area } Q/\text{area } S$.

3

Use the middle-square method to generate

a. 10 random numbers using $x_0 = 1009$.

b. 20 random numbers using $x_0 = 653217$.

c. 15 random numbers using $x_0 = 3043$.

d. Comment about the results of each sequence. Was there cycling? Did each sequence degenerate rapidly?

4

Use the linear congruence method to generate

a. 10 random numbers using $a = 5$, $b = 1$, and $c = 8$.

b. 15 random numbers using $a = 1$, $b = 7$, and $c = 10$.

c. 20 random numbers using $a = 5$, $b = 3$, and $c = 16$.

d. Comment about the results of each sequence. Was there cycling? If so, when did it occur?

Assignment **Deadline: 12 January, 2023**

1.

Using Monte Carlo simulation, write an algorithm to calculate that part of the volume of an ellipsoid

$$\frac{x^2}{2} + \frac{y^2}{4} + \frac{z^2}{8} \leq 16$$

that lies in the first octant, $x > 0$, $y > 0$, $z > 0$.

2.

Write a program to generate 1000 integers between 1 and 5 in a random fashion so that 1 occurs 22% of the time, 2 occurs 15% of the time, 3 occurs 31% of the time, 4 occurs 26% of the time, and 5 occurs 6% of the time. Over what interval would you generate the random numbers? How do you decide which of the integers from 1 to 5 has been generated according to its specified chance of selection?

3.

Write a computer program to generate uniformly distributed random integers in the interval $m < x < n$, where m and n are integers, according to the following algorithm:

Step 1 Let $d = 2^{31}$ and choose N (the number of random numbers to generate).

Step 2 Choose any seed integer Y such that

$$100000 < Y < 999999$$

Step 3 Let $i = 1$.

Step 4 Let $Y = (15625\,Y + 22221) \bmod(d)$.

Step 5 Let $X_i = m + \text{floor}[(n - m + 1)Y/d]$.

Step 6 Increment i by 1: $i = i + 1$.

Step 7 Go to Step 4 unless $i = N + 1$.

Here, floor $[p]$ means the largest integer not exceeding p.

For most choices of Y, the numbers $X_1, X_2, \ldots$ form a sequence of (pseudo)random integers as desired. One possible recommended choice is $Y = 568731$. To generate random numbers (not just integers) in an interval a to b with $a < b$, use the preceding algorithm, replacing the formula in Step 5 by

$$\text{Let } X_i = a + \frac{Y(b - a)}{d - 1}$$