Chapter 4 Experimental Modeling (TD4)

4.2. High-Order Polynomial Models

4.3. Smoothing: Low-Order Polynomial Models

Exercise in Class

1. In the following data, X is the Fahrenheit temperature and Y is the number of times a cricket chirps in 1 minute (see Problem 7, Section 4.1). Make a scatterplot of the data and discuss the appropriateness of using an 18th-degree polynomial that passes through the data points as an empirical model. If you have a computer available, fit a polynomial to the data and plot the results.

x	46	49	51	52	54	56	57	58	59	60
y	40	50	55	63	72	70	77	73	90	93

x	61	62	63	64	66	67	68	71	72
Y	96	88	99	110	113	120	127	137	132

2. In the following data, X represents the diameter of a ponderosa pine measured at breast height, and Y is a measure of volume-number of board feet divided by 10. Make a scatterplot of the data. Discuss the appropriateness of using a 13th-degree polynomial that passes through the data points as an empirical model. If you have a computer available,fit a polynomial to the data and graph the results.

x	17	19	20	22	23	25	31	32	33	36	37	38	39	41
y	19	25	32	51	57	71	141	123	187	192	205	252	248	294

3. For the data sets in Problems, construct a divided difference table. What conclusions can you make about the data? Would you use a low-order polynomial as an empirical model? If so, what order?

X	0	1	2	3	4	5	6	7
y	2	8	24	56	110	192	308	464

4. In the following data, X is the Fahrenheit temperature and Y is the number of times a cricket chirps in 1 min (see Problem 3, Section 4.2).

X	46	49	51	52	54	56	57	58	59	60
Y	40	50	55	63	72	70	77	73	90	93

X	61	62	63	64	66	67	68	71	72
Y	96	88	99	110	113	120	127	137	132

Assignment **Deadline: 28 December 2022**

1. In the following data, X is the Fahrenheit temperature and Y is the number of times a cricket chirps in 1 minute (see Problem 7, Section 4.1). Make a scatterplot of the data and discuss the appropriateness of using an 18th-degree polynomial that passes through the data points as an empirical model. If you have a computer available, fit a polynomial to the data and plot the results.

X	46	49	51	52	54	56	57	58	59	60
Y	40	50	55	63	72	70	77	73	90	93

X	61	62	63	64	66	67	68	71	72
Y	96	88	99	110	113	120	127	137	132

2. For the data sets in Problems, construct a divided difference table. What conclusions can you make about the data? Would you use a low order polynomial as an empirical model? If so, what order?

X	0	1	2	3	4	5	6	7
Y	23	48	73	98	123	148	173	198

3. In the following data, X represents the diameter of a ponderosa pine measured at breast height, and Y is a measure of volume-number of board feet divided by 10 (see Problem 4, Section 4.2)

X	17	19	20	22	23	25	31	32	33	36	37	38	39	41
Y	19	25	32	51	57	71	141	123	187	192	205	252	248	294

4.4. Cubic Spline Models

1. For each of the following data sets, write a system of equations to determine the coefficients of the natural cubic splines passing through the given points. If a computer program is available, solve the system of equations and graph the splines.

a.

x	2	4	7
y	2	8	13

b.

x	3	4	6
y	10	15	35

2. Find the natural cubic splines that pass through the given data points. Use the splines to answer the requirements.

x	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
y	20.08	22.20	24.53	27.12	29.96	33.11	36.60	40.45	44.70	49.40

a. Estimate the derivative evaluated at $x = 3.4$. Compare your estimate with the derivative of e^x evaluated at at $x = 3.45$.
b. Estimate the area under the curve from 3.3 to 3.6. Compare with

$$\int_{3.3}^{3.6} e^x dx$$

3. The Cost of a Postage Stamp—Consider the following data. Use the procedures in this chapter to capture the trend of the data if one exists. Would you eliminate any data points? Why? Would you be willing to use your model to predict the price of a postage stamp on January 1, 2010? What do the various models you construct predict about the price on January 1, 2010? When will the price reach $1? You might enjoy reading the article on which this problem is based: Donald R. Byrkit and Robert E. Lee, ''The Cost of a Postage Stamp, or Up, Up, and Away,'' Mathematics and Computer Education 17, no. 3 (Summer 1983): 184–190.

Date	First-class stam	
1885–1917	$0.02	
1917–1919	0.03	(Wartime increase)
1919	0.02	(Restored by Congress)
July 6, 1932	0.03	
August 1, 1958	0.04	
January 7, 1963	0.05	
January 7, 1968	0.06	
May 16, 1971	0.08	
March 2, 1974	0.10	(Temporary)
December 31, 1975	0.13	
July 18, 1976	0.13	
May 15, 1978	0.15	
March 22, 1981	0.18	
November 1, 1981	0.20	
February 17, 1985	0.22	
April 3, 1988	0.25	
February 3, 1991	0.29	
January 1, 1995	0.32	
January 10, 1999	0.33	
January 7, 2001	0.34	
June 30, 2002	0.37	
January 8, 2006	0.39	
May 14, 2007	0.41	
May 12, 2008	0.42	
May 11, 2009	0.44	
January 22, 2012	0.45	

Assignment **Deadline: January 05, 2023**

1. For each of the following data sets, write a system of equations to determine the coefficients of the natural cubic splines passing through the given points. If a computer program is available, solve the system of equations and graph the splines.

a.

x	0	1	2
y	0	10	30

b.

x	0	2	4
y	5	10	40

2. find the natural cubic splines that pass through the given data points. Use the splines to answer the requirements.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0.0	0.50	0.87	1.00	0.87	0.50	0.00

3. Construct a computer code for determining the coefficients of the natural splines that pass-through a given set of data points. See Burden and Fairs, cited earlier in this chapter, for an efficient algorithm.
4. The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania (adapted from J. Davidson, ‘‘On the Growth of the Sheep Population in Tasmania,’’ Trans. Roy. Soc. S. Australia 62(1938): 342–346). t (year) 1814 1824 1834 1844 1854 1864 P .t / 125 275 830 1200 1750 1650

x	1814	1824	1834	1844	1854	1864
y	125	275	830	1200	1750	1650