Institute of Technology of Cambodia

Assignment of Mathematical Modeling

Group: I3-AMS-A

Team 10: Members:

KRY SENGHORT	: e20200706
LENG MOURYHONG	: e20200413
LEAT SEANGLONG	: e20200971
LIM SUNHENG	: e20200807

Academic Year: 2022-2023

Exercise 1:

For each of the following data sets, write a system of equations to determine the coefficients of the natural cubic splines passing through the given points. If a computer program is available, solve the system of equations and graph the splines.

a.

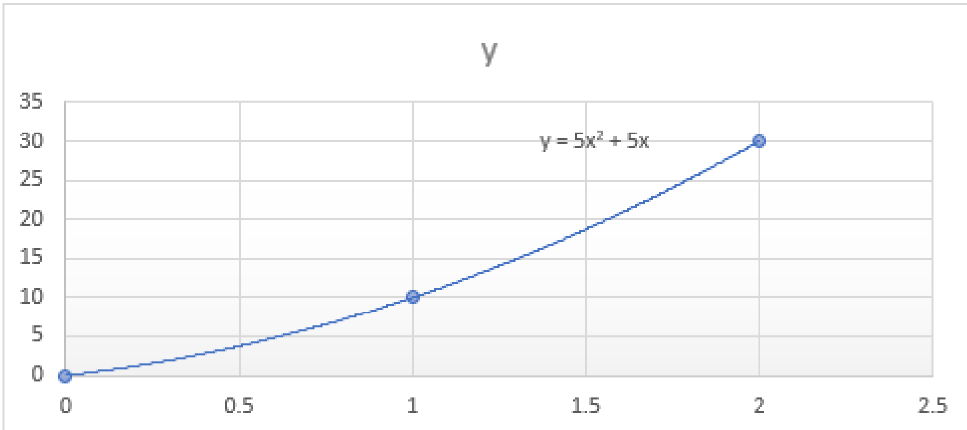
x	0	1	2
y	0	10	30

b.

x	0	2	4
y	5	10	40

Solution:

a	x	y
	0	0
	1	10
	2	30



From the above graph data representation, we have the fitting model to this data set is given by 2nd degree polynomial:

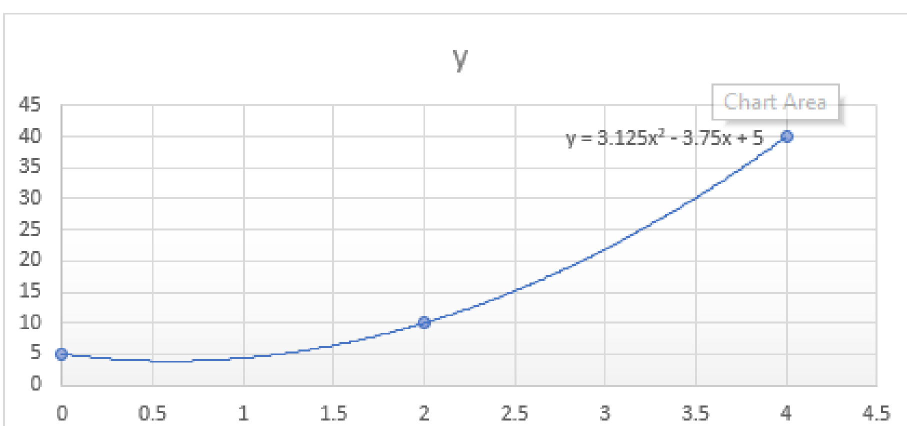
$$y = ax^2 + bx + c \quad (*)$$

- $y(0) = a(0)^2 + b(0) + c \Leftrightarrow c = 0$ (1)
- $y(1) = a(1)^2 + b(1) + c \Leftrightarrow a + b = 10$ (2)
- $y(2) = a(2)^2 + b(2) + c \Leftrightarrow 4a + 2b = 30$ (3)

After solving this problem, we get $a = b = 5,\ c = 0$

Therefore, the natural cubic splines of this data set are 2nd degree polynomial equation such that $y = 5x^2 + 5x$

b	x	y
	0	5
	2	10
	4	40



From the above graph data representation, we have the fitting model to this data set is given by 2nd degree polynomial:

$$y = ax^2 + bx + c \quad (*)$$

- $y(0) = a(0)^2 + b(0) + c \Leftrightarrow c = 5 \quad (1)$
- $y(1) = a(2)^2 + b(2) + c \Leftrightarrow 4a + 2b = 5 \quad (2)$
- $y(2) = a(4)^2 + b(4) + c \Leftrightarrow 16a + 4b = 35 \quad (3)$

After solving this problem, we have gotten the coefficients $a = 3.125,\ b = -3.75,\ c = 5$

Therefore, the natural cubic splines of this data set are 2nd degree polynomial equation such that

$$y = 3.125x^2 - 3.75x + 5$$

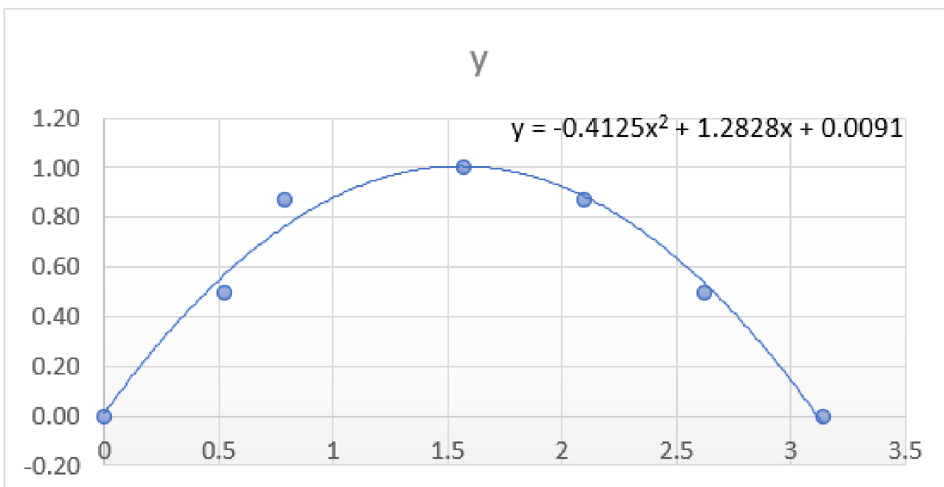
Exercise 2:

Find the natural cubic splines that pass through the given data points. Use the splines to answer the requirements.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0.0	0.50	0.87	1.00	0.87	0.50	0.00

Solution:

x	y
0	0.00
0.52359878	0.50
0.78539816	0.87
1.57079633	1.00
2.0943951	0.87
2.61799388	0.50
3.14159265	0.00



From the above graph data representation, we have the fitting model to this data set is given by 2nd degree polynomial:

$$y = ax^2 + bx + c \quad (*)$$

- $y(0) = a(0)^2 + b(0) + c \Leftrightarrow c = 0 \quad (1)$
- $y(0.523598776) = a(0.523598776)^2 + b(0.523598776) + c \Leftrightarrow 0.274155a + 0.523598b = 0.5 \quad (2)$
- $y(0.785398163) = a(0.785398163)^2 + b(0.785398163) + c \Leftrightarrow 0.616850a + 0.785398b = 0.87 \quad (3)$
- $y(1.570796327) = a(1.570796327)^2 + b(1.570796327) + c \Leftrightarrow 2.467401a + 1.570796b = 1 \quad (4)$
- $y(2.094395102) = a(2.094395102)^2 + b(2.094395102) + c \Leftrightarrow 4.386490a + 2.094395b = 0.87 \quad (5)$
- $y(2.617993878) = a(2.617993878)^2 + b(2.617993878) + c \Leftrightarrow 6.853891a + 2.617993b = 0.5 \quad (6)$

- $y(3.141592654) = a(3.141592654)^2 + b(3.141592654) + c \Leftrightarrow 9.869604a + 3.141592654b = 0 \quad (7)$
- After solving this problem, we have gotten the coefficients $a = -0.4125,\ b = 1.2828,\ c = 0.0091$

Therefore, the natural cubic splines of this data set are 2nd degree polynomial equation such that

$$y = -0.4125x^2 + 1.2828x + 0.0091$$

Exercise 3:

Construct a computer code for determining the coefficients of the natural splines that pass-through a given set of data points. See Burden and Fairs, cited earlier in this chapter, for an efficient algorithm.

Solution:

Computing z_i:

```
function z = cspline(t,y)
n = length(t);
z = zeros(n,1); h = zeros(n-1,1); b = zeros(n-1,1)
u = zeros(n,1); v = zeros(n,1);

h = t(2:n)-t(1:n-1); b = (y(2:n)-y(1:n-1))./h;
u(2) = 2*(h(1)+h(2)); v(2) = 6*(b(2)-b(1));

for i=3:n-1
  u(i) = 2*(h(i)+h(i-1))-h(i-1)^2/u(i-1);
  v(i) = 6*(b(i)-b(i-1))-h(i-1)*v(i-1)/u(i-1);
end

for i=n-1:-1:2
  z(i) = (v(i)-h(i)*z(i+1))/u(i);
end
```

Computing $S(x)$ for a given x:

```
function S = cspline_eval(t,y,z,x)
m = length(x);
n = length(t);
for i=n-1:-1:1
  if (x-t(i)) >= 0
    break
  end
end
h = t(i+1)-t(i);
S = z(i+1)/(6*h)*(x-t(i))^3 ...
   -z(i)/(6*h)*(x-t(i+1))^3 ...
  +(y(i+1)/h-z(i+1)*h/6)*(x-t(i)) ...
  -(y(i)/h-z(i)*h/6)*(x-t(i+1));
```

Use your functions:

```
>> t = [0.9,1.3,1.9,2.1]
t =
    0.9000    1.3000    1.9000    2.1000
>> y = [1.3,1.5,1.85,2.1]
y =
    1.3000    1.5000    1.8500    2.1000
>> z = cspline(t,y)
z =
         0
   -0.5634
    2.7113
         0
>> cspline_eval(t,y,z,1.5)
ans =
    1.5810
```

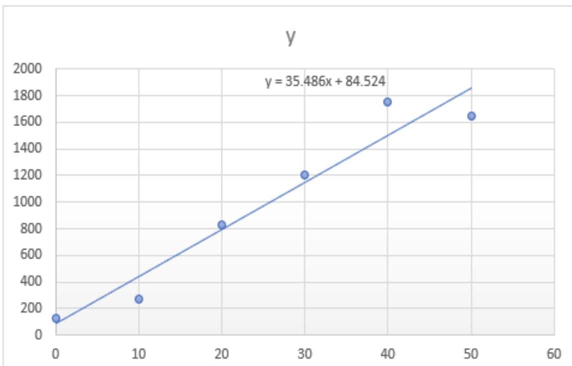
Exercise 4:

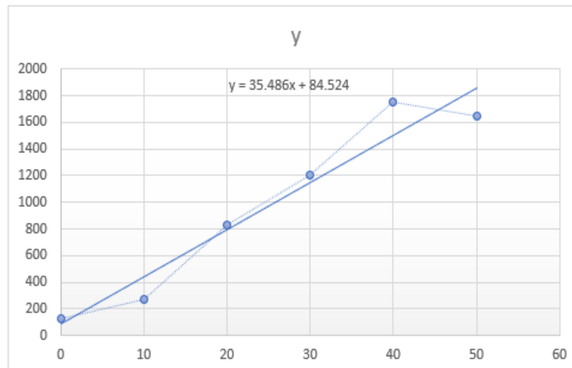
The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania (adapted from J. Davidson, ‘‘On the Growth of the Sheep Population in Tasmania,’’ Trans. Roy. Soc. S. Australia 62(1938): 342–346). t (year) 1814 1824 1834 1844 1854 1864 P .t / 125 275 830 1200 1750 1650

x	1814	1824	1834	1844	1854	1864
y	125	275	830	1200	1750	1650

Solution:

x	Time	y
1814	0	125
1824	10	275
1834	20	830
1844	30	1200
1854	40	1750
1864	50	1650





Based on the graph of data representation above we have the model fitting of cubic splines to this data set is linear equation: $y = 35.486x + 84.524$

Therefore, the natural cubic splines of this data set are linear equation such that $y = 35.486x + 84.524$