The Modeling Process, Proportionality, and Geometric Similarity

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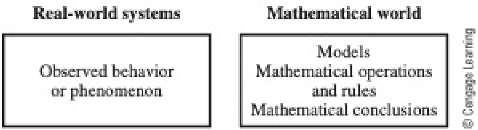
Course outline

Introduction

- We want to understand some behavior or phenomenon in the real world.
- We may wish to make predictions about that behavior in the future and analyze the effects that various situations have on it. (See Figure 2.1)

Figure 2.1

The real and mathematical worlds



Introduction

Goal. Draw conclusions about an observed phenomenon in the real world.

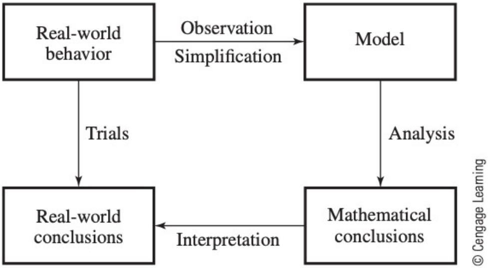
Two approaches to conclusions about the real world :

- **Approach 1.** One procedure would be to conduct some real-world behavior trials or experiments and observe their effect on the real-world behavior (See Figure 2.2).
  Drawback. This approach can be undoable, or too costly, or too harmful.

Introduction

Figure 2.2

Reaching conclusions about the behavior of real-world systems



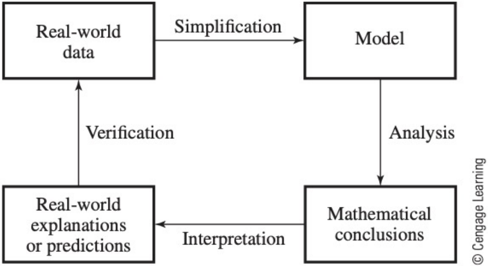
Introduction

- **Approach 2.** Use mathematical modeling (See Figure 2.3). We conduct the following rough modeling procedure:
    1. Through observation, identify the primary factors involved in the real-world behavior, possibly making simplifications.
    2. Conjecture tentative relationships among the factors.
    3. Apply mathematical analysis to the resultant model.
    4. Interpret mathematical conclusions in terms of the real-world problem.

Introduction

Figure 2.3

The modeling process as a closed system



Mathematical Models

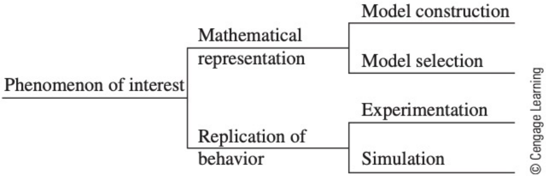
Definition

Mathematical model is a mathematical construct designed to study a particular real-world system or phenomenon. We include graphical, symbolic, simulation, and experimental constructs.

- Existing mathematical models that can be identified with some particular real-world phenomenon and used to study it.
- New mathematical models that we construct specifically to study a special phenomenon. See Figure 2.4.

Figure 2.4

The nature of the model



Mathematical Models

Characteristics

There are **three characteristics that a Mathematical model** can possess.

- **Fidelity:** The preciseness of a model?s representation of reality
- **Costs:** The total cost of the modeling process
- **Flexibility:** The ability to change and control conditions affecting the model as required data are gathered

Mathematical Models

Characteristics Cont.

Comparison of Characteristics of Mathematical Model.
Vertical axis = Degree of Effectiveness

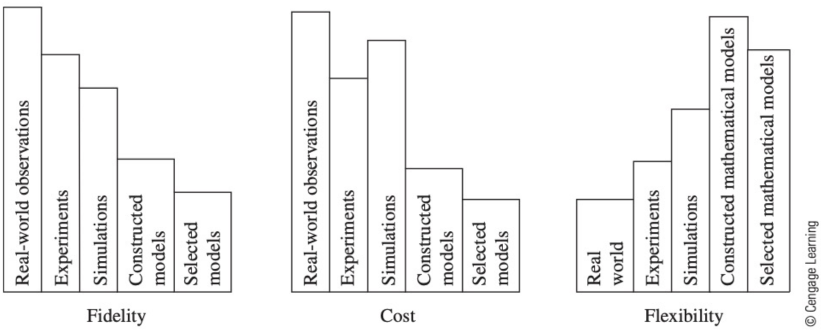


Figure 2.5

Comparisons among the model types

Mathematical Models

Construction of Model

Step 1: Identify the problem. What is the problem you would like to explore?

- Typically this is a difficult step because in real-life situations no one simply hands you a mathematical problem to solve.
- Usually you have to sort through large amounts of data and identify some particular aspect of the situation to study.
- Moreover, it is imperative to be sufficiently precise (ultimately) in the formulation of the problem to allow for translation of the verbal statements describing the problem into mathematical symbols. This translation is accomplished through the next steps.
- It is important to realize that the answer to the question posed might not lead directly to a usable problem identification.

Mathematical Models

Construction of Model Cont.

Step 2: Make assumptions.

- Generally, we cannot hope to capture in a usable mathematical model all the factors influencing the identified problem.
- The task is simplified by reducing the number of factors under consideration. Then, relationships among the remaining variables must be determined.
- Again, by assuming relatively simple relationships, we can reduce the complexity of the problem.

Mathematical Models

Construction of Model Cont.

Step 2: Make assumptions. The assumptions fall into two main activities:

- **Activity 1. Classifying the variables.** What things influence the behavior of the problem identified in Step 1? List these things as variables. The variables the model seeks to explain are the **dependent variables**, and there may be several of these. The remaining variables are the **independent variables**. Each variable is classified as dependent, independent, or neither.

Mathematical Models

Construction of Model Cont.

Step 2: Make assumptions. The assumptions fall into two main activities:

- **Activity 2. Determine relationships among variables selected for study.** Before we can hypothesize a relationship among the variables, we generally must make some additional simplifications. The problem may be so complex that we cannot see a relationship among all the variables initially. In such cases it may be possible to study submodels. That is, we study one or more of the independent variables separately. Eventually we will connect the submodels together. Studying various techniques, such as proportionality, will aid in hypothesizing relationships among the variables.

Mathematical Models

Construction of Model Cont.

Step 3: Solve or interpret the model.

- Now put together all the submodels to see what the model is telling us.
- In some cases the model may consist of mathematical equations or inequalities that must be solved to find the information we are seeking.
- Often, a problem statement requires a best solution, or *optimal solution*, to the model. Models of this type are discussed later.

Mathematical Models

Construction of Model Cont.

Step 4: Verifying the model.

- Before we can use the model, we must test it out.
- There are several questions to ask before designing these tests and collecting data?a process that can be expensive and time-consuming.
- First, does the model answer the problem identified in Step 1, or did it stray from the key issue as we constructed the model?
- Second, is the model usable in a practical sense? That is, can we really gather the data necessary to operate the model?
- Third, does the model make common sense?

Mathematical Models

Construction of Model Cont.

Step 5: Implement the model.

- We will want to explain our model in terms that the decision makers and users can understand if it is ever to be of use to anyone.
- Furthermore, unless the model is placed in a user-friendly mode, it will quickly fall into disuse. Expensive computer programs sometimes suffer such a demise. Often the inclusion of an additional step to facilitate the collection and input of the data necessary to operate the model determines its success or failure.

Mathematical Models

Construction of Model Cont.

Step 6: Maintain the model.

- Remember that the model is derived from a specific problem identified in Step 1 and from the assumptions made in Step 2. Has the original problem changed in any way, or have some previously neglected factors become important? Does one of the submodels need to be adjusted?

Mathematical Models

Example 1

Scenario. Consider the following rule often given in driver education classes:
Allow one car length for every 10 miles of speed under normal driving conditions, but more distance in adverse weather or road conditions. One way to accomplish this is to use the 2- second rule for measuring the correct following distance no matter what your speed. To obtain that distance, watch the vehicle ahead of you pass some definite point on the highway, like a tar strip or overpass shadow. Then count to yourself "one thousand and one, one thousand and two;" that is 2 seconds. If you reach the mark before you finish saying those words, then you are following too close behind.

The preceding rule is implemented easily enough, but how good is it? **Read the Text**

Mathematical Models

Example 1.

Step 1. Problem Identification. *Predict the vehicle's total stopping distance as a function of its speed.*

Step 2. Assumptions.

$$\text{total stopping distance} = \text{reaction distance} + \text{braking distance}$$

- *reaction distance:* the distance the vehicle travels from the instant the driver perceives a need to stop to the instant when the brakes are actually applied
- *braking distance*: the distance required for the brakes to bring the vehicle to a complete stop.

Mathematical Models

Example 1

Step 2. Assumptions.

Develop submodels:

- For reaction distance:

$$\text{reaction distance} = f(\text{response time, speed})$$

- For braking distance:

$$\text{braking distance} = h(\text{weight, speed})$$

- Thus for total stopping distance

$$\text{total stopping distance} = d(\text{response time, speed, weight})$$

Mathematical Models

Example 1.

Step 3. Solution.
Knowing response time, weight (constant), the speed as independent variable, we can predict the total stopping distance.
Step 4. Verification.

- To test the model, real-world data is needed.
- Do the predictions afforded by the model agree with real driving situations?
- If not, we would want to assess some of our assumptions and perhaps restructure one (or both) of our submodels. If the model does predict real driving situations accurately, then does the rule stated in the opening discussion agree with the model? The answer gives an objective basis for answering, How good is the rule?

Mathematical Models

Example 1.

Step 5. Implementation.
Whatever rule we come up with (to implement the model), it must be easy to understand and easy to use if it is going to be effective.
Step 6. Maintenance.
In this example, maintenance of the model does not seem to be a particular issue. Nevertheless, we would want to be sensitive to the effects on the model of such changes as power brakes or disc brakes, a fundamental change in tire design, and so on.

Mathematical Models

Iterative Nature of Model Construction

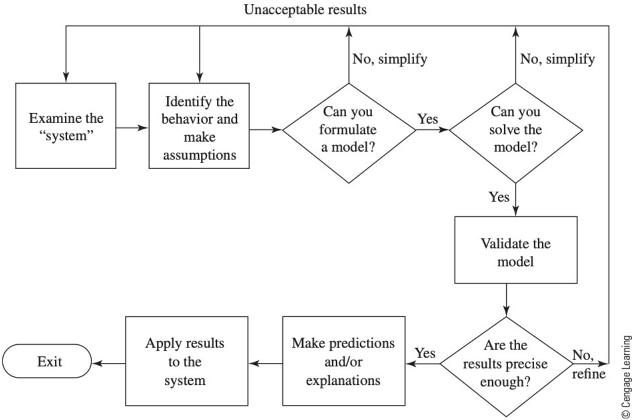


Figure 2.7

The iterative nature of model construction

Mathematical Models

Iterative Nature of Model Construction

Table 2.1 The art of mathematical modeling: simplifying or refining the model as required

Model simplification	Model refinement
1. Restrict problem identification.	1. Expand the problem.
2. Neglect variables.	2. Consider additional variables.
3. Conglomerate effects of several variables.	3. Consider each variable in detail.
4. Set some variables to be constant.	4. Allow variation in the variables.
5. Assume simple (linear) relationships.	5. Consider nonlinear relationships.
6. Incorporate more assumptions.	6. Reduce the number of assumptions.

Modeling Using Proportionality

Recalled proportionality definition

$$y \propto x \text{ if and only if } y = kx \text{ for some constant} k \neq 0 \qquad (3.1)$$

- By (3.1), since $k \neq 0$ if $y \propto x, \Rightarrow x \propto y \Rightarrow x = \left(\frac{1}{k}\right)y.$

The following are other examples of proportionality relationships:

$$y \propto x^2 \iff y = k_1 x^2 \text{ for } k_1 \neq 0 \text{ a constant} \qquad (3.2)$$
$$y \propto \ln x \iff y = k_2 \ln x \text{ for } k_2 \neq 0 \text{ a constant} \qquad (3.3)$$
$$y \propto e^x \iff y = k_3 e^x \text{ for } k_3 \neq 0 \text{ a constant} \qquad (3.4)$$

By (3.2), $y \propto x^2, \; k_1 \neq 0, \Rightarrow x \propto y^{1/2}$ becasue $x = \left(\frac{1}{k_1}\right)y^{1/2}$

Modeling Using Proportionality

- A transitive rule for proportionality

$$y \propto x \text{ and } x \propto z \text{ then } y \propto z$$

The geometric interpretation of proportionality

By (3.1) $y = kx \Rightarrow k = y/x$.

- k may be interpreted as the tangent of the angle θ depicted In Figure 2.8,
- $y \propto x$ defines a set of points along a line in the plane with angle of inclination θ.

Comparing the general form of a proportionality relationship $y = kx$ with $y = mx + b$, we can see that the graph of a proportionality relationship is a

Modeling Using Proportionality

Figure 2.8: Geometric interpretation of $y \propto x$

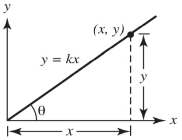
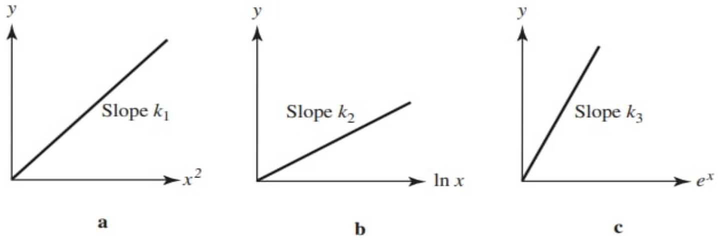


Figure 2.9: Geometric interpretation of Models (a) (3.2), (b) (3.3), and (c) (3.4)



Modeling Using Proportionality

Table 2.2 Famous proportionalities

Hooke's law: $F = kS$, where F is the restoring force in a spring stretched or compressed a distance S.

Newton's law: $F = ma$ or $a = \frac{1}{m}F$, where a is the acceleration of a mass m subjected to a net external force F.

Ohm's law: $V = iR$, where i is the current induced by a voltage V across a resistance R.

Boyle's law: $V = \frac{k}{p}$, where under a constant temperature k, the volume V is inversely proportional to the pressure p.

Einstein's theory of relativity: $E = c^2 M$, where under the constant speed of light squared c^2, the energy E is proportional to the mass M of the object.

Kepler's third law: $T = cR^{\frac{3}{2}}$, where T is the period (days) and R is the mean distance to the sun.

Table 2.3 Orbital periods and mean distances of planets from the sun

Planet	Period (days)	Mean distance (millions of miles)
Mercury	88.0	36
Venus	224.7	67.25
Earth	365.3	93
Mars	687.0	141.75
Jupiter	4331.8	483.80
Saturn	10,760.0	887.97
Uranus	30,684.0	1764.50
Neptune	60,188.3	2791.05

Modeling Using Proportionality

The data shown in Table 2.3 are from the 1993 *World Almanac*.

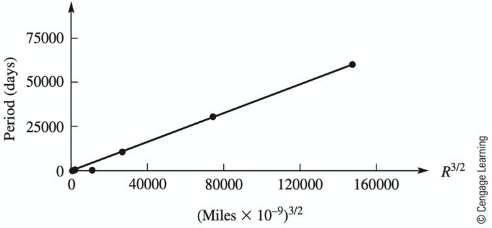
In Figure 2.12, we plot the period versus the mean distance to the $\frac{3}{2}$ power. The plot approximates a line that projects through the origin. We can easily estimate the slope (constant of proportionality) by picking any two points that lie on the line passing through the origin:

$$slope = \frac{60188 - 88}{147452 - 216} = .408.$$

So, we estimate the Model to be $T = 0.408R^{(3/2)}$.

Figure 2.12

Kepler's third law as a proportionality



Mathematical Models

Example 1. Modeling Vehicular Stopping Distance: Revisit

Scenario. Consider the following rule often given in driver education classes:

Allow one car length for every 10 miles of speed under normal driving conditions, but more distance in adverse weather or road conditions. One way to accomplish this is to use the 2-second rule for measuring the correct following distance no matter what your speed. To obtain that distance, watch the vehicle ahead of you pass some definite point on the highway, like a tar strip or overpass shadow. Then count to yourself "one thousand and one, one thousand and two;" that is 2 seconds. If you reach the mark before you finish saying those words, then you are following too close behind.

Discuss the example in the text. Page: 73–78.

Modeling Using Geometric Similarity

Definiton

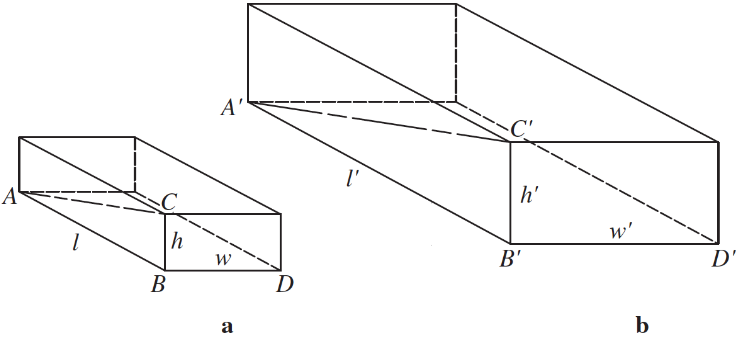
Two objects are said to be **geometrically similar** if there is a one-to-one correspondence between points of the objects such that the ratio of distances between corresponding points is constant for all possible pairs of points.

For example, consider the two boxes depicted in **Figure 2.18**.
Let $l =$ distance (A, B), $h =$ distance (B, C), and $w =$ distance (B, D) in **Figure 2.18 a,**
Similarly, let $l' =$ distance (A', B'), $h' =$ distance (B', C'), and $w' =$ distance (B', D') in **Figure 2.18 b.**
For the boxes to be geometrically similar, it must be true that,

$$\frac{l}{l'} = \frac{w}{w'} = \frac{h}{h'} = k, \quad \text{for some constant } k > 0$$

Modeling Using Geometric Similarity

Figure 2.18 Two geometrically similar objects



Consider the $\triangle ABC$ and $\triangle A'B'C'$.
If the two boxes are geometrically similar then
$\triangle ABC \sim \triangle A'B'C'$. So on, $\triangle CBD \sim \triangle A'B'C'$.
Since, when two objects are geometrically similar is a simplification in certain computations, such as volume and surface area.

Modeling Using Geometric Similarity

For the boxes depicted in **Figure 2.18,** consider the following argument for the ratio of the volumes V and V':

$$\frac{V}{V'} = \frac{lwh}{l'w'h'} = k^3 \tag{2.9}$$

Similarly, the ratio of their total surface areas S and S'i s given by

$$\frac{S}{S'} = \frac{2lh + 2wh + 2wl}{2l'h' + 2w'h' + 2w'l'} = k^2 \tag{2.10}$$

Let's select the length l as the characteristic dimension. Then with $\frac{l}{l'} = k$, we have

$$\frac{S}{S'} = k^2 = \frac{l^2}{l'^2}$$

Therefore,

$$\frac{S}{l^2} = \frac{S'}{l'^2} = \text{ constant}$$

holds for any two geometrically similar objects.

Modeling Using Geometric Similarity

That is, surface area is always proportional to the square of the characteristic dimension length:

$$S \propto l^2$$

Likewise, volume is proportional to the length cubed:

$$V \propto l^3$$

Thus, if we are interested in some function depending on an object's length, surface area, and volume, for example,

$$y = f(l, S, V)$$

we could express all the function arguments in terms of some selected characteristic dimension, such as length, giving

$$y = g(l, l^2, l^3)$$

Geometric similarity is a powerful simplifying assumption.

Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby

For conservation purposes, a sport fishing club wishes to encourage its members to release their fish immediately after catching them. The club also wishes to grant awards based on the total weight of fish caught: honorary membership in the 100 Pound Club, Greatest Total Weight Caught during a Derby Award, and so forth. How does someone fishing determine the weight of a fish he or she has caught? You might suggest that each individual carry a small portable scale. However, portable scales tend to be inconvenient and inaccurate, especially for smaller fish.

Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

Step 1. Problem Identification. *Predict the weight of a fish in terms of some easily measurable dimensions.*

Step 2. Assumptions. Many factors that affect the weight of a fish can easily be identified.

- *Species*: different shapes and different average weights per unit volume (weight density) based on the proportions and densities of meat, bone, and so on.
- *Gender*: male or female, especially during spawning season.
- *Season*: hot or cold, rainy or dry...

Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

Step 2. Assumptions.

Further restrictions. To simplify the problem, we assume

- Single species of fish only, say bass
- Within the species, the average weight density is constant (Later, it may be desirable to refine our model if the results prove unsatisfactory or if it is determined that considerable variability in density does exist.)
- Gender and season effects can be negligible
- All bass are geometrically similar, the volume of any bass is proportional to the cube of some characteristic dimension.

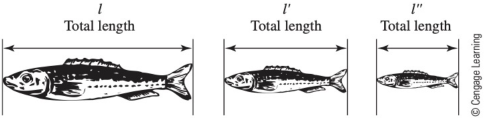
Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

Note that we are not assuming any particular shape, but only that the bass are scaled models of one another. The basic shape can be quite irregular as long as the ratio between corresponding pairs of points in two distinct bass remains constant for all possible pairs of points. This idea is illustrated in **Figure 2.19.**

Figure 2.19

Fish that are geometrically similar are simply scaled models of one another.



Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

Step 3. Model Construction and Solution.

Let length l be the fish as the characteristic dimension.

- Thus, the volume of a bass satisfies the proportionality:

$$V \propto l^3$$

- The weight W is volume times average weight density
- Average weight density is assumed to be constant
- It follows that

$$W \propto l^3$$

Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

Step 4. Verification. Let's test our model. Consider the following data collected during a fishing derby.

If our model is correct, then the graph of W versus l^3 should be a straight line passing through the origin. The graph showing an approximating straight line is presented in **Figure 2.20**.

Length, l (in.)	14.5	12.5	17.25	14.5	12.625	17.75	14.125	12.625
Weight, W (oz)	27	17	41	26	17	49	23	16

Modeling Using Geometric Similarity

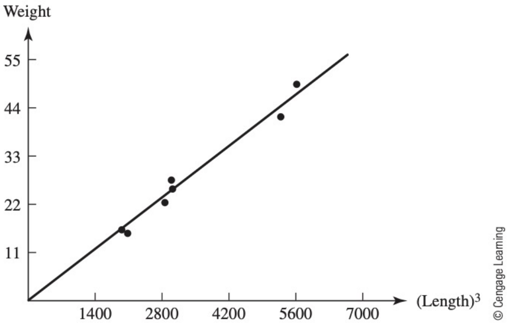
Example 2: Modeling a Bass Fishing Derby Cont.

The scatterplot of W vs l^3 appears to be a straight line, passing through the origin. We can estimate this equation to be:

$$W = 0.00853\, l^3$$

Figure 2.20

If the model is valid, the graph of W versus l^3 should be a straight line passing through the origin.

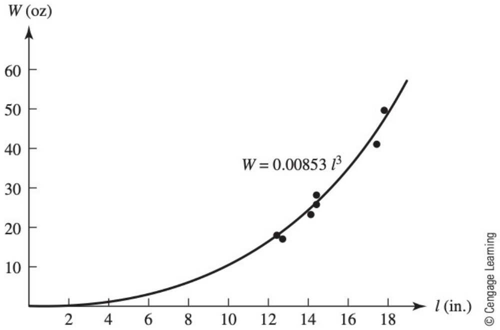


Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

The scatterplot of W vs l is shown in **Figure 2.21**. It appears that the model fits the data quite well.

Figure 2.21
Graph of the model
$W = 0.00853\, l^3$



Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

Step 4. Implementation. If we agree that the model is good, we can use it to predict some specific values.
Note: 1 ounce (oz) is equal to 0.0625 pounds (lb).

Length (in.)	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Weight (oz)	15	19	23	29	35	42	50	59	68	79	91	104	118	133	150
Weight (lb)	0.9	1.2	1.5	1.8	2.2	2.6	3.1	3.7	4.3	4.9	5.7	6.5	7.4	8.3	9.4

Can we refine the model?

- The above model treats fat and skinny fish alike. Let's address this dissatisfaction.

Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

- Instead of assuming that the fish are geometrically similar, assume that only their cross-sectional areas are similar.
- This does not imply any particular shape for the cross section, only that the definition of geometric similarity is satisfied.
- We choose the characteristic dimension to be the girth, g, defined subsequently.
- Major portion of the weight of the fish is from the main body; head and tail contribute relatively little to the total weight. (Constant terms can be added later if our model proves worthy of refinement.)
- The main body is of varying cross-sectional area. Then the volume can be found by $V \approx l_{\text{eff}} \times A_{\text{avg}}$, where A_{avg}: the average cross-sectional area and l_{eff}: the effective length.

Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

- Assume $l_{\text{eff}} \propto l$
- The girth, g, is the measurement of the circumference of the fish at its widest point
- Assume $A_{\text{avg}} \propto g^2$

Then

$$V \propto lg^2,$$

and

$$W \propto lg^2 \quad \text{or} \quad W = klg^2 \quad \text{as} \quad W \propto V.$$

Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

Verification. Let's test our model again. With the following data, we have:

$$W = 0 : 0187\, l\, g^2$$

Length, l (in.)	14.5	12.5	17.25	14.5	12.625	17.75	14.125	12.625
Girth, g (in.)	9.75	8.375	11.0	9.75	8.5	12.5	9.0	8.5
Weight, W (oz)	27	17	41	26	17	49	23	16

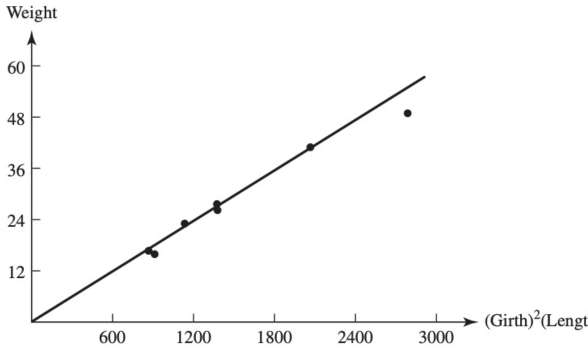
To see how well the model fits the model, see **Figure 2.22**.

Modeling Using Geometric Similarity

Example 2: Modeling a Bass Fishing Derby Cont.

Figure 2.22

Testing the proportionality between W and lg^2



Automobile Gasoline Mileage

Automobile Gasoline Mileage

Scenario During periods of concern when oil shortages and embargoes create an energy crisis, there is always interest in how fuel economy varies with vehicular speed. We suspect that, when driven at low speeds and in low gears, automobiles convert power relatively inefficiently and that, when they are driven at high speeds, drag forces on the vehicle increase rapidly. It seems reasonable, then, to expect that automobiles have one or more speeds that yield optimum fuel mileage (the most miles per gallon of fuel). If this is so, fuel mileage would decrease beyond that optimal speed, but it would be beneficial to know just how this decrease takes place. Moreover, is the decrease significant? Consider the following excerpt from a newspaper article (written when a national 55-mph speed limit existed):

Automobile Gasoline Mileage

Automobile Gasoline Mileage Cont.

Observe the 55-mile-an-hour national highway speed limit. For every 5 miles an hour over 50, there is a loss of 1 mile to the gallon. Insisting that drivers stay at the 55-mile-an-hour mark has cut fuel consumption 12 percent for Ryder Truck Lines of Jacksonville, Florid–a savings of 631,000 gallons of fuel a year. The most fuel-efficient range for driving generally is considered to be between 35 and 45 miles an hour.

Noted especially the suggestion that there is a loss of 1 mile to the gallon for every 5 miles an hour over 50 mph. How good is this general rule?

Automobile Gasoline Mileage

Automobile Gasoline Mileage Cont.

Step 1. Problem Identification. *What is the relationship between the speed of a vehicle and its fuel mileage?*
Step 2. Assumptions.

$$\text{fuel mileage} = f(\text{propulsion forces, drag forces, driving habits, ...})$$

Restricted Problem Identification. *Driving car on a given day on a level highway at constant highway speeds near the optimal speed for fuel economy, provide a qualitative explanation of how fuel economy varies with small increases in speed.*

$$\vdots$$

$$\text{mileage} \propto \frac{1}{v^2}$$

Read the discussion on Section 2.4 in the text.

Body Weight and Height, Strength and Agility

Body Weight and Height

How much should I weight? A rule often given to people desiring to run a marathon is 12 lb of body weight per inch of heigh, but shorter marathon runners seem to have a much easier time meeting this rule than taller ones.
Consider **Table 2.7,** which gives upper weight limits of acceptability for males between the ages of 17 and 21. (The table has no further delineators such as bone structure.)

Read the discussion on Section 2.5 in the text.

Body Weight and Height, Strength and Agility

Body Weight and Height Cont.

Table 2.7 weight versus height for males aged 17-21

Height (in.)	Weight(lb)	Height(in.)	Weight(lb)
60	132	71	185
61	136	72	190
62	141	73	195
63	145	74	201
64	150	75	206
65	155	76	212
66	160	77	218
67	165	78	223
68	170	79	229
69	175	80	234
70	180		

We examine qualitatively how weight and height should vary. Body weight depends on a number of factors, some of which we have mentioned. In addition to height, bone density could be a factor.

Body Weight and Height, Strength and Agility

Body Weight and Height Cont.

Is there a significant variation in bone density, or is it essentially constant? What about the relative volume occupied by the bones? Is the volume essentially constant, or are there heavy, medium, and light bone structures? And what about a body density factor? How can differences in the densities of bone, muscle, and fat be accounted for? Do these densities vary? Is body density typically a function of age and gender in the sense that the relative composition of muscle, bone, and fat varies as a person becomes older? Are there different compositions of muscle and fat between males and females of the same age?

Let' define the problem so that bone density in considered constant (by accepting an upper limit) and predict weight as a function of height, gender, age, and body density. The purpose or basis of the weight table must also be specified, so we base the table on physical appearance.

Body Weight and Height, Strength and Agility

Body Weight and Height Cont.

Step 1. Problem Identification: *For various heights, genders, and age groups, detemine upper weight limits that represent maximum levels of acceptability based on physical appearance.*

Step 2. Assumptios.

- The Weight of an adult is given by

$$W = k_1 + W_{in} + W_{out} \tag{2.17}$$

  where $k_1 > 0$ contant, W_{in} & W_{out} weight of inner and outer corse reapectively.

- The Sum of the components must be proportional to the cube of the height (h)

$$V_{in} \propto h^3 \tag{2.18}$$

Body Weight and Height, Strength and Agility

Body Weight and Height Cont.

- Consider the average weight density ρ_{avg} of $V = (v_1, v_2)$ each with a density ρ_1 and ρ_2. Then

$$V = V_1 + V_2$$

and

$$\rho_{avg} V = W = \rho_1 V_1 + \rho_2 V_2$$

yield

$$\rho_{avg} = W = \rho_1 V_1 + \rho_2 V_2$$

- Application of this result to the inner core implies that ρ_{in} is constant, yielding

$$W_{in} = V_{in} \rho_{in} \propto h^3$$

or

Body Weight and Height, Strength and Agility

Body Weight and Height Cont.

$$W_{in} = k_2 h^3 \tag{2.19}$$

- If τ represent this thickness, then the weight of the outer core:

$$W_{out} = \tau \rho_{out} S_{out},$$

  where the surface area of the outer core

- If the density of the outer core of fatty material is assumed to be constant for all individuals, then we have:

$$W_{out} \propto h^2$$

Body Weight and Height, Strength and Agility

Body Weight and Height Cont.

- If it is assumed that the thickess of the outer core is proportional to the height, then

$$W_{out} \propto h^3$$

- Allowing both these Assumption to reside in a single subodel gives

$$W_{out} = k_3 h^2 + k_4 h^3, \text{ where } k_3, k_4 \geq 0 \tag{2.20}$$

- By equation (2.17), (2.19) and (2.20) to determine a model for weight yields

$$W = k_1 + k_3 h^2 + k_5 h^3 \text{ for } k_1, k_5 > 0 \text{ and } k_3 \leq 0 \tag{2.21}$$

Body Weight and Height, Strength and Agility

Body Weight and Height Cont.

Step3. Model Interpretation: To allow constant weight increase, the trunk must increase in length while maintaining *the same cross-section area.*

- Comparing two model, we get the following data:

Height (in.)	Linear model (in., waist measure)	Geometric similarity model (in., wait measure)
66	30	30.0
72	30	32.7
78	30	35.5
84	30	38.2