Assignmet

Deadline: 20-October, 2022.

1). With the price of gas continuing to rise, you wish to look at cars that get better gas mileage. You narrow down your choices to the following 2012 models: Ford Fiesta, Ford Focus, Chevy Volt, Chevy Cruz, Toyota Camry, Toyota Camry Hybrid, Toyota Prius and Toyota Corolla. Each company has offered you their "best deal" as listed in the following table. You are able to allocate approximately \$500 for a car payment each month up to 60 months, although less time would be preferable. Use dynamical systems to determine which new car you can afford.

2012 Model	Best Deal Price	Cash Down	Interest and Duration
Ford Fiesta	\$14,200	\$500	4.5% APR for 60 months
Ford Focus	\$20,705	\$750	4.38% APR for 60 months
Chevy Volt	\$39,312	\$1,000	3.28% APR for 48 months
Chevy Cruz	\$16,800	\$500	4.4% APR for 60 months
Toyota Camry	\$22,955	0	4.8% APR for 60 months
Toyota Camry Hybrid	\$26,500	0	3% APR for 48 months
Toyota Corolla	\$16,500	\$900	4.25% for 60 months
Toyota Prius	\$19,950	\$1,000	44.3% for 60 months

2). You are considering a 30−year mortgage that charges 0.4% interest each month to pay off a \$250,000 mortgage.

a. Determine the monthly payment p that allows the loan to be paid off at 360 months.

b. Now assume that you have been paying the mortgage for 8 years and now have an opportunity to refinance the loan. You have a choice between a 20-year loan at 4% per year with interest charged monthly and a 15-year loan at 3.8% per year with interest charged monthly. Each of the loans charges a closing cost of \$2500. Determine the monthly payment p for both the 20-year loan and the 15-year loan. Do you think refinancing is the right thing to do? If so, do you prefer the 20-year or the 15-year option?

3). Consider the spreading of a highly communicable disease on an isolated island with population size N. A portion of the population travels abroad and returns to the island infected with the disease. Formulate a dynamical system to approximate the change in the number of people in the population who have the disease.

4). The data in the accompanying table show the speed n (in increments of 5mph) of an automobile and the associated distance a_n in feet required to stop it once the brakes are applied. For instance, $n = 6$ (representing $6 \times 5 = 30$ mph) requires a stopping distance of $a_6 = 47$ ft.

a. Calculate and plot the change Δa_n an versus n. Does the graph reasonably approximate a linear relationship?

b. Based on your conclusions in part (a), find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against n. Discuss the appropriateness of the model.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
a_n	3	6	11	21	32	47	65	87	112	140	171	204	241	282	325	376

Assignmet

Deadline: 27-October, 2022.

1. You plan to invest part of your paycheck to finance your children's education. You want to have enough in the account to draw \$1000 a month every month for 8 years beginning 20 years from now. The account pays 0.5% interest each month.
a. How much money will you need 20 years from now to accomplish the financial objective? Assume you stop investing when your first child begins college—a safe assumption.
b. How much must you deposit each month during the next 20 years?
2. Assume we are considering the survival of whales and that if the number of whales falls below a minimum survival level m, the species will become extinct. Assume also that the population is limited by the carrying capacity M of the environment. That is, if the whale population is above M, then it will experience a decline because the environment cannot sustain that large a population level. In the following model, a_n represents the whale population after n years. Build a numerical solution for $M = 5000, m = 100, k = 0.0001$, and $a_0 = 4000$.

$$a_{n+1} - a_n = k(M - a_n)(a_n - m)$$

Now experiment with different values for M, m, and k. Try several starting values for a_0. What does your model predict?
3. Mercury in Fish—Public officials are worried about the elevated levels of toxic mercury pollution in the reservoirs that provide the drinking water to your city. They have asked for your assistance in analyzing the severity of the problem. Scientists have known about the adverse affects of mercury on the health of humans for more than a century. The term mad as a hatter stems from the nineteenth-century use of mercuric nitrate in the making of felt hats. Human activities are responsible for most mercury emitted into the environment. For example, mercury, a by-product of coal, comes from the smokestack emissions of old, coal-fired power plants in the Midwest and South and is disseminated by acid rain. Its particles rise on the

smokestack plumes and hitch a ride on prevailing winds, which often blow northeast. After colliding with mountains, the particles drop to earth. Once in the ecosystem, microorganisms in the soil and reservoir sediment break down the mercury and produce a very toxic chemical known as methyl mercury.

Mercury undergoes a process known as bioaccumulation. This occurs when organisms take in contaminants more rapidly than their bodies can eliminate them. Therefore, the amount of mercury in their bodies accumulates over time. Humans can eliminate mercury from their system at a rate proportional to the amount remaining. Methyl mercury decays 50% every 65 to 75 days (known as the half-life of mercury) if no further mercury is ingested during that time.

Officials in your city have collected and tested 2425 samples of largemouth bass from the reservoirs and provided the following data. All fish were contaminated. The mean value of the methyl mercury in the fish samples was $0.43\mu g$ (microgram) per gram. The average weight of the fish was 0.817 kg.

a. Assume the average adult person (70 kg) eats one fish (0.817 kg) per day. Construct a difference equation to model the accumulation of methyl mercury in the average adult. Assume the half-life is approximately 70 days. Use your model to determine the maximum amount of methyl mercury that the average adult human will accumulate in her or his lifetime.
b. You find out that there is a lethal limit to the amount of mercury in the body; it is 50 mg/kg. What is the maximum number of fish per month that can be eaten without exceeding this lethal limit?
4. Complete the modules "The Growth of Partisan Support I: Model and Estimation" (UMAP 304) and "The Growth of Partisan Support II: Model Analytics" (UMAP 305), by Carol Weitzel Kohfeld. UMAP 304 presents a simple model of political mobilization, refined to include the interaction between supporters of a particular party and recruitable nonsupporters. UMAP 305 investigates the mathematical properties of the first-order quadratic-difference equation model. The model is tested using data from three U.S. counties.

1. You plan to invest part of your paycheck to finance your children's education. You want to have enough in the account to draw \$1000 a month every month for 8 years beginning 20 years from now. The account pays 0.5% interest each month.
a. How much money will you need 20 years from now to accomplish the financial objective? Assume you stop investing when your first child begins college—a safe assumption.
b. How much must you deposit each month during the next 20 years?

Answer:
Annuity at beginning of month required = 1000
No. of annuities or months= 8*12 96
Interest rate per month = 0.5% or 0.005
Present value of annuity due formula = Annuity + Annuity * (1- (1/(1+r) ^(n-1)))/r
=1000 + (1000*(1 - (1/ (1+0.005)) ^ (96-1))/0.005 =76475.69434
(a)We must have \$76475.69 at end of 20 years to withdraw \$1000 per month
(b) We have to invest for 20 years to accumulate \$76475.69 which is Future value of Annuity
no of months = 12*20=240
Future value of annuity formula =P *{(1+r) ^n - 1} / r
76475.69434= P*(((1+0.005) ^240)-1)/0.005
76475.69434= P*462.0408952
P=165.5171547
So We need to deposit \$165.52 every month to achieve the goal.

2. Assume we are considering the survival of whales and that if the number of whales falls below a minimum survival level m, the species will become extinct. Assume also that the population is limited by the carrying capacity M of the environment. That is, if the whale population is above M, then it will experience a decline because the environment cannot sustain that large a population level. In the following model, a_n represents the whale population after n years. Build a numerical solution for $M = 5000, m = 100, k = 0.0001$, and $a_0 = 4000$.

$$a_{n+1} - a_n = k(M - a_n)(a_n - m)$$

Now experiment with different values for M, m, and k. Try several starting values for a_0. What does your model predict?

Answer:
The variables, parameters & terms:

a_n = whale population after n years
k = growth rate
M = carrying capacity
m = minimum survival level

- If 0 < an <m, then k(Man) (an - m) < 0 and the population will decline.
- If m< an < M, then k(Man) (an - m) > 0 and the population will grow.
- If an > M, then k(Man) (an - m) < 0 and the population will decline.
- If an = m or an = M, then k(Man) (an - m) = 0 and the population will remain the same

(i.e., am and an = M are fixed points).

(b) To find fixed points, we set $a_{n+1} = a_n = a^*$:

$$\begin{aligned} a^* &= a^* + k(M - a^*)(a^* - m) \\ 0 &= k(M - a^*)(a^* - m) \\ a^* = M \quad &\text{OR} \quad a^* = m \end{aligned}$$

With $M = 5000$, $m = 100$, and $k = 0.0001$, we have fixed points $a^* = 5000$ and $a^* = 100$. To find the stability, let

$$\begin{aligned} f(a) &= a + 0.0001(5000 - a)(a - 100) \\ &= a + 0.0001(5100a - 500000 - a^2) \end{aligned}$$

Then,

$$f'(a) = 1 + 0.0001(5100 - 2a)$$

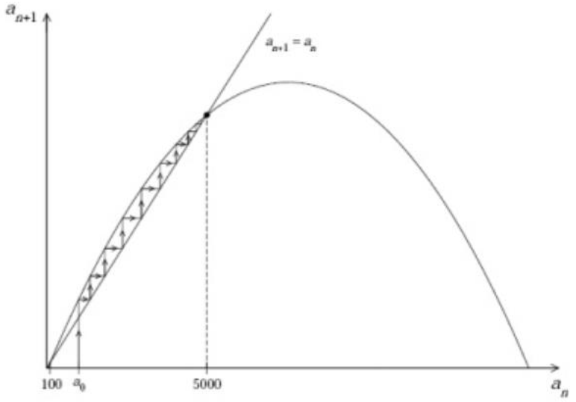
For $a^* = 100$:

$$\begin{aligned} |f'(100)| &= |1 + 0.0001(5100 - 200)| \\ &= 1.49 > 1 \end{aligned}$$

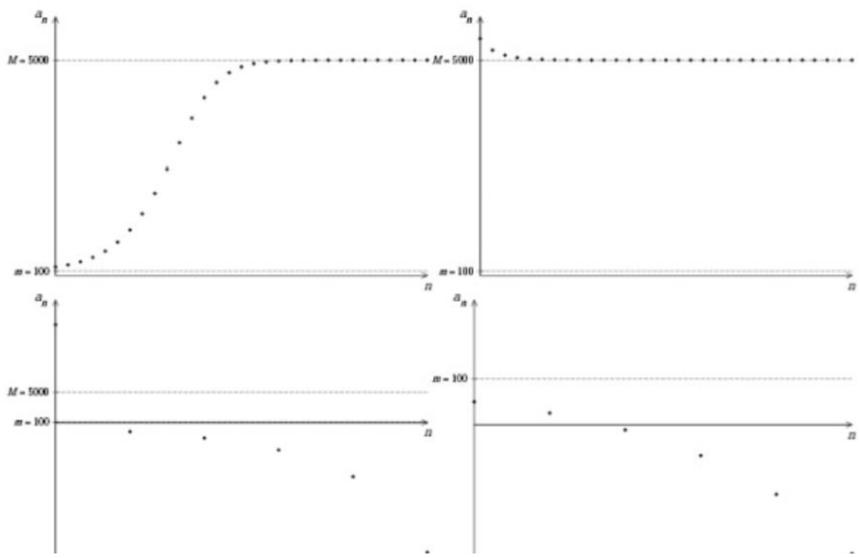
So, $a^* = 100$ is **unstable**.
For $a^* = 5000$:

$$\begin{aligned} |f'(100)| &= |1 + 0.0001(5100 - 10000)| \\ &= 0.51 < 1 \end{aligned}$$

(c) The following cobweb graphically says what we just found: $a^* = 100$ is unstable and $a^* = 5000$ is stable.



(d) The following represents solutions for different initial conditions:



When $a_0 < m$, the population declines and eventually becomes negative. When $a_0 >> M$, then $a_1 < 0$ and the population continues to decline. Both of these cases are problematic since we can't have a negative number of whales. I don't think the whales would appreciate that!!

2). You are considering a 30−year mortgage that charges 0.4% interest each month to pay off a $250, 000 mortgage.

a. Determine the monthly payment p that allows the loan to be paid off at 360 months.

b. Now assume that you have been paying the mortgage for 8 years and now have an opportunity to refinance the loan. You have a choice between a 20-year loan at 4% per year with interest charged monthly and a 15-year loan at 3.8% per year with interest charged monthly. Each of the loans charges a closing cost of $2500. Determine the monthly payment p for both the 20-year loan and the 15-year loan. Do you think refinancing is the right thing to do? If so, do you prefer the 20-year or the 15-year option?

(a)

Let S_n denote the Standing Capital at the end of n^{th} month. The change (decrement) in the amount owed at the end of each period (each month) increases by the amount of interest and decreases by the amount of monthly payment.

$\Delta S_n = S_{n+1} - S_n = rS_n - P$

Here,

Monthly rate of interest $r = 0.4\%$

Initial amount $S_0 = \$250,000$

P denotes the monthly payment that allows the loan to be paid off

Rearrange the equation:

$$\begin{aligned} S_{n+1} &= S_n + rS_n - P \\ &= (1+r)S_n - P \end{aligned}$$

Loan is to be paid off in 360 months

So the Standing capital at the end of 360^{th} month or S_{360} should be 0

To obtain a formula for S_n to equate S_{360} to 0

Put $n = 0,$ above expression gives

$$\begin{aligned} S_1 &= (1+r)S_0 - P \\ S_2 &= (1+r)S_1 - P \\ &= (1+r)\{(1+r)S_0 - P\} - P \\ &= (1+r)^2 S_0 - (1+r)P - P \end{aligned}$$

For $n = 3$ above expression gives

$$\begin{aligned} S_3 &= (1+r)S_2 - P \\ &= (1+r)\{(1+r)^2 S_0 - (1+r)P - P\} - P \\ &= (1+r)^3 S_0 - (1+r)^2 P - (1+r)P - P \end{aligned}$$

Similarly, for $n = 4$ expression gives

$$\begin{aligned} S_4 &= (1+r)S_3 - P \\ &= (1+r)\{(1+r)^3 S_0 - (1+r)^2 P - (1+r)P - P\} - P \\ &= (1+r)^4 S_0 - (1+r)^3 P - (1+r)^2 P - (1+r)P - PP \end{aligned}$$

Hence, write S_n as follows:

$$\begin{aligned} S_n &= (1+r)^n S_0 - P\{(1+r)^{n-1} + (1+r)^{n-2} + (1+r)^{n-3} + \ldots + 1\} \\ &= (1+r)^n S_0 - P\left\{\frac{1(1+r)^{n-1} - 1}{1+r-1}\right\} \\ &= (1+r)^n S_0 - P\left\{\frac{1(1+r)^{n-1} - 1}{r}\right\} \end{aligned}$$

Equate $S_{360}=0$ the following is obtained,

$$(1+r)^{360}S_0-P\left\{\frac{1(1+r)^{359}-1}{r}\right\}=0$$

Substitute the values of S_0 and *r*, above expression is equivalent to

$$(1+0.004)^{360}250000-P\left\{\frac{1(1+0.004)^{359}-1}{0.004}\right\}=0$$

Solve this expression for the value of *P*

$P=\$\boxed{1,318.5538}$