Homework 04 : Numerical Analysis.

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Given a table of value of x and y

x	3	-2	-3	0	4	2
y	238	13	88	1	697	?

1.) Construct the table of Newton Difference Divided :

Proof: • Sort arrange the given value in ascending order of x-values

x	-3	-2	0	2	3	4
y	88	13	1	?	238	697

• Compute the 1st order divided differences by substracting the y-values for adjacent x-values.

x	-3	-2	0	2	3	4
y	88	13	1	?	238	697

$\Delta 1$: -75 -12 ? 235 459

• Compute the 2nd order divided difference

$\Delta 1$: -75 -12 ? 235 459

$\Delta 2$: 63 -3 ? -224

• Do this process method until we reach and get all missing ? values in table

$\Delta 1$: -75 -12 ? 235 459

$\Delta 2$: 63 -3 ? -224

$\Delta 3$: 18 ? -274

$\Delta 4$: ? -256

$\Delta 5$: ? ?

Therefore, the table of Newton Divided Difference :

$\Delta 1$	-75	-12	?	235	459
$\Delta 2$	63	-3	?	-224	
$\Delta 3$	18	?	-274		
$\Delta 4$	?	-256			
$\Delta 5$	?	?			

2.) Write the Newton's Polynomial that approximates $f(x) = y$

Proof: we have

$$P_n(x) = y_0 + \Delta 1(x-x_0) + \Delta 2(x-x_0)(x-x_1)$$

$$+ \Delta 3(x-x_0)(x-x_1)(x-x_2) + - - - - - + \Delta n(x-x_0)(x-x_1)\dots(x-x_{n-1})$$

where $\Delta 1, \Delta 2, \Delta 3, \dots$ are the first; second; third ... and subsequence divided differences.

By using table above, we have the Newton Polynomial

$$P(x) = 88 - 75(x+3) + 63(x+3)(x+2) + 18(x+3)(x+2)(x-0)$$

$$\Leftrightarrow P(x) = 88 - 75x + 150x^2 - 50x^3 + 18x^2 - 36x^3 + 18x^4$$

Therefore, the Newton Polynomial that approximate $f(x) = y$ based on the given table above, we have

$$P(x) = 88 - 75x + 150x^2 - 50x^3 + 18x^2 - 36x^3 + 18x^4$$

$$= 88 - 75x + 168x^2 - 86x^3 + 18x^4$$

3.) Approximate $f(2)$ using the obtained Newton's Polynomial

we have $P(x) = 88 - 75x + 168x^2 - 86x^3 + 18x^4$

$\Rightarrow P(2) = 88 - 75\times 2 + 168\times 2^2 - 86\times 2^3 + 18\times 2^4 = 210$

$\boxed{\text{Therefore, the approximation of } f(2) \approx 210}$

Homework 05 : Numerical Analysis

We want to approximate a function f using natural cubic spline. The data point (x, y) is given by

x	-1	0	1	2
y	0.25	-1.25	-0.75	1.75

1.) Write the natural cubic spline $S(x)$

- Base on table above, we have the three intervals

  $[-1, 0], [0, 1], [1, 2]$

- Compute difference $h[i] = x[i+1] - x[i]$ for each interval

  - $h[0] = x[1] - x[0] = 0 - (-1) = 1$
  - $h[1] = x[2] - x[1] = 1 - 0 = 1$
  - $h[2] = x[3] - x[2] = 2 - 1 = 1$

- Coefficient of natural cubic spline that satisfying $Ac = B$ (c is coefficient)

  - tri-diagonal matrix A:

$$\begin{bmatrix} 2(h[0]+h[1]) & h[1] & 0 \\ h[1] & 2(h[1]+h[2]) & h[2] \\ 0 & h[2] & 2(h[2]+h[3]) \end{bmatrix}$$

  - vector B:

$$\begin{bmatrix} 3\left(\dfrac{y[2]-y[1]}{h[1]} - \dfrac{y[1]-y[0]}{h[0]}\right) \\ 3\left(\dfrac{y[3]-y[2]}{h[2]} - \dfrac{y[2]-y[1]}{h[1]}\right) \\ 3\left(\dfrac{y[4]-y[3]}{h[3]} - \dfrac{y[3]-y[2]}{h[2]}\right) \end{bmatrix}$$

from A, we have

$$= \begin{bmatrix} 5 \\ 3 \\ 3 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix} ; B = \begin{bmatrix} (\frac{3}{2} + \frac{3}{2})3 \\ (\frac{0.5}{2} - \frac{3}{2})3 \\ 3(\frac{7}{2} - \frac{5}{2}) \end{bmatrix}$$

⊛ Coefficients d, b for each interval

- $d[i] = \dfrac{c[i+1] - c[i]}{3h[i]}$
- $b[i] = \left(\dfrac{y[i+1] - y[i]}{h[i]} - \dfrac{h[i]}{3} \right)(2c[i] + c[i+1])$

⊛ Natural cubic splines:

- $S_1(x) = y[1] + b[0](x - x[1]) + c[0] \cdot (x - x[1])^2 + d[0](x - x[1])^3$

$\Rightarrow S_1(x) = -1.25 - 2.5(x+1) - 4.5(x+1)^2 - \frac{3}{2}(x+1)^3$

$= -1.25 - 2.5x - 2.5x^2 - 1.5x^3$

- $S_2(x) = y[2] + b[1](x - x[2]) + c[1](x - x[1])^2 + d[1](x - x[2])^3$

$= -0.75 + 0.75(x-1) - 2.25(x-1)^2 + 0.75(x-1)^3$

$= -0.75 + 0.75x - 0.75x^2 + 0.75x^3$

Therefore the natural cubic spline $S(x)$ is given by:

$$S(x) = \begin{cases} -1.25 - 2.5x - 2.5x^2 - 1.5x^3 ; & -1 \le x \le 0 \\ -0.75 + 0.75x - 0.75x^2 + 0.75x^3 ; & 0 \le x \le 1 \end{cases}$$

2.) Approximation $f(1.5)$, since 1.5 lie on the $[0,1]$

we have $S(1.5) = -0.75 + 0.75(1.5) - 0.75(1.5)^2 + 0.75(1.5)^3 = 0.953125$

$$\boxed{\text{Therefore, } f(1.5) \approx 0.953125}$$

3. Plot the natural cubic spline S(x).

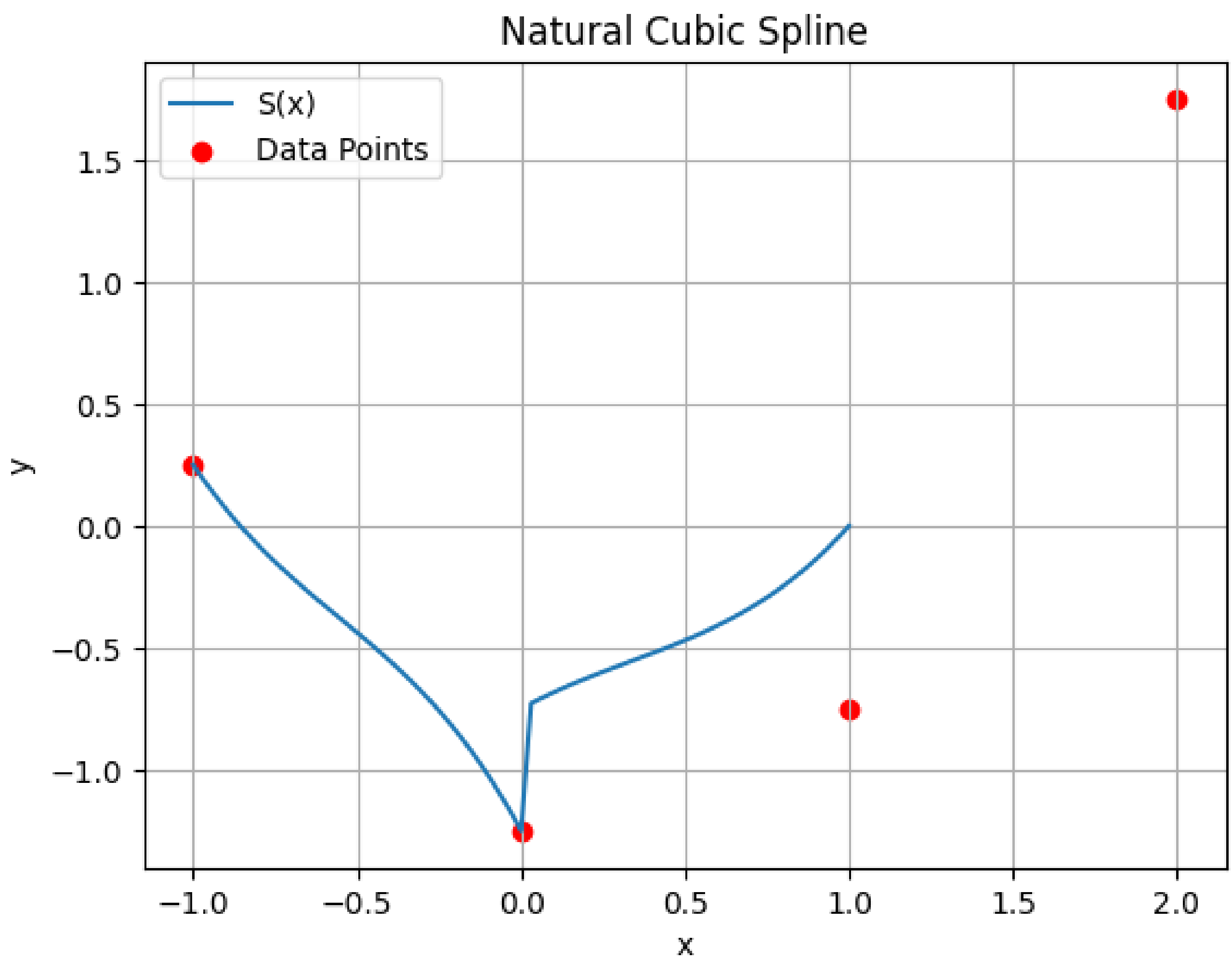
```
import numpy as np
import matplotlib.pyplot as plt

# Given data points
x = [-1.00, 0.00, 1.00, 2.00]
y = [0.25, -1.25, -0.75, 1.75]

# Define the natural cubic spline function S(x)
def S(x_val):
    if -1.00 <= x_val <= 0.00:
        return -1.25 - 2.50*x_val - 2.50*x_val**2 - 1.50*x_val**3
    elif 0.00 <= x_val <= 1.00:
        return -0.75 + 0.75*x_val - 0.75*x_val**2 + 0.75*x_val**3

# Generate points for plotting
x_plot = np.linspace(-1.00, 2.00, 100)
y_plot = [S(x_val) for x_val in x_plot]

# Plot the natural cubic spline
plt.plot(x_plot, y_plot, label='S(x)')
plt.scatter(x, y, color='red', label='Data Points')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Natural Cubic Spline')
plt.legend()
plt.grid(True)
plt.show()
```


Natural Cubic Spline
S(x)
Data Points
1.5
1.0
0.5
0.0
−0.5
−1.0
y
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x

Homework 06 : Numerical

Assume $A^* = A_0(h) + a_0h^2 + a_1h^4 + a_2h^6 + a_3h^8 + O(h^{10})$

. Complete the Richardson table and determine $A_3(h)$ provided that $A_0(h)$, $A_0(h/2)$, $A_0(h/4)$, $A_0(h/8)$ are the 1^{st} column.

$O(h^2)$	$O(h^4)$	$O(h^6)$	$O(h^8)$
0.7357558146			
0.7357586907	2.87612e-06		
0.7357588704	1.796597e-07	1.3971e-08	
0.735788816	1.11936e-08	8.796688e-10	7.54398e-11

Proof : We have Richardsons Extrapolation is given by

$$A^* = A_0(h) + a_0h^2 + a_1h^4 + a_2h^6 + a_3h^8 + O(h^{10})$$

. let denote values of $A_0(h)$, $A_0(h/2)$, $A_0(h/4)$, $A_0(h/8)$ as follow:

- $A_0(h) = 0.7357558146$
- $A_0(h/2) = 0.7357586907$
- $A_0(h/4) = 0.7357588704$
- $A_0(h/8) = 0.735788816$

We have

- $O(h^2) = A_0(h/2) - A_0(h) = 0.7357586507 - 0.7357558146$
  $= 2.8761e-06$
- $O(h^4) = A_0(h/4) - A_0(h/2) = 1.79657e-07$
- $O(h^6) = A_0(h/8) - A_0(h/4) = 1.11936e-08$
- $O(h^8) = A_3(h) - A_0(h/8) = A_3(h) - 0.7357588816$

Deduce from above computation, we have:

$A_3(h) - 0.7357588816 = O(h^8)$

$\Rightarrow A_3(h) = 0.7357588816 + O(h^8) = 0.7357588827$

Therefore, $A_3(h)$ is approximately 0.7357588827