

a.) Proof by using mathematical method

1. Let $f(x) = e^x - 2x - 2$.

(a) Show that $f(x) = 0$ has root in $[1, 2]$.

Proof: We have $f(x) = e^x - 2x - 2$

- $x = 1$: $f(1) = e - 2(1) - 2 = e - 4 < 0$
- $x = 2$: $f(2) = e^2 - 2(2) - 2 = e^2 - 6 > 0$

$\Rightarrow f(1) \times f(2) = (e - 4)(e^2 - 6) < 0$, then by using intermediate value theorem, we have the function $f(x) = 0$ has a root in $[1, 2]$

a.) Proof by using code implementation

In [24]:

```
1 import math as mt
2 def verify(a,b):
3     y1 = mt.exp(-a)-2*a-2
4     y2 = mt.exp(-b)-2*b-2
5     if(y1*y2<0):
6         print(f'f(x) = 0 has a root in the interval{a,b}')
7     else:
8         print(f'f(x) has no a root in the interval{a,b}')
9     return
10
11 if __name__ == "__main__":
12     verify(1,2)
```

f(x) has no a root in the interval(1, 2)

(b) Perform 10 iterations using Secant method.

In [25]:

```
1 from math import pi,cos,exp
2 def SecantMethod(f,p0,p1,TOL=1e-10,N=100):
3     print(f'i={0:2d}, x0={p0:0.16f}, f(x0)={f(p0):0.16f}')
4     for i in range(1,N,1):
5         p=p1-f(p1)*(p1-p0)/(f(p1)-f(p0))
6         print(f'i={i+1:2d}, x{i}={p:0.16f}, f(x{i})={f(p):0.16f}')
7         if(abs(p-p1)<TOL):
8             return p
9         p0 = p1
10        p1 = p
11    return None
12
13 if __name__ == "__main__":
14     def f(x): return exp(x)-2*x-2
15     p0 = 1
16     p1= 2
17     p=SecantMethod(f,p0,p1,TOL=1e-16)
18     print('The request root is : ',p)
```

```
i= 0, x0=1.0000000000000000, f(x0)=-1.2817181715409549
i= 2, x1=1.4799050918348966, f(x1)=-0.5672814093807768
i= 3, x2=1.6307176031026089, f(x2)=-0.1538966168898508
i= 4, x3=1.6868627146073407, f(x3)=0.0287794611761112
i= 5, x4=1.6780174072530005, f(x4)=-0.0011060175757591
i= 6, x5=1.6783447590574712, f(x5)=-0.0000074886339503
i= 7, x6=1.6783469906034445, f(x6)=0.0000000019696538
i= 8, x7=1.6783469900166597, f(x7)=-0.0000000000000036
i= 9, x8=1.6783469900166608, f(x8)=0.0000000000000004
i=10, x9=1.6783469900166605, f(x9)=0.0000000000000000
i=11, x10=1.6783469900166605, f(x10)=0.0000000000000000
The request root is :  1.6783469900166605
```

(c) Perform Secant iterations until $|f(x)| < 10^{-4}$.

In [26]:

```
1 from math import pi,cos,exp
2 import math as mt
3 def SecantMethod(f,p0,p1,TOL=1e-10,N=100):
4     print(f'i={0:2d}, x0={p0:0.16f}, f(x0)={f(p0):0.16f}')
5     for i in range(1,N,1):
6         p=p1-f(p1)*(p1-p0)/(f(p1)-f(p0))
7         if abs(f(p))<mt.pow(10,-4):
8             print(f'i={i+1:2d}, x{i}={p:0.16f}, f(x{i})={f(p):0.16f}')
9             if(abs(p-p1)<TOL):
10                 return p
11         p0 = p1
12         p1 = p
13     return None
14
15 if __name__ == "__main__":
16     def f(x): return exp(x)-2*x-2
17     p0 = 1
18     p1 = 2
19     p=SecantMethod(f,p0,p1,TOL=1e-16)
20     print('The request root is : ',p)
```

```
i= 0, x0=1.0000000000000000, f(x0)=-1.2817181715409549
i= 6, x5=1.6783447590574712, f(x5)=-0.0000074886339503
i= 7, x6=1.6783469906034445, f(x6)=0.0000000019696538
i= 8, x7=1.6783469900166597, f(x7)=-0.00000000000000036
i= 9, x8=1.6783469900166608, f(x8)=0.00000000000000004
i=10, x9=1.6783469900166605, f(x9)=0.0000000000000000
i=11, x10=1.6783469900166605, f(x10)=0.0000000000000000
The request root is : 1.6783469900166605
```

(d) Perform Secant iterations until $|x - x_1| < 10^{-4}$.

In [27]:

```
1 from math import pi,cos,exp
2 def SecantMethod(f,p0,p1,TOL=1e-10,N=100):
3     print(f'i={0:2d},p0={p0:0.16f},f(p0)={f(p0):0.16f}')
4     for i in range(1,N,1):
5         p=p1-f(p1)*(p1-p0)/(f(p1)-f(p0))
6         if abs(p-p0)<mt.pow(10,-4):
7             print(f'i={i+1:2d},p{i}={p:0.16f},f(p{i})={f(p):0.16f}')
8             if(abs(p-p1)<TOL):
9                 return p
10            p0 = p1
11            p1 = p
12    return None
13
14 if __name__ == "__main__":
15     def f(x): return exp(x)-2*x-2
16     p0 = 1
17     p1= 2
18     p=SecantMethod(f,p0,p1,TOL=1e-16)
19     print('The request root is : ',p)
```

```
i= 0,p0=1.0000000000000000,f(p0)=-1.2817181715409549
i= 8,p7=1.6783469900166597,f(p7)=-0.0000000000000036
i= 9,p8=1.6783469900166608,f(p8)=0.0000000000000004
i=10,p9=1.6783469900166605,f(p9)=0.0000000000000000
i=11,p10=1.6783469900166605,f(p10)=0.0000000000000000
The request root is :  1.6783469900166605
```