Project: Convex Optimization Promblem

I3_AMS_A

Optimization

2022-2023

3rd year Bachelor of Applied Mathematics and Statistics

Institute of Technology of Cambodia

Group_10 Members:

Nang Sreynich	ID: e20200447
Keo Vonmonyroth	ID: e20200759
Kry Senghort	ID: e20200706
Chea Rotha	ID: e20200702

Lecturers:

Mrs. TANN Chantara (Course)

Dr. MUTH Boravy (TD)

Contents

i. Introduction 1
ii. Convex Optimization Problem 1
i. Convex Function 1
ii. Methodology 3
a. Mathematical solution 3
b. Algorithm Coding (Excel, Python) 3
i. Using Microsoft Excel 3
ii. Using Python Programming 5
iii. Results 6

i. Introduction

In this project, we will focus on the *Convex Optimization Problem* which is considered in Chapter 4, and we will talk about the concern of the *Linear Program* with a few samples of exercises.

Linear programming a form of mathematical optimization that seeks to determine the best way of using limited resources to achieve a given objective and it is also a method of achieving the best outcome given a maximum or minimum equation with linear constraints. There are a few of essential conditions in a problem situation for linear programming.

- First, there must be limited resources (such as a limited number of workers, equipment, finances, and material); otherwise, there would be no problem.
- Second, there must be an explicit objective (such as maximize profit or minimize cost).

ii. Convex Optimization Problem

Convex Optimization is a subfield of mathematical optimization that studies the problem of minimizing convex functions over convex sets (or study about the equivalently, maximizing the concave functions over convex sets).

Standard Form

A convex optimization problem is in standard form if it is written as

minimize $f(x)$
subject to $g_i \leq 0, \; i = 1,2,3,\dots,m$
$h_i = 0, \quad i = 1,2,3,\dots,p$

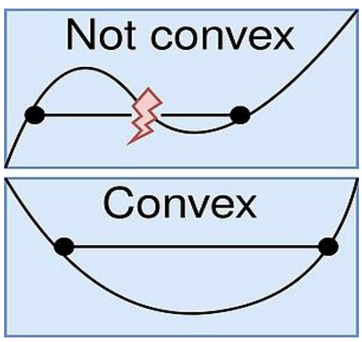
where

- $x \in \mathbb{R}^n$ is the optimization variable
- The objective function $f : D \subseteq \mathbb{R}^n$ is a convex function
- The inequality constraint functions $g_i : \mathbb{R}^n \rightarrow \mathbb{R}, \quad i = 1,2,\dots,m$ are convex functions
- The equality constraint functions $h_i : \mathbb{R}^n \rightarrow \mathbb{R}, \quad i = 1,2,\dots,p$ are affine transformation that is one of the forms of $h_i(x) = a_i . x - b_i$ where a_i is a vector and b_i is a scalar.

The solution to a convex optimization problem is any point $x \in C$ attaining $\mathbf{inf}\{\boldsymbol{f}(\boldsymbol{x}): \boldsymbol{x} \in \boldsymbol{C}\}$**.** In general, a convex optimization problem may have zero, one or many solutions.

i. Convex Function

A convex function is a continuous function whose value at the midpoint of every interval in its domain does not exceed the arithmetic mean of its values at the ends the interval.



$$f(tx_1 + (1+t)x_2) < tf(x_1) + (1-t)f(x_2)$$

$$[-\infty, \infty] = \mathbb{R} \cup \{\pm\infty\}$$

Where such a function takes 0 to 1 as a value. The map f is called strictly convex function if and only if for all 0<t<1 and all $x_1, x_2 \in X$ such that $x_1 \neq x_2$.

ii. **Problem (Linear Programming)**

Example 1: Puck and Pawn Company

We describe the steps involved in solving a simple linear programming model in the context of a sample problem, that of the Puck and Pawn Company, which manufactures hockey sticks and chess sets. Each hockey stick yields an incremental profit of $2, and each chess set $4. A hockey stick requires 4 hours of processing at Machine Center A and 2 hours at Machine Center B. A chess set requires 6 hours at Machine Center A, and 6 hours at Machine Center B, 1 hour at Machine Center C. Machine Center A has a maximum of 120 hours of available capacity per day, Machine Center B has 72 hours, and Machine Center C has 10 hours. If the company wishes to maximize profit, how many hockey sticks and chess sets should be produced per day?

Solution

Formulate the problem in mathematical terms.

If the company wishes to maximize profit, how many hockey sticks and chess sets should be produced per day?

Let H be the number of hockey sticks

C be the number of chesses sets

P be the profit

To maximize the profit the objective function may be stated as

Maximize P = 2$H + 4$C

The maximization will be subject to the following constraints as the following:

Constraint of Machine Center A $4H + 6C \leq 120$

Constraint of Machine Center B $2H + 6C \leq 72$

Constraint of Machine Center C $1C \leq 10$

Where $H, C > 0$

In order to formulate this problem, we have to satisfy the five requirements for standard Linear Programming as the following:

1) There are limited resources (a finite number of hours available at machine center A, B, C).
2) There is an objective function as we have mentioned above.
3) The equations are linear (no exponents or cross products).
4) The resources are homogenous (the machine has measure as hours).
5) The decision variables are divisible and nonnegative variables which are called as *slack variables*.

ii. Methodology

a. Mathematical solution

Maximize $P = 2\$H + 4\$C$

Constraint of Machine Center A $4H + 6C \leq 120$

Constraint of Machine Center B $2H + 6C \leq 72$

Constraint of Machine Center C $1C \leq 10$ where $H > 0, C > 0$

b. Algorithm Coding (Excel, Python)

i. Using Microsoft Excel

Example 1

	H	C		
Constraint 1	4	6	<=	120
Constraint 2	2	6	<=	72
Constraint 3	0	1	<=	10

	H	C
Decision Variables	24	4
Objective Value	64	
Constraint1	120	
Constraint2	72	
Constraint3	4	

Thus, we get the value of Decision Variables where H=24(number of hockey sticks) and C=4(number of chess sets) in order to get the maximize profit that should be produced per day.

The total maximize profit is $P = 2\$H + 4\$C = 48\$ + 16\$ = 64\$$

Example 2

1 Solve the following problem with Excel Solver:

$$\text{Maximize } Z = 3X + Y.$$
$$12X + 14Y \leq 85$$
$$3X + 2Y \leq 18$$
$$Y \leq 4$$

Solution

	X	Y		
Constraint 1	12	14	<=	85
Constraint 2	3	2	<=	18
Constraint 3	0	1	<=	4

	X	Y
Decision Variables	6	0
Objective Value	18	
Constraint 1	72	
Constraint 2	18	
Constraint 3	0	

Hence, we get the Decision Variables where X = 6 and Y = 0 in order to make the maximize the given objective.

2 Solve the following problem with Excel Solver:

$$\text{Minimize } Z = 2A + 4B.$$
$$4A + 6B \geq 120$$
$$2A + 6B \geq 72$$
$$B \geq 10$$

Solution

	A	B		
Constraint 1	-4	-6	<=	-120
Constraint 2	-2	-6	<=	-72
Constraint 3	0	-1	<=	-10

	A	B
Decision Variables	15	10
Objective Value	70	
Constraint 1	-120	
Constraint 2	-90	
Constraint 3	-10	

Hence, we get the Decision Variables where A = 15 and B = 10 in order to make the minimize the given objective.

ii. Using Python Programming

Example1

```
In [25]: from scipy.optimize import linprog
```

Maximize P = 2$H + 4$C

Constraint of Machine Center A 4H + 6C <=120 Constraint of Machine Center B 2H + 6C <=72 Constraint of Machine Center c 16C <=10

```
In [27]: #We need to convert it to Minimazation problem since Linprog() can't perform maximazation
#Min (-2,-4) obective function
objective_fun =[-2,-4]        #coeffcients of the objective funtion
coffient =[[4,6],[2,6],[0,1]] #coffcient of inequality constraint
value = [120,72,10]           ##coffcient of inequality constraint
boundary = [(0,float('inf')),(0,float('inf'))]
```

```
In [29]: optimazation = linprog(c=objective_fun,A_ub=coffient,b_ub=value, A_eq=None, b_eq=None ,bounds=boundary ,method='simplex')
optimazation

Out[29]:      con: array([], dtype=float64)
         fun: -64.0
     message: 'Optimization terminated successfully.'
         nit: 3
       slack: array([0., 0., 6.])
      status: 0
     success: True
           x: array([24.,  4.])
```

Hence, we get the value of H = 24(number of hockey sticks) and C = 4(number of chess sets) in order to make the maximize profit which is P = 4$H + 6$C = 48$ + 24$ = 64$.

Example 2

1 Solve the following problem with Excel Solver:

$$\text{Maximize } Z = 3X + Y.$$
$$12X + 14Y \leq 85$$
$$3X + 2Y \leq 18$$
$$Y \leq 4$$

Solution

```
Problem
                    Maximize Z = 3X + Y
                         12X + 14Y <=85
                         3X + 2Y <= 18
                         Y <= 4
```

```
In [30]: objective_fun1 =[-3,-1]        #coeffcients of the objective funtion
coffient1 =[[12,14],[3,2],[0,1]] #coffcient of inequality constraint
value1 = [85,18,4]           ##coffcient of inequality constraint
boundary1 = [(0,float('inf')),(0,float('inf'))]
```

```
In [31]: optimazation1 = linprog(c=objective_fun1, A_ub=coffient1 ,b_ub=value1, A_eq=None, b_eq=None ,bounds=boundary1 ,method='simplex')
optimazation1

Out[31]:      con: array([], dtype=float64)
         fun: -18.0
     message: 'Optimization terminated successfully.'
         nit: 4
       slack: array([13.,  0.,  4.])
      status: 0
     success: True
           x: array([6., 0.])
```

```
Thus, we obtain X=6 , Y= 0 and the objectiv value z = 18
```

Hence, we get the value of X = 6, Y = 0 in order to make the maximize objective value Z = 18.

2 Solve the following problem with Excel Solver:

$$\text{Minimize } Z = 2A + 4B.$$
$$4A + 6B \geq 120$$
$$2A + 6B \geq 72$$
$$B \geq 10$$

Solution

```
Problem2
                              Minimize Z = 2A + 4B
                                      4A + 6B >= 120
                                      2A + 6B >= 72
                                      B >= 4
        we convert it to
                              Minimize Z = 2A + 4B
                                      -4A - 6B <= -120
                                      -2A - 6B <= -72
                                      -B <= -10
```

```
In [35]: objective_fun2 =[2,4]        #coeffcients of the objective funtion
         coffient2 =[[-4,-6],[-2,-6],[0,-1]] #coffcient of inequality constraint
         value2 = [-120,-72,-10]          ##coffcient of inequality constraint
         boundary2 = [(0,float('inf')),(0,float('inf'))]

In [36]: optimazation2 = linprog(c=objective_fun2, A_ub=coffient2 ,b_ub=value2, A_eq=None, b_eq=None ,bounds=boundary2 ,method='simplex')
         optimazation2

Out[36]:      con: array([], dtype=float64)
              fun: 70.0
          message: 'Optimization terminated successfully.'
              nit: 3
            slack: array([ 0., 18.,  0.])
           status: 0
          success: True
                x: array([15., 10.])
```

Thus, optimal combination is B=10 , A15, Z=70

Hence, we get the value of A=15, B=10 in order to make the value of objective to be minimize.

iii. Results

Finally, we can solve the model used to achieve the best outcome given a maximum or minimum equation with linear constraints.

iv. Conclusion

To sum up, by using Microsoft Excel or Python Programming, we still get the same value of slack variables and the value of objective. From the examples above, worked throughout this document, it can be determined that the optimal objective value which was formulated above.

References

[1] Linear Programming by python
https://www.youtube.com/watch?v=yNyT9ZYYDdE&t=4s

Appendix – Excel and Python

[1] Reference for Project\example1.xlsx

[2] Reference for Project\example2.xlsx

[3]Reference for Project\optimazation.ipynb