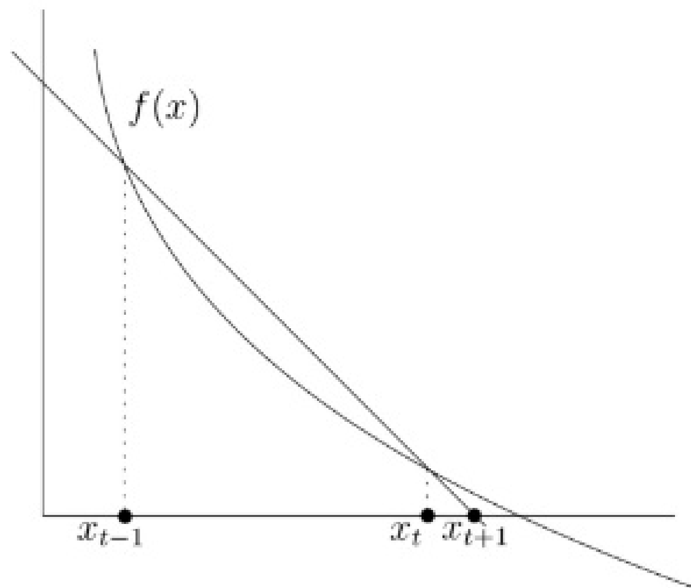
Exercise 2 (Geometric interpretation of the secant method)

Consider a step of the secant method

$$x_{t+1} = x_t - f(x_t)\frac{x_t - x_{t-1}}{f(x_t) - f(x_{t-1})}, \quad t \geq 1. \tag{2}$$

Assuming that $x_t \neq x_{t-1}$ and $f(x_t) \neq f(x_{t-1})$, prove that the line through the two points $(x_{t-1}, f(x_{t-1}))$ and $(x_t, f(x_t))$ intersects the x-axis at the point $x = x_{t+1}$



Exercise 3 (Affine invariance of the Newton method)

Consider $h(x) = g(Mx)$ where $M \in \mathbb{R}^{n\times n}$ is invertible and g is some convex function. Show that the Newton steps for h and g are also related by the same linear transformation, i.e., $M\Delta x_t = \Delta y_t$ where Δx_t and Δy_t are the Newton steps at the i^{th} iteration for h and g respectively. We assume $Mx_0 = y_0$ are the starting iterates for h and g respectively.