Quiz02: Eigendecomposition

1. Compute the determinant using the Laplace expansion

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 0 & 2 & 4 \end{bmatrix}$$

2. Compute the following determinant efficiently:

$$\begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 2 & -1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ -2 & 0 & 2 & -1 & 2 \\ 2 & 0 & 0 & 1 & 1 \end{bmatrix}$$

3. Find the eigenvector and eigenvalue of the following matrices:

$$X = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

4. Compute the eigendecomposition of

$$A = \frac{1}{2}\begin{bmatrix} 5 & -2 \\ -2 & 5 \end{bmatrix}$$

5. (a) Use PyTorch to confirm $Xv = \lambda v$ for the first eigenvector of X, where

$$X = \begin{bmatrix} 0 & -1 & 1 & 1 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

   (b) Confirm $Xv = \lambda v$ for the remaining eigenvector of X (You can use Numpy or PyTorch).

6. Use PyTorch to decompose the matrix P below into its component V, and V^{-1}. Then confirm that $P = V\Lambda V^{-1}$, where

$$P = \begin{bmatrix} 25 & 2 & -5 \\ 3 & -2 & 1 \\ 5 & 7 & 4 \end{bmatrix}$$

7. Use PyTorch to decompose the symmetric matrix Q below into its component Q, Λ and Q^T. Then confirm that $S = Q\Lambda Q^T$, where

$$S = \begin{bmatrix} 25 & 2 & -5 \\ 3 & -2 & 1 \\ 5 & 7 & 4 \end{bmatrix}$$