

```
>>> Initialize matrix, transpose, determinant size  
>>> inverse power add minus multiply rank multiply
```

```
In [1]: import numpy as np
```

```
In [2]: a1= np.matrix([[['a','b','c'],['d','e','f'],['g','h','i']])  
print('Initialize matrix :')  
print(a1)
```

```
Initialize matrix :  
[['a' 'b' 'c']  
 ['d' 'e' 'f']  
 ['g' 'h' 'i']]
```

```
In [3]: print('After transpose on matrix : ')  
print(a1.transpose())
```

```
After transpose on matrix :  
[['a' 'd' 'g']  
 ['b' 'e' 'h']  
 ['c' 'f' 'i']]
```

```
In [4]: print('#dimension of a1 : ',a1.ndim)  
print('#shape of a1 : ',a1.shape)  
print('#datatype of a1 : ',a1.dtype)
```



```
#dimension of a1 : 2  
#shape of a1 : (3, 3)  
#datatype of a1 : <U1
```

```
In [5]: print('size of element : ',a1.itemsize,'bits')  
print('total size of a1 : ',a1.nbytes,'bits')
```



```
size of element : 4 bits  
total size of a1 : 36 bits
```

```
In [6]: a2 = np.matrix([[['d','a','e'],['a','t','w'],['s','e','y']])  
print(a2)
```



```
[[['d' 'a' 'e']  
 ['a' 't' 'w']  
 ['s' 'e' 'y']]
```

```
In [7]: print('after transpose \n',a2.transpose())
```



```
after transpose  
[[['d' 'a' 's']  
 ['a' 't' 'e']  
 ['e' 'w' 'y']]
```

```
In [8]: print('after transpose\n',a2.T)
```



```
after transpose  
[[['d' 'a' 's']  
 ['a' 't' 'e']  
 ['e' 'w' 'y']]
```

```
In [9]: print('#dimension of a2 : ',a2.ndim)  
print('#shape of a2 : ',a2.shape)  
print('#size of element : ',a2.itemsize)  
print('#total size of a2 : ',a2.nbytes)
```

```
#dimension of a2 : 2
#shape of a2 : (3, 3)
#size of element : 4
#total size of a2 : 36
```

```
In [10]: a = np.array([[1,2],[4,6]])
b = a.transpose()
print('after transpose\n',b)
print("determinant(a)",np.linalg.det(a))

after transpose
[[1 4]
 [2 6]]
determinant(a) -2.0
```

```
In [11]: c = np.array([[1,1,3],[-2,2,1],[4,7,4]])
print('transpose of c\n',c.T)
print('det(c) : ',np.linalg.det(c))

transpose of c
[[ 1 -2  4]
 [ 1  2  7]
 [ 3  1  4]]
det(c) : -53.00000000000001
```

Inverse of matrix

```
In [12]: M = np.array([[1,1,1],[0,2,5],[2,5,-1]])
print(a)

[[1 2]
 [4 6]]
```

```
In [13]: # Display the inverse of matrix M
print(np.linalg.inv(M))

[[ 1.28571429 -0.28571429 -0.14285714]
 [-0.47619048  0.14285714  0.23809524]
 [ 0.19047619  0.14285714 -0.0952381 ]]
```

```
In [14]: A = np.array([[6, 1, 1, 3],
                  [4, -2, 5, 1],
                  [2, 8, 7, 6],
                  [3, 1, 9, 7]])
print(np.linalg.inv(A))

[[ 0.13368984  0.10695187  0.02139037 -0.09090909]
 [-0.00229183  0.02673797  0.14820474 -0.12987013]
 [-0.12987013  0.18181818  0.06493506 -0.02597403]
 [ 0.11000764 -0.28342246 -0.11382735  0.23376623]]
```

```
In [15]: N = np.array([[6, 1, 1],
                  [4, -2, 5],
                  [2, 8, 7]])
print(np.linalg.inv(N))

[[ 0.17647059 -0.00326797 -0.02287582]
 [ 0.05882353 -0.13071895  0.08496732]
 [-0.11764706  0.1503268   0.05228758]]
```

```
In [16]: O = np.array([[[1,2],[3,4]],
                  [[1,3],[3,5]]])
print(np.linalg.inv(O))
```

```
[[[-2. 1.  
 [ 1.5 -0.5 ]]]
```

```
[[-1.25 0.75]  
 [ 0.75 -0.25]]]
```

Rank and Nility of Matrix

In [181...]

```
print(a2)
```

```
[[1]  
 [2]  
 [3]  
 [4]  
 [5]  
 [6]  
 [7]  
 [8]]
```

In [182...]

```
# Display the rank of matrix a  
print(np.linalg.matrix_rank(a1))
```

```
2
```

In [183...]

```
# Display the nility of matrix a  
nility = a1.shape[1] - np.linalg.matrix_rank(a1)  
print(nility)
```

```
2
```

In [184...]

```
print(A)  
print(np.linalg.matrix_rank(A)) # is the rank of matrix A
```

```
[[ 1. 1.75 0.75 2.  
 [ 0. 1. -0.18181818 0.81818182]  
 [ 0. 0. 1. -1.22222222]]
```

```
3
```

In [185...]

```
nility2 = A.shape[1]-np.linalg.matrix_rank(A)  
print(nility2)
```

```
1
```

Eigenvalue of matrix

In [197...]

```
print(f)
```

```
[[1. 1. 1.]  
 [1. 1. 1.]]
```

In [203...]

```
X = np.matrix([[2,1,1],[1,2,1],[1,1,2]])
```

```
print(X)
```

```
u,v=np.linalg.eig(X)
```

```
print("Eigenvalue : ",u)  
print("Eigenvector : ",v)
```

```
[[2 1 1]  
 [1 2 1]  
 [1 1 2]]
```

```
Eigenvalue : [1. 4. 1.]
```

```
Eigenvector : [[-0.81649658 0.57735027 -0.23513651]  
 [ 0.40824829 0.57735027 -0.55958248]  
 [ 0.40824829 0.57735027 0.79471899]]
```

In [204...]

```
print(c)
```

```
w,v = np.linalg.eig(c)
```

```
print("Eigenvalue : \n",w)
print("Eigenvector : \n",v)
```

```

[[ 1  2  3  4  5  6  7  8  9 10  1  2  3  4  5]
 [ 4  1  5  5  6  0 -4 -2 -1  6  6  7  8  9  0]
 [ 1  8  0  7  8  9 -2 -1  3 -4 -8 -1  2  3  4]
 [ 2  4  5  6  4  3  8  9  0  7  2  3  4  5  6]
 [ 3  8  9  0  4  3  5  6  7  8  0  0  1  2  3]
 [ 1  2  3  4  5  6  7  2  3  4  5  6  8  9  0]
 [ 4  6  3  2  1 -9 -7  0  6  4  0  0  1 -2  3]
 [ 1  4  5  6  7  5  4  3  2  1  1  2  0  0  4]
 [ 1  2  3  4  5  6  7  8  9  0  9  9  5  4  8]
 [ 2  3  4  5  6  7  8  9  0  1  2  4  3  4  5]
 [ 0  0  7  0  5  4  3  9  0  1  0  0  8 -9 -2]
 [ 1  2  3  4  5  6  7  8  9  0  0  9  8  4  3]
 [ 1  2  3  4  5  6  7  8  9  0  9  7  5  4  3]
 [ 1  2  3  4  6 -9 -8 -6 -3 -1  2  3  5  6  7]
 [ 0  0  2  1  3  4  5  6  7  8  6  5  4  9  6]]

```

Eigenvalue :

```

[53.04437703+0.j      -9.89512937+0.j      10.29240605+1.8167192j
10.29240605-1.8167192j -5.22605172+6.44401115j -5.22605172-6.44401115j
-2.66438178+7.5917991j -2.66438178-7.5917991j -6.32688638+0.j
-1.93825917+4.67573756j -1.93825917-4.67573756j 2.45398749+1.69756522j
2.45398749-1.69756522j 3.67111849+1.48972388j 3.67111849-1.48972388j]

```

Eigenvector :

```

[[-0.32460509+0.j      0.21248937+0.j      -0.16223337+0.06765768j
-0.16223337-0.06765768j 0.26464457+0.13444776j 0.26464457-0.13444776j
-0.13366177-0.10350316j -0.13366177+0.10350316j -0.43277137+0.j
-0.19728067-0.15167203j -0.19728067+0.15167203j 0.28689463-0.32845063j
0.28689463+0.32845063j -0.23913006+0.25509201j -0.23913006-0.25509201j]
[-0.23050977+0.j      0.08346224+0.j      -0.04524259-0.04204774j
-0.04524259+0.04204774j -0.12989129-0.10688471j -0.12989129+0.10688471j
0.2972707 -0.07726197j 0.2972707 +0.07726197j -0.07670352+0.j
0.30757176+0.03654268j 0.30757176-0.03654268j -0.21505663+0.21696503j
-0.21505663-0.21696503j 0.13432175-0.18397474j 0.13432175+0.18397474j]
[-0.16810833+0.j      -0.54763667+0.j      0.06239871-0.14396095j
0.06239871+0.14396095j 0.61293052+0.j      0.61293052-0.j
-0.60446305+0.j      -0.60446305-0.j      -0.53547138+0.j
-0.58650064+0.j      -0.58650064-0.j      -0.03390758-0.08377834j
-0.03390758+0.08377834j 0.11106422+0.01440615j 0.11106422-0.01440615j]
[-0.28591382+0.j      0.11776843+0.j      -0.47253206+0.02911263j
-0.47253206-0.02911263j -0.07564522+0.05369239j -0.07564522-0.05369239j
0.0497595 -0.01097262j 0.0497595 +0.01097262j 0.24211259+0.j
-0.02316981-0.05869396j -0.02316981+0.05869396j 0.17929072+0.02010376j
0.17929072-0.02010376j -0.1167988 +0.01438655j -0.1167988 -0.01438655j]
[-0.26342002+0.j      0.48859899+0.j      -0.14684707-0.07275442j
-0.14684707+0.07275442j -0.19027972-0.20206811j -0.19027972+0.20206811j
-0.03067895+0.26497315j -0.03067895-0.26497315j 0.49839633+0.j
0.1132804 +0.26317243j 0.1132804 -0.26317243j -0.15915444+0.03410711j
-0.15915444-0.03410711j 0.04311578-0.05663244j 0.04311578+0.05663244j]
[-0.2755989 +0.j      0.07375507+0.j      0.09442989+0.00882312j
0.09442989-0.00882312j -0.02624721+0.22966973j -0.02624721-0.22966973j
0.04806179-0.13788186j 0.04806179+0.13788186j -0.03308407+0.j
-0.04773615-0.13224271j -0.04773615+0.13224271j 0.04664152+0.01340457j
0.04664152-0.01340457j -0.05169044-0.03058322j -0.05169044+0.03058322j]
[-0.09780142+0.j      0.14126987+0.j      -0.06725798+0.03737317j
-0.06725798-0.03737317j 0.01319449-0.22363154j 0.01319449+0.22363154j
-0.12264962-0.04567907j -0.12264962+0.04567907j -0.09685155+0.j
-0.0533944 -0.09738391j -0.0533944 +0.09738391j 0.0880277 -0.10950279j
0.0880277 +0.10950279j -0.0618458 +0.10207882j -0.0618458 -0.10207882j]
[-0.20726538+0.j      -0.29686603+0.j      -0.26129387-0.01579785j
-0.26129387+0.01579785j -0.12283175-0.05147774j -0.12283175+0.05147774j
0.10479717+0.18375755j 0.10479717-0.18375755j -0.13387315+0.j
0.16186315+0.23490967j 0.16186315-0.23490967j -0.09505445+0.11546107j
-0.09505445-0.11546107j 0.04835331-0.09095046j 0.04835331+0.09095046j]
[-0.36845366+0.j      -0.18627168+0.j      0.34195602+0.10116346j
0.34195602-0.10116346j -0.08556937+0.09463153j -0.08556937-0.09463153j
0.04112689-0.14808376j 0.04112689+0.14808376j 0.1393572 +0.j
-0.00796684-0.08694819j -0.00796684+0.08694819j 0.14697159-0.10327117j]
```

```

0.14697159+0.10327117j -0.09581584+0.10455756j -0.09581584-0.10455756j]
[-0.26705788+0.j -0.15183355+0.j -0.26472906-0.00210307j
-0.26472906+0.00210307j -0.09357968+0.06703549j -0.09357968-0.06703549j
0.1117755 +0.05415384j 0.1117755 -0.05415384j 0.31906064+0.j
0.11454829+0.02059329j 0.11454829-0.02059329j -0.07549083+0.06782478j
-0.07549083-0.06782478j 0.03693463-0.04226093j 0.03693463+0.04226093j]
[-0.14135194+0.j 0.12368902+0.j -0.22594995+0.1189259j
-0.22594995-0.1189259j -0.01256853-0.29369295j -0.01256853+0.29369295j
0.20969296+0.30415526j 0.20969296-0.30415526j -0.04132455+0.j
0.15959906+0.18216499j 0.15959906-0.18216499j 0.19830994+0.15995468j
0.19830994-0.15995468j -0.33815007-0.14351082j -0.33815007+0.14351082j]
[-0.33623656+0.j 0.08277256+0.j 0.49568968+0.j
0.49568968-0.j 0.18042828-0.02057634j 0.18042828+0.02057634j
-0.14642303+0.11014771j -0.14642303-0.11014771j -0.01071224+0.j
-0.14243623+0.12507215j -0.14243623-0.12507215j -0.60732865+0.j
-0.60732865-0.j 0.6986346 +0.j 0.6986346 -0.j ]
[-0.3289385 +0.j -0.01865242+0.j 0.17327572+0.12458759j
0.17327572-0.12458759j -0.12314057+0.24184066j -0.12314057-0.24184066j
0.15626728-0.22149481j 0.15626728+0.22149481j 0.08498075+0.j
0.16304712-0.2650756j 0.16304712+0.2650756j 0.27351275-0.01021154j
0.27351275+0.01021154j -0.27583088+0.02399704j -0.27583088-0.02399704j]
[-0.06513927+0.j -0.33530976+0.j 0.09722622-0.13459528j
0.09722622+0.13459528j -0.11392105-0.12211019j -0.11392105+0.12211019j
0.06617971+0.03408429j 0.06617971-0.03408429j -0.22417475+0.j
0.0259086 +0.07486243j 0.0259086 -0.07486243j 0.0038542 -0.02415779j
0.0038542 +0.02415779j 0.02626271+0.01530814j 0.02626271-0.01530814j]
[-0.28471683+0.j 0.29861381+0.j 0.15746036+0.01307082j
0.15746036-0.01307082j 0.07828234+0.21051688j 0.07828234-0.21051688j
0.00846041-0.25800265j 0.00846041+0.25800265j -0.06452182+0.j
-0.04330704-0.27900356j -0.04330704+0.27900356j 0.21242034-0.08863581j
0.21242034+0.08863581j -0.17128512+0.11278398j -0.17128512-0.11278398j]]

```

In [190...]

```

m = np.array([[1, 2, 3],
              [2, 3, 4],
              [4, 5, 6]])
print("Printing the Original square array:\n",m)

# find eigenvalues and eigenvectors
w, v = np.linalg.eig(m)
print("Printing the Eigen values of the given square array:\n",w)
print("Printing Right eigenvectors of the given square array:\n",v)

```

Printing the Original square array:

```

[[1 2 3]
 [2 3 4]
 [4 5 6]]

```

Printing the Eigen values of the given square array:

```
[ 1.08309519e+01 -8.30951895e-01 -1.09042184e-16]
```

Printing Right eigenvectors of the given square array:

```

[[ 0.34416959  0.72770285  0.40824829]
 [ 0.49532111  0.27580256 -0.81649658]
 [ 0.79762415 -0.62799801  0.40824829]]

```

~~Elementary row operation<row echelon form>~~

In [191...]

```

def fixRowTwo(A) :

    # Sets the sub-diagonal elements of row two to zero
    A[2] = A[2] - A[2,0] * A[1]
    A[2] = A[2] - A[2,1] * A[1]

    # Test if diagonal element is not zero.
    if A[2,2] == 0 :
        # Add a lower row to row two.
        A[2] = A[2] + A[3]

```

```

# Sets the sub-diagonal elements to zero again ???
A[2] = A[2] - A[2,0] * A[1]
A[2] = A[2] - A[2,1] * A[1]

if A[2,2] == 0 :
    print("S I N G U L A R")
    sys.Exit()

# Set the diagonal element to one
A[2] = A[2] / A[2,2]

return A

def fixRowThree(A) :

    # Sets the sub-diagonal elements of row two to zero
    A[3] = A[3] - A[3,0] * A[2]
    A[3] = A[3] - A[3,1] * A[2]
    A[3] = A[3] - A[3,2] * A[2]

    # Test if diagonal element is not zero.
    if A[3,3] == 0:
        print("S I N G U L A R")
        sys.Exit()

    # Set the diagonal element to one
    A[3] = A[3] / A[3,3]

    return A

A = np.array([
    [1, 7, 4, 3],
    [0, 1, 2, 3],
    [3, 2, 0, 3],
    [1, 3, 1, 3]
], dtype=np.float_)

fixRowTwo(A)
print("")
print("Row Two:")
print(A)

fixRowThree(A)
print("")
print("Row Three:")
print(A)

```

Row Two:

$$\begin{bmatrix} 1 & 7 & 4 & 3 \\ 0 & 1 & 2 & 3 \\ -0.75 & -0 & 1 & 0.75 \\ 1 & 3 & 1 & 3 \end{bmatrix}$$

Row Three:

$$\begin{bmatrix} 1 & 7 & 4 & 3 \\ 0 & 1 & 2 & 3 \\ -0.75 & -0 & 1 & 0.75 \\ 0.77777778 & 1.33333333 & 0 & 1 \end{bmatrix}$$

In [192...]

```

def row_echelon(A):
    """ Return Row Echelon Form of matrix A """

    # if matrix A has no columns or rows,
    # it is already in REF, so we return itself
    r, c = A.shape

```

```

if r == 0 or c == 0:
    return A

# we search for non-zero element in the first column
for i in range(len(A)):
    if A[i,0] != 0:
        break
else:
    # if all elements in the first column is zero,
    # we perform REF on matrix from second column
    B = row_echelon(A[:,1:])
    # and then add the first zero-column back
    return np.hstack([A[:,1:], B])

# if non-zero element happens not in the first row,
# we switch rows
if i > 0:
    ith_row = A[i].copy()
    A[i] = A[0]
    A[0] = ith_row

# we divide first row by first element in it
A[0] = A[0] / A[0,0]
# we subtract all subsequent rows with first row (it has 1 now as first element)
# multiplied by the corresponding element in the first column
A[1:] -= A[0] * A[1:,0:1]

# we perform REF on matrix from second row, from second column
B = row_echelon(A[1:,1:])

# we add first row and first (zero) column, and return
return np.vstack([A[:1], np.hstack([A[1:,:1], B]) ])

```

```
A = np.array([[4, 7, 3, 8],
             [8, 3, 8, 7],
             [2, 9, 5, 3]], dtype='float')
```

```
print(row_echelon(A))
```

```
[[ 1.          1.75         0.75         2.          ]
 [ 0.          1.          -0.18181818  0.81818182]
 [ 0.          0.          1.          -1.22222222]]
```

In []:

Summation, Multiplication, Subtraction, Power

```
In [23]: d = np.array([[5,4,6],[8,9,0],[3,4,5]])
print(c, '\n+\n',d, '\n=\n')
print(c+d)
```

```
[[ 1  1  3]
 [-2  2  1]
 [ 4  7  4]]
+
[[5 4 6]
 [8 9 0]
 [3 4 5]]
=
[[ 6  5  9]
 [ 6 11  1]
 [ 7 11  9]]
```

```
In [24]: print(c-d)
```

```
[[ -4 -3 -3]
 [-10 -7 1]
 [ 1 3 -1]]
```

```
In [25]: print('c^T-d^T = ')
print(c.T-d.T)
```

```
c^T-d^T =
[[ -4 -10 1]
 [-3 -7 3]
 [-3 1 -1]]
```

```
In [26]: print('det(c-d) = ')
print(np.linalg.det(c-d))
```

```
det(c-d) =
79.99999999999997
```

```
In [27]: print('det(c^T-d^T) = ')
print(np.linalg.det(c.T-d.T))
```

```
det(c^T-d^T) =
80.00000000000004
```

```
In [28]: print('(c-d)^T = ')
print((c-d).T)
```

```
(c-d)^T =
[[ -4 -10 1]
 [-3 -7 3]
 [-3 1 -1]]
```

```
In [29]: print('c*d = \n',c*d)
```

```
c*d =
[[ 5 4 18]
 [-16 18 0]
 [ 12 28 20]]
```

```
In [30]: print(c.T*d.T)
print(np.linalg.det(c.T*d.T))
```

```
[[ 5 -16 12]
 [ 4 18 28]
 [ 18 0 20]]
-8871.99999999998
```

```
In [31]: print('c^2 = \n',c**2)
```

```
c^2 =
[[ 1 1 9]
 [ 4 4 1]
 [16 49 16]]
```

```
In [32]: print('c^3 = \n',c**3)
```

```
c^3 =
[[ 1 1 27]
 [-8 8 1]
 [ 64 343 64]]
```

```
In [33]: print('Initialize matrix c \n',c)
for i in range(5):
    print("=> c^",i,"\\n",c**i,"\\n")
```

```
Initialize matrix c
[[ 1  1  3]
 [-2  2  1]
 [ 4  7  4]]
=> c^ 0
[[ 1  1  3]
 [-2  2  1]
 [ 4  7  4]]

=> c^ 1
[[ 0  0  2]
 [-1  3  0]
 [ 5  6  5]]

=> c^ 2
[[ 3  3  1]
 [-4  0  3]
 [ 6  5  6]]

=> c^ 3
[[ 2  2  0]
 [-3  1  2]
 [ 7  4  7]]

=> c^ 4
[[ 5  5  7]
 [-6  6  5]
 [ 0  3  0]]
```

```
In [34]: print('Initialize matrix c \n',c)
for i in range(11):
    print("=> pow(c,"+str(i)+")\n",c**i,"\\n")
```

```

Initialize matrix c
[[ 1  1  3]
 [-2  2  1]
 [ 4  7  4]]
=> pow(c,0)
[[1 1 1]
 [1 1 1]
 [1 1 1]]

=> pow(c,1)
[[ 1  1  3]
 [-2  2  1]
 [ 4  7  4]]

=> pow(c,2)
[[ 1  1  9]
 [ 4  4  1]
 [16 49 16]]

=> pow(c,3)
[[ 1  1  27]
 [-8  8  1]
 [64 343 64]]

=> pow(c,4)
[[ 1  1  81]
 [16 16  1]
 [256 2401 256]]

=> pow(c,5)
[[ 1  1  243]
 [-32 32  1]
 [1024 16807 1024]]

=> pow(c,6)
[[ 1  1  729]
 [64 64  1]
 [4096 117649 4096]]

=> pow(c,7)
[[ 1  1  2187]
 [-128 128  1]
 [16384 823543 16384]]

=> pow(c,8)
[[ 1  1  6561]
 [256 256  1]
 [65536 5764801 65536]]

=> pow(c,9)
[[ 1  1  19683]
 [-512 512  1]
 [262144 40353607 262144]]

=> pow(c,10)
[[ 1  1  59049]
 [1024 1024  1]
 [1048576 282475249 1048576]]

```

```

In [35]: print('initialize matrix d')
print(d)
for i in range(31):
    print('pow(d,'+str(i)+')\n',d**i)

```

```

initialize matrix d
[[5 4 6]
 [8 9 0]
 [3 4 5]]
pow(d,0)
 [[1 1 1]
 [1 1 1]
 [1 1 1]]
pow(d,1)
 [[5 4 6]
 [8 9 0]
 [3 4 5]]
pow(d,2)
 [[25 16 36]
 [64 81 0]
 [ 9 16 25]]
pow(d,3)
 [[125 64 216]
 [512 729 0]
 [ 27 64 125]]
pow(d,4)
 [[ 625 256 1296]
 [4096 6561 0]
 [ 81 256 625]]
pow(d,5)
 [[ 3125 1024 7776]
 [32768 59049 0]
 [ 243 1024 3125]]
pow(d,6)
 [[ 15625 4096 46656]
 [262144 531441 0]
 [ 729 4096 15625]]
pow(d,7)
 [[ 78125 16384 279936]
 [2097152 4782969 0]
 [ 2187 16384 78125]]
pow(d,8)
 [[ 390625 65536 1679616]
 [16777216 43046721 0]
 [ 6561 65536 390625]]
pow(d,9)
 [[ 1953125 262144 10077696]
 [134217728 387420489 0]
 [ 19683 262144 1953125]]
pow(d,10)
 [[ 9765625 1048576 60466176]
 [1073741824 -808182895 0]
 [ 59049 1048576 9765625]]
pow(d,11)
 [[ 48828125 4194304 362797056]
 [ 0 1316288537 0]
 [ 177147 4194304 48828125]]
pow(d,12)
 [[ 244140625 16777216 -2118184960]
 [ 0 -1038305055 0]
 [ 531441 16777216 244140625]]
pow(d,13)
 [[1220703125 67108864 175792128]
 [ 0 -754810903 0]
 [ 1594323 67108864 1220703125]]
pow(d,14)
 [[1808548329 268435456 1054752768]
 [ 0 1796636465 0]
 [ 4782969 268435456 1808548329]]
pow(d,15)
 [[ 452807053 1073741824 2033549312]

```

```

[      0 -1010140999      0]
[ 14348907 1073741824 452807053]]
pow(d,16)
[[[-2030932031      0 -683606016]
[      0 -501334399      0]
[ 43046721      0 -2030932031]]]
pow(d,17)
[[[-1564725563      0 193331200]
[      0 -217042295      0]
[ 129140163      0 -1564725563]]]
pow(d,18)
[[ 766306777      0 1159987200]
[      0 -1953380655      0]
[ 387420489      0 766306777]]
pow(d,19)
[[ -463433411      0 -1630011392]
[      0 -400556711      0]
[ 1162261467      0 -463433411]]
pow(d,20)
[[ 1977800241      0 -1190133760]
[      0 689956897      0]
[ -808182895      0 1977800241]]
pow(d,21)
[[1299066613      0 1449132032]
[      0 1914644777      0]
[1870418611      0 1299066613]]
pow(d,22)
[[ -2094601527      0 104857600]
[      0 51933809      0]
[ 1316288537      0 -2094601527]]
pow(d,23)
[[ -1883073043      0 629145600]
[      0 467404281      0]
[ -346101685      0 -1883073043]]
pow(d,24)
[[ -825430623      0 -520093696]
[      0 -88328767      0]
[ -1038305055      0 -825430623]]
pow(d,25)
[[ 167814181      0 1174405120]
[      0 -794958903      0]
[1180052131      0 167814181]]
pow(d,26)
[[ 839070905      0 -1543503872]
[      0 1435304465      0]
[ -754810903      0 839070905]]
pow(d,27)
[[ -99612771      0 -671088640]
[      0 32838297      0]
[2030534587      0 -99612771]]
pow(d,28)
[[ -498063855      0 268435456]
[      0 295544673      0]
[1796636465      0 -498063855]]
pow(d,29)
[[ 1804648021      0 1610612736]
[      0 -1635065239      0]
[ 1094942099      0 1804648021]]
pow(d,30)
[[ 433305513      0 1073741824]
[      0 -1830685263      0]
[ -1010140999      0 433305513]]

```

In [36]: `print('initialize matrix d')`
`print(d)`

```
for i in range(31):
    print(' \n==> d^'+str(i)+'\n',d^i)
```

```
initialize matrix d
[[5 4 6]
 [8 9 0]
 [3 4 5]]
```

```
==> d^0
[[5 4 6]
 [8 9 0]
 [3 4 5]]
```

```
==> d^1
[[4 5 7]
 [9 8 1]
 [2 5 4]]
```

```
==> d^2
[[ 7   6   4]
 [10 11   2]
 [ 1   6   7]]
```

```
==> d^3
[[ 6   7   5]
 [11 10   3]
 [ 0   7   6]]
```

```
==> d^4
[[ 1   0   2]
 [12 13   4]
 [ 7   0   1]]
```

```
==> d^5
[[ 0   1   3]
 [13 12   5]
 [ 6   1   0]]
```

```
==> d^6
[[ 3   2   0]
 [14 15   6]
 [ 5   2   3]]
```

```
==> d^7
[[ 2   3   1]
 [15 14   7]
 [ 4   3   2]]
```

```
==> d^8
[[13 12 14]
 [ 0   1   8]
 [11 12 13]]
```

```
==> d^9
[[12 13 15]
 [ 1   0   9]
 [10 13 12]]
```

```
==> d^10
[[15 14 12]
 [ 2   3 10]
 [ 9 14 15]]
```

```
==> d^11
[[14 15 13]
 [ 3   2 11]
 [ 8 15 14]]
```

```
==> d^12
```

```
[[ 9  8 10]
 [ 4  5 12]
 [15  8  9]]
```

```
==> d^13
 [[ 8  9 11]
 [ 5  4 13]
 [14  9  8]]
```

```
==> d^14
 [[11 10  8]
 [ 6  7 14]
 [13 10 11]]
```

```
==> d^15
 [[10 11  9]
 [ 7  6 15]
 [12 11 10]]
```

```
==> d^16
 [[21 20 22]
 [24 25 16]
 [19 20 21]]
```

```
==> d^17
 [[20 21 23]
 [25 24 17]
 [18 21 20]]
```

```
==> d^18
 [[23 22 20]
 [26 27 18]
 [17 22 23]]
```

```
==> d^19
 [[22 23 21]
 [27 26 19]
 [16 23 22]]
```

```
==> d^20
 [[17 16 18]
 [28 29 20]
 [23 16 17]]
```

```
==> d^21
 [[16 17 19]
 [29 28 21]
 [22 17 16]]
```

```
==> d^22
 [[19 18 16]
 [30 31 22]
 [21 18 19]]
```

```
==> d^23
 [[18 19 17]
 [31 30 23]
 [20 19 18]]
```

```
==> d^24
 [[29 28 30]
 [16 17 24]
 [27 28 29]]
```

```
==> d^25
 [[28 29 31]
```

```
[17 16 25]
[26 29 28]]
```

```
==> d^26
[[31 30 28]
[18 19 26]
[25 30 31]]
```

```
==> d^27
[[30 31 29]
[19 18 27]
[24 31 30]]
```

```
==> d^28
[[25 24 26]
[20 21 28]
[31 24 25]]
```

```
==> d^29
[[24 25 27]
[21 20 29]
[30 25 24]]
```

```
==> d^30
[[27 26 24]
[22 23 30]
[29 26 27]]
```

```
In [37]: e = np.matrix([[1,2,3,4,5],[6,7,8,9,10],[11,12,13,14,15],[0,1,2,3,3],[4,6,7,8,9]])
f = np.matrix([[4,5,6,7,9],[3,4,65,3,2],[5,7,67,54,3],[0,0,98,8,9],[5,-3,-4,-5,2]])
print('transpose(e)\n',e.T)
print('transpose(f)\n',f.T)
print('sum(e+f) = \n'+str(e+f))
print('sum(e.T+f.T) = \n',e.T+f.T)
```

```
transpose(e)
[[ 1  6 11  0  4]
 [ 2  7 12  1  6]
 [ 3  8 13  2  7]
 [ 4  9 14  3  8]
 [ 5 10 15  3  9]]
transpose(f)
[[ 4  3  5  0  5]
 [ 5  4  7  0 -3]
 [ 6 65  67  98 -4]
 [ 7  3 54  8 -5]
 [ 9  2  3  9  2]]
sum(e+f) =
[[ 5  7   9  11  14]
 [ 9  11  73  12  12]
 [ 16 19  80  68  18]
 [ 0  1 100  11  12]
 [ 9  3   3   3  11]]
sum(e.T+f.T) =
[[ 5   9  16   0   9]
 [ 7  11  19   1   3]
 [ 9  73  80 100   3]
 [ 11 12  68  11   3]
 [ 14 12  18  12  11]]
```

```
In [38]: print('det(e)  = ',np.linalg.det(e))
print('det(e.T) = ',np.linalg.det(e.T))
```

```
det(e)  =  2.5288413338684173e-15
det(e.T) =  3.330669073875456e-15
```

```
In [39]: print('det(f) = ',np.linalg.det(f))
print('det(f.T) = ',np.linalg.det(f.T))

det(f) = 1895118.000000004
det(f.T) = 1895118.000000005
```

```
In [40]: print(e.ndim)
print(f.ndim)

2
2
```

```
In [41]: g = np.array([[9.0,8.0,7.0],[6.0,5.0,4.0]])
h = np.array([[1,2,3,4,5,0],[-3,8,-5,0,8,8],[1,2,3,4,5,6]])
print('The 2x3 order of g is \n',g)
print('The 3x6 order of h is \n',h)
```

The 2x3 order of g is

```
[[9. 8. 7.]
 [6. 5. 4.]]
```

The 3x6 order of h is

```
[[ 1  2  3  4  5  0]
 [-3  8 -5  0  8  8]
 [ 1  2  3  4  5  6]]
```

```
In [46]: a11 = int(input("Enter a11 : "))
a12 = int(input("Enter a12 : "))
a13 = int(input("Enter a13 : "))
a14 = int(input("Enter a14 : "))

a21 = int(input("Enter a21 : "))
a22 = int(input("Enter a22 : "))
a23 = int(input("Enter a23 : "))
a24 = int(input("Enter a24 : "))

a31 = int(input("Enter a31 : "))
a32 = int(input("Enter a32 : "))
a33 = int(input("Enter a33 : "))
a34 = int(input("Enter a34 : "))

a41 = int(input("Enter a41 : "))
a42 = int(input("Enter a42 : "))
a43 = int(input("Enter a43 : "))
a44 = int(input("Enter a44 : "))
i = np.array([[a11,a12,a13,a14],
              [a21,a22,a23,a24],
              [a31,a32,a33,a34],
              [a41,a42,a43,a44]])
print(i)
```

```
Enter a11 : 2
Enter a12 : 2
Enter a13 : 2
Enter a14 : 3
Enter a21 : 4
Enter a22 : 5
Enter a23 : 6
Enter a24 : 7
Enter a31 : 8
Enter a32 : 6
Enter a33 : 5
Enter a34 : 4
Enter a41 : 3
Enter a42 : 2
Enter a43 : 3
Enter a44 : 4
[[2 2 2 3]
 [4 5 6 7]
 [8 6 5 4]
 [3 2 3 4]]
```

```
In [48]: print(i.T)
print(np.linalg.det(i.T))
print(np.linalg.det(i))
```

```
[[2 4 8 3]
 [2 5 6 2]
 [2 6 5 3]
 [3 7 4 4]]
25.000000000000007
24.99999999999996
```

```
In [49]: j = np.matrix([[1,2,3.9,4,5],[6,7,8,9,0],[-1,-3,4,5,6],[2,3,4,5,6],[0,8,6,5,4]])
print(j)
```

```
[[ 1.   2.   3.9  4.   5. ]
 [ 6.   7.   8.   9.   0. ]
 [-1.  -3.   4.   5.   6. ]
 [ 2.   3.   4.   5.   6. ]
 [ 0.   8.   6.   5.   4. ]]
```

```
In [50]: print("After transpose(j)\n",j.T,"\n")
print(np.linalg.det(j),"\n")
print(j**2,"\n")
print(j**100)
```

```
After transpose(j)
[[ 1.   6.  -1.   2.   0. ]
 [ 2.   7.  -3.   3.   8. ]
 [ 3.9  8.   4.   4.   6. ]
 [ 4.   9.   5.   5.   5. ]
 [ 5.   0.   6.   6.   4. ]]
```

```
888.5999999999988
```

```
[[ 17.1  56.3  81.5  86.5  72.4]
 [ 58.   64.  147.4 172.  132. ]
 [-13.   28.  44.1  44.   73. ]
 [ 26.   76.  103.8 110.   88. ]
 [ 52.   85.  132.  147.   82. ]]
```

```
[[1.46372009e+128 3.45731468e+128 5.56042406e+128 6.08504870e+128
 4.92050705e+128]
 [2.57952687e+128 6.09285628e+128 9.79918457e+128 1.07237352e+129
 8.67145315e+128]
 [8.74316042e+127 2.06513917e+128 3.32137818e+128 3.63474939e+128
 2.93913999e+128]
 [1.88349039e+128 4.44881438e+128 7.15506016e+128 7.83013832e+128
 6.33162571e+128]
 [2.25014271e+128 5.31484913e+128 8.54791007e+128 9.35440328e+128
 7.56418059e+128]]
```

```
In [51]: print(j.trace())
print(j.conjugate())
print("sort j",j.argsort())
```

```
[[21.]]
[[ 1.   2.   3.9  4.   5. ]
 [ 6.   7.   8.   9.   0. ]
 [-1.  -3.   4.   5.   6. ]
 [ 2.   3.   4.   5.   6. ]
 [ 0.   8.   6.   5.   4. ]]
sort j [[0 1 2 3 4]
[4 0 1 2 3]
[1 0 2 3 4]
[0 1 2 3 4]
[0 4 3 2 1]]
```

```
In [52]: k = np.array([[1,2,3,4,5,6,7,8,9,10],[4,1,5,5,6,0,-4,-2,-1,6],[1,8,0,7,8,9,-2,-1,3,-4],[2,4,5
[3,8,9,0,4,3,5,6,7,8],[1,2,3,4,5,6,7,8,9,0],[4,6,3,2,1,-9,-7,0,6,4],[1,4,5,6,7,
[1,2,3,4,5,6,7,8,9,0],[0,0,7,0,5,4,3,9,0,1]])
print(k)
```

```
[[ 1  2  3  4  5  6  7  8  9 10]
 [ 4  1  5  5  6  0 -4 -2 -1  6]
 [ 1  8  0  7  8  9 -2 -1  3 -4]
 [ 2  4  5  6  4  3  8  9  0  7]
 [ 3  8  9  0  4  3  5  6  7  8]
 [ 1  2  3  4  5  6  7  8  9  0]
 [ 4  6  3  2  1 -9 -7  0  6  4]
 [ 1  4  5  6  7  5  4  3  2  1]
 [ 1  2  3  4  5  6  7  8  9  0]
 [ 0  0  7  0  5  4  3  9  0  1]]
```

```
In [53]: print('After transpose')
print(k.T)
```

```
After transpose
```

```
[[ 1  4  1  2  3  1  4  1  1  0]
 [ 2  1  8  4  8  2  6  4  2  0]
 [ 3  5  0  5  9  3  3  5  3  7]
 [ 4  5  7  6  0  4  2  6  4  0]
 [ 5  6  8  4  4  5  1  7  5  5]
 [ 6  0  9  3  3  6 -9  5  6  4]
 [ 7 -4 -2  8  5  7 -7  4  7  3]
 [ 8 -2 -1  9  6  8  0  3  8  9]
 [ 9 -1  3  0  7  9  6  2  9  0]
 [10  6 -4  7  8  0  4  1  0  1]]
```

```
In [54]: print('det(k) = ',np.linalg.det(k))
print('det(k^T) = ',np.linalg.det(k.T))
```

```
det(k) =  0.0
det(k^T) =  1.3485847077899445e-08
```

```
In [55]: for i in range(1,11):
    print('\npow(k,'+str(i)+") = \n",pow(k,i))
```

```

pow(k,1) =
[[ 1 2 3 4 5 6 7 8 9 10]
[ 4 1 5 5 6 0 -4 -2 -1 6]
[ 1 8 0 7 8 9 -2 -1 3 -4]
[ 2 4 5 6 4 3 8 9 0 7]
[ 3 8 9 0 4 3 5 6 7 8]
[ 1 2 3 4 5 6 7 8 9 0]
[ 4 6 3 2 1 -9 -7 0 6 4]
[ 1 4 5 6 7 5 4 3 2 1]
[ 1 2 3 4 5 6 7 8 9 0]
[ 0 0 7 0 5 4 3 9 0 1]]

```

```

pow(k,2) =
[[ 1 4 9 16 25 36 49 64 81 100]
[ 16 1 25 25 36 0 16 4 1 36]
[ 1 64 0 49 64 81 4 1 9 16]
[ 4 16 25 36 16 9 64 81 0 49]
[ 9 64 81 0 16 9 25 36 49 64]
[ 1 4 9 16 25 36 49 64 81 0]
[ 16 36 9 4 1 81 49 0 36 16]
[ 1 16 25 36 49 25 16 9 4 1]
[ 1 4 9 16 25 36 49 64 81 0]
[ 0 0 49 0 25 16 9 81 0 1]]

```

```

pow(k,3) =
[[ 1 8 27 64 125 216 343 512 729 1000]
[ 64 1 125 125 216 0 -64 -8 -1 216]
[ 1 512 0 343 512 729 -8 -1 27 -64]
[ 8 64 125 216 64 27 512 729 0 343]
[ 27 512 729 0 64 27 125 216 343 512]
[ 1 8 27 64 125 216 343 512 729 0]
[ 64 216 27 8 1 -729 -343 0 216 64]
[ 1 64 125 216 343 125 64 27 8 1]
[ 1 8 27 64 125 216 343 512 729 0]
[ 0 0 343 0 125 64 27 729 0 1]]

```

```

pow(k,4) =
[[ 1 16 81 256 625 1296 2401 4096 6561 10000]
[ 256 1 625 625 1296 0 256 16 1 1296]
[ 1 4096 0 2401 4096 6561 16 1 81 256]
[ 16 256 625 1296 256 81 4096 6561 0 2401]
[ 81 4096 6561 0 256 81 625 1296 2401 4096]
[ 1 16 81 256 625 1296 2401 4096 6561 0]
[ 256 1296 81 16 1 6561 2401 0 1296 256]
[ 1 256 625 1296 2401 625 256 81 16 1]
[ 1 16 81 256 625 1296 2401 4096 6561 0]
[ 0 0 2401 0 625 256 81 6561 0 1]]

```

```

pow(k,5) =
[[ 1 32 243 1024 3125 7776 16807 32768 59049 100000]
[ 1024 1 3125 3125 7776 0 -1024 -32 -1 7776]
[ 1 32768 0 16807 32768 59049 -32 -1 243 -1024]
[ 32 1024 3125 7776 1024 243 32768 59049 0 16807]
[ 243 32768 59049 0 1024 243 3125 7776 16807 32768]
[ 1 32 243 1024 3125 7776 16807 32768 59049 0]
[ 1024 7776 243 32 1 -59049 -16807 0 7776 1024]
[ 1 1024 3125 7776 16807 3125 1024 243 32 1]
[ 1 32 243 1024 3125 7776 16807 32768 59049 0]
[ 0 0 16807 0 3125 1024 243 59049 0 1]]

```

```

pow(k,6) =
[[ 1 64 729 4096 15625 46656 117649 262144 531441
10000000]
[ 4096 1 15625 15625 46656 0 4096 64 1
46656]
[ 1 262144 0 117649 262144 531441 64 1 729
]

```

```

4096]
[ 64   4096   15625   46656   4096    729   262144   531441      0
117649]
[ 729   262144   531441      0   4096    729   15625   46656   117649
262144]
[ 1     64     729   4096   15625   46656   117649   262144   531441
0]
[ 4096   46656    729      64      1   531441   117649      0   46656
4096]
[ 1     4096   15625   46656   117649   15625    4096    729      64
1]
[ 1     64     729   4096   15625   46656   117649   262144   531441
0]
[ 0     0   117649      0   15625    4096    729   531441      0
1]]

```

```

pow(k,7) =
[[ 1     128     2187   16384   78125   279936   823543   2097152
4782969 100000000]
[ 16384     1     78125   78125   279936      0   -16384     -128
-1     279936]
[ 1     2097152      0   823543   2097152   4782969      -128     -1
2187   -16384]
[ 128     16384    78125   279936   16384    2187   2097152   4782969
0     823543]
[ 2187   2097152   4782969      0   16384    2187    78125   279936
823543 2097152]
[ 1     128     2187   16384    78125   279936   823543   2097152
4782969 0]
[ 16384   279936    2187     128      1   -4782969   -823543      0
279936 16384]
[ 1     16384    78125   279936   823543    78125   16384    2187
128     1]
[ 1     128     2187   16384    78125   279936   823543   2097152
4782969 0]
[ 0     0     823543      0   78125   16384    2187   4782969
0     1]]

```

```

pow(k,8) =
[[ 1     256     6561     65536   390625   1679616   5764801
16777216 43046721 100000000]
[ 65536     1     390625    390625   1679616      0   65536
256     1     1679616]
[ 1     16777216      0   5764801   16777216   43046721      256
1     6561     65536]
[ 256     65536    390625   1679616   65536     6561   16777216
43046721      0     5764801]
[ 6561     16777216   43046721      0     65536     6561   390625
1679616   5764801 16777216]
[ 1     256     6561     65536   390625   1679616   5764801
16777216 43046721 0]
[ 65536     1679616     6561     256      1   43046721   5764801
0     1679616 65536]
[ 1     65536    390625   1679616   5764801   390625     65536
6561     256     1]
[ 1     256     6561     65536   390625   1679616   5764801
16777216 43046721 0]
[ 0     0     5764801      0     390625     65536     6561
43046721      0     1]]

```

```

pow(k,9) =
[[ 1     512     19683   262144   1953125   10077696
40353607 134217728 387420489 1000000000]
[ 262144     1     1953125   1953125   10077696      0
-262144    -512     -1     10077696]
[ 1     134217728      0   40353607 134217728   387420489
]
```

```

      -512      -1    19683   -262144]
[ 512    262144  1953125  10077696  262144  19683
[ 134217728 387420489      0  40353607]
[ 19683  134217728 387420489      0  262144  19683
[ 1953125 10077696 40353607 134217728]
[ 1      512    19683   262144  1953125 10077696
[ 40353607 134217728 387420489      0]
[ 262144 10077696 19683   512      1 -387420489
[ -40353607      0 10077696 262144]
[ 1      262144 1953125 10077696 40353607 1953125
[ 262144 19683   512      1]
[ 1      512    19683   262144  1953125 10077696
[ 40353607 134217728 387420489      0]
[ 0      0    40353607      0  1953125 262144
[ 19683 387420489      0      1]]

```

```

pow(k,10) =
[[ 1      1024    59049   1048576  9765625 60466176
  282475249 1073741824 -808182895 1410065408]
[ 1048576      1    9765625  9765625 60466176      0
  1048576    1024      1  60466176]
[ 1 1073741824      0 282475249 1073741824 -808182895
  1024      1    59049  1048576]
[ 1024    1048576  9765625  60466176 1048576 59049
  1073741824 -808182895      0 282475249]
[ 59049 1073741824 -808182895      0 1048576 59049
  9765625 60466176 282475249 1073741824]
[ 1      1024    59049   1048576  9765625 60466176
  282475249 1073741824 -808182895      0]
[ 1048576 60466176    59049      1024      1 -808182895
  282475249      0 60466176 1048576]
[ 1 1048576  9765625  60466176 282475249 9765625
  1048576    59049    1024      1]
[ 1      1024    59049   1048576  9765625 60466176
  282475249 1073741824 -808182895      0]
[ 0      0    282475249      0  9765625 1048576
  59049 -808182895      0      1]]

```

In [56]: `print("power(k,0) = \n",pow(k,0))`

```

power(k,0) =
[[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]
[1 1 1 1 1 1 1 1 1 1]]
```

In [57]: `for j in range(1,10):
 print("\npower(k,"+str(j)+")\n",pow(k,1/j))`

```

power(k,1)
[[ 1.  2.  3.  4.  5.  6.  7.  8.  9.  10.]
 [ 4.  1.  5.  5.  6.  0. -4. -2. -1.  6.]
 [ 1.  8.  0.  7.  8.  9. -2. -1.  3. -4.]
 [ 2.  4.  5.  6.  4.  3.  8.  9.  0.  7.]
 [ 3.  8.  9.  0.  4.  3.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.  9.  0.]
 [ 4.  6.  3.  2.  1. -9. -7.  0.  6.  4.]
 [ 1.  4.  5.  6.  7.  5.  4.  3.  2.  1.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.  9.  0.]
 [ 0.  0.  7.  0.  5.  4.  3.  9.  0.  1.]]]

power(k,2)
[[1.          1.41421356 1.73205081 2.          2.23606798 2.44948974
 2.64575131 2.82842712 3.          3.16227766]
 [2.          1.          2.23606798 2.23606798 2.44948974 0.
  nan         nan         nan 2.44948974]
 [1.          2.82842712 0.          2.64575131 2.82842712 3.
  nan         nan 1.73205081      nan]
 [1.41421356 2.          2.23606798 2.44948974 2.          1.73205081
 2.82842712 3.          0.          2.64575131]
 [1.73205081 2.82842712 3.          0.          2.          1.73205081
 2.23606798 2.44948974 2.64575131 2.82842712]
 [1.          1.41421356 1.73205081 2.          2.23606798 2.44948974
 2.64575131 2.82842712 3.          0.          ]
 [2.          2.44948974 1.73205081 1.41421356 1.          nan
  nan 0.          2.44948974 2.          ]
 [1.          2.          2.23606798 2.44948974 2.64575131 2.23606798
 2.          1.73205081 1.41421356 1.          ]
 [1.          1.41421356 1.73205081 2.          2.23606798 2.44948974
 2.64575131 2.82842712 3.          0.          ]
 [0.          0.          2.64575131 0.          2.23606798 2.
 1.73205081 3.          0.          1.          ]]
power(k,3)
[[1.          1.25992105 1.44224957 1.58740105 1.70997595 1.81712059
 1.91293118 2.          2.08008382 2.15443469]
 [1.58740105 1.          1.70997595 1.70997595 1.81712059 0.
  nan         nan         nan 1.81712059]
 [1.          2.          0.          1.91293118 2.          2.08008382
  nan         nan 1.44224957      nan]
 [1.25992105 1.58740105 1.70997595 1.81712059 1.58740105 1.44224957
 2.          2.08008382 0.          1.91293118]
 [1.44224957 2.          2.08008382 0.          1.58740105 1.44224957
 1.70997595 1.81712059 1.91293118 2.          ]
 [1.          1.25992105 1.44224957 1.58740105 1.70997595 1.81712059
 1.91293118 2.          2.08008382 0.          ]
 [1.58740105 1.81712059 1.44224957 1.25992105 1.          nan
  nan 0.          1.81712059 1.58740105]
 [1.          1.58740105 1.70997595 1.81712059 1.91293118 1.70997595
 1.58740105 1.44224957 1.25992105 1.          ]
 [1.          1.25992105 1.44224957 1.58740105 1.70997595 1.81712059
 1.91293118 2.          2.08008382 0.          ]
 [0.          0.          1.91293118 0.          1.70997595 1.58740105
 1.44224957 2.08008382 0.          1.          ]]
power(k,4)
[[1.          1.18920712 1.31607401 1.41421356 1.49534878 1.56508458
 1.62657656 1.68179283 1.73205081 1.77827941]
 [1.41421356 1.          1.49534878 1.49534878 1.56508458 0.
  nan         nan         nan 1.56508458]
 [1.          1.68179283 0.          1.62657656 1.68179283 1.73205081
  nan         nan 1.31607401      nan]
 [1.18920712 1.41421356 1.49534878 1.56508458 1.41421356 1.31607401
 1.68179283 1.73205081 0.          1.62657656]
 [1.31607401 1.68179283 1.73205081 0.          1.41421356 1.31607401
 1.62657656]]

```

```

1.49534878 1.56508458 1.62657656 1.68179283]
[1. 1.18920712 1.31607401 1.41421356 1.49534878 1.56508458
1.62657656 1.68179283 1.73205081 0. ]
[1.41421356 1.56508458 1.31607401 1.18920712 1. nan
nan 0. 1.56508458 1.41421356]
[1. 1.41421356 1.49534878 1.56508458 1.62657656 1.49534878
1.41421356 1.31607401 1.18920712 1. ]
[1. 1.18920712 1.31607401 1.41421356 1.49534878 1.56508458
1.62657656 1.68179283 1.73205081 0. ]
[0. 0. 1.62657656 0. 1.49534878 1.41421356
1.31607401 1.73205081 0. 1. ]]
power(k,5)
[[1. 1.14869835 1.24573094 1.31950791 1.37972966 1.43096908
1.47577316 1.51571657 1.55184557 1.58489319]
[1.31950791 1. 1.37972966 1.37972966 1.43096908 0.
nan nan nan 1.43096908]
[1. 1.51571657 0. 1.47577316 1.51571657 1.55184557
nan nan 1.24573094 nan]
[1.14869835 1.31950791 1.37972966 1.43096908 1.31950791 1.24573094
1.51571657 1.55184557 0. 1.47577316]
[1.24573094 1.51571657 1.55184557 0. 1.31950791 1.24573094
1.37972966 1.43096908 1.47577316 1.51571657]
[1. 1.14869835 1.24573094 1.31950791 1.37972966 1.43096908
1.47577316 1.51571657 1.55184557 0. ]
[1.31950791 1.43096908 1.24573094 1.14869835 1. nan
nan 0. 1.43096908 1.31950791]
[1. 1.31950791 1.37972966 1.43096908 1.47577316 1.37972966
1.31950791 1.24573094 1.14869835 1. ]
[1. 1.14869835 1.24573094 1.31950791 1.37972966 1.43096908
1.47577316 1.51571657 1.55184557 0. ]
[0. 0. 1.47577316 0. 1.37972966 1.31950791
1.24573094 1.55184557 0. 1. ]]
power(k,6)
[[1. 1.12246205 1.20093696 1.25992105 1.30766049 1.34800615
1.38308755 1.41421356 1.44224957 1.46779927]
[1.25992105 1. 1.30766049 1.30766049 1.34800615 0.
nan nan nan 1.34800615]
[1. 1.41421356 0. 1.38308755 1.41421356 1.44224957
nan nan 1.20093696 nan]
[1.12246205 1.25992105 1.30766049 1.34800615 1.25992105 1.20093696
1.41421356 1.44224957 0. 1.38308755]
[1.20093696 1.41421356 1.44224957 0. 1.25992105 1.20093696
1.30766049 1.34800615 1.38308755 1.41421356]
[1. 1.12246205 1.20093696 1.25992105 1.30766049 1.34800615
1.38308755 1.41421356 1.44224957 0. ]
[1.25992105 1.34800615 1.20093696 1.12246205 1. nan
nan 0. 1.34800615 1.25992105]
[1. 1.25992105 1.30766049 1.34800615 1.38308755 1.30766049
1.25992105 1.20093696 1.12246205 1. ]
[1. 1.12246205 1.20093696 1.25992105 1.30766049 1.34800615
1.38308755 1.41421356 1.44224957 0. ]
[0. 0. 1.38308755 0. 1.30766049 1.25992105
1.20093696 1.44224957 0. 1. ]]
power(k,7)
[[1. 1.10408951 1.16993081 1.21901365 1.25849895 1.29170834
1.32046925 1.34590019 1.36873811 1.38949549]
[1.21901365 1. 1.25849895 1.25849895 1.29170834 0.
nan nan nan 1.29170834]
[1. 1.34590019 0. 1.32046925 1.34590019 1.36873811
nan nan 1.16993081 nan]
[1.10408951 1.21901365 1.25849895 1.29170834 1.21901365 1.16993081
1.34590019 1.36873811 0. 1.32046925]
[1.16993081 1.34590019 1.36873811 0. 1.21901365 1.16993081
1.32046925 1.16993081 0. 1.21901365 1.16993081]

```

```

1.25849895 1.29170834 1.32046925 1.34590019]
[1. 1.10408951 1.16993081 1.21901365 1.25849895 1.29170834
1.32046925 1.34590019 1.36873811 0. ]
[1.21901365 1.29170834 1.16993081 1.10408951 1. nan
nan 0. 1.29170834 1.21901365]
[1. 1.21901365 1.25849895 1.29170834 1.32046925 1.25849895
1.21901365 1.16993081 1.10408951 1. ]
[1. 1.10408951 1.16993081 1.21901365 1.25849895 1.29170834
1.32046925 1.34590019 1.36873811 0. ]
[0. 0. 1.32046925 0. 1.25849895 1.21901365
1.16993081 1.36873811 0. 1. ]
]]]

power(k,8)
[[1. 1.09050773 1.14720269 1.18920712 1.22284454 1.2510334
1.27537311 1.29683955 1.31607401 1.33352143]
[1.18920712 1. 1.22284454 1.22284454 1.2510334 0.
nan nan nan 1.2510334]
[1. 1.29683955 0. 1.27537311 1.29683955 1.31607401
nan nan 1.14720269 nan]
[1.09050773 1.18920712 1.22284454 1.2510334 1.18920712 1.14720269
1.29683955 1.31607401 0. 1.27537311]
[1.14720269 1.29683955 1.31607401 0. 1.18920712 1.14720269
1.22284454 1.2510334 1.27537311 1.29683955]
[1. 1.09050773 1.14720269 1.18920712 1.22284454 1.2510334
1.27537311 1.29683955 1.31607401 0. ]
[1.18920712 1.2510334 1.14720269 1.09050773 1. nan
nan 0. 1.2510334 1.18920712]
[1. 1.18920712 1.22284454 1.2510334 1.27537311 1.22284454
1.18920712 1.14720269 1.09050773 1. ]
[1. 1.09050773 1.14720269 1.18920712 1.22284454 1.2510334
1.27537311 1.29683955 1.31607401 0. ]
[0. 0. 1.27537311 0. 1.22284454 1.18920712
1.14720269 1.31607401 0. 1. ]]
]]]

power(k,9)
[[1. 1.08005974 1.12983096 1.16652904 1.19581317 1.22028494
1.24136582 1.25992105 1.27651801 1.29154967]
[1.16652904 1. 1.19581317 1.19581317 1.22028494 0.
nan nan nan 1.22028494]
[1. 1.25992105 0. 1.24136582 1.25992105 1.27651801
nan nan 1.12983096 nan]
[1.08005974 1.16652904 1.19581317 1.22028494 1.16652904 1.12983096
1.25992105 1.27651801 0. 1.24136582]
[1.12983096 1.25992105 1.27651801 0. 1.16652904 1.12983096
1.19581317 1.22028494 1.24136582 1.25992105]
[1. 1.08005974 1.12983096 1.16652904 1.19581317 1.22028494
1.24136582 1.25992105 1.27651801 0. ]
[1.16652904 1.22028494 1.12983096 1.08005974 1. nan
nan 0. 1.22028494 1.16652904]
[1. 1.16652904 1.19581317 1.22028494 1.24136582 1.19581317
1.16652904 1.12983096 1.08005974 1. ]
[1. 1.08005974 1.12983096 1.16652904 1.19581317 1.22028494
1.24136582 1.25992105 1.27651801 0. ]
[0. 0. 1.24136582 0. 1.19581317 1.16652904
1.12983096 1.27651801 0. 1. ]]
]]]

C:\Users\lh\AppData\Local\Temp\ipykernel_2376\4261224221.py:2: RuntimeWarning: invalid value
encountered in power
print("\npower(k,"+str(j)+")\n",pow(k,1/j))

```

```
In [58]: l = np.array([[1],[2],[1],[3],[5],[2],[0],[7],[-1],[-2]])
print(l)
```

```
[[ 1]
 [ 2]
 [ 1]
 [ 3]
 [ 5]
 [ 2]
 [ 0]
 [ 7]
 [-1]
 [-2]]
```

```
In [59]: print('j*1 = \n',j*1)
```

```
j*1 =
[[ 9]
 [18]
 [ 9]
 [27]
 [45]
 [18]
 [ 0]
 [63]
 [-9]
 [-18]]
```

```
In [60]: m = np.array([[1,2,3,4,5,6,7,8,9,10,1,2,3,4,5],[4,1,5,5,6,0,-4,-2,-1,6,6,7,8,9,0],[1,8,0,7,8,
[2,4,5,6,4,3,8,9,0,7,2,3,4,5,6],[3,8,9,0,4,3,5,6,7,8,0,0,1,2,3],[1,2,3,4,5,6,7,
[4,6,3,2,1,-9,-7,0,6,4,0,0,1,-2,3],[1,4,5,6,7,5,4,3,2,1,1,2,0,0,4],[1,2,3,4,5,6
[2,3,4,5,6,7,8,9,0,1,2,4,3,4,5],[0,0,7,0,5,4,3,9,0,1,0,0,8,-9,-2],[1,2,3,4,5,6,
[1,2,3,4,5,6,7,8,9,0,9,7,5,4,3],[1,2,3,4,6,-9,-8,-6,-3,-1,2,3,5,6,7],[0,0,2,1,1,3
print("Initialize matrix m")
print(m)
```

```
Initialize matrix m
[[ 1  2  3  4  5  6  7  8  9 10  1  2  3  4  5]
 [ 4  1  5  5  6  0 -4 -2 -1  6  6  7  8  9  0]
 [ 1  8  0  7  8  9 -2 -1  3 -4 -8 -1  2  3  4]
 [ 2  4  5  6  4  3  8  9  0  7  2  3  4  5  6]
 [ 3  8  9  0  4  3  5  6  7  8  0  0  1  2  3]
 [ 1  2  3  4  5  6  7  2  3  4  5  6  8  9  0]
 [ 4  6  3  2  1 -9 -7  0  6  4  0  0  1 -2  3]
 [ 1  4  5  6  7  5  4  3  2  1  1  2  0  0  4]
 [ 1  2  3  4  5  6  7  8  9  0  9  9  5  4  8]
 [ 2  3  4  5  6  7  8  9  0  1  2  4  3  4  5]
 [ 0  0  7  0  5  4  3  9  0  1  0  0  8  -9 -2]
 [ 1  2  3  4  5  6  7  8  9  0  0  9  8  4  3]
 [ 1  2  3  4  5  6  7  8  9  0  9  7  5  4  3]
 [ 1  2  3  4  6 -9 -8 -6 -3 -1  2  3  5  6  7]
 [ 0  0  2  1  3  4  5  6  7  8  6  5  4  9  6]]
```

```
In [61]: print(m.T)
```

```
[[ 1  4  1  2  3  1  4  1  1  2  0  1  1  1  0]
 [ 2  1  8  4  8  2  6  4  2  3  0  2  2  2  0]
 [ 3  5  0  5  9  3  3  5  3  4  7  3  3  3  2]
 [ 4  5  7  6  0  4  2  6  4  5  0  4  4  4  1]
 [ 5  6  8  4  4  5  1  7  5  6  5  5  5  6  3]
 [ 6  0  9  3  3  6 -9  5  6  7  4  6  6 -9  4]
 [ 7 -4 -2  8  5  7 -7  4  7  8  3  7  7 -8  5]
 [ 8 -2 -1  9  6  2  0  3  8  9  9  8  8 -6  6]
 [ 9 -1  3  0  7  3  6  2  9  0  0  9  9 -3  7]
 [10  6 -4  7  8  4  4  1  0  1  1  0  0 -1  8]
 [ 1  6 -8  2  0  5  0  1  9  2  0  0  9  2  6]
 [ 2  7 -1  3  0  6  0  2  9  4  0  9  7  3  5]
 [ 3  8  2  4  1  8  1  0  5  3  8  8  5  5  4]
 [ 4  9  3  5  2  9 -2  0  4  4 -9  4  4  6  9]
 [ 5  0  4  6  3  0  3  4  8  5 -2  3  3  7  6]]
```

```
In [62]: print(np.linalg.det(m.T))
print(np.linalg.det(m))
```

```
5787723947628.011
5787723947627.99
```

```
In [63]: print(m.size) # number of element
print(m nbytes) # size of total elements in array
print(m.shape) # the order of matrix
```

```
14
14
(4, 7)
```

```
↳ Accessing and changing specific elements
-
row and columns in matrix
```

```
In [64]: a = np.matrix([[1,2,3,4,5,6,7],[8,9,10,11,12,13,14],[2,4,7,8,9,6,4],[0,9,0,7,5,4,3]])
print(a)
```

```
[[ 1  2  3  4  5  6  7]
 [ 8  9  10 11 12 13 14]
 [ 2  4  7  8  9  6  4]
 [ 0  9  0  7  5  4  3]]
```

```
In [65]: # Get specific element[r,c]
print("a[0,0] =",a[0,0])
print("a[0,1] =",a[0,1])
print("a[0,2] =",a[0,2])
print("a[0,3] =",a[0,3])
print("a[0,4] =",a[0,4])
print("a[0,5] =",a[0,5])
print("a[0,6] =",a[0,6])
```

```
a[0,0] = 1
a[0,1] = 2
a[0,2] = 3
a[0,3] = 4
a[0,4] = 5
a[0,5] = 6
a[0,6] = 7
```

```
In [66]: for i in range(0,7):
    print("a[0,"+str(i)+"] =",a[0,i])
```

```
a[0,0] = 1
a[0,1] = 2
a[0,2] = 3
a[0,3] = 4
a[0,4] = 5
a[0,5] = 6
a[0,6] = 7
```

```
In [67]: # Get a specific row
print('0th row : ',a[0,:])
print('1st row : ',a[1,:])
print('2nd row : ',a[2,:])
print("3rd row : ",a[3,:])
```

```
0th row : [[1 2 3 4 5 6 7]]
1st row : [[ 8  9  10 11 12 13 14]]
2nd row : [[2 4 7 8 9 6 4]]
3rd row : [[0 9 0 7 5 4 3]]
```

```
In [68]: # Get a specific column
print('0th column : \n',a[:,0])
print('1st column : \n',a[:,1])
print('2nd column : \n',a[:,2])
print('3rd column : \n',a[:,3])
print('3rd column : \n',a[:,3])
print('4th column : \n',a[:,4])
print('5st column : \n',a[:,5])
print('6nd column : \n',a[:,6])
```

```
0th column :
```

```
[[1]
[8]
[2]
[0]]
```

```
1st column :
```

```
[[2]
[9]
[4]
[9]]
```

```
2nd column :
```

```
[[ 3]
[10]
[ 7]
[ 0]]
```

```
3rd column :
```

```
[[ 4]
[11]
[ 8]
[ 7]]
```

```
3rd column :
```

```
[[ 4]
[11]
[ 8]
[ 7]]
```

```
4th column :
```

```
[[ 5]
[12]
[ 9]
[ 5]]
```

```
5st column :
```

```
[[ 6]
[13]
[ 6]
[ 4]]
```

```
6nd column :
```

```
[[ 7]
[14]
[ 4]
[ 3]]
```

```
In [69]: print(k)
for i in range(10):
    print("row("+str(i)+") : ",k[i,:])
```

```

[[ 1  2  3  4  5  6  7  8  9 10]
 [ 4  1  5  5  6  0 -4 -2 -1  6]
 [ 1  8  0  7  8  9 -2 -1  3 -4]
 [ 2  4  5  6  4  3  8  9  0  7]
 [ 3  8  9  0  4  3  5  6  7  8]
 [ 1  2  3  4  5  6  7  8  9  0]
 [ 4  6  3  2  1 -9 -7  0  6  4]
 [ 1  4  5  6  7  5  4  3  2  1]
 [ 1  2  3  4  5  6  7  8  9  0]
 [ 0  0  7  0  5  4  3  9  0  1]]
row(0) : [ 1  2  3  4  5  6  7  8  9 10]
row(1) : [ 4  1  5  5  6  0 -4 -2 -1  6]
row(2) : [ 1  8  0  7  8  9 -2 -1  3 -4]
row(3) : [2 4 5 6 4 3 8 9 0 7]
row(4) : [3 8 9 0 4 3 5 6 7 8]
row(5) : [1 2 3 4 5 6 7 8 9 0]
row(6) : [ 4  6  3  2  1 -9 -7  0  6  4]
row(7) : [1 4 5 6 7 5 4 3 2 1]
row(8) : [1 2 3 4 5 6 7 8 9 0]
row(9) : [0 0 7 0 5 4 3 9 0 1]

```

```
In [70]: for j in range(10):
    print("column("+str(j)+") : ",k[:,j])
```

```

column(0) : [1 4 1 2 3 1 4 1 1 0]
column(1) : [2 1 8 4 8 2 6 4 2 0]
column(2) : [3 5 0 5 9 3 3 5 3 7]
column(3) : [4 5 7 6 0 4 2 6 4 0]
column(4) : [5 6 8 4 4 5 1 7 5 5]
column(5) : [ 6  0   9   3   3   6 -9   5   6   4]
column(6) : [ 7 -4  -2   8   5   7 -7   4   7   3]
column(7) : [ 8 -2  -1   9   6   8   0   3   8   9]
column(8) : [ 9 -1   3   0   7   9   6   2   9   0]
column(9) : [10  6 -4   7   8   0   4   1   0   1]

```

```
In [71]: print(k.trace()) # sum the diagonal values in matrix k
```

24

```
In [72]: print(a)
```

```

[[ 1  2  3  4  5  6  7]
 [ 8  9 10 11 12 13 14]
 [ 2  4  7  8  9  6  4]
 [ 0  9  0  7  5  4  3]]

```

```
# Get little more fancy[row-th,startIndex:endIndex:stepSize] where stepSize is a step of order
print('The fancy location of a is such that : ')
print(a[0,1:6:2])
print(a[1,0:6:1])
print(a[3,3:5:1])
print(a[2,2:-1:1])
```

The fancy location of a is such that :

```

[[2 4 6]
 [[ 8  9 10 11 12 13]]
 [[7 5]]
 [[7 8 9 6]]]
```

```
# Change element using specific index
b = np.matrix([[1,2,3,4,5,6,7],[8,9,10,11,12,13,14]])
print(b)
```

```

[[ 1  2  3  4  5  6  7]
 [ 8  9 10 11 12 13 14]]
```

```
# change b15
b[1,5] = 200
```

```
print(b)
[[ 1  2  3  4  5  6  7]
 [ 8  9 10 11 12 200 14]]
```

```
In [76]: # change b[1,2] and b[:,6] with the same value]
b[1,2] = 1200
b[:,6] = 3000
print(b)
```

```
[[ 1  2  3  4  5  6 3000]
 [ 8  9 1200 11 12 200 3000]]
```

```
In [77]: print(k)
```

```
[[ 1  2  3  4  5  6  7  8  9 10]
 [ 4  1  5  5  6  0 -4 -2 -1  6]
 [ 1  8  0  7  8  9 -2 -1  3 -4]
 [ 2  4  5  6  4  3  8  9  0  7]
 [ 3  8  9  0  4  3  5  6  7  8]
 [ 1  2  3  4  5  6  7  8  9  0]
 [ 4  6  3  2  1 -9 -7  0  6  4]
 [ 1  4  5  6  7  5  4  3  2  1]
 [ 1  2  3  4  5  6  7  8  9  0]
 [ 0  0  7  0  5  4  3  9  0  1]]
```

```
In [78]: k[:,0] = 200
k[:,2] = -6
print(k)
```

```
[[200  2 -6  4  5  6  7  8  9 10]
 [200  1 -6  5  6  0 -4 -2 -1  6]
 [200  8 -6  7  8  9 -2 -1  3 -4]
 [200  4 -6  6  4  3  8  9  0  7]
 [200  8 -6  0  4  3  5  6  7  8]
 [200  2 -6  4  5  6  7  8  9  0]
 [200  6 -6  2  1 -9 -7  0  6  4]
 [200  4 -6  6  7  5  4  3  2  1]
 [200  2 -6  4  5  6  7  8  9  0]
 [200  0 -6  0  5  4  3  9  0  1]]
```

```
In [79]: k[0,:] = 100
# change value at all rows that start from index(1) to index(9) with step size = 2 with the s
k[:,9] = 400
k[0,9] = 100
k[1,1:9:2] = 0 # change value at 1st row with index1 to 9 about stepsize=2
k[9,:] = 100
print(k)
```

```
[[100 100 100 100 100 100 100 100 100 100]
 [200  0  0  0  0  0  0  0  0 400]
 [200  8 -6  7  8  9 -2 -1  3 400]
 [200  4 -6  6  4  3  8  9  0 400]
 [200  8 -6  0  4  3  5  6  7 400]
 [200  2 -6  4  5  6  7  8  9 400]
 [200  6 -6  2  1 -9 -7  0  6 400]
 [200  4 -6  6  7  5  4  3  2 400]
 [200  2 -6  4  5  6  7  8  9 400]
 [100 100 100 100 100 100 100 100 100 100]]
```

```
In [80]: c = np.array([[[1,2],[3,4]],[[5,6],[7,8]]])
print(c)
```

```
# Get specific element(work outside in)
print('element[0,0]:'+str(c[0,1,1])) # c[order of matrix, row-th, column-th]
print('element[0,:] of two matrices :\n'+str(c[:,1,:]))
print('element[9,1] of two matrices :\n'+str(c[:,0,1]))
```

```

c[:,1,:] = [[9,9],[8,8]]
print('element[1:] of two matrix after change value :\n'+str(c))

[[[1 2]
 [3 4]]

 [[5 6]
 [7 8]]]
element[0,0]:4
element[0,:] of two matrices :
[[3 4]
 [7 8]]
element[9,1] of two matrices :
[2 6]
element[1:] of two matrix after change value :
[[[1 2]
 [9 9]]

 [[5 6]
 [7 8]]]

```

...> Initialize different types of matrix

-> Zero Matrix

In [81]: `a = np.zeros(5) # 1 row and 5 columns
print(a)`

[0. 0. 0. 0. 0.]

In [82]: `b = np.zeros(20)
print(b)`

[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]

In [83]: `c = np.zeros(100)
print(c)`

[0.
0.
0.
0.
0. 0. 0. 0.]

In [84]: `d = np.zeros((4,6)) # 4 rows and 6 columns
print(d)`

[[0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]]

In [85]: `e = np.zeros((20,20))
print(e)`

```
In [86]: # zero matrix with 15 rows and 10 columns
f = np.zeros((15,10))
print(f)
```

```
In [87]: f[0,:] = 1  
f[14,:] = 2  
f[:,0] = 4  
f[:,9] = 5  
print(f)
```

```
In [88]: # 3 zero matrices with 5 rows and 7 columns
g = np.zeros((3,5,7))
print(g)
```

```
[[[0. 0. 0. 0. 0. 0. 0.]  
 [0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]]
```

```
[[0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]]
```

```
[[0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]  
 [0. 0. 0. 0. 0. 0. ]]]
```

```
In [89]: # 20 zero matrices with 7 rows and 3 columns  
h = np.zeros((4,5,7,3))  
print(h)
```

```
[[[[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[0. 0. 0.]
```

```
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]]
```

```
[[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]]
```

```
[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]]
```

```
[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]]
```

```
[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]]
```

```
[[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]]]
```

```
[[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]]
```

```
[[0. 0. 0.]
```

```
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]  
[0. 0. 0.]
```

```
[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]
```

```
[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]
```

```
[[0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]  
 [0. 0. 0.]]]]
```

b.) One Matrix

```
In [90]: d = np.ones(20)  
print(d)
```

```
[1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
```

```
In [91]: e = np.ones(100)  
print(e)
```

```
[1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.  
 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.  
 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.  
 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.  
 1. 1. 1. 1.]
```

```
In [92]: # one matrix with 2 rows and 3 columns  
f = np.ones((2,3))  
print(f)
```

```
[[1. 1. 1.]  
 [1. 1. 1.]]
```

```
In [93]: # 2 one matrices with 3 rows and 5 columns  
g = np.ones((2,3,5))  
print(g)
```

```
[[[1. 1. 1. 1. 1.]  
 [1. 1. 1. 1. 1.]  
 [1. 1. 1. 1. 1.]]  
  
 [[1. 1. 1. 1. 1.]  
 [1. 1. 1. 1. 1.]  
 [1. 1. 1. 1. 1.]]]
```

```
In [94]: h = np.ones((4,6,10))
print(h)
```

```
In [95]: i = np.ones((10,10,10))
print(i)
```



```
In [96]: j = np.ones((4,2,2))
print(j)
print(j.dtype)
```

```
[[[1. 1.]  
 [1. 1.]]  
  
[[1. 1.]  
 [1. 1.]]  
  
[[1. 1.]  
 [1. 1.]]  
  
[[1. 1.]  
 [1. 1.]]]  
float64
```

```
In [97]: j = np.ones((4,2,2),dtype="int32")
print(j)
print(j.dtype)
```

```
[[[1 1]
 [1 1]]]
```

```
[[1 1]
 [1 1]]]
```

```
[[1 1]
 [1 1]]]
```

```
[[1 1]
 [1 1]]]
```

```
In [98]: x=np.full((2,2),99,dtype="float32")
print(x)
```

```
[[99. 99.]
 [99. 99.]]
```

```
In [99]: y = np.full((10,10,10),3)
print(y)
```



```
[[3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]]
```

```
[[3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]]
```

```
[[3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]]
```

```
[[3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]
 [3 3 3 3 3 3 3 3 3 3]]]
```

```
In [100]: z = np.full((20,20), -1)
print(z)
```

In [101...]

```
m = np.full((30,30),4)  
print(m)
```

In [102]:

```
n = np.array([[1,2,3,4,5,6,7],[8,9,10,11,12,13,14],[0,9,0,8,6,5,-3]])  
print(np.full_like(n,4))
```

```
[ [ 4 4 4 4 4 4 4 ]
  [ 4 4 4 4 4 4 4 ]
  [ 4 4 4 4 4 4 4 ] ]
```

In [103...]

```
print(np.full(n.shape,4))
```

```
[[4 4 4 4 4 4 4]  
 [4 4 4 4 4 4 4]  
 [4 4 4 4 4 4 4]]
```

```
In [104... print(np.full_like(n,5))
```

```
[[5 5 5 5 5 5]  
[5 5 5 5 5 5]  
[5 5 5 5 5 5]]
```

```
In [105... # np.full_like(n,a) = np.full(n.shape,a), where a is constant
```

```
#> Random number in matrix
```

```
In [106... # random number on(0,1) with 4 rows and 2 columns
```

```
print(np.random.rand(4,2))
```

```
[[0.97371859 0.42709182]  
[0.37206521 0.40848674]  
[0.14923774 0.12618286]  
[0.60493946 0.88358638]]
```

```
In [107... # random number on(0,1) with 2 rows and 3 columns for 4 matrices.
```

```
print(np.random.rand(4,2,3))
```

```
[[[0.22192885 0.72910252 0.56188822]  
[0.8479978 0.20278768 0.50940628]]  
  
[[0.44747659 0.95997382 0.85954142]  
[0.62031877 0.95796339 0.84424156]]  
  
[[0.56196644 0.34796444 0.01921259]  
[0.22538848 0.19908011 0.01685363]]  
  
[[0.37607774 0.77570189 0.08110065]  
[0.91295157 0.6643902 0.86017535]]]
```

```
In [108... # random on matrix n on the interval(0,1)
```

```
print(n)  
print(np.random.random_sample(n.shape))
```

```
[[ 1  2  3  4  5  6  7]  
[ 8  9 10 11 12 13 14]  
[ 0  9  0  8  6  5 -3]]  
[[0.06317738 0.72981118 0.19232798 0.53573423 0.96669776 0.24328439  
0.00586283]  
[0.72468763 0.03809205 0.16967673 0.44245594 0.29473533 0.38871269  
0.77857822]  
[0.1223513 0.6615598 0.79013274 0.40661738 0.76279336 0.55075034  
0.78897628]]
```

```
In [109... # random integer value on the interval(-4,8) with 3 matrices about 3 rows and 3 columns
```

```
print(np.random.randint(-4,8,size=(3,3,3)))
```

```
[[[ 6 -1  7]  
[-3  0 -1]  
[-2  1 -1]]  
  
[[ 0  2  7]  
[-1  1  4]  
[ 2 -4  6]]  
  
[[ 6 -2  3]  
[ 1  4  0]  
[ 4  7 -2]]]
```

```
In [110... # random integer value on the interval(1,100)
```

```
print(np.random.randint(1,100,size=(3,4,5)))
```

```
[[[55 54 47 83 24]
 [99 54 23 7 66]
 [48 99 15 41 74]
 [ 1 25 4 69 5]]
```

```
[[ 1 57 28 24 72]
 [85 40 95 64 43]
 [12 20 5 85 96]
 [87 73 77 15 92]]
```

```
[[34 15 72 86 86]
 [97 86 34 71 55]
 [19 59 7 89 92]
 [ 7 14 32 85 38]]]
```

```
In [111... print(np.random.randint(1,1000,size=(15,15)))
print("====")
print(np.linalg.det(np.random.randint(1,100,size=(15,15))))
```

```
[[850 403 148 543 676 18 288 524 869 441 372 561 586 551 244]
 [995 746 127 953 450 977 421 467 517 36 697 772 560 321 396]
 [353 958 407 711 766 485 88 272 140 903 8 653 451 875 852]
 [595 915 226 627 146 457 548 789 938 530 944 64 936 579 835]
 [431 100 336 334 704 978 391 756 673 408 496 214 85 988 550]
 [870 789 376 995 592 836 333 180 748 145 138 930 165 438 162]
 [930 954 123 230 579 454 286 807 575 718 128 51 143 664 727]
 [794 695 30 596 401 273 929 779 565 796 964 804 624 663 976]
 [163 259 345 597 838 350 806 849 547 250 449 347 487 772 74]
 [173 129 196 947 10 205 911 478 549 373 600 683 847 954 484]
 [394 621 121 616 921 946 445 663 682 357 532 607 994 239 699]
 [466 690 236 646 542 724 151 483 599 603 708 710 398 462 595]
 [449 571 336 573 517 892 992 960 254 246 268 120 777 956 489]
 [472 596 551 329 344 982 348 839 589 923 889 869 614 391 467]
 [714 64 621 832 148 112 717 771 205 117 594 516 366 50 156]]
=====
-5.068415188986818e+28
```

```
In [112... # np.random.rand(number of matrix,row-th,column-th), if no input number of matrix it will dis
# np.random.random_sample(m.shape) : this case we need input m.shape such the form (a,b) wher
# respectively in order.
# np.random.randint(startValue,endValue,size(#matrix,row-th,column-th))
```

e.) Identity Matrix

```
In [113... print(np.identity(3))
```

```
[[1. 0. 0.]
 [0. 1. 0.]
 [0. 0. 1.]]
```

```
In [114... print(np.identity(10))
```

```
[[1. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 1. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 1. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 1. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 1. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 1. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 1. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 1. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 1.]]
```

```
In [115... print(np.identity(20))
```

```
In [116]: print(np.identity(20).trace())
```

20.0

```
In [117]: print(np.identity(100).trace())
```

100.0

```
In [118]: print(pow(np.identity(20),2))
```

```
In [119]: print(np.identity(20)**100)
```

```
In [120]: print(np.identity(2))
```

```
[[1. 0.]  
 [0. 1.]]
```

```
In [121]: # Repeat an array
```

```
a = np.array([1,2,3])
print(np.repeat(a,10,0))
# 0 is default value for the axis where set of axis = {-1,0,1}
```

```
[1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3]
```

In [122...]

`# np.repeat(matrix, #repetition, axis) : it displays that has the repetition.`

```
print(x)
print("\n")
print(np.repeat(x, 7, 0))
```

```
[[99. 99.]  
 [99. 99.]]
```

```
In [123...]: table = np.ones((5,5))
          print(table)
```

```
[[1. 1. 1. 1. 1.]  
 [1. 1. 1. 1. 1.]  
 [1. 1. 1. 1. 1.]  
 [1. 1. 1. 1. 1.]  
 [1. 1. 1. 1. 1.]]
```

In [124...]

```
z = np.zeros((3,3))
z[1,1] = 9
print(z)

[[0.  0.  0.]
 [0.  9.  0.]
 [0.  0.  0.]]
```

In [125...]

```
table[1:4,1:4] = z
print(table)

[[1.  1.  1.  1.  1.]
 [1.  0.  0.  0.  1.]
 [1.  0.  9.  0.  1.]
 [1.  0.  0.  0.  1.]
 [1.  1.  1.  1.  1.]]
```

In [126...]

```
w = np.identity(4)
v = np.identity(3)
x = np.ones((10,10))
y = np.zeros((8,8))
y[4,4] = 2
x[1:9,1:9] = y
x[1:5,1:5] = w
x[6:9,6:9] = v
print(x)
print('\n')
print(x.T)
print('\n')
print(np.linalg.det(x))

[[1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [1.  1.  0.  0.  0.  0.  0.  0.  1.]
 [1.  0.  1.  0.  0.  0.  0.  0.  1.]
 [1.  0.  0.  1.  0.  0.  0.  0.  1.]
 [1.  0.  0.  0.  1.  0.  0.  0.  1.]
 [1.  0.  0.  0.  0.  2.  0.  0.  1.]
 [1.  0.  0.  0.  0.  0.  1.  0.  1.]
 [1.  0.  0.  0.  0.  0.  0.  1.  0.]
 [1.  0.  0.  0.  0.  0.  0.  0.  1.]
 [1.  1.  1.  1.  1.  1.  1.  1.  1.]]
```

```
[[1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [1.  1.  0.  0.  0.  0.  0.  0.  0.  1.]
 [1.  0.  1.  0.  0.  0.  0.  0.  0.  1.]
 [1.  0.  0.  1.  0.  0.  0.  0.  0.  1.]
 [1.  0.  0.  0.  1.  0.  0.  0.  0.  1.]
 [1.  0.  0.  0.  0.  2.  0.  0.  0.  1.]
 [1.  0.  0.  0.  0.  0.  1.  0.  0.  1.]
 [1.  0.  0.  0.  0.  0.  0.  1.  0.  1.]
 [1.  0.  0.  0.  0.  0.  0.  0.  1.  1.]
 [1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]]
```

0.0

f.) Copying matrix

In [127...]

```
a = np.array([1,2,3])
b = a
b[0] = 100
print(a)

[100 2 3]
```

```
In [128... b = a.copy()  
b[0] = 99  
print(b)
```

```
[99 2 3]
```

```
In [129... c = np.matrix([[1,2,3],[9,8,7],[0,7,6]])  
print(c)  
print("\n")  
c = b.copy()  
print(c)
```

```
[[1 2 3]  
 [9 8 7]  
 [0 7 6]]
```

```
[99 2 3]
```

IV.) Mathematics

```
In [130... y = np.array([3,4,5,6])  
print(y)
```

```
[3 4 5 6]
```

```
In [131... y= y+2  
print(y)
```

```
[5 6 7 8]
```

```
In [132... m = y-14  
print(m)
```

```
[-9 -8 -7 -6]
```

```
In [133... w = y*2  
print(w)
```

```
[10 12 14 16]
```

```
In [134... x = y/2  
print(x)
```

```
[2.5 3. 3.5 4.]
```

```
In [135... from math import sqrt  
a = np.matrix([1,-1,-1/2,1/2,sqrt(2)/2,sqrt(3)/2])  
print(np.arccos(a))  
print(np.arcsin(a))  
print(np.arctan(a))  
print(1/np.arctan(a))
```

```
[[0. 3.14159265 2.0943951 1.04719755 0.78539816 0.52359878]  
 [[ 1.57079633 -1.57079633 -0.52359878 0.52359878 0.78539816 1.04719755]  
 [[ 0.78539816 -0.78539816 -0.46364761 0.46364761 0.61547971 0.71372438]]  
 [[ 1.27323954 -1.27323954 -2.15681043 2.15681043 1.62474893 1.40110108]]
```

↳ Linear Algebra

↳ Matrix multiplication

```
In [136... a = np.ones((2,3))  
print(a)
```

```
[[1. 1. 1.]  
 [1. 1. 1.]]
```

```
In [137... b = np.full((3,2),2)  
print(b)  
  
[[2 2]  
 [2 2]  
 [2 2]]
```

```
In [138... result = np.matmul(a,b)  
print(result)  
  
[[6. 6.]  
 [6. 6.]]
```

```
In [139... c = np.array([[1,2,3,4,5,6,7,8,9,10,1,2,3,4,5],[4,1,5,5,6,0,-4,-2,-1,6,6,7,8,9,0],[1,8,0,7,8,  
 [2,4,5,6,4,3,8,9,0,7,2,3,4,5,6],[3,8,9,0,4,3,5,6,7,8,0,0,1,2,3],[1,2,3,4,5,6,7,  
 [4,6,3,2,1,-9,-7,0,6,4,0,0,1,-2,3],[1,4,5,6,7,5,4,3,2,1,1,2,0,0,4],[1,2,3,4,5,6,  
 [2,3,4,5,6,7,8,9,0,1,2,4,3,4,5],[0,0,7,0,5,4,3,9,0,1,0,0,8,-9,-2],[1,2,3,4,5,6,  
 [1,2,3,4,5,6,7,8,9,0,9,7,5,4,3],[1,2,3,4,6,-9,-8,-6,-3,-1,2,3,5,6,7],[0,0,2,1,3  
 d = np.array([[1,2,3,4,5,6,7,8,9,10,1,2,3,4,5]])  
result = np.matmul(c,d.T)  
print(c,"\\n")  
print(d,"\\n")  
print(result)  
  
[[ 1  2  3  4  5  6  7  8  9 10  1  2  3  4  5]  
 [ 4  1  5  5  6  0 -4 -2 -1  6  6  7  8  9  0]  
 [ 1  8  0  7  8  9 -2 -1  3 -4 -8 -1  2  3  4]  
 [ 2  4  5  6  4  3  8  9  0  7  2  3  4  5  6]  
 [ 3  8  9  0  4  3  5  6  7  8  0  0  1  2  3]  
 [ 1  2  3  4  5  6  7  2  3  4  5  6  8  9  0]  
 [ 4  6  3  2  1 -9 -7  0  6  4  0  0  1 -2  3]  
 [ 1  4  5  6  7  5  4  3  2  1  1  2  0  0  4]  
 [ 1  2  3  4  5  6  7  8  9  0  9  9  5  4  8]  
 [ 2  3  4  5  6  7  8  9  0  1  2  4  3  4  5]  
 [ 0  0  7  0  5  4  3  9  0  1  0  0  8 -9 -2]  
 [ 1  2  3  4  5  6  7  8  9  0  0  9  8  4  3]  
 [ 1  2  3  4  5  6  7  8  9  0  9  7  5  4  3]  
 [ 1  2  3  4  6 -9 -8 -6 -3 -1  2  3  5  6  7]  
 [ 0  0  2  1  3  4  5  6  7  8  6  5  4  9  6]]  
  
[[ 1  2  3  4  5  6  7  8  9 10  1  2  3  4  5]]
```

```
[[440]  
 [158]  
 [132]  
 [355]  
 [336]  
 [300]  
 [ 39]  
 [218]  
 [383]  
 [310]  
 [151]  
 [358]  
 [354]  
 [-53]  
 [369]]
```

```
In [140... e = np.array([[1,2,3,4,5,6,7,8,9,10,1,2,3,4,5],[4,1,5,5,6,0,-4,-2,-1,6,6,7,8,9,0]])  
result = np.matmul(c,e.transpose())  
print(result)
```

```
[[440 158]
 [158 390]
 [132 66]
 [355 193]
 [336 124]
 [300 277]
 [ 39 89]
 [218 107]
 [383 211]
 [310 148]
 [151 24]
 [358 181]
 [354 197]
 [-53 245]
 [369 226]]
```

In [141...]

```
g = np.array([[1,2,3,4,5,6,7,8,9,10,1,2,3,4,5],
 [1,2,3,4,5,6,7,2,3,4,5,6,8,9,0],
 [4,6,3,2,1,-9,-7,0,6,4,0,0,1,-2,3],
 [4,1,5,5,6,0,-4,-2,-1,6,6,7,8,9,0]])
result = np.matmul(c,g.transpose())
print(result)
```

```
[[440 300 39 158]
 [158 277 89 390]
 [132 113 17 66]
 [355 294 20 193]
 [336 210 109 124]
 [300 375 -41 277]
 [ 39 -41 262 89]
 [218 174 17 107]
 [383 358 10 211]
 [310 284 -51 148]
 [151 96 -7 24]
 [358 337 -2 181]
 [354 346 -5 197]
 [-53 47 168 245]
 [369 322 18 226]]
```

In [142...]

```
h = np.array([[1,2,3,4,5,6,7,8,9,10,1,2,3,4,5],[1,2,3,4,5,6,7,2,3,4,5,6,8,9,0],[4,6,3,2,1,-9,
 [4,1,5,5,6,0,-4,-2,-1,6,6,7,8,9,0],[0,0,7,0,5,4,3,9,0,1,0,0,8,-9,-2],[1,2,3,4,5
 result = np.matmul(c,h.transpose())
print(result)
```

```
[[440 300 39 158 151 358]
 [158 277 89 390 24 181]
 [132 113 17 66 38 175]
 [355 294 20 193 154 312]
 [336 210 109 124 156 255]
 [300 375 -41 277 96 337]
 [ 39 -41 262 89 -7 -2]
 [218 174 17 107 122 213]
 [383 358 10 211 151 446]
 [310 284 -51 148 170 331]
 [151 96 -7 24 330 185]
 [358 337 -2 181 185 455]
 [354 346 -5 197 161 413]
 [-53 47 168 245 -92 -13]
 [369 322 18 226 61 326]]
```

b.) Statistic

In [143...]

```
a = np.matrix([[1,2,3],[4,5,6],[7,8,9]])
print(a)
```

```
[[1 2 3]
 [4 5 6]
 [7 8 9]]
```

```
In [144... print(np.min(a[0,:])) # show minimum value at zero(th) row and any column
print(np.min(a[1,:]))
print(np.min(a[2,:]))
```

```
1
4
7
```

```
In [145... print(np.min(a[:,0]))
print(np.min(a[:,1]))
print(np.min(a[:,2]))
```

```
1
2
3
```

```
In [146... print(np.min(a))
print(np.max(a))
print(np.average(a))
```

```
1
9
5.0
```

```
In [147... print(a)
```

```
[[1 2 3]
 [4 5 6]
 [7 8 9]]
```

```
In [148... print(np.min(a, axis=-1))
```

```
[[1]
 [4]
 [7]]
```

```
In [149... print(np.min(a, axis=0))
```

```
[[1 2 3]]
```

```
In [150... print(np.min(a, axis=1))
```

```
[[1]
 [4]
 [7]]
```

```
In [151... print(np.max(a, axis=0))
```

```
[[7 8 9]]
```

```
In [152... print(np.average(a, axis=0))
```

```
[[4. 5. 6.]]
```

```
In [153... print(np.average(a, axis=1))
```

```
[[2.]
 [5.]
 [8.]]
```

```
In [154... Sum = np.sum(a)
print("sum all element in a = ",Sum)
```

```
sum all element in a = 45
```

```
In [155...]  
S = np.sum(a, axis=0)  
print(S) # show the total value for each column  
[[12 15 18]]
```

c.) Reorganizing array

```
In [156...]  
a1 = np.matrix([[1,2,3,4],[5,6,7,8]])  
print(a1)  
[[1 2 3 4]  
 [5 6 7 8]]
```

```
In [157...]  
a2 = a1.reshape((8,1))  
print(a2)  
[[1]  
 [2]  
 [3]  
 [4]  
 [5]  
 [6]  
 [7]  
 [8]]
```

```
In [158...]  
a3 = a1.reshape((4,2))  
print(a3)  
[[1 2]  
 [3 4]  
 [5 6]  
 [7 8]]
```

```
In [159...]  
a4 = a1.reshape((1,8))  
print(a4)  
[[1 2 3 4 5 6 7 8]]
```

```
In [160...]  
# vertically Stacking Vectors(Combine together)  
print("vertical stack")  
v1 = np.matrix([1,2,3,4])  
v2 = np.matrix([5,6,7,8])  
x = np.vstack([v1,v2])  
print(x)  
  
vertical stack  
[[1 2 3 4]  
 [5 6 7 8]]
```

```
In [161...]  
y = np.stack([v1,v2,v1,v2])  
print(y)  
[[1 2 3 4]  
 [5 6 7 8]  
 [1 2 3 4]  
 [5 6 7 8]]
```

```
In [162...]  
# Horizontal Stack  
print('Horizontal Stack')  
h1 = np.ones((2,4))  
print(h1)  
  
Horizontal Stack  
[[1. 1. 1. 1.]  
 [1. 1. 1. 1.]]
```

```
In [163... h2 = np.zeros((2,2))
print(h2)
```

```
[[0. 0.]
 [0. 0.]]
```

```
In [164... # After horizontal stack
print(np.hstack((h1,h2)))
```

```
[[1. 1. 1. 1. 0. 0.]
 [1. 1. 1. 1. 0. 0.]]
```

→ Boolean masking and advanced indexing

```
In [165... print(k)
```

```
[[100 100 100 100 100 100 100 100 100 100 100 100]
 [200 0 0 0 0 0 0 0 0 0 400]
 [200 8 -6 7 8 9 -2 -1 3 400]
 [200 4 -6 6 4 3 8 9 0 400]
 [200 8 -6 0 4 3 5 6 7 400]
 [200 2 -6 4 5 6 7 8 9 400]
 [200 6 -6 2 1 -9 -7 0 6 400]
 [200 4 -6 6 7 5 4 3 2 2 400]
 [200 2 -6 4 5 6 7 8 9 400]
 [100 100 100 100 100 100 100 100 100 100]]
```

```
In [166... a = k[k<10]
print(a)
```

```
[ 0 0 0 0 0 0 0 0 8 -6 7 8 9 -2 -1 3 4 -6 6 4 3 8 9 0
 8 -6 0 4 3 5 6 7 2 -6 4 5 6 7 8 9 6 -6 2 1 -9 -7 0 6
 4 -6 6 7 5 4 3 2 2 -6 4 5 6 7 8 9]
```

```
In [167... print(k[k>100])
```

```
[200 400 200 400 200 400 200 400 200 400 200 400 200 400 200 400]
```

```
In [168... print(k[k>200])
```

```
[400 400 400 400 400 400 400 400]
```

```
In [169... # Load data from file
file = np.genfromtxt('data2.txt',delimiter=',')
file.astype('int32')
print(file)
```

```
[ 1.  2.  3.  4.  5.  6.  7.  8.  9. 10. 11. 12. 13. 14.
 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28.
 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 38. 40. 78. 90.
 9. 87. 56. 78. 99. 900. 987. 865. 453. 100.]
```

```
In [170... print(file>50)
```

```
[False False False False False False False False False False
 False False False False False False False False False False
 False False False False False False False False False False
 False False False False True True False True True True True
 True True True True]
```

```
In [171... # test condition whether it is true or false with value < 59
print(file<50)
```

```
[ True  
 True  True  True  True  True  True  True  True  True  True  True  True  True  
 True  True  True  True  True  True  True  True  True  True  True  True  True  
 True  True  True  True  False  False  True  False  False  False  False  False  False  
 False  False  False  False]
```

In [172...]
`# Show the elements that are smaller than 10
print(file[file<10])`

```
[1. 2. 3. 4. 5. 6. 7. 8. 9. 9.]
```

In [173...]
`# show the element that are smaller than 50.
print(file[file<50])`

```
[ 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18.  
 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36.  
 37. 38. 39. 40. 9.]
```

In [174...]
`print(file[file>100])`

```
[900. 987. 865. 453.]
```

In [175...]
`x = np.all(file>50, axis=0)
print(x)`

```
False
```

In [176...]
`# Show the reality of element that are neither larger than 50 and less than 100.
y = (~(file>50)&(file<100))
print(y)`

```
[ True  
 True  True  True  True  True  True  True  True  True  True  True  True  True  
 True  True  True  True  True  True  True  True  True  True  True  True  True  
 True  True  True  True  False  False  True  False  False  False  False  False  False  
 False  False  False  False]
```

In [177...]
`# ~ = not. For example a & b = true then, ~(a & b) = false.
z = ~y
print(z)`

```
[False False False False False False False False False False  
 False False False False False False False False False False  
 False False False False False False False False False False  
 False False False False True  True False  True  True  True  True  
  True  True  True  True]
```

f.) Index with a list

In [178...]
`a = np.array([1,2,3,4,5,6,7,8,9])
print(a)
display the value at index[1], index[2] and index[8]
print(a[[1,2,8]])`

```
[1 2 3 4 5 6 7 8 9]  
[2 3 9]
```

In [179...]
`y = np.array([1,2,3,4,5])
for i in range(1,6):
 if(i==1):
 x = np.array([i,i+1,i+2,i+3,i+4])
 elif(i==2):
 x = np.array([i+4,i+5,i+6,i+7,i+8])
 elif(i==3):
 x = np.array([i+8,i+9,i+10,i+11,i+12])
 elif(i==4):
 x = np.array([i+13,i+14,i+15,i+16,i+17])`

```
elif(i==5):
    x = np.array([i+17,i+18,i+19,i+20,i+21])
print(x)
```

```
[1 2 3 4 5]
[ 6  7  8  9 10]
[11 12 13 14 15]
[17 18 19 20 21]
[22 23 24 25 26]
```