Hw #2

(a) Since $(A-\lambda I)v = 0$.

then $\det \begin{bmatrix} -4-\lambda & -7 \\ 4 & 2-\lambda \end{bmatrix} = 0$

$= (-4-\lambda)(2-\lambda) + 28 = 0$ 4

$= \lambda^2 + 2\lambda - 8 + 28 = 0$ 76 19

$= \lambda^2 + 2\lambda + 20 = 0$

$$\lambda_1 = \frac{-2+\sqrt{4-80}}{2} = \frac{-2+\sqrt{-76}}{2} = -1+\sqrt{19}\,i$$

$$\lambda_2 = \frac{-2-\sqrt{4-80}}{2} = \frac{-2-\sqrt{-76}}{2} = -1-\sqrt{19}\,i$$

Therefore $\sigma(A) = \{-1+\sqrt{19}\,i,\ -1-\sqrt{19}\,i\}$

Moreover, $|\lambda_1| = \sqrt{(-1)^2+(\sqrt{19})^2} = \sqrt{20}$

$|\lambda_2| = \sqrt{(-1)^2+(-\sqrt{19})^2} = \sqrt{20}$

So, $\rho(A) = \sqrt{20}$

(b) $\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{m} |a_{ij}|$

so $\|A\|_1 = \max(|-4| + |4|, |-7| + |2|)$

$= \max(8, 9)$

$= \boxed{9}$

$16+49$

$\|A\|_2 = \sqrt{\text{The largest eigenvalue of } AA^T}$ $-16-14$

$AA^T = \begin{bmatrix} -4, & -7 \\ 4, & 2 \end{bmatrix} \begin{bmatrix} -4 & 4 \\ -7 & 2 \end{bmatrix}$

$= \begin{bmatrix} 65, & -30 \\ -30, & 20 \end{bmatrix}$

$\det(AA^T - \lambda I) = 0$

so, $\det \begin{bmatrix} 65-\lambda, & -30 \\ -30, & 20-\lambda \end{bmatrix} = 0$

$(65-\lambda)(20-\lambda) + 900 = 0$

$\lambda^2 - 85\lambda + 1300 + 900 = 0$

$\lambda^2 - 85 + 2200 = 0$

$\lambda_1 = 80 \quad \lambda_2 = 5$

So $\|A\|_2 = \boxed{\sqrt{80}}$

$$\|A\|_\infty = \|A^T\|_1$$

$$A^T = \begin{bmatrix} -4 & 4 \\ -7 & 2 \end{bmatrix}$$

$$\|A^T\|_1 = \max(11, 6)$$
$$= 11$$

So $\|A\|_\infty = \boxed{11}$

1(c) right singular vector = eigenvector of A^TA

$$A^TA = \begin{bmatrix} -4 & 4 \\ -7 & 2 \end{bmatrix} \begin{bmatrix} -4 & -7 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 32 & 36 \\ 36 & 53 \end{bmatrix}$$

Since $\sigma(A^TA) = \sigma(AA^T)$, from (b), we know $\lambda_1 = 80$ $\lambda_2 = 5$,

then $\begin{bmatrix} 32-80 & 36 \\ 36 & 53-80 \end{bmatrix} v_1 = 0$

3

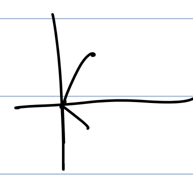
4

$$\begin{bmatrix} -48 & 36 \\ 36 & -27 \end{bmatrix} v_1 = 0$$

$$v_1 = \begin{bmatrix} \frac{3}{4} \\ 1 \end{bmatrix} = \frac{v_1}{\|v_1\|} = \begin{bmatrix} \frac{3}{5} \\ \frac{4}{5} \end{bmatrix}$$

$$\begin{bmatrix} 32-5 & 36 \\ 36 & 53-5 \end{bmatrix} v_2 = 0$$

$$\begin{bmatrix} 27 & 36 \\ 36 & 48 \end{bmatrix} v_2 = 0$$



$$V_2 = \begin{bmatrix} \frac{4}{3} \\ -1 \end{bmatrix} = \frac{v_2}{\|v_2\|} = \begin{bmatrix} \frac{4}{5} \\ -\frac{3}{5} \end{bmatrix}$$

Thus. we find right Singular vectors $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$

left Singular vector = eigenvectors of AA^T

$$AA^T = \begin{bmatrix} 65 & -30 \\ -30 & 20 \end{bmatrix}$$

calculating eigenvectors.

$$\begin{bmatrix} 65-80 & -30 \\ -30 & 20-80 \end{bmatrix} V_1 = 0$$

$$\begin{bmatrix} -15 & -30 \\ -30 & -60 \end{bmatrix} V_1 = 0$$

$$V_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \frac{V_1}{\|V_1\|} = \begin{bmatrix} -\frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}.$$

For V_2.

$$\begin{bmatrix} 65-5 & -30 \\ -30 & 20-5 \end{bmatrix} V_2 = 0$$

$$\begin{bmatrix} 60 & -30 \\ -30 & 15 \end{bmatrix} V_2 = 0$$

$$V_2 = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} = \frac{V_2}{\|V_2\|} = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}.$$

and thus, we find right singular vetors $\begin{bmatrix} \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$

Singular values are the square root of eigenvalues in (b). so

$$\begin{bmatrix} \sqrt{80} & 0 \\ 0 & \sqrt{5} \end{bmatrix}$$

$$A = U \Sigma V^T = \begin{bmatrix} \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{80} & 0 \\ 0 & \sqrt{5} \end{bmatrix} \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{-3}{5} \end{bmatrix}$$

(d) Since $B'B = A'A$, $BB' = AA'$, they should have the same eigenvalues. So switch the position of the two eigenvalues.

$$S' = \begin{bmatrix} \sqrt{5} & 0 \\ 0 & \sqrt{80} \end{bmatrix}.$$

in correspondence of the S', we have to switch the position of eigenvectors as well.

$$U' = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$

$$V' = \begin{bmatrix} \frac{4}{5}, & \frac{3}{5} \\ \frac{-3}{5}, & \frac{4}{5} \end{bmatrix}$$

$$B = U' \cdot S' \cdot (V')^T$$

$$= \begin{bmatrix} \frac{1}{\sqrt{5}}, & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}}, & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{5}, & 0 \\ 0, & \sqrt{80} \end{bmatrix} \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1, & -8 \\ 2, & 4 \end{bmatrix} \cdot \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{5} + \frac{24}{5} & \frac{3}{5} - \frac{32}{5} \\ \frac{8}{5} - \frac{12}{5}, & \frac{6}{5} + \frac{16}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{28}{5} & -\frac{29}{5} \\ -\frac{4}{5}, & \frac{22}{5} \end{bmatrix}$$

(e). The Symmetric matrix I choose is $B'B$.

① as we show above, one kind of Schur decomposition is

$$B'B = \begin{bmatrix} \frac{4}{5}, & \frac{3}{5} \\ -\frac{3}{5}, & \frac{4}{5} \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 80 \end{bmatrix} \begin{bmatrix} \frac{4}{5}. & -\frac{3}{5} \\ \frac{3}{5}, & \frac{4}{5} \end{bmatrix}$$

② or, we can change the sign of eigenvectors while still keeping orthogonality

$$B'B = \begin{bmatrix} \frac{4}{5}, & \frac{-3}{5} \\ -\frac{3}{5} & -\frac{4}{5} \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 80 \end{bmatrix} \begin{bmatrix} \frac{4}{5}, & \frac{-3}{5} \\ \frac{-3}{5}, & \frac{-4}{5} \end{bmatrix}.$$

(3). or, we can change the positin of eigenvalues, and change the positio of. eigenvector accordingly.

$$B'B = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix} \begin{bmatrix} 80 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{-3}{5} \end{bmatrix}$$