MM - ASSIGN — 4

DB

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Ans 1- Let A^i_j be a mixed tensor of rank 2 in (x^i) for $i = 1 - N$ coordinates

Let the component be transformed

(i). (ii). (iii).

$x^i \rightarrow x'^i \rightarrow x''^i$

$A^i_j \rightarrow A'^i_j \rightarrow A''^i_j$

In $x^i \rightarrow x'^i$, by law of transformation of mixed tensors, we have

$$A'^i_j = \frac{\partial x'^i}{\partial x^m} \cdot \frac{\partial x^n}{\partial x'^j} A^m_n \quad \text{— ①}$$

In $x'^i \rightarrow x''^i$, " " " " " "

$$A''^p_q = \frac{\partial x''^p}{\partial x'^i} \cdot \frac{\partial x'^j}{\partial x''^q} \cdot A'^i_j$$

Subst. ① in ②, we get -

$$A''^p_q = \frac{\partial x''^p}{\partial x'^i} \cdot \frac{\partial x'^j}{\partial x''^q} \cdot \frac{\partial x'^i}{\partial x^m} \cdot \frac{\partial x^n}{\partial x'^j} \cdot A^m_n$$

$$= \frac{\partial x''^p}{\partial x'^i} \cdot \frac{\partial x'^i}{\partial x^m} \cdot \frac{\partial x'^j}{\partial x''^q} \cdot \frac{\partial x^n}{\partial x'^j} A^m_n$$

$$\Rightarrow A''^{p}_{q} = \frac{\partial x''^{p}}{\partial x^{m}} \cdot \frac{\partial x^{n}}{\partial x''^{q}} \cdot A^{m}_{n}$$

This proves that if we make the transformation ~~for~~ from (i) to (iii), we get same law of transformation. Hence Proved

Ans 2. Let there be two tensors $A^{\bar{r}_1\bar{r}_2 \cdots \bar{r}_r}_{\bar{s}_1\bar{s}_2 \cdots \bar{s}_s}$ & $\bar{A}^{\bar{r}'_1\bar{r}'_2 \cdots \bar{r}'_{r'}}_{\bar{s}'_1\bar{s}'_2 \cdots \bar{s}'_{s'}}$ of rank (r,s) & (r',s')

When we take inner product of both, we get -

$$A^{\bar{r}_1\bar{r}_2 \cdots \bar{r}_r}_{\bar{s}_1\bar{s}_2 \cdots \bar{s}_s} \cdot \bar{A}^{\bar{r}'_1\bar{r}'_2 \cdots \bar{r}'_{r'}}_{\bar{s}'_1\bar{s}'_2 \cdots \bar{s}'_{s'}} = \bar{\bar{A}}^{\bar{r}_1\bar{r}_2 \cdots \bar{r}_r \bar{r}'_1\bar{r}'_2 \cdots \bar{r}'_{r'}}_{\bar{s}_1\bar{s}_2 \cdots \bar{s}_s \bar{s}'_1\bar{s}'_2 \cdots \bar{s}'_{s'}}$$

$\Rightarrow \bar{\bar{A}}$ has a rank of type $(r+r', s+s')$ Proved

Ans 3. There are two types of vector covarient $= A_i$ & contravariant $= A^i$

Now we have 3 possibilities -

Case ① Multiply 2 covarient tensors A_i & B_j

By open product of A_i & B_j, we get - $A_i \cdot B_j = C_{ij}$

So the open product ~~of~~ is a covarient tensor of rank type $(0,2)$ & order 2.

Case ② → Multiply 2 contravariant tensors A^i & B^j

$$A^i . B^j = C^{ij}$$

The open product of A^i & B^j is a contravariant tensor of rank type (2,0) & order 2.

Case ③ → Multiply one covariant tensor A_i & one contravariant tensor B^j.

$$A_i . B^j = C_i^j$$

Open Product is a mixed tensor of rank type (1,1) & order 2.

So, open product of 2 vectors is a tensor of order 2.

The converse of this may not be true as all tensors cannot be written as open product of 2 vectors.

⇒ The tensor of order 2 is not necessarily an open product of two vectors. Proved

Ans 4. Let $A^{i_1 i_2 \dots i_n}_{j_1 j_2 \dots j_m}$ be a mixed tensor of type (n, m), order $= n+m$

& $B^{p_1 p_2 \dots p_k}_{q_1 q_2 \dots q_l}$ be " " " " type (K, l) order $= K+l$

The outer product ~~of~~ by defination is equal to -

$$A^{i_1 i_2 \dots i_n}_{j_1 j_2 \dots j_m} \cdot B^{p_1 p_2 \dots p_k}_{q_1 q_2 \dots q_l} = C^{i_1 i_2 \dots i_n p_1 p_2 \dots p_k}_{j_1 j_2 \dots j_m q_1 q_2 \dots q_l}$$

Now C has type $(n+K, m+l)$

& Order of $C = (n+K+m+l)$

$= (n+m) + (K+l)$

$=$ Order of A + Order of B

Hence, the outer product of 2 tensors is a tensor whose order is the sum of orders of the two tensors. **Proved**

Ans 5. Contraction is defined as the process of reducing a higher order mixed ~~ten~~ tensor to a lower order tensor when the indices of covarient & contravarient part are same.

The inner product ~~of~~ is the outer product followed by contraction of the outer product.

$$A^p_r \cdot B^{rs}_t = C^{prs}_{rt}$$ (by open ~~product~~ product rule)

Now by doing contraction, we get -

$$C^{prs}_{rt} = C^{ps}_t$$

This is inner Product of A^p_r & B^{rs}_t of rank type (2, 1) & order 3.

Proved