ASSIGNMENT - 1

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Ans 1. Yes, $y=2x$ is a linear combination of the functions $y_1 = x$ & $y_2 = x^2$

$\therefore \quad \alpha y_1 + \beta y_2 = y$

$\alpha(x) + \beta(x^2) = 2x$

$\alpha = 2, \beta = 0$ is a soln.

Another Method, if $y=2x$ is a linear combination of y_1 & y_2, then y, y_1, y_2 must be linearly dependent.

$$\therefore W(y, y_1, y_2) = \begin{vmatrix} 2x & x & x^2 \\ 2 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix} = 2(2x-2x) = 0$$

As $W(y, y_1, y_2) = 0$, Hence they are linearly dependent.

Therefore, $y = 2x$ is a linear combination of $y_1 = x$ & $y_2 = x^2$.

Ans 2.
$$W(y, y_1, y_2) = \begin{vmatrix} \sin x & \cos x & \sin(x+1) \\ \cos x & -\sin x & \cos(x+1) \\ -\sin x & -\cos x & -\sin(x+1) \end{vmatrix}$$

$$= \sin x\left[\sin x \sin(x+1) + \cos x \cos(x+1)\right] - \cos x\left[-\cos x \sin(x+1) + \sin x \cos(x+1)\right] + \sin(x+1)\left[-\cos^2 - \sin^2 x\right]$$

$$= (\sin^2 x + \cos^2 x)(\sin(x+1)) - \sin(x+1)(\sin^2 x + \cos^2 x) + \sin x \cos x \cos(x+1) - \sin x \cos x \cos(x+1)$$

$$= 0$$

Therefore, y, y_1, y_2 are linearly dependent

$\therefore y_3$ is a linear combination of y_1 & y_2.

Ans 3. By the definition of linear independence / dependence

$$\alpha f_1 + \beta f_2 = 0$$

if $\alpha = 0, \beta = 0$, then $\underline{\underline{LI}}$ otherwise $\underline{\underline{LD}}$.

(i) (a) $f(x) = 9\cos 2x$, $g(x) = 2\cos^2 x - 2\sin^2 x$

$$\alpha f(x) + \beta g(x) = 9\alpha \cos 2x + 2\beta \cos^2 x - 2\beta \sin^2 x$$

$$= 9\alpha \cos 2x + 2\beta \cos 2x$$

$$= (9\alpha + 2\beta) \cos 2x = 0$$

$\Rightarrow 9\alpha + 2\beta = 0$

Let $\alpha = 1$, $\beta = -\frac{9}{2}$ $\therefore$ They are $\underline{\underline{L\cdot D}}$.

(b) $f(t) = 2t^2$, $g(t) = t^4$

$\alpha f(t) + \beta g(t) = 0$

$2\alpha t^2 + \beta t^4 = 0$

Only $\alpha = 0, \beta = 0$ is the soln. ~~they~~

Therefore they are $\underline{\underline{L\cdot I}}$

(ii) Verify with wronskian,

(a) $\begin{vmatrix} 9\cos 2x & 2\cos^2 x - 2\sin^2 x \\ -18\sin 2x & -4\cos x \sin x - 4\sin x \cos x \end{vmatrix}$

$= \begin{vmatrix} 9\cos 2x & 2\cos 2x \\ -18\sin 2x & -4\sin 2x \end{vmatrix} = -36\sin 2x \cos 2x + 36 \sin 2x \cos 2x$

$= 0$

Hence, they are $\underline{\underline{L\cdot D}}$.

(b) $\begin{vmatrix} 2t^2 & t^4 \\ 4t & 4t^3 \end{vmatrix} = 8t^5 - 4t^5 = 4t^5 \neq 0$

Hence they are $\underline{L.I.}$

Ans 5. Given, $\frac{dy}{dx} = \frac{4x^2 - 7x}{3y^2+2}$, $y(1) = 1$

This is I order differential eqn. & is seperable form.

By rearranging, we get, $3y^2 dy + 2dy = 4x^2 dx - 7x\, dx$

Integrating, $\int 3y^2 dy + 2\int dy = 4\int x^2 dx - 7\int x\, dx + C_1$

$$y^3 + 2y = \frac{4}{3}x^3 - \frac{7x^2}{2} + C_1$$

$$6y^3 + 12y = 8x^3 - 21x^2 + C_1$$

But $y(1) = 1$, $6(1) + 12(1) = 8(1) - 21(1) + C_1$

$$\boxed{C_1 = 31}$$

Hence the soln is – $\boxed{6y^3 + 12y = 8x^3 - 21x^2 + 31}$

Ans 6. Given, $\frac{d^2y}{dx^2} = 2$, on integrating –

$$\frac{dy}{dx} = 2x + C_1 \quad \text{—— (1)}$$

$$y = x^2 + C_1 x + C_2 \quad \text{— (2)}$$

$y'(0) = 6$, from (1) $\boxed{C_1 = 6}$

$y(0) = 0$, from (2) $\boxed{C_2 = 0}$

Solution $\rightarrow$ $\boxed{y = x^2 + 6x}$ Ans

Ans 7 Given $\frac{d^2y}{dx^2} + 4y = 0$

(i) To find CF. $m^2 + 4 = 0$

$m = \pm 2i$

$$y = c_1 e^{2ix} + c_2 e^{-2ix}$$

$PI = 0$, since it is Homogenous differential eq.

BVP, $y(0) = -2$ & $y(\pi/4) = 10$

$y(0) = c_1 + c_2$

$\boxed{c_1 + c_2 = -2}$ — ①

$y(\pi/4) = 10$

$c_1 e^{i\pi/2} + c_2 e^{-i\pi/2} = 10$

$ic_1 - ic_2 = 10$

$\boxed{c_1 - c_2 = 10}$ — ②

from ① & ②, $c_1 = 4$, $c_2 = -6$

solution → $y = 4e^{2ix} - 6e^{-2ix}$

(ii) $y(0) = -2$

$c_1 + c_2 = -2$

$y(2\pi) = -2$

$c_1 + c_2 = -2$

Infinitely many soln. exits, family → $\boxed{y = c_1 e^{2ix} + c_2 e^{-2ix}}$

(iii) $y(0) = -2$

$c_1 + c_2 = -2$

$y(2\pi) = 3$

$c_1 + c_2 = 3$

No soln. exists. Ans