Assignment-2

RAHUL SAINI
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Ans 1: (i) $2y'' + 18y = 6\tan(3t)$

G.S $\rightarrow$ $y'' + 9y = 0$

CF $\rightarrow$ $r^2 + 9 = 0$ $\quad r = \pm 3i$ $\quad y = C_1\cos 3t + C_2 \sin 3t$

$y_1 = \cos 3t$, $y_2 = \sin 3t$

$$W(y_1(t), y_2(t)) = \begin{vmatrix} \cos 3t & \sin 3t \\ -3\sin 3t & 3\cos 3t \end{vmatrix} = 3$$

, $y = u_1 y_1 + u_2 y_2$

$$u_1 = -\int \frac{3\sin 3t \tan 3t}{3}\, dt = \int(\cos 3t - \sec 3t)\, dt$$

$$= \frac{\sin 3t}{3} - \frac{1}{3}\ln|\sec(3t) + \tan 3t|$$

$$u_2 = \int \frac{3\cos 3t \tan 3t}{3} = \int \sin 3t = -\frac{\cos 3t}{3}$$

Final soln. $= G.S. + y_P$

$$= C_1\cos 3t + C_2 \sin 3t - \frac{\cos 3t}{3}\ln|\sec 3t + \tan 3t|$$

Ans 1. (ii) $y'' - 3y' + 2y = e^{3t}$

CF $\rightarrow$ $r^2 - 3r + 2$ $\quad r = 1, 2$

$y_1 = e^t$, $y_2 = e^{2t}$

$$W = \begin{vmatrix} e^t & e^{2t} \\ e^t & 2e^{2t} \end{vmatrix} = e^{3t}$$

$\rightarrow y = C_1 e^t + C_2 e^{2t}$

$y_P = u_1 y_1 + u_2 y_2$

$$\cancel{= \int \frac{e^{2t}\cdot e^{3t}}{e^{3t}}\,dt = \int e^{2t}\,dt = -\frac{1}{2}e^{2t}}$$

$$u_1' = -\frac{e^{2t}\cdot e^{3t}}{e^{3t}}, \quad u_1 = -\int e^{2t}\,dt = -\frac{1}{2}e^{2t}$$

$$u_2' = \frac{e^{t}\, e^{3t}}{e^{3t}}, \quad u_2 = \int e^{t}\,dt = e^{t}$$

$$y = c_1 e^{t} + c_2 e^{2t} + \frac{1}{2}e^{3t}$$ Ans

Ans 2:

$$ty'' - (t+1)y' + y = t^2$$

$$y'' - \left(1+\frac{1}{t}\right)y' + \frac{1}{t}y = t$$

$e^{t} \rightarrow y_1$

$t+1 \rightarrow y_2$

$$y = c_1 e^{t} + c_2 (t+1)$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$W = \begin{vmatrix} e^{t} & t+1 \\ e^{t} & 1 \end{vmatrix} = -e^{t}t$$

$$u_1' = -\frac{(t+1)t^2}{te^{t}} \Rightarrow u_1 = -\int t^2 e^{-t} - \int t e^{-t} = -(t+2)e^{-t}$$

$$u_2' = \frac{-e^{t}t}{te^{t}} \Rightarrow u_2 = -\int dt = -t$$

$$y_p = u_1 y_1 + u_2 y_2 = -(t^2+2t+2)$$

General soln $\rightarrow$ $y = c_1 e^{t} + c(t+1) - (t^2+2t+2)$ Ans

Ans 3. $y''' + A(x)y'' + B(x)y' + C(x)y = r(x)$ — ①

y_1, y_2 & y_3 are soln. of the homogenous eq. $y''' + A(x)y'' + B(x)y' + C(x)y = 0$

Then by variation of parameters, $y_P = u y_1 + v y_2 + w y_3$

y_P satisfies eq. ①

$y_P' = u' y_1 + u y_1' + v' y_2 + v y_2' + w' y_3 + w y_3'$ — ②

$= u y_1' + v y_2' + w y_3' + (u' y_1 + v' y_2 + w' y_3)$

Let $(u' y_1 + v' y_2 + w' y_3) = 0$

$\Rightarrow y_P' = u y_1' + v y_2' + w y_3'$ — ③

$y_P'' = u' y_1' + u y_1'' + v' y_2' + v y_2'' + w' y_3' + w y_3''$

Again, let $u' y_1' + v' y_2' + w' y_3' = 0$

$\Rightarrow y_P'' = u y_1'' + v y_2'' + w y_3''$ — ④

$y'''_P = u y_1''' + v y_2''' + w y_3''' + u' y_1'' + v' y_2'' + w' y_3''$ — ⑤

Putting ①, ②, ③, ④, ⑤ together, we get -

$u(y_1''' + A y_1'' + B y_1' + C y_1) + v(y_2''' + A y_2'' + B y_2' + C y_2)$

$+ w(y_3''' + A y_3'' + B y_3' + C y_3) + u' y_1'' + v' y_2'' + w' y_2'' = r$

Now, as y_1, y_2, y_3 are soln. of ①,

$\Rightarrow y_i''' + A y_i'' + B y_i' + C y_i = 0$ for $i = 1, 2, 3$

Therefore, $r = u' y_1'' + v' y_2'' + w' y_3''$

Solving for u', v', w',

$$\begin{bmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{bmatrix} \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix}$$

$$u' = \frac{W_1}{W}, \quad v' = \frac{W_2}{W}, \quad w' = \frac{W_3}{W}$$ (by Cramer's Rule)

$W =$ Wronskian (y_1, y_2, y_3)

so, $u = \int \frac{W_1}{W}\,dx$, $v = \int \frac{W_2}{W}\,dx$ $w = \int \frac{W_3}{W}\,dx$

and Particular Integral, $y_p = u y_1 + v y_2 + w y_3$

Hence Proved