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Course: B-tech	Sem/year: I/I	Branch: CSE

Subject Name: Engineering Calculus

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Assignment: 2 (Set: B)

1. Using Beta and Gamma functions evaluate the following:

(a) $\int_0^\infty x^{1/4} e^{-\sqrt{x}} dx$

Ans

$$\int_0^\infty x^{1/4} e^{-\sqrt{x}} dx$$

$$\Rightarrow \int_0^\infty e^{-x} x^{n-1} dx = \Gamma n$$

$$\begin{Bmatrix} \sqrt{x} = t^2 \\ dx = 2t dt \end{Bmatrix}$$

$$\Rightarrow 2\int_0^\infty t^{1/2} \cdot e^{-t} t dt$$

$$\Rightarrow 2\int_0^\infty t^{3/2} \cdot e^{-t} dt = 2 \times \Gamma\frac{5}{2} = 2 \times \frac{3}{2}\Gamma\frac{3}{2}$$

$$= 2 \times \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi} = \frac{3\sqrt{\pi}}{2} \text{ Ans}$$

(b) $\int_0^2 (8-x)^{-1/3} dx$

Ans let $x^3 = t \Rightarrow x = t^{-1/3}$

$$3x^2 dx = dt$$

$$dx = \frac{dt}{3x^2}$$

$$dx = \frac{dt}{3t^{1/3}}$$

$$\int_0^2 t^{1/3} (8-t)^{1/3} \frac{dt}{3t^{2/3}}$$

$$\frac{1}{3}\int_0^2 t^{-1/3} (8-t)^{1/3} dt$$

Name: Gaurav Mittal Sec: AT Roll no. 27

(c) $\int_0^\infty \frac{dx}{1+x^6}$

<u>Ans</u> $x^6 = y$

$x = y^{1/6}$

$dx = \frac{1}{6} y^{-5/6} dy$

$\frac{1}{6}\int_0^\infty \frac{y^{-5/6}}{(1+y)^1} = \frac{1}{6}\int_0^\infty \frac{y^{1/6-1}}{(1+y)^{1/6+1/5}}$

$\Rightarrow \frac{1}{6} B(1/6, 1/5) \Rightarrow \frac{\frac{1}{6}\Gamma\frac{1}{6}\Gamma\frac{5}{6}}{\pi} = \frac{1}{6} \times \frac{\pi}{\sin\pi/6}$

$\Rightarrow \frac{2}{6}\pi = \pi/3$ <u>A</u>

(d) $\int_0^{\pi/2} \tan^n x\, dx$

<u>Ans</u> $\int_0^{\pi/2} \frac{\sin^n x}{\cos^n x} dx = \int_0^{\pi/2} \sin^{n} x \cos^{-n} x\, dx$

$= \frac{\Gamma\frac{n+1}{2}\Gamma\frac{1-n}{2}}{2\Gamma\frac{n-n+2}{2}}$

$= \frac{\Gamma\frac{n+1}{2}\Gamma\frac{n-1}{2}}{2\pi} \Rightarrow \frac{1}{2}\Gamma\frac{n+1}{2}\Gamma\frac{1-n}{2} \quad \therefore \left(\frac{1-n}{2} = 1 - \left(\frac{n+1}{2}\right)\right)$

$\Rightarrow \frac{1}{2} \times \frac{\pi}{\sin\left(\frac{n+1}{2}\right)\pi} \quad \therefore \Gamma n\,\Gamma 1-n = \frac{\pi}{\sin n\pi}$

$\Rightarrow \frac{1}{2} \times \frac{\pi}{\cos n\pi/2} = \frac{\pi}{2}\sec\frac{n\pi}{2}$ <u>A</u>

2. <u>Using beta and Gamma functions Prove the following:</u>

(a) $\int_0^\infty x^{-1/2} e^{-x^2} dx \times \int_0^\infty x^2 e^{-x^4} = \frac{\pi}{4\sqrt{2}}$

Name: Haurav Mittal Sec: AT Roll no.: 27

Ans 2(a) $x^2 = t$ $x^4 = t$

$2x\,dx = dt$ $4x^3 = dt$

$$\Rightarrow \frac{1}{2}\int_0^\infty t^{-3/4} e^{-t} dt \times \frac{1}{4}\int_0^\infty t^{-1/4} e^{-t} dt$$

$$\Rightarrow \frac{1}{8}\Gamma\frac{1}{4}\Gamma\frac{3}{4} \Rightarrow \frac{1}{8}\frac{\pi}{\sin\frac{\pi}{4}} = \frac{\sqrt{2}\pi}{8} = \frac{\pi}{4\sqrt{2}}$$

(b) $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$

Ans $$\frac{\Gamma m\,\Gamma n}{\Gamma m+n} = \frac{\Gamma m+1\,\Gamma n}{\Gamma m+n+1} + \frac{\Gamma m\,\Gamma n+1}{\Gamma m+n+1}$$

$$= \frac{m\Gamma m\,\Gamma n + n\Gamma m\,\Gamma n}{\Gamma m+n\,(m+n)}$$

$$= \frac{\Gamma m\,\Gamma n}{\Gamma m+n} = \beta(m,n)$$

LHS = RHS

(c) $\int_0^\infty \frac{x^c}{c^x} dx = \frac{\Gamma(c+1)}{(\log c)^{c+1}}$; $c > 1$

Ans Let $c^x = e^t$

$t = x\log c$ $x = \frac{t}{\log c}$

$dt = \log c\, dx$

$dx = \frac{dt}{\log c}$

$$\Rightarrow \int_0^\infty \frac{t}{\log c} \times \left(\frac{t}{\log c}\right)^c \times \frac{1}{e^t} dt$$

$$\Rightarrow \int_0^\infty \frac{1}{(\log c)^{c+1}} + t^c e^{-t} dt$$

$$\Rightarrow \frac{1}{(\log c)^{c+1}} \int_0^\infty e^{-t} t^c dx$$

$$= \frac{\Gamma c+1}{(\log c)^{c+1}}$$ Hence Proved

Name: Gaurav Mittal Sec: AT Roll no. 27

3. Evaluate the following integrals:

(a) $\int_0^{\pi/2}\int_0^{a\cos\theta} r\sqrt{a^2-r^2}\,dr\,d\theta$

Ans $\int_0^{\pi/2}\int_0^{a\cos\theta} r\sqrt{a^2-r^2}\,dr\,d\theta$

$a^2-r^2=t^2$

$-2r\,dr = 2t\,dt$

$r\,dr = -t\,dt$

$\int_0^{\pi/2}\int_0^{a\cos\theta}\sqrt{t^2}\,(-t\,dt)\,d\theta$

$\int_0^{\pi/2}\int_0^{a\sin\theta} -t^2\,dt\,d\theta$

$\Rightarrow \int_0^{\pi/2}\left[-\frac{t^3}{3}\right]_0^{a\sin\theta} d\theta$

$\Rightarrow \int_0^{\pi/2} -\frac{a^3\sin^3\theta}{3}\,d\theta$

$\Rightarrow -\frac{a^3}{3}\int_0^{\pi/2}(1-\cos^2\theta)\sin\theta\,d\theta$

Let $\cos\theta = t$

$-\sin\theta\,d\theta = dt$

$a^3/3\int_0^1(1-t^2)\,dt$

$a^3/3\,(t-t^3/3)_0^1$

$\frac{a^3}{3}\left[-\left(1-\frac{1}{3}\right)\right] = \frac{2a^3}{9}$

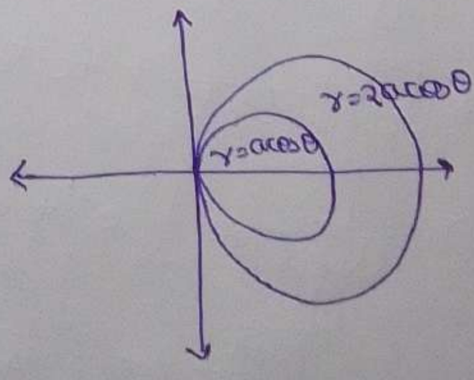
(b) $\iint r^2\,dr\,d\theta$ over the area between the circles $r=a\cos\theta$ and $r=2a\cos\theta$.

Ans $a\cos\theta \le r \le 2a\cos\theta$

$\frac{-\pi}{2}\le\theta\le\frac{\pi}{2}$

$\int_{-\pi/2}^{\pi/2}\int_{a\cos\theta}^{2a\cos\theta} r^2\,dr\,d\theta$

$\int_{-\pi/2}^{\pi/2}\left[\frac{r^3}{3}\right]_{a\cos\theta}^{2a\cos\theta} d\theta$



Name: Gaurav Mittal Sec: AT Roll no. 27

$$\Rightarrow \int_{-\pi/2}^{\pi/2}\left[\frac{8a^3\cos^3\theta}{3} - \frac{a^3\cos^3\theta}{3}\right]d\theta$$

$$\Rightarrow 2\times\frac{a^3}{3}\left[8\cos^3\theta - \cos^3\theta\right]_{-\pi/2}^{\pi/2} d\theta$$

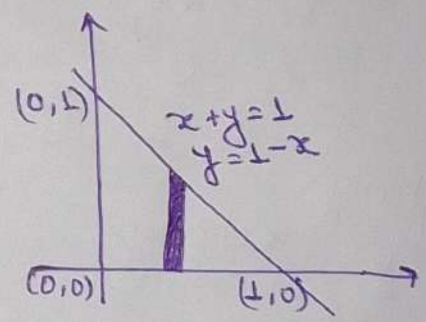
$$\Rightarrow 2\times 7a^3\left[\cos^3\theta\right]_{-\pi/2}^{\pi/2}$$

$$\Rightarrow 2\times\frac{7a^3}{3}\left[\frac{2}{3}\times 1\right]$$

$$\Rightarrow \frac{28a^3}{9} \text{ Ans}$$

4. Evaluate the $\iint e^{2x+3y}\,dx\,dy$, over the triangle bounded by $x=0$, $y=0$ and $x+y=1$.

Ans $\iint e^{2x+3y}\,dx\,dy$



$0 \le y \le 1-x$

$0 \le x \le 1$

$$\Rightarrow \int_0^1\int_0^{1-x} e^{2x+3y}\,dy\,dx$$

$$\Rightarrow \int_0^1\left[\frac{e^{2x+3y}}{3}\right]_0^{1-x} dx$$

$$\Rightarrow \frac{1}{3}\int_0^1\left[e^{2x+3-3x} - e^{2x}\right]dx$$

$$\Rightarrow \frac{1}{3}\int_0^1\left[e^{3-x} - e^{2x}\right]dx$$

$$\Rightarrow \frac{1}{3}\left[\frac{e^{3-x}}{-1} - \frac{e^{2x}}{2}\right]_0^1$$

$$\Rightarrow \frac{1}{6}(e-1)^2(2e+1) \text{ Ans}$$

Name: Gaurav Mittal Sec: AT Roll no. 27

5. Evaluate the following integrals by changing the order of integration :

(a) $\int_0^{\infty}\int_0^{\infty} \frac{e^{-y}}{y} dy dx$

Ans $y = x$ $y = \infty$

$x = 0$ $x = \infty$

R: $x=0$ $x=y$ $y=0$ $y=\infty$

$\Rightarrow \int_0^{\infty}\int_0^{y} \frac{e^{-y}}{y} dx dy$

$\Rightarrow \int_0^{\infty} \left[\frac{e^{-y}}{y} x\right]_0^y$

$\Rightarrow \int_0^{\infty} e^{-y} dy$

$\Rightarrow \left[e^{-y}\right]_0^{\infty}$

$\Rightarrow -e^{-\infty} + e^{0}$

$\Rightarrow 1$ Ans

(b) $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} x dx dy$

Ans $y=0$ $y = \frac{a}{\sqrt{2}}$ $x=y$ $x^2 = a^2 - y^2$

$x = \frac{a}{\sqrt{2}}$

$R_1 : 0 \le x \le \frac{a}{\sqrt{2}}$

$0 \le y \le x$

$R_2 : \frac{a}{\sqrt{2}} \le x \le a$

$0 \le y \le \sqrt{a^2-x^2}$

$\Rightarrow \int_a^{a/\sqrt{2}} \int_0^{x} x dy dx + \int_{a\sqrt{2}}^{a} \int_0^{\sqrt{a^2-x^2}} x dy dx$

$\Rightarrow \int_0^{a/\sqrt{2}} x [y]_0^x dx + \int_{a/\sqrt{2}}^{a} x [y]_0^{\sqrt{a^2-x^2}} dx$

$\Rightarrow \int_0^{a/\sqrt{2}} x^2 dx + \int_{a\sqrt{2}}^{a} x\sqrt{a^2-x^2}$

Name: Gaurav Mittal Sec: AT Roll no. 27

$$\Rightarrow \int_0^{a/\sqrt{2}} x^2 dx + \int_{a/\sqrt{2}}^{a} x\sqrt{a^2-x^2}$$

$a^2 - x^2 = t$

$-2x\,dx = dt$

$$\Rightarrow \left[\frac{x^3}{3}\right]_0^{a/\sqrt{2}} + \frac{1}{2}\int_{a^2/2}^{0} \sqrt{t}\,dt$$

$$\Rightarrow \frac{a^3}{6\sqrt{2}} + \frac{1}{2}\times\frac{2}{3}\left[t^{3/2}\right]_0^{a^2/2}$$

$$\Rightarrow \frac{a^3}{6\sqrt{2}} + \frac{a^3}{6\sqrt{2}} = \frac{2a^3}{6\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{2}a^3}{3} \text{ A}$$

6. Evaluate the following integrals by changing into polar coordinates.

(a) $$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$$

A $y = 0$ $y = \sqrt{2x - x^2}$

$x = 0$ $x = 2$

In polar form $x = r\cos\theta$, $y = r\sin\theta$

$r\cos\theta = 0 \longrightarrow \theta = \pi/2$

$$y^2 = 2x - x^2 - 1 + 1$$

$$y^2 + (x-1)^2 = 1$$

$$r^2 = 2r\cos\theta$$

$$\boxed{r = 2\cos\theta}$$

$$\boxed{r = 0}$$

$$0 \le \theta \le \pi/2$$

$$0 \le r \le 2\cos\theta$$

$$\int_0^{\pi/2}\int_0^{2\cos\theta} \frac{r\cos\theta}{r}\,dr\,d\theta$$

We know $dx\,dy = r\,dr\,d\theta$

$$\int_0^{\pi/2} \left[\frac{r^2}{2}\right]_0^{2\cos\theta} \cos\theta\,d\theta$$

$$2\int_0^{\pi/2} \cos^3\theta\,d\theta \Rightarrow 2\times\frac{2}{3}\times 1 = \frac{4}{3} \text{ A}$$

Name: Gaurav Mittal Sec: AI Rollno. 27

(b) $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy\,dx$

A: $x = r\cos\theta$

$y = r\sin\theta$

$\iint r e^{-r^2} dr\,d\theta$

$0 \le r \le \infty$

$0 \le \theta \le \pi/2$

$\Rightarrow \int_0^{\pi/2} \int_0^\infty r e^{-r^2} dr\,d\theta$ $\qquad r^2 = t$, $2r\,dr = dt$

$\Rightarrow \int_0^{\pi/2} \frac{1}{2} \int_0^\infty e^{-t} dt\,d\theta$

$\Rightarrow \int_0^{\pi/2} \left(\frac{e^0 - e^{-\infty}}{2} \right) d\theta$

$\Rightarrow \int_0^{\pi/2} \frac{1}{2} d\theta = \frac{\pi}{4}$ Ans

7. Using the transformations $x-y=u$, $x+y=v$, show that $\iint_R \sin\left(\frac{x-y}{x+y}\right) dx\,dy = 0$ where R is bounded by coordinate axes and $x+y=1$ in first quadrant.

A: $x-y=u$ $\qquad x+y=v$

$2y = v-u$ $\qquad 2x = v+u$

$\boxed{y = \frac{v-u}{2}}$ $\qquad \boxed{x = \frac{v+u}{2}}$

$$|J| = \begin{vmatrix} \partial x/\partial u & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -1/2 & 1/2 \end{vmatrix}$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

at $x=0$ $\qquad v+u=0$, $u=-v$ $\qquad$ at $x+y=1$

at $y=0$ $\qquad v-u=0$, $u=v$ $\qquad v=1$

$-v \le u \le v$

$0 \le v \le 1$

Name: Gaurav Mittal Sec: AT Roll no. 27

$\Rightarrow \iint \sin\left(\frac{u}{v}\right)\left(\frac{1}{2}\right) du\,dv$

$\Rightarrow \frac{1}{2}\iint \sin(u/v)\, du\,dv$

$\Rightarrow \frac{1}{2}\int_0^1 \int_{-v}^{u} \sin(u/v)\, du\,dv$

$\Rightarrow \frac{1}{2}\int_0^1 v\left[-\cos u/v\right]_{-v}^{v}$

$\Rightarrow \frac{1}{2}\int_0^1 0+0$

$= \frac{1}{2}\times 0 \Rightarrow 0$ Ans

Q. find the value of λ and μ so that the Surfaces $\lambda x^2 - \mu yz = (\lambda+2)x$ and $4x^2y + z^3 = 4$ intersect orthogonally at the Point $(1,-1,2)$.

Ans

$\phi_1 = \lambda x^2 - \mu y^2 - (\lambda+2)x$

$\nabla\phi_1 = [2\lambda x - (\lambda+2)]\hat{i} - \mu z\hat{j} - \mu y\hat{k}$

$\nabla\phi_1$ at $(1,-1,2)$

$= (\lambda-2)\hat{i} - 2\mu\hat{j} + \mu\hat{k}$

$\phi_2 = 4x^2y + z^3 - 4$

$\nabla\phi_2$ at $(1,-1,2) \Rightarrow -8\hat{i} + 4\hat{j} + 12\hat{k}$

since surfaces are orthogonal to each other at $(1,-1,2)$

$\nabla\phi_1 \cdot \nabla\phi_2 = 0$

$[(\lambda-2)\hat{i} - 2\mu\hat{j} + \mu\hat{k}][-8\hat{i} + 4\hat{j} + 12\hat{k}] = 0$

$-8(\lambda-2) - 8\mu + 12\mu = 0$ — (1)

at point $(1,-1,2)$ lies on surface

$\lambda + 1 + 2\mu = (\lambda+2)1$

$\boxed{\mu = 1}$

Putting in equation (i) we get

$-8(\lambda-2) - 8 + 12 = 0$

$\boxed{\lambda = 5/2}$

Name: Gaurav Mittal Sec: AT Rollno. 27

9. find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of line PQ where Q is the point $(5,0,4)$

Ans $f = x^2 - y^2 + 2z^2$

$\nabla f = 2x\hat{i} - 2y\hat{j} + 4z\hat{k}$

at $(1,2,3)$ $\nabla f = 2\hat{i} - 4\hat{j} + 12\hat{k}$

~~at~~ New vector $\vec{PQ} = (5-1)\hat{i} + (0-2)\hat{j} + (4-3)\hat{k}$

$= 4\hat{i} - 2\hat{j} + \hat{k}$

$\hat{a} = \dfrac{4\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{16+4+1}} \Rightarrow \dfrac{4\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{21}}$

So, directional derivative of f in the direction of

$\hat{a} = \nabla f \cdot \hat{a} = (2\hat{i} - 4\hat{j} + 12\hat{k}) \cdot \dfrac{(4\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{21}}$

$= 4/3\sqrt{21}$ A

10. A motion of a particle is given by $\vec{v} = yz\hat{i} + zx\hat{j} + xy\hat{k}$. Show that the field is irrotational and find the velocity potential.

Ans $\vec{v} = yz\hat{i} + zx\hat{j} + xy\hat{k}$

$$\nabla \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & zx & xy \end{vmatrix}$$

$$\nabla \vec{v} = \hat{i}\left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(zx)\right] - \hat{j}\left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz)\right] + \hat{k}$$
$$\left[\frac{\partial}{\partial x}(zx) - \frac{\partial}{\partial y}(yz)\right]$$

$\nabla \times \vec{v} = 0$

curl is 0 so vector is irrotational

11. find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at $(2,-1,2)$.

Name: Gaurav Mittal Sec: AT Roll no. 27

An 11 $\phi_1 = x^2+y^2+z^2-9$

$\phi_2 = x^2+y^2-z-3$

$\nabla\phi_1 = 2x\hat{i}+2y\hat{j}+2z\hat{k}$

$$\hat{n}_1 = \frac{\cancel{2}(x\hat{i}+y\hat{j}+z\hat{k})}{\cancel{2}\sqrt{x^2+y^2+z^2}}$$

$$\text{at } (2,-1,2) = \frac{2\hat{i}+(-1)\hat{j}+2\hat{k}}{\sqrt{(2)^2+(-1)^2+(2)^2}}$$

$$\Rightarrow \frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$$

Now $\nabla\phi_2 = 2x\hat{i}+2y\hat{j}-\hat{k}$

$$\hat{n}_2 = \frac{2x\hat{i}+2y\hat{j}-\hat{k}}{\sqrt{(2x)^2+(2y)^2+(-k)^2}}$$

$$\text{at point } (2,-1,2) = \frac{4\hat{i}-2\hat{j}-\hat{k}}{\sqrt{21}}$$

Now angle between ϕ_1 and ϕ_2 at $(2,-1,2)$

$$\cos\theta = \frac{\hat{n}_1 - \hat{n}_2}{|\hat{n}_1||\hat{n}_2|}$$

$$\cos\theta = \frac{1}{3\sqrt{21}}(8+2-2)$$

$$\theta = \cos^{-1}\left(\frac{4}{3\sqrt{21}}\right)$$

$$\boxed{\theta = 0.94}$$

12. Using Green's theorem, evaluate $\int_C [(y-\sin x)dx + \cos x\, dy]$ where C is the triangle formed by $y=0$, $x=\pi/2$, $y=2/\pi\, x$

An $\int_C (y-\sin x)dx + \cos x\, dy$

$y=0,\ x=\pi/2,\ y=\frac{2}{\pi}x$

using green theorem,

$$\int_C (y-\sin x)dx + \cos x\, dy = \int_C (P dx + Q dy)$$

$$\int_0^{\pi/2} dx \int_0^{2x/\pi} dy \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

Name: Gaurav Mittal Sec: AT Roll no. 27

$$\Rightarrow \int_0^{\pi/2} dx \int_0^{2x/\pi} dy \,(\sin x + 1)$$

$$\Rightarrow -\int_0^{\pi/2} dx \int_0^{2x/\pi} dy\,(\sin x + 1) = \frac{-2}{\pi}\int_0^{\pi/2} dx\,(x(\sin x) + x)$$

$$\Rightarrow \frac{2}{\pi}\left[x\cos x - \sin x - \frac{1}{2}x^2\right]_0^{\pi/2}$$

$$\Rightarrow \frac{2}{\pi}\left[0 - 1 - \frac{1}{2}\times\frac{\pi^2}{4}\right]$$

$$\Rightarrow -\left(\frac{\pi^2+8}{4\pi}\right) \quad \text{A}$$

13. verify green's theorem in the plane for $\int_C [(xy - y^2)dx + x^2 dy]$ where C is closed curve of the region bounded by $y = x$ and $y = x^2$.

A: Let $P = xy + y^2$ and $Q = x^2$

$\therefore \frac{\partial P}{\partial y} = x + 2y$ and $\frac{\partial Q}{\partial x} = 2x$

Given, $y = x$ and $y^2 = x$

solving simultaneously $x^2 = x$

$\therefore x^2 - x = 0$

$x = 0$ or $x = 1$

but $y = x$

$\therefore y = 0$ or $y = 1$

two curves intersect at $(0,0)$ and $(1,1)$

Part 1: Consider,

$$I = \int_C P\,dx + Q\,dy = \int_C (xy + x^2)\,dx + x^2 dy$$

along line OA equation: $y = x$

$\therefore dx = dy$

$$I_{OA} = \int_0^1 (x\cdot x + x^2)\,dx + x^2 dx$$

$$\int_0^1 3x^2\,dx$$

$$= \left[\frac{3x^3}{3}\right]_0^1 \Rightarrow 1 - 0 = 1$$

Name: Gaurav Mittal Sec: AI Roll no. 27

Along AO parabola equation $x = y^2$

$$\therefore dx = 2y\,dy$$

$$\therefore I_{AO} = \int_1^0 (y^2 \cdot y + y^2) \cdot 2y\,dy + (y^2)^2 dy$$

$$= \int_1^0 (2y^4 + 2y^3 + y^4)\,dy$$

$$\Rightarrow \int_1^0 (3y^4 + 2y^3)\,dy$$

$$\Rightarrow \left[\frac{3y^5}{5} + \frac{2y^4}{4}\right]_1^0 = \frac{-11}{10} \quad \text{—(i)}$$

$$\int_C P\,dx + Q\,dy = I_{AO} + I_{OA} = 1 - \frac{11}{10} = -\frac{1}{10}$$

Part 2:

Consider

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$\iint_R [2x - (x + 2y)]\,dx\,dy$$

$$= \int_0^1 \int_x^{\sqrt{x}} (x - 2y)\,dx\,dy$$

$$\Rightarrow \int_0^1 \{(x\sqrt{x} - x] - [x^2 - x^2]\}\,dx$$

$$\Rightarrow \int_0^1 [x^{3/2} - x]\,dx = \left(\frac{x^{3/2}}{5/2} - \frac{x^2}{2}\right]_0^1$$

$$\Rightarrow \frac{2}{5} - \frac{1}{2} = -1/10 \quad \text{—(ii)}$$

from eq (i) & (ii)

$$\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$\therefore$ Theorem verified