ECS132 HOMEWORK 1

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Problem 2.(Matloff): Consider the ALOHA example. Suppose it is known that X1 !=X2. Find the probability that there were 0, 1 or 2 collisions during those two epochs analytically and confirmed via R simulation.

Let C be the number of collisions in the first two epochs.
Assume that X0 = 2, so at the start of the first epoch, there are two active nodes.
By using p = 0.4 and q = 0.8 from the material:
Since that X1 !=X2, so we need to calculate all probabilities of 0, 1, or 2 collisions that occur conditioned on the event where X1 != X2. P(C = 0|X1 !=X2), P(C = 1|X1!=X2),
and P(C = 2|X1!= X2). By the definition of conditional probability we got:
P(C=a|X1!=X2) =P=(C=a, X1!= X2)\P(X1!=X2) to each case
P(X1!= X2)= P((X1 =2, X2 =1) or (X1 =1, X2 =2) or (X1 =1,X2 =0)
=P(X1 =2,X2 =1)+P(X1 =1,X2 =2)+P(X1 =1,X2 =2)
Case X1 = 2,X2 = 1: In the first epoch: (p^2 + (1-p)^2))and 2.
In the second epoch one node sent (2p(1-p)) . so P(X1 =2,X2 =1)=(p^2 +(1-p)^2)2p(1−p)
Case X1 = 1, X2 = 2: When 1 in the first epoch, one node sent (2p(1-p)) and 2
The second epoch, both nodes are active at the end: (p^2 + (1-p)^2)):
P(X1 =1, X2 =2)=2p(1-p)q(p^2 +(1-p)^2)
Case X1 = 1, X2 = 0: When 1 in the first epoch, one node sent (2p(1-p)) and 2.
The second epoch: the inactive is still inactive,the the active node sent p
P(X1 =1, X2 =2)=2p(1-p)(1-q)p
From above: P(X1!=X2)= (p^2 +(1−p)^2)2p(1-p)+2p(1-p)q(p^2+(1−p)^2)+2p(1-p)(1-q)p approximately is 0.48768. Now we see three scenarios: C = 2, X1!= X2:
there is no way to get C = 0 and X1!=X2 at the same time.Thus, P(C=2|X1 != X2)= P(C=2, X1!= X2)\P(X1 != X2)= 0\P(X1!=X2)=0. C=1, X1!=X2:
Case 1:The collision is in the first epoch (p2), making X1 = 2, and in the second epoch, an active node must be sent, so this case has probability p^2[2p(1-p)]
Case 2: The collision is in the second epoch, indicating it should be X2 = 2 so X1 = 1 to get them unequal. so, one node sent in the first epoch is (2p(1-p)), in the second epoch generates q, so both nodes are tended to send messages as p^2, thus its probability is 2p(1-p)qp^2
Thus: P(C=1|X1!=X2)=P(C=1,X1! =X2)\P(X1!=X2)=
(p^2[2p(1-p)]+2p(1-p)qp^2)\(p^2+(1-p)^22p(1-p)+2p(1-p)q(p^2+(1-p)^2)+2p(1-p)(1-q)p)
Approximately equals 0.2834646. Under the scenario C = 0, X1!=X2: Case 1: In the first epoch neither nodes were sent ((1-p)^2). In the second epoch, one node sent 2p(1-p), its probability here is thus (1-p)^2[2p(1-p)]. Case 2: in the first epoch, one node sent, so X1 = 2p(1-p), in the second epoch, one node generated (1-q) the active node sent p. So its probability is 2p(1-p)(1-q)p
Case 3: in the first epoch one node sent, X1 = 2p(1-p). In the second epoch, q is generated, so X2 = 2, its probability: 2p(1-p)q(1-p)^2. In summary of all three scenarios with their cases above:
P(C=0|X1!= X2)=(P(C=0, X1!= X2))\(P(X1!= X2))
=[(1-p)^2]2p(1-p)+2p(1-p)(1-q)p+2p(1-p)q(1-p)^2\(p^2+(1-p)^2)2p(1-p)+2p(1-p)q(p^2+(1-p)^2)+2p(1-p)(1-q)p) approximately equals to 0.7165354. <u>Therefore, the probability that there were 0 collisions during those two epochs is **0.7165354**, the probability that there were 1 collisions during those two epochs is **0.2834646**, the probability that there were 2 collisions during those two epochs is **0**.</u>

```r
SimulateFunc <- function (p, q, nreps) {
  #count for x1 not equal to x2
  resultval  <-  0
  num_col_0 <-  0
  num_col_1 <-  0
  num_col_2 <-  0

  for ( i in 1 : nreps) {

    # at epoch  1
    #no message attempted to send
    NumSent <- 0
    # number of collisions here
    num_col <- 0
    #  simulate
    for ( j in 1 : 2 ) {
      if ( runif( 1 ) < p) NumSent <- NumSent + 1
    }
    if ( NumSent == 1 ) X1 <- 1
    if ( NumSent == 0 ) X1 <- 2
    if ( NumSent == 2 ) {
      X1  <- 2
      num_col <- num_col  + 1
    }

```

```r
    # at epoch  2
    # if X1 = 1
    # one node generate a new message
    ActiveNum  <-  X1
    if ( X1 == 1 &&  runif( 1 ) < q)
      ActiveNum <- ActiveNum  + 1

    # check if send or not
    if  ( ActiveNum  ==  1 )
    {
      if  ( runif( 1 )  < p)  X2  <- 0
      else  X2  <- 1
    }
    else  {  #  is the active number is 2
      NumSent <- 0
      for ( i in 1 : 2 ) {
        if ( runif( 1 ) < p)
          NumSent <- NumSent + 1
      }
      if ( NumSent == 1 ) X2 <- 1
      if ( NumSent == 0 ) X2 <- 2
      if ( NumSent == 2 ) {
        X2 <- 2
        num_col <- num_col + 1
      }
```

```r
        num_col <- num_col + 1
      }
    }
    if ( X1 != X2 ) {
      # increment our result as when x1!=x2
      resultval  <-  resultval  +  1

      if  ( num_col ==  0 ) num_col_0 <-  num_col_0 + 1
      if  ( num_col ==  1 ) num_col_1 <-  num_col_1 + 1
      if  ( num_col ==  2 ) num_col_2 <-  num_col_2 + 1
    }
  }
  # print our results

  cat(" P(X1!=X2) = ", resultval / nreps , "\n")
  cat(" P( C= 0|X1!=X2 ):"  , num_col_0 / resultval , "\n")
  cat(" P( C= 1|X1!=X2 ):"  , num_col_1 / resultval , "\n")
  cat(" P( C= 2|X1!=X2 ):"  , num_col_2 / resultval , "\n")
}

SimulateFunc(0.4,0.8,100000)

```

```
> source("~/Downloads/problem2.R", echo=TRUE)

> SimulateFunc <- function (p, q, nreps) {
+   #count for x1 not equal to x2
+   resultval  <-  0
+   num_col_0  <-      0
+   num_col_1  <-      0
+   num_col_2 .... [TRUNCATED]

> SimulateFunc(0.4,0.8,100000)
 P(X1!=X2) =     0.48719
 P( C=  0|X1!=X2 ): 0.7148135
 P( C=  1|X1!=X2 ): 0.2851865
 P( C=  2|X1!=X2 ): 0
>
```