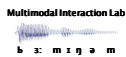


# HMM Theory and Practice



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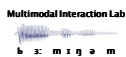
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## Objectives

- To understand how to apply HMMs
- To understand the basic HMM algorithms
- *Notes: pp 46 - 55*



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## Training & Recognition

- Major advantage of HMMs is the availability of a 'toolkit' of powerful, well-founded mathematical methods for HMM manipulation
- The **Baum-Welch** algorithm is used to train the parameters of a set of HMMs given a set of training data
- **Viterbi Decoding** is used to classify an unknown speech pattern in terms of the sequence of HMMs which is most likely to have produced it



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## The Recognition Problem

- Given a sequence of acoustic feature vectors

$$Y = \{y_1, \dots, y_T\}$$

we want to find the sequence of words

$$W = \{w_1, \dots, w_L\}$$

such that the probability

$$P(W|Y)$$

is maximized.

- If  $M = \{M_1, \dots, M_K\}$  is the sequence of HMMs which represents  $W$ , then  $P(W|Y) = P(M|Y)$

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## Bayes' Theorem

- Computation of the probability  $P(M|Y)$  is made possible using **Bayes' Theorem**

$$P(W|Y) = \frac{p(Y|W)P(W)}{p(Y)}$$

- $P(W)$  is the “language model probability”
- $p(Y|W)$  is the “acoustic model probability”
- Bayes Theorem has been referred to as the “fundamental theorem of speech recognition”!

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## The Baum-Welch Algorithm

- The Baum-Welch algorithm is the method which is normally used for HMM parameter estimation
- Given a set of HMMs  $M_0$  and a set of speech patterns  $Y$ , Baum's theorem defines how to produce a new model set  $M_1$  such that

$$P(Y|M_1) \geq P(Y|M_0)$$

- Baum-Welch algorithm applies this method repeatedly until a HMM  $M_n$  is found which (locally) maximizes  $P(Y|M)$
- Baum's theorem only valid for particular classes of state output PDF

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## Notes on B-W Reestimation

- The Baum-Welch algorithm is only guaranteed to find a **locally** optimal HMM set - hence choice of  $M_0$  can be important
- Baum-Welch is a **supervised** training algorithm which requires labelled speech data
- The labelling need **not** be at the same level as the HMM set
  - phoneme level HMMs can be trained using data labelled orthographically at the phrase or sentence level
- For large applications B-W reestimation can be **very** computationally expensive

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## Viterbi Decoding

- Viterbi Decoding is the algorithm which is used to find the sequence of HMMs which is most likely to have generated a given speech pattern
- Based on **Dynamic Programming**
- Viterbi Decoding illustrates the type of computation typically done with HMMs

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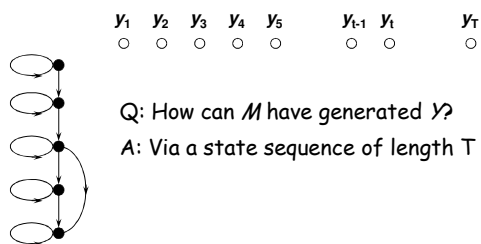


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## Viterbi Decoding (1)



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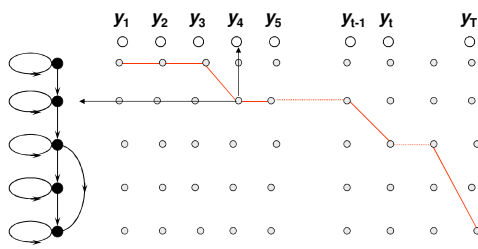


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## Function of State Sequence



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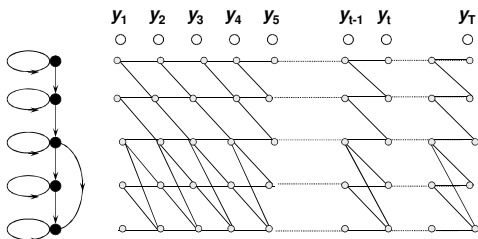
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## Viterbi Decoding (2)

### Construction of 'state-time trellis'



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## Constructing the State-Time Trellis

- Simple Rule:
  - Connect node  $(i, t)$  of the trellis to node  $(j, t+1)$  if and only if there is a transition between state  $i$  and state  $j$  in the HMM with probability  $a_{ij}$  greater than zero

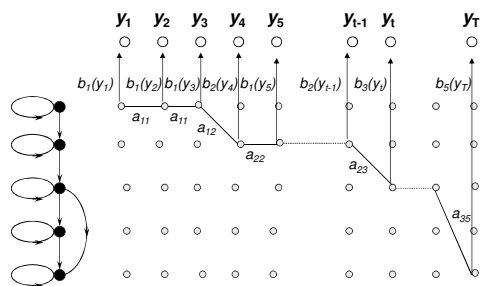
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## Basic Probability Calculation



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## Viterbi Decoding (3)

- Let  $X = \{x_1, \dots, x_T\}$  be a state sequence of length  $T$
- The joint probability of  $Y$  and  $X$  is given by:

$$p(Y, X) = b_{x_1}(y_1) \prod_{t=2}^T a_{x_{t-1}x_t} b_{x_t}(y_t)$$

- i.e. the product of the state-output and state transition probabilities along the state sequence
- The optimal state sequence is the sequence  $X$  such that  $p(Y, X)$  is maximized
- $p(Y)$  is the sum of  $P(Y, X)$  over all sequences  $X$

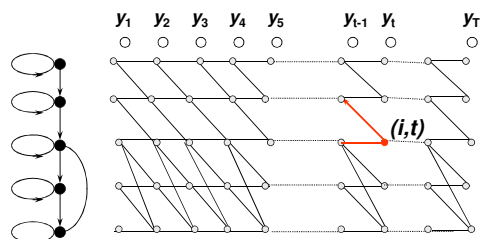
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## Viterbi Decoding (4)



$$p_t(i) = \text{Prob}(y_1, \dots, y_t, \text{opt sequence to } (i, t))$$

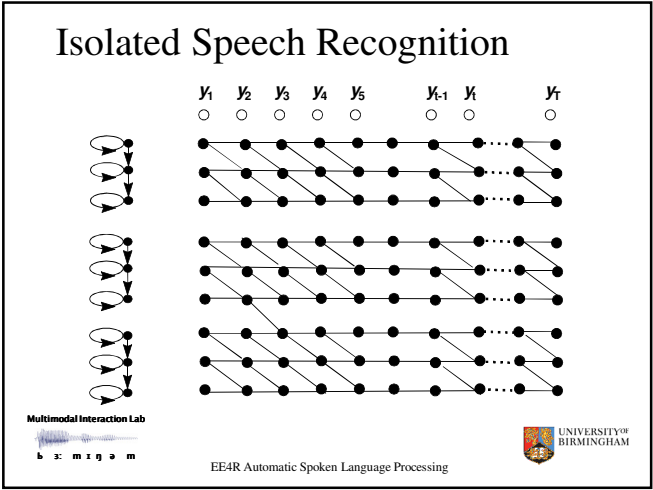
$$p_t(i) = \max \{p_{t-1}(i-1)a_{i-1,i}, p_{t-1}(i)a_{i,i}\} b_i(y_t)$$

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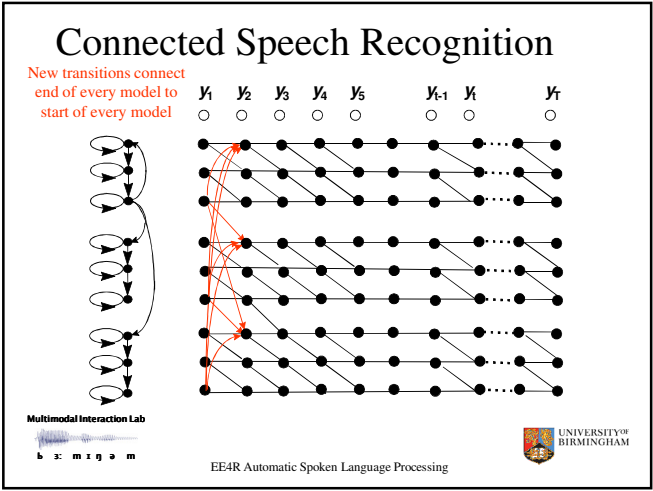
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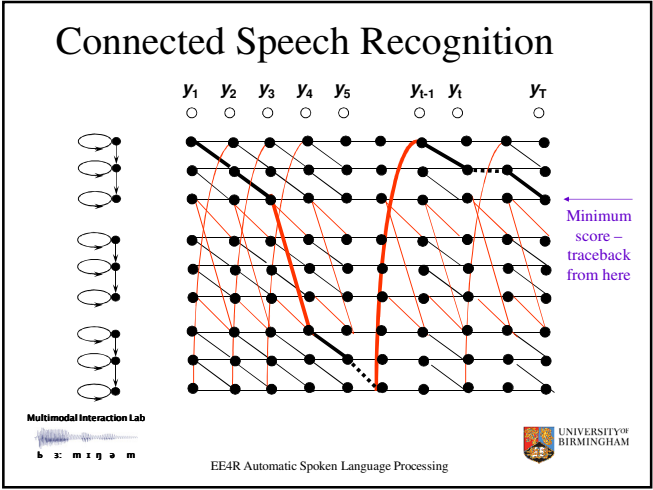
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## Partial Traceback (1)

- In continuous speech recognition, cannot trace-back from the end of the utterance (there is no end!)
- Instead **partial traceback** operates as follows:
  - For each time  $t$  and state  $i$  a *word link record* describes the sequence of words on the best path to  $(t, i)$ .
  - At regular intervals all active paths are traced back to see if they converge at some time  $s$  in the past
  - If so, the best path up to time  $s$  cannot change, and the sequence of words up to  $s$  can be output

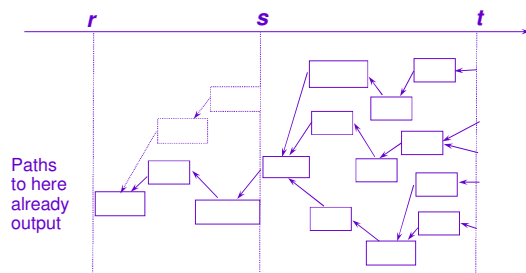
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## Partial Traceback (2)



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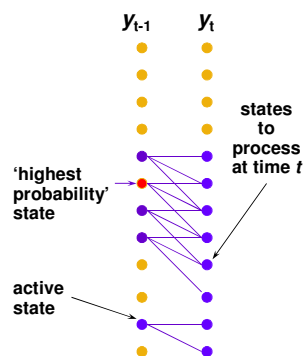
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## Beam Pruning

- Choose threshold  $T$
- The active states at time  $t-1$  are those whose partial path scores are within  $T$  of the best path score at time  $t-1$
- At time  $t$ , only process those states which link back to an active state



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## Partial Traceback, Beam Pruning & Recognition ‘Speed’

- Partial traceback introduces a ‘lag’ into recognition process - **not** due to inadequate processor speed
- Lag worse when models are poor
- Beam Pruning less effective for ambiguous input
- Severe Beam Pruning will degrade performance
- Proper management of Partial Traceback and Beam Pruning is essential for optimal performance

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## Summary

- HMM Theory and Practice
- HMM Training
  - The Baum-Welch algorithm
- Recognition
  - Viterbi decoding
  - Beam pruning
  - Partial traceback

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