EE4R Automatic Spoken Language Processing, February 2015

Question sheet 2

1. A Hidden Markov Model (HMM) M has four emitting states: s_1, s_2, s_3, and s_4 each of which can emit the symbol a, b or c. The state output probabilities for these states are given by:

$$\begin{array}{lll} p(a \mid s_1) = 0.6 & p(b \mid s_1) = 0.2 & p(c \mid s_1) = 0.2 \\ p(a \mid s_2) = 0.2 & p(b \mid s_2) = 0.7 & p(c \mid s_2) = 0.1 \\ p(a \mid s_3) = 0.2 & p(b \mid s_3) = 0.4 & p(c \mid s_3) = 0.4 \\ p(a \mid s_4) = 0.1 & p(b \mid s_4) = 0.1 & p(c \mid s_4) = 0.8 \end{array}$$

The initial state probability vector is given by $\pi = (1,0,0,0)$ and the state transition probability matrix A is given by:

$$A = \begin{bmatrix} 0.5 & 0.2 & 0.3 & 0 \\ 0 & 0.6 & 0.4 & 0 \\ 0 & 0 & 0.6 & 0.4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let y be the observation sequence *ababbcc*

a. Draw the state-time trellis which relates the model M to the observation sequence y.

b. Use the Viterbi algorithm to calculate the probability $P(y, \hat{s} \mid M)$ and the optimal state sequence $\hat{s}$.

2. Consider a 2 class classification problem, where each class is modelled using a 3 component multivariate (3-dimensional) GMM with the parameters (means, variances and weights) as below:

Class C_1: Component 1: $\mu_1 = (0,0,0)$, $\sigma_1^2 = (1,2,1)$

Component 2: $\mu_2 = (1,0,0)$, $\sigma_2^2 = (1,1,1)$

Component 3: $\mu_3 = (1,1,1)$, $\sigma_3^2 = (2,2,1)$

Weights: $w_1=0.5$; $w_2=0.3$; $w_3=0.2$

Class C_2: Component 1: $\mu_1 = (0,-1,0)$, $\sigma_1^2 = (1,1,1)$

Component 2: $\mu_2 = (1,-1,0)$, $\sigma_2^2 = (1,1,2)$

Component 3: $\mu_3 = (0,-1,-1)$, $\sigma_3^2 = (2,2,1)$

Weights: $w_1=0.4$; $w_2=0.3$; $w_3=0.3$

Calculate to which class does the sequence of feature vectors $Y=y_1,\ldots,y_T$ given below corresponds to (i.e., you should calculate P($Y|C_1$) and P($Y|C_2$)).

$$Y = y_1, y_2, \ldots, y_5 = \begin{pmatrix} 0 & 0.5 & 0.5 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -2 & 0 \end{pmatrix}$$

3. Consider modelling of 1-dimensional data using a Gaussian Mixture Model (GMM) with 2 mixture components. The parameters of the GMM components at the current iteration of the E-M training procedure are:

   Gaussian component 1: $\mu_1 = 6,\ \sigma_1^2 = 1,\ w_1 = 0.5$ and

   Gaussian component 2: $\mu_2 = 8,\ \sigma_2^2 = 1,\ w_2 = 0.5$.

   Consider the sequence of data y=(7, 5, 10, 9, 4). Calculate the new values for the means and variances of the GMM components after one iteration of the E-M training procedure.