ASTRO 100/G – Experiment 2

Telescopes: Area & Resolution

Links to Lectures 4 and 15

Name: ID: UPI/Username:	**Grade** /10

Heads up!

This experiment has done all of the unit conversions for you.
Do not convert any units into any other units.

Introduction

A reflecting telescope like the one in front of you is effectively a system of two mirrors. There is one large mirror, called the *primary*, at the back of the telescope, and one smaller mirror called the *secondary*, both shown in blue in the figure below. When light comes in (the purple and green lines), it reflects off both mirrors and into an eyepiece (shown as a red cross on the right), enabling us to see the image it is magnifying.

This lab focuses on the mathematics of telescopes, walking you through calculating the resolutions for the telescope in the room in front of you and the Hubble Space Telescope (HST). You will then compare the two in order to discover why a bigger telescope has a better resolution, and what this means.

In lectures, we learned that the amount of light that can be collected by a telescope depends on the surface area of the telescope. Because telescope mirrors are typically circular, the area is given by

$$\text{Area} = \pi \times (\text{radius})^2, \quad \text{and therefore,} \quad \text{Area} = \pi \times \left(\frac{\text{diameter}}{2}\right)^2$$

Thus, if we double the radius of the telescope, we quadruple (as $2^2 = 4$) the area of the telescope. With more collecting area, we collect more light – so we can see fainter objects.

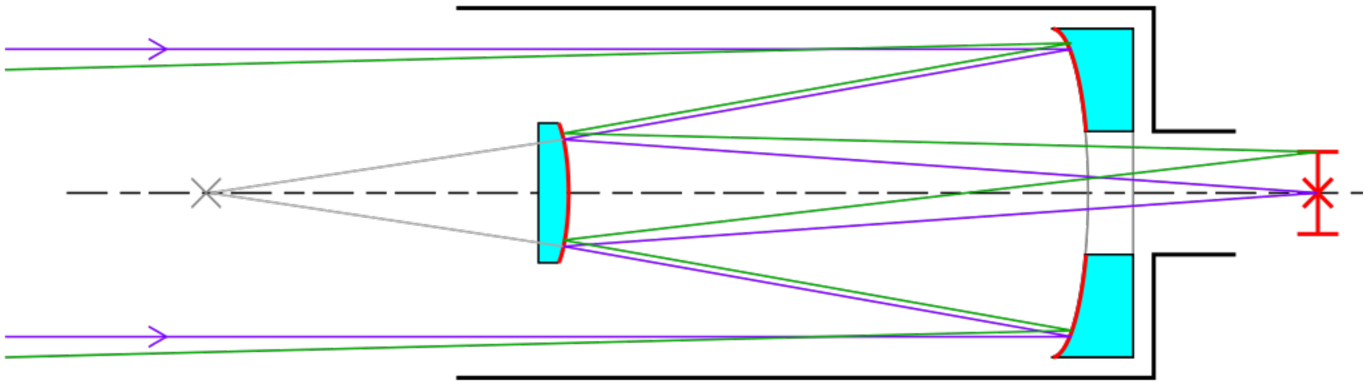

Figure 1: Diagram of a Cassegrain-Schmidt telescope by Krishna Vedala on Wikicommons, licensed under CC-BY-SA 4.0. https://commons.wikimedia.org/wiki/File:Cassegrain_Telescope.svg

At the end of this lab, students will be able to:

- **Compute** the ratio of light collected by two different optical instruments.
- **Qualitatively describe** the resolution of a telescope.
- **Recall** how the resolution of a telescope is related to its diameter.

1. (1 mark) The telescope in front of you has a diameter of 300mm. If the diameter of the pupil of a dark adapted eye is about 6mm, what is the **ratio** of the amount of light collected by the telescope to the amount of light collected by the eye?

   Start by writing down the ratio as

$$\text{ratio} = \frac{\text{Light collected by telescope}}{\text{Light collected by eye}} = \frac{\text{Area of telescope}}{\text{Area of eye}}$$

Solution:

$$\begin{aligned}\frac{\text{Light collected by telescope}}{\text{Light collected by eye}} &= \frac{\pi(\text{telescope radius})^2}{\pi(\text{eye radius})^2} \\ &= \frac{(\text{telescope radius})^2}{(\text{eye radius})^2} \\ &= \frac{300^2}{6^2} \\ &= 2500\end{aligned}$$

The ratio is 2500:1.

Resolution

There is another reason why we build big telescopes that is a bit more subtle. Light is a wave and whenever a wave tries to go through a hole or gap it spreads out. This is called *diffraction*. The spreading/blurring is greater the more similar the wavelength of light is to the size of the gap – or in this case the size of the mirror in the telescope.

For a circular aperture – that is, our mirror – there is a relatively simple equation that can tell us how much the light spreads. This equation is *Rayleigh's Criterion*, and it gives the limit of angular resolution θ (in arcseconds, symbol $''$) for a telescope under perfect conditions. It depends on the telescope diameter d (in millimetres), and the wavelength of light λ (in nanometres, symbol nm):

$$\theta = 0.252 \times \left(\frac{\lambda}{d}\right)$$

The 0.252 out the front does double-duty: it converts all the units for you (so you can plug in nm for λ and mm for d and have the numbers work out right), and it also encodes the fact that the telescope is circular. The exact maths underlying this is very deep, but is not required for this course.

2. (1 mark) Using blue light (which has a wavelength of 300nm), calculate the angular resolution limit for the telescope in Q1 (300mm diameter) in arcseconds.

Solution:

$$\begin{aligned}\theta &= 0.252 \times \left(\frac{\lambda}{d}\right) \\ &= 0.252 \times \left(\frac{300\text{ nm}}{300\text{ mm}}\right) \\ &= 0.252\text{ arc seconds}\end{aligned}$$

The distance (D), between the Earth and the Moon is approximately 4×10^8 metres. The angular resolution can be turned into a distance at which two objects, for example two mountains on the Moon, can be seen as separate (i.e. 'resolved') and not blurred into one. The resolution (R) is given by:

$$R = \theta \times \left(\frac{D}{206265} \right)$$

This means that two objects that are D meters away from us can be resolved as separate if they are separated by a distance of R meters. The units of the resolution are the same as what the distance was measured in. The 206265 converts arcseconds to radians so the maths all works out nicely for you.

3. (1 mark) Use your value of θ (in arcseconds) from above to find the size of the smallest feature visible on the Moon (the resolution) with this telescope. Remember to include units!

Solution:

$$\begin{aligned} R &= \theta \times \left(\frac{D}{206265}\right) \\ &= 0.252 \times \left(\frac{4 \times 10^8}{206265}\right) \\ &= 0.252 \times 1939.2529 \\ &= 489 \text{ metres} \end{aligned}$$

4. (2 marks) The Hubble Space Telescope (HST) collects about 64 times more light than the telescope in front of you does. If the diameter of this telescope (present in room) is 300mm, what is the diameter of the HST?

(Hint: Start by writing down the ratio: $64 = \frac{\textit{area of the HST}}{\textit{area of the telescope in the room}}$. You'll need to do a tiny bit of rearranging the equation to figure out the diameter of the HST – ask for help if you haven't done this before!)

Solution:

$$\begin{aligned} 64 &= \frac{\text{Area of HST}}{\text{Area of Telescope}} \\ &= \frac{\pi\left(\frac{d_{\text{HST}}}{2}\right)^2}{\pi\left(\frac{d_{\text{tel}}}{2}\right)^2} \\ &= \frac{\pi\left(\frac{d_{\text{HST}}}{2}\right)^2}{\pi\left(\frac{300}{2}\right)^2} \\ &= \frac{\left(\frac{d_{\text{HST}}}{2}\right)^2}{\left(\frac{300}{2}\right)^2} \\ 8 &= \frac{\left(\frac{d_{\text{HST}}}{2}\right)}{\left(\frac{300}{2}\right)} \\ 8 &= \frac{d_{\text{HST}}}{300} \\ 2400 &= d_{\text{HST}} \end{aligned}$$

Award full marks for correct answer. Award 1 mark for mostly correct answer (e.g. the derivation went weird).

5. (1 mark) Calculate the angular resolution θ (in arcseconds) for the HST, if the wavelength of light it detects is 600nm, using $\theta = 0.252 \times \left(\frac{\lambda}{d}\right)$

(Hint: You're repeating Q2 for a new telescope)

Solution:

$$\begin{aligned}\theta &= 0.252 \times \left(\frac{\lambda}{d}\right)\\ &= 0.252 \times \left(\frac{600}{2400}\right)\\ &= 0.063 \text{ arc sec}\end{aligned}$$

6. (1 mark) Now use $R = \theta \times \left(\frac{D}{206265}\right)$ to find the size of the smallest feature visible on the Moon with the Hubble Space Telescope (same D as in question 3). Remember to include units!

Solution:

$$\begin{aligned}R &= \theta \times \left(\frac{D}{206265}\right)\\ &= 0.063 \times \left(\frac{4 \times 10^8}{206265}\right)\\ &= 122.2 \text{ metres}\end{aligned}$$

7. (2 marks) Compare your answers to questions 3 and 6. Which of the two telescopes is better, and why?

(Hint: you've implicitly assumed in question 6 that the HST is located on Earth, not in space, so the answer is not "the HST because it is in space".

Solution: HST is better (the resolution is smaller, so it can see finer detail).

1 mark for saying HST is better, 1 mark for correct reason.

You are now finished. Please hand your sheet to the demonstrator for marking.