Relativistic Limitations of Quantum Mechanics

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Motivation & History

1. Relativity and modern Quantum Mechanics were developed more or less simultaneously
2. Historic attempts were made to integrate them, both fell short in several regards
3. We will explore the ways physicists of yonderyear attempted to integrate them
4. We will ponder the question; can the two be fully unified?

A quick aside about notation

I will be making use of Einstein Summation Notation:

$$\underbrace{\sum_{i=1}^{3} a_i b^i = a_i b^i}_{\text{Latin indices}} \qquad \underbrace{\sum_{\mu=0}^{3} a_\mu b^\mu = a_\mu b^\mu}_{\text{Greek indices}}$$

My metric signature will be $(+ - --)$. Operators will typically be boldface and have carets, e.g., $\hat{\mathbf{p}}$ for momentum.

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Is Quantum Mechanics relativistic?

For a theory to be relativistic, it has to be *Lorentz invariant*, i.e., invariant under the transformations

$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Is the T.D.S.E:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

Lorentz Invariant?

Can we *make* it relativistic?

The relativistic Hamiltonian is

$$H = \sqrt{p^2c^2 + m^2c^4}$$

Can we work with this?

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$$H = \sqrt{p^2c^2 + m^2c^4}$$

Can we work with this? **Dirac couldn't.**
You would have $\sqrt{\nabla}$ which would involve expansion in a power series, which is not simple.

Can we *make* it relativistic?

Let's work with the *square* of that Hamiltonian, in operator form:

$$\hat{\mathbf{H}}^2 = \hat{\mathbf{p}}^2 c^2 + m^2 c^4$$

$$\implies ((-i\hbar\nabla)^2 c^2 + m^2 c^4)\psi = \left(i\hbar\frac{\partial}{\partial t}\right)^2 \psi$$

The Klein-Gordon Equation

$$(\Box + \mu^2)\psi = 0$$

Think of the *D'Alembertian* as the four-dimensional analogue of the Laplacian.

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In natural units, where $\hbar = c = 1$, $\mu = m$ and hence the Klein-Gordon equation becomes

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In natural units, where $\hbar = c = 1$, $\mu = m$ and hence the Klein-Gordon equation becomes

$$(\Box + m^2)\psi = 0$$

Or for a massless particle

$$\Box\psi = 0$$

The Dirac Equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

with

$$\gamma^1 = \begin{pmatrix} \mathbb{I}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & -\mathbb{I}_2 \end{pmatrix} \qquad \gamma^\mu = \begin{pmatrix} \mathbf{0}_2 & \sigma^\mu \\ -\sigma^\mu & \mathbf{0}_2 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} \mathbf{0}_2 & \mathbb{I}_2 \\ \mathbb{I}_2 & \mathbf{0}_2 \end{pmatrix}$$

and

$$\partial_\mu = \begin{pmatrix} \frac{\partial}{\partial t} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{pmatrix}$$

This is a relativistic Schrödinger equation!

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Where to from here?

Mankind has uncovered two extremely efficient theories: one that describes our universe's structure (Einstein's gravity: the theory of general relativity), and one that describes everything our universe contains (quantum field theory), and these two theories won't talk to each other.

– Christophe Galfard

Where to from here?

Conformal theory of everything

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Abstract

By using conformal symmetry we unify the standard model of particle physics with gravity in a consistent quantum field theory which describes all the fundamental particles and forces of nature.

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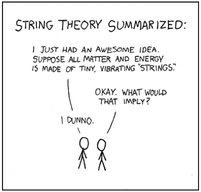

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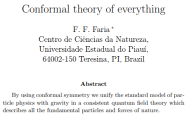
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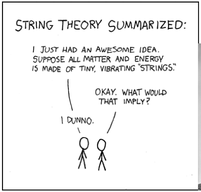

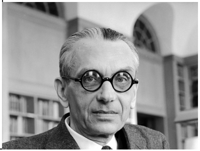

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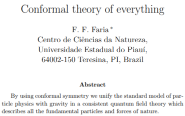

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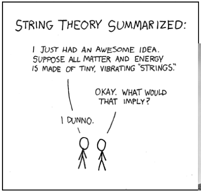

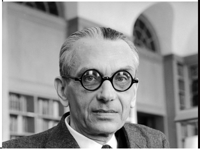

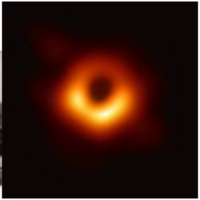

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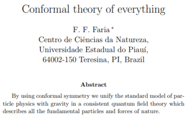

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