Experiment 231: Diffraction at Slits

Aim

To investigate quantitatively the far-field diffraction pattern for coherent light from a system of parallel slits and to compare the results with Fraunhofer diffraction theory.

Caution: Do not look directly into the laser beam or point it at another person.

References

1. F.L. Pedrotti and L.S. Pedrotti, "Introduction to Optics" (2nd Edition), Prentice-Hall (1993)
2. E.F. Hecht, "Optics", (2nd Edition), Addison Wesley (1987)

Notes: This experiment involves the *quantitative* study of diffraction patterns observed when light passes through various systems of parallel slits. You should have some familiarity with the expected *qualitative* diffraction behaviour. Before beginning this experiment, you should review the general properties of multiple-slit diffraction discussed in the references and the Appendix.

Before attempting this experiment, you **must** be familiar with PYTHON, preferably by having done experiment 213 Data Analysis with PYTHON first.

Theory

As described in the Appendix, the far-field distribution of light intensity diffracted at angle θ from a system of N regularly spaced slits of width w spaced a distance s apart is given by:

$$I(\theta) \propto w^2 \left[\frac{\sin u}{u}\frac{\sin Nv}{\sin v}\right]^2 \tag{1}$$

where

$$u = \frac{1}{2}kw\,\sin\theta \qquad v = \frac{1}{2}ks\,\sin\theta \qquad k = \frac{2\pi}{\lambda}$$

and λ is the wavelength of the light.

The detector is in the far-field (Fraunhofer condition) provided that the distance D between the slits and the detector satisfies:

$$D > \frac{2\xi_{\max}^2}{\lambda} \tag{2}$$

where $\xi_{\max}$ is the half distance between the extreme slits.

Equipment

The light source is a helium-neon laser with an adjustable collimator. To adjust the collimator for plane waves, shine the beam on a distant wall and slide the barrel to minimize the spot size. The wavelength of the laser light is 632.8 nm.

A variety of diffraction obstacles are available on three slides labelled 9165-A, B and C. Each slide has four configurations of slits labelled A through D. On each slide, the number of slits N is marked. As part of the experiment, you will work out the slit width w and the slit separation s for the multiple slits. The slide is mounted in the adjustable holder *with the label away from the laser.* Ensure that the slide is fully inserted so that the lower edge rests on the two locating pins.

The light detection is achieved here with a digital camera mounted on a tripod. The photos you take will be processed by a Python function provided, giving you the measured intensity pattern as an array to be plotted.

The Camera

Ask your demonstrator to get the camera equipment for you from the store. Please handle the camera responsibly as it is expensive and is to be used by all students on this experiment. For example, be careful not to trip over the tripod whilst taking photographs in the dark! The camera is a commercial model so you may be proficient with its use. If not, trial and error with pushing all of the different buttons should familiarise yourself with its functions. Failing that, the instruction manual can be provided if you get stuck, or ask your demonstrator. The only controls necessary for this experiment are the zoom, focus (the camera should be pre-set on auto-focus), exposure time, and the timer. You also have to make sure the camera is outputting its photos in large j-peg format in order for them to be read using Python. Hopefully the last student has cleared the memory. If not, do not be tempted to use their photos. This is *your* experiment and perhaps you can take better photos anyway.

Procedure

Before beginning to take your photographs, switch on the laser and set up a screen on the optical rail (a white piece of paper will suffice) and have a careful look at the diffraction pattern that you observe. Do this for several different slit configurations, and relate the changes that you see to the results that you expect based on diffraction theory. Once you understand the patterns that you see, then you can begin to accumulate quantitative results using the camera. If you are unsure of exactly what it is you are seeing, consult your demonstrator before proceeding further.

(1) Adjust the collimator on the laser to produce plane waves as described above.

(2) Make sure the diffraction pattern is centred on the screen above the optical rail. Check that the distance between the plane of the slide and the screen is about one metre. Measure and record this distance which will be used to calculate the angle θ.

(3) Screw the camera onto the tripod mount piece and place it on the tripod. Adjust the legs until they are level, and the camera as horizontal as possible. Place the camera pointing over the laser barrel directly toward the diffraction pattern.

(4) Turn the camera on. You may set the focus to auto by selecting AF on the lens-barrel switch. Look through the viewfinder and point straight towards the centre of the screen. Zoom in so the image is encompassed by the screen and covers only the diffraction pattern.

(5) Choose an exposure time using the dial near the capture button. A longer exposure will capture a fuller pattern with a higher signal-to-noise ratio.

    Note: the colour response of the camera is obviously not the same as our eyes. Even though the laser light is pure red, it is sufficiently bright under dark surroundings that the red channel in the ouput 8-bit RGB image is saturated at 255. Beyond this point the camera begins adding blue and green while keeping proportion to the brightness, but it can not express higher brightness than full white (255,255,255). In the dark, the colour of the central maximum begins turning white for an exposure time longer than 0.5 s. A longer exposure is not recommended.

(6) **Important:** you will need to take a calibration shot either before or after your shots in order to graph the physical angular width of the diffraction pattern. With the lights on, rule a horizontal measured line across the screen, or tape a ruler to the screen itself in line with the diffraction pattern. Capture a photo of the ruler/ruled line and then *do not change* the camera's position or zoom, lest you take another calibration shot.

(7) Having a longer exposure increases the chance of the image being blurred if the camera wobbles during a take. In particular, you should be careful not to put the camera into vibration as you release the shutter. If you do not have a steady hand, you can set the camera to do a delayed shot after button-press so it is not vibrating during its exposure.

(8) The experiment provides a total of 12 configurations of slits, but you do not need to use them all. Discuss with a demonstrator a suitable selection of 6 slit configurations for quantitative analysis. With your slit selected, cover over the rear face of the slide to conceal the glare of the laser from the camera, turn all the lights out, and pull the curtains to make the background as dark as possible. You may take a couple of shots of each pattern just in case one is blurry for any reason.

Analysis

(9) Write a Python function that evaluates the right-hand side of equation (1). A suggested function prototype is:

```
def diffract(theta,w,s,N,lmbda):
    ⋮
    return y
```

where `theta` is a vector of angles and `y` is the vector of corresponding intensities. (Note that `lambda` is a reserved word in Python. Do not use it as a variable name). Test out your function by plotting out a few theoretical diffraction patterns and satisfying yourself that they are as you would expect.

Note: For `theta` equal to 0 degrees, Python will give a divide-by-zero error! You should code this special case appropriately.

(10) We now wish to process the experimental data so that we can compare them with the theory. Connect the camera to the computer via USB, or use an SD-card reader, and import your .jpg files to your python workspace folder.

(11) The functions needed for this analysis are contained in the phylab module which you can import by the command

```
from phylab import *
```

These functions require figures to be displayed separately and not in-line, so use the regular console instead of IPython if you're using Spyder, or run them in a script. When your photos are displayed they may appear behind the Spyder window.

First, import the horizontal distance scale of your photographs by using

```
pixwidth = photocalib('rulerphoto.jpg', scale)
```

Give the physical length of your ruler or reference line as `scale`, eg. `0.15` for a 15 cm ruler. You will be prompted to click the two ends of your scale. The function returns the physical horizontal width of each pixel.

Next, begin loading the intensity data by calling the function

```
intens = photoproc('imagefile.jpg')
```

You will be prompted to click two diagonal corners of a box around your diffraction pattern. The function then sums the pixel colours by column and transforms the data into linear intensity (although it is an arbitrary scale, with no units). Make sure you crop around the whole diffraction pattern, but no more. Avoid including any other light artefacts above and below the pattern. The cropped image is displayed next for your review.

(12) The measured data is now in the NumPy array `intens`. We wish to plot this data against $\sin\theta$, where θ is the angle from the centre line. In all of the diffraction patterns, the intensity is maximum at the centre of the pattern. We can exploit this by adding the following PYTHON code to calculate θ:

```
intens = intens.tolist()
#casting as a list is convenient and allows the index() function

peakval = max(intens)
ipeak = intens.index(peakval)
Ndata = len(intens)
dist =                          # Distance between slits and screen
theta = arctan(pixwidth*(arange(Ndata)-ipeak)/dist)
```

Run the script file after having loaded one of your image files.

Note: In the code above, do not forget to set `dist` based on your measurements.

(13) Plot out your result as a function of $\sin\theta$, by entering

```
subplot(211); plot(sin(theta),intens); grid
```

Your graph should look like the diffraction patterns familiar from theory. Using the `plot` command in PYTHON produces a linear plot, but much of the important structure in the diffraction pattern has a much lower intensity than the peak, and is not clearly shown. In this case, it is better to use a logarithmic scale on the y-axis to display the results. To compare the linear and logarithmic displays, enter:

```
subplot(212); semilogy(sin(theta),intens)
```

Include the plot in your report.

Note: With large slit separation s there is a chance the central peak may not be the maximum in your data, thanks to camera bloom or otherwise. If this occurs you will have to find the central index manually.

(14) For a single slit, we wish to determine the width w. We note that the theoretical intensity distribution simplifies to

$$I(\theta) \propto w^2 \left[\frac{\sin u}{u}\right]^2$$

where $u = \frac{1}{2}kw\sin\theta$. From this expression, the intensity is zero whenever u is a non-zero integer multiple of π. This corresponds to $\sin\theta$ being a non-zero integer multiple of λ/w since $k = 2\pi/\lambda$. Thus if we measure the values of $\sin\theta$ at which the intensity is zero, we can determine w.
From your data, find the values of $\sin\theta$ at which the intensity is very small. These correspond to the theoretical zeros. Use them to deduce the value of w.

(15) Check that the experimental and theoretical curves are close to each other by plotting them on top of each other using

```
N = 1                   # Number of slits
w =                     # Enter your calculated value
s = 0.001               # Slit spacing -- arbitrary for a single slit
lmbda = 632.8e-9
scale = max(intens)/(N*w)**2
semilogy(sin(theta),scale*diffract(theta,w,s,N,lmbda),sin(theta),intens)
```

The variable `scale` is used so that the peak intensities coincide.

(16) In the same way, draw superimposed theoretical and experimental graphs for all the slit configurations. You will need to devise a similar procedure (based on locating the zeros of the theoretical intensity distribution) for determining the slit separation s as well as the slit width w for configurations with more than one slit.

(17) Once you have signed off the experiment, delete your images from the camera in order to conserve space. Also take care to switch back any odd settings you may have changed.

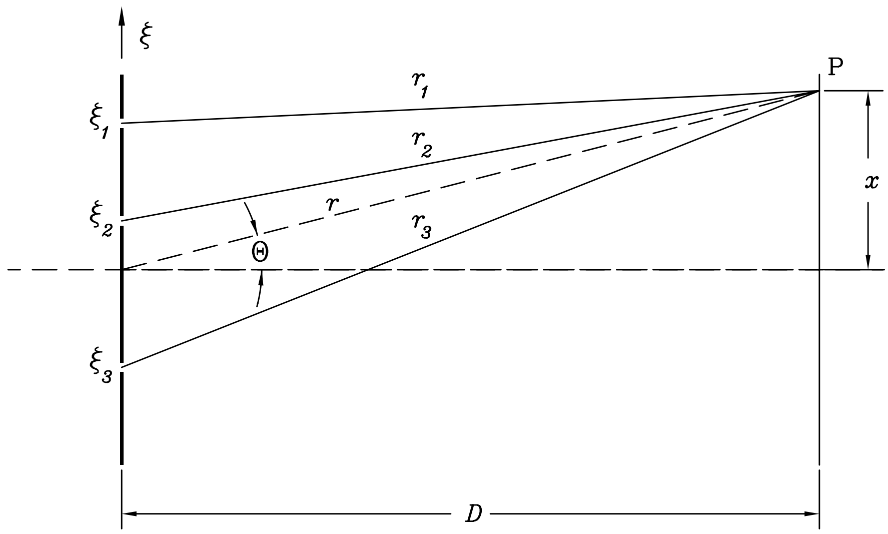
Write-up

Together with your results, your write-up should include:

1. A discussion of the physical effects on the diffraction pattern of changing the number of slits N, the slit width w and the slit separation s.
2. A derivation of equation (1). One method of doing this is outlined clearly in the Appendix, but other approaches to the problem can be taken. If you intend to use the Appendix, ensure that you include answers to all the intermediate questions that are listed. If you intend to use another method of calculation, then discuss this with your demonstrator first.

APPENDIX

Fraunhofer diffraction from slits



Diffraction theory is based on the idea that the oscillating electric field at a point P is the sum of contributions due to effective sources which make up the diffracting obstacle. In Figure 1, three effective sources are shown with complex amplitudes A_1, A_2, and A_3. The amplitude at P is:

$$A_P = \exp(i\,\omega t)\left[\frac{A_1}{r_1}\exp(-ikr_1) + \frac{A_2}{r_2}\exp(-ikr_2) + \frac{A_3}{r_3}\exp(-ikr_3)\right] \tag{A.1}$$

where $\omega = 2\pi c/\lambda$. The presence of r_m in the denominators gives the decrease of the amplitude with distance which falls as the inverse square law in intensity (inverse first power in amplitude). Usually, the distance between P and the sources is so large that we can take $r_1^{-1} \approx r_2^{-1} \approx \ldots \approx D^{-1}$. Thus

$$A_P = \frac{\exp(i\,\omega t)}{D}\left[A_1\exp(-ikr_1) + A_2\exp(-ikr_2) + A_3\exp(-ikr_3)\right] \tag{A.2}$$

Question 1: Why can we not make a similar approximation in the exponents?

From the diagram we see that

$$r_m = \sqrt{D^2 + (x - \xi_m)^2} \tag{A.3}$$

If we expand this using the binomial theorem to second order in ξ_m, we get:

$$r_m \approx r - \xi_m \sin\theta + \frac{\xi_m^2}{2r}\cos^2\theta \tag{A.4}$$

where $r = \sqrt{D^2 + x^2}$ and $\sin\theta = x/r$.

Question 2: Verify the above expansion.

If D is so large that $\xi_m^2 \cos^2\theta / (2r) < \lambda/4$, the additional phase due to the quadratic term may be neglected. This leads to the Fraunhofer condition (2). We then have:

$$A_P = \frac{\exp(i[\omega t - kr])}{D} \sum_m A_m \exp(ik\xi_m \sin\theta) \tag{A.5}$$

The intensity is given by $|A_P|^2$.

We now consider the case of N equal sources with the same amplitude A separated by distance s. This corresponds to the situation of having N infinitesimally wide slits. Substituting $\xi_m = ms$ for $m = 0, 1, 2, \ldots, N-1$ into equation (A5) yields

$$A_p = \frac{A\exp(i[\omega t - kr + (N-1)v])}{D}\left(\frac{\sin Nv}{\sin v}\right) \tag{A.6}$$

where $v = \frac{1}{2}ks\ \sin\theta$.

Question 3: Verify equation (A6) using the result for the sum of a geometric series.

The case of a single wide slit of width w can be derived from the above result by letting $N \to \infty$, $A \to 0$ and $s \to 0$ in such a way that $Ns \to w$ and $NA \to wA_0$. If this is done, we obtain

$$A_P = \frac{wA_0 \exp(i[\omega t - kr + u])}{D}\left(\frac{\sin u}{u}\right) \tag{A.7}$$

where $u = \frac{1}{2}kw\sin\theta$ and A_0 depends only on the intensity of the incident beam.

Question 4: Verify equation (A7) using the result that $\sin v \to v$ as $v \to 0$.

The result for N wide slits of width w separated by s can be found by summing together N amplitudes of the form of equation (A7), where the contribution from slit m is multiplied by a phase factor $\exp(ikms\sin\theta)$. The resulting intensity (absolute square of the amplitude) has the form given in equation (1).

Question 5: Verify equation (1).

List of Equipment

1. 1 x Canon EOS 1200D
2. 1 x Tripod
3. 1 x MiniUSB to USB connector
4. 1 x HeNe 1mW Laser
5. 1 x 1.5 m Angle Aluminium Bracket
6. 1 x Diffraction Grating
7. 2 x Translation Platforms

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