Experiment 254: Ramsauer-Townsend Effect

Aim

To demonstrate quantum mechanical scattering by a potential well by studying the scattering of electrons from xenon atoms.

Caution: This experiment involves the handling of liquid nitrogen. Read the safety notice situated by the apparatus.

References

1. S.G. Kukolich, American Journal of Physics, 36 p 701 (1968)
2. C. Ramsauer, Annalen der Physik, 64 p 513 (1921)
3. H.S.W. Massey, Atomic and Molecular Collisions, [539.754 M41a] (Introduction and Chapter 5)
4. E.W. McDaniel, Atomic Collisions, [539.754 M13] (pp 116, 181, 233)
5. R.A. Serway, Modern Physics, [530 S49m], or any standard quantum mechanics text

Cross section and mean free path

Both of these concepts are needed in order to understand the scattering of a beam of particles (or a beam of radiation) by a collection of scattering centres (for instance in the present case, the scattering of low energy electrons by the xenon atoms in the 2D21 xenon thyratron used in this experiment.) The scattering cross section is a hypothetical area, σ_s, centred on the xenon atom. If an electron passes through this area it is scattered. It is related to the probability of scattering in the following manner. Consider a thin slice of gas of thickness dx with a density of N atoms per m^3. Consider, further a beam of electrons distributed over an area A and incident on this gas. The electrons encounter $NAdx$ atoms. Their aggregate scattering cross section area is then $NAdx\sigma_s$ m^2. Now the probability, dP, of scattering is equal to this aggregate cross section divided by the total area over which the beam of electrons is distributed:

$$dP = NA\sigma_s dx/A = N\,\sigma_s\,dx$$

Hence if J is the number of electrons incident on this thin slice, the number of electrons removed from the beam is

$$dJ = -J\,dP = -JN\sigma_s dx$$

or

$$\frac{dJ}{J} = -N\sigma_s dx \tag{1}$$

By integrating both sides of this equation, the total number of electrons left in the beam after traversing a distance x of the gas is thus

$$J = J_0 \exp\left(-N\sigma_s x\right)$$

or

$$J = J_0 \exp\left(-x/\lambda_p\right)$$

where λ_p is the mean free path of the electrons in the gas and $\lambda_p = 1/N\sigma_s$. The probability of scattering for a given value of x is thus

$$P\left(x\right) = 1 - \frac{J}{J_0} = 1 - \exp\left(-x/\lambda_p\right) \tag{2}$$

The Ramsauer-Townsend effect

If the scattering of electrons from atoms is described using classical mechanics in which both the electron and atom are considered to have a well defined size, the cross section for scattering will be independent of the energy of the bombarding electrons (Think of scattering of a ping-pong ball from a billiard ball). It was therefore a great surprise when in 1921 Ramsauer in Germany and Townsend in England showed that at an energy of around 0.7 eV, atoms of argon appeared to be almost transparent to the electrons; that is, their scattering cross section was very low and was lower than for energies both above and below 0.7 eV. Figure 1, taken from Massey, shows the collision cross section for three of the noble gases. Each shows a clear minimum in the cross section at energies below 1 eV. An understanding of this Ramsauer-Townsend effect demands the use of quantum mechanics in which the electron is treated as a wave motion. Indeed for an electron of momentum p the wavelength of the associated wave is $\lambda = h/p$ where h is Planck's constant. The wavelength λ (in nm) of an electron of kinetic energy K (in eV) is thus given by $\lambda = 1.226/\sqrt{K}$.

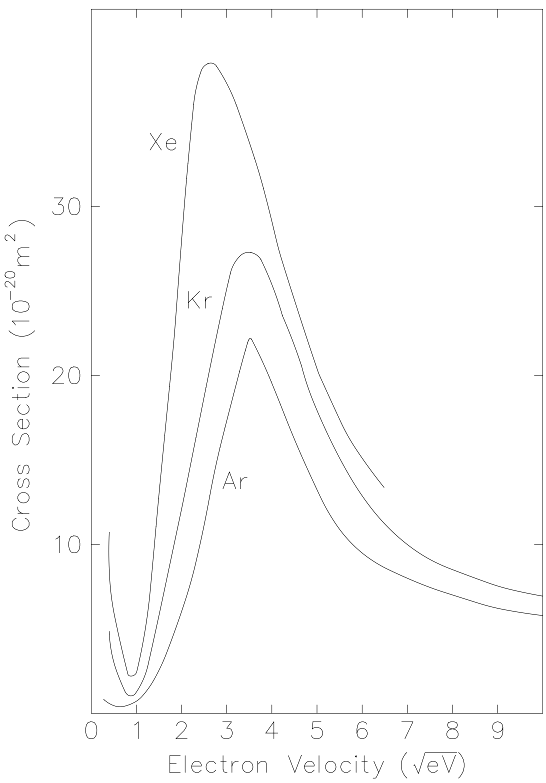

Figure 1:

A proper description of the scattering process needs to use the quantum mechanical scattering theory for scattering from a three dimensional potential. However the important quantum mechanical features of the problem can be understood by modelling the interaction between the electron and the xenon atom in terms of a one dimensional problem in which the xenon atom is treated as a square well of depth V_0 and width D equal to the diameter of the atom. (See for instance the discussion in Serway). In this case, the probability of scattering can be taken to be given by the reflection coefficient R:

$$R = \frac{V_0^2 \sin^2 (2\pi D/\lambda)}{V_0^2 \sin^2 (2\pi D/\lambda) + 4E\,(E + V_0)}$$

where V_0 is the well depth in eV, E is the incident electron energy in eV, while λ is the electron wavelength in the well, $\lambda = 1.226/\sqrt{E + V_0}$. Convenient units for both D and λ are nm (nanometre = 10^{-9} metres). Clearly, the scattering becomes zero when $2\pi D/\lambda = n\pi$, where n is an integer. Rewriting this last equation slightly gives the condition as $D = n\lambda/2$: the scattering goes to zero when D contains a half integral or integral number of wavelengths of the electron in the potential well. The diameter of a xenon atom can be taken as 0.434 nm and the complete treatment suggests that the potential well is quite deep so that V_0 can be taken to be about 50 eV. The full three dimensional treatment explains why there is actually only one minimum in the scattering cross section rather than the large number predicted by the simple one dimensional theory. For further details, consult the texts by Massey or by McDaniel.

Procedure

The experiment uses a 2D21 xenon thyratron. This device has four electrodes inside the glass envelope. Three of the electrodes are enclosed in the rectangular shield which is what you see when you examine the thyratron. Figure 2 shows a cross section. A hot filament heats the cathode which is thus a source of electrons. For the purposes of this experiment, the grid is connected to the shield. The shield is at a positive potential with respect to the cathode and it is arranged that the plate is a few volts more positive than the shield. Almost all of the electrons emitted by the cathode are attracted to the shield. This means that the shield current gives a good measure of the total number of electrons emitted from the cathode per second. A small fraction of the emitted electrons actually travel through the aperture nearest the cathode and enter the field free region between the two apertures in the shield. In this region, the electrons therefore have a constant energy which (in eV) is eV_{acc} where V_{acc} is the potential of the shield relative to the cathode. This potential is labelled V_{acc} because it is the accelerating voltage of the electrons. All electrons which reach the second aperture are attracted to the plate. The plate current is therefore a measure of the electrons per second which have not been scattered in the field free region.

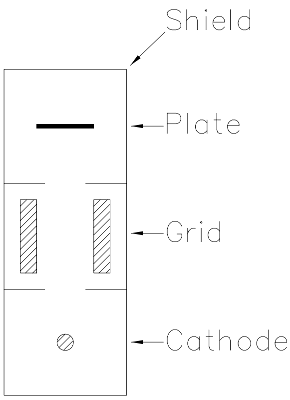

Figure 2:

At any given value of accelerating voltage of the electrons, the plate current and shield current depend on three factors: the geometry of the thyratron, the accelerating voltage and on the scattering in the field free region. Measurements are made with the thyratron at room temperature and with the thyratron cooled by liquid nitrogen. When cooled by liquid nitrogen, the xenon freezes and falls out of the path of the electron beam. As a result no scattering occurs and hence any difference between the results at room temperature and at liquid nitrogen temperature must be caused by the presence of scattering by the xenon gas at room temperature.

(1) The collision cross section in xenon is measured by measuring the attenuation of the electron current as a function of the accelerating voltage and applying equation (2). Let I_P and I_S be the plate current and the shield current for a given V_{acc}. The currents I_P^* and I_S^* denote the same quantities with the xenon frozen out. To determine $P(x)$ for a given V_{acc} it is necessary to measure I_P/I_S and I_P^*/I_S^*. At any given value of V_{acc}, the probability that no scattering has occurred is the double ratio $(I_P/I_S)\,/\,(I_P^*/I_S^*)$. Hence the probability of scattering is

$$P(x) = 1 - \frac{(I_P/I_S)}{(I_P^*/I_S^*)} \tag{3}$$

Here x is the distance from the first aperture to the plate. In the 2D21 thyratron, $x = 0.7$ cm. The apparatus is arranged so that it is easy to choose a sequence of accelerating voltages which is the same in each case (i.e with and without the xenon frozen out). This makes subsequent analysis using PYTHON quite straightforward.

(2) Use the DVM in its ohm-meter mode to determine accurate values for R_P and R_S. Check that their nominal values are close to your measured values.

(3) You will need to choose a sequence of values for V_{acc} which will give you around 30 points and adequately defines the cross section from the lowest measurable electron momentum (which is proportional to $\sqrt{V_{\mathrm{acc}}}$) to a value of around 3.5 $\sqrt{\mathrm{volt}}$.

(4) I_P and I_S are obtained by measuring the voltage across resistors which connect them to the common positive voltage as in Figure 3. The voltages are measured using a DVM which is multiplexed to make it easy to read the different voltages required. Probability of scattering is given by equation (3).

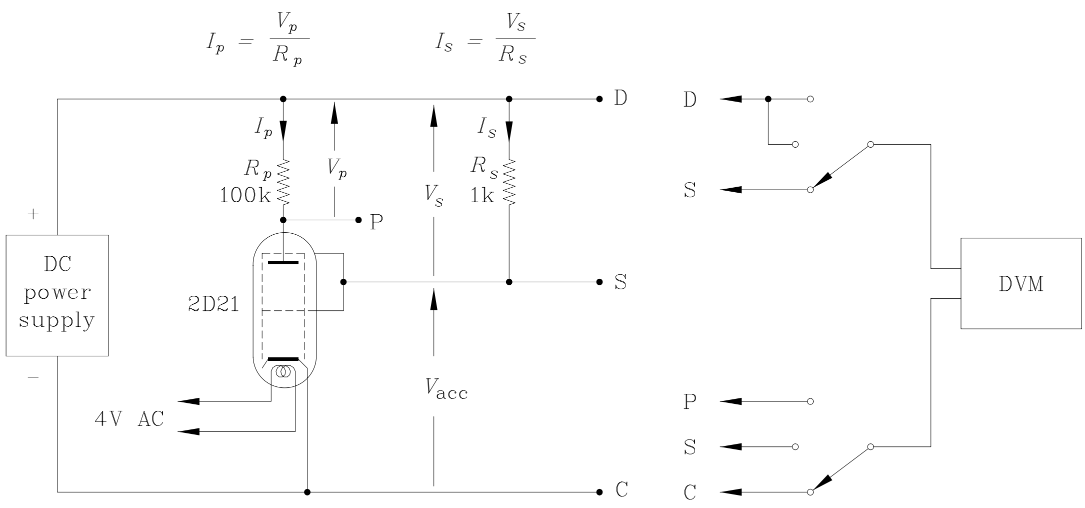

Figure 3:

(5) Record a sequence of readings of V_{acc}, V_S and V_P with the thyratron at room temperature.

(6) Disconnect the heater supply and allow the thyratron to cool. Then freeze the xenon gas by immersing the end of the thyratron tube (black area) in liquid nitrogen. Reconnect the heater supply and repeat your measurements using the same values of V_{acc} as in procedure (5) to obtain V_S^* and V_P^*. Note that as the experiment proceeds liquid nitrogen evaporates and its level lowers. The height of the thyratron assembly will need to be adjusted to maintain the black area of the tube in the liquid nitrogen.

(7) Enter your observations in a PYTHON script. Plot $10 \times I_P$ versus V_{acc} and I_P^* versus V_{acc} on the same graph (remember that $I_P = V_P/R_P$ etc.)

(8) The probability of scattering is given by equation (3) where all quantities are recorded at the same value of V_{acc}. Calculate this ratio and plot it as a function of $\sqrt{V_{\mathrm{acc}}}$.

(9) From equations (2) and (3), $(I_P/I_S)\,/\,(I_P^*/I_S^*) = \exp{(-x/\lambda_P)}$where $\lambda_P = 1/N\sigma_s$. Use these relations to calculate the scattering cross section σ_s at each value of V_{acc} and plot σ_s against a quantity which is proportional to the momentum of the electrons.

Data

Radius of xenon atom	= 2.17 angstrom (0.217 nm)
Ionisation potential of xenon	= 12.129 eV
Melting point of xenon	= −111.8° C
Boiling point of xenon	= −108.0° C
Boiling point of nitrogen	= −195.8° C
Xenon pressure in 2D21 thyratron	= 0.05 torr
First aperture to plate to 2D21	= 0.7 cm
DVM reading errors for voltage	= ±0.1% of reading ±1 digit
DVM reading errors for resistance	= ±0.1% of reading ±1 digit
DVM input resistance	= 100 MΩ on 200 mV range
	= 1000 MΩ on 2 V range
	= 10 MΩ on 20 V, 200 V and 1000 V ranges

Questions

1. Verify that the wavelength (in nm) of an electron of kinetic energy K (in eV) is given by

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{1.226}{\sqrt{K}}$$

2. Calculate the wavelength of electrons of energy $0.1, 1.0$ and 10 eV and compare these to the diameter of a xenon atom.
3. In the expression for R, the reflection coefficient given above, set $V_0 = 49.28$ eV and use PYTHON to calculate R for E ranging from 0 to 16 eV. Plot R versus $\sqrt{E}$ and determine the wavelength, both inside and outside the potential well, of the electron at the energy which corresponds to the minimum scattering. How many wavelengths of the electron are contained in the well?
4. Will the xenon be a solid, liquid or a gas when the thyratron is immersed in the liquid nitrogen?
5. Convert the quoted pressure of the xenon in the thyratron to pressure in SI units (Pascal).
6. Why is it suggested that the independent variable for plotting can be taken as $\sqrt{V_{\mathrm{acc}}}$?
7. Is it necessary for the plate to be at a higher positive potential relative to the cathode than the shield?
8. What errors does the input resistance of the DVM introduce into the determination of I_P?
9. What factors influence the choice of values of R_P and R_S?
10. Compare your measured cross section to the cross section for xenon as given in Figure 1.

List of Equipment

1. Thyratron assembly
2. 2D21 thyratron
3. Dewar flask
4. 0 – 20 V DC power supply
5. 4 V AC power supply
6. Weston 1241 DVM

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