Experiment 334: Phonon Excitation Model

Aim

To demonstrate the theory of lattice interactions in a crystal by means of a simple mechanical model.

References

1. C. Kittel "Introduction to Solid State Physics" p176, Wiley 1971
2. M. Born & K. Huang "Dynamical Theory of Crystal Lattices" p55, Clarendon press 1954

Introduction

The theory of lattice vibrations (i.e. phonons) is fundamental to solid state theory, especially with regard to sound propagation, electromagnetic radiation, absorption, and specific heat. The object of this experiment is to show the simple relationship between the frequency ω and the wave number k in a simple mechanical model of a diatomic crystal lattice. The model consists of a stretched wire with regularly spaced masses on it, excited into transverse vibration by a magnet and an alternating current passing down the wire.

Acoustic and Optical Vibrations

A real single crystal consists of a large number of identical three-dimensional cells, each containing a fixed number of atoms. If each cell contains one atom, the crystal only supports vibrations of the acoustic type, which are called acoustic because they correspond in the limit of long wavelengths to the ordinary elastic vibrations of a continuum.

If there are two (or more) atoms per cell, the acoustic vibrations are those in which the atoms in a cell are approximately in phase with each other. However, optical vibrations, in which atoms in the same cell are out of phase with each other, can also occur: these have a higher frequency and are usually separated from the acoustic frequencies by a band gap.

Suppose we consider an ionic diatomic crystal; with two ions of opposite charge vibrating out of phase, large electric moments are produced, and phonons in the optical mode can interact strongly with infrared electromagnetic radiation. This is the reason for calling the second mode of vibration the "optical" mode.

Theory

Consider point masses on a (massless) wire at tension T, each mass equally spaced at a distance a. Masses M and m $(M > m)$ are located alternately. The masses M are located at the even sites $x = 2na$, the masses m at the odd sites $x = (2n+1)\,a$ (n is an integer). Let the transverse displacement at the r'th site be y_r. If we consider small enough displacements, we have:

$$M\frac{\mathrm{d}^2}{\mathrm{d}t^2}(y_{2n}) = \frac{T}{a}\left[y_{2n+1} + y_{2n-1} - 2y_{2n}\right] \tag{1}$$

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}(y_{2n+1}) = \frac{T}{a}\left[y_{2n+2} + y_{2n} - 2y_{2n+1}\right] \tag{2}$$

This set of coupled differential equations has travelling wave solutions of type:

$$y_{2n} = A_1 \exp\left[i\left(\omega t \pm 2nak\right)\right]$$
$$y_{2n+1} = A_2 \exp\left[i\left(\omega t \pm (2n+1)\,ak\right)\right]$$

where the same sign must occur in both exponentials, and the displacement is understood to be the real part of the complex term.

Equation (2) yields:

$$-A_1\omega^2 M = \frac{T}{a}\left[2A_2 \cos ka - 2A_1\right]$$
$$-A_2\omega^2 m = \frac{T}{a}\left[2A_1 \cos ka - 2A_2\right]$$

Therefore

$$A_1\left(\omega^2 M - \frac{2T}{a}\right) + A_2\left(\frac{2T}{a}\cos ka\right) = 0 \tag{3}$$
$$A_1\left(\frac{2T}{a}\cos ka\right) + A_2\left(\omega^2 m - \frac{2T}{a}\right) = 0 \tag{4}$$

Two simultaneous homogeneous linear equations in two unknowns can only have a non-zero solution if the determinant of the coefficients is zero. In other words, waves can only propagate along the wire if:

$$\omega^2 = \frac{T}{a}\left[\frac{1}{M} + \frac{1}{m}\right] \pm \frac{T}{a}\left[\left(\frac{1}{M} + \frac{1}{m}\right)^2 - \frac{4\sin^2 ka}{Mm}\right]^{1/2} \tag{5}$$

This gives a dispersion diagram (a plot of ω against k) with two branches — the one with the "+" sign describes waves in the optical mode, the one with the "−" sign describes waves in the acoustic mode.

Now consider stationary solutions for a finite number of masses on a wire with fixed ends. For example, with the wire fixed at the origin and at $x = 16a$; with 8 masses m at odd sites $x = (2n+1)\,a$; and with 7 masses at even sites $x = 2na$. The solutions for the displacements are:

$$y_{2n} = A_1 \sin 2nka \sin \omega t$$
$$y_{2n+1} = A_2 \sin (2n+1)\, ka \sin \omega t$$

where ω is as given in equation (5). These are obtained by superposing two waves of equal amplitude travelling in opposite directions. We also require:

$$A_2 2T \cos ka = A_1\left(2T - \omega^2 Ma\right) \tag{6}$$
$$y_0 = y_{16} = 0$$

and so

$$ka = \frac{N\pi}{16} \text{ where integer } N \text{ is the mode number} \tag{7}$$

We are interested in the number of distinct resonances which can be observed, i.e., the number of distinct solutions for ω and y_j which can be obtained by varying the integer N. Since $ka = N\pi/16$, and ka appears in all equations as "$\sin ka$" or "$\sin nka$" etc., there are at most 32 solutions for $N = 0, 1, 2, ..., 15$, if both optical and acoustic modes are taken into account. However, it is easy to show that solutions for $N = 9, 10, ..., 15$ are the same as those for $N = 7, 6, ..., 1$; also the solution for $N = 0$ and the acoustic branch solution for $N = 8$ are zero displacement (trivial) cases. Thus the system has 15 resonant frequencies, 8 in the optical mode, 7 in the acoustic mode (see Appendix A and B).

Equipment

An alternating current from a 60 W (into 8Ω) transistorised audio amplifier, with a frequency response of about 15 Hz − 50 kHz ±1 dB and an "Eclipse Major" permanent magnet are used to excite the nichrome resistance wire.

The current needed to drive the wire in practice varies from 100 mA to 1 A depending on the mode being driven. The pole pieces have slots milled in them so that the magnet can be used in the same position as one of the masses. If the magnet is placed between two masses, then because of the change of phase between successive masses vibrating in an optical mode, phase reversal can occur within the length of the pole piece. Between the pole pieces of the magnet the maximum field is 0.58 tesla (1 tesla = 10^4 gauss = 1 weber/m^2).

Equipment Data

Data for each set is provided with the apparatus.

Experiment

(1) No trouble should be experienced in exciting the acoustic modes and measuring their frequency. The frequency calibration of the oscillator is sufficiently accurate for this experiment. The optical modes are less easy to separate. Try to find ways to excite the required mode but not the adjacent higher and lower modes.

A useful way of finding resonance is to observe the beats between the resonant frequency and the driving frequency which occur in the transient regime. The period of these beats increases as resonance is approached.

Ensure that the wave form is sinusoidal by means of the monitor oscilloscope. Measure the frequency of the driving using the oscilloscope, as the frequency dial on the signal generator is not sufficiently accurate. Note that in the optic mode the amplitude of vibration of the large masses will seem very small and the node very close to the large mass. This is because the amplitudes are inversely proportional to the masses (see question 2).

(2) A theoretical dispersion relationship has been computed for the values of T, a, M and N appropriate to this experiment. No correction was made for the thickness of the masses. The results of the computation are plotted in Appendix C.

Compare your experimental resonant frequencies with the theoretical ones and try to account for discrepancies. You may find it useful to calculate the expected resonant frequencies (especially for the optical modes) before starting the experiment.

Questions

1. Show that the acoustic mode does indeed correspond to elastic vibrations.

   Hint: Show first that for small k, ω is proportional to k. Compare the constant of proportionality with the usual expression for the phase velocity of elastic waves on a uniform wire.

2. Show that for $k \to 0$, the ratio of the amplitudes A_1 and A_2 for the optical mode is given by:

$$\frac{A_1}{A_2} = -\frac{m}{M}$$

   i.e., the motions of the two particles in every cell are opposed to one another.

List of Equipment

1. Tensioned nichrome wire with masses spaced at equal separations
2. Noy-Tronics 300 MSTPC/02 unction Generator
3. 60 W Power amplifier
4. Permanent magnet

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July 1997

APPENDIX A

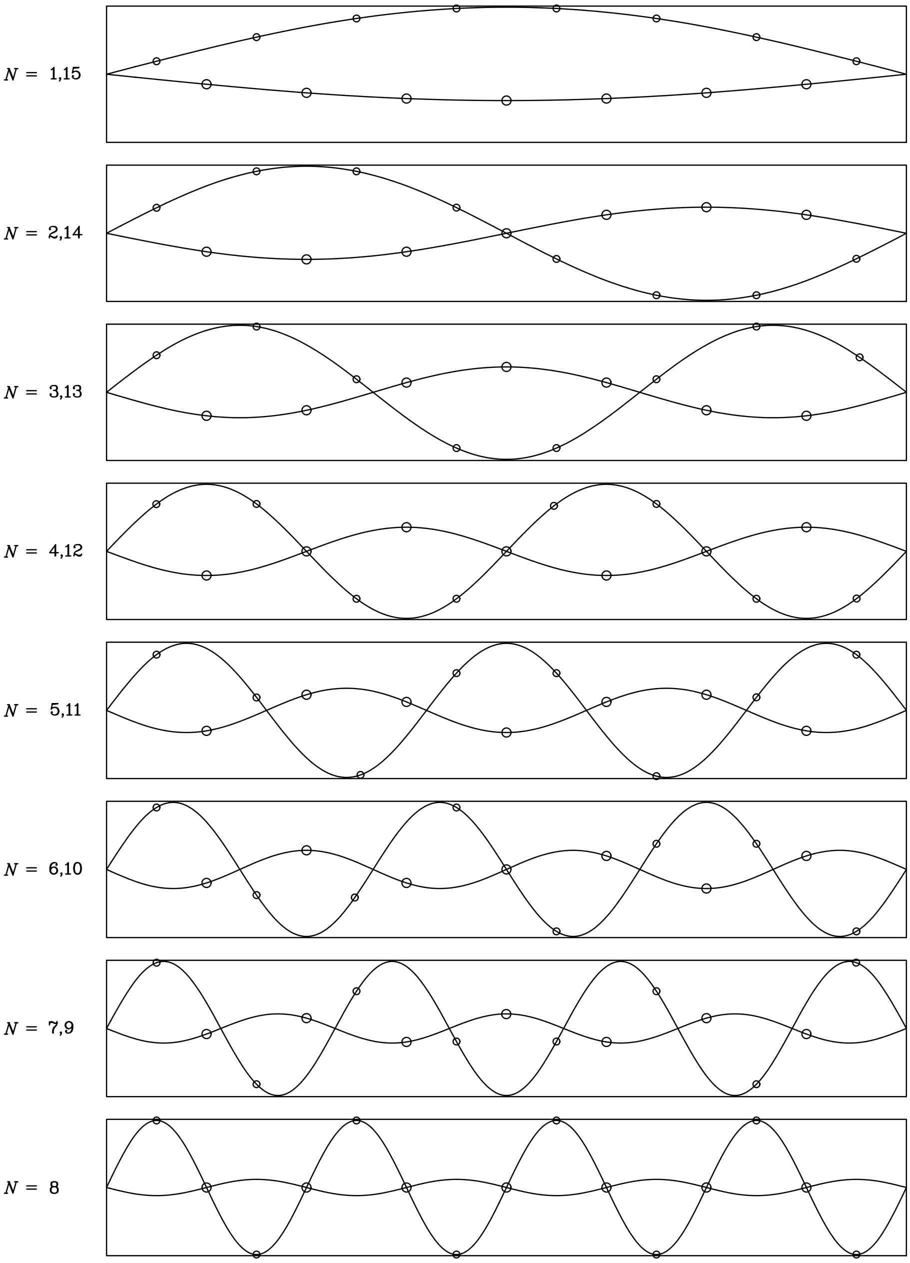

Figure 1: Optical mode resonance frequencies, plots of y against x

APPENDIX B

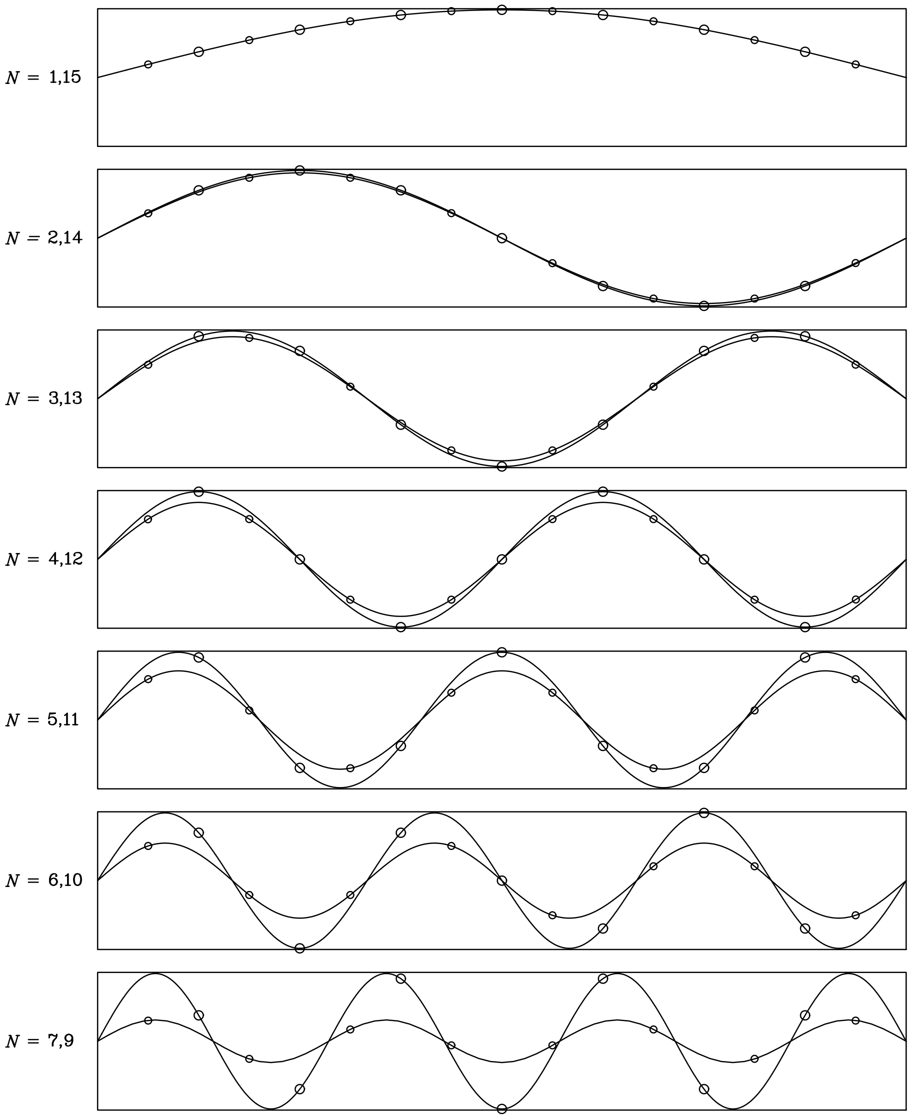

Figure 2: Acoustic mode resonance frequencies, plots of y against x.

APPENDIX C

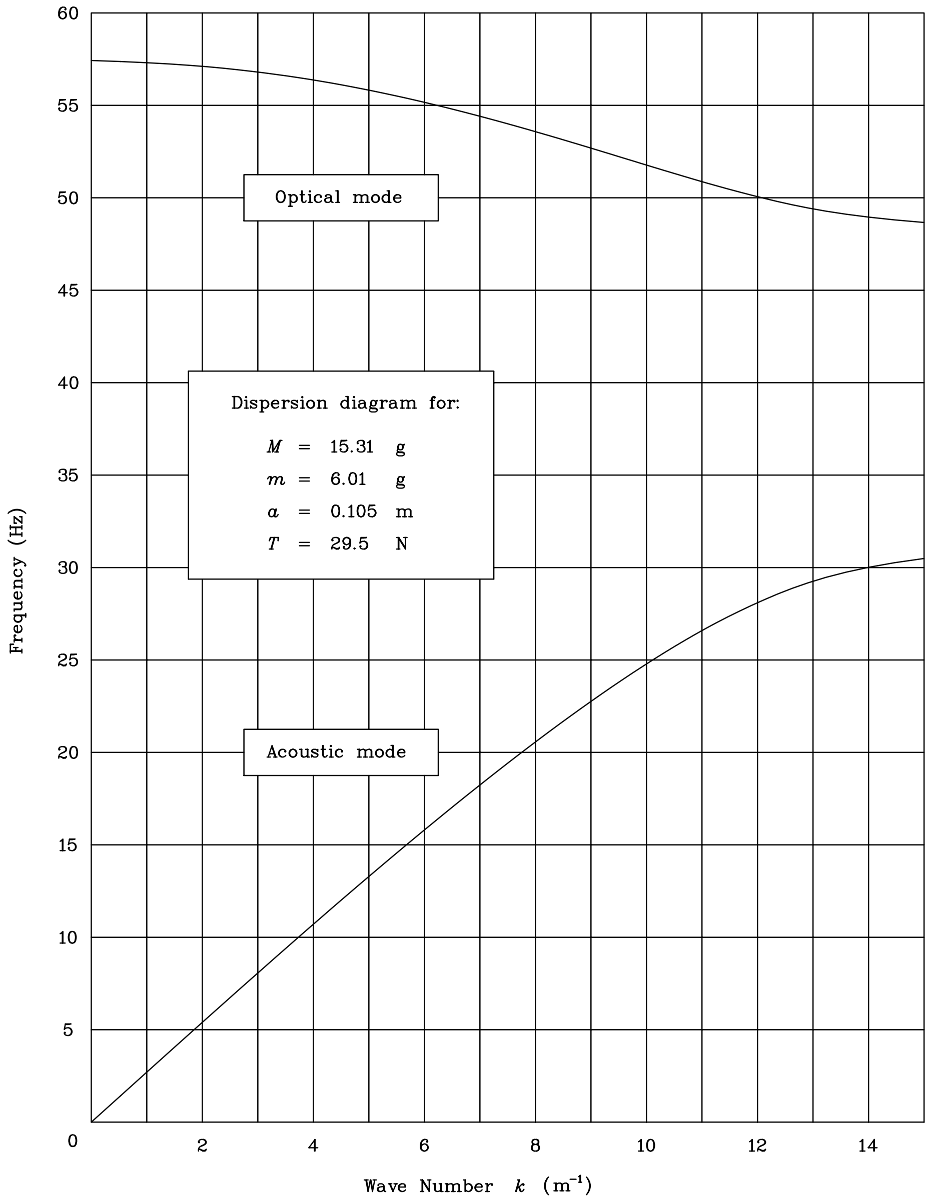

Figure 3: Dispersion diagram