Experiment 242: Optical Spectrum of Sodium using the Diffraction Grating

Aim

To become familiar with the techniques of using a diffraction grating spectrometer and to study the spectrum of sodium.

References

1. A.C. Melissinos: "Experiments in Modern Physics", Academic Press.

2. A.P. French & E.F. Taylor: "An Introduction to Quantum Physics", Nelson.

3. Eisberg & Resnick: "Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles", Wiley.

4. C.E. Moore: "Atomic Energy Levels", Vol.1, National Bureau of Standards, Circular 467 (1949).

5. K.A. MacFadyen: "A Physics Laboratory Handbook for Students", University of London Press.

6. Jenkins & White: "Fundamentals of Optics" (4th Ed.) McGraw-Hill.

7. NBS Monograph 32: Tables of Spectral-Line Intensities

References 1, 5 and 6 are available in the Advanced Laboratory library.

Introduction

Understanding the way in which atoms absorb and emit light is of fundamental importance to physics. In particular, the study of emission from excited atoms has lead to major advances in physics in the twentieth century. In this experiment you will study the properties of light emitted by excited sodium atoms using a diffraction grating spectrometer. You will see that unlike white light from the sun or an incandescent lamp, the spectrum of the emitted radiation consists of emission only at discrete wavelengths, and leads to a spectrum of discrete lines when viewed with the spectrometer. You will also see that each "line" in the spectrum is actually split into two, forming a doublet. The observation of this *fine structure* in atomic emission spectra was a great puzzle to physicists in the early years of the century, and was only resolved in 1925 when Uhlenbeck and Goudsmit discovered that electrons in atoms possess an intrinsic quantum number which they referred to as *spin*.

The first part of this experiment involves learning how to align the spectrometer correctly to observe a line spectrum, and to determine the spacing of grooves in the grating assuming that the wavelength of one particularly strong line in the sodium spectrum is known. Using this value for the grating spacing, the wavelengths of other fainter lines in the sodium spectrum will be determined. You will then compare the results of your measurements with known results, and match the emitted wavelengths with particular electronic transitions in sodium. This will allow you to confirm the transition *selection rules* predicted by quantum mechanics.

The diffraction grating

The grating formula

$$d\left(\sin i \pm \sin\theta\right) = n\lambda$$

gives the angular position θ of the lines of the spectrum (see Figure 1).
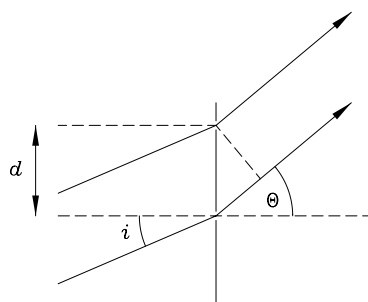
Figure 1:

If the light is incident normally on the grating ($i = 0°$), the grating formula takes the simpler form:

$$d \sin \theta = n\lambda$$

If light of known wavelength λ is used, measurement of θ enables the grating spacing d to be evaluated. If d is known, measurements of θ determine λ. The chromatic resolving power of a grating (see Section 17.9 of Reference 6) is defined as $\lambda/\Delta\lambda$. It can be shown on certain assumptions that:

$$\frac{\lambda}{\Delta\lambda} = n\,N = \frac{Nd \sin \theta}{\lambda}$$

where N is the number of lines of the grating illuminated, n is the order of the spectrum and Nd is the width of the illuminated grating surface.

Procedure

It is **essential** that the adjustment of the spectrometer be carried out meticulously if accurate results are to be obtained.

Adjusting the Spectrometer

I sometimes regret that much of our modern apparatus, even for students, has all the interesting difficulties removed beforehand. If a student is going to work with a spectrometer, I think it is highly desirable that he should go through the process of adjusting the collimator, the telescope and the eyepiece himself. It is desirable that he shall go through the process of getting the grating lines parallel with the axis of rotation. It is desirable that he shall know how to set the axis of the telescope perpendicular to the axis of rotation. Once the spectrometer is adjusted, all of the good of the experiment has been utilised. I do not think that the student learns much in the last act of measuring the wavelength of light.

W.F.G. Swann, "The Teaching of Physics", Am. J. Phys. **19**(3), 182 – 187, 1951.

Focussing the eyepiece

(1) Direct the telescope towards a uniformly illuminated surface to give a suitable background. Slide the eyepiece in and out till the cross-wires are clear and sharp when the eye is relaxed. (i.e. focussed on infinity).

Focussing the telescope and collimator for parallel light

(2) The most straightforward method of doing this is to use a light source at "infinity". Take the spectrometer into the main laboratory and focus the telescope on the most distant object visible. Then point the telescope at the collimator and adjust the collimator until a sharp image of the slit is seen. Alternatively a procedure known as Schuster's method may be used. Details of this are in Ref.5.

Adjustment of grating for normal incident light

(3) Set the telescope carefully in the straight-through position. Then using the scale of degrees turn the telescope through exactly 90° and clamp it. Without touching the ruled surface of the grating, place the grating centrally in its holder on the table and clamp it. Using the grating as a plane mirror turn the table till the slit image is on the cross-wires and note the vernier reading. Using the scale of degrees turn the table back through exactly 45° and clamp in position. Release the telescope.

Any slight errors in the last setting can be corrected in subsequent experiments by measuring all diffraction angles on both sides of the straight-through position and averaging.

Read page 60 of Ref. 5 on how to set the cross-wire on a spectral line and how to avoid eye fatigue.

Preliminary Consistency Check

Before proceeding to make any detailed measurements on the sodium spectrum, you should check that the spectrometer has been aligned correctly.

(4) Set the spectrometer so that the slit is evenly lit by the sodium lamp. Note that you can alter the position of the aperture in the source holder to get the maximum brightness. Now by rotating the telescope you should be able to see the different discrete lines in the sodium spectrum. If the spectrometer is correctly aligned, careful observation will reveal that each "line" is in fact a *doublet* of two closely spaced lines. If you cannot resolve this doublet structure, then try reducing the width of the spectrometer slit until you can. If this doesn't work, consult a demonstrator, since the observation of the doublet structure of the sodium lines is one of the main points of this experiment.

Also note that as you increase the angle of observation, each line in the spectrum repeats itself. This is what is meant by the observation of different *orders* in the spectrum, corresponding to different values of n in the grating equation. The straight-through position is the case when $n = 0$, and there is no angular separation of different wavelengths. The first observation of each separate line is the first order $(n = 1)$, the second observation of each separate line is the second order etc. Ensure that you understand what is meant by the different orders of diffraction before proceeding further.

For one particular well defined line in the spectrum, line up the cross-wires and take readings on both verniers. Repeat the measurement on the same line several times and compare your measurements. If they agree to within 1 or 2 minutes then you are achieving consistency in setting. If not, then attempt to determine why this is not so **before** proceeding with the experiment.

Calibration of the grating

(5) To calibrate the grating we will use the bright sodium D-line. The two doublets in this line have wavelengths $\lambda_{D_1} = 589.00$ nm and $\lambda_{D_2} = 589.59$ nm. Measure the diffraction angle of each component of the doublet in both the first and second order, and calculate the grating spacing by averaging your results.

Wavelengths of sodium lines

(6) Measure the wavelengths of the visible lines in the sodium spectrum in both first and second order. Ensure that for each line, you measure the wavelength of each doublet component. With dark adaptation, and patience, you should be able to see at least up to 10 different lines in the first order, and a bit less in the second order. It is usually possible to see more lines than this, and you should attempt to record as many as possible since it makes the interpretation of your results easier. Compare your experimental values with those given in Reference 7.

Questions

These must be answered as part of your write-up.

1. Why do we observe the spectral components as *lines*, and not for example, as circles?

2. Explain in words, what is meant by the resolving power of the grating.

3. Why is it suggested that both verniers of the spectrometer be read in procedure (4)?

4. Determine whether the resolving power formula correctly predicts whether the yellow lines are resolved in 1st, 2nd, 3rd order.

5. Identify the electronic transitions which give rise to the sodium spectral lines and verify that the selection rules given in the theory are obeyed for the lines you observe.

   (a) Explain in your own words why the energy levels for sodium as given in Figure 2 are different from those for hydrogen even though sodium is described as a "hydrogen-like atom".

   (b) Explain why all the lines of the sodium spectrum are split into doublets.

List of Equipment

1. Spectrometer

2. Sodium lamp

3. Grating (600 lines per millimeter)

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Revised, S. Coen, March 2014

APPENDIX

Theory of sodium spectrum

Sodium has 11 electrons, so that the $n = 1$ and $n = 2$ shells are completely filled leaving just one electron ($n = 3$) outside the closed shells. In this respect the sodium spectrum should be equivalent to that of hydrogen except for the central charge that the electron sees. Since the nucleus with $Z = 11$ is "screened" by 10 negative charges (the $n = 1$ and $n = 2$ electrons), the free electron sees a potential $-e/4\pi\varepsilon_0 r$ when far from the nucleus and a potential $(-Ze/4\pi\varepsilon_0 r) + C$ when close to it, where C is the potential generated at the nucleus by the other electrons. However, whereas for hydrogen only one energy level was found for each value of n, (see Reference 1), a more complex situation arises in sodium, with several energy levels arising for each value of n. This splitting is attributed to the fact that the time-independent Schroedinger equation for the hydrogen-like atom,

$$\nabla^2\Psi + \frac{2m}{\hbar^2}\left(E - V\right)\Psi = 0$$

where $\hbar = h/2\pi$, h being Planck's constant, admits solution with a principal quantum number n, and angular momentum quantum number l, such that $n > l + 1$. When the potential that the electron sees is exactly that of the Coulomb type (as in the case of hydrogen, where $V = -Ze/4\pi\varepsilon_0 r$) the energy eigenvalues:

$$E_n = \frac{1}{2}\frac{Z^2 e^4}{\left(4\pi\varepsilon_0\right)^2}\frac{m}{\hbar^2}\frac{1}{n^2}$$

are independent of l, and agree with the Bohr theory. However, the screened potential that the outer sodium electron sees is no longer of the simple Coulomb type, and the energy of the level depends on both n and l. Orbits with smaller values of l are expected to come closer to the nucleus and thus be bound with greater strength; as a consequence their energy will be depressed. The energy-level diagram of sodium is shown in Figure 2 where the levels have been grouped according to their l value. The customary notation is used, namely:

$$
\begin{array}{ll}
l = 0 & \text{S state} \\
l = 1 & \text{P state} \\
l = 2 & \text{D state} \\
l = 3 & \text{F state} \\
l = 4 & \text{G state}
\end{array}
$$

and so on, alphabetically.

The last column in Figure 2 gives the position of the levels of a hydrogen-like atom. The left-hand scale is in 10^3 cm^{-1} referred to 0 for the singly-ionized sodium atom; the right-hand scale is in electron volts referred to 0 at the ground state of the sodium atom.

We note that the higher the value of l, the smaller the departures from the hydrogen-like levels (as suggested qualitatively previously), and that for a given l the energy levels for different n follow the same ordering as the hydrogen-like atom.

Selection rules

As we know, the spectral lines that we observe are due to transitions from one energy state to a lower one; however, in analyzing the spectrum of sodium, it becomes immediately evident that not all possible transitions occur. Thus certain "selection rules" for atomic transitions must be operative and it is found that (with some exceptions) for all spectral lines:

$$\Delta l = \pm 1$$

This selection rule is readily explained by the quantum-mechanical theory of radiation; it then means that only "electric dipole" transitions occur. Indeed, the "electric dipole" transition probability is larger by a factor of c/v (where c is the velocity of light, v the velocity of electron) from the next order, while under no

conditions can a transition be had in which the angular momentum does not change at all ($\Delta l = 0$). The exceptions to the selection rule (as quadrupole transitions) are found in stellar spectra, but they occur so seldom that the resultant spectral lines are very faint.
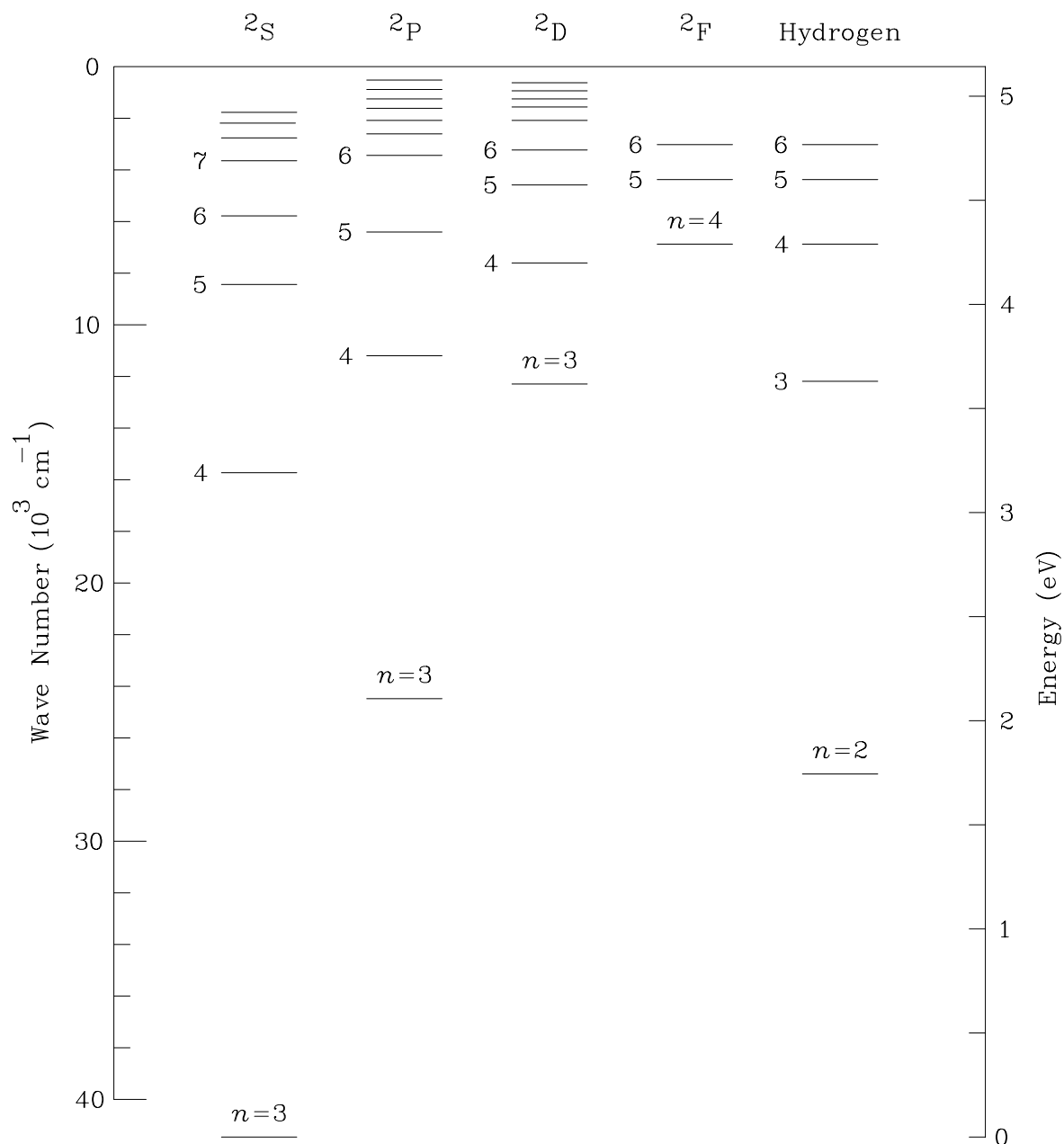

Figure 2: Energy Diagram

By applying the selection rule $\Delta l = \pm 1$ to the energy-level diagram of Figure 2 we obtain Figure 3, which gives the principal lines of the sodium spectrum. Since l must change by one unit, transitions will always occur between adjacent columns and never within the same one.

You should be able to determine that in each case the prominent lines of the spectrum occur as doublets. This splitting of spectral lines was named "fine structure" and must reflect a splitting of the energy levels of sodium; if we express the wave-lengths of sodium in wave numbers ($\widetilde{v} = 1/\lambda = v/c$ in a scale proportional to energy), it becomes evident that the spacing in all doublets is exactly the same and equal to 17.3 cm^{-1}. Indeed, the doublet structure of all the above lines is due to the effect of the electron "spin" and its coupling to the orbital angular momentum (designated by l). According to the Dirac theory, the electron possesses an

additional degree of freedom, called the "spin", which has the properties of angular momentum of magnitude $s = \hbar/2$. The electron thus possesses two possible orientations with respect to any axis, $m_s = +1/2$ or $m_s = -1/2$ where m_s is the electron-spin magnetic quantum number.
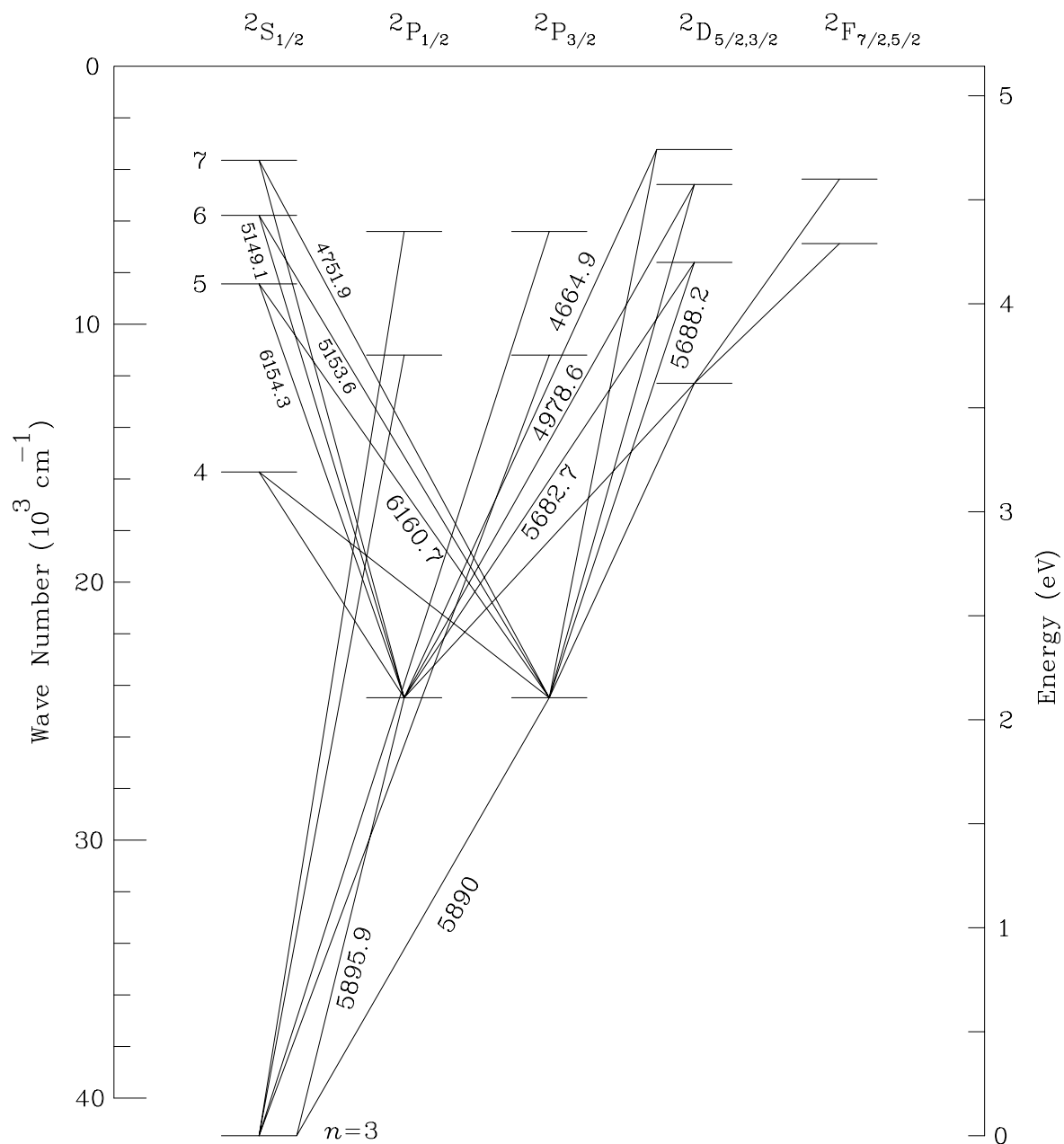

Figure 3: Principal lines of the sodium spectrum.

The spin s can then be coupled to l according to the quantum-mechanical rules of addition for angular moments; this will result in a total momentum of magnitude $J = l + 1/2$ or $J = l - 1/2$ and the energy of the state will depend on J. In the case of sodium, the ^{3}P level splits into two levels, with $J = 1/2$ and $J = 3/2$ designated as $^3P_{1/2}$ and $^3P_{3/2}$ and separated by 17.3 cm^{-1}.