Experiment 317: Millikan's Oil-Drop Experiment

Aim

To determine the charge on the electron using Millikan's oil-drop method.

References

1. R. A. Millikan, "The isolation of an ion, a precision measurement of its charge, and the correction of Stokes's law", Physical Review (Series I) **32**, 349–397 (1911) [DOI: 10.1103/PhysRevSeriesI.32.349].
2. R. A. Millikan, "On the elementary electrical charge and the Avogadro constant", Physical Review **2**, 109–143 (1913) [DOI: 10.1103/PhysRev.2.109].
3. R. A. Millikan, "Electrons, Protons, Photons, Neutrons, Mesons and Cosmic Rays", Uni. of Chicago Press (1957).
4. W. M. Rohsenow, J. P. Hartnett, and Y. I. Cho, "Handbook of heat transfer", 3rd ed., McGraw-Hill (1998).
5. T. H. Brown, "Taylor Manual of Advanced Undergraduate Experiments in Physics", Addison-Wesley.
6. Pasco Scientific, "Instruction manual and experiment guide for the PASCO scientific Millikan oil-drop apparatus".

Introduction

The natural unit of charge is not the coulomb but the charge of the electron; this constant, referred to as e, is of fundamental importance in physics. The quantity may be determined by a method employed by the well-known American physicist Robert A. Millikan (1868–1953). His experiment was started in 1909 and published in 1913 [R. A. Millikan, "On the elementary electrical charge and the Avogadro constant", Physical Review **2**, 109–143 (1913)]. Millikan went on to win the 1923 Nobel Prize for Physics, in part for this work. The method, known as the "oil-drop experiment", is the study of the vertical motion of a tiny charged oil drop in an electric field. The beauty of the oil-drop experiment is that as well as allowing quite accurate determination of the fundamental unit of charge, Millikan's apparatus also provides a "hands on" demonstration that charge is actually quantized.

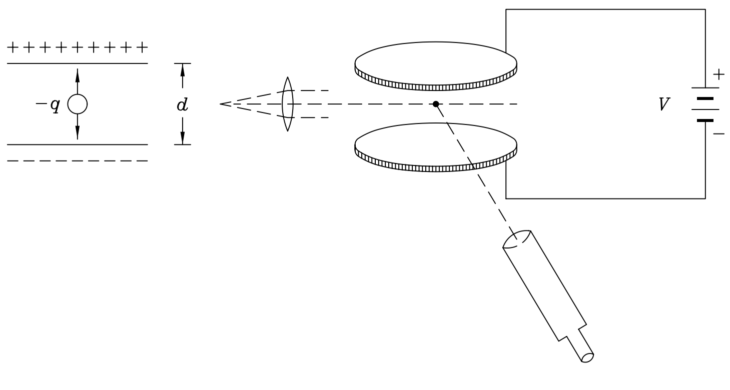

Figure 1: Experimental setup

Apparatus

The electric charge carried by a particle may be calculated by measuring the force experienced by the particle in an electric field of known strength. However, the force exerted by, e.g., a field of 1000 V/cm on a particle bearing one excess electron is only 1.6×10^{-14} N and is comparable to the gravitational force on a particle with a mass of 10^{-12} gram. The success of the Millikan Oil Drop experiment depends on the ability to measure forces this small.

Essentially the apparatus consists of a parallel plate capacitor whose plates are separated by a layer of air of known thickness (see Fig. 1). Oil is sprayed by an atomiser and a few droplets enter the capacitor chamber through a small hole. Measuring the terminal velocity of the drop in free-fall in air and rising in an electric field permits a calculation of the force on, and hence, the charge carried by the oil drop. A microscope is used to observe the drops moving vertically at the centre of the capacitor.

Theory

The motion of a selected drop is observed through the microscope. With no electric field between the plates of the capacitor, the drop will reach a constant terminal velocity u_{g} as it falls through the resisting air.

When an electric field E is applied by connecting a voltage source across the capacitor plates, the drop will travel with a different terminal velocity u_{E}. By using the plate voltage reversing switch, the field between the plates may either oppose or assist the gravitational field and the drop may either rise or fall.

If the terminal velocity of the drop through the air does not exceed about 10^{-3} m s^{-1}, we may use Stokes' law that gives the drag or frictional force exerted on a spherical object in a continuous viscous fluid. By equating opposing forces, we then get:

- When no electric field is applied:

  $$\text{Viscous retarding force} = \text{Gravitational force} - \text{Buoyancy force}$$
  $$6\pi r\eta u_{\mathrm{g}} = \frac{4}{3}\pi r^3 (\sigma - \rho)\, g \tag{1}$$

  where r is the radius of the drop, η is the coefficient of dynamic viscosity of air, u_{g} is the terminal velocity of the drop due to gravity only, σ is the density of the drop and ρ is the density of air.

- When an electric field E is applied giving rise to an upward terminal velocity u_{E}, we get:

  $$6\pi r\eta u_{\mathrm{E}} = qE - \frac{4}{3}\pi r^3 (\sigma - \rho)\, g \tag{2}$$

  where q is the charge on the drop.

From (1) and (2) we get:

$$q = \frac{6\pi\eta\,(u_E + u_g)}{E} r \tag{3}$$

The radius of the drop can be derived from (1):

$$r = \left[\frac{9\eta u_g}{2\,(\sigma - \rho)\, g}\right]^{1/2} \tag{4}$$

Substituting (4) in (3) and replacing E by V/d where V is the applied voltage and d is the distance between the capacitor plates, we get:

$$q = 9\sqrt{2}\,\pi \frac{d}{V} \left[\frac{\eta^3}{(\sigma - \rho)\, g}\right]^{1/2} (u_E + u_g)\, u_g^{1/2} \tag{5}$$

Correction to Stokes' law

The above equations assume that the medium is homogeneous. When the size of the drop is comparable to the mean free path of the air molecules, the medium is no longer homogeneous and a correction has to be applied to the terminal velocity in Stokes' law. This occurs for droplets with small velocities of less than 0.1 cm/s. The correction is in the form of a factor containing the ratio of mean free path to the radius of the drop. Now since the mean free path is inversely proportional to the pressure of the medium, the factor applied is $1/\left[1 + k/(rP)\right]$ where P is the pressure of the medium and k is a constant. If each terminal velocity u_{g} and u_{E} in (5) is multiplied by $1/\left[1 + k/(rP)\right]$, the expression for q becomes:

$$q = 9\sqrt{2}\,\pi \frac{d}{V} \left[\frac{\eta^3}{(\sigma - \rho)\,g}\right]^{1/2} (u_{\mathrm{E}} + u_{\mathrm{g}})\, u_{\mathrm{g}}^{1/2} \left[1 + \frac{k}{rP}\right]^{-3/2} \tag{6}$$

where P is atmospheric pressure in pascals $\left(\mathrm{N\,m^{-2}}\right)$ and $k = 8.226 \times 10^{-3}\ \mathrm{N\,m^{-1}}$. This correction to Stokes' law has actually been obtained through careful experimentation by Millikan himself, who noticed that the value he obtained for e varied with the mass of the drop [R. A. Millikan, "The isolation of an ion, a precision measurement of its charge, and the correction of Stokes's law", Physical Review (Series I) **32**, 349–397 (1911)].

Experiment

The experimental equipment is depicted in Fig. 2. Before you start, make sure the apparatus is level using the bubble level. The Millikan oil-drop chamber is illuminated by a halogen lamp through a window in its side. The interior of the chamber is viewed through a second window almost diametrically opposite to the first using a microscope. To adjust the microscope focus to the centre of the droplet viewing chamber, unscrew the focusing wire from its storage place on the right of the viewing chamber and carefully insert it into the hole in the centre of the top capacitor place (Lift off the clear perspex cover lid to do this). View the focusing wire through the microscope and bring the wire into sharp focus by turning the droplet focusing ring. The eyepeice of the microscope can also be adjusted to bring the reticle into sharp focus. The line separation is 0.5 mm between the major dark divisions. Return the focusing wire to its storage location on the platform.

Using the digital voltmeter, monitor the voltage applied to the plate voltage connectors as you turn on the high voltage DC power supply. Adjust it to deliver about 500 V. Beware that the adjustment knob is very coarse. Make sure you DO NOT EXCEED 500 V.

The temperature of the droplet viewing chamber can be obtained by measuring the resistance between the thermistor connectors using the digital multimeter and by referring to the Thermistor Resistance Table located on the platform. That temperature is that of the lower brass plate. For the best accuracy, you should check it every 15 minutes.

(1) Move the ionization source lever to the "Spray Droplet" position to allow air to escape from the chamber during the introduction of oil droplets.

(2) Place the nozzle of the atomiser provided into the hole on the lid of the droplet viewing chamber with its tip pointing down. While observing through the microscope, squeeze the atomiser bulb with one quick squeeze. Then squeeze it slowly to force the droplets through the hole and into the space between the two capacitor plates. Atomise sparingly or you will coat your optics with oil and see nothing.

(3) When you see a shower of drops through the microscope, move the ionization source lever to the OFF position. If you cannot see the drops, the most common cause is scattered light caused by oil on optical surfaces, or that the hole in the top plate or in the droplet hole cover may be clogged. If there is scattered light, the view through the microscope has a background which is not truly black. So, if you cannot see drops, try to determine if the background light is significant. You may have to clean the droplet viewing chamber. **Before you disassemble it, make sure to turn off the DC power supply**.

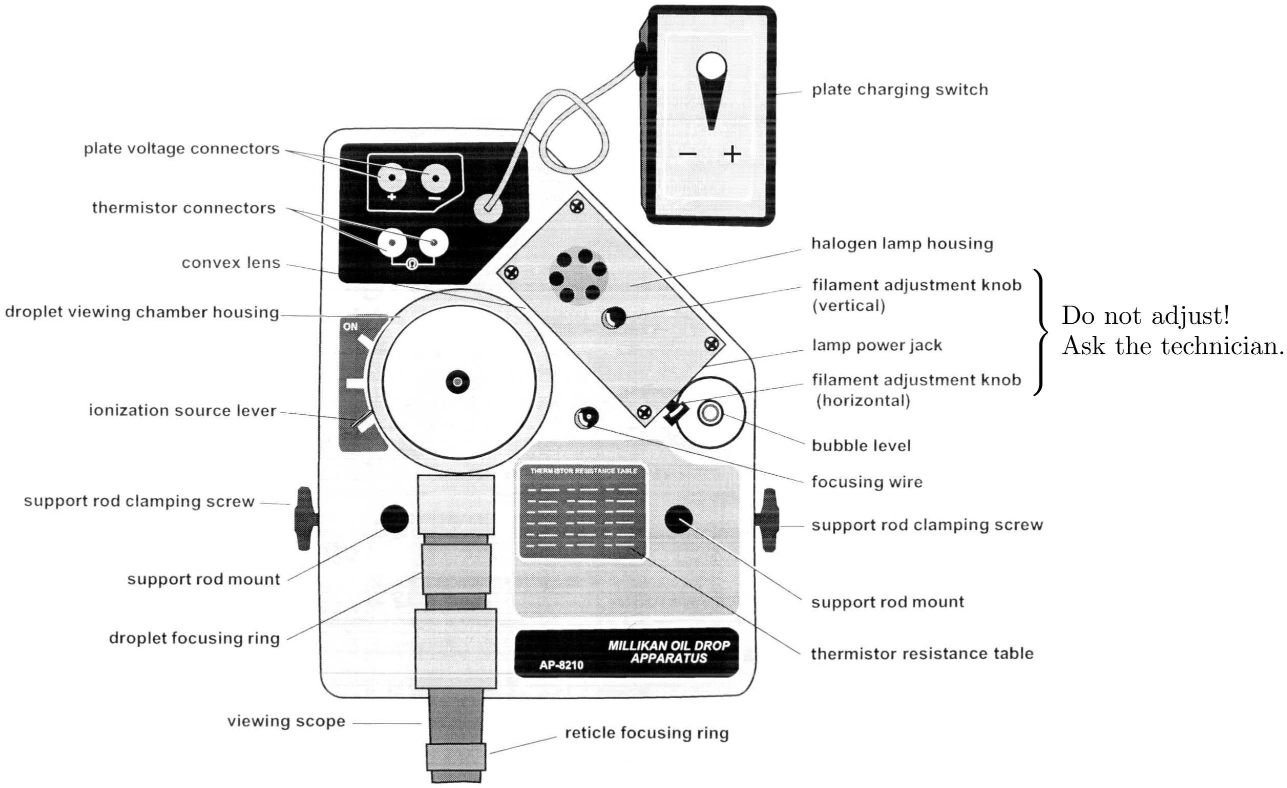

Figure 2: Experimental equipment and controls

(4) The drops will be randomly charged and may be distinguished by applying the field E. Repeatedly reverse the field several times using the plate charging switch so that the very large or uncharged drops fall to the bottom plate. Select a single drop of suitable size (approximately $1\,\mu$m diameter) for observation. The drop should have a small charge for its velocity to be small under the influence of the field. As the velocity of the same drop has to be measured several times in the absence and in the presence of the electric field, it would be a good idea to rehearse procedures (4) to (7) before commencing measurements. Note that the microscope does NOT invert the field of view.

(5) When you find an appropriately sized and charged oil droplet, fine tune the focus of the microscope.

(6) Switch off the electric field so that the selected drop is falling under the influence of gravity only. Time its motion over a set number of graticule divisions in the microscope by pressing the start/stop button. Store the measured time interval by pressing the store button. This also resets the timer. Reverse the direction of motion of the drop by applying the electric field. Time its motion over the same number of graticule divisions. Press the store button to store the measured time interval.

(7) Repeat procedure (6) two more times to obtain a total of three pairs of time intervals (you can store up to 10 measures in the digital timer). Remember to press the store button after each timing. Press the scroll button and read off the three pairs of stored time intervals. Calculate an average value from the three time intervals for motion under the influence of gravity only and an average value from the three time intervals for motion in the presence of the applied electric field E. Record the applied voltage V.

Note: The drop may acquire additional charge during an observation. This will be reflected in a significant change in time of travel. When this happens in the middle of procedures (6)–(7) discard the whole set of time measurements. It is actually desirable to observe as many different charges on a single drop as possible. You can attempt to change the charge of a droplet on purpose by setting the ionization level to ON for a few seconds as the droplet falls from the top of the field of view. The ionization source is a Thorium-232 naturally-occurring, low level, alpha-particle emitter.

In one case, Millikan was able to observe the same drop for about 4 hours, during which the drop carried all possible multiples of the elementary charges from 4 to 17, save only 15. For your experiment, try to use drops with maximum 5 units of charge.

(8) Knowing that the major dark graticule divisions seen in the microscope are separated by 0.5 mm, determine the distance over which the timing measurements have been made. Calculate the average velocities u_{g} and u_{E} for the drop.

(9) From the barometer and thermometer located next to the allocation benches, record the barometric pressure P and the temperature T of the droplet viewing chamber. The barometric pressure is required in the correction factor for the terminal velocity in Stokes' law. The temperature enables the value of the dry air viscosity to be extrapolated from that given for 23 °C using (derived from W. M. Rohsenow, J. P. Hartnett, and Y. I. Cho, "Handbook of heat transfer", 3rd ed., McGraw-Hill: 1998):

$$\eta = 18.35 + 0.0476\,(T - 23)\ \mu\mathrm{N\,s\,m^{-2}} \qquad [T \text{ in } {}^{\circ}\mathrm{C}]$$

This value can be corrected to take into account the relative humidity of the air (RH, in %). An empirical correction valid between 10 and 30 °C is to subtract $(0.015T^2 - 0.25T + 4)\mathrm{RH}/1850$ from the above value [extrapolated from Fig. 2 of P. T. Tsilingiris, "Thermophysical and transport properties of humid air at temperature range between 0 and 100 °C", Energy Conversion and Management **49**, 1098–1110 (2008)].

(10) Measure the density σ of the oil using the density bottle provided and the precision scales in the lab. Measure the separation d of the capacitor plates by measuring the thickness of the plastic spacer. **Do not forget to turn OFF the DC power supply as you disassemble the droplet viewing chamber**. Make sure you do not include the raised rim of the spacer in your measurement. The accuracy of this measurement is critical to the degree of accuracy of your experimental results.

(11) Repeat the velocity measurements for a total of at least ten drops. Record your observations in the form shown in the table:

Drop No.	Free fall times				Field applied times			
j	t_{j1}	t_{j2}	t_{j3}	av.= t_{g}	t_{j1}	t_{j2}	t_{j3}	av.= t_{E}
1								
2								
⋮								

(12) For each drop calculate the velocities u_{g}, u_{E}, radius r and the correction factor $[1 + k/(rP)]$. Determine q using equation (6). Assume that the fundamental (smallest) unit of charge q_0 is $\approx 1.6 \times 10^{-19}$ C [see procedure (14) for verification] and divide q by q_0. This should yield a value very close to an integer number. Take the integer number as n. Calculate the electronic charge e by dividing q by n. Present your results in the tabular form shown.

Drop j	u_{g} $\times 10^{-6}\,\mathrm{ms^{-1}}$	u_{E} $\times 10^{-6}\,\mathrm{ms^{-1}}$	r $\times 10^{-7}\,\mathrm{m}$	$[1 + k/(rP)]$	q $\times 10^{-19}\,\mathrm{C}$	q/q_0	n	$e = q/n$ $\times 10^{-19}\,\mathrm{C}$
1								
2								
⋮								

(13) Obtain an average value for the electronic charge e and determine its error.

Verification of the existence of a fundamental unit of charge

(14) Plot points representing the values of q of the various drops and whole fractions of q (e.g. $q/2$, $q/3$, $q/4$ and $q/5$ etc.) in a diagram as shown in Fig.3. The fundamental unit will show up as a common factor (dashed line in Fig. 3).

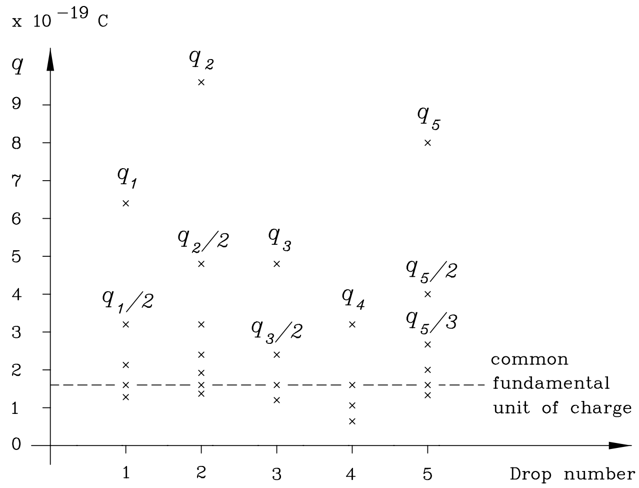

Figure 3: Determination of e from charges of several oil drops

Questions

1. Calculate the mass of one the droplets you observed.
2. Why is the drop viewed almost opposite the illuminating beam?
3. Why is there such a large space between the top of the housing chamber and the capacitor?
4. Why do very small drops give poor results?
5. Why must the oil used possess a very low vapour pressure?

List of Equipment

1. Pasco Millikan oil-drop chamber platform with integrated light source and 12 V DC supply
2. Pasco kiloVolt Power Supply
3. High Voltage leads (with banana-to-banana connectors)
4. Plate voltage reversing switch
5. Digital multimeter and leads
6. Atomiser
7. Digital Timer
8. Density measuring equipment

R. Garrett

A. Chisholm

Z.C. Tan

Revised by S. Coen, March 2009. This version, July 10, 2014.