Experiment 264: A-C Bridge Measurements

Aim

To make accurate measurements of capacitance, inductance, mutual inductance and frequency using simple *ac* bridges.

Reference

1. Bold and Earnshaw, "Linear Steady-state Network Theory", Appendix C.

Theory

The basic *ac* bridge consists of four impedances arranged as for a Wheatstone bridge, with an *ac* source in one diagonal and an *ac* detector in the other (see Figure 1).

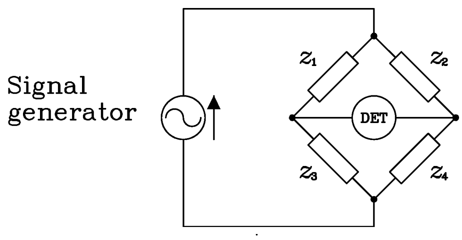

Figure 1: Basic *ac* bridge circuit.

The bridge is *balanced* when the instantaneous voltage across the detector is always zero, and this occurs when:

$$\frac{z_1}{z_3} = \frac{z_2}{z_4}$$

where z_1, z_2, z_3 and z_4 and are the impedance operators of the four arms of the bridge.

In general the impedance operators are complex so that *two separate* conditions must be satisfied. That is, the real and imaginary coefficients of both sides must be equal.

Thus, unlike *dc* bridges where only one adjustment is necessary, *two* adjustments must be made on most *ac* bridges to obtain balance. These are rarely completely independent so they must be made alternately to diminish the indication on the detector until a final balance is reached.

Detectors and Sources

Where the detector has a finite impedance and its indication depends upon the power delivered to it, the optimum detector impedance is equal to the impedance seen looking back into the output terminals of the bridge.

Here, a pair of headphones are used to detect the null. The headphones should be plugged into the black plastic box which contains an audio amplifier. The inputs to the amplifier should be connected to the bridges as the detector shown in the circuit diagrams. The input impedance of the amplifier is high compared to the impedances used in the bridges. The ear is most sensitive to changes in sound level when these are close to the threshold of hearing. Thus, when searching for a null, adjust the amplitude of the source so that the sound just disappears at the position of the null.

The *ac* source can be any type of oscillator delivering the required power over a suitable frequency range. If the waveform is not purely sinusoidal, harmonics will be present. But, as we will see, the balance condition can be usually satisfied at only one frequency, and hence when the *fundamental* waveform is balanced, the harmonics will not be. Thus the total voltage across the detector will never be completely zero.

Some bridges have balance conditions which *are* nominally frequency independent.[1] But even here, the effective values of the components themselves are always slightly frequency dependent, so the same applies. By listening carefully to the pitch of the signal slightly away from the null point, you should be able to notice if the fundamental tone vanishes at the null (the pitch of the small residual signal at the null will usually be higher than that heard while the bridge is unbalanced).

Marginal Balancing and Error Estimation

When a balance is ill-defined, or *broad*, e.g. settings of, say, 384.1 and 384.9 produce no noticeable change at the detector, use the following procedure:

With an initial setting of 384.4 (say), alternate the settings between 383.4 and 385.4. If the true setting *should* be 384.4, then *equal* amounts of unbalance will occur at the upper and lower settings, giving equal indications on the detector. Try different upper and lower settings until the indications *are* equal.

This procedure also allows a crude error estimation. Find the "equal indication" settings of both controls at which the difference from minimum indication is *just detectable.* If this happens in the above case for readings of 383.6 and 385.2, then the value can be quoted as 384.4 ± 0.8.

Sometimes the balance will be *sharp*, and changing a balance component's value by the smallest step available will give a definite unbalance. In this case, quote this smallest step-size as the error.

This estimation is crude (and statistically incorrect) because the two balance settings *interact.* It's better than nothing, so use it here when requested.

Primary and Secondary Balance Conditions

The *primary* balance determines the unknown quantity the bridge is designed to measure, whilst the *secondary* balance is usually related to the component loss resistance. Normally the primary balance condition is determined more accurately than the secondary one. For example, in Maxwell's L/C bridge (see Figure 3) the balance conditions are:

$$\begin{aligned} L_1 &= R_2 R_3 C_4 && \text{primary balance} \\ r_1 &= \frac{R_2 R_3}{R_4} && \text{secondary balance} \end{aligned}$$

The true value of R_4 is not known accurately since the loss resistance of the standard capacitor appears in parallel with it. However R_4 does not appear in the primary balance equation. This is a good design feature.

Capacitance Measurements: Wien's Capacitance Bridge

A simple *ac* bridge for measuring capacitance is Wien's Capacitance Bridge, shown in Figure 2.

[1] The Robinson-Wien Frequency Bridge is an example of a bridge whose balance condition *is* frequency dependent.

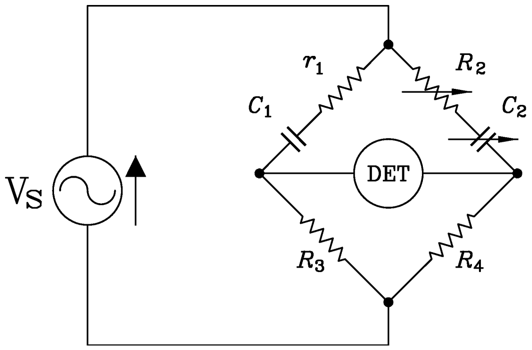

Figure 2: Wien's capacitance bridge

The balance conditions are:

$$C_1 = \frac{R_4}{R_3} C_2 \qquad \text{primary balance}$$

$$r_1 = \frac{R_3}{R_4} R_2 \qquad \text{secondary balance}$$

The capacitor under test is regarded as an ideal capacitance in series with a resistance. This is compared with a variable standard capacitor, which is assumed to have negligible loss resistance, connected in series with a variable resistance.

The balance conditions are satisfied semi-independently by varying C_2 and R_2 alternately.

The capacitance and the equivalent loss resistance of the capacitor provided is to be determined at 1 kHz, in terms of a variable decade capacitance standard box. The value of r_1 has been artificially increased by adding a series resistor, since the loss resistance of a modern high quality capacitor is too small to measure with accuracy.

The capacitance is approximately $0.22\,\mu$F, so that at 1 kHz its reactance is approximately 725 Ω. The optimum arrangement exists when 10 kΩ decade resistance boxes are used for R_3 and R_4 and the 1 kΩ decade resistance box for R_2.

(1) Set R_3 and R_4 to 725 Ω to minimise the effects of stray capacitance.

(2) Adjust C_2 and R_2 for balance using the marginal balancing technique.

(3) Find C_1 and r_1 and estimate the probable errors of measurements from the sensitivity of the balance adjustments.

(4) Tabulate your results.

Inductance Measurements: Maxwell's L/C Bridge

Maxwell's L/C bridge allows an *inductance* to be determined in terms of a standard *capacitance*. This is better than comparing with a standard inductance, since standard capacitors approximate ideal components much more closely than the very best inductors. The circuit is shown in Figure 3.

The balance conditions are:

$$L_1 = R_2 R_3 C_4 \qquad \text{primary balance}$$

$$r_1 = \frac{R_2 R_3}{R_4} \qquad \text{secondary balance}$$

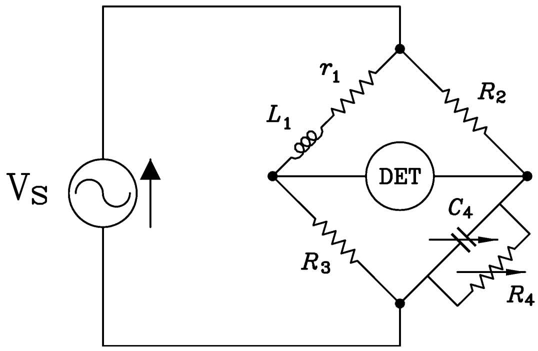

Figure 3: Maxwell's L/C Bridge

The inductor under test is regarded as an ideal inductor in series with a resistance. It is compared with a variable standard capacitor which is in parallel with a variable resistance. The balance conditions may be satisfied semi-independently by varying C_4 and R_4 alternately.

We determine the primary and secondary inductances, L_p and L_s, and series resistances, r_p and r_s of an audio-frequency transformer.

The transformer supplied has the following approximate parameter values:

$L_p \approx 15$ mH $\quad$ $L_s \approx 50$ mH $\quad$ $M \approx 20$ mH $\quad$ (not required at this point)

The reactance of the primary inductance at 1 kHz is approximately 100 Ω. Ideally R_2 and R_3 should also be set at 100 Ω, but this value is too low to obtain a balance when the maximum value of the variable capacitance is 1.111 μF, since, from the primary balance condition, the corresponding maximum value of L_1 is only 11.11 mH!

(5) Set R_2 and R_3 to 200 Ω using two of the 10 kΩ resistance boxes. Use the other 10 kΩ resistance box for R_4.

(6) Find L_p and r_p and estimate the probable errors of measurement.

(7) Find L_s and r_s with R_2 and R_3 set to 300 Ω and estimate the probable errors of measurement.

(8) Tabulate your results.

(9) Use the automatic RCL meter (left-hand rear side wall of lab) to check your measurements. If you haven't used this before, ask a demonstrator. This meter gives the parameters of both series and parallel circuit models of inductors and capacitors. Here, the series model is relevant. Tabulate your measurements against those obtained using the bridges. Major discrepancies should be investigated.

Mutual Inductance Measurements

Mutual inductance is a property of two inductors in proximity. It determines the voltage induced in one of the inductors by a changing current in the other.[2] The *coupling coefficient* is a property of two separate inductors and their mutual inductance. It is conventionally used to characterise the performance of closely-coupled transformers.

The RCL meter may be used to determine both parameters of the transformer indirectly, by making use of the equivalent circuits of Figures 4 and 5.

Figure 4 shows that when terminals 3 and 4 are connected together, the series inductance, L_{12} between terminals 1 and 2 is equal to

[2]See *Linear Steady-state Network Theory*, Chapter 7 (240 course text). Also *Electromagnetism*, Gary E.J. Bold, Chapter 7 (220 course text)

$$L_{12} = L_p + L_s - 2M$$

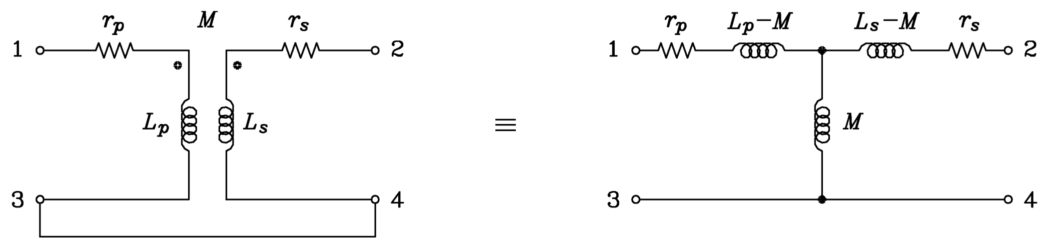

Figure 4: Equivalent *tee* circuit of a mutual inductance when *like* windings are connected

Figure 5 shows that when terminals 2 and 3 are connected together the series inductance, L_{14}, between terminals 1 and 4 is equal to

$$L_{14} = L_p + L_s + 2M$$

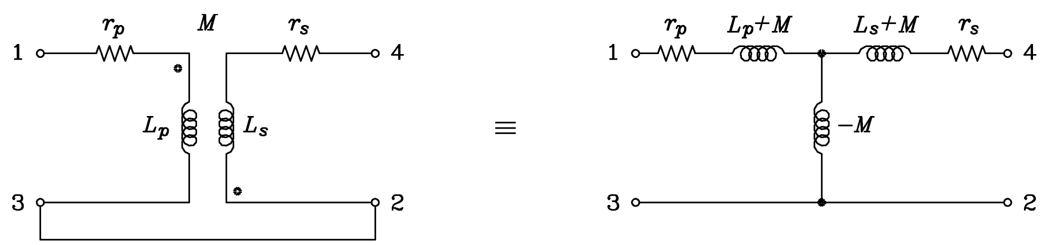

Figure 5: Equivalent *tee* circuit of a mutual inductance when *unlike* windings are connected

We then find

$$M = \frac{L_{14} - L_{12}}{4} \qquad k = \frac{M}{\sqrt{L_p L_s}} \qquad \text{(this is a definition)}$$

(10) Determine both M and k using the principle set out above.

Frequency Measurements: The Robinson-Wien Frequency Bridge

The bridges used so far determine unknown impedances and the balance conditions have been nominally frequency-independent. In Wien's capacitance bridge, as modified by Robinson, the balance conditions are frequency dependent. This enables the source frequency to be measured in terms of known capacitances and resistances. Figure 6 shows the Robinson-Wien bridge.

The balance conditions are:

$$\omega^2 = \frac{1}{R_3 R_4 C_3 C_4} \qquad \text{primary balance}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} + \frac{C_4}{C_3} \qquad \text{secondary balance}$$

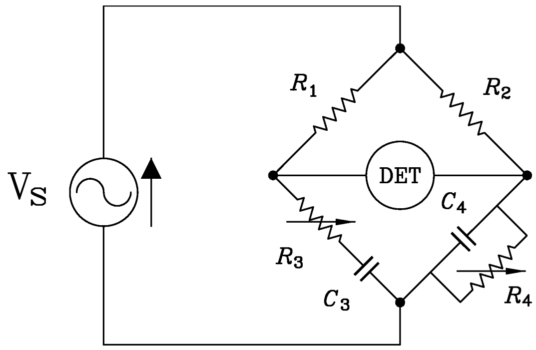

Figure 6: The Robinson-Wien frequency bridge.

If we choose $R_3 = R_4 = R$ and $C_3 = C_4 = C$, then the balance conditions simplify to:

$$\omega = \frac{1}{RC} \qquad \text{and} \qquad R_1 = 2R_2$$

The second condition is chosen to be always satisfied. We now invariably measure frequency using a *frequency counter*, and this bridge is rarely used as a measuring device.

However, the configuration is often used as a frequency selective feedback network in RC oscillators, and so is worthwhile being familar with it.

The actual frequency of the oscillator is to be determined using a Robinson-Wien bridge.

(11) Set the oscillator frequency to 1 kHz. We shall see how accurate this is.

(12) Use the nominal 0.22 μF capacitor for the capacitance C_3. Note that the loss resistance of this component (which should be negligible anyway) constitutes part of R_3.

(13) Set C_4 equal to the value of C_3 as measured previously.

(14) Set R_1 to 1 kΩ and R_2 to 500 Ω.

(15) Adjust R_3 and R_4 to obtain a balance.

(16) Calculate the frequency of the oscillator and estimate the probable error.

List of Equipment

1. 1 x Low Distortion Audio Oscillator
2. 1 x Stereo headphones and headphone amplifier with power supply
3. 1 x Muirhead Decade Capacitance Box (type B-21-F)
4. 3 x Muirhead Decade Resistance Box (type D-825-L)
5. 1 x Muirhead Decade Resistance Box (type D-825-K)
6. 1 x 0.22 μF Capacitor (mounted)

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