Experiment 235: Microwave Optics A

Aims

To show that electro-magnetic radiation in the form of microwaves exhibits the same phenomena as light and to compare the observations made with predictions from optical wave theory.

References

1. Hecht 2nd. ed., "Optics", Addison-Wesley
2. Pedrotti & Pedrotti, "Introduction to Optics", Prentice-Hall

Experimental Set-up

Transmitter

The transmitter uses a Gunn diode oscillator to produce 15 mW of coherent, linearly polarised microwave radiation. The output is linearly polarised along the axis of the attached horn. The wavelength of the microwaves is about 3 cm. The transmitter is powered from a wall socket.

The output power of the microwave transmitter is well within safety levels. However, one should never look directly into the beam at close range

Receiver

Four ranges are available on the receiver unit, including a variable sensitivity dial. This can be used to offset an initial reading to a convenient value. However, in order to make comparative quantitative measurements, this offset must not be altered between readings. The receiver is powered from internal batteries. To conserve battery power, **do not leave the receiver unit on for long periods of time.**

Note the effective points of transmission and reception in the transmitter and receiver respectively, as shown in Figure 1.

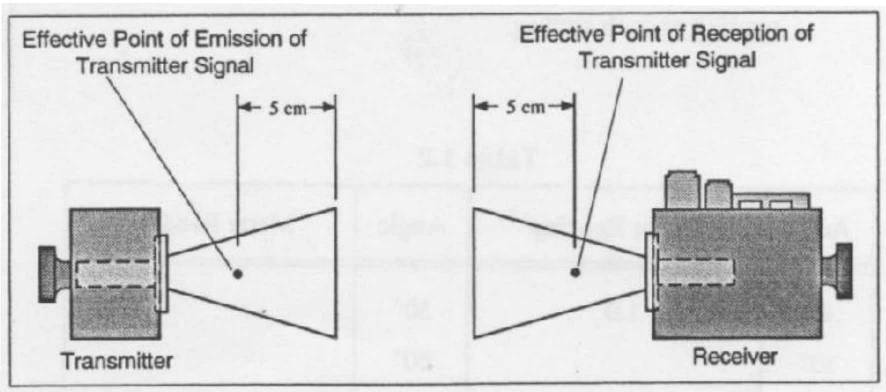

Figure 1: Effective transmission and reception points.

It will become apparent that microwaves are easily reflected off a variety of large scale surfaces. For this reason, it is important to note and minimise the effect of reflection of planar surfaces, including walls, the bench-top, the body and any nearby metallic objects.

Basic Operation

Before proceeding to more sophisticated experiments, you should become familiar with the basic transmitter and receiver system.

1. Place the transmitter and receiver on the *goniometer* (the hinged ruler) with the transmitter on the fixed arm, see Figure 2. Set the distance between the transmitter and receiver at 40cm. Turn on the transmitter and receiver, and set the receiver controls to show a near full scale deflection. Record the meter reading as the transmitter-receiver distance is increased. Make use of the various meter scales, and remember not to adjust the variable sensitivity dial between readings. Plot a graph of the meter reading against distance.

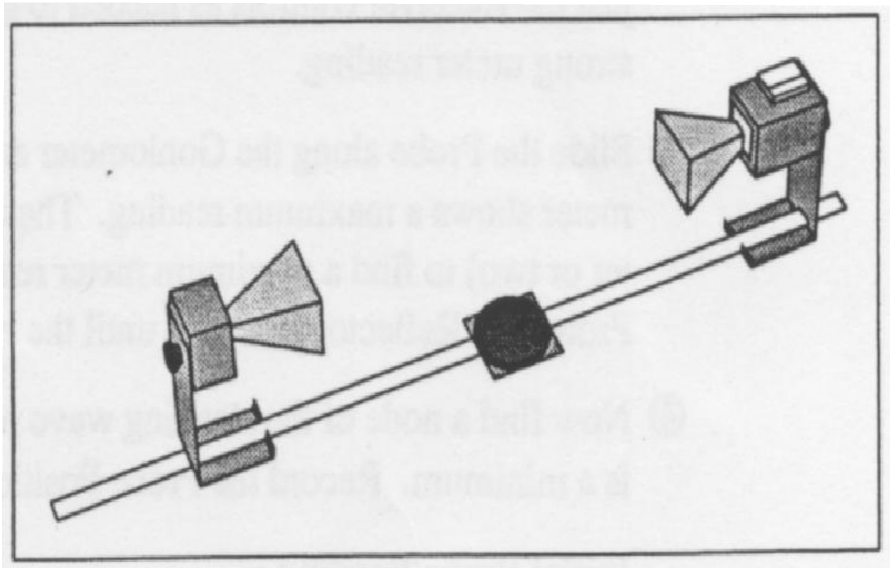

Figure 2: Transmitter and receiver units setup on the goniometer.

   Question 1: Using your data, can you determine if the meter reading is proportional to the *intensity* or the *electric field* of the microwaves?

2. Set the transmitter-receiver distance to about 80cm. Slowly decrease the distance between the transmitter and receiver.

   Question 2: Does the meter reading increase smoothly? Describe and explain what you observe.

3. Set the transmitter and receiver at about 80cm apart. Hold a metal reflector parallel to the beam axis and perpendicular to the table-top. Slowly move the reflector towards the beam axis, keeping the reflector plane parallel to the beam axis. Describe and explain the meter reading.

4. Place the transmitter such that the end of the output horn is directly over the degree plate. Set the transmitter-receiver distance to about 50cm. Offset the receiver from the beam axis by various angles. Record the meter reading at each angle, maintaining the same receiver-transmitter distance. Plot your results. From your plot, find the angle where the meter reading drops to $1/e$ of the on-axis maximum.

5. Using a metal reflector, a rotating component holder, and the goniometer, verify that the law of reflection holds for microwaves, see Figure 3. Compare the relative reflectivities of the metal and wood reflectors. Is all the incident energy reflected? Is any energy absorbed?

Double-Slit Interference

Recall from Young's double slit experiment that for light incident on two thin slits separated by a distance d, an interference pattern will be produced with maxima such that $d \sin\theta = n\lambda$. Here θ is the angle of detection, λ is the wavelength of the radiation and n is an integer.

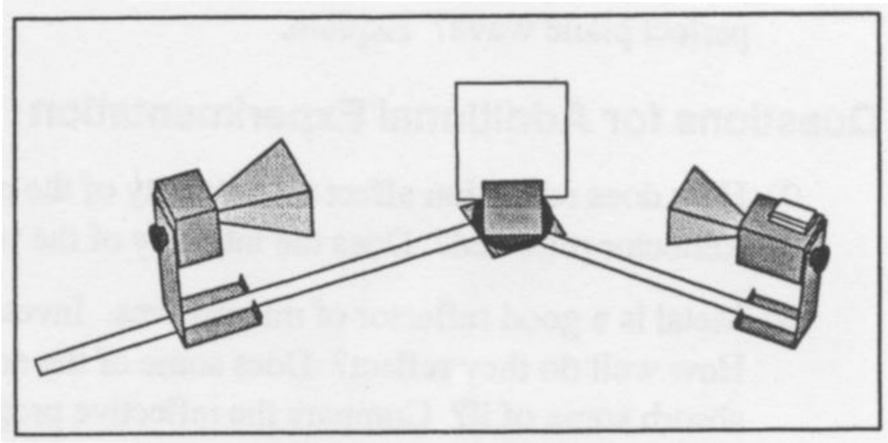

Figure 3: Setup to verify the principle of reflection.

6. Create a double slit using a component holder, the two wide metal reflectors and the narrow reflector. Make the slit width about 1.5 cm. The distance from the centre of one slit to the centre of the other should be about 7.6 cm. Ensure that the slits are as symmetric as possible. Place the component holder on the degree plate of the goniometer and set the transmitter and receiver units directly facing each other, see Figure 4.

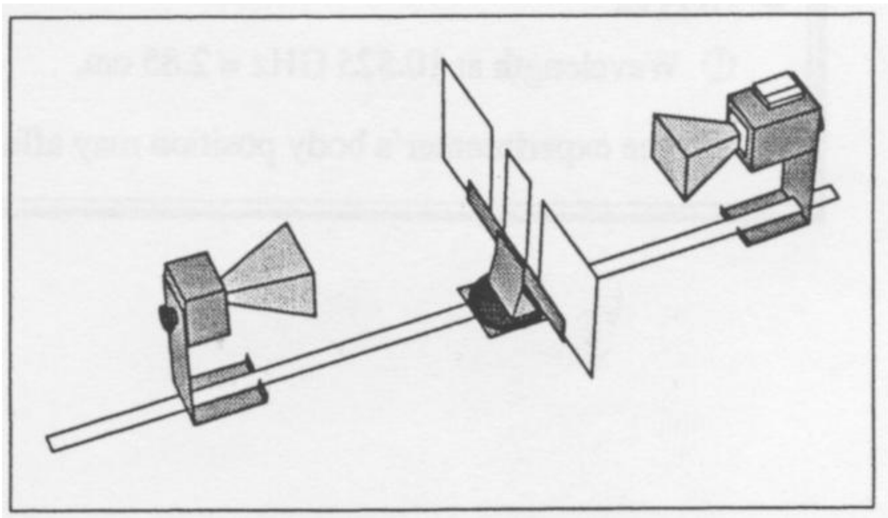

Figure 4: Set-up for double slit interference.

7. Set the transmitter and receiver units for vertical polarisation. To do this, make sure that both units are rotated so that they read $0°$ on their angular scales[1]. Set the receiver controls for a full scale deflection at an angle of detection $\theta = 0$, *ie* the receiver is pointed directly at the slits. Carefully rotate the receiver unit on the goniometer arm so that the angle of detection θ varies. Ensure that the slit-receiver distance remains constant. Record the meter readings for a range of values of θ. Plot the results and find the positions of all maxima and minima. Find the wavelength, λ, of the microwave radiation.

8. Repeat Procedure 7 using the wide spacer reflector to make the two slits. Keep the slit width at 1.5 cm. The distance between slit centres should now be 10.6 cm. Move the transmitter unit so that the on axis reading is the same as that measured in Procedure 7. Plot the results on the same graph.

   Question 3: What assumptions are made in the derivation of the diffraction formula? To what extent are they met in this experiment?

[1] You can check the polarisation by using the wire grid polariser. **E** parallel to the wire is **not** transmitted.

Fabry-Perot Interferometer

A Fabry-Perot interferometer consists of two partial reflectors facing one another, see Figure 5. A maximum signal at the receiver indicates that all the waves passing through the reflectors are in phase. This occurs when the spacing of the two reflectors is such that a wave bouncing back and forth between the reflectors accumulates a phase change of $2n\pi$ over one round trip, where n is an integer.

Question 4: What is the relationship between the inter-reflector distance, d, and the wavelength, λ for a Fabry-Perot interferometer in order to yield an output maximum?

Question 5: What must d be to yield an output minimum?

9. Set two partial (i.e. wooden) reflectors between the transmitter and receiver units on the goniometer. Set the transmitter-receiver distance to about 80 cm, see Figure 5. Adjust the receiver controls to obtain a near full scale reading. Become familiar with the effects of changing the distance between the two reflectors. Observe the relative maxima and minima. Set the distance between the reflectors to give a maximum reading. Record this initial inter-reflector distance, d_{i}.

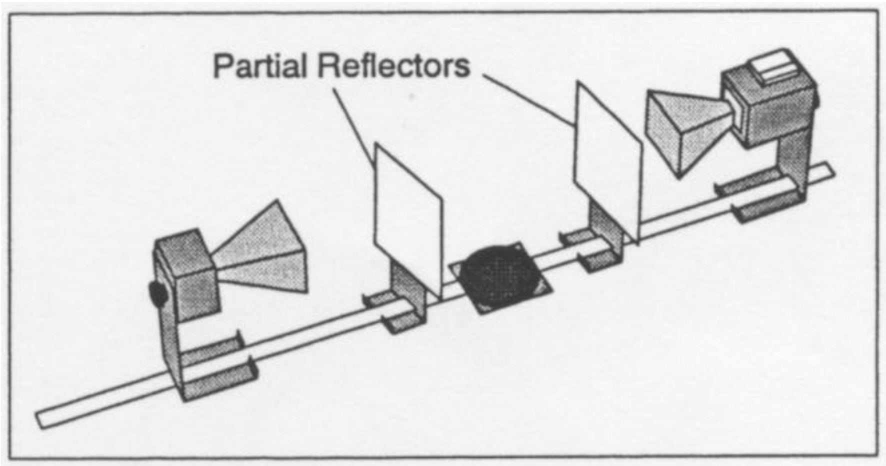

Figure 5: Fabry-Perot interferometer.

10. Slowly move one of the reflectors away from the other while closely watching the receiver meter. Move the reflector until at least 10 minima have been observed and returned to a maximum. Note the number of minima traversed and the final inter-reflector distance, d_{f}. Repeat with a different d_{i} *at least* 4 times, for a total of 5 data sets. Use your data to compute the wavelength of the microwave radiation. Ensure that you use a proper error analysis.

Michelson Interferometer

The Michelson interferometer, like the Fabry-Perot interferometer, splits an incoming wave, and then recombines the two components to produce an interference pattern.

11. Set up the fixed arm and goniometer assembly as in Figure 6. Place a partial reflector (C) on a component holder on the degree plate. Place two metal reflectors (A and B) on the arms facing the transmitter and receiver units. Check the orientation of the partial reflector with respect to the transmitter and receiver units. Set the receiver controls for a near full-scale reading. Ensure that you can move reflector A back by about 25cm on the ruler. Slowly move the receiver unit and observe the relative maxima and minima of the meter readings. Set the receiver meter to a position where a maximum is observed. Record the initial position of reflector A.

Question 6: How far must a reflector be moved to produce a 2π change in the phase between the two beams?

12. While watching the receiver meter, slowly move reflector A away from the partial reflector. Record the position of the reflector at each minima. Move the reflector until at least 10 minima have been

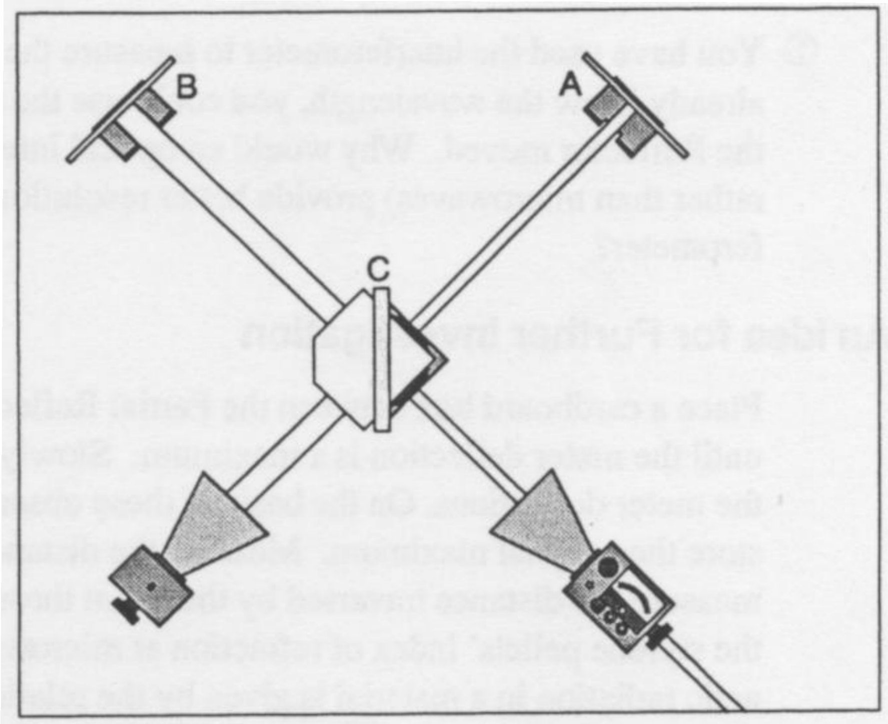

Figure 6: Michelson interferometer.

traversed and returned to a maximum. Plot the reflector position against fringe number. Using the graph of your data, find the wavelength of the microwave radiation. Include an error estimate with your result.

Question 7: If the wavelength of the radiation used with a Michelson interferometer is known, distance measurements can be made. What is the approximate distance resolution that is possible with a Michelson interferometer using (i) the microwave source in this experiment and (ii) He-Ne visible light ($\lambda = 632$ nm)? Justify your answer.

Bragg Diffraction

The phenomenon of diffraction can be used to infer the inter-planar spacings in crystals. By investigating the diffraction patterns of x-rays from crystals, the crystalline structure can be inferred. If the following two criteria are met for an incident wave, constructive interference at an angle θ will be seen, due to difffraction: (i) the angle of incidence equals the angle of reflection and (ii) Bragg's equation is satisfied, that is: $2d \sin\theta = n\lambda$, where n is an integer and θ is the grazing angle of the incident wave (Ensure you understand the relationship between the incident angle and the grazing angle). d is the inter-planar distance. For further information, see Reference 2.

Question 8: Figure 7 shows radiation incident on a lattice of reflectors. Prove Bragg's equation.

In this experiment, the principles of Bragg diffraction will be demonstrated using microwaves. A foam cube containing a lattice of metal spheres simulates a "crystal". By investigating the diffraction pattern from the cubic lattice, the inter-planar distance will be determined.

In a crystal lattice, there are several families of parallel planes that will produce diffraction effects. Each family is denoted by a Miller index (after the British crystallographer W.H. Miller) and has a different values of d. The Miller indices for some of the simple parallel planes are shown in Figure 8. The grazing angle is recorded with respect to the plane under investigation, not the face of the cubic lattice, see Figure 9.

13. Arrange the transmitter and receiver units on the goniometer with the receiver unit on the rotatable arm. Place the foam rotating table on the degree plate. Place the cubic lattice on the rotating table

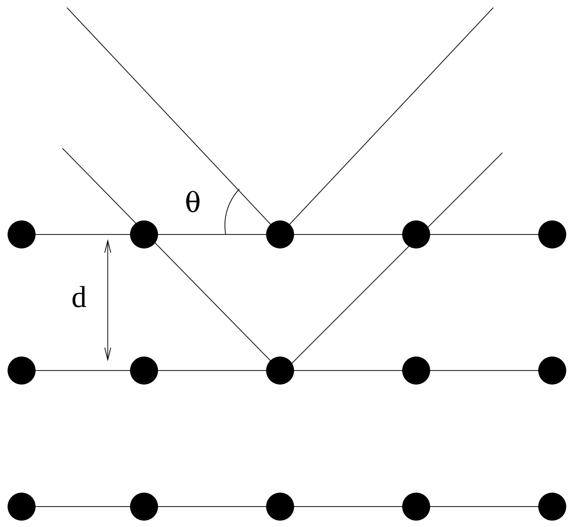

Figure 7: Incident EM radiation on a lattice of reflectors. The inter-planar distance is d.

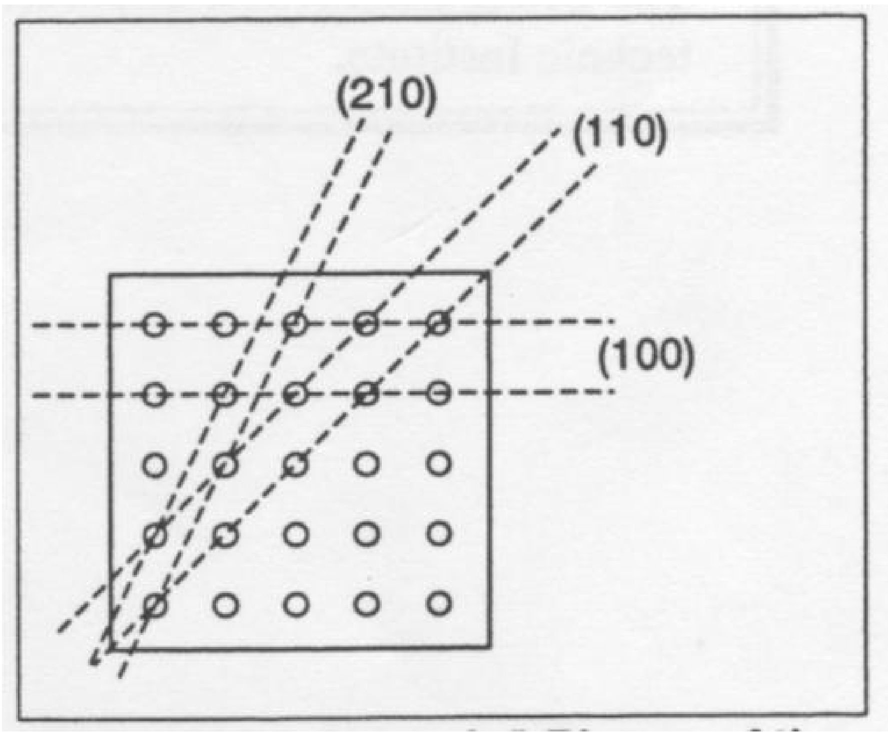

Figure 8: Miller indices for the "crystal" planes.

such that the (100) planes are normal to the incident beam, see Figure 10. Set the receiver unit opposite the transmitter and set the receiver controls for a mid-range reading.

14. Rotate the crystal with the rotating table one degree clockwise. Move the receiver unit on the goniometer arm two degrees clockwise. Ensure that the crystal-receiver distance remains constant. Record the grazing angle of the incident beam and the meter reading. Repeat, rotating the receiver goniometer arm two degrees for every degree of rotation of the cubic lattice. Find the meter reading for grazing angles up to about 60 degrees. Graph the relative intensity of the diffracted signal as a function of grazing angle. Determine at what angles peaks in the signal are found. Using your data, determine the spacing between the (100) planes of the cubic lattice. Measure the inter-planar distance directly and compare your results. Assume the wavelength of the microwave source to be $\lambda = 2.85$cm.

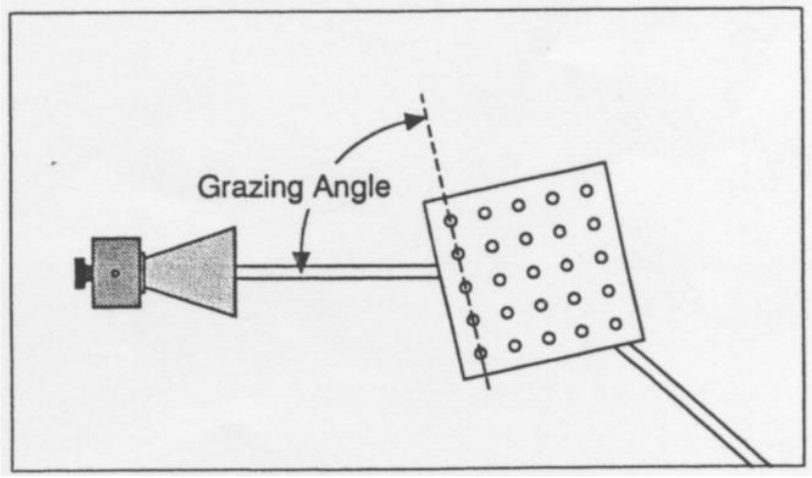

Figure 9: Miller plane grazing angle.

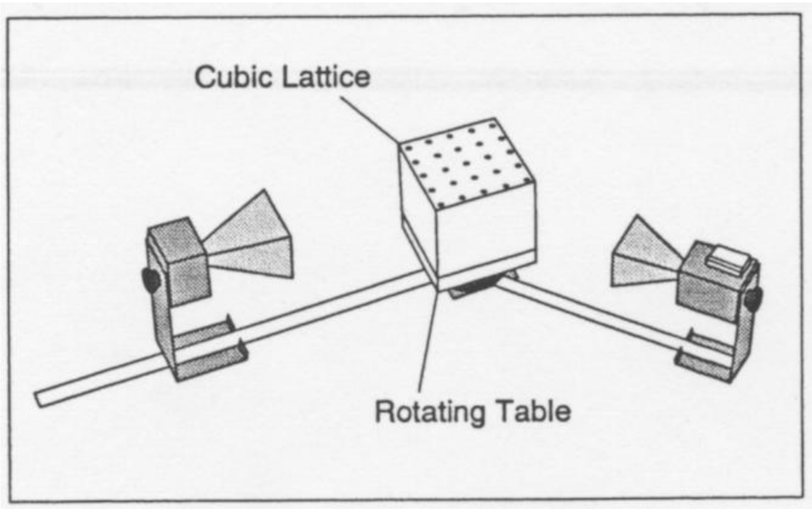

Figure 10: Set-up for Bragg diffraction.

Fresnel Zones

In this section, the phenomena of Fresnel diffraction and Fresnel zones will be investigated.

Consider a spherical wave incident on a circular aperture. In the theory of Fresnel diffraction, the wavefront passing through the aperture can be divided up into a number of annular regions. Consider the path length of a point on the spherical wavefront to a point P. The difference in path length between neighbouring zone boundaries to P is $\lambda/2$ (see Reference 1 and Figure 11 for more information). These are called Fresnel zones.

A sensor at point P detects the combination of all zones that pass through the aperture.

Question 9: Give a qualitative argument why the field at P will be approximately zero if there are an *even* number of zones admitted by the aperture.

Question 10: Consider a source and sensor placed an equal distance, x from a Fresnel zone plate. The aperture in the Fresnel zone plate has radius R. Show that the *maximum* path difference produced by diffraction is: $2(\sqrt{x^2 + R^2} - x)$.

15. Measure the radii of the Fresnel zone plates. For the larger Fresnel zone plate, compute the smallest distance, x at which destructive interference will occur.

16. Set up the large Fresnel zone plate using the two stands. Ensure that the zone plane is perpendicular. Set up the transmitter and receiver at the distance x from the large Fresnel zone. Ensure that the

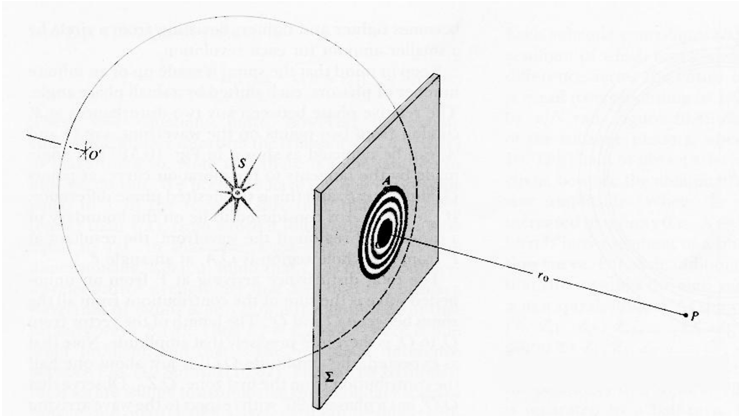

Figure 11: Fresnel zone diffraction.

transmitter and receiver units are at the same height, and that they are collinear with the center of the aperture. Move the receiver unit slowly until a minimum is found. Record the distance of the receiver to the zone plate. Comment on any differences between your measurement and the theoretical value you computed.

17. Without moving the transmitter and receiver units, remove the large Fresnel zone plate. Set the meter reading to half full scale. Carefully place the smaller aperture zone plate between the transmitter and receiver. Record and explain the meter reading.

List of Equipment

1. Gunn Diode Transmitter and Receiver units
2. Goniometer
3. Set square ruler
4. 3 component holders
5. 2 metal reflectors (15 cm)
6. 2 partial (wood) reflectors
7. 7.6 cm wide slit spacer
8. 10.6 cm wide slit spacer
9. Rotating foam table
10. Foam cubic lattice
11. 2 Fresnel zone plates and metal stands
12. Metre ruler

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