Experiment 382: Feedback and Oscillation (Elvis edition)

Aims

To present the basic principles of feedback theory.

To demonstrate the concept of stability in linear systems.

To illustrate the use of feedback to produce oscillators and active filters.

To illustrate the use of feedback in digital circuits to produce periodic sequences.

References

1. Kuo, "Network Analysis and Synthesis," Chapter 8.
2. Millman & Halkias, "Integrated Electronics," Chapters 13 & 14

Basic Feedback Theory

Feedback is said to be present in a circuit when a portion of the output signal of an active circuit is "fed back" and combined with the input signal. For a sinusoidal signal of a given frequency, the signal fed back may either increase or decrease the input amplitude giving rise to positive or negative feedback respectively. Under suitable conditions, the amount of feedback may be such as to cause the circuit to oscillate (i.e. to produce an output signal in the absence of an input signal).

Natural Response and Forced Response in Feedback Circuits

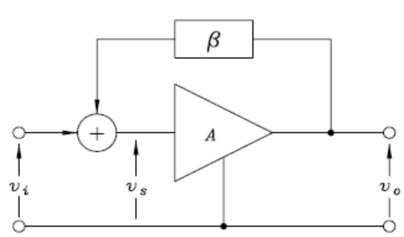


Figure 1: Amplifier with feedback

Consider the feedback circuit shown in Fig. 1 in which a fraction of the output signal of an amplifier with voltage gain is added to the input signal. In general, both A and β are complex functions of the complex frequency s. The equation which gives the response of the system to an input or forcing function $V_i(s)$ may be derived as follows:

$$V_s(s) = V_i(s) + \beta(s)\,V_o(s)$$
$$V_o(s) = A(s)\,V_s(s)$$

whence

$$V_o(s) = \frac{A(s)}{1 - A(s)\,\beta(s)} V_i(s)$$

where $A(s)$ is the **open loop gain**, $\beta(s)$ is the **feedback fraction**, $A(s)\,\beta(s)$ is called the **loop gain** and $A(s)\,/\,[1 - A(s)\,\beta(s)]$ is called the **closed loop gain**.

For a given frequency, the feedback is said to be **positive** if $|1 - A(j\omega)\beta(j\omega)| < 1$ and **negative** if $|1 - A(j\omega)\beta(j\omega)| > 1$. With positive feedback, the magnitude of the closed loop gain exceeds the magnitude of the open loop gain, whereas for negative feedback, the reverse is true.

Next, we define the **natural response** of the system to be any signal which can be sustained within the system in the absence of an input signal. Setting $V_i(s) = 0$ in the above equations, we obtain $V_s(s) = \beta(s)V_o(s)$ and $V_o(s) = A(s)V_s(s)$ which can be written in matrix form as:

$$\begin{pmatrix} A(s) & -1 \\ 1 & \beta(s) \end{pmatrix} \begin{pmatrix} V_s(s) \\ V_o(s) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This has nontrivial solutions [$V_s(s)$ and $V_o(s)$ not both zero] iff the determinant of the above matrix is zero, i.e. iff $|1 - A(s)\beta(s)| = 0$. In general, this is a polynomial equation in called the **characteristic polynomial** of the system. If the roots of this equation are $s_1, s_2, \ldots, s_n$, the natural response of the system can be shown to have the form:

$$v_o(t) = \mathrm{Re}\left[c_1 \exp(s_1 t) + c_2 \exp(s_2 t) + \cdots + c_n \exp(s_n t)\right]$$

for some arbitrary constants $c_1, c_2, \ldots, c_n$. Hence:

(a) If all the roots are in the left-hand half plane (i.e. $\mathrm{Re}\, s_k < 0$, for each $k = 1, \ldots, n$) then any natural response decays to zero and vanishes in the steady-state.

(b) If there is a conjugate pair of roots $\pm j\omega_0$ on the imaginary axis, there is a natural response consisting of sinusoidal oscillations at the frequency $\omega_0/(2\pi)$.

(c) If there are roots in the right-hand half plane, there are natural responses consisting of exponentially expanding oscillations. In practice, the amplitude of such oscillations is limited by nonlinearities in the circuit.

If (a) holds, the system is said to be **stable**, otherwise it is **unstable**. Oscillators are examples of unstable systems. We note that the condition $1 - A(s)\beta(s) = 0$ is precisely the condition for the loop gain to be equal to unity. This observation is the basis of the Nyquist criterion for stability.

1 National Instruments Elvis II

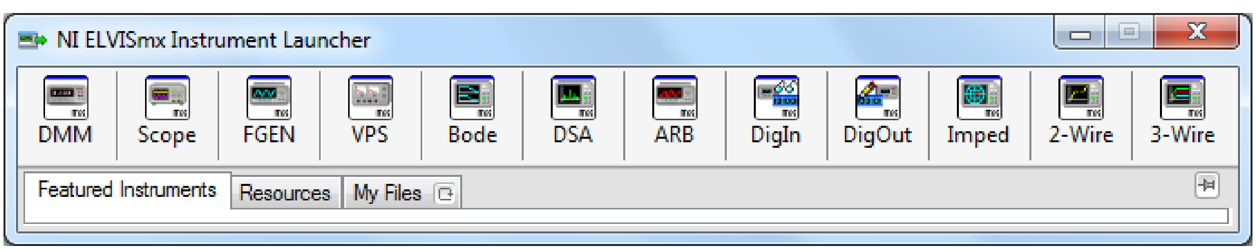


Figure 2: The instrument launcher for the NI ELVIS II, containing such useful devices as the digital multimeter (DMM), an oscilloscope (Scope), a function generator (FGEN) and a Bode analyser (Bode)

This experiment designed to make use of the Elvis II platform from National Instruments. **All the circuits that you will study in this lab need to be built and tested on the Elvis. The electronic components and wires you need to complete this experiment can be found in the components cabinet next to the LCR meter in the corner of the lab.** The layout of the Elvis II allows circuits to be built without having to use solder to connect components, and also contains devices such as an oscilloscope and a function generator, all accessible via a USB connection to a PC with the appropriate software installed.

To access these components, ensure that the USB plug is connected to a PC and the Elvis II is powered on using the switch on the side of the device. The LED indicator for the USB should display ‘READY’. You can now launch the ‘NI ELVISmx Instrument Launcher’, shown in Fig. 2. It is important to understand the layout of the NI Elvis to ensure that the circuits that you build are wired correctly. An overview is shown in Fig. 3, and some important features are numbered as follows:

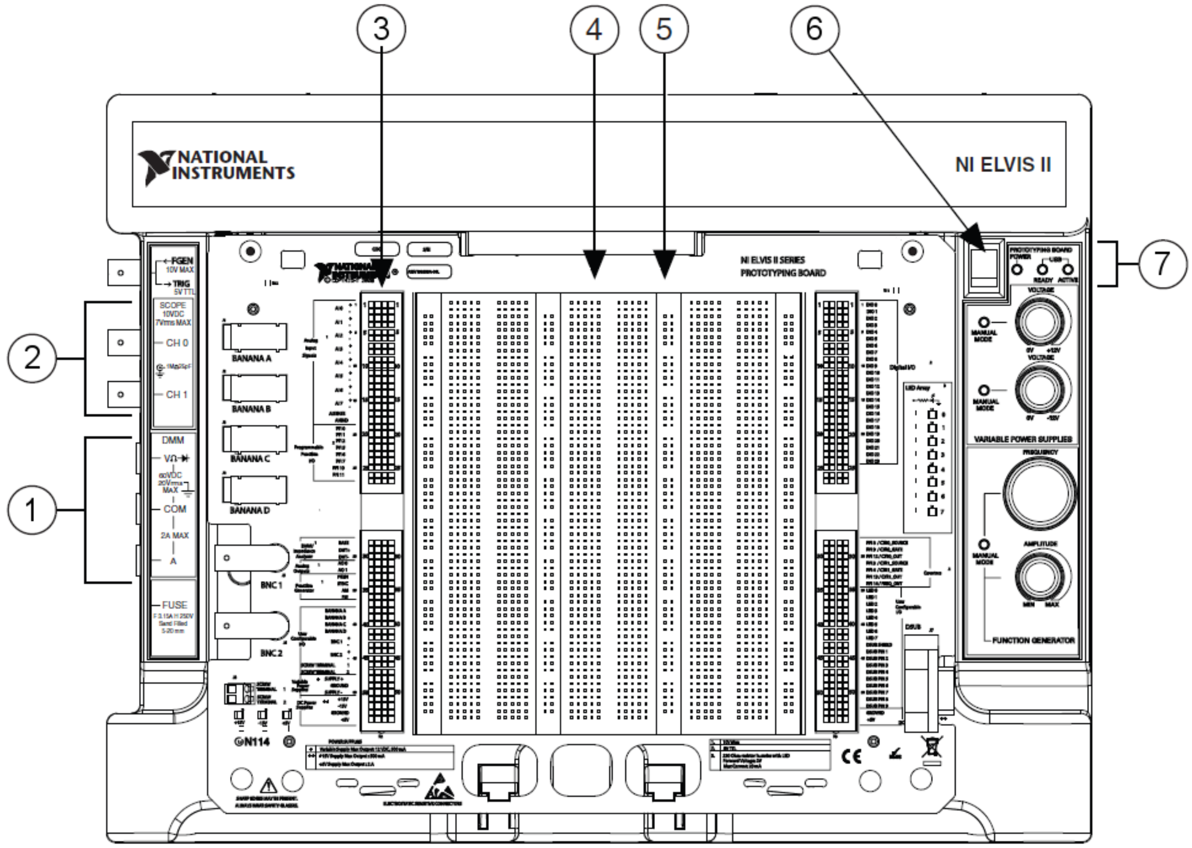


Figure 3: The Elvis II from National Instruments.

1. Connections for the digital multimeter (DMM) leads. To use this instrument, select DMM from the instrument launcher, select the appropriate function and set it to ‘Run’.

2. Connections for two oscilloscope (Scope) channels, labeled CH 0 and CH 1. The oscilloscope can be accessed from the instrument launcher.

3. Various connections that can be wired to circuits on the prototyping board. In this experiment we shall use the DC power supplies (pins 51-54) and the function generator (FGEN, pin 33).

4. The prototyping board, where circuits can be constructed. Ensure you understand how the pinholes are connected before you begin to wire up your circuits. An IC chip can be inserted so that its legs occupy columns E and F of the prototyping board, and wires can be taken from neighbouring pinholes to connect to other parts of your circuit.

5. Two columns of pinholes denoted by a red + and a blue −. It is good practice to take a wire from a power supply and connect it to the appropriate rail. Subsequent wires can then be used to power the components of your circuit.

6. Power switch for the prototyping board. This must be active for the power supplies and function generator to operate.

7. LED status indicators to show USB activity, as well as the prototyping board’s power status.

Note: Be sure to include oscilloscope plots and Bode plots in your report where appropriate. You should do this by importing your data into Python.

2 An R-C Oscillator

Consider the natural response of the circuit shown in Fig. 4 in which there is no input signal.

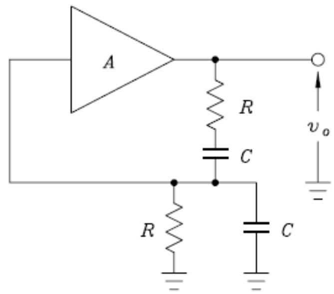


Figure 4: An RC oscillator

In this case, we have:

$$A(s) = A \quad \text{and} \quad \beta(s) = \frac{\omega_0 s}{s^2 + 3\omega_0 s + \omega_0^2}$$

where $\omega_0 = 1/(RC)$. The characteristic polynomial is (check!):

$$s^2 + (3 - A)\,\omega_0 s + \omega_0^2 = 0$$

which has roots at:

$$\left[\frac{A-3}{2} \pm \mathrm{j}\sqrt{1 - \left(\frac{A-3}{2}\right)^2}\right]\omega_0$$

These lie on the imaginary axis (condition (b) above) when $A = 3$. In this case the roots are $\pm \mathrm{j}\omega_0$ so that the system acts as a sinusoidal oscillator of frequency $\omega_0/(2\pi)$.

For $A < 3$, the roots are in the left-hand half plane and the steady-state natural response decays to zero. For $A > 3$, the roots are in the right-hand half plane and the natural response consists of an expanding sinusoid. In practice, the amplitude limitation gives rise to a non-sinusoidal waveform.

Procedure

1. We wish to implement the amplifier section of Fig. 4 as a non-inverting op-amp circuit. The op-amp we will use is the LF356, which is a modern, high-gain, low-noise, op-amp. The pin-out for this chip is shown in Fig. 5. The Balance pins can be left unconnected. The V_+ and V_- pins require $+15V$ and $-15V$ respectively. We require an amplifier with a variable gain between 1 to 4. This can be achieved using a 3.3 kΩ fixed resistor and a 10 kΩ potentiometer as shown in Fig. 5. The gain is controlled by the potentiometer with maximum gain obtained with the shaft fully clockwise. Use the signal generator and oscilloscope functions of the Elvis to verify the gain of your op-amp amplifier at frequencies around 1 kHz (Note: if you prefer you can use the Elvis' Bode analyzer function to measure the full frequency response of the amplifier).

2. Build the full oscillator circuit (amplifier and feedback loop) as shown in Fig. 6. Note: It is possible to break the feedback loop for open-loop gain measurements. Use $R_1 = R_2 = 33\,\mathrm{k}\Omega$ and $C_1 = C_2 = 6.8\,\mathrm{nF}$. Advance the gain control gradually from 1, monitoring the output of the amplifier using the Elvis' built-in oscilloscope . Note that the oscillation commences suddenly as the system becomes unstable. Comment on the shape of the waveform and the oscillation frequency when the gain is increased past the onset of oscillation.

3. Adjust the gain carefully until oscillation is just present and measure the frequency of the output waveform using the oscilloscope. Next, break the feedback loop and measure the open-loop gain by measuring the output for a known input signal applied directly to pin3 of the LM356. Compare the frequency and gain measured with the calculated values.

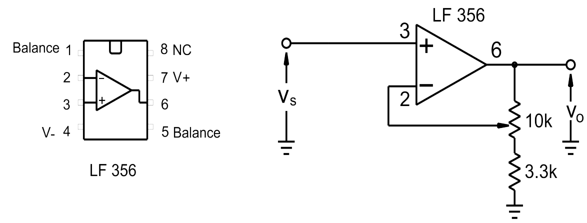


Figure 5: Non-inverting amplifier of variable gain constructed from an opamp

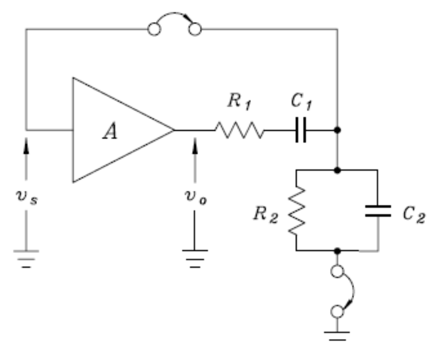


Figure 6: Non-inverting amplifier of gain A with feedback

4. Repeat procedure (3) with $C_1 = C_2 = 680\,\mathrm{nF}$, noting the new oscillation frequency.
5. Reformulate the theory for the situation in which C_1 and C_2 are unequal. Try using $C_1 = 680\,\mathrm{nF}$ and $C_2 = 6.8\,\mathrm{nF}$ in the experimental setup. Compare the frequency and open-loop gain at the onset of oscillation with those theoretically calculated.

3 A High Q Low Frequency Tuned Amplifier

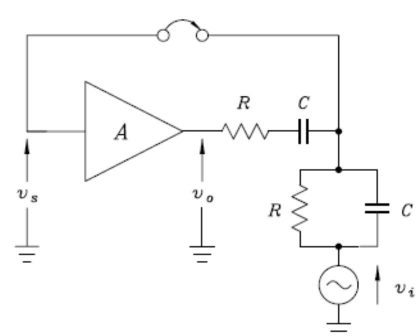


Figure 7: Applying an input signal to the amplifier with feedback

Next we consider the forced response of the system in Fig.6 to an input signal applied as shown in Fig.7. Finding $V_s(s)$ using Millman's theorem, we deduce that:

$$\frac{V_o(s)}{V_i(s)} = A\left[\frac{(s+\omega_0)^2}{s^2 + (3-A)\,\omega_0 s + \omega_0^2}\right]$$

where $\omega_0 = 1/(RC)$. For $A > 1$, the function has a double zero at $s = -\omega_0$ and a pair of conjugate poles at:

$$\left[\frac{A-3}{2} \pm \mathrm{j}\sqrt{1 - \left(\frac{A-3}{2}\right)^2}\right]\omega_0$$

As A is increased, verify that the poles move on a circle of radius as shown in Fig. 8.

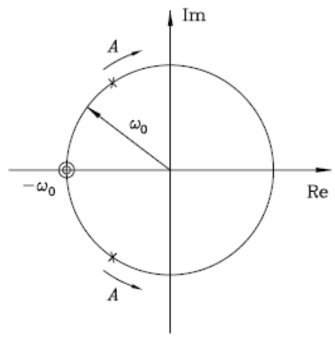


Figure 8: Locus of the poles of the transfer function as the amplifier gain A is varied

Putting $s = \mathrm{j}\omega$, we find that for sinusoids the previous expression reduces to:

$$\frac{V_o(\mathrm{j}\omega)}{V_i(\mathrm{j}\omega)} = \frac{2A}{3-A}\left[\frac{1+\mathrm{j}QX}{1+\mathrm{j}Q'X}\right]$$

where $Q = \frac{1}{2}$, $Q' = \frac{1}{3-A}$ and $X = (\omega/\omega_0) - (\omega_0/\omega)$.

Note that Q and Q' are the Q-values associated with the numerator and denominator polynomials. If A is just slightly less than 3, $Q' \gg Q$ and if we consider frequencies such that ω is close to ω_0, the transfer function may be well-approximated by:

$$\frac{V_o(\mathrm{j}\omega)}{V_i(\mathrm{j}\omega)} = \frac{2A}{3-A}\left[\frac{1}{1+\mathrm{j}Q'X}\right]$$

This has the form of a bandpass filter with centre frequency ω_0 and effective Q given by Q'. Note however that the approximation is valid only near the peak in the response.

Procedure

1. Using the same circuit you built in the previous section, disconnect R_2 and C_2 from ground and connect a signal from the Elvis function generator v_i (FGEN, pin33) shown in Fig. 7. Using $C_1 = C_2 = 6.8\,\mathrm{nF}$ and $A < 3$, verify that the circuit acts as a tuned amplifier. Keep the input amplitude sufficiently small so that the output signal is less than 6 V p-p in order to avoid slew-rate limitations.

2. Plot the magnitude of the transfer function (in dB) against log frequency for $A = 2$ and $A = 2.7$. This is best done using the Bode analyzer function of the Elvis board. Again, be sure to keep your input signal small enough that the output signal is not distorted. Compare these plots with the theoretical results, noting especially the value of the transfer function at ω_0 and the effective Q. Are the graphs symmetrical about $\log f_0$? Why?

3. Adjust A so that the system is just below the verge of oscillation and estimate the effective Q. Observe and discuss the output waveform in this case for triangular and square wave inputs of frequencies around $\omega_0/(2\pi)$. What factors limit the highest Q that can reliably be achieved?

4 The Biquadratic Filter

The biquadratic filter circuit is an example of an active filter which provides both low-pass and band-pass outputs. The coefficients of its characteristic polynomial can be independently set by the various components making it possible to implement a wide range of Q's and ω_0's.

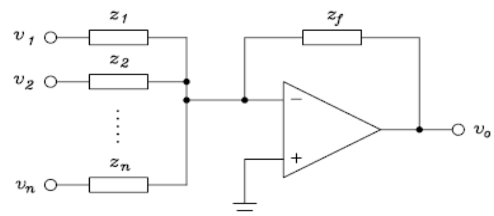


Figure 9: An inverting summing amplifier made from an opamp

For an ideal operational amplifier with infinite gain, the configuration of Fig. 9 can be shown to have an output voltage given by:

$$v_0 = -\left[\left(\frac{z_f}{z_1}\right)v_1 + \left(\frac{z_f}{z_2}\right)v_2 + \cdots + \left(\frac{z_f}{z_n}\right)v_n\right] = -z_f\left[\frac{v_1}{z_1} + \frac{v_2}{z_2} + \cdots + \frac{v_n}{z_n}\right]$$

The circuit of the biquadratic filter is shown in Fig. 10.

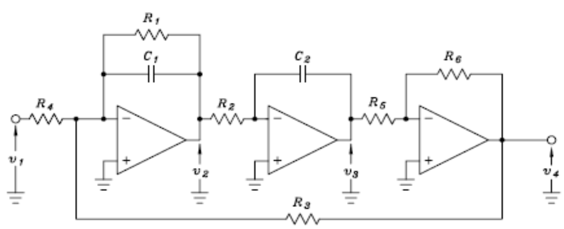


Figure 10: The biquadratic filter

For convenience, we define ω_1, ω_2, ω_3, ω_4 and k as follows:

$$\omega_1 = \frac{1}{R_1C_1}, \quad \omega_2 = \frac{1}{R_2C_2}, \quad \omega_3 = \frac{1}{R_3C_1}, \quad \omega_4 = \frac{1}{R_4C_1}, \quad k = \frac{R_6}{R_5}$$

We may obtain

$$\begin{aligned} V_2(s) &= -\left(\frac{\omega_4 s}{s^2 + \omega_1 s + k\omega_2\omega_3}\right)V_1(s) \\ V_3(s) &= \left(\frac{\omega_2\omega_4}{s^2 + \omega_1 s + k\omega_2\omega_3}\right)V_1(s) \\ V_4(s) &= -\left(\frac{k\omega_2\omega_4}{s^2 + \omega_1 s + k\omega_2\omega_3}\right)V_1(s) \end{aligned}$$

Verify that the output v_2 gives a band-pass filter characteristic and that the output v_3 (or v_4) gives a low-pass filter characteristic where:

$$\omega_0 = \sqrt{k\omega_2\omega_3} = \sqrt{\frac{R_6}{R_2R_3R_5C_1C_2}} \quad \text{and} \quad Q = \frac{\omega_0}{\omega_1} = R_1\sqrt{\frac{R_6C_1}{R_2R_3R_5C_2}}$$

Hence Q can be adjusted by R_1 independently of ω_0.

Procedure

1. Set up the biquadratic filter using three LF356 op-amps and set $R_2 = R_3 = R_4 = R_5 = R_6 = 47\,\mathrm{k}\Omega$ and $C_1 = C_2 = 4.7\,\mathrm{nF}$. Use the Bode analyzer function to plot the magnitude of $V_2\,(\mathrm{j}\omega)\,/V_1\,(\mathrm{j}\omega)$ and of $V_3\,(\mathrm{j}\omega)\,/V_1\,(\mathrm{j}\omega)$ for each of $R_1 = 470\,\mathrm{k}\Omega$, $33\,\mathrm{k}\Omega$ and $10\,\mathrm{k}\Omega$.

   Measure ω_0 and Q and compare them with the calculated values. Of these values of R_1, which gives the best performance as a low-pass filter?

2. Observe and comment on the response of the circuit to a low frequency (50Hz) square wave. (note: this signal may be noisy but you will still be able to see the characteristics of the response.

3. What happens if R_1 is removed? Why?

5 A Digital Sequence Generator (Optional, see your demonstrator for logic chips, and to negotiate extra credit)

Feedback may also be used to generate digital sequences. A shift-register is used with a combinational logic circuit as shown in Fig. 11 so that the data fed back into the input of the shift-register is dependent on the state of the shift-register outputs.

Note: The three rectangles with the letters D and Q shown in Fig. 11 are D flip-flips - one bit elements of memory. On a falling clock edge (the clock signal falling from high to low) the input of the flip-flip, D, is loaded into the output of the flip-flop, Q. This output Q will then remain unchanged until the next clock edge when the new value of D is loaded into Q. Both D and Q are digital signals with $0V$ representing LOW, and $+5V$ representing HIGH.

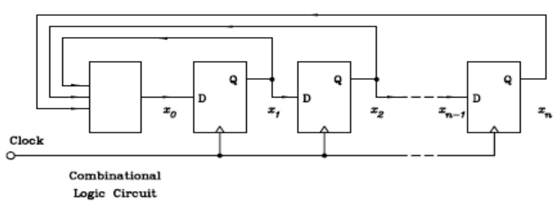


Figure 11: Circuit of a generic digital sequence generator

The output of the combinational circuit can be expressed as:

$$x_0 = f\,(x_1, x_2, \ldots, x_n)$$

where f is the logic function of the combinational circuit. In the case of a linear shift-register feedback circuit the function f defines a linear combination of its arguments, i.e.

$$f\,(x_1, x_2, \ldots, x_n) = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

Note that this expression involves **binary quantities** which can only take on the values 0 or 1. The “multiplications” represent the logical AND operation, while the “additions” represent modulo-2 addition (i.e., the XOR operation).

On the t^{th} clock pulse, the feedback equation for a linear circuit then is:

$$x_0\,[t] = c_1 x_1\,[t] + c_2 x_2\,[t] + \cdots + c_n x_n\,[t]$$

For a D flip-flop (say the j^{th} flip-flop in the shift-register), the relationship between the output x_j and the input x_{j-1} may be written formally as:

$$x_j\,[t] = Z^{-1} x_{j-1}\,[t]$$

where Z^{-1} is a **delay operator**, which acts on a **sequence** to produce an output sequence which has been delayed by one point relative to the input, i.e., the above is simply a notation which means

$$x_j[t] = x_{j-1}[t-1]$$

If $x_0[t]$ is the sequence at the input of the shift register, we see that $x_j[t] = Z^{-j}x_0[t]$. The feedback equation for a linear circuit may thus be alternatively written as:

$$x_0[t] = c_1 Z^{-1}x_0[t] + c_2 Z^{-2}x_0[t] + \cdots + c_n Z^{-n}x_0[t]$$

Since modulo-two addition and modulo-two subtraction are identical, this equation may be further written as:

$$x_0[t] + c_1 Z^{-1}x_0[t] + c_2 Z^{-2}x_0[t] + \cdots + c_n Z^{-n}x_0[t] = 0$$

where the zero on the right-hand side represents the **sequence** which is zero for all values of the index. Applying Z^n to both sides, we find

$$\left(Z^n + c_1 Z^{n-1} + c_2 Z^{n-1} + \cdots + c_n\right) x_0[t] = 0$$

The polynomial within the parentheses in the last equation is known as the **characteristic polynomial** of the circuit since it completely defines the characteristics of the circuit. For a given characteristic polynomial we can draw the corresponding circuit and deduce the periodic sequence which may be obtained from the output of any of the flip-flops. As an example, for the characteristic polynomial Z^3+Z+1, the corresponding feedback equation may be derived as follows:

$$\left(Z^3 + Z + 1\right) x_0[t] = 0$$

This is equivalent to

$$\left(1 + Z^{-2} + Z^{-3}\right) x_0[t] = 0$$

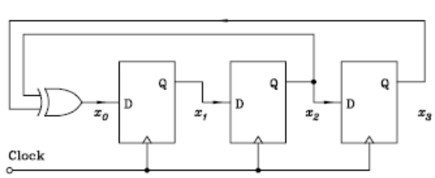
and to

$$x_0[t] = Z^{-2}x_0[t] + Z^{-3}x_0[t]$$

If we use a shift register whose input is $x_0[t]$, the output of the j'th flip-flop is $x_j[t] = Z^{-j}x_0[t] = x_0[t-j]$. Thus we may implement the sequence generator by setting

$$x_0 = x_2 + x_3$$

where it should be remembered that the + sign means an exclusive-OR operation. The corresponding circuit and the periodic sequence which will appear at each of the flip-flops is shown in Fig. 12.



Clock	x_0	x_1	x_2	x_3
0	1	0	0	1
1	0	1	0	0
2	1	0	1	0
3	1	1	0	1
4	1	1	1	0
5	0	1	1	1
6	0	0	1	1
7	1	0	0	1
8	0	1	0	0
9	1	0	1	0
10	1	1	0	1
⋮	⋮	⋮	⋮	⋮

periodic
period = 2^3-1

Figure 12: Shift-register based implementation of a digital sequence generator associated with the characteristic polynomial $Z^3 + Z + 1$

For certain characteristic polynomials, called primitive polynomials, the shift-register feedback circuit will step through all the combinations of the flip-flop states except the all zeroes combination which results in a null sequence at each flip-flop output. The periodic sequence generated at each flip-flop output by circuits based on primitive polynomials is of maximal length, namely $2^n - 1$ where n is the number of flip-flops in the shift-register. Such binary sequences have statistical properties similar to those of random "noise" and are called maximal-length pseudo-random binary sequences (or m-sequences for short). They are often used as sample data sequences to test the performance of communications systems.

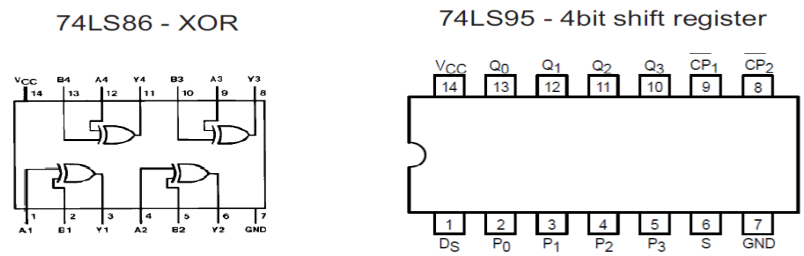


Figure 13: Pin-out for the 74LS86 (XOR) and 74LS95 (D flip-flop)

Figure 13: Pin-out for the 74LS86 (XOR) and 74LS95 (D flip-flop)

Procedure

1. To implement the digital sequence generator shown in Fig. 12 we will use the 74LS86 chip for an XOR gate and the 74LS95 chip as a D flip-flop. The pin-outs for these chips are shown in Fig. 13. The V_{cc} pins should be connected to $+5V$ and the GND pins to ground for both chips. To operate the 74LS95 as a D flip-flop you will need to supply $+5V$ to the S pin, then the P_0 pin is the input D, and the Q_0 pin is the output Q. The $\overline{CP}2$ pin is the Clock input. Your circuit will need three D flip-flops and one XOR gate to operate. For the clock signal use a 1kHz square wave (0 to 5V amplitude) from the Elvis function generator.

2. Connect up the circuit generating the m-sequence associated with $Z^3 + Z + 1$ as shown in Fig. 12 and observe the outputs of the shift-register on the Elvis' oscilloscope, verifying that the above sequence is generated. Note the output x_0 is located at the input to the first D flip-flop, the outputs x_1, x_2, and x_3 are located at the outputs of the first, second, and third D flip-flops respectively. Sketch all waveforms in time synchronisation.

3. Connect up the circuit to implement the m-sequence associated with $Z^4 + Z^3 + 1$. Draw the outputs of the shift-register in time synchronisation and compare these with a theoretical determination of the states.

G.E.J. Bold, S.M. Tan, Z.C. Tan, 1999

Revised for Elvis, May 2014 SGM

Revised for Python: 24 November 2014