Experiment 216: Surface Tension of Water

Aim

To evaluate the surface tension of water by studying the behaviour of ripples on its surface using a single source and multiple sources.

Reference

General background reading on the surface tension of liquids may be found in:
Newman and Searle, "General Properties of Matter", Arnold.

For more information on interference, refer to Serway & Jewett's "Physics for Scientists and Engineers".

Method

The speed c at which very short wavelength waves, or ripples, travel across the surface of a liquid depends on restoring forces arising both from gravity g and from surface tension T. Provided the wavelength is sufficiently short in comparision to the depth ($\lambda \leq 3h$ – see Experiment 218, "Gravity waves and dispersion"), the speed is given by:

$$2\pi c^2 = \lambda \left(g + \frac{4\pi^2 T}{\lambda^2 \rho} \right) \tag{1}$$

where λ is the wavelength and ρ is the liquid density. Since wave speed c and wave frequency f are related by:

$$c = \lambda f \tag{2}$$

the surface tension may be expressed as:

$$T = \frac{\lambda\rho}{2\pi} \left(f^2 \lambda^2 - \frac{\lambda g}{2\pi} \right) \tag{3}$$

Error Analysis

It will be necessary to appreciate the relative magnitudes of the errors inherent in the method, so that you will know which factor will limit the accuracy with which the surface tension may be derived, and hence with what accuracy other factors should be determined. We may make an error analysis in the usual way.

Take equation (3) in the form $T = T(f, \lambda, \rho, g)$; we can then write

$$(\Delta T)^2 = \left(\frac{\partial T}{\partial f} \right)^2 (\Delta f)^2 + \left(\frac{\partial T}{\partial \lambda} \right)^2 (\Delta \lambda)^2 + \left(\frac{\partial T}{\partial \rho} \right)^2 (\Delta \rho)^2 + \left(\frac{\partial T}{\partial g} \right)^2 (\Delta g)^2$$

Evaluation of the various derivatives will thus lead us to an estimate of the error ΔT in surface tension due, say, to an error Δp in some parameter p. Some preliminary experimenting with the ripple tank will give you approximate figures to use for f and λ in these calculations; other figures will need to come from the published tables.

Question 1: From an error analysis, which experimentally determined quantity will limit the accuracy of your value of surface tension?

Question 2: With this limit on the accuracy of T, how precisely should values of ρ and g be derived? For example, is it appropriate to allow for the variation of standard gravity with latitude, or for the variation of water density with temperature?

Question 3: How much does the surface tension of water vary with temperature? Will your determination of T be sufficiently precise to make it necessary to measure the associated water temperature?

Part 1

1. Thoroughly clean the tank, the agitator and anything else you intend to put into the tank (including your fingers) using the detergent solution provided. Rinse thoroughly with clean tap water.
2. Fill the tank with tap water to a suitable level. Remember that theory makes an assumption about the depth of water relative to the wavelength in the derivation of equation (1).
3. Set up the apparatus so that well-defined shadows of as extensive a pattern of surface waves as possible are cast on the bottom of the tank.

   You need to optimize the depth of water, the position of the agitator, the position of the stroboscope, and the gain setting of the generator which drives the agitator. Note that the stroboscope, with the trigger switch in the "external" position, is synchronised to the driving generator.

   Hint: You may (or may not) find the wooden blocks and/or graph paper useful.
4. When you have produced a well-defined pattern of stationary waves, measure the length of as long a train of waves as possible, and hence the wavelength of a single wave.
5. Determine the surface tension.
6. Estimate the probable error of the wavelength measurement, and hence the probable error in the surface tension.
7. Repeat the experiment using a range of frequencies from roughly 20 Hz to 60 Hz. Take your data using a flat paddle and then a spherical one.

Question 4: Why is cleanliness important in this experiment?

Question 5: Why do the waves appear stationary?

Question 6: When you use a spherical paddle, you may see interference patterns even though there's only a single point source. How can this happen?

Question 7: Does the tabulated value of T for water lie within the range of your estimated error? If not, what could account for the discrepancy?

Part 2

Two point sources, oscillating at the same frequency, produce a pair of concentric wave fronts. These wave fronts superimpose to give areas of constructive and destructive interference.

The path difference between two interfering waves, be they electromagnetic, sound, or water, determines the amplitude of the resultant wave.

In our case, an area of constructive interference occurs when the distance between point source 1 and that area is an integer number of wavelengths. Let this be

$$r_1 = n_1 \lambda$$

and the distance between point source 2 and that area is

$$r_2 = n_2\lambda$$

Taking the path difference, it is easy to see that

$$\Delta r \ = (n_2 - n_1)\lambda \ = m\lambda \tag{4}$$

1. The experimental arrangement is similar to that in part 1, but position the two spherical paddles so that they are roughly 2 cm apart. Set the generator to any frequency that you used in part 1.
2. Take a piece of grid paper and draw a set of axes, with an appropriate scale, and put this underneath the transparent tank.
3. Before you increase the gain setting on the generator, note the coordinates of the shadows of the spherical paddles. (How accurately can you measure this?)
4. Slowly increase the amplitude of the waves until you see a fine interference pattern on the grid paper.
5. Determine the coordinates of several areas of constructive interference. Then you can draw the coordinates on your grid to help measure and tabulate the values for r_1 and r_2.
6. Repeat the above for at least two different frequencies that you used in part 1.

Question 8: Since the path difference in equation (4) is an integer number of wavelengths (m), review the corresponding value of λ that you obtained in part 1. What is a good estimate of m? Then divide Δr by m. What do you get?

Question 9: Calculate values for T, the surface tension, and compare these with the results obtained in part 1. Are there any major discrepancies?

Diffraction Effects (Optional)

While the ripple tank is set up, you might like to demonstrate to yourself various diffraction effects such as the effect of the edge of a barrier, and diffraction at slits.

Write-up

The experiment write-up must include:

1. Brief answers to the questions.
2. The experimenter's conclusions regarding the accuracy required for the individual parameters in equation (3).
3. A brief discussion of any methods the experimenter may have used to obtain the best possible results.
4. An account of the experimenter's understanding of the sensitivity of surface tension to influences such as contaminants and temperature, and hence the need for cleanliness and temperature control.
5. A set of measurements of frequency and corresponding wavelength.
6. A discussion of errors.
7. An estimate of T with confidence limits. A comparison of this value of T with a reference value.

List of Equipment

1. 1 x Microprocessor-controlled Frequency Generator
2. 1 x LED Stroboscope
3. 1 x Plastic tank 29 cm x 39 cm x 6 cm
4. 1 x Agitator paddle (2x spherical, 1x flat)
5. 1 x Co-axial cable with BNC-to-Banana Plugs
6. 1 x 10 cm Glass ruler.

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