

Experiment 252: Relativistic Electrons

Aims

1. To show the need for relativistic kinematics when the speed of a particle approaches that of light.
2. To obtain a rough value for the rest energy of the electron.
3. To demonstrate mass increase with velocity.
4. To show the focussing action of a 180° spectrometer.

References

1. A.P. French “Special Relativity” Norton.
2. K. Siegbahn “Beta and Gamma Ray Spectroscopy” North Holland Publishing.
3. H. Enge “Introduction to Nuclear Physics” Addison-Wesley.

Chapter III of Reference 2 discusses the 180° focussing property. A brief discussion is also given in Sections 10-2 and 11-2 of Reference 3.

Introduction

The main purpose of this experiment is to give you direct experience of measurements that require the kinematics of special relativity for their interpretation.

Although you should by now be familiar with relativistic kinematics the apparatus used is quite sophisticated and you will need to know only the broad principles of how each unit works. Only one aspect of the instrumentation will be studied in any detail, namely the 180° focussing action of the spectrometer.

Description of Apparatus

The black box between the magnet poles is the spectrometer itself. A schematic diagram and photograph of its internal parts are displayed with the apparatus.

A radioactive source containing 74 MBq (2 mCi) of ^{90}Sr emits a continuous spectrum of β particles of kinetic energy from zero to about 2 MeV. In the presence of a magnetic field B , perpendicular to the electron velocity vector, the electrons move in a circular arc of radius r where the momentum is given by:

$$p = Ber$$

where e is the electronic charge.

The source and detector are on opposite ends of a diameter of 90 mm so that r is fixed at 45 mm. Thus any momentum p may be selected by adjusting B . The strength of the field B is measured by inserting the Hall probe into the hole on the side of the spectrometer.

The electrons are detected by means of a Lithium-drifted-Silicon (Si-Li) semiconductor detector. This detector consists of a 5 mm wafer of silicon between two electrodes across which an electric field is applied. As the electrons travel through the silicon, they lose energy and produce electron-hole pairs; the number of electron-hole pairs produced being proportional to the electron energy loss in the silicon. These electron-hole pairs are separated in the electric field and are collected at the electrodes, resulting in the accumulation of charge on the electrodes. The detector preamplifier produces an output electrical pulse whose height

(voltage) is proportional to the accumulated charge and hence to the incident electron kinetic energy. After shaping and amplification, these pulses are digitised and displayed by the pulse height analyser (PHA) which sorts pulses by height and plots the number of pulses of a given height on the ordinate (y-axis) against height on the abscissa (x-axis). Since pulse height is proportional to electron energy this plot may be regarded as an electron energy spectrum. Since only a narrow band of electron momenta are selected by the spectrometer, we expect a fairly well defined energy peak on the pulse height analyser.

These independent measurements of the kinetic energy T and the momentum p allow us to check the two kinematical theories. Classical Newtonian theory asserts that the kinetic energy T is related to the momentum p by:

$$T = \frac{p^2}{2m}$$

whereas special relativity theory predicts that:

$$T = \sqrt{E_0^2 + p^2 c^2} - E_0$$

where E_0 is the rest energy of the electron and c is the speed of light.

A note about units

Although S.I. units are used in most branches of physics, it is conventional to use electron volts and its multiples for energy, MeV/c or keV/c for momentum, and MeV/c² or keV/c² in atomic, nuclear and particle physics. (Watch out for the cs when making calculations.)

Calibrating the energy scale

- (1) It is first necessary to calibrate the energy scale of the PHA. Clearly this calibration is dependent on the amplifier gains used. Thus once the calibration has been done, these settings must be kept constant.

As the detector can be damaged if it is on when the vacuum is changed, pump the spectrometer down now, making sure that the small air inlet tap at the top of the pump is screwed shut. Leave the vacuum pump running for the rest of the experiment. When the experiment is completed, switch off the pump and open the air inlet tap two turns. (Make sure you turn the detector off first!)

You can now power the detector and set the electronics. The following settings give good results:

Detector bias:	+750 V	main amp gain:	coarse:	×8, ×2
			fine:	2.6
		output pulse:	bipolar	(clip = 2)

Important: Severe damage can result to the semiconductor detector if it is subjected to abrupt voltage changes or electrical discharges. Therefore:

- (a) Always turn the detector bias on or off very slowly!
- (b) Always ensure that the detector bias has been reduced to zero before either pumping down the spectrometer or returning it to atmospheric pressure.

To calibrate the system place a ²²Na source near the detector, but outside the apparatus (on top of the black box and at the right-hand side); and observe the energy spectrum on the PHA. If you have not used the PHA before you may need to ask a demonstrator for assistance. A brief manual can be found in the Appendix which should cover what you need to know (and more).

Note: 256 channels is sufficient for this experiment. If the maximum channel shown on the display is more than this, see 'Changing the Number of Channels' in the Appendix for how to change it.

The ^{22}Na calibration source emits 0.511 MeV and 1.275 MeV gamma rays. In this detector, peaks corresponding to these gamma rays are not directly observed, but rather what one sees are the Compton spectra produced by Compton interactions of the gamma rays with the silicon. The high-energy side of the Compton distributions provides two well-defined “edges” that may be used for calibration purposes. Figure 1 shows what the spectrum looks like and gives the energies of these Compton edges. The Compton edge corresponding to the 0.511 MeV gamma ray lies at 0.341 MeV and the Compton edge corresponding to the 1.275 MeV gamma ray is at 1.062 MeV. So these two energies provide two calibration points.

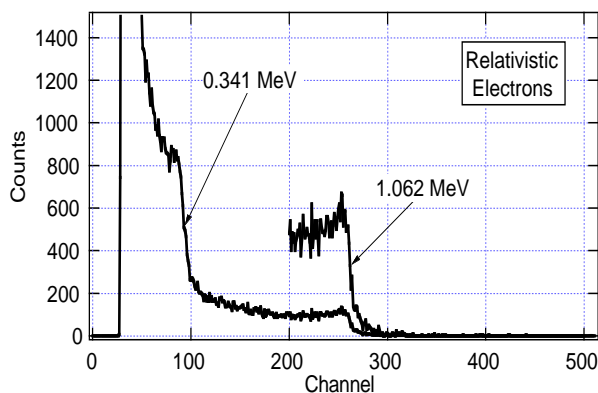


Figure 1: Energy spectrum of ^{22}Na .

To determine the channel number corresponding to the Compton edges, simply move the cursor on the screen until it is centrally located within the rise at each Compton edge as indicated by the arrows in Figure 1, and read off the channel number from the display. (Ignore the energy values — these will be from someone else’s calibration). Plot the calibration line on a graph of channel number against energy in keV so that electron energies may be read off.

When calibration is completed, remove the ^{22}Na source.

Note: If you take data over more than one session, you will need to carry out a calibration at the beginning of each session.

Measuring the kinetic energy T and the momentum p of electrons.

- (2) Measuring T and p is now very straightforward. Determine the channel number corresponding to the centre of the energy peak as a function of magnetic field. For each setting of the field a counting time of 100 s will give adequate statistics in the peak.

The magnetic field is measured by inserting the Hall probe into the hole just below the detector and moving it around so as to maximise the reading (rotation has the greatest effect). Since the trajectory of the electrons is partly in the fringe field regions of the pole pieces it is necessary to reduce this measured field by 4% in calculating p . You should zero the Hall Probe/Teslameter before starting measurements (make sure the probe is away from the magnet when you do this).

The quantities directly measured are the magnetic field strength B and the channel number of the peak. From B the momentum p may be calculated. From the channel number the kinetic energy T (keV) of the electrons may be determined from the calibration graph. Tabulate these quantities as well as the corresponding derived quantities E (total energy), E^2 , v/c and m/m_0 . The expressions for

calculating these quantities are:

$$p \text{ (keV/c)} = \frac{B \text{ (Tesla)}}{0.741 \times 10^{-4}} \quad (1)$$

$$E \text{ (keV)} = T \text{ (keV)} + 511 \quad (2)$$

$$E^2 = E_0^2 + p^2 c^2 \quad (3)$$

$$\frac{v}{c} = \sqrt{1 - \left(\frac{E_0}{E}\right)^2} \quad (4)$$

$$\frac{m}{m_0} = \frac{E}{E_0} \quad (5)$$

Graphs

- (3) Plot total energy in keV versus momentum in keV/c. On this graph the theoretical line for massless particles, $p = E/c$, should be shown as well as the relativistic calculation:

$$E = \sqrt{E_0^2 + p^2 c^2}$$

using the electron rest mass energy $E = 511$ keV. For comparison plot also the classical curve:

$$E = E_0 + \frac{p^2}{2m}$$

- (4) Plot the experimental values of E^2 versus p^2 . Include error bars, and use the **regress** function in PYTHON to determine E_0 and its associated error. Calculate chi-squared and provide a plot of the data with its errors and the best fitting line. Comment on this result.
- (5) Plot m/m_0 against v/c .

Focussing action of 180° spectrometer.

Two extreme trajectories for the apparatus are shown in Figure 2. Although these trajectories are diverging from the source it may be shown that they converge again in the vicinity of the detector.

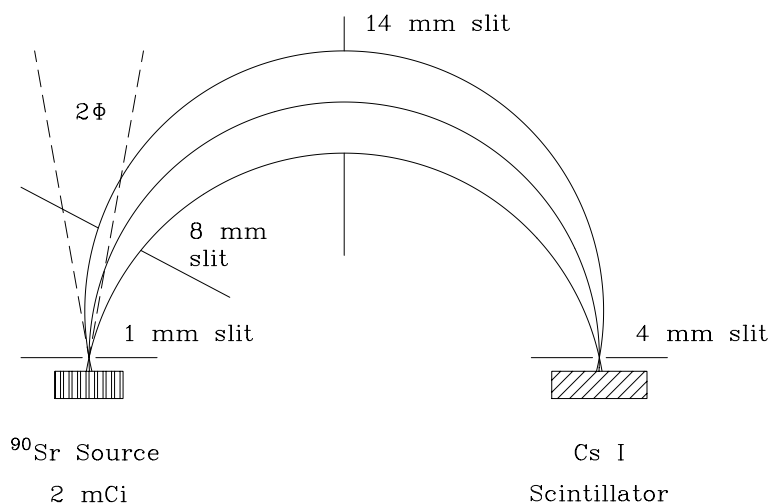


Figure 2: Spectrometer trajectories

If the angular spread at the source is 2ϕ and the source width is s , it can be shown that the image of the source at the detector has a width Δx given by:

$$\Delta x = 2\rho(1 - \cos\phi) + s \quad (6)$$

where ρ is the radius of curvature.

(6) For this apparatus:

$$\rho = 45 \text{ mm}$$

$$s = 1 \text{ mm}$$

$$\phi = 0.16 \text{ radians}$$

The image width Δx corresponds to a momentum change Δp . The resolving power is then defined as:

$$R = \frac{p}{\Delta p} = \frac{\rho}{\Delta\rho} = \frac{2\rho}{\Delta x} = \frac{2\rho}{s + \rho\phi^2} \quad (7)$$

As part of the experiment you should draw some electron trajectories (on graph paper) with a compass and verify by “experimental geometry” the existence of the 180° focussing action. Measure Δx and compare with equation(6). (You could, as an alternative, use PYTHON).

Questions

The answers to the following questions are a necessary part of the write-up.

1. Using the known dimensions of the apparatus, derive equation (1).
2. Starting from the equations:

$$m = \gamma m_0 \quad \text{and} \quad E = \gamma m_0 c^2$$

where:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

derive equations (4) and (5).

3. Show that equation (7) follows from equation (6).
4. In the calibration procedure, why is it unnecessary to place the ^{22}Na source inside the apparatus?

List of Equipment

1. Vacuum pump
2. Preamplifier
3. Magnet power supply
4. Beta-ray spectrometer
5. Supply for detector bias
6. Si-Li semiconductor detector
7. ORTEC model 435 main amplifier
8. Canberra Multiport II MCA

9. Group 3 Digital Teslameter for measuring magnetic field.

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(Updated by I. C. Barnett, October 2007)

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APPENDIX

Using the 'Gamma Acquisition' Software A Rough Guide to PHA Mode (Pulse Height Analysis)

Basic Features/Getting Started

Note: The maximum input pulse height that can be processed is a little less than 8V. Make sure the maximum height of the pulses you are interested in is a little less than this.

Finding the Software

You should find an icon called 'Gamma Acquisition' on the Windows desktop.

The Data Acquisition Window

The window you will usually be looking at is shown in Figure 3.

Things to note:

- **Energy Spectrum** is a histogram showing number of counts versus channel number (a number which is proportional to the height of the pulse, and, therefore, energy). In other words, each channel corresponds to a small range of energies (an energy bin) and the software plots a histogram showing how many times it received a pulse falling within that range.

The number of bins/channels can be selected (see 'Changing the Number of Channels' below). In Figure 3 there are 512 channels.

- **Log scales** are good for seeing small peaks when large ones are present, i.e. they emphasise small scale features.
- **Channel:** the channel number where the cursor is. The energy value should be disregarded unless **you** have recently calibrated the energy scale. Note that channel number is (should be) linearly related to pulse height and, thus, energy.

The energy calibration facility should generally not be used as mistakes are difficult to correct and you have no record of the accuracy of your calibration. If you really want to use it, read the help on 'Energy Only Calibration' — make sure you still record channel numbers, just in case.

- **Counts:** The number of counts in the channel where the cursor is, i.e. the number of times a pulse of that height (energy) was detected.

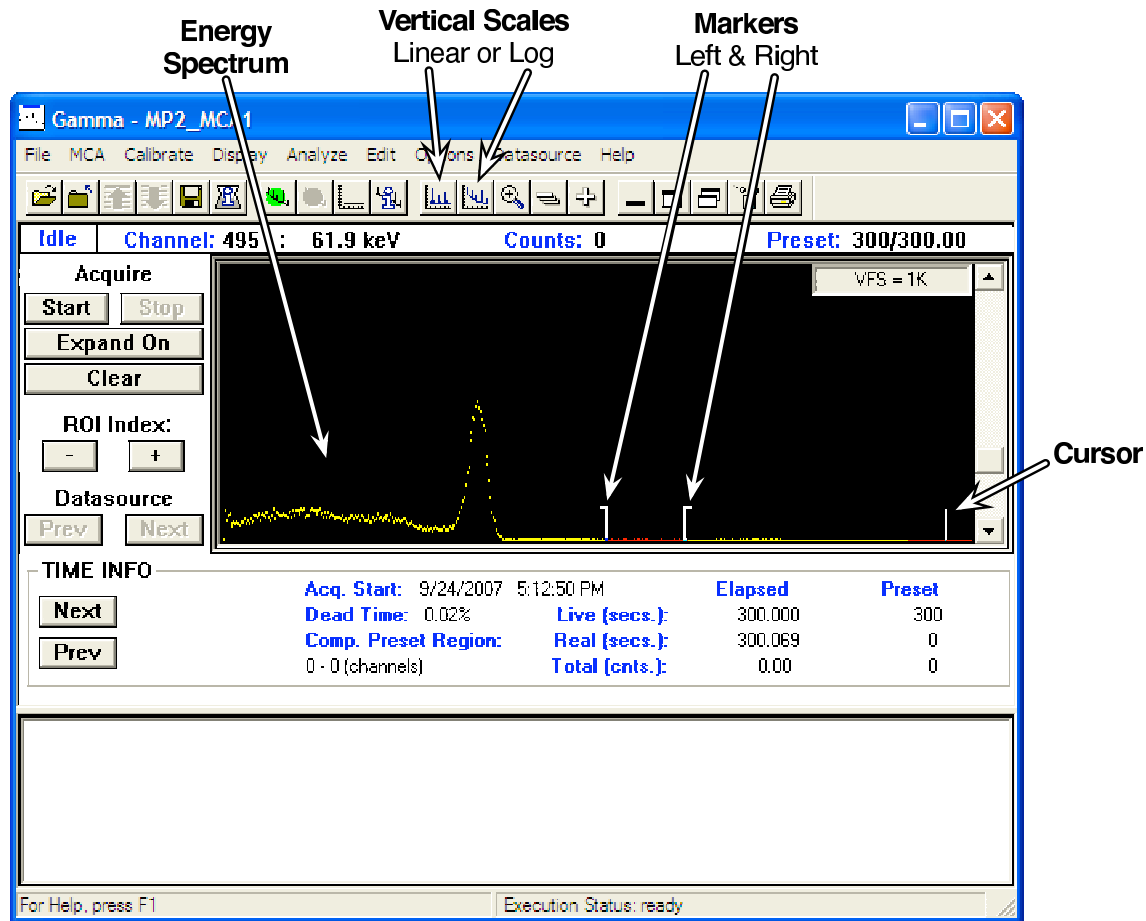


Figure 3: Data Acquisition program window.

- **Preset:** The preset time for which data collection will run, after which it will automatically stop. The display also indicates how far through this time it is. In this case, it has completed. (See ‘Preset Times’ for more information.)
- **Acquire:** Button controls for starting and stopping data acquisition, clearing the energy spectrum, and enabling you to expand a portion of the display (selectable using the mouse) for closer inspection.
- **ROI Index:** Button controls which enable you to cycle the markers through the defined Regions Of Interest (ROIs). ROIs have their channels coloured red instead of yellow. (See ‘Using ROIs’ for more information.)
- **TIME INFO:** This portion of the screen can display various collections of information, cycled through by clicking the Next and Prev buttons. In this case it shows information related to time. Note that ‘Live’ time excludes Dead Time, i.e. it is the actual number of seconds during which data was actively collected (300, in this case). ‘Real’ time indicates how long data acquisition has been active, i.e. it equals ‘Live’ plus ‘Dead’ time, (300.069s, in this case, This indicates the system was unable to respond to incoming signals for 0.069s.)

The other collection of interest is **MARKER INFO**. This tells you, among other things, the total number of counts (Integral) in the channels between the markers. *Do not use Area as this includes an attempt to subtract background via a method not applicable to the experiments in this laboratory.*

- The **cursor** can be moved using the left and right arrows, page up and page down keys, and by clicking the mouse on/near the channel of interest.
- **Markers** can be moved by picking them up with the mouse (the mouse indicator will change to signify you are on a marker) or, if ROIs are set up, by pressing the +/- buttons under ROI Index (see ‘Using ROIs’ below).

Enabling the Collection of Data

In order to collect data, you must open a detector data source:

- Select File – Open Datasource ... from the menus.
- Click the Source: Detector radio button.
- Select MP2_MCA1 and then click Open.

The energy spectrum will now display the last data collected.

You can now use the buttons under Acquire to collect data.

Advanced Features

Changing the Number of Channels

Although the ADC (Analogue to Digital Converter) is capable of digitising pulses (max. height just below 8V) into 16382 channels, this level of resolution is seldom required. If you wish to change the number of channels, do the following:

1. Select the number of channels you require. This can be found under the menu item MCA – Adjust. If you are not there already, use the Next/Prev buttons until you find the window shown in Figure 4. Select the Conv. Gain (Conversion Gain) you require.

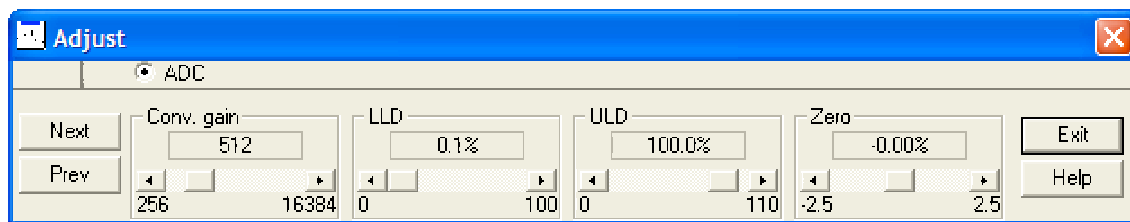


Figure 4: Changing the conversion gain.

2. Ensure that the input size is at least as large as the number of channels you have selected. This can be found under the menu item MCA – Acquire Setup.

Note: MCA stands for Multi-Channel Analyser, a term describing all of the functions of this hardware/software combination.

Setting Up The Preset Acquisition Time

It is sometimes useful to be able to collect data for a known, repeatable, amount of time. To set this up, choose MCA – Acquire Setup from the menu. You can now modify the settings in the Time Preset box.

Saving Energy Spectra

As always, it is a good idea to save your data, just in case something goes wrong.

There are two file types which you may find useful: *.CNF (CAM Files) which can be loaded back into this program, and *.TKA (Toolkit Files) which are text files and can be used in MATLAB.

Using ROIs (Regions Of Interest)

Regions of interest are useful because they enable you to set up a group of channels which are both highlighted (they display in red instead of yellow) and easily selected and reselected for obtaining information such as the integral of counts in that region. Thus, for example, if you wish to obtain the total number of counts in a peak of a particular energy over several measurements, setting up a ROI is a good way to ensure you always select the same number of channels over which to integrate. They may also help you to detect gain shifts, i.e. shifts in the amplification of the pulses. (Expt 352: Interaction of γ Rays with Matter is an experiment for which ROIs should be used.)

The following are the commands you will require:

- **Delete All ROIs:** Select Display – ROIs – Clear All from the menus.
- **Delete One ROI:** Place the cursor within the ROI and press the <Delete> key.
- **Create a ROI:** Place the left and right markers on either side of the region and press the <Insert> key. If it took effect, the region will change colour.
- **Finding the counts integral:** Select the ROI by using the ROI Index buttons until the markers are either side of it. Then, if necessary, use the Next/Prev buttons to select the MARKER INFO *window*.

Note: Do not use Area as this includes an attempt to subtract background via a method not applicable to the experiments in this laboratory.

Note that ROIs can overlap. The only way to find out where they are is to use the ROI Index buttons to cycle the markers through the current ROIs.

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