Experiment 238: Spatial Fourier transform

Aim

In this experiment you will acquire a basic understanding of spatial Fourier transforms and how you can realize Fourier transformations in optics.

References

1. S. Jutamulia and T. Asakura: "Optical Fourier-transform theory based on geometrical optics", Opt. Eng. **41**(1), pg. 13-16, (2002)
2. S. Lehar: "An Intuitive Explanation of Fourier Theory", `http://sharp.bu.edu/~fourier/fourier.html`
3. E. Hecht: "Optics", Addison Wesley

Background

Fourier theory is probably one of the most important mathematical tools in signal processing today. Fourier transformations are used in applications ranging from cellphones and mp3 players to Magnetic Resonance Imaging. In particular the spatial Fourier transform has proven to be an indispensable tool in image processing, for everything from image enhancements to pattern recognition. However many people often find Fourier theory quite difficult when they first encounter the mathematical formulation. However, it is possible to gain some very intuitive insights into Fourier transforms and signal processing by looking at the spatial Fourier transform.

Simply speaking the Fourier theorem states, that any function can be expressed as a superposition or sum of sinusoidal functions. That means we can write any (continuous) function by simply writing a sum of sine and cosine functions. Thus any function can either be described by its behaviour in time (the so-called time domain) or by its frequency composition (the frequency domain). The time and frequency domain of a simple sine wave is depicted in Figure 1 and 2 respectively. The transformation between the two domains

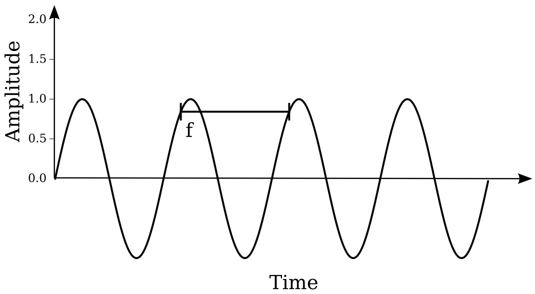

Figure 1: Time domain

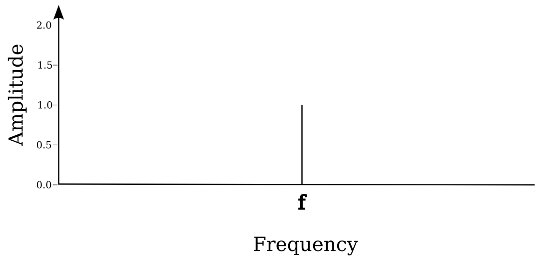

Figure 2: Frequency domain

is called the Fourier transform:

$$\tilde{\mathcal{F}}(\omega) = \int_{-\infty}^{\infty} F(t) e^{\mathrm{i}\omega t} \mathrm{d}t \quad (1)$$

$$F(t) = \int_{-\infty}^{\infty} \tilde{\mathcal{F}}(\omega) e^{-\mathrm{i}\omega t} \mathrm{d}\omega \,. \quad (2)$$

The above can easily be extended into two dimensions and be rewritten as a transformation between the spatial dimensions $\vec{x}$ and spatial frequency dimensions $\vec{k}$.

$$\tilde{\mathcal{F}}(\vec{k}) = \int_{-\infty}^{\infty} F(\vec{x}) e^{\mathrm{i}\vec{k}\vec{x}} \mathrm{d}\vec{x} \tag{3}$$

$$F(\vec{x}) = \int_{-\infty}^{\infty} \tilde{\mathcal{F}}(\vec{k}) e^{-\mathrm{i}\vec{k}\vec{x}} \mathrm{d}\vec{k}\,. \tag{4}$$

A two dimensional spatial function is just a mathematical formulation of an image. We can visualize the function by a grayscale image. Every point (position) of the image has a brightness value corresponding to the value of the function. Vis-versa, we can formulate every image by a two dimensional function. Thus we have the means to calculate the Fourier transform of any given image.

Let us have a closer look at the two dimensional Fourier transform. Figure 3 shows an image depicting a simple sinusoid along the horizontal axis and Fig. 4 shows its Fourier transform. The Fourier transform con-

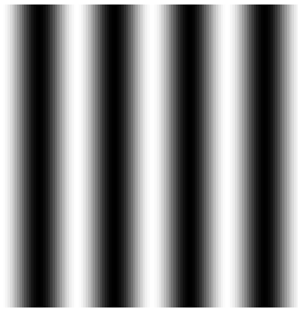

Figure 3: A horizontal sinusoid

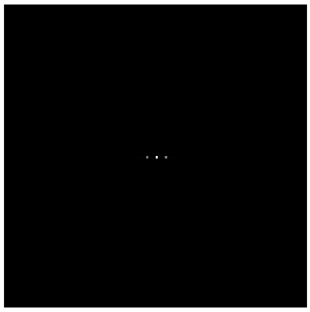

Figure 4: The Fourier transform of the sinusoid

sists of three dots, one in the middle and the other two to its left and right respectively. Out of mathematical reasons, beyond the scope of this manual, the Fourier transform is symmetrical around the origin. Thus the point in middle of the Fourier transform image is the "DC term" which is at zero frequency. The two points two the left and right thus correspond to the frequency of the variation, just like in Fig. 2. The point at the zero frequency (the DC term) corresponds to the average brightness of the image. Can you explain why there is no DC term in Fig. 2?

Experiment

The spatial Fourier transform experiment is divided into two parts. A computer based experiment and an optical experiment. Each one is designed for one lab session and you can begin with either one.

Computer Fourier filtering

ImageFFT is a program written to help you understand the concept of Fourier transforms and to give you a feel for the use of Fourier filtering in image manipulation. It allows you to open and manipulate images using filters in the Fourier domain.

Interface

The ImageFFT software interface is shown in figure 5. Although the appearance of the program might vary slightly on different operating systems, the layout will be the same. The window is divided into two main panels. The upper panel contains the currently open image, its Fourier transform and the filtered image obtained from the filtered Fourier transform. Initially these will be empty until you open an image. The

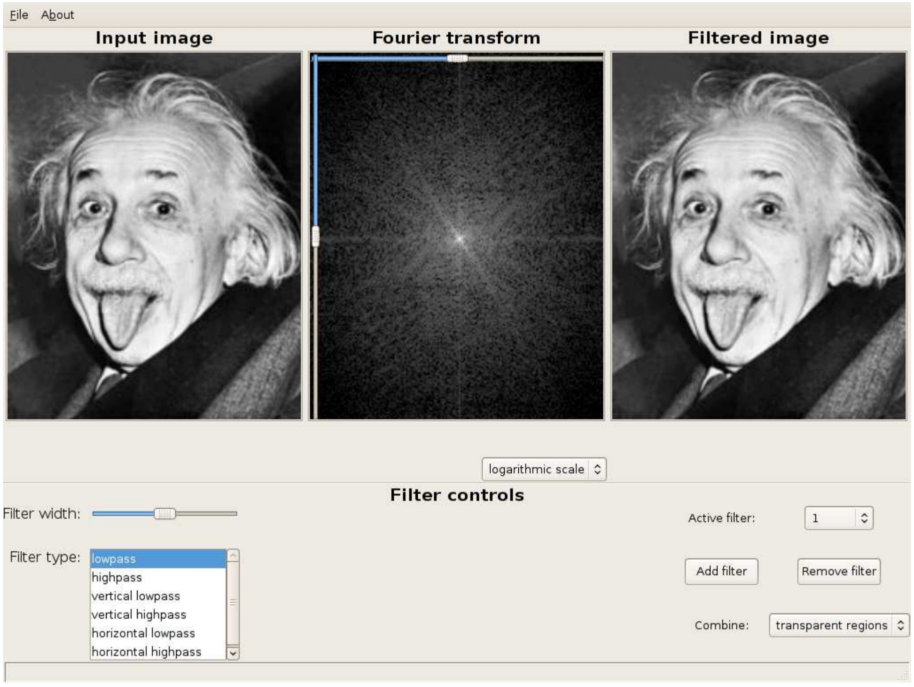

Figure 5: The ImageFFT interface

lower panel contains the filter controls. Finally at the top you will find a menu which lets you open and save images. Currently the program supports PNG (Portable Network Graphics) and JPEG format images. The images reside in `C:\Program Files\ImageFFT\Images`.

Image Panel The image panel consists of three different frames. The input image, the Fourier transform and the filtered image. Once you open an image, the Fourier transform and the output image are calculated automatically. You can switch the scale of the Fourier transform between a linear and a logarithmic scale. Generally for most of your exercises you probably want to use the logarithmic scale, at it makes the higher order modes more easily distinguishable. It is also similar to the way the human eye behaves. However it is important to remember that we are dealing with a computer. Thus the Fourier transform is applied to a discrete (digital) image, that means it will contain components which are relics of the discretization. Additionally we are applying the Fourier transform to a finite image, although ideally it would be infinite. Thus we might see additional Fourier components. The use of a logarithmic scale makes these more apparent, so you might want to switch to to the linear scale in some cases.

You can save the Fourier transform image and the filtered image by choosing the appropriate actions in the menu.

Filter control The lower part of the screen is made up by the controls for the different Fourier filters. The box on the left-hand side allows you to choose between the different filters, and the slider allows you to control the width of the filter. The filter possibilities are high- and lowpass filters, as well as horizontal and vertical high- and lowpass filters. A lowpass filter blocks all spatial frequencies higher than a threshold value, while a highpass filter will block all frequencies below the threshold. The vertical and horizontal filters act the same way but in only one spatial dimension.

Once you have chosen a filter, a red region will appear superimposed on the Fourier transform. You can control the width of the filter by moving the width slider, and the position of the filter by using the two sliders in the Fourier transform panel.

It is also possible to apply more than one filter, on the right-hand side of the filter control panel you will find control which will allow you add more than one filter. There is two ways of superpositioning the filters.

Either the blocked (opaque) regions are added, or the transparent regions are added. Play with the filter controls using several filters until you are sure that you understand the difference.

Exercises

First open a few of the images, get familiar with the program interface and the effects of the different filters. Let's have a look at some basic Fourier transforms. Open the images `stripes1.png - stripes5.png` and examine the Fourier transforms. Answer the following questions:

(1) If you have a sinusoidal variation along one image dimension, how is the direction of the variation reflected in the Fourier transform?

(2) How does the frequency of the variation effect the Fourier transform?

(3) What does the Fourier transform of the beating of two or more waves look like? Explain!

(4) What happens when you filter out the DC component of an image?

(5) How many different elemental sinusoids can you produce by filtering `stripes5.png`. For every one save the Fourier transform with its filter, and the resulting image.

Now you should be ready to perform some filtering operations on some "real" images:

(6) Open the image `einstein_horizontal_stripes.png`. Remove the stripes from the image without significantly altering its appearance. Save the Fourier transform & filter, and the output image.

(7) Open the image `einstein_vertical_stripes.png`. Remove the stripes from the image without significantly altering its appearance. Save the Fourier transform & filter, and the output image.

(8) Open the image `einstein.png` and create a relief type image (an image which shows only the sharp edges). Save the filter and output.

(9) Answer the following questions:
   - Describe what happens if you apply a lowpass filter to an image.
   - Describe what happens if you apply a highpass filter to an image.
   - What image features correspond to the high spatial frequencies?
   - What is the Fourier transform of a periodic feature in an image. How can you distinguish this feature from a similar feature with a longer period?

(10) Finally open the file `gaussian.png`. It shows a two-dimensional Gaussian with some additional noise. Remove the noise, so that your output image shows a smooth Gaussian. Explain how this could be useful in free-space optics and how it can be implemented experimentally.

Optical Fourier Transform

One intriguing aspect of free space optics is, that it can very easily produce the Fourier transform of an image in real-time, all we need is a lens. If we place an image, such as a slide or transparency into the focal plane of a lens and illuminate the slide with a coherent, collimated light beam (as coming from a laser), the lens will produce the Fourier transform in its other focal plane (see Fig. 6). To perform the inverse transform, we simply do the reverse (see Fig. 7).

In order to understand the mechanism behind the optical Fourier transform, we have to know that we can regard every point of the image in the focal plane as a point source. The light from each of these sources will follow a slightly different path and thus will have a slightly different path length from the other. If the light is coherent, the light from the sources will interfere constructive and destructively. By doing the maths we can see that the resulting interference pattern is the Fourier transform of the image (see Ref. 1 for the mathematical proof).

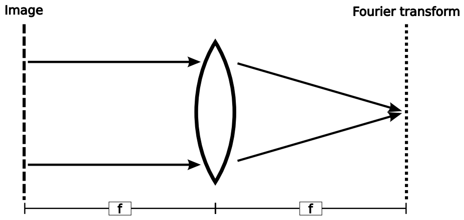

Figure 6: Producing the optical Fourier transform

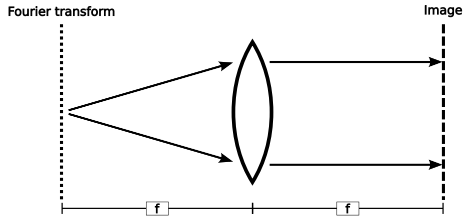

Figure 7: Producing the inverse optical Fourier transform

Being able to produce the forward and backward Fourier transform (the normal and inverse transform) enables us to to perform Fourier filtering operations by simply placing filters such as pinholes, slits etc. in the focal plane between two lenses. At the end of this experiment you should be able to perform some basic Fourier filtering operations, and have gained some experience in basic optical experimental techniques.

Safety guidelines

You will be using a HeNe laser. Although the optical power of this laser is relatively low (0.8 mW), and can therefore be considered “safe”, it is still important to observe some basic rules when working with a laser:

- Do not look directly into the collimated beam. Although your natural reflex to close your eyelid should protect you from any damage, it is not worth the try.
- In particular do not look into the beam at the or close to the focal spot of the beam. Although the power of the beam is considered to small to be able to damage your eye before your eyelid shuts, this might not be the case when it is focused and the intensity (power per unit area) is a lot higher.
- Do not place any highly reflective objects (metallic objects, mirrors etc.) into the propagation path unless you are sure where the reflection goes. Before placing any mirrors etc. into the path, block the beam and unblock it once you have placed the mirror. In particular be careful with watches as the metallic wristbands or the displays might reflect the beam when you are working. It is generally considered better to take these items off when working with lasers.

Experimental setup

At first think about what you want to achieve in this experiment. You want to encode some image onto a beam, perform a Fourier transformation, do some filtering in the Fourier domain and finally observe the image after a reverse Fourier transformation. The given equipment is,

1. 1 optical rail $L = 1\,\mathrm{m}$
2. 1 HeNe laser with $P = 0.8\,\mathrm{mW}$ and $\lambda = 633\,\mathrm{nm}$ on rail mount
3. 1 lens $f = 10\,\mathrm{mm}$ mounted
4. 1 lens $f = 200\,\mathrm{mm}$ mounted
5. 2 lenses $f = 750\,\mathrm{mm}$ mounted
6. 1 mounted slide holder
7. 1 translation stage with translatable post
8. 1 screen
9. 1 variable iris

10. several image and filter slides
11. a transparency marker pen

You now need to design the experimental setup. First, in order to encode a significant portion of the image onto the beam we need to expand the beam. So that instead of the $0.5 - 1$ mm beam diameter from the HeNe laser, we have a beam with around 10 mm diameter.

You can expand a beam by a telescope system consisting of two lenses, a short focal length lens (f_s) and a long focal length lens (f_l). The expansion ratio is given by the lens ration f_l/f_s. Behind lens f_l the beam should be collimated again. This means, it should not vary significantly in diameter during propagation, this is equivalent to its focal spot being at infinity. To achieve this place the two lenses a distance of $f_s + f_l$ apart, however make some adjustments, by shining the light at some distant surface and observing the beam diameter when moving the lens. The diameter at the distant surface should be approximately the same as just behind the second lens, and there should be no focal point in between.

Choose two lenses of appropriate focal length and set up the beam expansion directly behind the laser. To estimate the focal length of a lens by use the ceiling lights or one of the desk lights to find the focal point, the distance between lens and focal point is the focal lens. You can now place the image slide into the beam path and adjust the slide position so that the image is centred to the beam. The full image should be clearly visible if you place the screen into the beam path.

To perform the actual Fourier transform, choose one of the remaining lenses and place it in the beam path. As the extent of the Fourier transform is proportional to the focal length, you want to choose the longest focal length possible. You should be able to see the different spatial frequencies of the image in the focal plane of the lens. Now perform the inverse transform. It is done in exactly the same way as the previous transform, so take the last lens, and place it at the correct position. Place the screen into the correct position behind the lens, so that you can see the image sharp and clear. Remember that you want to filter the image, so you should place a filter mount in the Fourier plane.

Experiments and questions

Before performing any experiments draw a sketch of your experimental setup. This sketch should include the different focal lengths of the lenses and the separation between the different components. You should also indicate the beam in your drawing, especially if it is converging, diverging or collimated. Document your work, during your experiments. You can use any of the filters, and there is also a transparency marker and a clear slide to make your own filters, but please do not write on any of the other slides. Also, Make sure to write down which filters and images you used.

(11) Choose one of the slides with a grid pattern. Observe the image on the screen, you should be able to clearly see the grid pattern. Now choose the appropriate filter and try to eliminate the vertical lines from the pattern.

(12) Take a different grid image and try to eliminate the horizontal lines.

(13) Choose the slide depicting Albert Einstein. Try to create a relief type image, i.e. an image with only the edges showing.

(14) Finally take the Einstein image with superimposed horizontal lines. Try to eliminate the lines from this image.

Comparison

Once you have finished both parts of the experiment, please answer the following questions:

(15) Please compare the two methods of Fourier filtering, what are the advantages, what are the disadvantages?

(16) In what situations would you use computer based Fourier transformations?

(17) What are the situations where it would be advantageous to use the optical based Fourier setup. Can you think about applications?

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