Experiment 385: Frequency Modulation and Demodulation

Aim

To demonstrate the principles of frequency and phase modulation and demodulation.

Outline of Angle Modulation

In angle modulation, the general form of the modulated signal is $f(t) = A\cos\theta(t)$, where $\theta(t)$ is made to depend on the modulating (or message) signal $m(t)$. The **instantaneous frequency** $\nu(t)$ is defined to be

$$\nu(t) = \frac{1}{2\pi}\frac{\mathrm{d}\theta}{\mathrm{d}t},$$

so that for an unmodulated carrier signal of frequency ν_c, we have $\theta(t) = 2\pi\nu_c t + \phi$, and the instantaneous frequency is ν_c at all times. If the instantaneous frequency of the modulated signal is made to vary with the message signal according to

$$\nu(t) = \nu_c + k_f m(t),$$

for some constant k_f,the result is called **frequency modulation** (FM), whereas if vary the angle term according to

$$\theta(t) = 2\pi\nu_c t + k_p m(t)$$

for some constant k_p, the result is called **phase modulation** (PM). We thus obtain the following forms for the modulated signal:

$$f_{\mathrm{FM}}(t) = A\cos\left[2\pi\nu_c t + 2\pi k_f\int^t m(\tau)\,\mathrm{d}\tau\right]$$
$$f_{\mathrm{PM}}(t) = A\cos\left[2\pi\nu_c t + k_p m(t)\right].$$

The two forms of modulation are seen to be closely related, and are collectively referred to as **angle modulation**. Frequency modulation with $m(t)$ is equivalent to phase modulation with $\int^t m(\tau)\,\mathrm{d}\tau$, and conversely, phase modulation with $m(t)$ is equivalent to frequency modulation with $m'(t)$. Thus a modulator designed to perform either form of modulation may be used for the other with suitable processing of the message signal.

In this experiment, we shall mainly be concerned with the case in which the message signal is sinusoidal, as the general case is not easily analyzed. Setting $m(t) = B\cos 2\pi\nu_m t$ yields the following modulated signals:

1. For FM,

   $$f_{\mathrm{FM}}(t) = A\cos\left[2\pi\nu_c t + \frac{k_f B}{\nu_m}\sin 2\pi\nu_m t\right] = A\cos\left[2\pi\nu_c t + \frac{\Delta\nu}{\nu_m}\sin 2\pi\nu_m t\right]$$

   where $\Delta\nu = k_f B$. The instantaneous frequency varies between $\nu_c - \Delta\nu$ and $\nu_c + \Delta\nu$, so that $\Delta\nu$ is referred to as the **maximum frequency deviation**. We define the **modulation index** $\beta = \Delta\nu/\nu_m$, so that

   $$f_{\mathrm{FM}}(t) = A\cos\left[2\pi\nu_c t + \beta\sin 2\pi\nu_m t\right].$$

   For FM, the modulation index is directly proportional to the message amplitude, and inversely proportional to the message frequency.

2. For PM,

   $$f_{\mathrm{PM}}(t) = A\cos\left[2\pi\nu_c t + k_p B\cos 2\pi\nu_m t\right],$$

   and we define the modulation index $\beta = k_p B$, so that this can be written in a form similar to the above, i.e.,

   $$f_{\mathrm{PM}}(t) = A\cos\left[2\pi\nu_c t + \beta\cos 2\pi\nu_m t\right].$$

   For PM, the modulation index is directly proportional to the message amplitude, but is independent of the message frequency. The maximum frequency deviation for PM is $\Delta\nu = k_p B\nu_m$.

Narrow-band FM and PM

From the expressions for the modulated signal $f(t)$, it is clear that the modulation index β may be interpreted as the **maximum phase deviation**. An important special case arises when the maximum phase deviation is much less than one radian, since it is then possible to make several approximations. The resulting forms are known as **narrow-band** FM and PM respectively. Considering the case of narrow-band PM, we see that

$$\begin{aligned} f_{\mathrm{NBPM}}(t) &= A\cos\left[2\pi\nu_c t + k_p m(t)\right] = A\left\{\cos(2\pi\nu_c t)\cos\left[k_p m(t)\right] - \sin(2\pi\nu_c t)\sin\left[k_p m(t)\right]\right\} \\ &\approx A\cos(2\pi\nu_c t) - Ak_p m(t)\sin(2\pi\nu_c t), \end{aligned}$$

where we have set $\cos\left[k_p m(t)\right] \approx 1$ and $\sin\left[k_p m(t)\right] \approx k_p m(t)$, since the maximum phase deviation is small. We see that f_{NBPM} may be regarded as the sum of two terms:

(a) A **double sideband** (DSB) modulated signal

$$-Ak_p m(t)\sin(2\pi\nu_c t),$$

which is essentially the product of the message signal $m(t)$ and a carrier of frequency ν_c, and

(b) The unmodulated **carrier** signal

$$A\cos(2\pi\nu_c t).$$

In the AM experiment, we learned that the sum of a DSB signal and a carrier is just an AM signal. However, for angle modulation, there is an important difference. Instead of the added carrier being of the **same phase** as the carrier used to produce the DSB signal (via multiplication by the message $m(t)$), the added carrier for narrow-band angle modulation is 90° out of phase**.** Thus for the NBPM example above, the carrier used to produce the DSB signal is $-\sin(2\pi\nu_c t)$, whereas the added carrier is proportional to $\cos(2\pi\nu_c t)$.

A block diagram of a narrow-band phase modulator based on these ideas is shown in figure. The value of k_p is set by changing the amplitude of the DSB signal relative to the added carrier. This forms the basis of the **Armstrong modulator** which we shall examine later in the experiment.

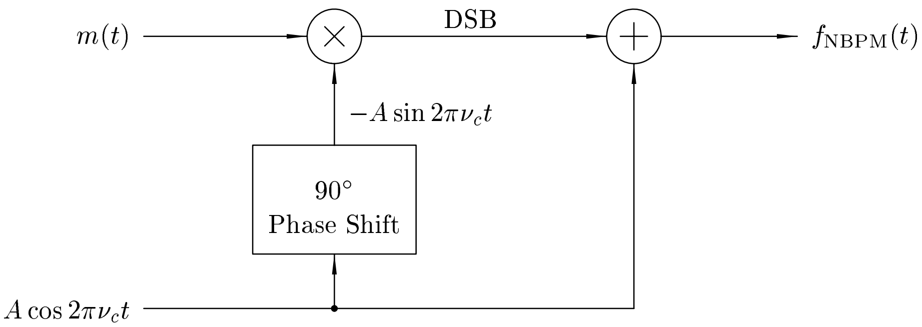

Figure 1: Narrow-band phase modulator

The FM Spectrum

When the message signal is sinusoidal, it is possible to obtain analytic expressions for the spectrum of the PM or FM signal. This involves writing the modulated signal as a sum of discrete frequency components with the help of the identities

$$\begin{aligned} \cos(\beta\sin\omega t) &= \mathrm{J}_0(\beta) + 2\sum_{k=1}^{\infty}\mathrm{J}_{2k}(\beta)\cos(2k\omega t) \\ \sin(\beta\sin\omega t) &= 2\sum_{k=1}^{\infty}\mathrm{J}_{2k-1}(\beta)\sin\left[(2k-1)\omega t\right], \end{aligned}$$

where $\mathrm{J}_k(\beta)$ denotes the k'th order Bessel function of the first kind, which may be written in various ways, such as

$$\mathrm{J}_k(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m (\beta/2)^{2m+k}}{m!\,(m+k)!} = \frac{1}{\pi} \int_0^{\pi} \cos(\beta \sin\theta - k\theta)\, \mathrm{d}\theta.$$

We may thus write the sinusoidally modulated FM signal as

$$\begin{aligned}
f_{\mathrm{FM}}(t) &= A\cos(2\pi\nu_c t + \beta \sin 2\pi\nu_m t) \\
&= A\cos 2\pi\nu_c t \cos(\beta \sin 2\pi\nu_m t) - A \sin 2\pi\nu_c t \sin(\beta \sin 2\pi\nu_m t) \\
&= A\{\mathrm{J}_0(\beta)\cos 2\pi\nu_c t + \\
&\quad \mathrm{J}_1(\beta)\left[\cos 2\pi(\nu_c + \nu_m)t - \cos 2\pi(\nu_c - \nu_m)t\right] + \\
&\quad \mathrm{J}_2(\beta)\left[\cos 2\pi(\nu_c + 2\nu_m)t + \cos 2\pi(\nu_c - 2\nu_m)t\right] + \\
&\quad \mathrm{J}_3(\beta)\left[\cos 2\pi(\nu_c + 3\nu_m)t - \cos 2\pi(\nu_c - 3\nu_m)t\right] + \\
&\quad \cdots\} \\
&= A\left\{\mathrm{J}_0(\beta)\cos 2\pi\nu_c t + \sum_{k=1}^{\infty} \mathrm{J}_k(\beta)\left[\cos 2\pi(\nu_c + k\nu_m)t + (-1)^k \cos 2\pi(\nu_c - k\nu_m)t\right]\right\}.
\end{aligned}$$

Although the mathematical details are not important for this experiment, the final result gives us the desired FM spectrum for a sinusoidal message signal of frequency ν_m. The spectrum consists of an infinite number of sidebands centred about ν_c and separated by ν_m. The amplitudes are completely specified by β, the modulation index. If we ignore the signs of the various terms, the amplitude of the sideband pair of frequency $\nu_c \pm k\nu_m$ is simply $A\mathrm{J}_k(\beta)$. Extensive tables of the Bessel function have been compiled, and the function is called `besselj` in Matlab.

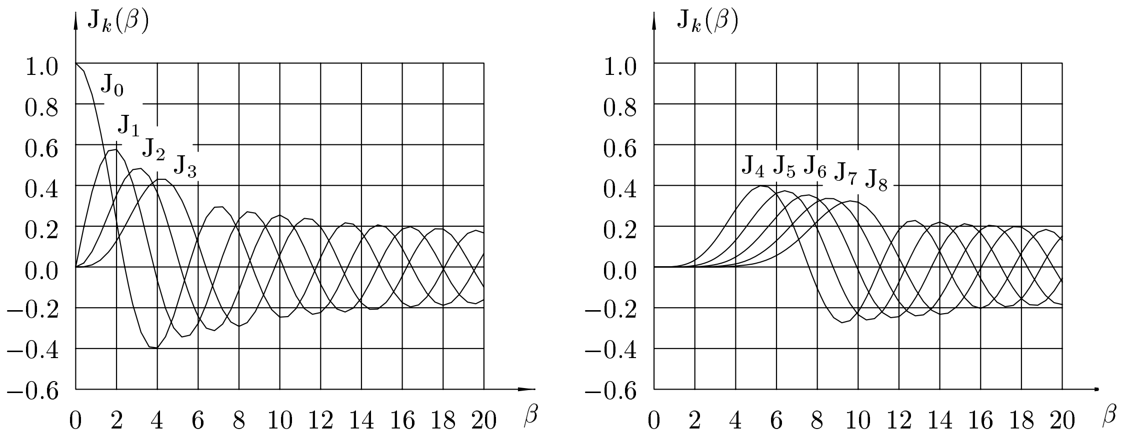

Figure 2: Graphs of some Bessel functions

As an example, if we want to find the sideband structure for $\beta = 1$, we see that $\mathrm{J}_0(1) = 0.77$, $\mathrm{J}_1(1) = 0.44$, $\mathrm{J}_2(1) = 0.11$, $\mathrm{J}_3(1) = 0.02$ and $\mathrm{J}_k(1) < 0.01$ for $k \geq 4$. Expressed in decibels relative to the amplitude of the unmodulated carrier, the carrier is at −2.3 dB and the first three sideband pairs are at −7.1 dB, −18.8 dB, −34.2 dB, as shown in figure 3.

We see that although there are theoretically an infinite number of sidebands, the amplitudes of those distant from ν_c are small. We can define the effective bandwidth of the signal by considering only those sidebands whose amplitudes are greater than 1% (−40 dB) of the unmodulated carrier amplitude. The following table shows the first value of β for which $\mathrm{J}_k(\beta) = 0.01$, so that effective bandwidths may be estimated.

First β value for which $\mathrm{J}_k(\beta) = 0.01$							
$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$
0.02	0.28	0.79	1.44	2.16	2.93	3.73	4.56

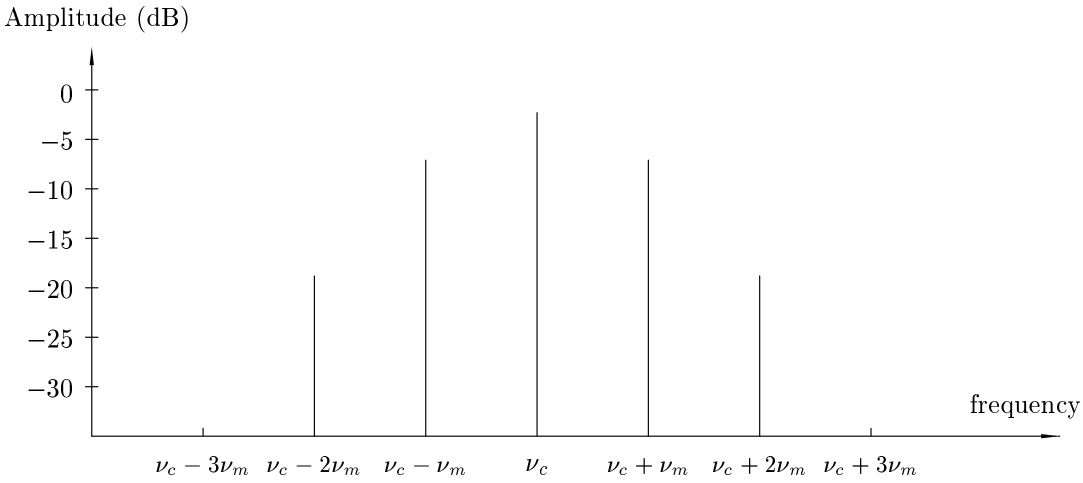

Figure 3: Sideband structure for $\beta = 1$

The Voltage Controlled Oscillator

A voltage controlled oscillator (VCO) is a device whose output frequency is dependent on an input control voltage. A common form of VCO is based on a multivibrator as shown in figure 4.

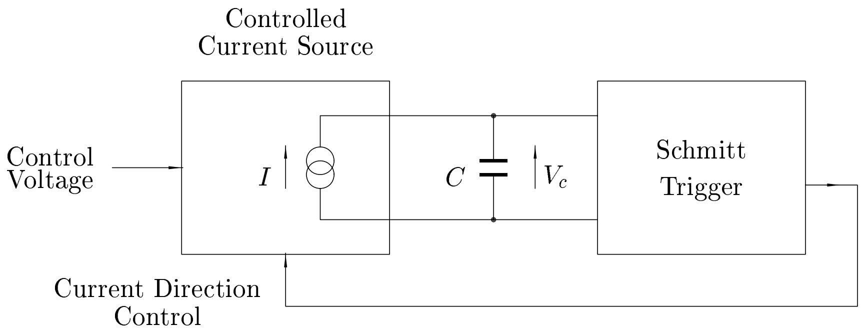

Figure 4: Multivibrator-based voltage controlled oscillator

The current source is designed to produce a current whose magnitude I is linearly related to the control voltage and whose direction (or sign) is controlled by the output of a Schmitt trigger. Suppose initially that the current is flowing in the direction indicated. The capacitor C then charges up so as to increase V_C at the rate $\mathrm{d}V_C/\mathrm{d}t = I/C$. This continues until V_C reaches the upper threshold voltage V_{tH} of the Schmitt trigger. At this time, the Schmitt trigger reverses the current direction so that C discharges at the rate $\mathrm{d}V_C/\mathrm{d}t = -I/C$. This continues until V_C reaches the lower threshold voltage V_{tL} whereupon the cycle repeats. It is easy to see that the oscillation frequency is given by

$$\nu = \frac{I}{2\,(V_{tH} - V_{tL})\,C},$$

which is linearly dependent on I (see figure 5).

Inexpensive VCO integrated circuits based on the above principles are readily available. It is often possible to vary the oscillation frequency over a wide range, and the variation of frequency with voltage is remarkably linear. A sinusoidal output formed by shaping the triangular voltage V_C is also often provided.

Unfortunately, multivibrator-based VCOs are not suitable for generating FM in modern communications systems, because of the relative instability of their centre frequencies. In a communications transmitter, the mean carrier frequency must typically be maintained to within a few parts per million or to a few hertz. We shall later consider alternative modulation techniques which can provide adequate stability. The VCO provided can directly produce a wide-band frequency modulated signal suitable for examining the FM spectrum.

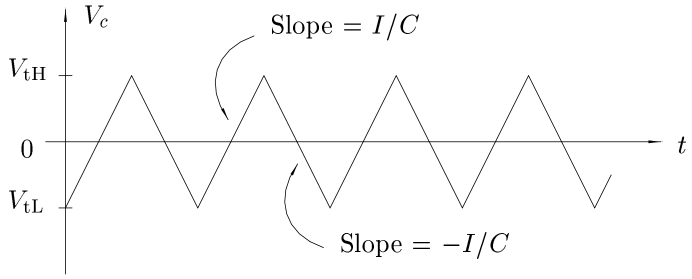

Figure 5: Multivibrator-based voltage controlled oscillator

Procedure

We first wish to set up a VCO module so that it generates a 100 kHz signal when the control voltage is zero, and has a deviation sensitivity k_f of $10\,\mathrm{kHz\,V^{-1}}$. In the setup shown in Figure 6, the control voltage of the VCO is obtained by adding together a DC voltage from the VARIABLE DC module and an audio signal from the AUDIO OSCILLATOR module. The ADDER module has two gain controls labelled G and g which enable the relative proportions of the two signal sources to be adjusted.

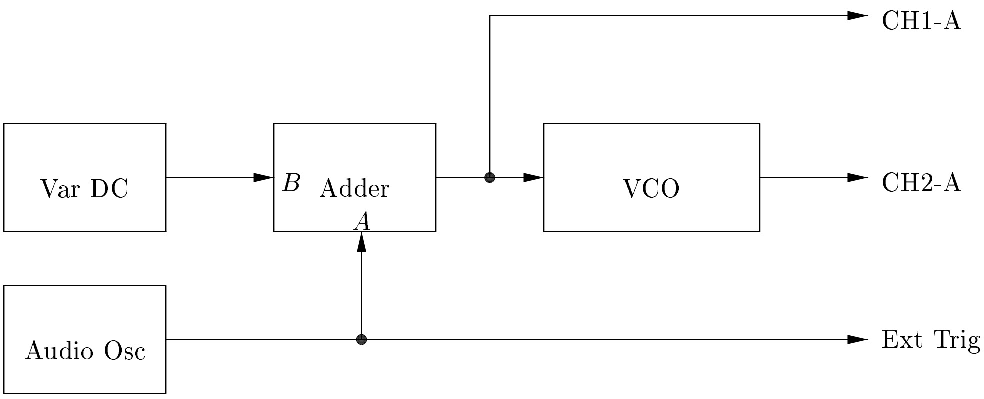

Figure 6: FM modulator using a VCO

(1) Connect up the modules shown in Figure 6. Ensure that the switch SW2 on the circuit board of the VCO module is in the VCO position. Set the front panel switch of the VCO to HI and rotate the front panel GAIN control fully anticlockwise. Monitor the VCO output frequency using the FREQUENCY COUNTER module and adjust the f_0 control so that the output is precisely 100 kHz.

(2) Turn the gain control G (for channel A of the ADDER) fully anticlockwise, and gain control g to approximately mid-scale. While monitoring the input to the VCO on the oscilloscope (CH1A), adjust the voltage from the VARIABLE DC module so that the input to the VCO is exactly $-1.0\,\mathrm{V}$. With the GAIN control of the VCO set fully anticlockwise, this should have no effect on the VCO frequency. Increase the GAIN control from zero until the frequency changes by 10 kHz, thus setting the VCO sensitivity $|k_f|$ to $10\,\mathrm{kHz\,V^{-1}}$. What is the sign of k_f for the VCO module? Subsequently, **do not readjust** the GAIN and f_0 controls on this VCO.

(3) Plot a graph of the VCO output frequency versus the DC control voltage at the V_{IN} input over a sufficiently wide range to show the onset of non-linearity of the characteristics. This is best done by plotting the graph as the results are taken. Notice that a warning light turns on when the control voltage exceeds the linear region of the VCO characteristic.

(4) Monitor CH2A on the Tektronix oscilloscope, configuring it as a spectrum analyzer by depressing the MATH button and selecting FFT CH2. Use a vertical sensitivity of 5 dB per division, and a sampling rate of 500 kilosamples per second. Adjust the horizontal position control on the oscilloscope so that

100 kHz appears at the centre of the screen. Vary the DC control voltage to the VCO and check that the carrier peak on the spectrum analyzer moves to the position indicated by the frequency counter.

We now wish to replace the DC voltage input to the VCO by a low-frequency (audio) message signal. This can be done by turning the gain control g on the ADDER module fully anticlockwise. The AUDIO OSCILLATOR module is used to provide a sinusoidal signal over a frequency range of approximately 250 Hz to 10 kHz. Since the output voltage of the audio oscillator is not adjustable, the gain control G on the ADDER module is used to vary the modulation amplitude to the VCO. Use the frequency counter to measure the message frequency from the audio oscillator for this part.

(5) Observe the modulated signal on the oscilloscope while triggering the oscilloscope using the signal itself. Notice that the amplitude is constant, and that the trace has the appearance of a compressed and expanded spring, showing that the carrier frequency is being modulated. You should learn to recognize this waveform, which is characteristic of an angle-modulated signal.

(6) Sketch the spectra of the resulting modulated signals when the following signals are at the VCO input (as monitored on CH1A of the oscilloscope). Note that the modulation index β is the same in all cases. Adjust the vertical position control on the spectrum analyzer so that when the message signal is zero, the level of the unmodulated carrier peak (at 100 kHz) is aligned with the top line of the screen.

  (a) 10 kHz sinusoidal modulation at 4 V peak-to-peak,

  (b) 5 kHz sinusoidal modulation at 2 V peak-to-peak,

  (c) 2.5 kHz sinusoidal modulation at 1 V peak-to-peak,

  (d) 1.25 kHz sinusoidal modulation at 0.5 V peak-to-peak.

  Measure the levels of the carrier and the first few sidebands in each case. Comment on the similarities and differences between the spectra. How do the spectra compare with the theoretical spectrum for the value of β deduced from the message amplitude and the known value of k_f?

(7) Using an 10 kHz sinusoidal message, vary the modulation amplitude (at the VCO input) and observe how the spectrum changes when β is successively set to 0.1, 0.2, 0.5 and 1.0. Measure the value of β at which the second order sidebands exceed -40 dB of the unmodulated carrier amplitude and check that this is close to the expected value of 0.28.

(8) Using a 2.5 kHz sinusoidal message function, draw the spectra for $\beta = 0.5$, 1.0, 2, 5 and 10. Compare these with the theoretically predicted spectra. Estimate the effective bandwidth in each case, and plot the effective bandwidth against the maximum frequency deviation $k_f B$. How well does the graph fit the rule that the bandwidth is approximately $2\,(\nu_m + k_f B)$?

(9) Using a very high modulation index (but avoiding VCO non-linearity) with the lowest frequency output from the audio oscillator, estimate the effective bandwidth of the modulated signal. Keeping the modulation amplitude constant, increase the modulation frequency up to its maximum value, noting the bandwidth and general form of the spectrum in each case. Which frequencies give rise to the greatest bandwidths?

(10) It is possible to obtain an independent determination of k_f by noting that the Bessel functions $J_k\,(\beta)$ have zeros at specific values of β. Thus at these values of β, the k'th order sidebands (of frequency $\nu_c \pm k\nu_m$) vanish, giving a means of measuring β from examination of the spectrum. Using a 2.5 kHz sinusoidal message signal, plot β against the amplitude of the message function at the VCO input terminals by using the table of zeros of $\mathrm{J}_k\,(\beta)$ given in the table as follows. Increase the value of β (slowly) from zero by adjusting G, and notice when the carrier amplitude as seen on the spectrum analyzer first falls to zero. Measure the message amplitude, which corresponds to $\beta = 2.40$. As G is increased further, the amplitudes of the sidebands at $\nu_c \pm \nu_m$ fall to zero when $\beta = 3.83$. Check that the various sidebands vanish in the correct sequence as G (and hence β) is increased. Record

the message amplitudes corresponding to each of the values of β in the table, and plot the graph of β against amplitude.

k	β values for which $\mathrm{J}_k(\beta) = 0$		
	1 st	2 nd	3 rd
0	2.40	5.52	8.65
1	0.00	3.83	7.02
2	0.00	5.14	8.42
3	0.00	6.38	9.76
4	0.00	7.59	11.06

Measure the message frequency (nominally 2.5 kHz) using the frequency counter. How does the value of k_f obtained compare with the expected value of $10\,\mathrm{kHz\,V^{-1}}$? (Recall that $\beta = k_f B/\nu_m$).

The Zero-Crossing-Counter Demodulator

A simple FM demodulator is one which generates a pulse of fixed width in response to each zero-crossing of the FM signal, as shown in Figure 7. The time-average of the train of zero crossing pulses is then proportional to the frequency of the FM signal, and hence to the message signal.

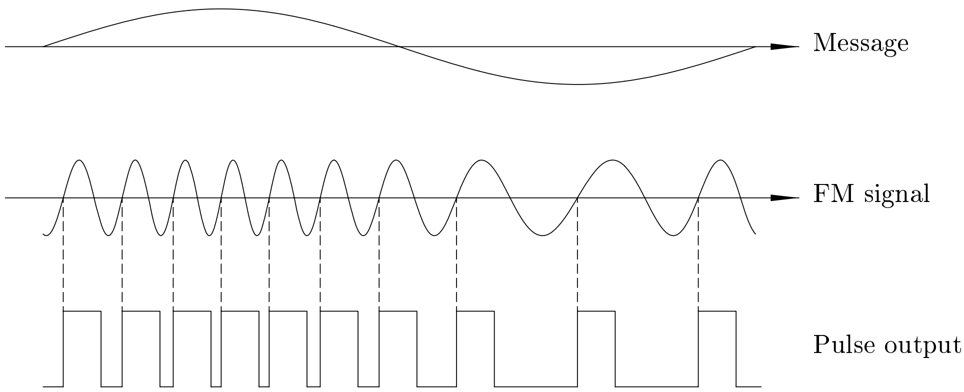

Figure 7: FM demodulator using a zero-crossing counter

In the experimental setup shown in Figure 8, a comparator from the Utilities module is used to detect zero crossings. It is important for correct operation to connect the reference input of the comparator to zero volts. The output of the comparator is a TTL signal which is used as a clock for the Twin Pulse Generator. The twin pulse generator can be configured so that it produces a single pulse of fixed width for each positive going edge of the clock. The width of the pulse may be varied using a front panel control. The Q_1 analogue output of the twin pulse generator is then low-pass filtered using the RC filter which is also part of the Utilities module.

Note: The colour convention used in the TIMS kit for front-panel sockets is that yellow sockets are for analogue signals, while red sockets are for TTL logic signals. Do not connect one type of sockets to the other unless you are sure of what you are doing.

Procedure

(11) Do not disconnect your FM modulator of Figure 6, as you will need to use the VCO output as the FM input for the setup of Figure 8. Use the Twin Pulse Generator and Utilities modules to set up the configuration shown in Figure 8. Check that the on-board mode switch of the Twin Pulse Generator module is set to SINGLE. This will cause it to produce a single pulse at the Q_1 outputs in response to each positive transition on the CLK input. Note that the comparator has two yellow input sockets on the left hand side of the Utilities module panel. The upper socket is the reference

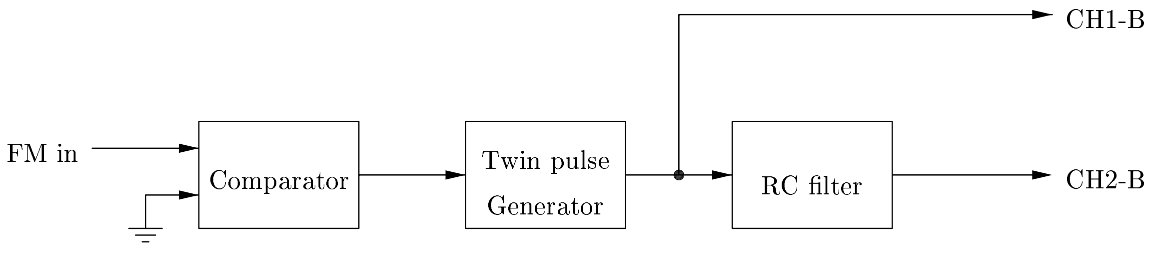

Figure 8: Experimental setup for zero-crossing counter

voltage which should be connected to ground (available on the VARIABLE DC module) while the lower socket is the actual comparator input. The comparator output is the red socket on the right hand side of the panel, and this should be connected to the digital clock input of the TWIN PULSE GENERATOR module labelled CLK. There are two Q_1 outputs from the twin pulse generator. We need to use the upper (yellow) analogue output, which should be connected to the input of the RC filter.

(12) Use the frequency counter to monitor the VCO output. Generate a 100 kHz unmodulated carrier (e.g. by turning down both g and G on the ADDER module fully anticlockwise). Monitor the output of the pulse generator using CH1B on the oscilloscope, and adjust the WIDTH control on the TWIN PULSE GENERATOR so that the output pulses have a 1 : 1 mark/space ratio. Also record the amplitude of the pulses.

(13) Using the variable DC voltage and the g knob on the adder, plot a graph showing the output of the demodulator (at CH2B) as a function of the FM frequency. Calculate what you expect the result to be, making use of the known amplitude and width of the pulses produced by the TWIN PULSE GENERATOR.

(14) Use the AUDIO OSCILLATOR to modulate the VCO, and show that the demodulator recovers the message signal. Note that the RC lowpass filter has a -3 dB corner frequency at about 2.8 kHz. Show that the amplitude of the demodulator output:

   (a) varies with the message amplitude into the VCO. Is this a linear variation?

   (b) varies with the pulse width from the TWIN PULSE GENERATOR. Is this a linear variation?

   (c) remains constant as the message frequency from the AUDIO OSCILLATOR is varied, so long that the oscillator frequency is low. Does this confirm that the VCO is producing frequency modulation, rather than phase modulation?

(15) Increase the message amplitude into the VCO until distortion is visible at the demodulator output. Can you identify the source of this distortion? Does the point at which distortion occurs (and the type of distortion) vary with the pulse width setting of the TWIN PULSE GENERATOR?

The Phase Locked Loop as an FM demodulator

Another common way of demodulating FM is to use a phase-locked loop (PLL). In its basic form, a PLL is illustrated in figure 9.

In the absence of an input signal, the VCO oscillates at some preset frequency called the free-running or rest frequency ν_r. When a signal is applied, the phase comparator compares the frequency and phase of the input signal and the VCO output to generate an error voltage $e(t)$ which is related to the phase difference between the two signals. The error voltage is filtered and used to control the VCO. If the input frequency is sufficiently close to ν_r, the feedback nature of the PLL causes the VCO to synchronize or lock with the incoming signal. Once in lock, the VCO frequency is identical with the input signal, except for a finite phase difference. When the loop is in lock, the VCO control voltage $c(t)$ is proportional to the VCO output frequency, and hence to the instantaneous input frequency. If the input is a frequency modulated signal, $c(t)$ will be proportional to the original modulating function.

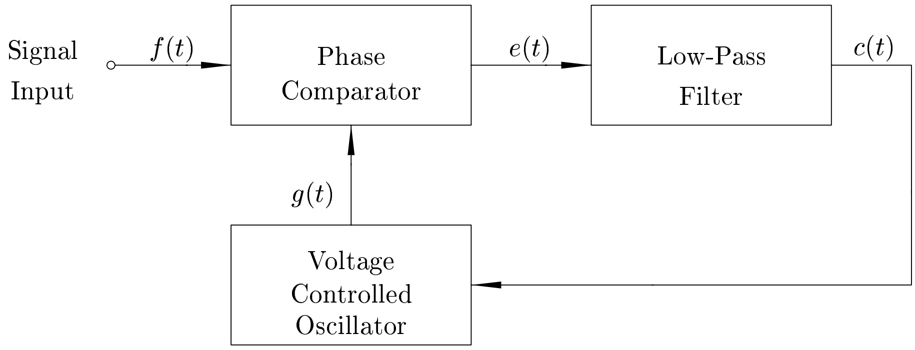

Figure 9: Basic phase-locked loop

Two fundamental parameters associated with phase-locked loops are the lock range (or hold range) and the capture range (or acquisition range). The lock range is defined to be the range of frequencies around ν_r over which the PLL can maintain lock with an input signal, if the frequency of the input signal is varied slowly. The capture range is defined to be the range of frequencies around ν_r over which the PLL can establish or acquire lock with an input signal. Hence if the input signal frequency is first increased slowly and then decreased, the VCO output frequency will behave as shown in figure 10. The capture and lock ranges are indicated.

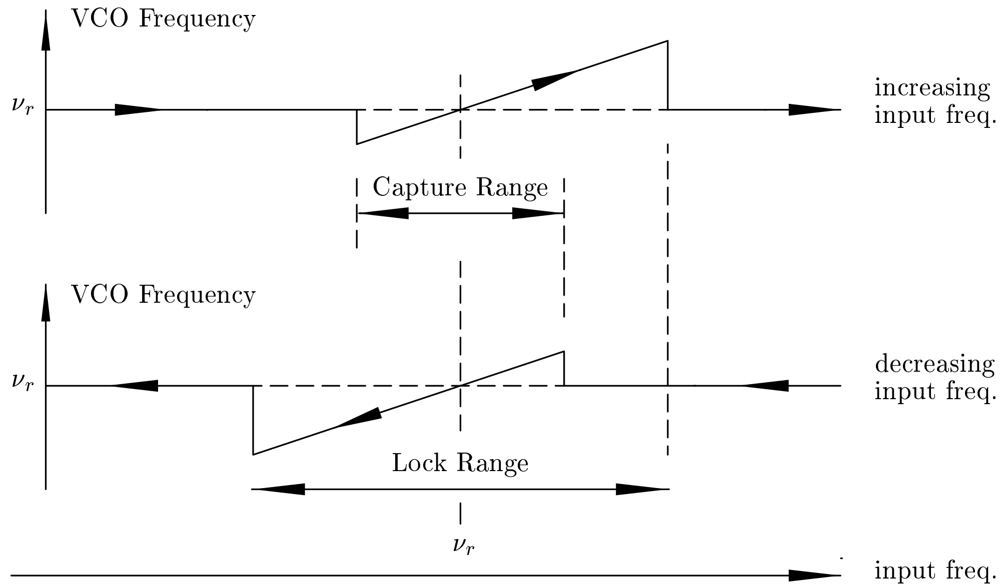

Figure 10: Capture and lock ranges of a phase locked loop

In the experiment, we shall use a multiplier as a phase comparator. If we feed two sinusoidal input signals $v_1(t) = A_1 \cos(\omega t + \phi)$ and $v_2(t) = A_2 \cos(\omega t)$ into a multiplier which produces an output $K v_1(t) v_2(t)$, we may write the output as

$$\begin{aligned} K v_1(t) v_2(t) &= K A_1 A_2 \cos(\omega t + \phi) \cos(\omega t) \\ &= \frac{K A_1 A_2}{2} [\cos(2\omega t + \phi) + \cos\phi] \end{aligned}$$

The low frequency component of the output is $\frac{1}{2} K A_1 A_2 \cos\phi$. This is clearly not a linear function of ϕ, but if ϕ is close to $\pi/2$ or $3\pi/2$, the output voltage is approximately linearly related to ϕ, the "gain" being $\frac{1}{2} K A_1 A_2 \,\mathrm{V\,rad}^{-1}$. Note that if the two inputs are of different frequencies, e.g., if $v_1(t) = A_1 \cos(\omega' t)$, we can still use the above analysis by setting $\phi = (\omega' - \omega) t$.

Procedure

A phase locked loop may be constructed using a MULTIPLIER module, a VCO module and the RC lowpass filter from the UTILITIES module. We shall use the FM modulator of Figure 6 to test the operation of the phase locked loop, so ensure that this is still set up. Figure 11 shows the way the phase locked loop will be implemented. We shall be setting up this block diagram in stages during the following steps.

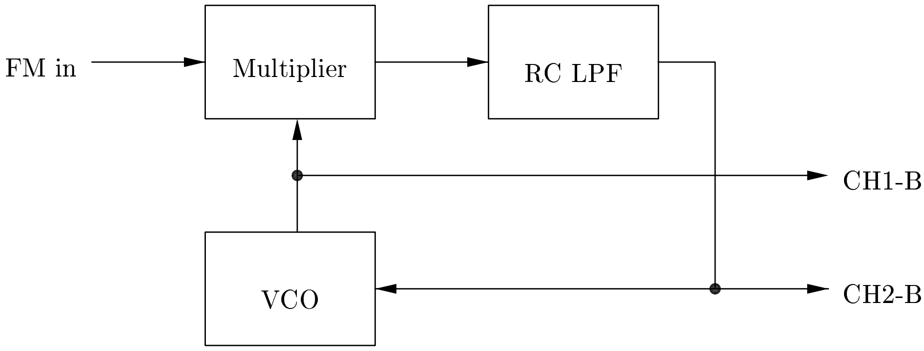

Figure 11: Experimental implementation of a phase locked loop

(16) Ensure that the switch on the VCO module to be used for the phase locked loop is set to the VCO position. Set the frequency range to HIGH and turn the GAIN control fully anticlockwise. Set the centre frequency using the f_0 control to approximately 100 kHz. Connect the VCO output to one of the inputs of a MULTIPLIER module and connect the other input of the multiplier to the output of the FM modulator of Figure 6 (which is in fact the output of another VCO module). Connect the output of the multiplier to the input of the RC low-pass filter in the UTILITIES module and monitor the output of the filter on CH2B of the oscilloscope.

(17) Set up the modulator so that its output frequency is controlled by the VARIABLE DC source (i.e., set $G = 0$ and g to midrange on the ADDER module). Adjust the DC voltage so that the output frequency is also approximately 100 kHz. You should see a low frequency beat signal corresponding to the difference between the two VCO frequencies at the output of the RC low pass filter. Verify that this beat frequency changes as you would expect as the frequency of the FM signal is varied.

(18) View the output of the FM modulator on CH2A and the output of the VCO of the phase locked loop on CH1B in time synchronization on the oscilloscope, and trigger the oscilloscope sweep using CH2. As the frequency of the FM signal is varied using the variable DC source, you should find that the frequency of CH2A varies, but the frequency of CH1B remains unchanged.

(19) Close the phase locked loop by connecting the output of the RC low pass filter to the control voltage input of the VCO. With the FM input signal frequency close to the VCO free-running frequency, gradually increase the GAIN of the VCO. You should find that the loop suddenly locks, and the VCO frequency becomes equal to the FM input frequency. If the GAIN is increased further, the loop should remain in lock, but the VCO waveform may become distorted.

(20) Set the gain of the VCO to approximately the middle of its range and use the variable DC source to sweep the FM input signal slowly up and down in frequency around the resting frequency ν_r. Confirm that the loop goes in and out of lock as shown in figure 10. In particular, it should be apparent from your results that the capture range is smaller than the lock range. Carefully look at the phase relationship between the signals CH1B and CH2A (which enter the phase comparator) as the frequency is swept through the lock range. What is the phase relationship is at the centre of the lock range, when the input frequency is equal to the resting frequency, and what is the phase relationship at the extremes of the lock range when the lock is just about to be lost? Explain your results.

(21) Measure the lock and capture ranges when the GAIN of the VCO in the phase locked loop is $5\,\mathrm{kHz\,V^{-1}}$ and when it is $10\,\mathrm{kHz\,V^{-1}}$. (You will need to break the loop temporarily and use the variable DC source to calibrate the VCO gain). Can you account for the numerical values of the lock range?

(22) Verify that the phase locked loop demodulates an FM signal when the message is a sinusoidal signal from the audio oscillator. Account for how the amplitude of the demodulated output changes as the GAIN of the VCO in the phase locked loop is varied. Check that the loop can lose lock if the maximum frequency deviation is too large.

(23) Set the GAIN $\mathcal{G}$ of the VCO in the phase locked loop to $10\,\mathrm{kHz\,V^{-1}}$ and feed in a sinusoidally modulated FM signal. The frequency deviation should be adjusted so that the loop remains in lock. Sweep the message frequency over the range of the audio oscillator and notice how the amplitude of the demodulated signal at the output of the phase locked loop changes. If we use a single pole RC filter with corner frequency ν_0 in the phase-locked loop, a linearized analysis of the phase-locked loop shows that the frequency response of the demodulator is that of a two pole low-pass filter (see figure 12). The corner frequency ν_d of the demodulator is proportional to $\sqrt{\mathcal{G}\nu_0}$ and the demodulator Q is equal to ν_d/ν_0 (which is thus also proportional to $\sqrt{\mathcal{G}}$). From the frequency response of a two-pole low-pass filter shown in figure 12, we can measure A, $A_{\max}$ and $\nu_{\max}$. From these Q and ν_d may be found since

$$\nu_{\max} = \nu_d\sqrt{1-\frac{1}{2Q^2}} \quad \text{and} \quad A_{\max} = AQ\left(1-\frac{1}{4Q^2}\right)^{-1/2}.$$

Determine the values of Q and ν_d from your measurements and check that $Q = \nu_d/\nu_0$. Repeat your observations for $\mathcal{G} = 5\,\mathrm{kHz\,V^{-1}}$, verifying that Q and ν_d are proportional to $\sqrt{\mathcal{G}}$.

Note: If control of the frequency response of the phase locked loop to different message signals is important, it is possible to use a more sophisticated filter than an RC network at the output of the phase comparator.

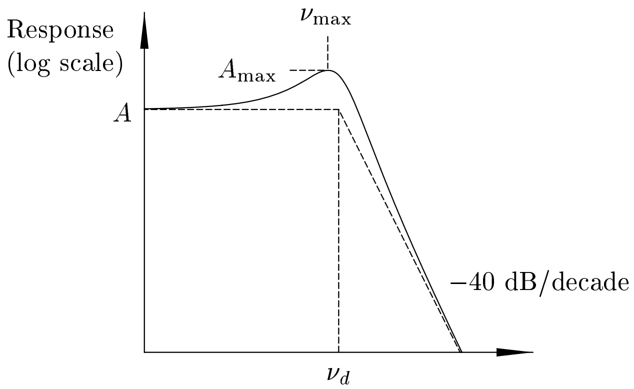

Figure 12: Amplitude response of a two-pole low-pass filter

The Armstrong modulator

The Armstrong modulator consists of a narrow-band phase modulator followed by a series of frequency multiplication stages which serve to increase the frequency deviation. It may also be used as a frequency modulator by first integrating the message signal before feeding it into the phase modulator. Unlike the voltage controlled oscillator considered above, the carrier frequency can be made very accurate, and Armstrong modulators are thus often used for radio communications.

As discussed earlier, the narrow band phase modulator is based on the approximation:

$$\begin{aligned} f_{\mathrm{NBPM}}(t) &= A\cos\left[2\pi\nu_c t + k_p m(t)\right] = A\left\{\cos(2\pi\nu_c t)\cos\left[k_p m(t)\right] - \sin(2\pi\nu_c t)\sin\left[k_p m(t)\right]\right\} \\ &\approx A\cos(2\pi\nu_c t) - Ak_p m(t)\sin(2\pi\nu_c t), \end{aligned}$$

which is simply obtained by adding together a DSB signal $-Ak_p m(t)\sin(2\pi\nu_c t)$ and the carrier $A\cos(2\pi\nu_c t)$. In order to understand better the nature of the approximation, let us write the sum in terms of an amplitude and a phase:

$$A\cos(2\pi\nu_c t) - Ak_p m(t)\sin(2\pi\nu_c t) = A\sqrt{1+k_p^2 m(t)^2}\cos\left(2\pi\nu_c t + \tan^{-1}\left[k_p m(t)\right]\right)$$

The phase of the carrier is actually modulated by $\tan^{-1}[k_p m(t)]$ instead of $k_p m(t)$. Distortion will occur unless $k_p m(t) \ll 1$, so that $\tan^{-1}[k_p m(t)] \approx k_p m(t)$. This means that the added carrier should be of greater amplitude than the DSB signal for proper operation. If the inequality is satisfied, the amplitude may be expanded by the binomial theorem

$$A\sqrt{1 + k_p^2 m(t)^2} \approx A\left(1 + \tfrac{1}{2}k_p^2 m(t)^2\right)$$

It is important to adjust the phase of the carrier correctly in order to obtain NBPM, and avoid amplitude modulation. If we consider adding the carrier with a phase error of ϕ to the DSB signal, we obtain $A\cos(2\pi\nu_c t + \phi) - Ak_p m(t)\sin(2\pi\nu_c t)$. If $k_p m(t) \ll 1$, the amplitude of the output may be shown to be

$$A\left(1 + k_p\, m(t)\sin\phi + \tfrac{1}{2}k_p^2 m(t)^2 \cos^2\phi\right)$$

Thus when the carrier phase is adjusted correctly $(\phi = 0)$, the envelope of the Armstrong NBPM modulator varies as $m(t)^2$ rather than as $m(t)$. If we consider a sinusoidal message signal $m(t) = \cos 2\pi\nu_m t$, we see that the envelope contains a component of frequency ν_m while ϕ is non-zero, but this component vanishes when $\phi = 0$ and the envelope contains components of frequency $2\nu_m$ and above. This provides a convenient method for adjusting the phase of the carrier in the Armstrong narrow-band phase modulator.

For true angle modulation, the amplitude is constant. A limiting amplifier (or *clipper*) is often used which preserves the times of the zero-crossings at its input, but whose output amplitude that is essentially independent of the input amplitude. The output of the limiting amplifier is essentially a frequency-modulated square wave. This is followed by a bandpass filter which only passes through the desired frequencies. We shall consider how these work in more detail after observing their effects experimentally.

Procedure

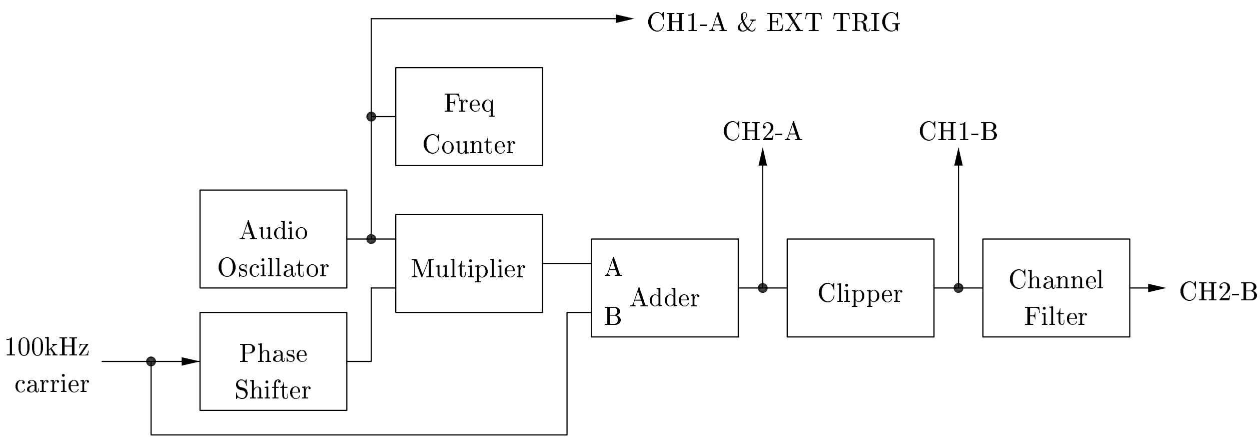

Figure 13: Experimental setup of narrow-band phase modulator

(24) We wish to set up the circuit shown in Figure 13. Before doing so, check that the frequency range switch on the circuit board of the PHASE SHIFTER module is set to HI and that the switches controlling the clipper (which is part of the UTILITIES module) are set to give hard clipping (SW1a and b should be ON, and SW2a and b should be ON). Note that the uppermost section of the UTILITIES module is labelled COMPARATOR but this section actually contains a clipper as well as a comparator. The input is the lower yellow socket on the left-hand side of the module and the clipper output is the upper yellow socket on the right-hand side. The input labelled REF (the comparator reference voltage) need not be connected when the clipper is used.

(25) Set the AUDIO OSCILLATOR output frequency to 1.25 kHz. We wish to adjust the gains G and g on the ADDER module so that the amplitude of the DSB signal and the amplitude of the injected carrier are equal. To do this, first rotate g fully anticlockwise, so no carrier is present. Adjust G so that the

DSB signal at CH2A of the oscilloscope is 3 V peak-to-peak. In order to adjust g, temporarily pull out the plug connected to the A input of the adder. Advance the g knob until the carrier at CH2A of the oscilloscope is also 3 V peak-to-peak. Reconnect the A input of the adder.

(26) We now need to adjust the PHASE SHIFTER so that the added carrier is of the correct phase. Look at the sum on CH2A of the oscilloscope, paying attention to the envelope, while adjusting the phase. As discussed above, when the phase is correct, the ripple on the envelope should be least and should be at *twice* the message frequency. In the audio range, another way of making this adjustment is to connect the output of the adder to the RECTIFIER in the utilities module, and the output of the rectifier to the HEADPHONE AMPLIFIER module. This models an envelope detector. Turn on the 3 kHz low pass filter in the headphone amplifier module by switching the LPF SELECT switch to the IN position. Listen to the signal in the headphones as the phase control on the PHASE SHIFTER module is adjusted. When the phase is correct, the 1.25 kHz tone should vanish and you should hear a 2.5 kHz tone. Notice how sensitive the ear is to this point — if the FINE phase control is rotated even slightly, the fundamental 1.25 kHz tone will be very apparent.

(27) Observe the spectrum at CH2A using the Tektronix oscilloscope. Since the amplitudes of the carrier and the DSB signals are equal, we have

$$A\cos(2\pi\nu_c t) - A\cos(2\pi\nu_m t)\sin(2\pi\nu_c t) = A\cos(2\pi\nu_c t) - \frac{A}{2}\sin[2\pi(\nu_c+\nu_m)t] - \frac{A}{2}\sin[2\pi(\nu_c-\nu_m)t],$$

so the sideband amplitudes at $\nu_c \pm \nu_m$ should be half (-6 dB) that of the carrier.

(28) We now consider the effects of the limiting amplifier (clipper) and band-pass filter. Set the selector switch on the 100 KHZ CHANNEL FILTER to position 3 in order to activate the 100 kHz band-pass filter. Successively observe the waveforms at CH1B and at CH2B, synchronizing the oscilloscope to the waveform being observed, making sure you see how the clipper removes the amplitude fluctuations of the envelope. The output of the bandpass filter at CH2B should be sinusoidal in shape, and have constant amplitude. Observe the spectrum at CH2B, noting how the clipping process has introduced many more sidebands at $\nu_c \pm k\nu_m$ for integers $k > 1$.

(29) When a sinusoidal message signal with frequency ν_m is used, the Armstrong phase modulator with an amplitude limiter produces

$$f_{\text{Armstrong}}(t) = A\cos\left[2\pi\nu_c t + \tan^{-1}(\beta\cos 2\pi\nu_m t)\right]$$

where β is the ratio of the amplitude of the DSB signal to the added carrier. By contrast, a true phase modulator produces

$$f_{\text{PM}}(t) = A\cos[2\pi\nu_c t + \beta\cos 2\pi\nu_m t].$$

The two are approximately equal only if β is small. The sideband amplitudes (expressed in decibels relative to the unmodulated carrier) are given in the table for $f_{\text{Armstrong}}$ and for f_{PM} when $\beta = 1$ and $\beta = 0.5$. Confirm the amplitudes for $f_{\text{Armstrong}}$ with your experimental setup.

	$\beta = 1$		$\beta = 0.5$	
Component	Armstrong PM	True PM	Armstrong PM	True PM
ν_c	-1.5	-2.3	-0.5	-0.56
$\nu_c \pm \nu_m$	-8.4	-7.1	-12.8	-12.3
$\nu_c \pm 2\nu_m$	-22.9	-18.8	-31.6	-30.3
$\nu_c \pm 3\nu_m$	-30.1	-34.2	-44.0	-51.8
$\nu_c \pm 4\nu_m$	-40.7	-52.1	-59.2	-75.9

Deviation Multiplication

As discussed above, the Armstrong modulator is only able to produce narrow-band modulation before excessive distortion is introduced by the failure of the approximations involved. Fortunately, it is possible to increase the frequency deviation following the modulator in a way which does not compromise the stability of the carrier frequency. We shall examine this process of *deviation multiplication,* which is based on a sequence of steps involving amplitude limiting followed by bandpass filtering.

Suppose that we have a signal which can be expressed in terms of an amplitude function $A(t)$ and a phase function $\phi(t)$ as

$$f(t) = A(t)\cos\phi(t).$$

We assume that the function $A(t)$ is always positive (and hence never goes to zero). The function $f(t)$ is zero if and only if $\cos\phi(t)$ is zero, i.e., if and only if $\phi(t)$ is an odd integer multiple of $\pi/2$. We now pass $f(t)$ through an ideal limiting amplifier. The output of such a limiter is a square wave $g(t)$ whose zero-crossings coincide with those of $\cos\phi(t)$. Since A is always positive, we may suppose (without loss of generality) that:

$$g(t) = \begin{cases} 1 & \text{if } \cos\phi(t) > 0 \\ -1 & \text{if } \cos\phi(t) < 0 \end{cases}.$$

We now wish to calculate the spectrum of $g(t)$. One way of doing this is to use the Fourier series of a square wave. If we define the function

$$s(x) = \frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^n \frac{\cos[(2n+1)x]}{2n+1} = \frac{4}{\pi}\left[\cos x - \frac{\cos 3x}{3} + \frac{\cos 5x}{5} - \frac{\cos 7x}{7} + \dots\right],$$

it is not difficult to see that, as we take a large number of terms,

$$s(x) = \begin{cases} 1 & \text{if } \cos x > 0 \\ -1 & \text{if } \cos x < 0 \\ 0 & \text{if } \cos x = 0 \end{cases}$$

For example, in Figure 14, the functions $\cos x$ and $s_5(x)$ (which is the sum for $s(x)$ truncated after the fifth term) are shown.

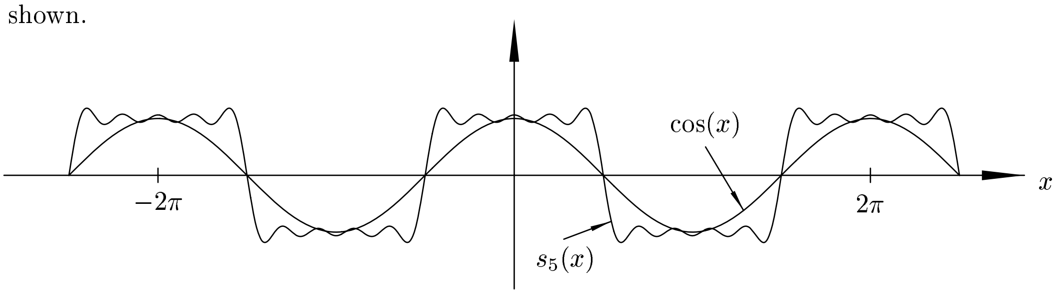

Figure 14: The functions $\cos(x)$ and $s_5(x)$.

From this definition, we see that

$$g(t) = s[\phi(t)] = \frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^n \frac{\cos[(2n+1)\phi(t)]}{2n+1}$$

If the limiter is followed by a band-pass filter, this filter selects out some of the terms in the sum.

We may apply this theory to the case of a sinusoidally angle-modulated signal with modulation index β. In this case,

$$f(t) = A(t)\cos[2\pi\nu_c t + \beta\cos 2\pi\nu_m t]$$

for which $\phi(t) = 2\pi\nu_c t + \beta\cos 2\pi\nu_m t$. After ideal limiting, the amplitude function $A(t)$ is removed and

$$g(t) = \frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^n \frac{\cos[(2n+1)(2\pi\nu_c t + \beta\cos 2\pi\nu_m t)]}{2n+1}.$$

Consider the terms in the sum for various values of n. Firstly for $n = 0$, we have

$$\frac{4}{\pi}\cos(2\pi\nu_c t + \beta\cos 2\pi\nu_m t)$$

which is just the original phase-modulated signal. As discussed previously, this has a cluster of spectral lines around ν_c with spacing ν_m.

Next consider $n = 1$. The term of interest is

$$-\frac{4}{3\pi}\cos\left[2\pi\left(3\nu_c\right)t + \left(3\beta\right)\cos 2\pi\nu_m t\right].$$

This is also a phase-modulated signal, but the carrier frequency is now $3\nu_c$. The message **frequency** is **unchanged** at ν_m, but the **modulation index** has been multiplied to 3β. The spectrum consists of a cluster of lines around $3\nu_c$ with spacing ν_m.

More generally, the n'th term is

$$\frac{\left(-1\right)^n \times 4}{2n+1}\cos\left[2\pi\left(2n+1\right)\nu_c t + \left(2n+1\right)\beta\cos 2\pi\nu_m t\right]$$

which is a phase modulated signal with carrier frequency $\left(2n+1\right)\nu_c$, message frequency ν_m and modulation index $\left(2n+1\right)\beta$.. The spectrum consists of a cluster of lines around $\left(2n+1\right)\nu_c$ with spacing ν_m.

Provided that $\nu_m \ll \nu_c$, which is almost always the case in practice, the clusters of spectral lines around the carriers of frequency $\left(2n+1\right)\nu_c$ do not overlap, and a suitable bandpass filter centred around one of these carriers may be used to extract one of the clusters. From the above analysis, the modulation index (and hence the phase and frequency deviations) are multiplied by the same factor as the carrier frequency, so that narrow-band modulation can be converted into wide-band modulation via a sequence of such steps of deviation multiplication.

Procedure

The FM Utilities module contains two amplitude limiting (clipper) subsystems, a 33.333 kHz bandpass filter and a sinusoidal carrier generator at 11.111 kHz. In order to produce an angle-modulated signal at 100 kHz using an Armstrong modulator and deviation multiplication, we must start with a lower frequency carrier, and multiply this carrier frequency and the deviation using the limiters and bandpass filters. The 11.111 kHz carrier is derived by frequency division from the 100 kHz signal produced by the Master Signals module, and is chosen so that multiplication by nine will recover the 100 kHz frequency.

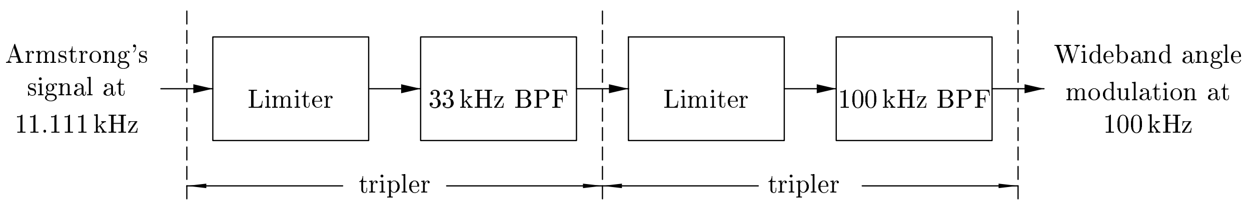

Figure 15: Deviation multiplication by nine

The subsystems should be combined as shown in figure 15. The Armstrong modulator should be connected up as before, but with the carrier replaced by the 11.111 kHz signal from the FM Utilities module. The output of the Adder module which combines the carrier and the DSB signal is fed into the first limiter shown in figure 15. The 100 kHz bandpass filter is provided by the 100kHz Channel Filter module with its selector set to the third position.

Since the 11.111 kHz carrier frequency used is actually quite low (and much smaller than would be used in practice), we cannot use a message frequency above about 1 kHz, if we are to separate out the clusters of sidebands produced during the frequency multiplication. It is also necessary to keep the modulation index before multiplication at the Armstrong modulator small ($\beta_1 \ll 1$) in order to get a good approximation to NBPM.

(30) For the experiment, use a message frequency of 625 Hz. Temporarily disconnect the wire from the Multiplier module to the A input of the adder, and adjust the carrier amplitude (using the g control

on the adder) so that its amplitude at the adder output is 3 V peak-to-peak. Reconnect the multiplier output to the A input of the adder, and temporarily disconnect the lead from the 11.111 kHz carrier to the B input of the adder, so that the DSB signal amplitude can be adjusted (using the G control on the adder) to be 3 V peak-to-peak as well. Reconnect the carrier, and adjust the PHASE SHIFTER by monitoring the envelope of the output of the adder as discussed previously. We see that when G is set to give a 3 V peak-to-peak DSB signal, the modulation index $\beta_1 = 1$. Subsequently, do not adjust the carrier amplitude (the g control on the adder) or the phase shift.

(31) Reduce the value of G on the adder so that $\beta_1 \approx 0.3$. Using the oscilloscope, look at the signals at the output of the adder and after the first limiter. This should be a narrow-band angle modulated signal at 11.111 kHz. Next observe the signal at the output of the 33.333 kHz bandpass filter. This filter selects those frequency components around $3\nu_c$ which are present in the clipped 11.111 kHz angle-modulated signal. Look at the signal after the second limiter and the 100 kHz band-pass filter. You should observe the characteristic "compressed and expanded spring" waveform of constant amplitude.

(32) We wish to observe the spectrum at the output of the 100 kHz band-pass filter, and to compute the modulation index there using the method of Bessel function zeros discussed earlier. Since the spectral lines are expected to be spaced only 625 Hz apart, it may be preferable to use a sample rate of 250 kS/s and an FFT zoom factor of 10 for the spectrum analyser, so that the dispersion on the screen is 1.25 kHz per division. Reduce the G control to zero and ensure that you see just the 100 kHz carrier line. Gradually increase G and notice the sidebands appearing spaced at 625 Hz. Using the table of Bessel zeros, check that you can obtain modulation indicies β_9 (after deviation multiplication) of 2.40 and 3.83 (corresponding to zeros of the carrier and first sideband pair). Measure the amplitudes of the DSB signal in the Armstrong modulator and confirm that the values of β_1 (before deviation multiplication) are as you would expect. Besides looking for zeros, it is possible to locate several other values of β_9 by comparing the relative amplitudes of various spectral components. In the table, we show the values of β_9 at which the amplitudes of various spectral components are equal.

Components	β_9
Carrier (ν_c) and first ($\nu_c \pm \nu_m$)	1.43
Carrier (ν_c) and second ($\nu_c \pm 2\nu_m$)	1.85
First ($\nu_c \pm \nu_m$) and second ($\nu_c \pm 2\nu_m$)	2.6

Set up the modulator to produce these values of β_9, measure the amplitudes of the DSB signal in the Armstrong modulator and confirm that the values of β_1 (before deviation multiplication) are as you would expect.

(33) Using your results and the table for the next few Bessel zeros, plot a graph of β_9 against the peak-to-peak amplitude of the DSB signal. Notice how the results deviate from the expected values as the modulation index is increased and the approximations break down.

List of Equipment

1. TIMS (Telecommunications Instructional Modelling System) kit with plug-in modules.
2. Tektronix TDS 210 oscilloscope with plug-in spectrum analyser module installed.
3. Hitachi dual trace oscilloscope

S.M. Tan

July 2001