Experiment 324: Optical Frequency Doubling

Aim

The aims of this experiment are to construct a laser, illustrate the concept of laser modes, demonstrate optical frequency doubling and to investigate factors influencing the efficiency of this phenomenon.

WARNING Some common sense is required:

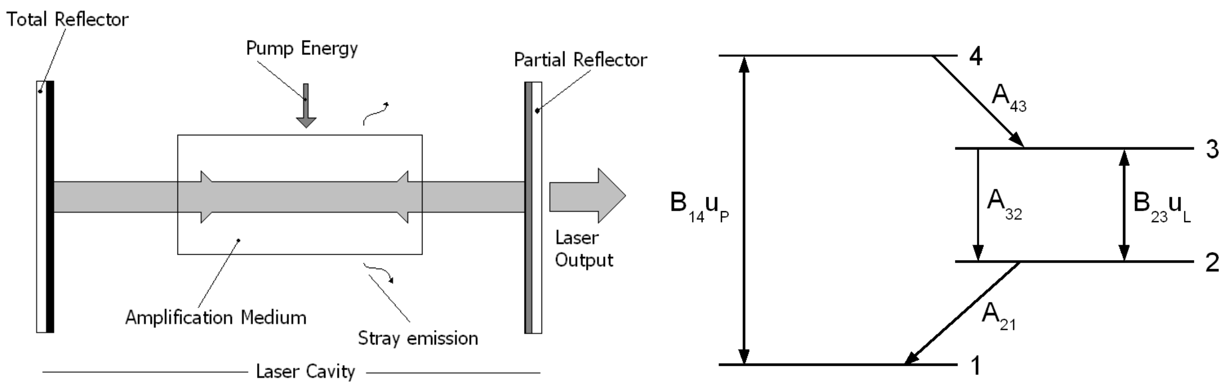
- Never look directly into a laser.
- Do not touch any of the actual optical surfaces of the provided components with your hands (only handle by the posts/holders).

Theory

Below we will cover the necessary theory to understand the physics in the experiment. Some background knowledge is inevitably assumed. Needless to say, if you come up against ideas, terminology or mathematics you have not seen before, you are expected to read up independently until the matter is clear in your mind.

Lasers

By now you should know that three basic interactions are possible between light and matter: absorption, spontaneous emission, and stimulated emission. You should also have a fair idea regarding how lasers work: we have a pumping source that puts in photons into a resonant cavity which reflects these photons and "traps" them for many, many oscillations. The output mirror is partially transmitting – otherwise no light would ever come out. Inside the cavity we have a gain medium – a material the atoms of which undergo population inversion due to pumping by the source at some appropriate frequency. By multiple reflections of the photons through the gain medium, the signal is amplified (via stimulated emission). Now, all lasers experience losses as some atomic spontaneous decay is inevitable, but these spontaneous photons will have a random k-vector (and phase), and will exit the cavity off the optical axis. When steady state is reached (the output intensity cannot grow infinitely!), the gain in the medium is equal to the losses of the cavity. A simple laser is illustrated in Fig. **??** (a).



(a) (b)

Figure 1: (a) Schematic of a laser, (b) energy level diagram of a four-level laser.

In this experiment, you will use a gain medium with the active atomic structure of a four-level laser such as that shown in Fig. **??** (b). A_{ij} is the spontaneous decay rate from level i to j, B_{mn} is the absorption and stimulated emission rate between levels m and n, and u_P, u_L are the energy densities of the pump and laser fields, respectively. Note that $u_{P/L}$ is proportional to the intensity of the relevant field in the cavity and that we are taking A_{42}, A_{41}, A_{31} to be negligible. We need to have $A_{43}, A_{21} \gg A_{32}, B_{14}u_P$; this basically says that

- as soon as an atom is excited from level 1 to 4, it immediately decays to level 3, so that we may take level 4 to be unoccupied,
- any atoms that find themselves in state 2 immediately decay to 1, so that level 2 may be assumed unoccupied.

Let N_j denote the number of atoms in state j per unit volume in the gain medium, and $N = \sum_j N_j$. Under the above approximations, the rate equation for the population of level 3 is

$$\frac{dN_3}{dt} = u_P N_1 - u_L N_3 - A_{32} N_3. \tag{1}$$

The first term on the right represents pumping from level 1 to 4, followed by almost instantaneous decay to level 3, the second term represents stimulated emission from 3 to 2 (note that absorption from 2 to 3 is also possible, but we assumed level 2 is empty!), and the final term accounts for spontaneous emission from 3 to 2. Taking into account that $N_1 = N - N_3$ and solving (**??**) in steady state, gives

$$N_3 = \frac{u_P N}{u_P + A_{32} + u_L}. \tag{2}$$

Now, the rate of amplification of the laser field in the medium is proportional to $N_3 - N_2 = N_3$. Meanwhile, some of the laser power is lost through the output mirror. Let us denote this depletion rate of the intra-cavity laser field by κ. In steady state, gain equals loss, or mathematically: $N_3 \propto \kappa$. If we absorb the proportionality constant into κ, this leads to

$$u_L = u_P\left(\frac{N}{\kappa} - 1\right) - A_{32}. \tag{3}$$

Observe that while $u_P < A_{32}\kappa/(N - \kappa)$, (**??**) predicts a negative laser energy density. This is of course not possible – below this threshold pump power, there is no lasing as there is no population inversion. The threshold in a four-level laser is much smaller than in a three-level system, but is still finite. Finally, recalling that $u \propto I$, we see that the intensity of a laser above threshold is proportional to the pump intensity.

Optical frequency doubling

As the name suggests, optical frequency doubling is the phenomenon of generating photons with double the frequency of those you have started with. Crudely speaking, two photons are absorbed simultaneously, and one photon – with double the frequency – is emitted. This sort of thing is possible in non-linear crystals; in fact quantization is not even strictly necessary for its description.

Since light is an electormagnetic (EM) wave, when light is incident on a medium, atoms/molecules in the medium are aligned with the instantaneous electric field vector at their location, and since the electric field is an oscillating one, the dipoles in the material also oscillate. Accelerating charges radiate energy, and so the oscillating polarization field P radiates photons. When the incident field E is weak compared to the induced polarization, we get a linear EM response: $P = \varepsilon_0 \chi E$, where ε_0 and χ are the permittivity of free space and electric susceptibility of the medium, respectively. However, this is only an approximation! In reality, the polarization field is non-linear in E:

$$P = \varepsilon_0 \left(\chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots\right). \tag{4}$$

In other words, the susceptibility is a *tensor*[1], not a scalar. The reason a linear approximation can often be made is that if E is small, E^2 is smaller still etc.

[1]Basically, a tensor is like a multi-dimensional matrix.

Let us assume that the incoming light is a plane monochromatic wave propagating along the z-axis and that the radiated field at double the frequency is also propagating in the same direction. Assume that the electric field is linearly polarized, say along the x-direction. Consider the E^2 term:

$$\underset{\sim}{E}_{in} = E_o \cos(\omega t) e^{ikz} \hat{x}, \tag{5}$$

$$\underset{\sim}{P}^{(2)} = \varepsilon_0 \chi^{(2)} E_o^2 \cos^2(\omega t) e^{2ikz} \hat{x} = \frac{1}{2} \varepsilon_0 \chi^{(2)} E_o^2 \left[1 + \cos(2\omega t)\right] e^{2ikz} \hat{x}. \tag{6}$$

This term has twice the temporal frequency and twice the wave-vector magnitude – this is the light we are interested in.

We will now derive the mathematical relation to be verified experimentally, equation (??). We start from Maxwell's equations in matter (standard notation, SI units throughout):

$$\underset{\sim}{\nabla} \times \underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t}, \tag{7}$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{H} = \underset{\sim}{J} + \frac{\partial \underset{\sim}{D}}{\partial t}, \tag{8}$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{D} = \rho, \tag{9}$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{B} = 0, \tag{10}$$

$$\underset{\sim}{D} = \varepsilon_0 \underset{\sim}{E} + \underset{\sim}{P}, \tag{11}$$

$$\underset{\sim}{B} = \mu_0 \underset{\sim}{H} + \underset{\sim}{M}, \tag{12}$$

and consider a medium where $\rho = 0$, $\underset{\sim}{J} = 0$, $\underset{\sim}{M} = 0$. Now take the curl on both sides of (??), substitute in (??), (??) and (??) in that order, and recall that $\mu_0 \varepsilon_0 = 1/c^2$, to get

$$\underset{\sim}{\nabla} \times \underset{\sim}{\nabla} \times \underset{\sim}{E} + \frac{1}{c^2} \frac{\partial^2 \underset{\sim}{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \underset{\sim}{P}}{\partial t^2}. \tag{13}$$

This is the non-linear wave equation of EM. Using the vector-calculus identity

$$\underset{\sim}{\nabla} \times \underset{\sim}{\nabla} \times \underset{\sim}{E} = \underset{\sim}{\nabla} \cdot \left(\underset{\sim}{\nabla} \cdot \underset{\sim}{E}\right) - \nabla^2 \underset{\sim}{E}, \tag{14}$$

and (??) with $\rho = 0$, we may write

$$\nabla^2 \underset{\sim}{E} - \frac{1}{c^2} \frac{\partial^2 \underset{\sim}{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \underset{\sim}{P}}{\partial t^2}, \tag{15}$$

or, taking the linear part of the polarization over to the LHS and denoting $1 + \chi^{(1)} \equiv n^2$ (incidentally, n is the refractive index of the medium), we arrive at

$$\nabla^2 \underset{\sim}{E} - \frac{n^2}{c^2} \frac{\partial^2 \underset{\sim}{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \underset{\sim}{P}_{NL}}{\partial t^2}, \tag{16}$$

where $\underset{\sim}{P}_{NL}$ is the non-linear part of the polarization. For our purposes, we will take $\underset{\sim}{P}_{NL} = \underset{\sim}{P}^{(2)}$.

Next, we will assume that the electric field is made up of components oscillating at two frequencies: ω and 2ω (this is more than sensible). In addition, we will make the assumption that we are dealing with plane-waves, so that we shall have no dependence on x and y. However, be aware of the fact that this dependence very much exists and can be neglected only in the crudest of approximations – we will deal with it in the section on optical modes. Thus, we write

$$\underset{\sim}{E} = \left[E_1(z) e^{i\omega t} + E_2(z) e^{2i\omega t} + c.c.\right] \hat{x}, \tag{17}$$

where $c.c.$ stands for complex conjugate and $E_{1,2}$ are possibly complex functions. Now substitute (??) into (??) with $\underset{\sim}{P}^{(2)} = \varepsilon_0 \chi^{(2)} \underset{\sim}{E} \cdot \underset{\sim}{E} \hat{x}$. After carrying out the derivatives with respect to time we get

$$\begin{aligned} &\left(\frac{d^2 E_1}{dz^2} e^{i\omega t} + \frac{d^2 E_2}{dz^2} e^{2i\omega t} + c.c.\right) + \frac{n^2 \omega^2}{c^2} \left(E_1 e^{i\omega t} + 4 E_2 e^{2i\omega t} + c.c.\right) \\ &= -\frac{2\omega^2}{c^2} \chi^{(2)} \left(2 E_1^2 e^{2i\omega t} + 8 E_2^2 e^{4i\omega t} + 9 E_1 E_2 e^{3i\omega t} + E_1^* E_2 e^{i\omega t} + c.c\right). \end{aligned} \tag{18}$$

We now notice that all the time dependence in the equation is explicit and exponential. Clearly, the terms on the RHS oscillating at 3ω and 4ω cannot be balanced by any of the terms on the LHS. Because we have truncated the power-series expansion of $\underset{\sim}{P}$ at the second term, we must keep our approximations consistent and discarding all terms in (**??**) oscillating faster than 2ω. Once this is done, we also notice that the equation is *hermitian* – *i.e.* every term comes with its complex conjugate. Consider: let $\zeta, \xi \in \mathbb{C}$, and assume that $\zeta = \xi$, then clearly also $\zeta + \zeta^* = \xi + \xi^*$ is true – in other words, the "*c.c.*" bits give us no new information. In light of this, let us focus on the positive frequency components of (**??**). Finally, again using our knowledge of the explicit time-dependence, we may separate the equation into *two* equations: one containing terms oscillating at ω, the other only terms oscillating at 2ω. This process leads to

$$\frac{d^2E_1}{dz^2} = -\frac{\omega^2}{c^2}\left(2\chi^{(2)}E_1^*E_2 + n^2E_1\right), \quad (19)$$

$$\frac{d^2E_2}{dz^2} = -\frac{4\omega^2}{c^2}\left(\chi^{(2)}E_1^2 + n^2E_2\right). \quad (20)$$

Next, assume that

$$E_1(z) = A_1(z)e^{-ik_1z}, \quad k_1 = \omega n_1/c, \quad (21)$$

$$E_2(z) = A_2(z)e^{-ik_2z}, \quad k_2 = 2\omega n_2/c, \quad (22)$$

where the A's are slowly varying envelopes (as a function of z) and although thus far not emphasized, χ is a frequency-dependent tensor, and so the refractive index depends on frequency (hence the subscripts). Substitute (**??**-**??**) into (**??**-**??**), discard the terms proportional to the second spatial derivative of the A's (*slowly* varying envelopes!), simplify and get

$$\frac{dA_1}{dz} = -i\frac{\omega}{cn_1}\chi^{(2)}A_1^*A_2e^{i\Delta kz}, \quad (23)$$

$$\frac{dA_2}{dz} = -i\frac{\omega}{cn_2}\chi^{(2)}A_1^2e^{-i\Delta kz}, \quad (24)$$

where $\Delta k = 2k_1 - k_2$. We will now crudely solve the equation for A_2 by assuming that A_1 is roughly constant over z. Physically, this says that the "fundamental" field in the cavity is so strong that when a small proportion of it is frequency-doubled, the detriment is negligable compared to the power left over in the fundamental mode. Thus, assuming that A_1 is constant, the two equations decouple. Take $A_2(z = 0) = 0$ and integrate the RHS over the crystal length from $z = 0$ to $z = \ell$. After several convenient multiplications by 1 in various forms (with the purpose of arriving at a particular form for the solution), we get

$$A_2 = -i\frac{\omega}{cn_2}\chi^{(2)}A_1^2\ell e^{-i\Delta k\ell/2}\mathrm{sinc}\left(\Delta k\ell/2\right). \quad (25)$$

This is the frequency-doubled field at the output of the non-linear crystal, and its intensity is

$$I_2 = |A_2|^2 \propto I_1^2\mathrm{sinc}^2\left(\Delta k\ell/2\right). \quad (26)$$

Phase matching

We see that I_2 depends quadratically on I_1 and is very sensitive to Δk. This Δk is the "phase mismatch" and the critical dependence upon it can be easily understood from basic physics: as we've mentioned earlier, frequency doubling is the process of "combining" two photons into a new one. But we know that energy and momentum are conserved in all processes! The two initial photons are identical, so the new photon must have twice the energy and momentum of the old photons. That is, the new photon must have frequency 2ω and wave-number $k_2 = 2k_1$ (as $p = \hbar k$). This is precisely equivalent to $\Delta k = 0$ – the condition required for the sinc function to attain its maximum.

Now, recall that $\Delta k = \frac{2\omega}{c}(n_1 - n_2) = \frac{2\omega}{c}\left[n(\omega) - n(2\omega)\right] \neq 0$ in general, as the refractive index in a medium is frequency-dependent. Therefore we need some sort of method to achieve phase-matching. Luckily, we can exploit the optical properties of the non-linear crystal which is used for frequency doubling.

The non-linear crystal, apart from its second-harmonic-generation capability, is also *birefringent* because it is anisotropic and has an axis of symmetry (called the optical axis). This means that when light refracts in this medium it is split into two beams: the *ordinary* ray that obeys Snell's law, and the *extraordinary* ray, which does not. The basic geometry is illustrated in Fig. ??. Both beams are linearly polarized orthogonally to each other, with the ordinary beam polarized orthogonally to the optical axis. The two rays have different phase-velocities and indices of refraction. Whereas n_o is a constant, n_e depends on the direction of propagation due to its dependence on the direction of polarization with respect to the optical axis. Thus one can imagine that if the crystal was rotated about the incoming-beam axis, the ordinary ray would stay fixed while the extraordinary would precess about the axis of rotation.

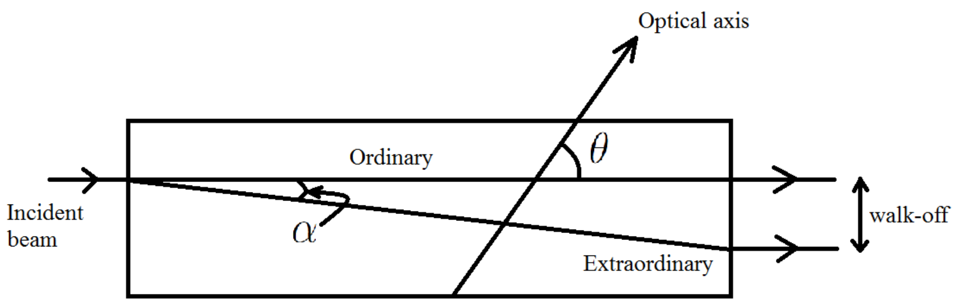


Figure 2: Schematic of a birefringent crystal.

Furthermore, $n_e(\theta)$ varies between its maximal value, n_o, which occurs if the extraordinary ray travels parallel to the optical axis, and its minimal value, denoted by n_e (without the θ dependence), when the extraordinary beam propagates perpendicular to the optical axis. At an arbitrary angle θ,

$$n_e(\theta) = \frac{n_o n_e}{\sqrt{n_o^2 \sin^2(\theta) + n_e^2 \cos^2(\theta)}}. \tag{27}$$

We can obtain phase-matching if we choose θ such that $n_o(\omega) = n_e(\theta, 2\omega)$ where we have re-introduced the dependence on frequency. In other words, it is possible to vary Δk – and even reduce it to zero – by rotating the non-linear crystal about the beam axis. This is precisely how we shall achieve phase matching in this experiment.

Optical modes

In the frequency doubling section we have assumed that the electric field was a plane-wave, with no dependence on x and y. However you should know that the most common mode produced by lasers has a Gaussian intensity profile, which certainly implies that x and y must feature in the solution (in fact a plane-wave laser output is quite inconceivable!). Let us see how the *shape* of the mode may be incorporated into the theory, as you will be required to produce, record and identify several mode-shapes during the experiment.

We go back to (**??**), still assuming the electric field is polarized along the x-axis. When we get to (**??**), $E_{1,2}$ are no longer assumed to be functions of z only – we take $E_{1,2}(x, y, z)$. This change is reflected in (**??**): instead of $\frac{d^2 E_{1,2}}{dz^2}$, we have $\nabla^2 E_{1,2}$, which is also carried through to (**??**-**??**). When we get to (**??**-**??**), $A_{1,2}$ are no longer merely functions of z, but are replaced with $A_{1,2}(x, y, z)$ where the dependence on z is still assumed to be weak. Analogous working to that described in the frequency doubling section leads us to the new equivalents of (**??**-**??**):

$$\frac{dA_1}{dz} = i\frac{c}{2\omega n_1}\left(\frac{d^2 A_1}{dx^2} + \frac{d^2 A_1}{dy^2}\right) - i\frac{\omega}{cn_1}\chi^{(2)} A_1^* A_2 e^{i\Delta k z}, \tag{28}$$

$$\frac{dA_2}{dz} = i\frac{c}{4\omega n_2}\left(\frac{d^2 A_2}{dx^2} + \frac{d^2 A_2}{dy^2}\right) - i\frac{\omega}{cn_2}\chi^{(2)} A_1^2 e^{-i\Delta k z}. \tag{29}$$

These coupled partial second-order differential equations are very difficult to solve. Luckily, we can get all the necessary insight by solving a much simpler equation: the equation describing the electric field inside a laser cavity, without a non-linear crystal and therefore without a second-harmonic component. In other words, if we discard equation (**??**) and set $A_2 = 0$ in (**??**), we arrive at the paraxial wave equation

$$\frac{d^2 A}{dx^2} + \frac{d^2 A}{dy^2} + 2ik\frac{dA}{dz} = 0, \tag{30}$$

where we have dropped the indices as they are no longer necessary. We will now concentrate on finding the solutions permitted by (**??**).

First of all, take the trial solution

$$A(x,y,z) = \mathcal{A}(z)\exp\left\{-ik\frac{x^2+y^2}{2q(z)}\right\}. \tag{31}$$

and substitute into (**??**). This leads to

$$\mathcal{A}\left[\frac{2ik}{q}\left(\frac{d\mathcal{A}}{dz}\frac{q}{\mathcal{A}} - 1\right) - \frac{k^2(x^2+y^2)}{q^2}\left(1 + \frac{dq}{dz}\right)\right] = 0, \tag{32}$$

which is only possible in general if two conditions are satisfied:

$$\frac{1}{\mathcal{A}}\frac{d\mathcal{A}}{dz} = \frac{1}{q}, \tag{33}$$

$$\frac{dq}{dz} = -1. \tag{34}$$

Now let us make the trial solution more complicated:

$$A(x,y,z) = \mathcal{A}(z)E_m(x)E_n(y)\exp\left\{-ik\frac{x^2+y^2}{2q(z)}\right\}. \tag{35}$$

Upon substitution of (**??**) into (**??**), we arrive at

$$\begin{aligned}&\frac{1}{E_m}\frac{d^2E_m}{dx^2} - \frac{2ikx}{q}\frac{1}{E_m}\frac{dE_m}{dx} - \frac{k^2x^2}{q^2}\left(1+\frac{dq}{dz}\right) + \\ &\frac{1}{E_n}\frac{d^2E_n}{dy^2} - \frac{2iky}{q}\frac{1}{E_n}\frac{dE_n}{dy} - \frac{k^2y^2}{q^2}\left(1+\frac{dq}{dz}\right) + \\ &2ik\left(\frac{1}{\mathcal{A}}\frac{d\mathcal{A}}{dz} - \frac{1}{q}\right) = 0.\end{aligned} \tag{36}$$

Since (**??**) is a *trial solution*, we can assume whatever we like regarding the terms featuring in it – if the trial solution, such as it may be, satisfies the differential equation, it is *the* solution. Hence, we will assume that $q(z)$ and $\mathcal{A}(z)$ are such that conditions (**??**) and (**??**) are satisfied. This simplifies matters considerably. We now have

$$\begin{aligned}&\frac{1}{E_m}\frac{d^2E_m}{dx^2} - \frac{2ikx}{q(z)}\frac{1}{E_m}\frac{dE_m}{dx} + \\ &\frac{1}{E_n}\frac{d^2E_n}{dy^2} - \frac{2iky}{q(z)}\frac{1}{E_n}\frac{dE_n}{dy} = 0.\end{aligned} \tag{37}$$

For the following argument, denote the first line of (**??**) by $L_1(x,z)$, the second by $L_2(y,z)$. Consider: if we vary x, only L_1 changes, and yet the sum remains zero. Therefore, $L_1(x,z)$ must in fact be independent of x. Similarly, if we vary y, only L_2 changes, but the sum is still zero and hence $L_2(y,z)$ cannot actually depend on y. Therefore, it must be the case that $L_1(z) = -\lambda(z) = -L_2(z)$. From now on, we will only work with the x-equation, as the y-equation has the same exact form. At this point, we have

$$\frac{d^2E_m}{dx^2} - \frac{2ikx}{q}\frac{dE_m}{dx} + \lambda(z)E_m = 0. \tag{38}$$

Define $w(z)$ such that

$$\frac{1}{q} = -\frac{2i}{kw^2}, \tag{39}$$

and substitute into (**??**):

$$\frac{d^2 E_m}{dx^2} - \frac{4x}{w^2}\frac{dE_m}{dx} + \lambda E_m = 0. \tag{40}$$

Finally, define

$$u = \frac{\sqrt{2}x}{w}, \tag{41}$$

and change variables in (**??**) from x to u, to get

$$\frac{d^2 E_m}{du^2} - 2u\frac{dE_m}{du} + \lambda\frac{w^2}{2}E_m = 0. \tag{42}$$

This has the same form as the differential equation for the m^{th} Hermite polynomial:

$$\frac{d^2 H_m}{dx^2} - 2x\frac{dH_m}{dx} + 2mH_m = 0, \tag{43}$$

and in fact $E_m = H_m$ if $\lambda(z) = \lambda_m(z) = 4m/w^2(z)$. Thus, the solutions for $E_m(x)$ and $E_n(y)$ are Hermite polynomials in $\frac{\sqrt{2}x}{w}$ and $\frac{\sqrt{2}y}{w}$, respectively[2]. Incidentally, from (**??**) we get $q(z) = -z + c_1$ and from (**??**), $\mathcal{A}(z) = c_2/q(z)$, so the solution is fully determined. These laser modes are called Hermite-Gaussian modes and occur when the laser cavity has boundary conditions with cartesian symmetry. In practice, one laser mode always dominates – *which* one depends quite sensitively on the alignment and geometry of the components.

Experiment

Our first goal is to build an infra-red (IR) laser. For this purpose, we are provided with an 808 nm laser diode – this is our pump source which will create population inversion in the gain medium. A 3 mm focal length lens is attached to the laser diode downstream of the output 808 nm light so that we can focus the pumping power to a point inside the gain medium for better efficiency. The amplification medium itself is a Nd:YVO_4 crystal, and will eventually lase in the IR at 1064 nm. The first of the cavity mirrors is attached to the crystal; this is a dichroic mirror so that pump light is transmitted *into* the cavity easily, but laser light generated inside the cavity is reflected and cannot get *out*. We are also given a non-linear KTP crystal for second-harmonic generation – it will convert some of the 1064 nm light into 532 nm green light. An output mirror is available which is highly reflective to 808 nm and 1064 nm light but highly transparent to 532 nm light. As we intent to put the non-linear crystal inside the resonant cavity of the IR laser, this ensures that nearly all frequency-doubled green light comes out of the cavity while the pump and fundamental light remain trapped.

We are also given a He-Ne laser and an aperture to facilitate alignment. The other equipment available is a detector with an optical power meter and a controller box for the laser diode which controls the pump power. All of these are easily mounted on the optical rail.

Set-up

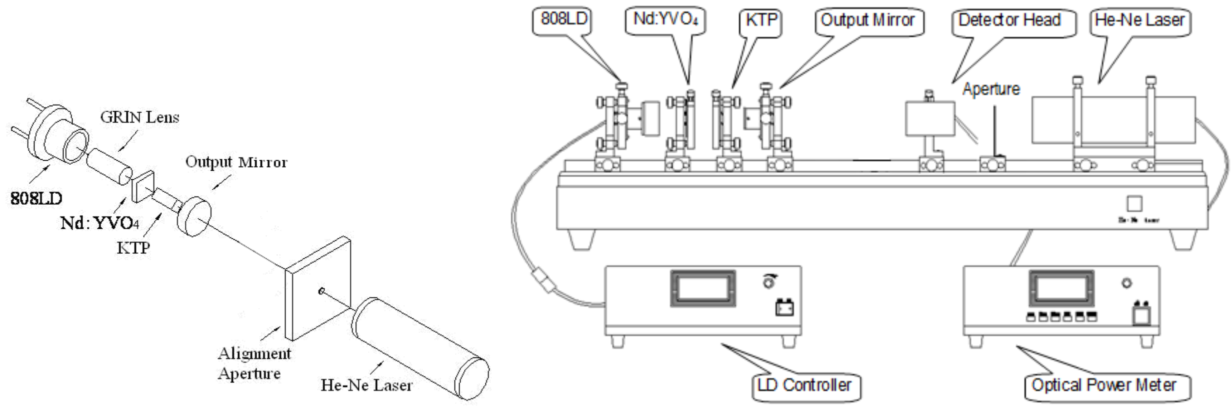
First, you need to identify the components. The He-Ne laser is fixed to the optical rail on the far-right and should not be touched (switch on and off by the red button on the optical rail beneath it). The power meter and laser diode controller (they are labeled) should be equally conspicuous; the detector head and laser diode are connected to their respective "boxes". The alignment aperture is the metal plate with a *small* hole in it. The remaining components are conveniently numbered, as in Table **??**.

The set-up is illustrated in Fig. **??**; the detector is used later in the experiment and is absent during alignment. All components, once mounted on the rail, are fixed by tightening a knob at the base of the holder. Make sure these knobs are always facing towards you, as shown in panel (b) [this is actually very important!].

[2]Note that, contradictory to the implications of our derivation, the mode order is not necessarily the same in both directions [because assuming (**??**-**??**) is not strictly necessary].

Component	Number
Output mirror	4
KTP crystal	5
Nd:YVO_4 crystal	6
Laser diode	7
Detector	8

Table 1: Part numbers for component identification. Actual serial numbers are LEOI-50-#, where # is one of the numbers in the table.



(a) (b)

Figure 3: The set-up.

Always push optical components towards you flat against the opposite side of the rail so that they are not tilted before fastening these knobs.

The main components are installed in “translation stages” – these allow you to move the optical component horizontally and vertically, as well as tilting it. The translating knobs are only available on the laser diode and the output mirror: they are the ones that are sticking out vertically up from the top and horizontally out from the side of the holders. DO NOT TOUCH THESE – at least not yet! – as you may have a hard time finding the signal if you do. The tilting knobs are always at the back of the holder, one in the top left corner, one in the bottom right (looking from the back). The top left knob will generally tilt the component vertically, and the bottom right horizontally.

At all times during this experiment be very careful not to knock components against each other or bump them with your hands, or drop them on the table etc. Before you begin, there is a piece of paper with a pen-mark taped to the top of the KTP holder. Make sure that the pen-mark is somewhere around 110° on the angle-scale glued to the rotating front part of the KTP holder. If it is not so, loosen the knob sticking out towards you out of the “cut-off” top corner and gently rotate the KTP. Re-tighten the knob when you’re done.

Carry out the following steps to obtain green light:

1. Begin by turning on the He-Ne laser and setting up the aperture about 5 cm to the left of it. Then mount the laser diode on the far-left of the optical rail. A reflection off the front surface of the laser diode should be now visible on the aperture plate. Tilt the laser diode until you have moved the red spot on to the actual aperture (*i.e.* the hole). Always try to center the spot as well as you can.

2. Next, add the Nd:YVO_4 (‘Nd’ for short) crystal right after the laser diode. While viewing directly from above, adjust the distance between them so that the top rubber padding hemisphere on the laser diode and the Nd crystal holder *almost* touch. Now if you look from the side, you ought to see a small gap between the actual optical components – it is hard to judge, but this should be about 1-2 mm.

There will now be a new spot on the aperture which is not centered. Tilt the Nd crystal [this is tricky – be delicate so as not to bump the laser diode] and center the spot. You will see some interference fringes when the two spots overlap – this is normal.

3. Now insert the KTP crystal after the Nd, as close as possible without touching the Nd (about 1 mm gap between the holders). Note the holder orientation in Fig. **??**(b). Make sure you can rotate the KTP freely when the appropriate knob is loosened – if you can't go around the full 360 degrees (you might feel the rotation dial get stuck – if you do, don't force it but gently turn it back), move the KTP further away from the Nd. Again, tilt the KTP to center the reflected red dot on the aperture.

4. Finally, insert the output mirror. Place it close to the KTP – so that the closest protruding bits on the holders are about 1-2 mm apart [these happen to be the screws in the front of the mirror holder and the tilting knobs of the KTP]. You will notice that there is now a lot of scattered light in a concentric pattern about the actual small spot in the middle. Again, this is normal. Do not tilt the mirror to center the spot yet (it is pointless – see later).

5. Turn on the laser diode and gradually increase the current by turning the knob on the controller box (when it's time to turn it off, likewise turn down the current gradually before flipping the switch). Get to about, but no higher than, 500 mA (*e.g.* 495 mA). Note that the current knob is somewhat dysfunctional and does not monotonically increase/decrease the current as you turn the knob one way. It is a nuisance but not a serious hindrance to the experiment – be patient and you'll get there.

6. If you see green light right away, this is your lucky day! If not, look at the large "scattered" red light spot that appeared after we inserted the output mirror. It ought to be more or less centered on the aperture. If it is, play with the tilt of the mirror only (move the little red dot around quite widely) and hopefully you'll see green light before long. If the large scattered spot is considerably off to one side, play with the translation of the output mirror by turning the vertical and horizontal knobs in the holder. The control is very dodgy here, and if you find yourself struggling, you may wish to ask a demonstrator to help. Your goal is to move the large spot around (roughly towards the middle) until you see a ghost of green light. Once you do, play with both the translation and the tilt of the mirror and try to get as bright and as Gaussian-like a beam as you can.

For your convenience, a sketch of the light patterns on the aperture is given in Fig. **??** when a bright, Gaussian, green beam is visible. This is only meant to help you get started. Once you get green light, optimize *that* (instead of trying to reproduce the picture), as things will be different each time the laser is assembled.

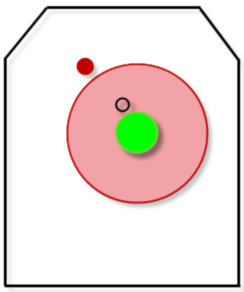


Figure 4: View of the aperture when everything is properly aligned (guide-line only). Black empty circle is the aperture hole, small filled red circle is the "concentrated" reflected He-Ne spot, filled green circle is the 532 nm light, and the large light-red circle is the scattered He-Ne reflection (which in reality will have fringes and patterns on it).

The reason we have to spoil our "perfect alignment" by off-setting the mirror alignment is that the mirror translation stage is faulty and cannot do its job properly, so we have to make do. Once you have a bright green spot on the aperture, turn off the red laser (or block it with a piece of paper). Rotate the KTP crystal and find the setting which gives the brightest light – 110° is just a rough estimate and depends on many things that could have changed since this value was obtained. It is possible that you will not be able to find a stable, bright Gaussian mode. If a high-power Gaussian cannot be obtained you should find and select the next-best-thing, *i.e.* brightest, most concentrated other mode you can find. Gently and *slightly* tune the tilt of all the other components. You may find it useful to use the power meter for this optimization (see below), or you may prefer to do it by eye.

The laser diode will need about half an hour to warm up and stabilize its output power.

We will soon need to measure optical power. The power meter gives you a choice of scales, labeled by the maximum power they can detect. For starters, select the 2 mW scale. Turn on the power meter and cover the detector completely by a piece of paper. Then turn the zeroing knob on the power meter box until the reading is actually zero (if it wasn't already).

Now, the detector face is actually rather large, while the beams we want are usually quite small with a significant amount of scattered light around them. Alternatively, we may want to only measure the central bright spot in a mode. To make sure we are really measuring the beam of interest only, we need to introduce a "clean-up" aperture. This will be custom-made for each beam you want to measure due to lack of suitable equipment. Find the metal plate with the *large* hole in it – this has not been used so far. Tape a bit of thick-ish paper over the large aperture and make a hole of suitable size and in the right place to only let through the light you want to measure. Insert this clean-up aperture at a suitable distance in front of the detector. Note that beams diverge the farther they travel, which might prove useful in isolating the "good" part of the beam from the "bad".

Put the detector on the optical rail, about half-way between the aperture and the output mirror. Twist the detector holder laterally a little (before fastening the rail-attachment knob) while observing the light falling on the clean-up aperture. If there are reflections off the detector, tilt the detector so that these reflections end up on the paper and do not go through the aperture. With the beam cleanly isolated, draw the curtains, turn on the power meter and monitor the reading for a while. Make sure it is not erratically changing, though perfect stability is not possible (so don't waste your time). Sometimes, the power jumps abruptly by a significant amount even after the laser diode has warmed up. If this happens, you will have to decide whether to continue taking data and correct for the jump later, or if retaking your data set might be better [up to you].

When taking data, you may need to change scales in order to maintain good precision. It is always good practice to note down what scale you're working on and the points at which you change scales.

Experimental procedures

1. The curtain should remain drawn as you take data (you may like to have a small torch handy). Carefully rotate the KTP in 3° increments in a suitably-chosen range of angles around your optimal, brightest-intensity reference value. Record the intensity of the beam at each position. Note that the rotating part of the KTP has the capacity for wobble and you should try to always reduce this unwanted tilt. Once you have taken down the data, plot it in Matlab to make sure it makes for a *roughly* smooth and continuous curve (but don't be pedantic – we just want to make sure the experiment is working!). Return the KTP to the initial angular position where we have the brightest beam.

2. Reduce the current driving the laser diode in small steps (10 mA is suggested) and record the power reading at each current value. Take the current down until the fluctuations on the readings suggest to you that further data is useless, and remember to change scales when the power falls sufficiently for higher accuracy. Plot the data in Matlab to ensure it is a roughly smooth line. Increase the current back up to 495 mA.

3. Sketch the shape of the mode you have used for the above two steps. Play with the tilts of each component (one knob at a time, and only turn them a little bit or you will rapidly lose alignment) and rotate the KTP crystal a full revolution to see the effect each component has upon the mode shape. Which components are most important in this respect? Why? Sketch several of the low-order shapes you see.

Analysis and write-up

Plot your data from procedure 2 of the experiment. With solid references to the theory section, describe what you expect this graph to look like and explain why. Fully interpret your plot of laser power *vs.* pump power. Explain all interesting features and the shape of the curve[3]. If the graph does not agree with the theoretical prediction, discuss possible causes for the discrepancy: what could be going on with the equipment? does your proposed idea account for the deviation of the data from theory? how could you test this? what additional equipment would you need? and so on.

Plot your data from procedure 1 of the experiment. How do you expect the second harmonic intensity to depend on the KTP angle? On a different set of axes, plot the qualitative functional form you expect to see. Comment on your results: are they similar? if not, why? are there imperfections in the data? what causes these? how could you test the idea? etc.

During procedure 3 of the experiment you have observed the output of the frequency-doubled light as you rotated the KTP through 360°. What were your general observations and how do they compare with our theoretical prediction? Explain what is actually occurring and why it is in disagreement with theory.

Look up, sketch and label several of the low-order Hermite-Gaussian modes. How can you tell the Hermite-polynomial order in each direction by visually inspecting the pattern? Does this add up with what you know in general about *polynomials*? With this in mind, sketch (in the report) and identify several of the modes observed during step 3 of the experiment.

References

These sources have been used to put together the theory section:

- *Physics of nonlinear optics*, by Guangsheng He and Song H. Liu, World Scientific, 1999 (section 3.1).
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- *Enhancement and analysis of nonlinear optical processes in designed microstructures*, by Guy Klemens, PhD thesis, University of California, San Diego, 2007 (chapter 2).
- `http://en.wikipedia.org/wiki/Second-harmonic_generation`
- *Introduction to Laser Spectroscopy*, by Halina Abramczyk, Elsevier, 2005 (section 5.2).
- *Laser Beam Diagnostics in a Spatial Domain*, by Tae Moon Jeong and Jongmin Lee, available from `http://www.intechopen.com/source/pdfs/12545/InTech-Laser-beam-diagnostics-in-a-spatial-domain.pdf`
- *A single-element plane-wave solid-state laser rate equation model*, by E.H. Bernhardi *et. al.*, South African Journal of Science, **104**, 9-10, (2008).

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[3]Note that it is not acceptable to simply claim that the data has a certain trend by inspection – *show* that it does scientifically.