Experiment 384: Amplitude Modulation and Demodulation

Aim

The object of this experiment is to demonstrate some of the characteristics of amplitude modulation, and to illustrate techniques for modulation and demodulation.

Introduction

Modulation is the process by which some characteristic of a carrier (usually high frequency) signal is varied by a message (usually low frequency) signal so that the latter may be transmitted by the carrier. The recovery of the message signal from a modulated carrier is achieved in a reverse process called demodulation.

For a sinusoidal carrier signal, $f_{\mathrm{c}}(t) = A_{\mathrm{c}} \cos(\omega_c t + \phi)$, the three parameters, A_{c}, ω_c and ϕ, which can be varied by a message signal, give rise to the terms amplitude, frequency and phase modulation respectively. The last two, collectively referred to as angle modulation, have similar characteristics and will be dealt with in a separate experiment.

In this experiment, we shall implement circuits using a communications systems modelling kit called TIMS (Telecommunications Instructional Modelling System). In communications texts, models are usually represented by block diagrams which represent the mathematical operations which need to be performed on various signals. Each block performs a single operation, such as a multiplication, an addition or a voltage to frequency conversion. With the TIMS kit, a variety of plug-in modules are provided, which perform such operations. Although it is possible to move these modules around between the slots, as they only receive power from the backplane, it should not be necessary to change the configuration during the course of the experiment. However, if you are interested in the components needed to make a module work, feel free to pull out a card (carefully!) and have a look. The functional description of each module is included in the bench copy of the pamphlet.

Equipment Familiarization

The lower portion of the kit contains the power supply and a collection of fixed modules. Of particular interest are:

- The Scope Selector module, which is used to connect signals to the two channels of an oscilloscope. Each of the channels may be switched between two inputs labelled A and B, so that two of four signals may be viewed simultaneously. A trigger signal may also be connected for synchronization.
- The Frequency Counter module, which measures the frequencies of signals, or counts pulses, depending on the position of the selector switch.

In this part of the experiment, we patch up a simple model to become familiar with using the TIMS kit.

Adding Two Sinusoids

Consider adding two sine waves of the same frequency ν but different phases:

$$y(t) = V_1 \cos(2\pi\nu t) + V_2 \cos(2\pi\nu t + \phi)$$

This can be done using phasors or complex number techniques. If the reference phase is taken to be that of the signal $\cos(2\pi\nu t)$, the complex amplitude of the result is $V_1 + V_2 \mathrm{e}^{\mathrm{j}\phi}$. The modulus of this gives the real amplitude, and the argument gives the phase of the result. It is apparent that if $V_1 = V_2$ and ϕ is 180°, the sum will be zero.

This may be investigated by using the block diagram shown in Figure 1, which can be set up using the Audio Oscillator, Phase Shifter and Adder modules. The audio oscillator provides a sinusoidal output of frequency between 300 Hz and 10 kHz, the phase shifter produces a phase shift variable over 360°, and the Adder calculates the sum $GA + gB$, where A and B are the two input signals and the gains G and g are variable from zero (knob fully anticlockwise) to approximately -2. Notice that none of the knobs on the modules is calibrated, so it will be necessary to make measurements to establish the parameter values. The following steps will illustrate how we can adjust the controls to obtain a zero output sum by first setting up the amplitudes and phase to be approximately correct, and then finally trimming the values for the best null. You should always proceed in small stages in which one or at most two knobs are adjusted, so that the effects of each step are clearly seen.

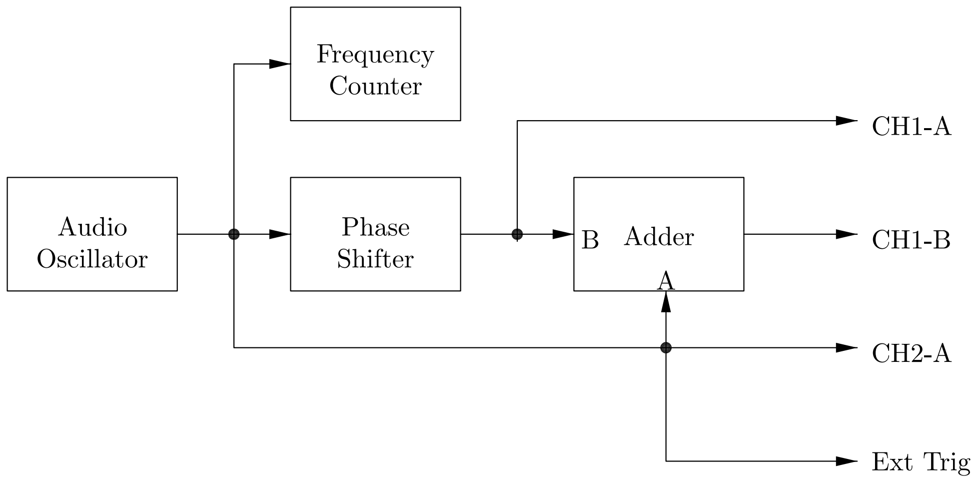

Figure 1: Setup for adding two sinusoids

Procedure

(1) Ensure that the oscilloscope inputs are connected to the appropriate outputs on the Scope Selector module, and that the Audio Oscillator, Phase Shifter and Adder modules are available in the upper rack. Pull out the Phase Shifter card and ensure that the switch on the card is in the LOW position. The gate time on the Frequency Counter module should be set to 1 s, so that frequencies are measured with a resolution of 1 Hz .

(2) Set up the block diagram shown in Figure 1. Adjust the audio oscillator frequency to approximately 1 kHz . CH2-A of the oscilloscope (i.e., CH2 on the oscilloscope, when the lower selector switch on the Scope Selector module is in the A position) monitors the output of the audio oscillator. CH1-A of the oscilloscope is connected to the output of the Phase Shifter module. Adjust the controls of the phase shifter module (including the ±180° toggle switch) to verify that phase shifts from 0° to 360° may be obtained without a change in the amplitude.

(3) We now wish to set up the gains G and g on the Adder module so that the two sinusoids to be added together have the same amplitude. To set up g, turn the G knob fully anticlockwise (so that $G = 0$). While monitoring the sum on CH1-B, adjust g so that the voltage is about 4 V peak-to-peak. Notice that the gain g is in fact negative. To set up G, temporarily disconnect the signal to input B of the adder. Do this by pulling out the plug(s) connected to that input. The input impedance of the adder is such that this effectively sets the voltage at input B to zero. Turn up the G knob while monitoring the sum, so that the voltage is also about four volts peak-to-peak. Reinsert the plug(s) into input B of the adder.

(4) If $V_1 = V_2$, the amplitude of the sum $\left|V_1 + V_2 e^{j\phi}\right|$ will vary from zero to twice the amplitude of either signal as ϕ is varied. Check that this is in fact the case. In order to get the best null, set the FINE control of the phase shifter to the middle position. Adjust the COARSE control (perhaps using the ±180° toggle switch as well) to get the minimum amplitude of the sum. Increase the sensitivity of the oscilloscope as the null is approached.

(5) Adjust the G control on the adder to further improve the null. Since the voltages V_1 and V_2 are already almost equal, only a small correction should be needed.

(6) By going backwards and forwards between the phase adjustment (using the FINE control as necessary) and the amplitude adjustment, try to get the best possible null. Examine the remaining signal at the best null, and comment on what may be causing what you see.

(7) Check that when the $\pm 180°$ toggle switch on the phase shifter is thrown from the null condition, the result is of the expected amplitude.

Double Sideband Suppressed Carrier (DSBSC) Modulation

Perhaps the simplest form of amplitude modulation consists of multiplying together the message signal $f_{\mathrm{m}}(t)$ and the carrier signal $f_{\mathrm{c}}(t) = A_{\mathrm{c}} \cos(2\pi\nu_{\mathrm{c}} t)$. The result is called DSBSC (double sideband suppressed carrier) or just DSB modulation for reasons which will become apparent. If we first suppose the message signal also to be sinusoidal, i.e., $f_{\mathrm{m}}(t) = A_{\mathrm{m}} \cos(2\pi\nu_{\mathrm{m}} t)$, then the product $f_{\mathrm{DSB}}(t)$ is just

$$\begin{aligned} f_{\mathrm{DSB}}(t) &= A_{\mathrm{m}} A_{\mathrm{c}} \cos(2\pi\nu_{\mathrm{m}} t) \cos(2\pi\nu_{\mathrm{c}} t) \\ &= \frac{A_{\mathrm{m}} A_{\mathrm{c}}}{2} \left\{ \cos\left[2\pi(\nu_{\mathrm{c}} + \nu_{\mathrm{m}}) t\right] + \cos\left[2\pi(\nu_{\mathrm{c}} - \nu_{\mathrm{m}}) t\right] \right\}. \end{aligned}$$

Usually, the carrier frequency is much higher than the frequencies present in the message signal, so that $\nu_{\mathrm{c}} \gg \nu_{\mathrm{m}}$. The spectrum of the DSB signal for sinusoidal modulation at frequency ν_{m} thus consists of a pair of peaks, one at frequency $\nu_{\mathrm{c}} - \nu_{\mathrm{m}}$ below the carrier frequency and one at frequency $\nu_{\mathrm{c}} + \nu_{\mathrm{m}}$ above the carrier frequency. For a more complicated message signal, which may be decomposed into a sum of components at different frequency, we obtain a pair of peaks for each component. The result is two *sidebands* of spectral components, the *upper sideband,* consisting of frequencies higher than ν_{c} and the *lower sideband* consisting of frequencies lower than ν_{c}. If the carrier frequency is much larger than the highest frequency present in the message signal $\nu_{\max}$, the entire spectrum of the DSB signal (which extends from $\nu_{\mathrm{c}} - \nu_{\max}$ to $\nu_{\mathrm{c}} + \nu_{\max}$) has a bandwidth which is small compared to ν_{c}. The DSB signal is thus said to be a *narrowband signal.*

Double sideband modulation is performed using a Multiplier module in the TIMS kit. A 100 kHz carrier signal may be obtained from the Master Signals module (on the lower rack), and the Audio Oscillator module may be used to supply the message signal.

Procedure

(8) First verify the operation of a Multiplier module by connecting a 100 kHz signal from the $\cos(\omega t)$ output of the Master Signals module to the Y input of the multiplier, and a DC voltage from the Variable DC module (on the lower rack) to the X input of the multiplier. The switch on the multiplier front panel should be set to DC to allow quantities with non-zero DC components to be multiplied together. Monitor the X and Y multiplier inputs using CH1-A and CH2-A, and the output of the multiplier kXY using CH2-B on the Scope Selector module, respectively. Synchronize the oscilloscope to the 100 kHz output from the Master Signals module. Take enough measurements to determine the value of the constant k in the definition of the multiplier characteristic. Check that the multiplier works for negative as well as positive values of the DC voltage.

(9) Replace the DC voltage connected to the X input of the multiplier with the sinusoidal output from the Audio Oscillator module. Since the message and carrier signals now have no DC component, it is permissible to turn the switch on the multiplier to either the DC or AC position. Connect the external trigger of the oscilloscope to the output of the audio oscillator so that the sweep is synchronized to the message signal. Observe the message signal and the DSB signal in time synchronization on the oscilloscope. Sketch your results. You should find that the envelope of the signal remains stable, but that the carrier oscillations will appear as a blur or will drift within the confines of the envelope.

(10) Using the spectrum analyzer facilities of the Tektronix oscilloscope (press MATH, followed by FFT CH1 or FFT CH2), observe the spectrum of the DSB signal. Choose a sampling rate of 500 kS/s

(kilo-samples per second) so that frequency components up to 250 kHz may be displayed. The FFT ZOOM sets the dispersion. At the above sampling rate, the $\times 5$ setting gives a dispersion of 5 kHz per division. Adjust the horizontal position control so that a frequency of 100 kHz is in the centre of the screen (the centre frequency appears in a readout above the trace). A "flat-top" window is useful for getting accurate amplitudes of the spectral lines. Note that the vertical scale is *logarithmic*, and calibrated in terms of a number of decibels per division. This can be adjusted using the vertical sensitivity switch. A setting of 5 dB/div is useful to start with. You may have to adjust the vertical position controls to get the spectrum onto the screen. Connect the message signal to the frequency counter, and observe how the spectrum changes as the message frequency is adjusted. Explain why it is that you see a small component at the carrier frequency even though there should theoretically be no component there (hence the term "suppressed carrier").

(11) In order to obtain a time-domain waveform in which both the message and the carrier signals appear locked on the oscilloscope screen, it is necessary that the carrier frequency be an exact multiple of the message frequency. There is an output on the Master Signals module which is labelled "2 kHz MESSAGE" which provides a signal that is phase locked to the 100 kHz carrier. If you measure the message frequency on the frequency counter, you will find that its actual value is $2083\frac{1}{3}$ Hz which is $\frac{100}{48}$ kHz. Use this signal in place of the audio oscillator output, and carefully sketch the resulting waveform, paying particular attention to what happens near the zero crossings of the message signal.

(12) We now wish to consider message signals which are more complicated than a single sinusoid. A sinusoidal signal may be converted into an approximate square wave by using the Comparator portion (the uppermost four sockets) of the Utilities module. Disconnect the 2 kHz MESSAGE from the multiplier and connect it instead to the analogue signal input of the Comparator (the lower of the two yellow sockets on the left-hand side). The yellow socket on the upper right of the Comparator is the output of a clipper, which should be approximately a square wave (check this using the oscilloscope). If the sides of the waveform are not very steep, it may be because some switch settings on the board are not set correctly to provide hard clipping (consult with a demonstrator if you need help). Connect the output of the clipper to the X input of the multiplier as the message signal. Once again sketch the message signal and the output of the multiplier carefully and observe the spectrum of the resulting DSB signal. Measure the amplitudes of the sidebands (remembering the logarithmic vertical axis) and check their relative values against the Fourier series expansion for a square wave.

Question 1: Verify that the Fourier series for a square wave of unit amplitude and frequency ν is

$$\frac{4}{\pi}\left[\sin(2\pi\nu t) + \frac{1}{3}\sin(2\pi[3\nu]t) + \frac{1}{5}\sin(2\pi[5\nu]t) + \cdots\right]$$

(13) Replace the 2 kHz MESSAGE signal with the output of the Audio Oscillator module and check that the result of modulating with square waves of different frequencies are as you would expect, both in terms of the time waveform and the spectrum.

Amplitude Modulation (AM)

Historically, amplitude modulation was developed before DSBSC modulation, and it is still the most widely used modulation technique for commercial broadcasting on the medium and short wave bands, mainly because of the ease with which it can be produced and demodulated. From a theoretical point of view, AM results from adding a component at the carrier frequency to the DSB signal, so that the signal does not vanish when the message signal goes to zero. If the message signal $f_{\mathrm{m}}(t)$ always lies between -1 and $+1$, the equation for an AM signal is:

$$f_{\mathrm{AM}}(t) = (1 + f_{\mathrm{m}}(t))\, A_{\mathrm{c}} \cos(2\pi\nu_{\mathrm{c}} t) = f_{\mathrm{DSB}}(t) + A_{\mathrm{c}} \cos(2\pi\nu_{\mathrm{c}} t)$$

The "instantaneous amplitude" or envelope of the AM signal is $1 + f_{\mathrm{m}}(t)$. The restriction that $|f_{\mathrm{m}}(t)| \leq 1$ is made so that the instantaneous amplitude is always non-negative. We shall see later that this considerably simplifies the recovery of $f_{\mathrm{m}}(t)$ from $f_{\mathrm{AM}}(t)$.

If we consider the case of a sinusoidal message signal, written as $f_{\mathrm{m}}(t) = m\cos(2\pi\nu_{\mathrm{m}}t)$,

$$\begin{aligned} f_{\mathrm{AM}}(t) &= (1 + m\cos(2\pi\nu_{\mathrm{m}}t))\, A_{\mathrm{c}}\cos(2\pi\nu_{\mathrm{c}}t) \\ &= \frac{A_{\mathrm{c}}m}{2}\cos[2\pi(\nu_{\mathrm{c}} + \nu_{\mathrm{m}})t] + A_{\mathrm{c}}\cos[2\pi\nu_{\mathrm{c}}t] + \frac{A_{\mathrm{c}}m}{2}\cos[2\pi(\nu_{\mathrm{c}} - \nu_{\mathrm{m}})t]. \end{aligned}$$

The quantity m is called the *modulation index*, which usually satisfies $m \le 1$. If $m > 1$, the AM signal is said to be *overmodulated*. The spectrum of AM consists of the upper and lower sidebands, together with the carrier. For a sinusoidal message signal at frequency ν_{m}, there are three spectral lines, the carrier frequency ν_{c} having amplitude A_{c} and the side frequencies $\nu_{\mathrm{c}} \pm \nu_{\mathrm{m}}$ each having amplitudes $\frac{1}{2}A_{\mathrm{c}}m$.

Question 2: Show that for 100% AM (i.e., $m = 1$) with a sinusoidal message signal, $\frac{2}{3}$ of the total power is in the carrier, and $\frac{1}{6}$ is in each of the two sidebands.

Question 3: Show that if P and Q are the maximum and minimum instantaneous amplitudes of an AM signal, $m = (P - Q)/(P + Q)$, assuming that the signal is not overmodulated.

With the TIMS kit, AM may be produced by using a MULTIPLIER module to multiply together $1 + m\cos(2\pi\nu_{\mathrm{m}}t)$ and the carrier signal $A_{\mathrm{c}}\cos(2\pi\nu_{\mathrm{c}}t)$ from the MASTER SIGNALS module. The AUDIO OSCILLATOR may be used to produce $\cos(2\pi\nu_{\mathrm{m}}t)$, and the ADDER module may be used in conjunction with a DC voltage from the VARIABLE DC module to form $1 + m\cos(2\pi\nu_{\mathrm{m}}t)$. Of course, in practice the constant "1" actually means a quantity sufficiently large that $1 + m\cos(2\pi\nu_{\mathrm{m}}t)$ never goes negative. The block diagram in Figure 2 illustrates the setup.

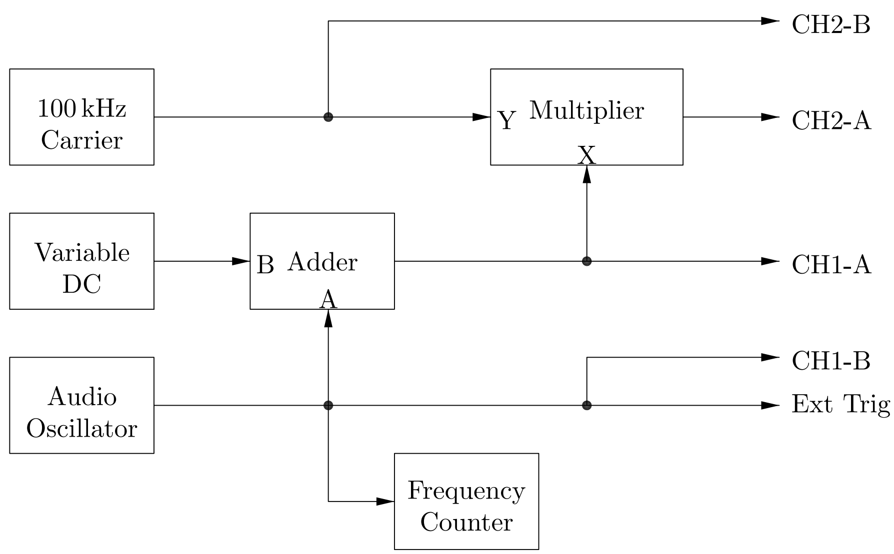

Figure 2: Setup for amplitude modulation

Procedure

(14) First connect up the AUDIO OSCILLATOR and VARIABLE DC source to the ADDER, and monitor the adder output on CH1-A. Use the FREQUENCY COUNTER to set the oscillator frequency to 1 kHz. Turn both the g and G knobs on the adder fully anticlockwise (i.e., $g = G = 0$), and the variable knob on the VARIABLE DC source almost fully anticlockwise so that it produces about -2 V. At this stage, the oscilloscope (set to DC coupling, and a vertical sensitivity of 0.5 V / cm) should be reading zero, so align the trace with a convenient graticule line.

(15) Advance the g gain control so that the output of the adder is 1 V, as recorded on the oscilloscope. (Note that the gains of the adder are negative). Next, advance the G knob so that the bottoms of the waveform exactly touch the zero line. This sets up a signal with $m = 1$, since $1 + m\cos(2\pi\nu_{\mathrm{m}}t)$ now ranges from zero to 2 V.

(16) Connect the output of the adder and the 100 kHz carrier signal to the inputs of the multiplier as shown in the figure. Ensure that the switch on the multiplier is set to DC. Monitor the output of the multiplier on CH2-A, synchronizing the oscilloscope with the message signal. The result should be a 100% modulated AM waveform. Measure the peak-to-peak amplitude of the AM signal and confirm that its value is as predicted, knowing the signal levels into the multiplier and the value of k determined in (8).

(17) By observing the message signal on CH1-A and the modulated signal CH2-A in time synchronization, notice that the envelope of the AM signal follows the message signal. Adjust the value of the modulation index m by varying G, first decreasing m below one and then raising m above one so as to cause overmodulation. Sketch your results, and notice that for all $m < 1$, the envelope of the AM has the same shape as the message, but for $m > 1$, the envelope does *not* have the shape of the message.

(18) A speech signal is available as SIGNAL 1 on the Trunks Panel. Rewind the cassette tape if necessary and switch it on to the play the tape. You can listen to the output by connecting the signal to the input of the Headphone Amplifier module in the lower rack. In the amplitude modulator, replace the output of the Audio Oscillator with the speech as the message signal. By looking at the AM waveform, how easy is it to adjust G so that the peak modulation index never exceeds 100%?

(19) The *modulation trapezoid* is a convenient way of measuring the modulation index for complex signals so that the gain may be adjusted to avoid overmodulation. In order to display the modulation trapezoid, the oscilloscope is placed in XY mode. The message signal on CH1-A is connected to the horizontal deflection plates, while the modulated signal on CH2-A is connected to the vertical deflection plates. Observe the modulation trapezoid and describe how it may be used to determine the modulation index.

(20) Use the spectrum analyzer facility on the Tektronix oscilloscope with the settings outlined in step (10) to observe the spectrum of the AM signal, with a sinusoidal message signal of frequency of 5 kHz from the Audio Oscillator. Make measurements of the spectrum for a variety of modulation indices, e.g., $m = 0.5$, $m = 1$ and $m = 1.5$, as determined from the AM waveform, or the modulation trapezoid. Check that the amplitudes and frequencies of the lines are as you would expect.

Important Note: Circuits within the TIMS system start behaving in a non-linear fashion once peak-to-peak voltages exceed about 4 V, so you should keep voltage levels everywhere within these limits. This is especially important at the output of the adder module, where an excessive voltage leads to distortion which is not easily visible on the oscilloscope, but which can give confusing extra peaks in the spectrum.

Envelopes and Envelope Recovery

The Envelope of a Signal

The concept of the envelope makes most sense when we consider narrowband signals, such as the AM and DSB signals observed above. A narrowband signal $y(t)$ with frequency components around ν_0 can be written in the form

$$y(t) = a(t)\cos[2\pi\nu_0 t + \phi(t)]$$

where $a(t)$ and $\phi(t)$ contain only frequency components much lower than ν_0. When viewed in the time domain, a narrowband signal consists of rapid oscillations at frequency ν_0 with instantaneous phase $\phi(t)$. The envelope $e(t) = |a(t)|$ is an imaginary line drawn such that the oscillations are contained within the boundaries specified by $\pm|a(t)|$. In terms of a phasor description, the envelope is the length of the resultant phasor. The process of envelope recovery attempts to extract $|a(t)|$ from $y(t)$. As we have seen, the envelope of an AM signal has the same shape as the message signal (so long that the modulation index is not more than one), so that a circuit which can recover the envelope of a signal is an AM demodulator. In this section, we shall investigate the concept of the envelope in more detail, leaving a discussion of circuits for envelope recovery to the next section.

From the definition of an AM signal with sinusoidal modulation, we have that

$$f_{\mathrm{AM}}(t) = (1 + m\cos(2\pi\nu_{\mathrm{m}}t))\,A_{\mathrm{c}}\cos(2\pi\nu_{\mathrm{c}}t)$$

Comparing this with the above expression for $y(t)$, we may identify $\nu_0 = \nu_c$, $\phi(t) = 0$ and $a(t) = (1 + m\cos(2\pi\nu_m t))A_c$. The envelope of $f_{AM}(t)$ is thus

$$e_{AM}(t) = |(1 + m\cos(2\pi\nu_m t))A_c| .$$

Provided that $m \leq 1$, the absolute values are superfluous, and $e(t) = (1 + m\cos(2\pi\nu_m t))A_c$. The envelope of a DSB signal can be found by discarding the "1" in the expression for $a(t)$, and so

$$e_{DSB}(t) = |A_c m\cos(2\pi\nu_m t)| ,$$

which has the shape of $|\cos(2\pi\nu_m t)|$.

Procedure

(21) Set up (or restore) the TIMS kit to conform to the block diagram in Figure 2 used above for generating AM. Note that it can also generate DSB by disconnecting the lead from the Variable DC source to the Adder. Set the message frequency to 1 kHz, and satisfy yourself that you can produce DSB and AM with modulation indices both less than and greater than one. Sketch the signals and their envelopes, showing that these are consistent with the theoretical predictions for e_{AM} and e_{DSB}.

(22) We shall now investigate how the idea of an envelope is still useful for wideband signals, by changing the carrier frequency so that it is no longer always much greater than the message frequency. In order to do this, we use a VCO module to generate a carrier with a variable frequency instead of the fixed 100 kHz carrier signal from the Master Signals module. A VCO is a voltage controlled oscillator, whose output frequency may be controlled by an input voltage. In this part, however, we do not use the voltage control capability, but instead just use it as an oscillator whose frequency may be varied over a wide range. Select the HI frequency range on the VCO and adjust the output frequency (using the frequency counter) to 100 kHz. Use the analogue output of the VCO instead of the carrier signal from the Master Signals module, and adjust the adder gains for an AM signal with modulation depth somewhere between 50% and 100%. Observe the offset message signal (at the output of the adder) on CH1-A and the modulated signal (at the output of the multiplier) on CH2-A. Trigger the oscilloscope using the message signal. Notice that the envelope of the AM signal has the same shape as the message signal, and use the vertical sensitivity and shift controls to overlay the message signal on top of the upper boundary of the AM signal. It is also instructive to monitor the spectrum of the multiplier output as well on the Tektronix oscilloscope.

(23) Monitor the frequency of the VCO output on the frequency counter. Gradually reduce the carrier frequency by turning the f_0 knob on the VCO down. Notice how the carrier oscillations may drift within the envelope, but they always fit within the same boundary. Turn the f_0 knob fully clockwise, and switch the VCO to the low frequency range. Now turn the f_0 knob down and observe how the modulated signal moves with respect to the envelope. Sketch your results when f_0 is around $3\nu_m$ and when f_0 is around $2\nu_m$.

Envelope Recovery

Provided that we are dealing with a narrowband signal, an ideal envelope recovery system consists of a precision rectifier followed by a lowpass filter. The lowpass filter must be able to reject the high frequency carrier without substantially modifying the envelope. A practical envelope detector (such as found in a broadcast AM radio) uses a diode in place of the precision rectifier and a simple single pole RC circuit as the low pass filter. In this part of the experiment we shall examine the operation of the "ideal" and the practical envelope detectors in terms of their ability to recover the message from an AM signal.

For AM with a modulation index $m \leq 1$, the envelope is of the same shape as the message, and so the frequency components of the envelope should be just those of the message. Once $m \geq 1$ however, or in the case of DSB, the frequency components in the envelope can be quite different from those of the message.

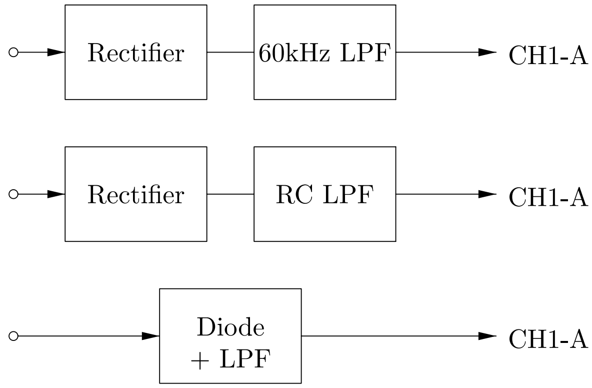

Figure 3: Three trial envelope detectors

Procedure

(24) Set up a circuit (e.g. based on the diagram in Figure 2) which allows AM or DSB to be generated with a message frequency of 2 kHz and a carrier frequency of 100 kHz . In this part, we shall feed the modulated signal through each of the envelope detectors shown in Figure 3. Observe the modulated signal on CH2-A, the demodulated signal in time-synchronization on CH1-A and trigger the oscilloscope using the message signal. First use the "ideal" envelope detector which consists of the RECTIFIER part of the UTILITIES module followed by the 60 KHZ LOWPASS FILTER module. Notice that when the modulator is set up to produce DSB, the output of the lowpass filter is essentially $|\cos 2\pi\nu_{\mathrm{m}}t|$, which is the expected envelope of the DSB signal. Use the Tektronix oscilloscope in spectrum analyzer mode to look at the spectrum of the demodulator output. You should still use a 500 kS/s sample rate, but this time adjust the horizontal position so that DC (0 Hz) appears on the left-hand side of the screen. Observe that the spectrum consists of lines at multiples of 4.166 kHz, which is twice the message frequency.

Question 4: What do you expect the relative amplitudes of the spectral lines to be given by for the envelope of the DSB signal?

(25) Gradually increase the amount of carrier. Ensure that the peak-to-peak amplitude of the multiplier output does not exceed 4 V, by reducing the amplitude of the message if necessary. Notice how spectral lines at the message frequency 2.083 kHz appear once some carrier is present. As the carrier amplitude is increased past a certain point, you should notice that all the harmonics of 2.083 kHz in the demodulated output essentially vanish, leaving only the fundamental. At what value of m (the modulation index) does this occur? At this stage, the envelope detector is essentially recovering the message signal perfectly.

(26) Restore the 2 kHz message signal and replace the 60 KHZ LOWPASS FILTER module by the RC LPF found on the UTILITIES module. This provides a single pole RC lowpass filter with a corner frequency of about 2.8 kHz . With the modulator set first to provide DSB, comment on the shape of the output signal and its spectrum as observed on the spectrum analyzer. Gradually increase the amount of carrier. At what value of m do you find that the spectrum becomes essentially a single line at the message frequency? Notice that there are still small ripples visible on the demodulator output waveform when the spectrum looks pure. These are at high frequency, and would not be audible in any case.

Question 5: Provided that we only consider AM signals with $m \leq 1$, what are the design constraints on the low-pass filter following an ideal rectifier? Does it matter whether the ideal rectifier is a half-wave or a full-wave device?

(27) Finally consider the DIODE+LPF found on the UTILITIES module in which the precision rectifier is replaced by a simple diode. A diode suffers from a knee voltage of about $0.6 - 0.7$ V, which means that the voltage across the diode must exceed this "knee" before the device will conduct. Observe the output demodulator in time synchronization with the modulated signal, making sure that the zero volt baselines are aligned. Notice the effect of the knee voltage, then shift the demodulator output

vertically on the screen to observe how it tracks the peaks of the modulated signal. Starting first with DSB and then gradually increasing the carrier amplitude so that AM is obtained, again find the value of m at which the spectrum of the output becomes essentially a single line at the message frequency. Compare your results with that for the precision rectifier followed by the RC lowpass filter.

(28) Replace the 2 kHz message signal with a voice signal from the tape recorder, and listen to the demodulated output using the headphones for each of the three envelope detectors. Use the modulation trapezoid in order to get an idea of the modulation index. Adjust the amount of carrier present in the modulated signal and listen to the results of envelope detection of AM with various modulation indices, overmodulated AM and DSB. You may also wish to use a message signal from the AUDIO OSCILLATOR module to investigate the frequency response of the demodulators.

Product Demodulation

We have seen that AM signals may be demodulated using an envelope detector, but that this method fails for DSB, or when the modulation index m exceeds unity. Since much of the power in an AM signal is due in the carrier, which in itself carries no information, it is often desirable to use methods in which the carrier amplitude is reduced or suppressed. Demodulation of such signals requires more sophisticated methods, usually based on a product demodulator.

Consider a DSB signal with a sinusoidal message of the form considered above, i.e.,

$$f_{\mathrm{DSB}}(t) = A_{\mathrm{m}} A_{\mathrm{c}} \cos(2\pi\nu_{\mathrm{m}} t)\cos(2\pi\nu_{\mathrm{c}} t)$$

Suppose that at the receiver it is somehow possible to produce a local copy of the carrier signal $\cos(2\pi\nu_{\mathrm{c}} t)$. If the DSB signal is multiplied by the local carrier, we get

$$\begin{aligned} f_{\mathrm{DSB}}(t)\cos(2\pi\nu_{\mathrm{c}} t) &= A_{\mathrm{m}} A_{\mathrm{c}} \cos(2\pi\nu_{\mathrm{m}} t)\cos^2(2\pi\nu_{\mathrm{c}} t) \\ &= \frac{A_{\mathrm{m}} A_{\mathrm{c}}}{2}\cos(2\pi\nu_{\mathrm{m}} t) + \frac{A_{\mathrm{m}} A_{\mathrm{c}}}{2}\cos(2\pi\nu_{\mathrm{m}} t)\cos(2\pi[2\nu_{\mathrm{c}}]t). \end{aligned}$$

If we now consider the frequency components present, we see that there is a component at ν_{m}, the original message frequency. The remaining term $\cos(2\pi\nu_{\mathrm{m}} t)\cos(2\pi[2\nu_{\mathrm{c}}]t)$ may be written as the sum of two signals at frequencies $2\nu_{\mathrm{c}} + \nu_{\mathrm{m}}$ and $2\nu_{\mathrm{c}} - \nu_{\mathrm{m}}$. If we assume that $\nu_{\mathrm{m}} \ll \nu_{\mathrm{c}}$, both of these frequencies are very much higher than the message frequency of interest and may be removed by a simple low-pass filter. A product demodulator thus consists of a multiplier followed by a low-pass filter.

It should be apparent that if we started with a more complicated message signal with many frequency components, the product detector will still recover the message provided that all frequencies in the message are much less than ν_{c} so that the unwanted high-frequency terms can be removed by the lowpass filter. Furthermore, if we start with AM, rather than DSB, the only change would be that a DC term is added to the output, proportional to the amplitude of the carrier. It is thus possible to demodulate an AM signal even if the modulation index exceeds unity with a product demodulator.

The problem with product demodulation is the assumption that we can produce a local copy of the carrier at the receiver. We may produce a local carrier which is incorrect in phase or frequency.

Question 6: Suppose for example that the local carrier does not have quite the correct *phase*, i.e. that we actually compute $f_{\mathrm{DSB}}(t)\cos(2\pi\nu_{\mathrm{c}} t + \phi)$. Show that after this product is filtered, the recovered signal is $\frac{1}{2}A_{\mathrm{m}} A_{\mathrm{c}} \cos(2\pi\nu_{\mathrm{m}} t)\cos\phi$. If ϕ is chosen to be an odd multiple of $\pi/2$, the output is reduced to zero.

Question 7: Suppose for example that the local carrier does not have quite the correct *frequency*, i.e. that we actually compute $f_{\mathrm{DSB}}(t)\cos(2\pi\nu'_{\mathrm{c}} t)$. Show that after this product is filtered, the recovered signal is $\frac{1}{2}A_{\mathrm{m}} A_{\mathrm{c}} \cos(2\pi\nu_{\mathrm{m}} t)\cos(2\pi[\nu_{\mathrm{c}} - \nu'_{\mathrm{c}}]t)$.

If it is important that the local carrier be of exactly the correct frequency (and phase), the product demodulator is said to be *synchronous*, otherwise, it is said to be *asynchronous*. For DSB demodulation, it is important to have synchronous product demodulation.

In this experiment, we shall use a *stolen carrier* from the modulator in order to illustrate the operation of product demodulators (i.e., we will just connect a wire from the carrier signal the modulator, and pass it through a phase shifter to simulate the effect of a phase error in the local carrier). In practical circuits, one must reconstruct the carrier from the received signal, which can itself be a non-trivial task. The output of the multiplier is passed through a low-pass filter, which may be the 60 kHz LOWPASS FILTER module, the RC LPF found on the UTILITIES module (which is a single-pole filter with a corner at 2.8 kHz) or the 3 kHz fifth-order elliptic lowpass filter in the HEADPHONE AMPLIFIER. The block diagram of the product demodulator is shown in Figure 4.

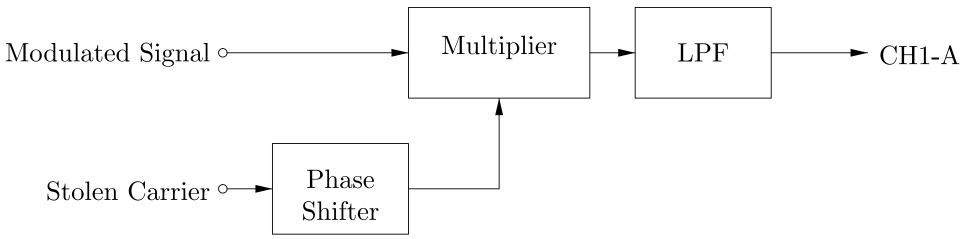

Figure 4: The product demodulator

Procedure

(29) Set up a circuit (e.g. based on the diagram in Figure 2) which allows AM or DSB to be generated with a message frequency of 2 kHz and a carrier frequency of 100 kHz. Connect the output of the modulator to the product demodulator, ensuring that the switch on the PHASE SHIFTER card is set to the HIGH position. Use the filter in the HEADPHONE AMPLIFIER module as the lowpass filter, noting that the output of the filter is accessible at the socket directly above the headphone jack. This will allow you to listen to as well as see the output of the demodulator. Look at the modulated signal and the demodulator output in time synchronization, triggering the oscilloscope with the message signal.

(30) Adjust the modulator to produce a DSB signal, and observe the output of the demodulator as the phase of the stolen carrier is varied. It should be possible to find a minimum and maximum in the demodulated signal. Since the oscilloscope is triggered using the message signal, it should be possible to see if the phase of the demodulated signal varies as the phase of the carrier is adjusted.

(31) Increase the amplitude of the carrier, converting the DSB signal into AM. Note how the output of the demodulator changes as this is done.

Question 8: In practice, there can be several radio stations which may be transmitting at frequencies quite close to each other. In the medium wave band, for example, radio stations can have carriers 9 kHz apart. Suppose that there are two AM signals with carriers 9 kHz apart, so that the input to the demodulator is the sum of these two signals. Explain how a demodulator based on an envelope detector will respond to this composite signal, and how a product demodulator will respond to the signal.

(32) In order to examine the effect of an incorrect local carrier frequency, we can use a VCO module with the switch set to the high frequency range. By adjusting the f_0 knob, the output frequency may be adjusted coarsely, but we need to be able to carry out finer frequency adjustments as well. This can be done by connecting the VARIABLE DC source to the control input of the VCO, and turning up the GAIN knob on the VCO just a few degrees clockwise from the minimum position. By adjusting the DC voltage, it is possible to change the VCO frequency by a few hertz. You may alternatively find it convenient to leave the DC voltage set to some small constant value and vary the frequency using the GAIN knob on the VCO. You may use the frequency counter to determine the VCO frequency or use the oscilloscope to compare the VCO and carrier frequencies.

(33) Set the VCO frequency as precisely as possible to 100 kHz and use this in place of the stolen carrier in the product demodulator. Observe the demodulated output on the oscilloscope and listen to it with the headphones. Comment on the need for having synchronous product demodulation for DSB.

Single Sideband Modulation

With double sideband modulation, each frequency component of frequency ν_{m} in the message produces a pair of frequencies $\nu_{\mathrm{c}} + \nu_{\mathrm{m}}$ in the upper sideband and $\nu_{\mathrm{c}} - \nu_{\mathrm{m}}$ in the lower sideband. This means that the bandwidth of the DSB signal is twice as wide as the message signal. The information in each sideband "mirrors" the information in the other so that one of them is actually redundant. By discarding one of the sidebands, we obtain SSB or single sideband modulation. With SSB, the modulated signals have the same bandwidth as the message (and half that of AM or DSB). This allows more SSB signals to share a given band of frequencies. Another advantage of SSB over DSB which we shall soon see is that it is possible to use an asynchronous product demodulator for recovering speech messages. This makes it much easier to produce a local carrier signal at the receiver.

Producing SSB is somewhat more difficult than DSB and AM. There are three main methods used. The simplest of these involves the use of a filter with an approximately rectangular passband which simply allows the desired sideband to pass through while rejecting the other sideband. This is perhaps the most commonly used technique at present, although the design of such filters can be problematic. The second method is called the phasing method, which is investigated in this experiment, while the third, called Weaver's method requires more hardware than we have available in the TIMS kit.

Consider the DSB signal resulting from modulating a cosinusoidal carrier with a cosinusoidal message. The sidebands arise from the trigonometric identity

$$\cos\left(2\pi\nu_{\mathrm{c}}t\right)\cos\left(2\pi\nu_{\mathrm{m}}t\right) = \frac{1}{2}\left\{\cos\left(2\pi\left[\nu_{\mathrm{c}}+\nu_{\mathrm{m}}\right]t\right)+\cos\left(2\pi\left[\nu_{\mathrm{c}}-\nu_{\mathrm{m}}\right]t\right)\right\}$$

If we consider the product of the carrier phase shifted by 90° and the message also phase shifted by 90°, we find that

$$\sin\left(2\pi\nu_{\mathrm{c}}t\right)\sin\left(2\pi\nu_{\mathrm{m}}t\right) = \frac{1}{2}\left\{\cos\left(2\pi\left[\nu_{\mathrm{c}}-\nu_{\mathrm{m}}\right]t\right)-\cos\left(2\pi\left[\nu_{\mathrm{c}}+\nu_{\mathrm{m}}\right]t\right)\right\}$$

Adding these two signals, we find

$$\cos\left(2\pi\nu_{\mathrm{c}}t\right)\cos\left(2\pi\nu_{\mathrm{m}}t\right)+\sin\left(2\pi\nu_{\mathrm{c}}t\right)\sin\left(2\pi\nu_{\mathrm{m}}t\right) = \cos\left(2\pi\left[\nu_{\mathrm{c}}-\nu_{\mathrm{m}}\right]t\right)$$

We thus have eliminated the upper sideband, leaving only the lower sideband. If we had subtracted rather than added the two products, we could have eliminated the lower sideband.

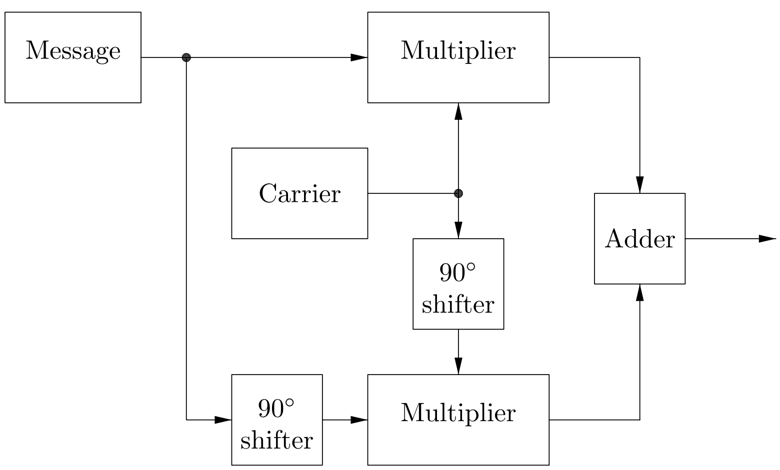

Figure 5: The phasing method for generating SSB

Figure 5 shows an implementation of the phasing method for generating SSB. There are two 90° phase shifters required, one for the carrier and one for the message. It is easy to make the phase shifter for the carrier, because the carrier is a pure signal of a single frequency. However, the message signal will generally contain a wide range of frequencies, and it is necessary in principle to produce a phase shift of 90° over the entire range. This is actually very difficult to do accurately. In the TIMS kit, there is a module called the

Quadrature Phase Splitter which has two inputs and two outputs. If the same signal is fed into the two inputs, the module introduces a frequency-dependent phase shift on each signal. However, the *difference* between the two phase shifts is close to 90° over a wide frequency range. Thus we may connect the same message signal to the inputs of the module and use the outputs from the phase splitter as inputs to the two multipliers. It turns out that for speech signals, the introduction of such a phase shift does not appreciably affect the intelligibility, although the technique is not suitable for data transmission.

Procedure

(34) First check out the operation of the Quadrature Phase Splitter module by connecting the output of the Audio Oscillator to both inputs, and monitoring the two outputs of the phase splitter in time synchronization. As the oscillator frequency is varied, you should check that the two outputs remain at 90° relative to each other, with a negligible change in amplitude. It is also possible to check this phase relationship by switching the oscilloscope to XY mode and checking that the resulting Lissajous figure (use equal sensitivities!) remains a circle over the entire range of frequencies.

(35) Set up a single sideband modulator based on the block diagram of figure 5, using the quadrature phase splitter for the message signal and the Phase Shifter module for the carrier. Check that the switch on the phase shifter card is in the HIGH position. The message signal should be provided by the Audio Oscillator module. We set up the modulator (adjusting G, g on the Adder and the phase shift on the Phase Shifter module) by making use of the fact that the SSB modulated output for a sinusoidal message is a sine wave of a single frequency. Start off by setting the audio oscillator frequency to 2 kHz. By disconnecting each signal in turn from the adder, adjust G and g (near the middle of their ranges) so that the two inputs to the summer have the same amplitudes. Unless the phase shift happens to be exactly correct, you should find that the output of the adder has a non-constant amplitude. Adjust the phase shift (and possibly trim one of the gains on the adder slightly) so as to minimize the size of the ripple, giving an essentially pure sine wave at the output of the adder.

(36) Measure the frequency of the output of the modulator. Have you generated an upper sideband or a lower sideband signal? Work out how to generate the opposite sideband and check that you can do this. Use the spectrum analyzer in the Tektronix oscilloscope to look at the spectra at the outputs of the two multipliers and also at the output of the adder. Vary the message frequency and ensure you understand what is going on.

(37) Notice that on the Master Signals module, the carrier is available both as $\sin \omega t$ and as $\cos \omega t$. Use both of these signals in order to free up the Phase Shifter module, and trim the gains on the adder if necessary to produce the best SSB signal. Set up the product demodulator shown in Figure 4, using a stolen carrier from the Master Signals module. Feed in the SSB signal into the demodulator and check that you can recover the message at the output of the product demodulator. Adjust the phase of the stolen carrier using the Phase Shifter module and comment on what happens at the demodulator output. (Note: The oscilloscope should be triggered using the message signal).

(38) Replace the stolen carrier with the output of a VCO module connected to the Variable DC source as discussed in (32) above. Adjust the VCO frequency as accurately as possible to 100 kHz and view the demodulated signal. Adjust the frequency of the VCO slightly and check that the demodulator output changes as you would expect.

(39) Replace the message signal with a voice signal and look at the SSB signal both on the oscilloscope and on the spectrum analyzer. Listen to the output of the product demodulator, and comment on what happens as the local carrier (VCO) frequency is tuned through the correct value of 100 kHz. Notice how the intelligibility of the signal changes as the local carrier frequency is changed.

List of Equipment

1. TIMS (Telecommunications Instructional Modelling System) kit with plug-in modules.
2. Tektronix TDS 210 oscilloscope with plug-in spectrum analyser module installed.

3. Hitachi dual trace oscilloscope
4. Portable radio and cassette tape recorder.

S.M. Tan

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