Experiment 219: Young's modulus by bending of a cantilever beam

Aim

To study the bending of a cantilever beam and determine Young's modulus.

References

1. Worsnop and Flint: "Advanced Practical Physics for Students", Methuen
2. Champion and Davy: "Properties of Matter", Blackie

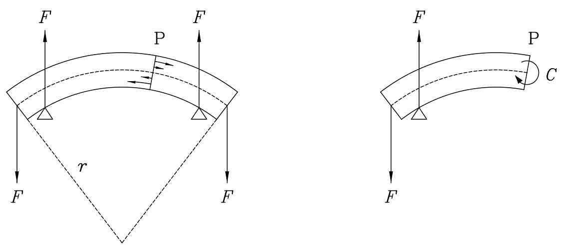
Theory

The Young's modulus of a material measures its elasticity under stretching and compression. For a piece of material of uniform cross-section, the amount of force that must be applied per unit cross-sectional area to produce a given fractional change in its length is given by:

$$\frac{\text{Force}}{\text{Area}} = \text{Young's modulus} \times \frac{\text{change in length}}{\text{original length}}$$

End-loaded Beam

In this experiment we shall determine Young's modulus by an indirect method involving the bending rather than the stretching of a beam. Consider a beam loaded as shown in Figure 1(a). Any portion of the beam, for example that to the left of P, is in linear and rotational equilibrium under the action of the forces and couples which act on it. In Figure 1(b), which shows the left-hand portion as a free body, we see that the right-hand portion of the beam need only provide a couple C to maintain both linear and rotational equilibrium. Physically the couple C arises from the stretching of the material above the dashed curved line (the neutral surface) and the compression of the material below it. The couple C is the bending moment of the beam at position P.



(a) (b)

Figure 1:

The amount by which the beam bends is measured by its radius of curvature. The following formula relates the amount of bending for a beam to the bending moment at that particular position:

$$C = \frac{YI}{r} \tag{1}$$

where

Y	is Young's modulus of the material
r	is the radius of curvature of the neutral surface
I	is the geometrical moment of inertia of the cross section of the beam
C	is the bending moment

The quantities r, I and C can in general vary along the beam. For a rectangular beam of thickness t and width w, the moment of inertia I is given by:

$$I = \frac{wt^3}{12} \tag{2}$$

For the situation in Figure 1(a) the bending moment at P due to the applied force is independent of the position of P so long as it lies between the knife edges. Since C is constant between the supports, the portion of the beam between the supports forms an arc of a circle if it is of uniform cross-section.

Centre-loaded Beam

In the experiment, we do not load the beam in the manner shown in Figure 1(a). Instead, a single load is applied in the middle, as shown in Figure 2(a), and the angle of deflection of the ends is measured. The bending moment now depends on the position of P along the beam and an integral is required to compute the deflection in terms of the variable radius of curvature. The problem may be simplified by thinking of the beam in two halves as shown in Figure 2(b). Each half is an inverted cantilever of length $L/2$ loaded with forces $F/2$ and we analyse this situation below.

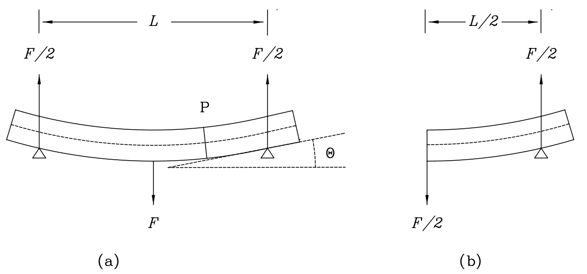


Figure 2:

Cantilever

In Figure 3, a cantilever of length L loaded at the end by a force F is shown. The wall exerts an upwards force F and an anti-clockwise moment FL in order to keep the cantilever in equilibrium.

If we consider a position P at a distance x from the load and imagine the beam to be cut at that position, the left-hand portion of the bar applies an upward force F and an anti-clockwise moment of Fx to maintain the right-hand portion in equilibrium. Using equation (1) to relate this moment to the radius of curvature of the beam at P, we find:

$$Fx = \frac{YI}{r} \tag{3}$$

Over a section of the beam of length dx, the incremental angular deflection $d\theta$ is given by:

$$r\,d\theta = dx \tag{4}$$

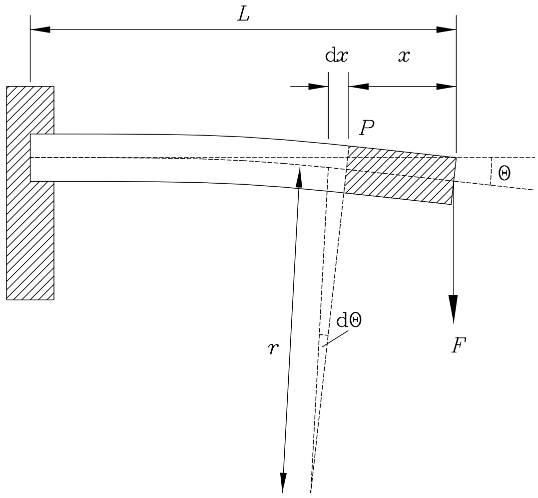


Figure 3:

Combining equations (3) and (4) and integrating over the length L of the cantilever yields the following expression for the total angular deflection θ:

$$\theta = \int_0^L \frac{dx}{r} = \frac{F}{YI}\int_0^L x\,dx = \frac{F}{YI}\frac{L^2}{2} \tag{5}$$

Returning to the centre-loaded beam used in the experiment, we simply substitute $L/2$ for L and $F/2$ for F in equation (5), giving:

$$\theta = \frac{F}{YI}\frac{L^2}{16} \tag{6}$$

Koenig's Apparatus

The angle of deflection θ may be determined using Koenig's Apparatus (see Figure 4). The mirrors may be placed at or beyond the knife-edge. Since the overhanging parts of the beam remain straight, θ is the same in either case. From the geometry of the light paths, θ may be shown to be given by:

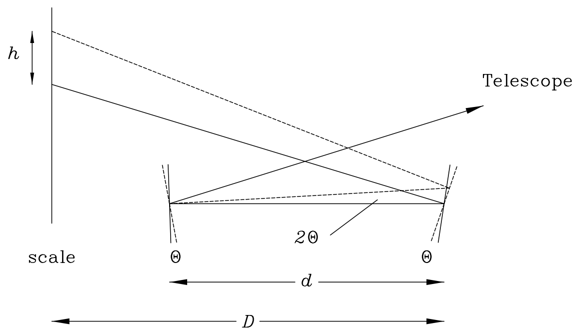


Figure 4:

$$\theta = \frac{h}{2\,(d + 2D)} \tag{7}$$

Suggestion:

Since t appears in equation (2) as t^3, make the side of shortest dimension t, and the other side (which is less uniform) w.

Question 1: Prove, for the situation in Figure 1(a), that the bending moment is constant.

Question 2: Prove, for the situation as in Figure 2 that the bending moment does depend on the position of P. If the beam is over loaded where would it fail?

Question 3: Derive equation (1).

Question 4: Arrange the following three beams in the order of their radius of curvature when Y, C and A and are equal:
(i) a round beam (ii) a square beam (iii) an H-shaped beam.

Question 5: Derive equation (2).

Question 6: Derive equation (7).

Procedure

1. Adjust the apparatus, and make preliminary tests with a full load using the steel beam. Note that, with the telescope, sharp images of both the scale and the cross-wires can be obtained simultaneously by exploiting the two mechanisms by which the telescope can be focussed (turning the knob and sliding the eyepiece).
2. Apply the load in stages and plot a graph to test Hooke's law. It is desirable to take a set of measurements while the beam is being loaded and another set while it is being unloaded.
3. Measure w and t at a number of points between the knife-edges and take the average. You could use either a metre rule, vernier calipers or a micrometer. Choose the most appropriate.
4. Use PYTHON to deduce the average value of Young's modulus for the steel beam from your data.
5. Estimate your standard error.
6. Repeat for the brass beam.

Write-up

The experiment write-up must include:

1. Answers to questions 1 to 6.
2. Plots of θ against F for the steel and brass beams.
3. Measurements of w and t for both beams.
4. Estimates of Young's modulus (with confidence limits) for both beams and a comparison with published data.

List of Equipment

1. 2 x Metal beams
2. 2 x Mirrors
3. 2 x Supporting stands
4. 6 x 1 kg Weights
5. 1 x Metre rule
6. 1 x 50 cm Viewing screen

7. 1 x Telescope
8. 1 x Lamp
9. 3 x Retort stands.

A.R. Poletti, S.M. Tan: February 10, 2011.

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