Experiment 342: Zeeman Effect Using Fabry-Pérot Etalon

Note: This experiment should only be attempted by students who have completed or are attending 453350FC: Advanced Quantum Physics.

Aim

The aim of this experiment is two-fold:

1. To acquaint you with an optical instrument of extremely high resolving power by which the wavelengths of two spectral lines may be accurately compared.
2. To measure the Zeeman splitting of spectral lines due to a magnetic field.

References

An excellent primer for the Zeeman effect is Griffiths' text (Ref. 1), but more detailed theory in Eisberg (Ref. 2) is strongly recommended. For experimental details and some theory see Melissinos (Ref. 3). For a detailed discussion of spectroscopy in general see Kuhn (Ref. 4). For a detailed discussion of the Fabry-Pérot etalon see Tolansky (Ref. 5).

1. D.J. Griffiths, "Introduction to Quantum Mechanics", Prentice-Hall, 1995
2. R.M. Eisberg & R. Resnick, "Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles", 2nd Ed., Wiley, 1985
3. A.C. Melissinos, "Experiments in Modern Physics", Academic Press, 1969
4. H.G. Kuhn, "Atomic Spectra", 2nd Ed., Longman, 1969
5. S. Tolansky, "An Introduction to Interferometry", 2nd Ed., Longman, 1973
6. Python plotting tutorial, `http://matplotlib.org/1.3.0/users/image_tutorial.html`

Refs. 3, 4 and 5 are available in the laboratory.

Procedure

(1) Familiarise yourself with the theory of Zeeman splitting and the Fabry-Pérot etalon.

(2) Familiarise yourself with the constant deviation spectrometer. A demonstrator must be consulted at this stage.

(3) Check out the photo camera and the card reader from the laboratory technician. The camera and reader need to be returned at the end of the session. The card reader will work on any of the Laboratory PCs.

(4) The Fabry-Pérot etalon should not need to be adjusted for parallelism. If it does, ask your demonstrator or a technician.

Note: Adjustments must be done *in situ* as moving the etalon can put the plates out of alignment. The simplest method is to adjust the clamping screws to give the best quality interference pattern of circular rings using a wide slit. When making these adjustments observe the Hg 5461 (green) line. When parallelism has been achieved, the fringes should be uniformly sharp around their circumferences. It is important to have the telescope correctly focussed and the collimator optimally positioned with

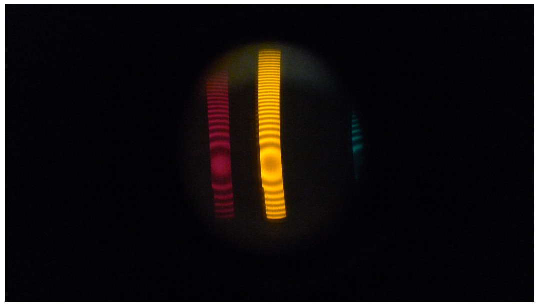

Figure 1: Example image

respect to the source. When correctly adjusted, a fraction of a turn of one of the three adjusting screws on the etalon should be sufficient to destroy perfect parallelism. If not, your technique is suspect. As with most optics experiments, the precision of the results is determined by the quality of your setting up procedure.

(5) Measure the approximate resolution of the instrument from the width of the fringes. This gives the dimensionless value, Δn, the fraction of order that one pattern must be separated from the other to be distinguishable if both are present simultaneously. Do this for both Hg and He.

Hint: Use Python to read the `.jpg` files stored on the memory card of the camera. You need to select a part of the image, analyse a single colour (eg. red, green, blue), and turn it into an intensity profile. See Ref. 6.

(6) Determine the exact spacing, t, of the Fabry-Pérot plates in terms of known wavelengths. Use the following neon lines as standards:

$$\begin{aligned} \lambda_{\mathrm{red}} &= 640.2246 \text{ nm} \\ \lambda_{\mathrm{yellow}} &= 585.2488 \text{ nm} \end{aligned}$$

Plot ring number against radius squared to determine the fractional order ε (see Refs. (3) or (5)). Determine the intercept graphically or better from a least squares fit. Then refer to the appendix to calculate t.

Observe another prominent line in the Ne spectrum and determine its wavelength using the known spacing.

(7) Observe the Zeeman splitting of He and natural Hg.

(a) The $\lambda = 501.5\,\text{nm}$ line of He is due to the transition ${}^1\mathrm{P}_1 \rightarrow {}^1\mathrm{S}_0$. From theory, predict the expected splitting pattern and the polarization of the lines. Confirm your prediction by measuring the splitting and using the polaroid filter supplied.

Due to the difficulty of measuring the splitting of the fringes by direct observation (as opposed to photographically), it is sufficient to determine the field for "half-order" splitting. In Fig. 2(c), you can see that the upper and lower components of adjacent fringes have merged. This condition is easy to observe and corresponds to "half-order" splitting.

(b) Now observe the Zeeman effect on the Hg 546.1 nm line. This is a ${}^3\mathrm{S}_1 \rightarrow {}^3\mathrm{P}_2$ state transition. Predict the number of lines that should be observed for this transition with the polariser parallel and perpendicular to the magnetic field.

The experimental situation is complicated by the fact that natural Hg contains a mixture of isotopes. Look up the nuclear spins, mass numbers and abundances of these isotopes and decide to what extent your observations should accord with the prediction.

(c) By choosing the appropriate experimental arrangement, measure the "half-order" splitting for the Hg 546.1 nm line. From this measurement and the similar measurement for He, determine the value of the universal constant μ_o/hc.

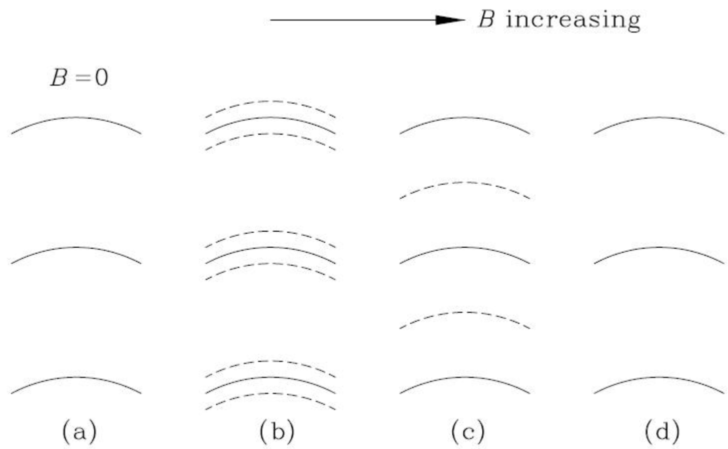

Figure 2: Zeeman splitting of spectral lines

Questions

1. Why is the Hg 5461 line used for getting parallelism?
2. Why is the effective resolution greater if one is comparing two wavelengths separately rather than simultaneously?
3. Why did you need to use the polarizer?
4. Calculate the ratio of the Doppler broadening of Hg to that of He and compare with the results of your fringe width measurements (see Melissinos p 306 in 1^{st} edition or p 236 in 2^{nd} edition). What do you conclude from these results?
5. The splitting of spectral lines can be due to one or more of the following causes:
   (a) Fine structure. This is due to electron spin. The classic example is the sodium D line splitting.
   (b) Zeeman splitting due to the application of an external magnetic field.
   (c) Hyperfine splitting due to the interaction of the nuclear moments with the magnetic field of the electrons at the nucleus.
   (d) Hyperfine structure due to isotopic shift.

   Calculate the order of magnitude of each of the above effects and decide which will be observable with the apparatus supplied.

Appendix

It is stated that the condition for an intensity maximum to occur in a Fabry-Pérot etalon is

$$2t = n\lambda \tag{1}$$

with thickness t, order number n and wavelength λ. We are given two wavelengths,

$$\begin{aligned} \lambda_1 &= 609.6\,\text{nm} \\ \lambda_2 &= 585.2\,\text{nm} \end{aligned}$$

We also deduced the corresponding fractional orders from our graph,

$$\begin{aligned} \epsilon_1 &= 0.20 \\ \epsilon_2 &= 0.90 \end{aligned}$$

Now, suppose the maker's value for the spacing $t = 10.040\,\text{mm}$. Using (1), it follows that the order for λ_1 is

$$n = \frac{2t}{\lambda_1} = 32939.6325... \approx 32940$$

and we know by observations that the fractional order for this line is $\epsilon_1 = 0.20$. Since the real n lies in the vicinity of 32940, we can say that

$$n_{\text{hypothetical}} = 32940 + \text{integer} + \epsilon_1 = 32940.20 + \text{integer}$$

We take a couple of different hypothetical order numbers such as 32941.20, 32942.20, 32943.20, etc. to find a range of corresponding t.

Then we use these t to find n_2 for the other known λ_2. There also exists a fractional order for this wavelength, ϵ_2. By inspecting your n_2, one (or more) of these values lies close to

$$\text{integer} + \epsilon_2$$

If there is more than one n_2 value, calculate t from these n_2 and compare them with the maker's value. The t which lies closest to 10.040 mm gives the exact etalon spacing. (If there is a chance of ambiguity it is a good idea to use a third standard line.)

Hypothetical order for λ_1	Corresponding order calculated for λ_2
32941.20	34314.688
32942.20	34315.730
32943.20	34316.772
32944.20	34317.813
32945.20	34318.855
32946.20	**34319.897**
32947.20	34320.938

Thus, $t = 10.04200186$ mm.

List of Equipment

1. Geissler tubes and high voltage supply (this is potentially lethal)
2. Electromagnet and power supply
3. Gaussmeter for magnetic field measurements
4. Optical bench equipped with:
   collimator, Fabry-Pérot etalon, constant deviation prism, telescope
5. Polaroid filter and stand
6. Digital photo camera
7. SD card reader

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