Experiment 225: Measurement of Magnetic Susceptibility

Aim

To determine the magnetic susceptibility χ_m of a paramagnetic solution of ferric chloride using Quincke's method for liquids.

Reference

1. Reitz, Milford and Christy, "Foundations of Electromagnetic Theory" (1992).

An excerpt from this text is with the apparatus.

Introduction

The magnetic field at any point in a material is given by:

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

where

$\mathbf{B}$	is the magnetic field	(Tesla, T)
$\mathbf{H}$	is the magnetic intensity	($\mathrm{A\,m^{-1}}$)
$\mathbf{M}$	is the magnetic moment per unit volume	($\mathrm{A\,m^{-1}}$)
μ_o	is the magnetic permeability in vacuo	($\mathrm{T\,m\,A^{-1}}$)

The magnetic moment density $\mathbf{M}$ is sometimes called the "intensity of magnetisation". In a class of materials, in particular isotropic non-ferromagnetic materials, $\mathbf{M}$ and $\mathbf{H}$ are related by the following approximate relationship:

$$\mathbf{M} = \chi_m \mathbf{H}$$

where the dimensionless scalar quantity χ_m is called the volume-susceptibility. It is a measure of three things:

(i) the strength of magnetic dipoles present at the atomic level,

(ii) how readily these dipoles will align to an external magnetic field and

(iii) the number of these dipoles per unit volume in the particular sample of the material.

If χ_m is positive, the material is called paramagnetic, and the magnetic field is strengthened by the presence of the material.

If χ_m is negative, the material is called diamagnetic, and the magnetic field is weakened by the presence of the material.

The magnetic field at any point in a sample of an isotropic linear material is then given by:

$$\mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H} = \mu \mathbf{H}$$

where $\mu = \mu_0 (1 + \chi_m)$ is the magnetic permeability of the sample. The magnetic susceptibility is a measure of the fractional change in the permeability of the material relative to free space:

$$\chi_m = \frac{\mu}{\mu_0} - 1$$

Note: Many sources of data, for instance Handbook of Chemistry and Physics, list magnetic susceptibility in Gaussian units (one of a family of cgs units used in electromagnetism). The relationship between χ_{SI} and χ_{cgs} is $\chi_{\mathrm{SI}} = 4\pi\,\chi_{\mathrm{cgs}}$. Furthermore these references quote magnetic susceptibility per unit concentration $\chi_{m,\mathrm{molar}}$ of a given substance within a sample. The units of concentration are then important. They are often cited as moles per cubic centimetre ($\mathrm{mol\,cm^{-3}}$).

The most straightforward method for measuring paramagnetic and diamagnetic susceptibility was invented by Faraday. It depends upon measuring the resultant mechanical force on a specimen of the material *partly* immersed in a magnetic field. Quincke's method for a liquid is particularly elegant. The net force (due to magnetization) on a liquid in a tube between the poles of an electromagnet is balanced by hydrostatic pressure. The latter is measured by the difference in levels when one surface of the liquid (in our case a solution of ferric chloride) is in a uniform field and the other surface is in a region of zero field.

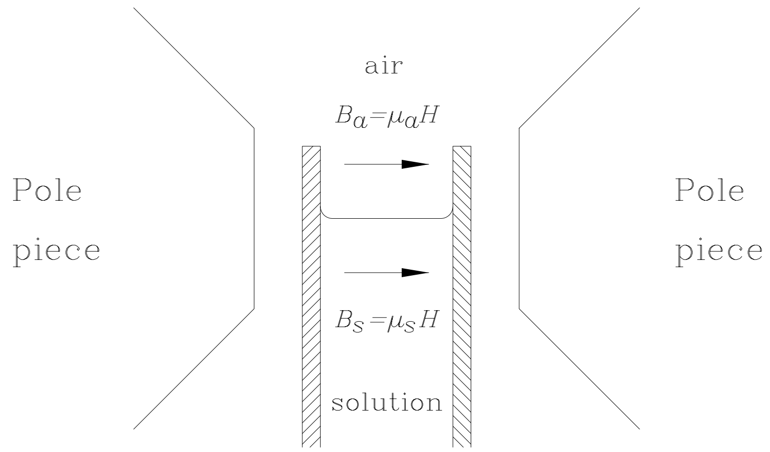

Figure 1:

Theory

Stress in matter due to the application of electric and magnetic fields is treated in detail in advanced texts on electromagnetism such as the reference given. The development is complex but fortunately the results can be summarised in a simple geometric form. It is this simple form, due originally to Maxwell, which is used here for a purely magnetic field.

The magnetic force on a body can be accounted for by assuming that stress is applied over an enclosing surface, conveniently taken to be the boundary surface of the body. Consider the element dS of the surface shown in Figure 2.

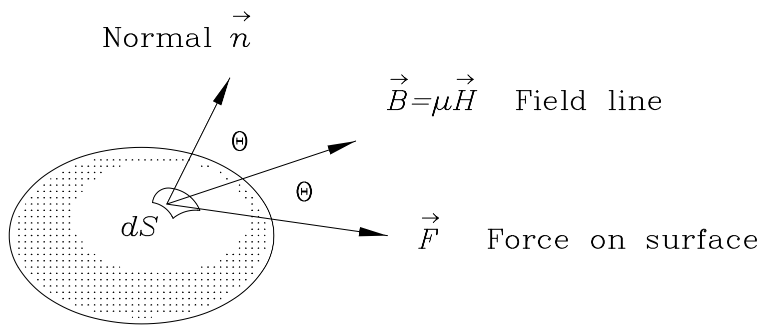

Figure 2:

The magnitude of the resultant force across dS due only to the presence of a magnetic field is given by:

$$|\mathbf{F}| = \frac{1}{2}\frac{\mathbf{B}\cdot\mathbf{B}}{\mu} = \frac{1}{2}\mu\left(\mathbf{H}\cdot\mathbf{H}\right) dS$$

The force $\mathbf{F}$ acts in the plane containing the outward normal $\mathbf{n}$ and the field lines. It is independent of the sense of the field. The field line bisects the angle between $\mathbf{F}$ and $\mathbf{n}$ so that when $\mathbf{B}$ makes an angle θ with $\mathbf{n}$, the angle between $\mathbf{F}$ and $\mathbf{n}$ is 2θ. Hence when the field is tangential to the boundary, $\mathbf{F}$ acts in the opposite direction to $\mathbf{n}$, i.e. along the inward normal.

Consider now the surface of a liquid in a tube between the poles of an electromagnet. The magnetic field is applied parallel to the surface. Then B/μ $(= H)$ is continuous across the surface (see Figure 1).

Downwards the field applies a force per unit area given by (the subscript "a" refers to "air"):

$$\frac{1}{2}\frac{B_a^2}{\mu_a} = \frac{1}{2}\mu_a H^2$$

and upwards, within the liquid, a force per unit area given (subscript "s" refers to "solution"):

$$\frac{1}{2}\frac{B_s^2}{\mu_s} = \frac{1}{2}\mu_s H^2$$

If $\mu_a \neq \mu_s$ these forces do not balance and to maintain equilibrium there must be a hydrostatic pressure difference:

$$\frac{1}{2}H^2\left(\mu_a - \mu_s\right)$$

In the experiment we further assume that the liquid density ρ is constant and that the liquid in the tube is connected to a reservoir which is in a region of zero field. The hydrostatic pressure due to the application of the field is conveniently measured by adjusting the height of the reservoir (field-free) surface to hold the meniscus in the fixed tube at the level before the field is applied. If h is the change in height of the surface in the reservoir, then:

$$\frac{1}{2}H^2\left(\mu_a - \mu_s\right) + \rho g h = 0$$

For paramagnetic materials, $\mu_s > \mu_a$. Hence h is positive, i.e. a decrease in height occurs. For diamagnetic materials, h is negative and an increase in height occurs. The sample supplied is a saturated aqueous solution of ferric chloride that is strongly paramagnetic in comparison with air (water is weakly diamagnetic) and:

$$\mu_a = \left(1 + 3.8\times10^{-7}\right)\mu_0 \approx \mu_0$$

then

$$\begin{aligned}\rho g h &= -\frac{1}{2}H^2\left(\mu_a - \mu_s\right)\\ &\approx \frac{1}{2}\mu_0 H^2\left(\frac{\mu_s}{\mu_0} - 1\right) = \frac{1}{2}\frac{B^2}{\mu_0}\chi_m\end{aligned}$$

Procedure

1. The electro magnets are powered by a high current three phase D.C. supply. Current to the electro magnets is controlled by turning the handles and compressing the stack of carbon blocks, thus changing the series resistance. A breaker switch is in series with the current meter enabling the current to the magnets to be turned on and off.

   CAUTION: Reduce current to the magnets first before opening or closing the switch. Failure to do so will cause arcing on the switch contacts and may lead to them being welded together. Ensure you reduce the current after taking your measurements as overheating will take place. You must turn the equipment off after use.

2. For a series of suitable current settings, measure h using a travelling microscope and B using the Hall probe supplied. Note that the scales are in Gauss (1 Gauss = 10^{-4} Tesla). You must use the probe carefully to get good results. Consult your demonstrator if you are in doubt.

3. Use PYTHON and the `regress` function to determine the slope of the plot of B^2 versus h.

   Alternatively use a graphical method if you have not learnt how to use PYTHON. Determine the slope k of the graph and estimate the standard error σ_k as follows:
   For each point estimate the standard error σ_i in the vertical value y_i and draw an error bar joining $y_i - \sigma_i$ to $y_i + \sigma_i$. If necessary, error bars can be included in the horizontal values. Draw the lines of greatest and least slope which give an acceptable fit to the points on the graph. If the slopes of these lines are k_1 and k_2 then:

$$k = \frac{1}{2}(k_1 + k_2) \qquad \text{and} \qquad \sigma_k = \pm\frac{1}{4}(k_1 - k_2)$$

4. Calculate χ_m and its standard error using the values:

$$\rho = (1.33 \pm 0.01) \times 10^3 \quad \mathrm{kg\,m^{-3}} \text{ (density of a saturated solution of } \mathrm{FeCl_3} \text{ at } 20^\circ\mathrm{C})$$
$$g = 9.799 \pm 0.001 \quad \mathrm{N\,kg^{-1}}$$

5. Consult the section on "Concentrative Properties of Aqueous Solutions" in the Handbook of Chemistry and Physics (page D-237 of the 60^{th} Edition). Interpolate between entries in the table to find the concentration ρ_m of $FeCl_3$ in $\mathrm{mol\,cm^{-3}}$ for the given density of the $FeCl_3$ solution. Also find the density of solid $FeCl_3$.

6. Consult the CRC Handbook of Chemistry and Physics (page E-125 of the 60^{th} Edition) and obtain values of the magnetic susceptibility of $FeCl_3$, water, oxygen (gas) and nitrogen (gas). Note that the values given in the Handbook are molar susceptibilities $\chi_{m,\text{molar}}$ (in Gaussian units which they refer to rather loosely as 'cgs units'). To convert to volume susceptibility (in SI units) use:

$$\chi_m = 4\pi\chi_{m,\text{molar}}\,\rho_m$$

   In this equation ρ_m is the molar density of the substance (in moles per cubic centimetre) and the factor of 4π makes the conversion from Gaussian to SI units.

7. Use the results you obtained in Procedures 5 and 6 to calculate the susceptibility of the $FeCl_3$ solution (recall from page 225-1 the three factors upon which volume-susceptibility depends). Compare with the result of your experiment.

Questions

These must be answered as part of the write-up.

1. Suppose the liquid sample was replaced by a solid rod. What measurements would you make to determine χ_m for the rod? Would this technique be satisfactory for iron or steel? If not, why not? How would you determine χ_m for a ferromagnetic material?

2. Estimate the value of h (in the apparatus of this experiment) for pure water using the values obtained in Procedure 6.

List of Equipment

1. 1 x Electromagnet

2. 2 x DC Power Supply 20-50 V

3. 1 x Carbon Stack Variable Resistance
4. 2 x Travelling microscope
5. 1 x Ammeter 0-50 A
6. 1 x Switch
7. 1 x Adjustable Manometer containing Ferric Chloride
8. 1 x Hall Probe (Type STG 3-0404) with Bell 600 Fluxmeter

P.R. Wills
April 2000

Updated: M. Brett, 10 May 2013

This version: October 1, 2014