Experiment 316: Seismic Ray Tracing

Aim

To trace the ray propagation of seismic body waves using a microcomputer and to estimate the epicentral distance of two earthquakes from recorded seismograms.

References

1. K.E. Bullen & B.A. Bolt "An Introduction to the Theory of Seismology" (1985) (Sci.Lib. 551.22 B93)
2. H. Jeffreys & K.E. Bullen "Seismological Tables" (1958) (available in Adv. Lab.)
3. F.D. Stacey "Physics of the Earth" (1977) (Sci.Lib. 551 S77)

1 Introduction

A microcomputer is used to trace the ray propagation of seismic body waves (P and S waves) generated by a hypothetical earthquake having a surface focus. Ray intersections with the earth's surface, core-mantle boundary and core-innercore boundary are detected and the student can select from a menu of options (reflection, refraction, possible mode change) for the continuation of the ray. For each ray path, the angular distance and travel time are printed and may be compared with observed values in the Jeffreys-Bullen tables. By plotting a variety of ray paths, the student should be able to demonstrate the existence of a "shadow zone" for P waves.

The computer program uses seismic ray velocity profiles given by Bullen in the 1963 edition of Ref. 1. Among the reasons for discrepancies between observed times and the computer simulation are errors in the velocity profiles and their computer simulation.

Copies of two seismograms recorded at Scott Base are available for student interpretation. The task is to estimate the epicentral distance of the earthquake from the observatory using the Jeffreys-Bullen travel time tables.

2 Elastic wave theory

The elastic waves which may propagate through a homogeneous, isotropic, perfectly elastic medium are of two types: P (primary or "push") waves and S (secondary or "shake") waves. Wave equations for these two types can be derived from the equations of motion for such a medium (see Bullen & Bolt sections 2.1-2.4 and 4.1). The P waves are dilational-compressional (i.e., sound) waves and are described by the equation:

$$\rho \frac{\partial^2 \theta}{\partial t^2} = \left(K + \frac{4}{3}\mu \right) \nabla^2 \theta \tag{1}$$

where ρ is the mass density, K is the bulk modulus, μ is the rigidity modulus and θ is the dilatation:

$$\theta = \nabla \cdot \mathbf{u}$$

where $\mathbf{u}$ is the particle displacement vector. The S waves are rotational waves and are described by the equation:

$$\rho \frac{\partial^2 \xi}{\partial t^2} = \mu \nabla^2 \xi \tag{2}$$

where

$$\xi = \nabla \times \mathbf{u}$$

By comparison with the wave equation for waves at speed c:

$$\frac{\partial^2 A}{\partial t^2} = c^2 \nabla^2 A$$

the speeds of the two types of waves are thus:

$$\text{P waves: } \alpha = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \text{ and for S waves: } \beta = \sqrt{\frac{\mu}{\rho}}$$

For bodily elastic waves in the earth the above theory holds to a good approximation and the variation in the wave velocities in the various regions of the earth's interior (mantle, outer core, inner core) is generally slow enough for ray theory to be applicable. At the boundaries of these regions both reflection and refraction occur; also there is partial conversion of P waves to S waves and vice versa.

At sufficient distances from a point earthquake source, the seismic waves are effectively planar. The P waves are longitudinal but the S waves are transverse and may be plane-polarised. S waves with a horizontal plane of polarisation are denoted by SH while waves with a vertical plane of polarisation are denoted by SV. Assuming spherical stratification of the earth's interior the interchange of energy between P and S modes at a reflecting boundary is as follows: both P and SV waves (in general) give rise to reflected P, reflected SV, refracted P and refracted SV waves, but SH waves are reflected and refracted only in the SH mode. In general, an S wave has both SH and SV components, but tends to SH if the SV component is diminished by conversion to P. Snell's Law is applicable to elastic P and S waves in the form:

$$\frac{\sin i_p}{\alpha_1} = \frac{\sin i_s}{\beta_1} = \frac{\sin j_p}{\alpha_2} = \frac{\sin j_s}{\beta_2} \tag{3}$$

where i_p and i_s are angles of incidence between two media in which the P and S velocities are α_1 and β_1 respectively and j_p, j_s are angles of refraction (see Fig. 1). For further details, see Bullen & Bolt (1985). In particular, note that if $\beta_1 < \beta_2$ and $\sin i_s > \beta_1/\beta_2$ there is total reflection of the S waves; similarly, if $\alpha_1 < \beta_2$ and $\sin i_p > \alpha_1/\beta_2$ there is total reflection of the P waves.)

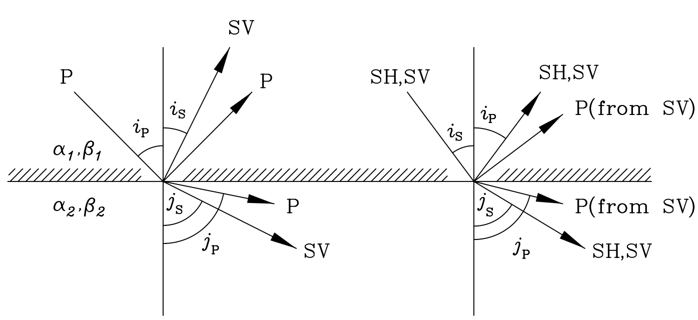


Figure 1: Reflection and refraction of P and S waves at an interface

3 Seismic ray theory

Consider a seismic P or S wave propagating through a spherically stratified earth, i.e. one composed of an indefinitely large number of thin, homogeneous, concentric spherical cells, and consider a portion of the ray path which crosses two such shells in which the wave velocities are v_1 and v_2 respectively.

By Snell's law:

$$\frac{\sin i_1}{v_1} = \frac{\sin j}{v_2}$$

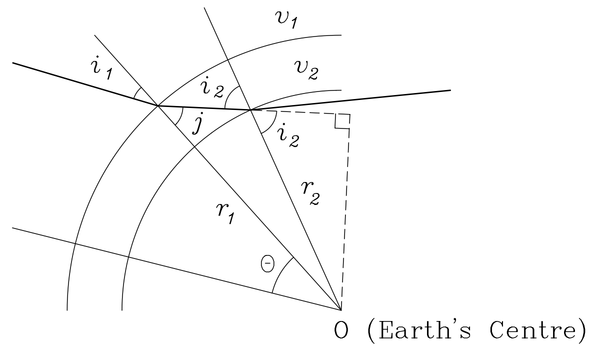


Figure 2: Propagation of a ray through a spherically stratified earth

and from the sine rule:

$$r_1 \sin j = r_2 \sin i_2$$

Therefore

$$\frac{r_1 \sin i_1}{v_1} = \frac{r_1 \sin j}{v_2} = \frac{r_2 \sin i_2}{v_2}$$

or, in general a ray with angle of incidence i and speed v at radius r,

$$p = \frac{r \sin \iota}{v} = \text{constant} \tag{4}$$

$\mathrm{d}r$, $r\mathrm{d}\theta$ and $\mathrm{d}s$ are the sides of a right-angle triangle with $\mathrm{d}s$ as the hypotenuse and i as the angle between $\mathrm{d}r$ and $\mathrm{d}s$. We then have:

$$\sin i = r\frac{\mathrm{d}\theta}{\mathrm{d}s} \tag{5}$$

$$(\mathrm{d}s)^2 = (\mathrm{d}r)^2 + (r\,\mathrm{d}\theta)^2 \tag{6}$$

where measures the path length. From (4) and (5) we get:

$$\mathrm{d}s = \frac{r^2}{pv}\mathrm{d}\theta \tag{7}$$

and substituting (7) into (6) we have:

$$\left(\frac{r}{pv}\right)^2 = 1 + \left(\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2 \tag{8}$$

Differentiation of (8) with respect to θ gives:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right) = \left(\frac{r}{pv}\right)^2\left(1 - \frac{r}{v}\frac{\mathrm{d}v}{\mathrm{d}r}\right) \tag{9}$$

Note also that

$$\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta} = \cot i \text{ and } \left(\frac{r}{pv}\right)^2 = \operatorname{cosec}^2 i$$

The Parameters Δ, T

The Δ-value of a ray path is the angle subtended at the earth's centre by the ray path (see Fig. 3). Clearly

$$\Delta = \int \mathrm{d}\theta$$

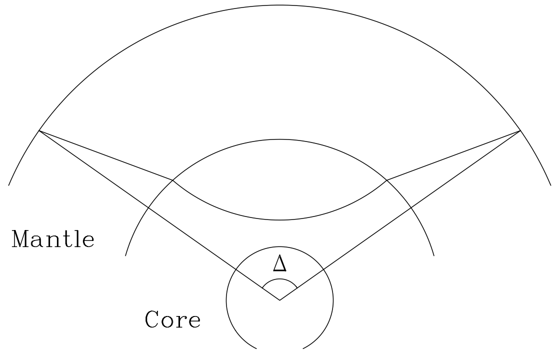


Figure 3: The Δ value of a ray path

The travel time along the path is denoted by T:

$$T = \int \frac{\mathrm{d}s}{v}$$

By (7) we have the relation:

$$T = \frac{1}{p} \int \frac{r^2}{v^2}\, \mathrm{d}\theta \tag{10}$$

4 Ray tracing program

The computer solves numerically the following set of first order differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right) = \left(\frac{r}{pv}\right)^2 \left(1 - \frac{r}{v}\frac{\mathrm{d}v}{\mathrm{d}r}\right) \tag{11}$$

From equation (10):

$$\frac{\mathrm{d}T}{\mathrm{d}\theta} = p\left(\frac{r}{pv}\right)^2 \tag{12}$$

and writing

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = r\left(\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right) \tag{13}$$

Equations (11), (12) and (13) can be written in the form:

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}\theta} = \mathbf{f}\left(\theta, \mathbf{X}\right)$$

In the present set, does not appear on the right-hand side, hence:

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}\theta} = \mathbf{f}\left(\mathbf{X}\right) \tag{14}$$

in which

$$\begin{aligned} X_1 &= \frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta} = \cot i \\ X_2 &= T \\ X_3 &= r \end{aligned}$$

On the right-hand side of (11):

$$\left(\frac{r}{pv}\right)^2 = \operatorname{cosec}^2 i = 1 + X_1^2$$

is derived from X_1. The symbol p is a constant for the ray, evaluated before entry to the integration procedure, and remaining constant through reflection, refraction and mode changes.

$1-(r/v)\,(\mathrm{d}v/\mathrm{d}r)$ is a function only of the mode and the radial distance from the earth's centre. This function is simulated in the program by segments of third-order polynomials fitted to data taken from Bullen's velocity profiles.

Fourth-order Runge-Kutta procedures are used to integrate the equation set (14).

5 Running the program

The program is written in the Visual Basic language and runs on an IBM-compatible computer. It is available on any of the Laboratory's IBM-compatible computers and can be run by double-clicking on the "Seismic" icon on the Windows 95 desktop. Ask for help if you are not familiar with running programs on an IBM-compatible computer.

The program provides two windows, one showing the currently selected parameters together with a graphical plot of rays traced, and the other listing ray histories. To trace a ray, set the start angle for the ray (measured in degrees from vertically upwards), select P or S wave type, and click on the go box. As the ray path is incremented, the program checks to see whether a boundary has been crossed. If it has, the program pauses to allow the user to select whether the ray is to be reflected or refracted, and to change wave type if desired.

A new ray can be started at any stage by editing the start angle box, and selecting the desired wave type. The plot of rays is not cleared until the Reset box is clicked. The resulting accumulation of rays can give a clearer picture of phenomena such as shadow zones.

At any stage, the screen plot and the ray history for the displayed rays can be printed by clicking on the Print box. The total range and travel time to the end of each segment is shown for a ray. Note that seismologists concatenate the labels for the individual segments to describe a ray, so that a refracted ray through the inner core of P wave type throughout is denoted PKIKP, and an S wave reflected off the outer core is denoted ScS. The full nomenclature is found in the references, but you can deduce it by tracing various rays and reading their histories.

To a large extent students may "do their own thing" with the program. An obvious exercise is to demonstrate the "shadow zone" for P waves using a bundle of rays narrowly missing the earth's core and a suitable bundle refracted through the core. The angular limits of the shadow zone should be compared with the observed limits. The velocity structure in the upper mantle can lead to three different P arrivals at some ranges, and students may wish to examine this phenomenon. Other exercises are to generate synthetic travel time plots and find what modes can be received at a particular location.

6 Interpretation of seismograms

The purpose of this part of the experiment is to acquaint students with an important practical application of seismic ray theory - the location of the foci of earthquakes using seismograms. A seismogram from the D.S.I.R. Scott Base observatory is provided and students can use the Jeffreys-Bullen travel time tables to estimate the epicentral distance of the earthquake from the observatory.

Question:

How much data is needed in order to determine the geographic position of the epicentre and the focal depth of the earthquake?

For the earthquakes treated here, the focus may be assumed to be at the base of the crustal layers (33 km depth or $h = 0.00$ in the J-B tables). The various seismic "phases" appear on the seismogram as trains of waves, rather than as individual pulses as might be expected on the basis of simple earthquake theory (Bullen & Bolt, 1985, Ch 4). This complicates the task of interpreting of the seismograms, and in general some skill is required to identify particular phases and their starting times. However, the Scott Base seismograms used in this experiment are relatively clear and noise-free; also, the order of arrival of the phases is given to assist the student (see below).

Copies of either of two Scott Base seismograms are under the glass on one of the Laboratory tables. These seismograms are from a long-period seismograph which records the vertical (Z) component of ground vibrations. (This is usually more useful than the NS or EW components.) They are made up of 24 1-hr traces (each corresponding to a complete rotation of the recording drum) with time marks every minute (c.f. Figure 6.3 in Stacey; the time scale is 1 mm = 4 sec.). Each seismogram contains one set of fairly large-amplitude wave trains which are due to the arrival of seismic waves from a distant earth quake. The order of arrival of the seismic phases is as follows:

Jan. 5, 1969	P, PP, PPP, S, SS, SSS
March 31, 1969	PKIKP, PP, PKS, PPP, SKS, PS, PPS, SKKS, SSS

Surface waves are also present (in fact they are the part of the wave train with largest amplitude) but there is no point in trying to use the arrival times of these waves as their velocity depends on the propagation path; the second, smaller set of wave trains on the March 31 seismogram is due to a distant deep-focus earthquake – note that this does not excite significant surface waves.

Reduced copies of the relevant parts of each seismogram are available in the Laboratory. Include these sheets, suitably annotated, with your write-up. For each case, examine the seismogram carefully, mark in the starting times of the seismic phases listed above, and calculate Δ for the earthquake.

In order to estimate Δ, you may use the numerical values of T in the tables, or you may find it more convenient to plot the arrivals to scale along the edge of a strip of paper for direct comparison with Figure 5 of the J-B tables. This can aid you in identifying phase and problems where arrivals have been incorrectly "picked". Note also that the travel time for a given phase on a seismogram may differ from the globally averaged travel-time tables by many seconds, reflecting local geology and deeper departures of the Earth from spherical symmetry.

Hints for the Jan 5 Earthquake:

The P and S arrivals are quite well-defined (note that S usually exhibits lower frequency oscillations than P. The P-S time difference and the J-B travel time curves may be used to obtain a rough value for Δ; hence accurate values of T_p, T_s, Δ and other starting times.

Hints for the March 31 Earthquake:

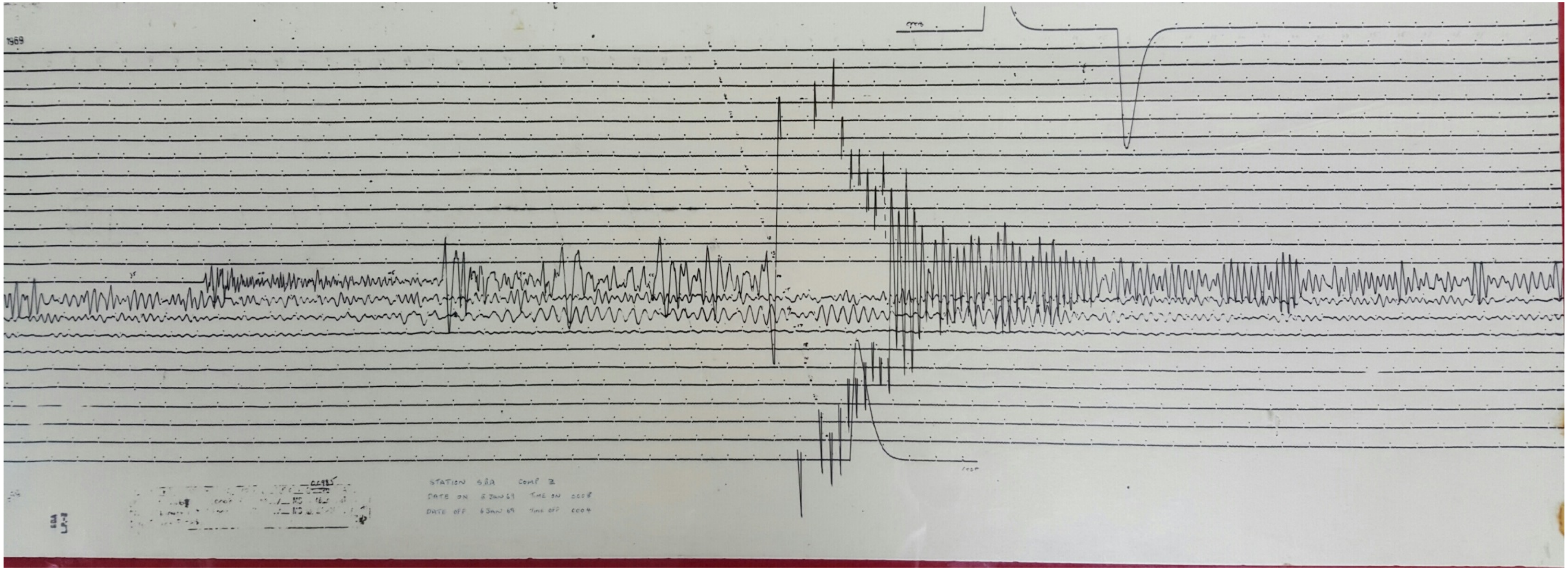
The first three arrivals, while not of large amplitude, are well-defined and may be used in the same way as the P and S starting times are used in the case of the Jan. 5 earthquake. The PKIKP arrival occurs very near a timing mark. (Note: this phase is labelled PKP in the J-B tables).

List of Equipment

1. IBM Compatible PC
2. SEISMIC program
3. Inkjet Printer

B.J. Brennan

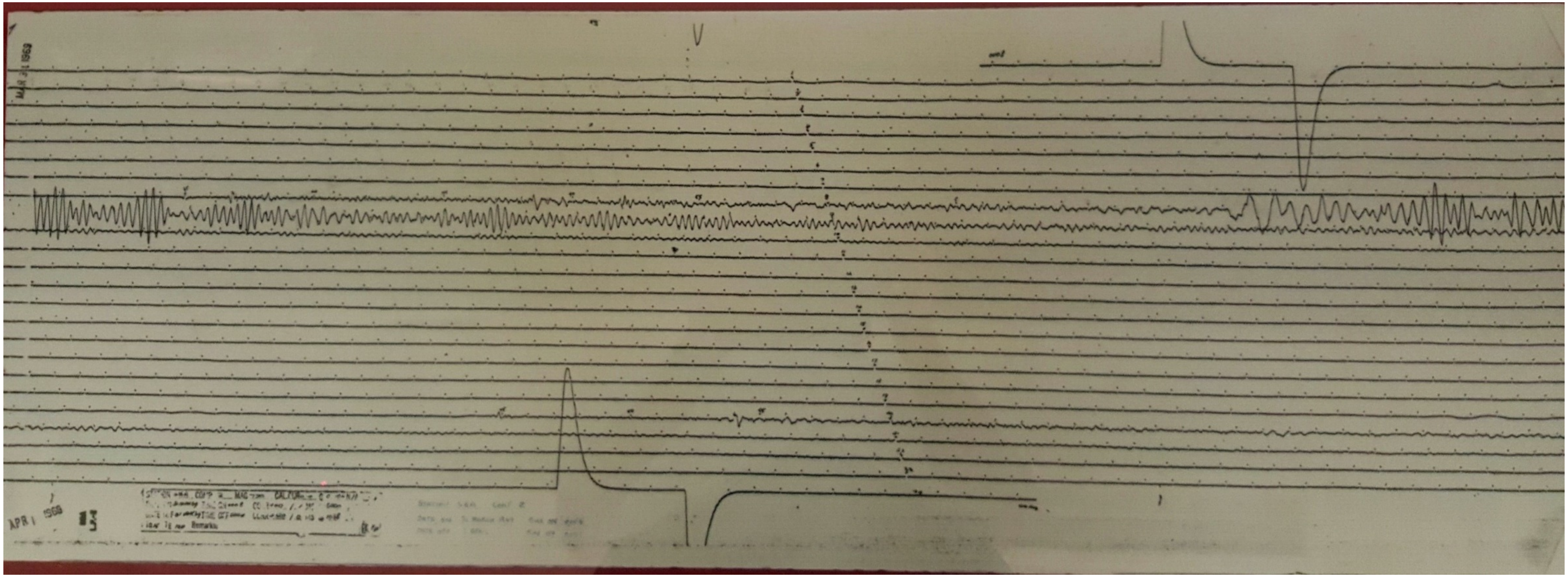
February 1997



Station S.B.A Comp Z

Date on 5th January 1969 Time on 0008

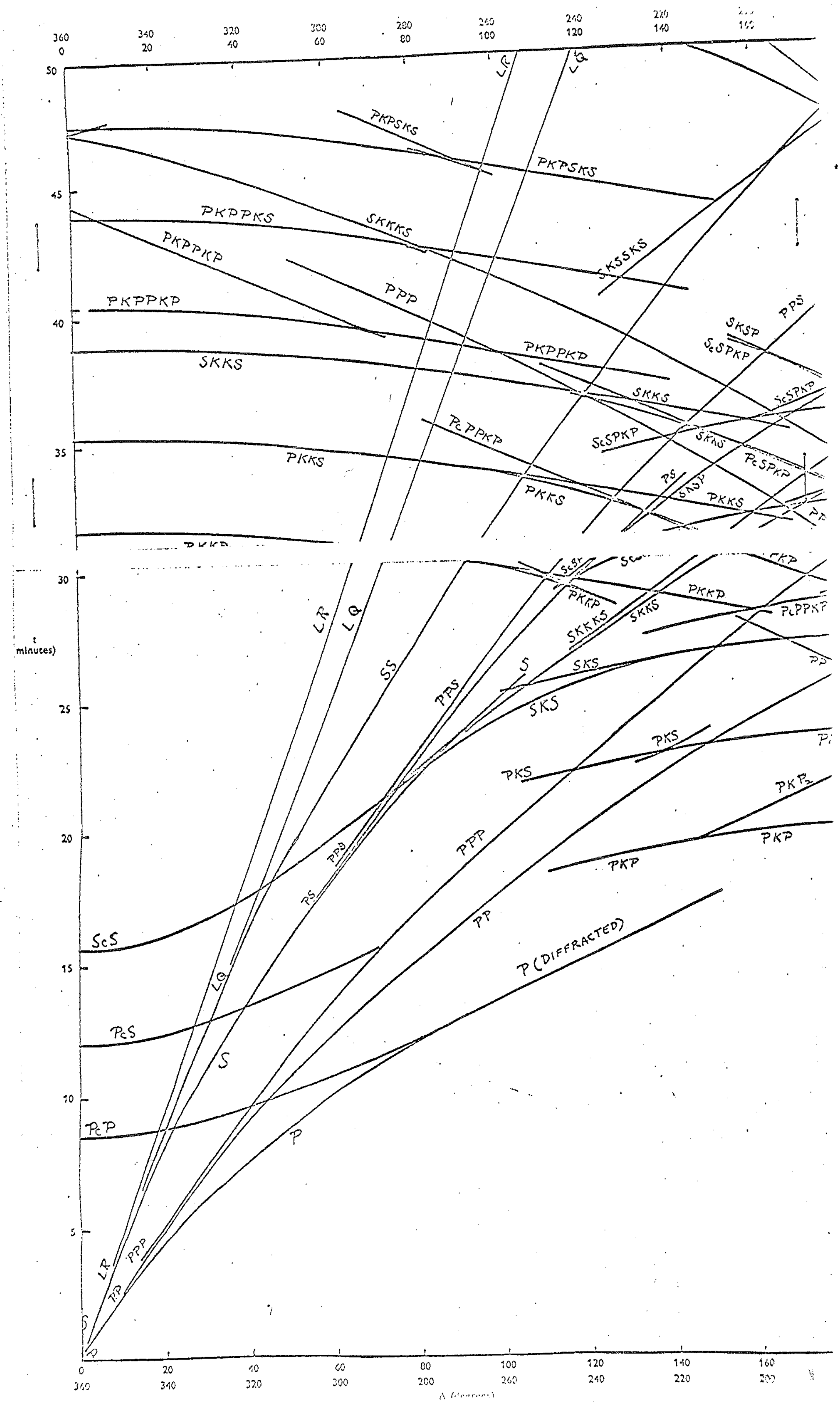
Date off 6th January 1969 Time off 0004



Station S.B.A Comp Z

Date on 31st March 1969 Time on 0008

Date off 1st April 1969 Time off 0001



PKPSKS
PKPSKS
PKPPKS
SKKKS
PKPPKP
SKSSKS
PPP
PKPPKP
SKSP
PPS
SKKS
PKPPKP
SKKS
PcPPKP
ScSPKP
PKKS
PKKS
PKKS
LR
LQ
SS
PPS
PKKP
SKKKS
SKKS
PKKP
S
SKS
SKS
PKS
PKS
PKP
PKP
PPP
PP
P (DIFFRACTED)
ScS
PcS
S
PcP
P
minutes)