Experiment 391: Passive Pulse Circuits

Aim

The aim of this experiment is to acquaint students with the response of simple linear passive electrical networks to transient voltage waveforms and to introduce them to the techniques of handling pulses in more complex electronic circuits.

Theory

All the relevant theory is presented in standard texts on linear networks, but very little mathematics, beyond that required to determine the response of a series R-C circuit to an input, is needed for the first five sections of the experiment. In the circuit of Fig. 3(a), the charge $q(t)$ on the capacitor is related to the voltage $v_o(t)$ across it by $q(t) = Cv_o(t)$. Kirchoff's voltage law gives:

$$v_i(t) = i(t)\,R + \frac{q(t)}{C}$$

Since the current $i(t)$ is the time-derivative of $q(t)$, this leads to the differential equation:

$$v_i = R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{q}{C}$$

which may in turn be written in terms of v_0 as:

$$\frac{\mathrm{d}v_o}{\mathrm{d}t} + \frac{1}{RC}v_o = \frac{1}{RC}v_i.$$

This is the differential equation which must be solved in order to find $v_o(t)$, for any given $v_i(t)$. Either elementary methods, or Laplace transforms can be used to solve such differential equations. In this experiment a Pulse Function Generator (PFG) is used to generate a variety of input excitations $v_i(t)$, and we seek to explain the form of $v_o(t)$.

It can rapidly become laborious to formulate and solve differential equations, and so several useful short-cuts may be employed. All the circuits we consider in this experiment give rise to **linear** differential equations with **constant coefficients**. For such circuits we can readily show that if we know that a particular input function $v_i(t)$ gives rise to an output function $v_o(t)$, then:

1. The response to a scaled version $av_i(t)$ of the input will be $av_o(t)$, i.e., the correspondingly scaled version of the original output.
2. If we know how the network responds to each of two input signals applied individually, the response to the sum of these inputs will be the sum of the corresponding responses.
3. If the input is delayed by an interval T, so that the new input is $v_i(t-T)$, the output is also delayed by the same interval, and so becomes $v_o(t-T)$.
4. If the derivative of the original input $v_i'(t)$ is fed into the network, the output will be $v_o'(t)$, the derivative of the original output.
5. If the integral of the original input $\int_{-\infty}^{t} v_i(\tau)\,\mathrm{d}\tau$ is fed into the network, and all the initial conditions at $t = -\infty$ are zero, the output will be $\int_{-\infty}^{t} v_o(\tau)\,\mathrm{d}\tau$, the integral of the original output.

These properties are **extremely useful** for obtaining new solutions from old, and should be used whenever possible in explaining your experimental results.

Experimental Set-up

The waveforms used are “slow” by modern standards and events happen about 1000 to 10,000 times slower than those which are often encountered in present-day systems. However, the use of a slow time-scale avoids many of the difficulties due to stray capacitances and inductances thereby making it possible to carry out fundamental studies with very nearly “ideal” waveforms. With the custom-designed equipment provided it is possible to make a rapid study of the fundamental circuits from which sophisticated electronic systems are built.

The Pulse Function Generator (PFG), the circuit under investigation and the dual-trace oscilloscope are interconnected as shown in Fig. 1.

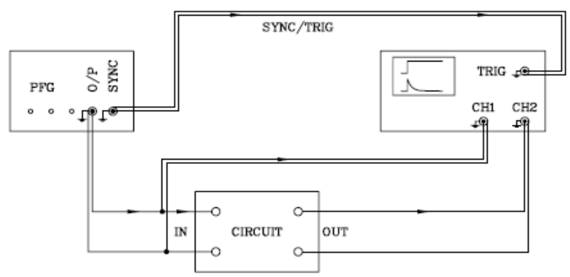


Figure 1: Experimental set-up

Oscilloscope

For this experiment any oscilloscope will suffice, but to observe the best results you should preferably use the Cleverscope CS328 Digital Oscilloscope that is available from about 10 Advanced Lab computers. Alternatively, you could use the Hitachi VC-6545 20 MHz Solid State Oscilloscope but there is only one of these available in the laboratory. For both oscilloscopes the cables from the rear of the PFG should be connected to the two channel inputs and also to the External Trigger. The following control settings should be used to give clearly observable outputs:

CH1 & CH2	DC
VOLTS/DIV	0.5 V
TIME/DIV	0.1 ms
TRIGGER SOURCE	EXT

The oscilloscope should be set to display in real time for the clearest outputs.

If you choose to use the Cleverscope then you will need to start the Cleverscope software from the start menu in the physics folder. This works in a similar manner to a normal oscilloscope and many of the buttons and functions should already be familiar, even if the layout is not immediately obvious. The following settings should be used:

Chan A & Chan B	DC
BW	25
TRIGGER 1	160 ms
TRIGGER SOURCE	Ext Trigger
ACQUIRE MODE	Triggered

The Cleverscope display can be resized to suit and the axes adjusted by clicking the larger or smaller square wave buttons and moved by clicking the arrows or simply dragging the display. It is advisable to spend a little while familiarising yourself with the operation of the scope. A manual is available in the lab. The Cleverscope has an added advantage in that the data from the graph can be saved and then imported to

PYTHON where it can be plotted. To do this select `File > Save Graph as Text`. There will be 3 columns of data — one for time and each of the the two oscilloscope traces. If you use the analogue oscilloscope you will need to draw the responses by hand.

Pulse Function Generator

Six different voltage waveforms are available at the output terminals of the PFG and each voltage waveform is generated with an "earth" base-line. A graphic description of the six waveforms together with the sync pulse is provided by Fig. 2.

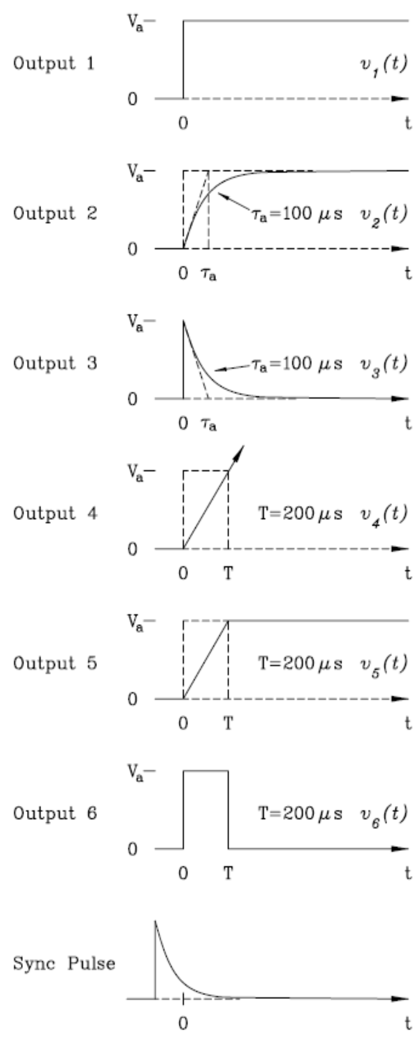


Figure 2: Output waveforms of the Pulse Function Generator

Step function

Ideally the step-function $v_1(t)$ rises from 0 to V_a instantaneously, but in practice the step is never instantaneous and the two most common front-edges are the exponential-step $v_2(t)$ and the ramp-step $v_3(t)$. Where the rise-times of the exponential-step and the ramp-step are very much less than the time resolution of the measuring instruments they can both be considered approximate to the ideal step-function. The rise-time of the step-function produced by the PFG is less than 1 μs, and this is considerably less than the resolution of the oscilloscope when the time-base is set at 0.1 ms/div. The step function and its Laplace transform are:

$$v_1(t) = V_a \mathrm{u}(t), \qquad V_1(s) = \frac{V_a}{s}$$

where $\mathrm{u}(t)$ is called the unit step (or Heaviside step) function which is zero for $t < 0$, and one for $t > 0$.

Exponential step

The exponential-step $v_2(t)$ rises from zero to V_a with a time constant τ_a. The time constant is the time at which the step rises to $1 - \mathrm{e}^{-1} \approx 0.63$ of its final value. As a rule of thumb, after five time constants, the step is essentially at its final value, since $1 - \mathrm{e}^{-5} = 0.99$. The step function and its Laplace transform are:

$$v_2(t) = V_a \mathrm{u}(t)\left[1 - \exp\left(-\frac{t}{\tau_a}\right)\right], \qquad V_2(s) = \frac{V_a}{s} - \frac{V_a}{s + (1/\tau_a)}$$

Notice how we use the Heaviside step function to ensure that $v_2(t) = 0$ for $t < 0$.

Exponential pulse

The exponential pulse or "pip" $v_3(t)$ is one of the commonly encountered waveforms in electronic pulse circuits. It occurs whenever a step-function is "passed" from one circuit to another via a R-C network.

$$v_3(t) = V_a \mathrm{u}(t) \exp\left(-\frac{t}{\tau_a}\right), \qquad V_3(s) = \frac{V_a}{s + (1/\tau_a)}$$

Notice that $v_2(t) = v_1(t) - v_3(t)$. This relationship is useful for predicting the output when $v_2(t)$ is fed into the network, once we know the responses to $v_1(t)$ and to $v_3(t)$.

Linear ramp

The linear ramp $v_4(t)$ is proportional to the integral of the ideal step-function and its own derivative is clearly a constant. This waveform is basic to all analogue-type time to pulse-height converters. The linear ramp is arranged to rise to 1.5 volts in 200 μs so that it has a slope that is identical with that of the initial section of the ramp-step.

$$v_4(t) = \frac{V_a t}{T} \mathrm{u}(t), \qquad V_4(s) = \frac{V_a}{Ts^2}$$

Notice that

$$v_4(t) = \frac{1}{T} \int_{-\infty}^{t} v_1(\tau)\, \mathrm{d}\tau$$

so that if the response of a network to $v_1(t)$ is $v_o(t)$, the response to $v_4(t)$ will be

$$\frac{1}{T} \int_{-\infty}^{t} v_o(\tau)\, \mathrm{d}\tau$$

Ramp-step

The ramp-step $v_5(t)$ is a composite waveform and may be considered as the sum of a positive ramp followed, after an interval equal to the rise-time T, by a negative ramp. It is also proportional to the integral of the rectangular pulse $v_6(t)$. i.e.,

$$v_5(t) = v_4(t) - v_4(t - T), \qquad V_5(s) = \frac{V_a}{Ts^2}\left[1 - \exp(-Ts)\right]$$

Rectangular pulse

The rectangular pulse $v_6\,(t)$ is also a composite waveform since it is equivalent to a positive step followed, after an interval equal to the pulse width , by a negative step. This waveform is also proportional to the derivative of the ramp-step.

$$v_6\,(t) = v_1\,(t) - v_1\,(t - T)\,, \qquad V_6\,(s) = \frac{V_a}{s}\,[1 - \exp\,(-Ts)]$$

The amplitude of all the waveforms, except the linear ramp, is pre-set at 1.5 volts. This is because $1.5/\mathrm{e} \approx 0.5$, and $(1 - 1/\mathrm{e}) \times 1.5 \approx 1$ and with the vertical sensitivities of the two channels of the oscilloscope set at 0.5 V/div these values are easy to observe.

Sync pulse

The sync pulse occurs 50 times per second, i.e., once every 20 ms, and it is timed to occur 100 μs ahead of each output waveform. In this way the oscilloscope time-base is triggered in sufficient time for the front edge of the output waveform to be observed conveniently on a display at 0.1 ms/div.

The duration of the time-base is 1 ms and after the end of the sweep there is an interval of 19 ms before the time-base is repeated. During the “flyback” and the waiting period the trace is “blacked out” to remove the confusing waveforms which would otherwise be displayed in these periods. With the dual-trace facility, channel 1 and channel 2 are displayed alternately and triggered by alternate sync pulses.

The main purpose of the 1 : 19 mark-space ratio of the output waveforms is to provide a relatively long settling period between transients, and in this way the response of a circuit to one waveform is reduced to negligible size before the succeeding one arrives. The display on the oscilloscope is exactly the same as that for a single waveform, and it is distinctly easier to observe.

Comment

It is instructive to observe the six waveforms available from the PFG with the TIME/DIV control on the oscilloscope set at either 2 ms/div or 5 ms/div, but once this has been done it must be returned to 0.1 ms/div for subsequent observations.

In this experiment the six different output waveforms from the PFG are “passed through” various shaping circuits, and in their journeys they undergo a series of interesting transformations. It is essential for students to master the mechanisms of each transformation before proceeding on to the next, and those students who experience any difficulties whatsoever must consult a demonstrator.

A reasonable test of one’s understanding of a circuit’s pulse shaping characteristics is to be able to predict changes in the response waveform due to changes in the values of components in a network.

It has been explained earlier that the finite rise and fall-times of the step and pulse waveforms may be ignored since they are much less than the time resolution of the oscilloscope when the time-base is set at 0.1 ms/div.

The output impedance of the PFG is 100 Ω and the input impedances of the two input channels of the oscilloscope (1 MΩ in parallel with 50 pF) may also be ignored. The validity of these approximations should become clear to students as they progress.

1 R-C Quasi-Integration Circuit

The R-C quasi-integration circuit (Fig. 3(a)) arises both unintentionally and intentionally in electronic circuits. Its unintentional occurrence is due to the fact that in practice every signal source has a certain output impedance and there is always some stray capacitance present. The effect of the circuit is to “round off” any rapid changes in the signal waveform, i.e. any high-frequency components originally present are

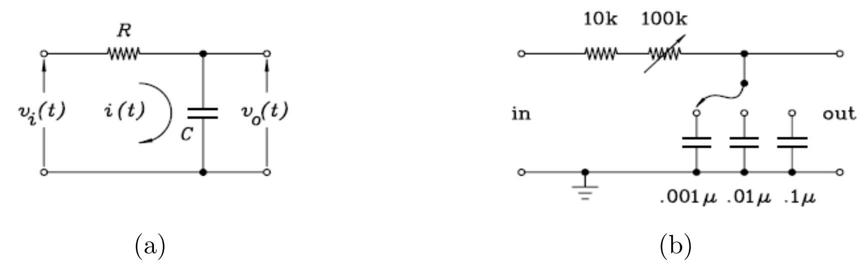


Figure 3: RC quasi integration circuit

progressively attenuated in amplitude as the time-constant is increased. When the circuit is used intentionally in pulse applications it may be for any or all three of the following possible purposes:

(i) to provide smoothing, i.e. to remove irregularities and discontinuities in a waveform,

(ii) to provide an output proportional to the integral of the input when $t \ll \tau$ where $\tau = RC$, or

(iii) to introduce a time-delay proportional to τ between the output and the input waveforms.

As discussed above, the differential equation for the circuit in Fig. 3(a) is:

$$\frac{\mathrm{d}v_o}{\mathrm{d}t} + \frac{1}{RC}v_o = \frac{1}{RC}v_i,$$

whose Laplace transform (with initial condition $v_o(0) = 0$) is:

$$\left(s + \frac{1}{RC}\right) V_o(s) = \frac{1}{RC} V_i(s) \quad \text{so that} \quad V_o(s) = \frac{1/(RC)}{s + 1/(RC)} V_i(s)$$

The current $i(t)$ through the circuit is given by

$$i(t) = q'(t) = Cv_o'(t)$$

whose Laplace transform is:

$$I(s) = sCV_o(s) = \frac{V_i(s)}{R + \frac{1}{sC}}$$

The actual circuit used in this experiment (Fig. 3(b)) is provided with continuously variable values of time-constant in three overlapping ranges: $10\,\mu$s to $110\,\mu$s, 0.1 ms to 1.1 ms, and 1 ms to 11 ms. With this facility students can quickly gain an understanding of the functioning of the circuit by relating changes in the shape of the output waveform to corresponding changes in the circuit time-constant.

(1) Apply the six basic waveforms to the circuit and observe $v_o(t)$ for each waveform as R and C are varied. Relate your observations to the time-constant, e.g. $10\,\mu$s, $100\,\mu$s and 1 ms. Note that:

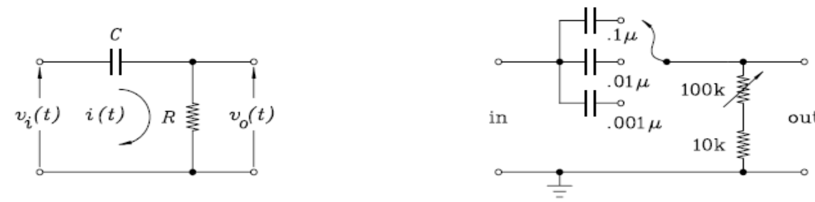
   (a) For $t \ll \tau/10$ the output is an integral of the input.
   (b) With the step input, the half-height delay of the exponential-step output is $\tau \log_e 2 \approx 0.69\tau$.
   (c) With the ramp input, the output "ramp" is delayed by τ for $t > 5\tau$, i.e., $v_o(t) \approx (V_a/T)(t - \tau)$
   (d) With the "pip" and pulse inputs, the output waveforms are of negligible amplitude for $\tau \gg \tau_a$ or.$\tau \gg T$

   Questions:

1. Solve the differential equation for $v_o(t)$ analytically (either directly, or by using Laplace transform methods) for the input signals $v_1(t)$ and $v_3(t)$. Use these solutions to find the responses for the other input signals using the properties discussed in the theory section. Also calculate the current $i(t)$ in each case. Show that your observations (a) through (d) in the above step are derivable from your solutions.

2. Under what conditions is a waveform passed without appreciable change and under what conditions is it significantly distorted?

2 R-C Quasi-Differentiation Circuit



(a) (b)

Figure 4: RC quasi-differentiation circuit

The R-C quasi-differentiation circuit (Fig. 4(a)) is one of the most commonly encountered circuits in modern electronic equipment and it is therefore most important that its properties should be thoroughly understood. Essentially it provides signal coupling from one electronic circuit to another whilst at the same time providing d-c isolation between the two.

The current $i(t)$ in this circuit is the same as for the quasi-integration circuit, but in thid case $v_o(t) = i(t)R$. It is interesting to note that the shape of the $v_o(t)$ waveform is identical to that of the $i(t)$ waveform. In fact, a common way of observing a current waveform is to observe the voltage waveform which results from passing the current waveform through a resistance. Alternatively, we may note that for any given input waveform $v_i(t)$, the output waveform of the R-C quasi-differentiation circuit is equal to the difference between the input waveform and the output waveform of the R-C quasi-integration circuit:

$$v_o(t)\,[\text{Quasi-differentiator}] = v_i(t) - v_o(t)\,[\text{Quasi-integrator}]$$

The actual circuit (Fig. 4(b)) is identical with that of Fig. 3(b) except that the positions of the resistors and capacitors are interchanged. Thus the range of available time-constants is the same as before.

(2) Apply the six basic waveforms to the circuit and observe for each waveform as and are varied. Relate your observations to the time-constants of the circuit and make accurate sketches of the input and output waveforms of the circuits for three values of the time-constant: 10 μs, 100μs and 1 ms. Note that:

   (a) The exponential pulse (pip) produced by the quasi-differentiation of the step-function has the same amplitude as the step.
   (b) Quasi-differentiation of the exponential-step results in the progressive reduction in amplitude of the output pip as the circuit time-constant is reduced. (Short pips can only be generated by the quasi-differentiation of a fast rising step.
   (c) Quasi-differentiation of the linear ramp results in an output waveform which is similar in shape to that obtained from the quasi-integration of the step-function.
   (d) Quasi-differentiation of the ramp-step results in an output waveform which is is similar in shape to that obtained from the quasi-integration of the rectangular pulse.
   (e) The positive and negative pips resulting from the quasi-differentiation of the rectangular pulse are completely independent when the circuit time-constant is less than $T/5$.
   (f) Quasi-differentiation of the exponential-pulse results in an output waveform with a negative overshoot which is particularly noticeable when the circuit time-constant is less than that of the pip. The maximum value of the negative overshoot occurs after a time equal to twice that required for the output waveform to pass through zero.

Question:

3. Obtain expressions for the output of the quasi-differentiator to each of the input waveforms using your results for the quasi-integrator. Show that you can explain observations (a) through (f) above. In particular, prove theoretically the statement concerning the negative overshoot in observation (f).

3 Pulse Shaper based on a Quasi-Integrator

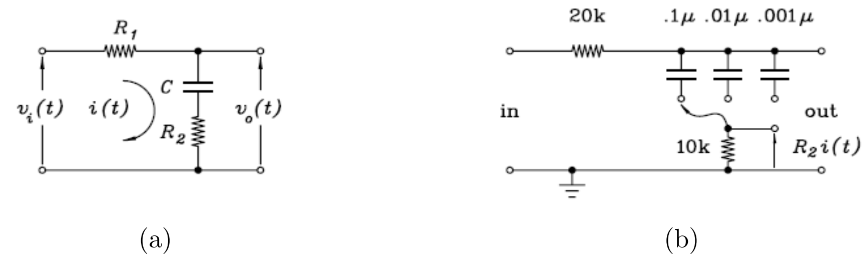


Figure 5: Pulse shaper based on a quasi-integrator

The circuit in Fig. 5(a) is often used in amplifier design to provide a gain characteristic which has unit gain at low frequencies, and a gain of $R_2/(R_1+R_2)$ at high frequencies and for that reason it is presented for study here. The circuit is similar to the quasi-integration circuit in Fig. 5(a) except for the inclusion of the resistance R_2 in series with the capacitance C, and the solution for $i(t)$ obtained for the quasi-integration circuit is again valid after making the substitution of R_1+R_2 for R. The value of $v_o(t)$ is most simply obtained from the relationship:

$$v_o(t) = v_i(t) - R_1 i(t)$$

Taking the Laplace transform:

$$\begin{aligned} V_o(s) &= V_i(s) - R_1 I(s) = V_i(s) - R_1 \frac{V_i(s)}{R_1 + R_2 + \frac{1}{sC}} \\ &= \frac{R_2}{R_1+R_2}\left[\frac{s+\frac{1}{R_2 C}}{s+\frac{1}{(R_1+R_2)C}}\right] V_i(s) = \frac{\tau_1}{\tau_2}\left[\frac{s+1/\tau_1}{s+1/\tau_2}\right] V_i(s) \end{aligned}$$

The two time-constants which arise naturally from an analysis of the circuit are:

$$\tau_1 = R_2 C \quad \text{and} \quad \tau_2 = (R_1+R_2)\,C$$

and of these, τ_2 is the circuit response time-constant. Note $\tau_2 > \tau_1$.

The actual circuit (Fig. 5(b)) has three alternative values for corresponding to $\tau_1 = 10\,\mu$s, $100\,\mu$s and 1 ms; and $\tau_2 = 30\,\mu$s, $300\,\mu$s and 3 ms respectively. Facilities are also provided to make it possible to observe the shape of the current waveform by observing the voltage across R_2.

(3) Apply the step waveform to the circuit for each of the three values of C. Observe both $v_o(t)$ and $i(t)$ and relate your observations to the appropriate time-constants. Make accurate sketches of the waveforms.

Note that the initial step in $v_o(t)$ is equal to $[R_2/(R_1+R_2)]\,V_a$ and that the shape of $i(t)$ is that of an exponential pulse (pip) with initial amplitude $V_a/(R_1+R_2)$ and time-constant τ_2.

(4) Apply the pip waveform to the circuit for each of the three values of C. Observe and make accurate sketches as before.

Note that when τ_1 is equal to the time-constant of the input pip, i.e. $100\,\mu$s, the output is a pip with time-constant equal to τ_2. The factor by which the pip is “stretched” under these conditions is exactly equal to the factor by which its amplitude has been reduced.

Question:

4. The frequency response of the circuit is such that it has corner frequencies at $f_1 = 1/(2\pi R_2 C)$ and $f_2 = 1/[2\pi(R_1+R_2)C]$. Why is the time-constant τ_1 not involved in the transient response of the circuit?

4 Pulse Shaper based on a Quasi-Differentiator

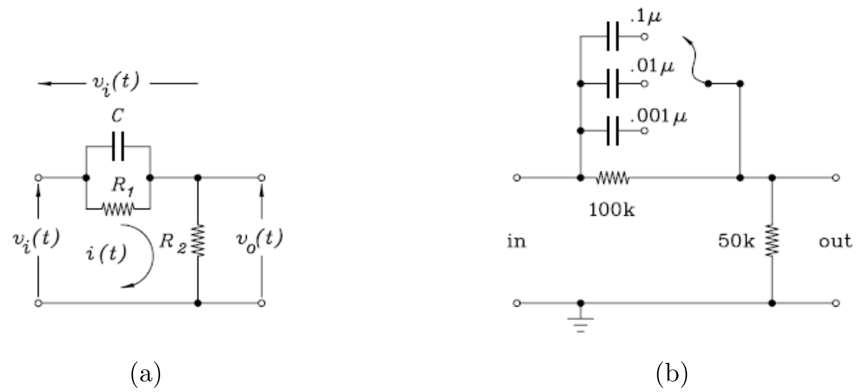


Figure 6: Pulse shaper based on a quasi-differentiator

The circuit of Fig. 6(a) is often used in amplifier design to provide a gain characteristic that is complementary to that of the previous circuit (Fig. 5(a)), i.e. the gain is $R_2/(R_1 + R_2)$ at low frequencies and increases to unity at high frequencies. It is also used for "sharpening" the front edges of pulses and for that reason it is presented for study here. The circuit is similar to the quasi-differentiation circuit in Fig. 4(a) except for the inclusion of the resistance R_1 in parallel with the capacitance C. The response of the circuit may best be determined by first obtaining a solution for $v_p(t)$ by the use of Thévenin's theorem together with a knowledge of the solution for the quasi-integration circuit in Fig. 3(a). The output may then be obtained from the relationship:

$$v_o(t) = v_i(t) - v_p(t)$$

As with the previous circuit (Fig. 5(a)) there are two time-constants which arise during the analysis of the circuit, and these are:

$$\tau_1 = R_1 C \quad \text{and} \quad \tau_2 = RC \quad \text{where} \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

and of these, is the circuit response time-constant. Note that $\tau_1 > \tau_2$. The Laplace transforms of the input voltage $V_i(s)$ and the output voltage $V_o(s)$ satisfy:

$$V_o(s) = \left[\frac{s + 1/\tau_1}{s + 1/\tau_2}\right] V_i(s)$$

The actual circuit (Fig. 6(b)) has three alternative values for C corresponding to $\tau_1 = 100\,\mu$s, 1 ms and 10 ms; and $\tau_2 = 33\,\mu$s, $330\,\mu$s and 3.3 ms respectively. The values of the circuit response time-constant τ_2 have been specifically selected to be approximately equal to those for the previous circuit (Fig. 5(b)).

(5) Apply the step waveform to the circuit for each of the three values of C. Observe $v_o(t)$ and relate your observations to the appropriate time-constants. Make accurate sketches of the input and output waveforms. Note that the output consists of a step of amplitude $[R_2/(R_1 + R_2)]\, V_a$ upon which is superimposed a pip of amplitude $[R_1/(R_1 + R_2)]\, V_a$ with a time-constant τ_2.

(6) Apply the exponential-step waveform to the circuit and repeat the observations and sketches as before. Note that when τ_1 is equal to the time-constant of the input exponential-step, i.e., $100\,\mu$s, the output is a faster rising exponential-step with a reduced time-constant equal to τ_2. The factor by which the rise-time is improved is equal to the factor by which the amplitude has been reduced.

Question:

5. Suppose that it is possible to cascade the quasi-integrator pulse shaper circuit of Fig 5(a) and the quasi-differentiator pulse shaper circuit of Fig 6(a) in such a way that they do not load one another, what are the conditions for the output waveform to have exactly the same shape as the input?

5 Attenuator with Compensation for Stray Capacitance

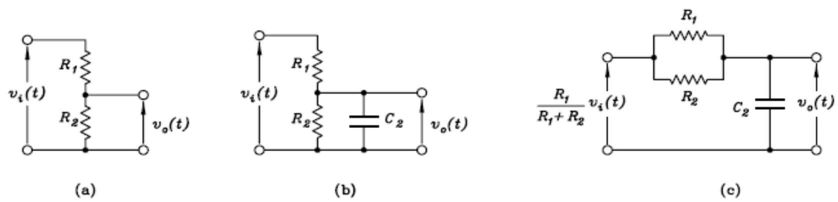


Figure 7: (a) Voltage divider (b) Voltage divider with stray capacitance (c) Thévenin equivalent

The attenuation of voltage waveforms without the introduction of distortion is not as straightforward as it initially appears, particularly when stray capacitance is present. Fig. 7(a) shows a simple resistive attenuator for which:

$$v_o(t) = \frac{R_2}{R_1 + R_2} v_i(t) \quad \text{and} \quad V_o(s) = \frac{R_2}{R_1 + R_2} V_i(s)$$

Clearly no distortion is introduced as there are no differentiator operator terms in the gain factor. When there is some stray capacitance across the output, as illustrated in Fig. 7(b), the attenuator behaves as a quasi-integrator with time-constant equal to $[R_1 R_2 / (R_1 + R_2)]\, C_2$, as shown in Fig. 7(c). The d-c level of the input signal is reduced by the factor $R_2/(R_1 + R_2)$ but the higher frequency components present are attenuated by progressively increasing amounts, i.e. distortion occurs.

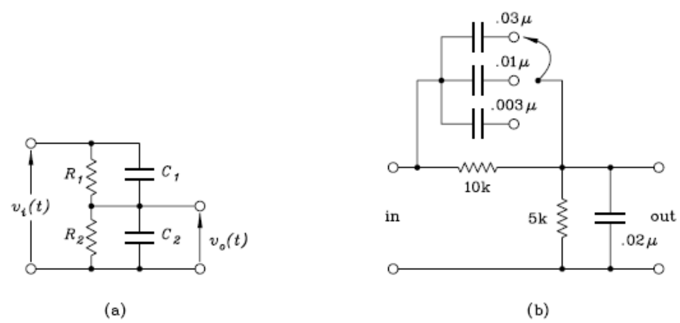


Figure 8: (a) Attenuator with compensation for stray capacitance (b) Experimental set-up

Fig. 8(a) shows the basic circuit of a resistive attenuator with compensation for stray capacitance across the output. The condition for distortionless attenuation is:

$$R_1 C_1 = R_2 C_2$$

(7) Apply a step waveform to the actual circuit shown in Fig. 8(b) and observe the output waveform for each of the three values of C_1.

Note that when $R_1 C_1 < R_2 C_2$ quasi-integration is present and that when $R_1 C_1 > R_2 C_2$ quasi-differentiation is present.

Although the input circuit of the oscilloscope has insignificant loading effects on the circuits in this experiment there are many instances in practice where it does. In order to reduce this loading, attenuator probes are used. The circuit of one of these probes is shown in Fig. 9(a); it introduces an attenuation in the signal of -times but at the same time it reduces the loading on the circuit under observation by -times as indicated in Fig. 9(b).

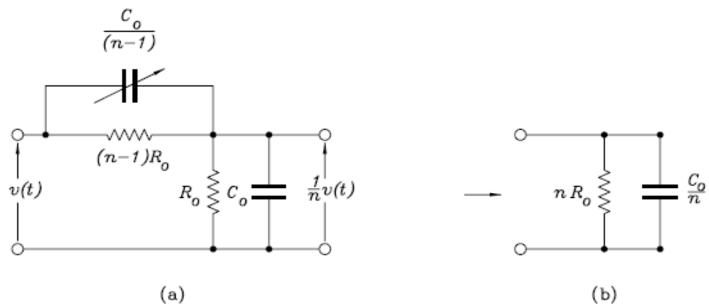


Figure 9: (a) Compensated attenuator found in oscilloscope probe (b) Equivalent input impedance

6 Series L-R-C Oscillatory Circuit

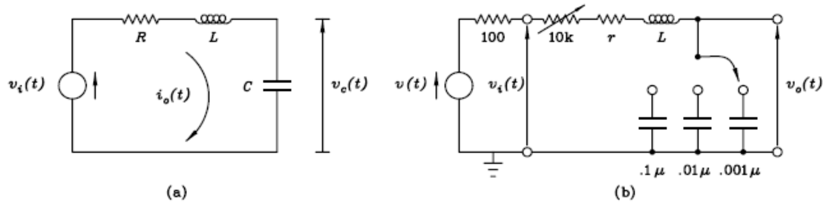


Figure 10: Series L-R-C Oscillatory Circuit

In practical circuits resistors, capacitors and wire leads have stray inductances and these can result in "ringing" on the front and back edges of fast pulses. The circuit illustrated in Fig. 10(a) looks like a quasi-integrator with added series inductance, but when the resistance is sufficiently small the circuit has oscillatory properties. The differential equation for the current and its Laplace transform (for zero initial conditions) are:

$$L\frac{\mathrm{d}^2 i}{\mathrm{d}t^2} + R\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{i}{C} = \frac{\mathrm{d}v_i}{\mathrm{d}t} \quad \text{and} \quad \left(sL + R + \frac{1}{sC}\right) I\left(s\right) = V_i\left(s\right)$$

Notice how the equation for the Laplace transform could have been written down by inspection using Ohm's law and the impedances for the components. This illustrates how it is possible to derive the differential equations for circuits from elementary network theory.

The natural response consists of decaying oscillations with frequency:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

provided that $R < 2\sqrt{L/C}$, and the damping factor is given by $\exp\left[-R/\left(2L\right)t\right]$. In the actual circuit shown in Fig.10(b), damping is provided by the $100\,\Omega$ output impedance of the PFG, the self-resistance r of the inductor and the $0 - 10\,\mathrm{k}\Omega$ variable resistance.

(8) Apply a voltage step to the circuit starting with $C = 0.001\,\mu$F and the variable resistance set at zero. It may be necessary to make some adjustments to the oscilloscope controls to view the output waveform satisfactorily. Determine the value of L from your observations and estimate r.

(9) Increase the damping resistance and note the changes in the response. Repeat with $C = 0.01\,\mu$F and $0.1\,\mu$F. Note that the value of the damping resistance for critical damping decreases as C increases.

(10) With minimum damping and $C = 0.001\,\mu$F, apply the pulse and the ramp-step input. Note the superposition principles in operation and the tendency to"ring" decreases as the rise-time increases.

7 Lumped Constant Delay Line

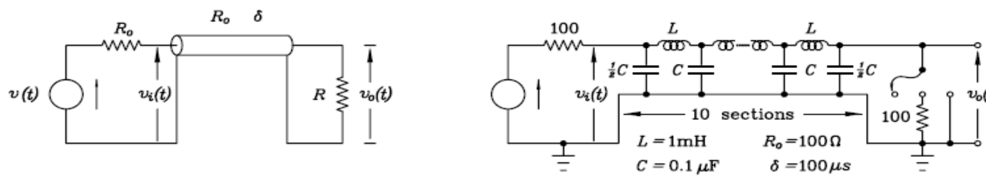


Figure 11: (a) Delay line (b) Lumped constant model

The circuit of Fig. 11(a) shows an ideal lossless delay line with a characteristic impedance of R_o and a delay of δ. The sending end is matched to the PFG and the receiving end is terminated in a resistance R. When $R = R_o$ then $v_i(t) = \frac{1}{2} v(t)$ and $v_o(t) = v_i(t - \delta)$. The actual circuit provided is a lumped constant approximation to an ideal delay line consisting of 10 sections.

(11) Apply a step input for the three values of the terminating resistance and observe $v_i(t)$ and $v_o(t)$ in all three cases. Explain your results.

(12) Repeat with the ramp-step input and then with the rectangular pulse input. Explain your observations in terms of the transmitted and reflected signals propagating along the line.

List of Equipment

1. 1 x Cleverscope CS328 Digital Oscilloscope and PC

   Or

   1 x Hitachi Digital Oscilloscope Model VC-6545
2. 1 x Pulse Function Generator
3. 1 x Circuit Board Housing
4. 4 x Circuit Boards

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