# Experiment 217: Viscosity and Critical Velocity of Water

## Aim

The object of the experiment is to measure the viscosity of water at a particular temperature, and to find the value of the critical Reynolds number $\mathcal{R}$ at which the flow becomes turbulent.

## References

1. Worsnop and Flint, "Advanced Practical Physics for Students", Methuen.
2. Champion and Davy, "Properties of Matter", Blackie.

## Theory

Figure 1 shows parallel flow of a liquid over a fixed surface. The drag force $F$ on the surface depends on $A$, the area of the surface, and $dv/dy$, the rate at which the velocity $v$ increases as we proceed along the direction of the $y$-axis, normal to the fluid flow. The drag force $F$ then is given by:

$$F \propto A\frac{dv}{dy} \qquad \text{that is,}$$

$$F = \eta A\frac{dv}{dy} \tag{1}$$

where $\eta$ is a constant of proportionality which varies with different liquids. For a particular liquid, it is called the coefficient of viscosity of the liquid and is given by:

$$\eta = \frac{F}{A\,(dv/dy)} \tag{2}$$

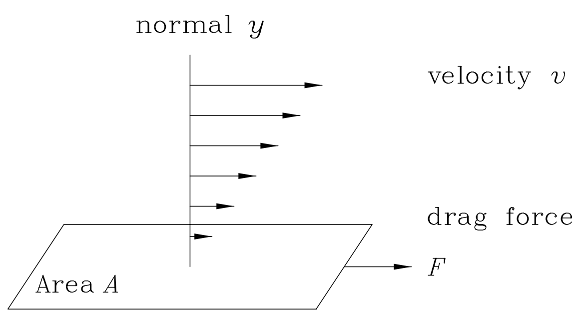


Figure 1:

For streamline flow through a tube of radius $R$, the volume of liquid $Q$ flowing per second is given by Poiseuille's equation:

$$Q = \frac{\pi R^4}{8\eta}\left(\frac{dp}{dl}\right) \tag{3}$$

where $dp/dl$ is the pressure gradient down the tube. (See Worsnop and Flint, or Champion and Davy). You should understand the detailed derivation of this formula, especially the definition of $Q$ in terms of the equation of continuity

$$Q = \pi R^2\,\overline{v} \tag{4}$$

where $\overline{v}$ is the mean velocity of fluid flowing through the vessel and $\pi R^2$ is the cross-sectional area of the vessel.

Equation (3) gives us a method of determining the coefficient of viscosity of water and is the basis of the present experiment.

# Procedure

In the apparatus provided (see Figure 2), there are two very small holes drilled in the tube a distance $L$ apart. These are connected to a manometer so that the pressure difference between these points can be read as a height of water $h$. The actual pressure difference $\Delta p$ can then be calculated ($\Delta p = \rho g h$); the pressure gradient $dp/dl$ along the tube, assumed to be uniform, is $\Delta p/L$.

**Question 1:** The radius $R$ is measured using a travelling microscope but $h$ is measured just with a metre rule. Is the value of $h$ obtained in this way sufficiently accurate? Justify your answer.

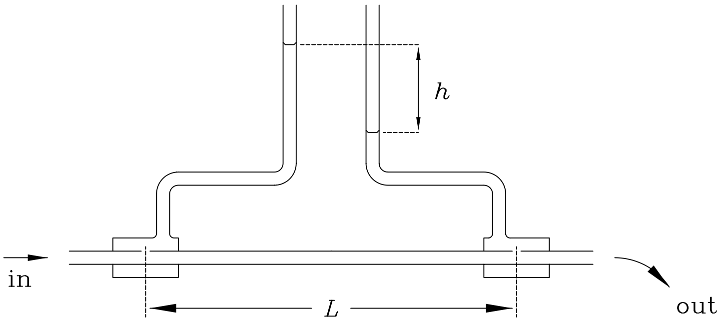


Figure 2:

(1) Before commencing the experiment, ensure that the tap supplying the reservoir is on. Water should be issuing from the overflow tube. To maintain a steady pressure, the tap should be adjusted so that the water level in the reservoir half covers the outlet pipe.

(2) The radius $R$ of the tube occurs in the formula to the fourth power and must be found with as great an accuracy as possible[1]. In this particular experiment a sample of the tube is provided so that its internal diameter may be measured with a travelling microscope.

(3) A stop-watch and measuring cylinder are used to find the rate of flow $Q$, in $m^3$ per second. When the pressure gradient is steady, the flow is timed for a certain length of time and the corresponding volume of water collected in the cylinder. The viscosity $\eta$ could now be found from equation (3).

(4) In practice a graph of $Q$ versus manometer reading $h$ is plotted and $\eta$ (in mks units) is determined from the slope of the straight portion of the graph. It is equally acceptable to plot $Q$ versus $\Delta p$ (measured in Pascals).

About 15 points should be plotted on this graph for $h$ in the range 0 – 40 cms. Plotting the graph during the experiment will allow departure from linearity to be detected and extra readings can then be taken about the region of nonlinearity. If you find that you cannot reach large enough values of $h$ even with the valve fully open, you probably need to **purge air bubbles trapped in the pipe** bringing the water. To this end, open the valve that is in the sink, and let some water flow until all the air is gone (the flow should then be steady and regular).

[1] A useful method is to weigh a sample of tube empty and weigh it again when filled with mercury. The mean radius of the sample can then be calculated from the volume of mercury enclosed.

(5) Since viscosity varies rapidly with temperature it is essential to note the temperature of the water during each run.

(6) Determine the gradient of your graph and from that value determine $\eta$ (note: the gradient is NOT equal to $\eta$ but $\eta$ can be derived from it). Compare your value of $\eta$ with accepted standards.

**Note:**

Although we use mks units here, the cgs unit (the Poise $\equiv$ dyne s cm$^{-2}$ $\equiv$ g cm$^{-1}$ s$^{-1}$) is still often used for viscosity. The mks unit is the "deka-poise" and 1 deka-poise = 10 Poise.

**Question 2:** Use equation (3) for $Q$ and the equation of continuity (4) to obtain an expression for the mean velocity of the liquid in the tube in terms of $\eta$, $R$, and $dp/dl$.

**Question 3:** Show how you can get the same result by considering the equation for the radially symmetric velocity field for fluid in a cylindrical vessel

$$v(r) = \frac{dp}{dl}\frac{1}{4\eta}\left(R^2 - r^2\right) \tag{5}$$

**Question 4:** What is the relationship between the mean value and maximum value of the fluid velocity?

## Turbulence and Reynolds number

(7) At high rates of flow the linear relation between $Q$ and $\Delta p/L$ breaks down. This is due to the flow becoming turbulent.

**Question 5:** What does this mean?

(8) In studying turbulence it is convenient to introduce the dimensionless constant $\mathcal{R}$ called the Reynolds number as follows:

$$\mathcal{R} = \frac{\overline{v}\rho R}{\eta} \tag{6}$$

It is found that for all liquids turbulence sets in when $\mathcal{R}$ exceeds a critical value of about 1160.

Verify this for water by finding the point on your graph of $Q$ versus $\Delta p/L$ where the departure from linearity occurs.

**Question 6:** What is the best technique of determining this point on the graph?

**Note:**

The mean velocity $\overline{v}$ at the point where $Q$ versus $\Delta p/L$ departs from linearity may be calculated using the equation of continuity (4) and the critical value of $\mathcal{R}$ can then be obtained from equation (6). Sometimes two distinct points can be seen on the curve where the nature of the flow of water alters. If this occurs calculate $\mathcal{R}$ for each point.

## Results

These should be summarized in a table:

| | | |
|---|---|---|
| $L$ | = | metre |
| $R$ | = | metre |
| $T_{\text{water}}$ | = | °C |
| $\eta$ | = | (mks units) |
| $\Delta p/L$ | = | Pa m$^{-1}$ for turbulence |
| $\mathcal{R}_{\text{critical}}$ | = | |

## Write-up

The experiment write-up must include:

1. The answers to all questions.
2. The measured radius, with confidence limits, of the sample of capillary tube.
3. A set of measurements of rate of flow and corresponding pressure gradients.
4. A plot of rate of flow against pressure gradient.
5. A short explanation of any non-linearity in the plot.
6. An estimate, with confidence limits, of the viscosity of water.
7. An estimate of the critical Reynolds number for the break-up of laminar flow.
8. Derivation of an equation giving the velocity profile across the tube. (Equation (5)).
9. Proof that the fluid velocity on the axis of the tube is twice the mean velocity.

## List of Equipment

1. Apparatus on wooden stand
2. Stop-watch
3. Constant pressure tank on wall
4. Metre rule
5. Sample of stainless steel tube
6. Thermometer (0–50 °C)
7. Measuring cylinder (0–200 ml)
8. Travelling microscope

P.R. Wills, March 2000
D.A. Wardle
S. Coen — May 27, 2016.