Experiment 263: Electrical Resonance

Aim

To investigate the phenomenon of electrical resonance, plot an accurate resonance curve, and deduce values of circuit parameters.

Reference

1. Gary E.J. Bold and J. Brian Earnshaw, "Linear Steady-state Network Theory", 6th ed. or later, Chapter 6.

Theory

The measurements made in this experiment relate to an ideal series resonant circuit as shown in Figure 1(a). The resulting rms current as a function of angular frequency is given in Figure 1(b).

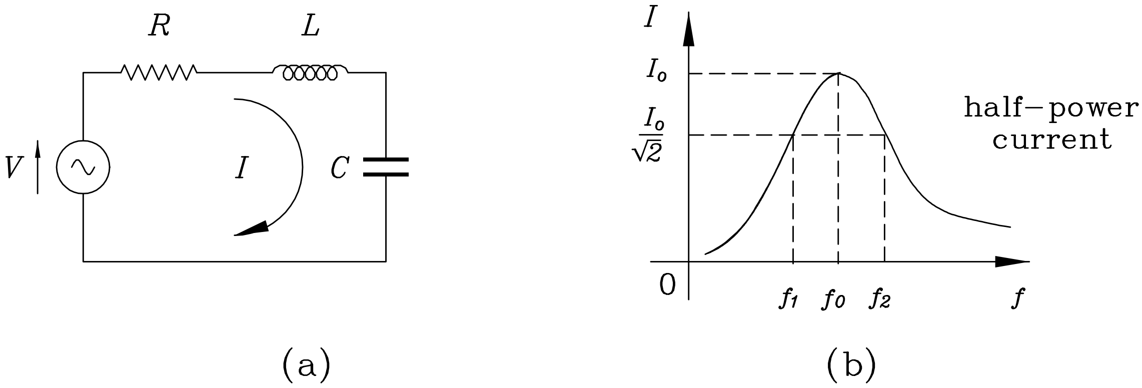

Figure 1: (a) A series resonant circuit. (b) A typical resonance curve.

The current phasor is given by:

$$\mathbf{I} = \frac{\mathbf{V}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{\mathbf{V}}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)}$$

The *rms* current is the magnitude of the current phasor:

$$I = \frac{V}{\left(R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right)^{1/2}}$$

Here, V is the *rms* voltage. The circuit is *resonant* when I has its maximum value I_0. This occurs when the second term in the denominator is zero, at angular frequency ω_0, where

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \text{or} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

At this frequency, I is in phase with V, and maximum power is dissipated in the resistor R. At frequencies f_1 and f_2, on either side of f_0, the power drops to *half* this maximum value, and hence the current drops to $I_0/\sqrt{2}$. (Power varies as current squared).

Quality Factor

The *quality factor*, Q, describes the *sharpness* of the resonance curve. For a series resonant circuit, Q is given by:

$$
\begin{aligned}
Q &= \frac{\text{reactance of } L \text{ or } C \text{ at resonance}}{\text{series resistance of circuit}} \\
&= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}
\end{aligned}
$$

In terms of Q, the sharpness of the resonance curve as measured by its half-power width $\omega_2 - \omega_1$ or $f_2 - f_1$ is given by:

$$
\begin{aligned}
\omega_2 - \omega_1 &= \frac{\omega_0}{Q} \\
\text{or } f_2 - f_1 &= \frac{f_0}{Q}
\end{aligned}
$$

It can also be shown that $f_0 = \sqrt{f_1 f_2}$, so there is no need to measure f_0. This is fortunate, since the resonance curve is slowly changing at f_0, making an exact estimation difficult.

Experimental Set-up

Figure 2 shows the experimental set-up. A constant-amplitude *ac* signal is provided by a signal generator with a digital frequency readout. Detachable leads allow different capacitances and resistances to be connected to the fixed inductor and the $100\,\Omega$ (1%) current-monitoring resistor. The voltage $v_m(t)$ across the current-monitoring resistor is proportional to the current $i(t)$ in the resonant circuit. An oscilloscope is provided for observing the variation of $v_m(t)$ with frequency. Its *rms* value at any frequency may be measured by connecting $v_m(t)$ to the *ac*-to-*dc* converter inside the circuit housing and reading the output *rms* level with a digital multimeter. The *rms* current I is then equal to the *rms* voltage divided by $100\,\Omega$.

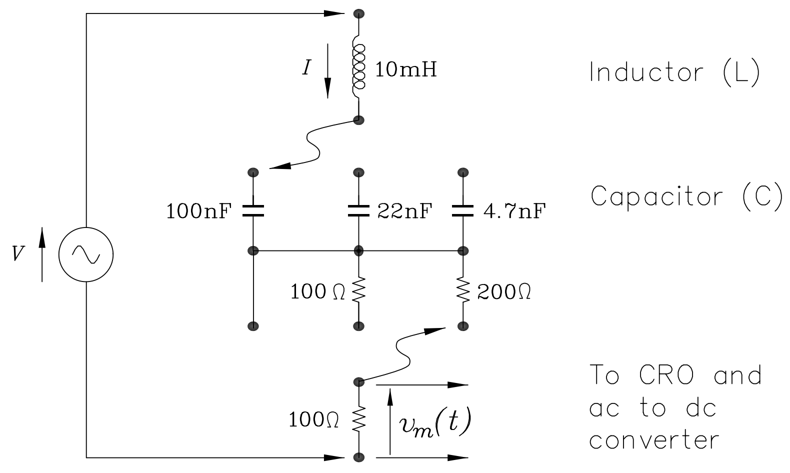

Figure 2: The experimental setup

A "real" (non-ideal) inductor may be represented by an equivalent circuit consisting of an inductance L in series with a resistance r_L. Similarly, a real capacitor may be represented by a capacitance C in series with a resistance r_C. The series resonance circuit therefore consists of the following series components:

$$
\begin{aligned}
L &= \text{equivalent series inductance of inductor (10 mH)} \\
C &= \text{equivalent series capacitance of capacitor (100 or 22 or 4.7 nF)} \\
R &= \text{total series resistance} = r_g + r_L + r_C + r_s + r_m
\end{aligned}
$$

where

$$
\begin{aligned}
r_g &= \text{resistance of signal generator } (50\,\Omega) \\
r_L &= \text{equivalent series resistance of inductor } (20\,\Omega) \\
r_C &= \text{equivalent series resistance of capacitor (zero)} \\
r_s &= \text{series resistance inserted } (0,\ 100\,\Omega \text{ or } 200\,\Omega\ (1\%)) \\
r_m &= \text{resistance for measuring current } (100\,\Omega\ (1\%))
\end{aligned}
$$

The values given in brackets above are nominal values. Actual values will be obtained by measurement.

Procedure

Note: Read this twice! Plot the required graphs *as the experiment proceeds*! Only by doing this will you see whether the results you are getting are "reasonable".

(1) Before connecting the signal generator to the resonance circuit, turn the amplitude control knob of the generator fully clockwise and, using the oscilloscope, check that the amplitude of the signal generator is independent of frequency.

(2) Connect up the circuit in Figure 2 with $C = 22$ nF and $r_s = 0$. Observe and get a qualitative feel for how $v_m(t)$ and hence $i(t)$ in the series resonant circuit varies with frequency. Vary the frequency until $v_m(t)$ is a maximum. Reduce the amplitude of the signal generator until the digital multimeter reads 3.0V. This ensures that the maximum *rms* voltage of $v_m(t)$ is 3.0 V and allows the full range of the 2-cycle linear-logarithmic paper (provided) to be used for plotting a symmetrical graph of the *rms* voltage of $v_m(t)$ versus frequency.

(3) Take readings in the vicinity of both of the half-power points, i.e where the *rms* voltage of $v_m(t)$ is about 0.7 of the maximum value. This ensures sufficient measurements for determining f_1 and f_2 to be made at nearly the same time, minimizing the effects of drift. Then take measurements to fill in the rest of the curve. You *must* plot the graph as you take the points, so that you can use your judgement — taking points closer together where the curve is most rapidly changing, or at other points of interest.

(4) Without changing the amplitude setting of the oscillator and staying with the 22 nF capacitor, plot resonance curves (using the same axes as before) for $r_s = 100\,\Omega$ and $r_s = 200\,\Omega$.

(5) Again using the *same axes*, and keeping r_s at $100\,\Omega$, plot two more resonance curves for $C = 100$ nF and $C = 4.7$ nF.

(6) Take the circuit board to the RCL meter and measure L and r_L. Write the values in the bracketed spaces in the following table (which you will complete and hand in as part of your report). Also measure C and r_c and record in the spaces provided (you might find that r_c is too small to measure in which case record it as zero). There is no need to measure the resistances of r_s and r_m as their nominal values are accurate to within 1%.

(7) The resistance r_g of the signal generator may be measured by applying the signal directly to the current measuring resistance r_m. Adjust the amplitude of the signal generator until the digital multimeter reads 1V. Disconnect the signal generator from r_m and measure the open-circuit *rms* voltage of the signal generator by connecting it directly to the input of the *ac*-to-*dc* converter. The generator resistance r_g then is given by the difference between the open-circuit *rms* voltage and 1V, divided by 1V and multiplied by $100\,\Omega$.

(8) From your resonance plots and measured values of C determine f_0, Q and R. Calculate the expected values of f_0, Q and R from the *measured* values of the components. Hence complete the table. Any significant discrepancies between measured and expected values should be investigated and resolved.

Table of results

Note: For the expected values, use $R = \sum r = r_g + r_L + r_C + r_s + r_m$.

$L = 10$ mH [ ] $r_L = 20\Omega$ [ ] $r_g = 50\Omega$ [ ]				Measured values			Expected values		
Nominal values		Measured values		f_0	Q	R	f_0	Q	R
C (nF)	r_s (Ω)	C (nF)	r_c (Ω)	$\sqrt{f_1 f_2}$	$\dfrac{f_0}{f_2 - f_1}$	$\dfrac{1}{\omega_0 C Q}$	$\dfrac{1}{2\pi\sqrt{LC}}$	$\dfrac{1}{R}\sqrt{\dfrac{L}{C}}$	$\sum r$
100	100								
22	0								
22	100								
22	200								
4.7	100								

List of Equipment

1. 1 x Newtronics Model 200PC or 200MSPC Pulse/Function Generator
2. 1 x Model V552 Oscilloscope
3. 1 x / Oscilloscope Probe
4. 1 x Circuit Board Housing with AC-to-DC Converter
5. 1 x Circuit Board
6. 1 x Fluke Model 77 Digital Multimeter
7. 1 x Coaxial cable - BNC to alligator clips
8. 1 x Short Wire pair - Banana plugs to alligator clips
9. 1 x Long Wire pair - Banana plugs to banana plugs

Gary E.J. Bold, January 2000.

This version: June 10, 2008.