Experiment 343: Compton Scattering and Relativistic Kinematics

Aim

To investigate two aspects of relativistic kinematics:

(a) the scattering of a photon on a free electron (Compton scattering)

(b) the relationship between the momentum and the total energy of a fast electron, and the dependence of the mass of an electron on its velocity,

and to measure the relative differential cross-section and compare with classical and quantum mechanical predictions.

References

The kinematics of Compton scattering is dealt with in many texts on "Modern Physics", e.g. Refs. 1 and 2. These two books also give the kinematical relationships of special relativity. For information on the gamma-ray spectra of the radio-active sources see Refs. 3 and 4.

1. R. D. Evans, "The Atomic Nucleus" McGraw-Hill
2. A. C. Melissinos, "Experiments in Modern Physics" Academic Press
3. C. M. Lederer, "Table of Isotopes"
4. CRC Handbook of Chemistry and Physics.

Ref. 1 still gives the best and most complete description of Compton scattering. All of these references are available in the laboratory.

Introduction

There are three main points to this experiment.

(i) Determination of the scattered photon energy as a function of scattering angle using a ^{137}Cs source. This is done using the pulse height analyser to determine the energy shift of the photoelectric peak in the scintillation detector.

(ii) Determination of the relative differential cross section from the count rate (corrected for geometry and detector efficiency) as a function of angle.

(iii) A study of the energy-momentum relations in special relativity.

Suggestions

- Read through the entire pamphlet before starting the experiment, since the results for parts 1 and 2 of the experiment can be taken simultaneously.
- Turn on the equipmment and allow it to warm up and stabilize before making any measurements (this can take up to an hour).

- Calibration of the equipment should be repeated each day, and monitored throughout the duration of the experiment.
- Think about how you set the regions of interest for the multi-channel analyser so as to obtain good counting statistics.
- The Multi-Channel Analyser programme `Nucleus` has an option to save the data collected. This can be useful for finding appropriate regions of interest.

1 Compton kinematics

The kinematics of Compton scattering were important in the development of quantum theory. This scattering showed clearly that an energetic X-ray or gamma-ray could be treated as a particle, the "photon", travelling at the speed of light c.

Consider the Compton scattering of a photon with energy E_γ and momentum P_γ on an electron, rest mass m_o, which is initially at rest. The photon scattered at an angle θ then has energy E'_γ and momentum P'_γ. Application of the conservation laws for (total) energy and momentum to the photon and electron both treated as particles yields the result:

$$\frac{1}{E'_\gamma} - \frac{1}{E_\gamma} = \frac{1}{m_o c^2}\,(1 - \cos\theta) \tag{1}$$

In the purely classical picture of the scattering of an electromagnetic wave on an electron, the energy (or frequency) of the scattered wave is unchanged from that of the incident wave — Thomson scattering.

In this part of the experiment we investigate this dependence of E'_γ on θ for the 662 keV gamma rays from the strong ^{137}Cs source.

Experiment

(1) Set up the apparatus as follows:

Set photomultiplier HT = +500 V
Connect output of photomultiplier to preamplifier input
Connect negative preamplifier output to 485 amplifier
Connect bipolar output to analyser

The amplifier gain and the analyser controls should be set to get the ^{137}Cs photopeak suitably positioned on the analyser display. Note that as the scattering angle θ is changed, the value of E'_γ will change. The peak should remain on the screen as θ is varied.

(2) Calibrate the multichannel analyser using a suitable selection of the 10 μCi sources.

Some of the gamma-ray energies are listed in the Appendix. For more details look at the references listed earlier.

(3) Set up each source in turn on the holder in front of the detector. Accumulate the spectra and plot the energy of the photopeaks against channel number.

(4) Insert one of the flat scatterers and unplug the strong ^{137}Cs source. Measure E'_γ, the energy of the scattered photon as a function of the scattering angle. At each angle orient the scatterer so that the amount of material traversed by the photons is minimised. Measure on both sides of the beam to reduce misalignment errors.

(5) Measure at a few wisely chosen angles with the other flat scatterer.

Analysis

(6) Plot a suitable graph to test the validity of equation (1).

(7) Determine the rest energy of the electron. In these analyses, you might use the MATLAB routine `regress` which gives error estimates on the fitted parameters.

Questions

1. The scattered photon energy has been measured using both aluminium and copper scatterers. What conclusion relating to the nature of the scattering objects do you draw from the results?
2. When viewing the unscattered spectrum of the source at least three prominent peaks are seen. Explain briefly the origin of these.

2 Compton Cross-section

The probability of scattering (the "cross-section") is given

(i) classically (very low energy) by the Thomson formula:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = r_o^2 \left(\frac{1 + \cos^2 \theta}{2} \right)$$

where r_o = classical electron radius = $e^2 / \left(4\pi\varepsilon_o mc^2\right) = 2.82 \times 10^{-15}\,\mathrm{m}$

(ii) in quantum mechanics (for any energy) by the Klein-Nishina formula:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = r_o^2 \left(\frac{1 + \cos^2 \theta}{2} \right) \frac{1}{\left[1 + \gamma \left(1 - \cos\theta\right)\right]^3} \left(1 + \frac{\gamma^2 \left(1 - \cos\theta\right)^2}{\left(1 + \cos^2\theta\right)\left[1 + \gamma\left(1 - \cos\theta\right)\right]} \right)$$

where $\gamma = h\nu / \left(mc^2\right)$.

Experiment

(8) Investigate the differential cross-section for Compton scattering. Set the L.L.D. to as low a value as possible (i.e., just above the noise level). Operate the pulse height analyser in the integrate mode (see manual supplied with apparatus). Give some thought to the range of integration used. These measurements as a function of angle will be used to determine the relative differential cross-section.

Use one of the flat scatterers and orient it as shown in Fig. 1 to avoid passing the scattered photons through too much scattering material. Since the volume of the scatterer irradiated is determined by the size of the gamma ray beam, rotation of the scatterer through an angle ϕ from the normal makes the amount of scattering material in the beam path vary as $1/\cos\phi$. Note that with the equipment provided, it is easiest to set the orientation of the *plane* of the scatterer. This should be done carefully so that the *normal* is as shown in the figure.

For $\theta < 90^\circ$ the resultant counts must be weighted by $\cos\left(\theta/2\right)$ and for $\theta > 90^\circ$ by $\cos\left[\left(\pi - \theta\right)/2\right]$, where θ is the scattering angle.

Background arises mainly from uncollimated beam leaking out of the source, beam scattering off material near the experiment, and at very forward angles ($20^\circ - 30^\circ$), from beam scattering off the detector shielding. The scatterer attenuates the beam by only $20 - 30\%$, hence a reasonable estimate of background can be obtained by removing the scatterer and counting as before.

(9) Weight your results with 1/(detector efficiency) (see graph in appendix) and compare with the Klein-Nishina and Thomson formulae. Repeat at a few selected angles using the other flat scatterer.

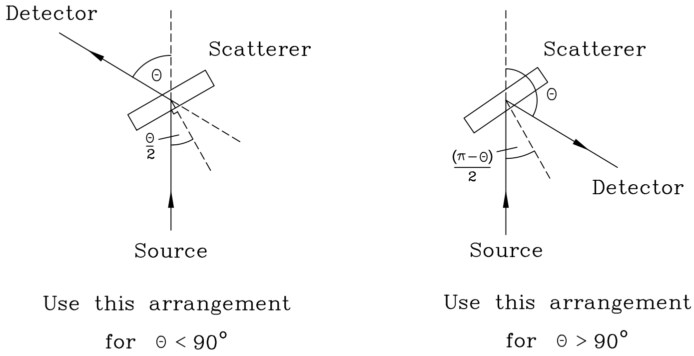

Figure 1: Orientation of scatterer and detector for various values of θ

3 Relativistic energy-momentum relations

The classical mechanics of Newton is inaccurate for particles whose velocity is an appreciable fraction of the speed of light, and special relativity is required for an accurate description of the kinematics.

For example, the relationship between momentum P and kinetic energy T for a particle of (rest) mass m_o is different. Classically:

$$\frac{P^2}{2m_o} = T \tag{2}$$

In special relativity the total energy E is given by:

$$E = T + m_o c^2$$

where m_o represents the "rest energy" of the mass m_o. The energy E and the momentum P are related by:

$$E^2 = P^2c^2 + m_o^2c^4 \tag{3}$$

We find then:

$$\frac{P^2}{2m_o} = T + \frac{T^2}{2m_oc^2} \tag{4}$$

Also, in special relativity, the mass varies with the velocity according to:

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}} \tag{5}$$

In the following we shall investigate these equations experimentally.

Since electrons have a small rest energy ($m_oc^2 = 0.511\,\text{MeV}$) they become highly relativistic at quite low energies. For example, an electron of kinetic energy $1.5\,\text{MeV}$ has $\beta = v/c = 0.97$. In this particular experiment we study the recoil electrons produced in the Compton scattering of gamma-rays of energy in the region of $1\,\text{MeV}$; such electrons will show quite large relativistic effects. By using as a target for the scattering of the gamma-rays a detector device such as a sodium iodide scintillator or a germanium Ge (Li) semiconductor, we can measure both the gamma-ray energy and the electron energy in terms of pulse height from the detector and deduce from these, using well established principles independent of special relativity, the momentum of the electrons. We thus have an experimental determination of both the kinetic energy and the momentum of the electrons.

In the following, a subscript e will denote quantities belonging to an electron, and a γ those for a photon. According to Maxwell's classical electromagnetic theory, the linear momentum P_γ carried in an electromagnetic wave by a photon of energy E_γ is given by:

$$P_\gamma = \frac{1}{c}E_\gamma \tag{6}$$

We now consider the Compton scattering between a photon and an electron. One might picture Compton scattering as the "radiation pressure" of an electromagnetic wave on a single electron.

Consider a head-on collision between the photon (of energy E_γ) and the electron, so that the photon is scattered backwards (180°) with energy E'_γ and momentum P'_γ (see Fig. 2). The electron recoils (at 0°) with momentum P_e, kinetic energy T_e and total energy E_e.

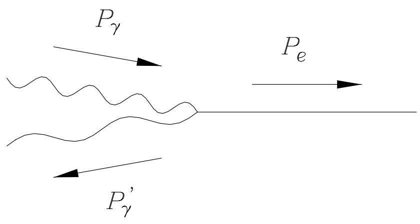

Figure 2: Geometry of Compton scattering

Momentum conservation gives:

$$P_e = \frac{1}{c}\left(E_\gamma + E'_\gamma\right)$$

and total energy conservation gives:

$$E_\gamma + m_o c^2 = E'_\gamma + T_e + m_o c^2$$

Hence, the momentum of the recoiling electron is given by:

$$P_e = \frac{1}{c}\left(2E_\gamma - T_e\right) \tag{7}$$

Since both E_γ and T_e can be determined experimentally, as will be explained below, the electron momentum can also be determined from experimental data, through the use of equation (7). We can therefore examine experimentally the relationship between an electron's momentum and kinetic energy, thus allowing us to find which of the equations (2) or (4) gives the better description.

The mass variation with velocity (see equation (5)) can also be measured using the measured E_γ and T_e values, as follows: First

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{m}{m_o} = \frac{mc^2}{m_o c^2} = \frac{E_e}{m_o c^2} \tag{8}$$

Then using equations (3) and (7), we find that is given by both:

$$\gamma = \left[\left(\frac{2E_\gamma - T_e}{m_o c^2}\right)^2 + 1\right]^{1/2} \tag{9}$$

$$\gamma = \frac{\left(2E_\gamma - T_e\right)^2}{T_e m_o c^2} - 1 \tag{10}$$

Experiment

In order to obtain fast electrons (with energies in the region of an MeV) we scatter gamma rays off the electrons in a sodium iodide scintillation crystal. By selecting the electrons recoiling at 0°, we get electrons of a single energy, which is determined from the energy of the incident gamma-ray.

The spectrum of pulse heights in a sodium iodide scintillator has the form shown in Fig. 1.

The important features are:

E_γ : The energy of the gamma-ray corresponds to the full energy peak ("photo-peak").

T_e : The maximum Compton electron recoil energy, from a 180° scattering of the photon, corresponds to the "edge" of the Compton recoil spectrum.

The spectrum above is that from a mono-energetic gamma-ray source.

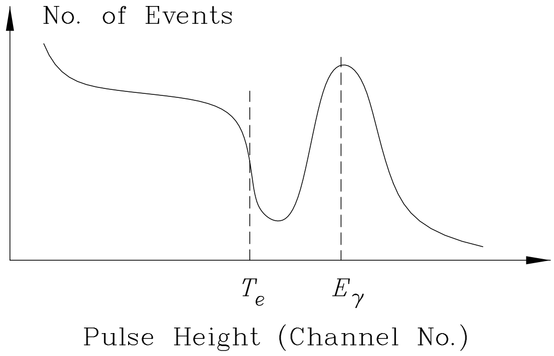

Figure 3: Energy spectrum of electrons recoiling from NaI scintillation crystal

(10) Calibrate the pulse height scale of the analyser in MeV, as in Part A using the full-energy peaks.

(11) For a given gamma-ray energy E_γ, determine the energy T_e of the "zero-degree" electrons from the position of the Compton edge (using the calibration graph to convert "channel number" to MeV).

Calculate the momentum of the electrons using equation (7), which is based on (Maxwellian) electromagnetism and conservation laws.

(12) Obtain an experimental value for the quantity $P_e^2/(2m_o)$ using the equation:

$$\frac{P_e^2}{2m_o} = \frac{(2E_\gamma - T_e)^2}{2m_o c^2} \tag{11}$$

(13) Repeat procedures (11) and (12) for a series of gamma rays, using a selection of radio-active sources.

These measurements of T_e together with the known values of E_γ, provide us also with experimental values of γ through equation (9) or (10).

Analysis

(14) Examine the momentum–kinetic energy relationship by plotting the measured values of $P_e^2/(2m_o)$ against T_e. Compare this experimental relationship with the theoretical predictions of classical mechanics and of special relativity, as given by equations (2) and (4).

(15) Plot experimental values of γ against T_e. (Use equation (9) rather than equation (10).) Compare this with the curve given by special relativity, viz.

$$\gamma = 1 + \frac{T_e}{m_o c^2} \tag{12}$$

(16) Using your measured values of γ, calculate β and plot β against T_e.

Questions

3. Prove equations (4), (9), (10), (11) and (12).

4. In calculating the experimental value of γ from the measured T_e (and the given, assumed exact, value of E_γ), why is it preferable to use equation (9) rather than equation (10)?

   Assume that the error in γ arises from T_e only; calculate the resulting error in γ using each of the equations (9) and (10).

5. The sodium iodide scintillation crystal used in this experiment is quite small – diameter and depth about 25 mm. Why, for this experiment, Part C in particular, is this small crystal preferable to a larger one, such as a 50 mm one?

6. The "Compton Edges" in your measured spectra are smeared out over a number of channels. Explain your choices of channels for T_e.

7. Why does ^{60}Co not provide a suitable source for Part C of this experiment?

8. The peak at 0.075 MeV from the ^{207}Bi source gives a very useful calibration point. However, if you find the decay scheme of ^{207}Bi in the "Table of Isotopes" you will not find a γ ray of 0.075 MeV. What is the source of the peak at 0.075 MeV?

Appendix

Decay schemes of (i)^{137}Cs, (ii)^{22}Na and (iii)^{60}Co

Note: These schemes are for a few representative sources. If they are unavailable, you may need to choose suitable alternatives and look up their decay schemes.

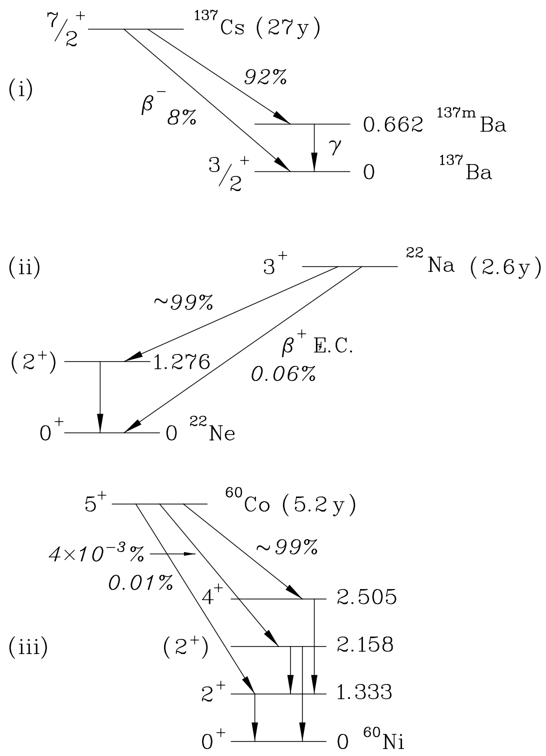

Figure 4: Decay schemes

The energies of the gamma-ray sources used are:

^{137}Cs	0.662 MeV
^{22}Na	0.511 MeV
	1.275 MeV
^{207}Bi	0.075 MeV
	0.570 MeV
	1.063 MeV

Table of the Klein-Nishina cross-section

θ	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°
σ	.815	.645	.488	.364	.282	.221	.180	.164	.149	.148	.146	.146	.147

Table of σ_m relative to zero degrees as a function of scattering angle for $E = 662\,\mathrm{keV}$.

Graph of detector efficiency

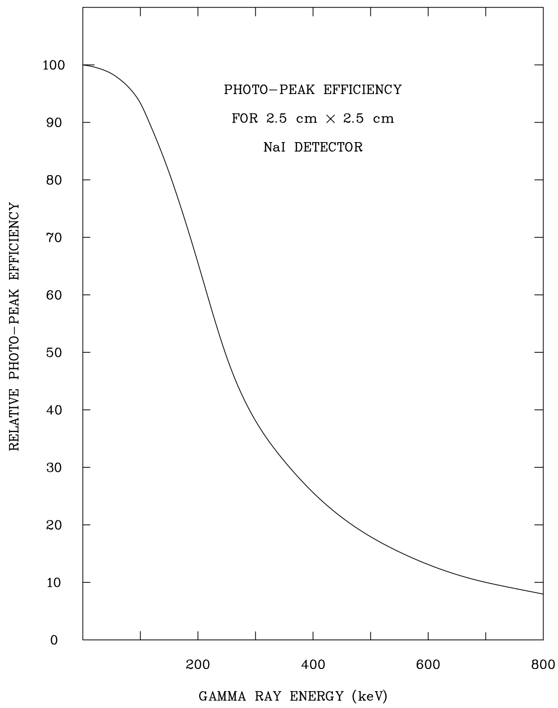

Figure 5: Detector Efficiency

List of Equipment

1. A strong (30 mCi) ^{137}Cs gamma ray source in a lead shield with a collimating hole
2. Weak (10 μCi) gamma ray sources: ^{137}Cs, ^{22}Na and ^{207}Bi
3. Scatterers of aluminium and copper
4. A sodium iodide scintillation detector mounted, with shielding, on a rotating arm
   (Size of scintillator crystal: 25 mm diameter x 25 mm long)
5. Brandenburg Model 472R high voltage supply
6. ORTEC Model 485 amplifier

APPENDIX

Using the 'Gamma Aquisition' Software
A Rough Guide to PHA Mode (Pulse Height Analysis)

Basic Features/Getting Started

Note: The maximum input pulse height that can be processed is a little less than 8V. Make sure the maximum height of the pulses you are interested in is a little less than this.

Finding the Software

You should find an icon called 'Gamma Acquisition' on the Windows desktop.

The Data Acquisition Window

The window you will usually be looking at is shown in Figure 6.

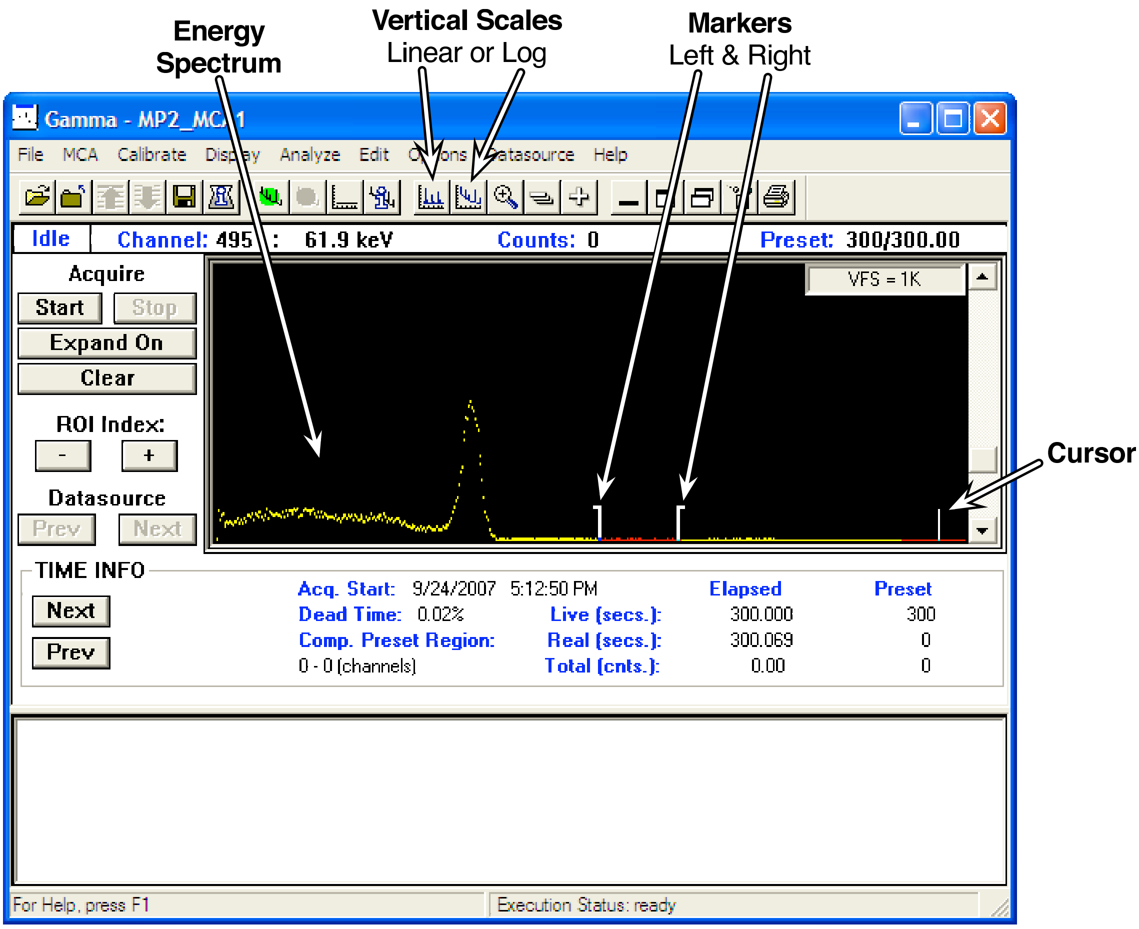

Figure 6: Data Acquisition program window.

Things to note:

- **Energy Spectrum** is a histogram showing number of counts versus channel number (a number which is proportional to the height of the pulse, and, therefore, energy). In other words, each channel corresponds to a small range of energies (an energy bin) and the software plots a histogram showing how many times it received a pulse falling within that range.

  The number of bins/channels can be selected (see 'Changing the Number of Channels' below). In Figure 6 there are 512 channels.

- **Log scales** are good for seeing small peaks when large ones are present, i.e. they emphasise small scale features.

- **Channel:** the channel number where the cursor is. The energy value should be disregarded unless **you** have recently calibrated the energy scale. Note that channel number is (should be) linearly related to pulse height and, thus, energy.

  The energy calibration facility should generally not be used as mistakes are difficult to correct and you have no record of the accuracy of your calibration. If you really *want to use it, read the help on 'Energy Only Calibration' — make sure you still record channel numbers, just in case.*

- **Counts:** The number of counts in the channel where the cursor is, i.e. the number of times a pulse of that height (energy) was detected.

- **Preset:** The preset time for which data collection will run, after which it will automatically stop. The display also indicates how far through this time it is. In this case, it has completed. (See 'Preset Times' for more information.)

- **Acquire:** Button controls for starting and stopping data aquisition, clearing the energy spectrum, and enabling you to expand a portion of the display (selectable using the mouse) for closer inspection.

- **ROI Index:** Button controls which enable you to cycle the markers through the defined Regions Of Interest (ROIs). ROIs have their channels coloured red instead of yellow. (See 'Using ROIs' for more information.)

- **TIME INFO:** This portion of the screen can display various collections of information, cycled through by clicking the Next and Prev buttons. In this case it shows information related to time. Note that 'Live' time excludes Dead Time, i.e. it is the actual number of seconds during which data was actively collected (300, in this case). 'Real' time indicates how long data acquisition has been active, i.e. it equals 'Live' plus 'Dead' time, (300.069s, in this case, This indicates the system was unable to respond to incoming signals for 0.069s.)

  The other collection of interest is **MARKER INFO**. This tells you, amoung other things, the total number of counts (Integral) in the channels between the markers. *Do not use* Area *as this includes an attempt to subtract background via a method not applicable to the experiments in this laboratory.*

- The **cursor** can be moved using the left and right arrows, page up and page down keys, and by clicking the mouse on/near the channel of interest.

- **Markers** can be moved by picking them up with the mouse (the mouse indicator will change to signify you are on a marker) or, if ROIs are set up, by pressing the +/− buttons under ROI Index (see 'Using ROIs' below).

Enabling the Collection of Data

In order to collect data, you must open a detector data source:

- Select File – Open Datasource ... from the menus.
- Click the Source: Detector radio button.
- Select MP2_MCA1 and then click Open.

The energy spectrum will now display the last data collected.

You can now use the buttons under Acquire to collect data.

Advanced Features

Changing the Number of Channels

Although the ADC (Analogue to Digital Converter) is capable of digitising pulses (max. height just below 8V) into 16382 channels, this level of resolution is seldom required. If you wish to change the number of channels, do the following:

1. Select the number of channels you require. This can be found under the menu item MCA – Adjust. If you are not there already, use the Next/Prev buttons until you find the window shown in Figure 7. Select the Conv. Gain (Conversion Gain) you require.

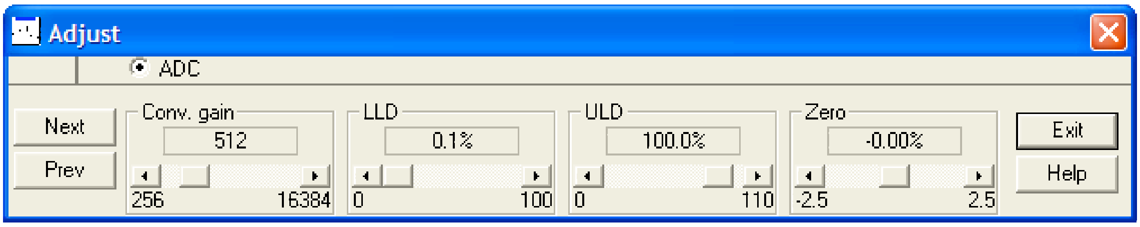

Figure 7: Changing the conversion gain.

2. Ensure that the input size is at least as large as the number of channels you have selected. This can be found under the menu item MCA – Acquire Setup.

Note: MCA stands for Multi-Channel Analyser, a term describing all of the functions of this hardware/software combination.

Setting Up The Preset Acquisition Time

It is sometimes useful to be able to collect data for a known, repeatable, amount of time. To set this up, choose MCA – Acquire Setup from the menu. You can now modify the settings in the Time Preset box.

Saving Energy Spectra

As always, it is a good idea to save your data, just in case something goes wrong.

There are two file types which you may find useful: *.CNF (CAM Files) which can be loaded back into this program, and *.TKA (Toolkit Files) which are text files and can be used in MATLAB.

Using ROIs (Regions Of Interest)

Regions of interest are useful because they enable you to set up a group of channels which are both highlighted (they display in red instead of yellow) and easily selected and reselected for obtaining information such as the integral of counts in that region. Thus, for example, if you wish to obtain the total number of counts in a peak of a particular energy over several measurements, setting up a ROI is a good way to ensure you always select the same number of channels over which to integrate. They may also help you to detect gain shifts, i.e. shifts in the amplification of the pulses. (Expt 352: Interaction of γ Rays with Matter is an experiment for which ROIs should be used.)

The following are the commands you will require:

- **Delete All ROIs:** Select Display – ROIs – Clear All from the menus.
- **Delete One ROI:** Place the cursor within the ROI and press the <Delete> key.

- **Create a ROI:** Place the left and right markers on either side of the region and press the <Insert> key. If it took effect, the region will change colour.
- **Finding the counts integral:** Select the ROI by using the ROI Index buttons until the markers are either side of it. Then, if necessary, use the Next/Prev buttons to select the MARKER INFO *window*.

  Note: Do not use Area as this includes an attempt to subtract background via a method not applicable to the experiments in this laboratory.

Note that ROIs can overlap. The only way to find out where they are is to use the ROI Index buttons to cycle the markers through the current ROIs.

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