Super-resolution microscopy using speckle illumination

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Outline

1. Image formation model
2. From SIM to blind-SIM
3. Marginal estimation

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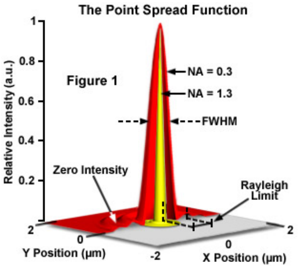
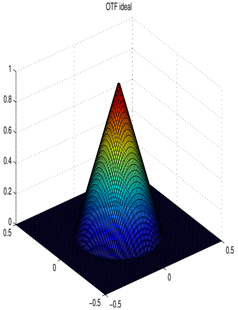
Conventional microscopy is a diffraction limited system:

$$y = h \otimes (\rho \times I) + \epsilon$$



(a) Point spread function h



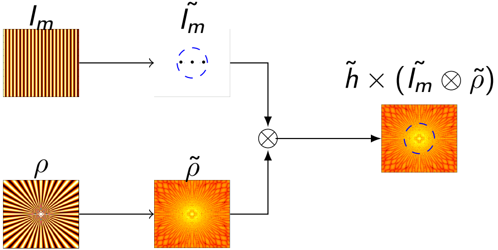
(b) $\tilde{h}$ for circular pupil

Outline

Surpassing the resolution limit
Structured illumination microscope

The image formation model of SIM [2][4] is given by:

$$y = h \otimes (\rho \cdot I_{structured}) + \epsilon$$



From SIM to Blind-SIM

- Sensitive to the distortions of the Illumination
- Blind-SIM [3] is a alternative to SIM proposed in 2012. It replaces structured illumination by M random speckles $I_1, \cdots, I_M$

$$
\begin{cases}
y_I = h \otimes (\rho \cdot I_1) + \epsilon_1 \\
\vdots \\
y_M = h \otimes (\rho \cdot I_M) + \epsilon_M
\end{cases}
$$

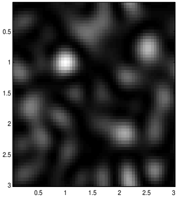


Figure : speckle illumination

Joint estimation in blind-SIM

The super resolution image is obtained by minimizing the joint least square criterion:

$$J(\rho, I_1, \cdots, I_M) = \sum_{m=1}^{M} ||y_m - h \otimes (\rho \cdot I_m)||^2$$

subject to:

$$\begin{cases} \rho >= 0, \quad I_m >= 0, \quad \forall m \\ \dfrac{1}{M} \displaystyle\sum_{m=1}^{M} I_m = I_0 \end{cases} \tag{1}$$

Outline

- Good asymptotic properties
- Known super resolution capacity (2 times OTF support)[5]

$$y_m = h \otimes (\rho \cdot I_m) + \epsilon_m$$

in which:

$$I_m \sim \mathcal{N}(I_0, \Sigma_i)$$

$$\epsilon_m \sim \mathcal{N}(0, \upsilon I)$$

Marginal approach

Maximum Likelihood estimation

$$y_m \sim \mathcal{N}(h \otimes (\rho \cdot I_0), \Sigma_y)$$

in which:

$$\Sigma_y = HR\Sigma_i RH^t + \upsilon I$$

Then the marginal (anti-log) likelihood function is a function of ρ and does not depend on the illumination values

$$J(\rho) = -\sum_{m=1}^{M} \log(f(y_m|\rho))$$

Dealing with covariance matrix Σ_y is difficult for real size images, hence we introduce a patch model:

$$y_{mp} = H_p R I_m \tag{2}$$

In this case, the anti-log likelihood function $J(\rho)$ becomes:

$$J(\rho) = \sum_p J_p(\rho) \tag{3}$$

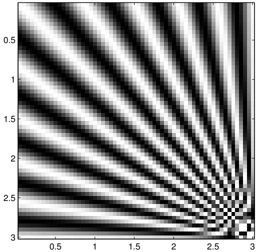
- A star like object is used as the true sample: $\rho \propto (1 + \cos(40\theta))$
- The point spread function is given by the Bessel function

$$h(r, \theta) = \left(\frac{J_1(NAk_0 r)}{k_0 r} \right)^2 \frac{k_0^2}{\pi} \tag{4}$$

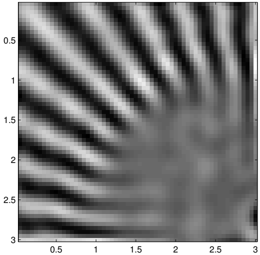
  in which NA is the objective numerical aperture set to 1.49 and $k_0 = \frac{2\pi}{\lambda}$ is the free-space wavenumber with λ the emission and the excitation wavelengths. The sampling step is $\frac{\lambda}{20}$.
- Covariance speckle $\Sigma_I = H$
- Noise level 40dB

Simulations

Original object



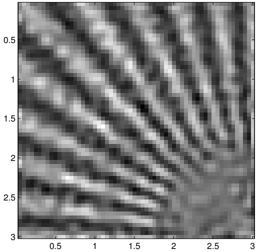
(a) Original object



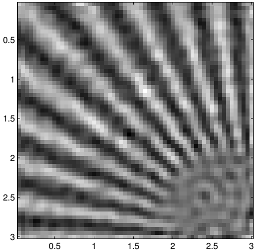
(b) Deconvolution of wide-field image

Simulations

Marginal estimator without patch



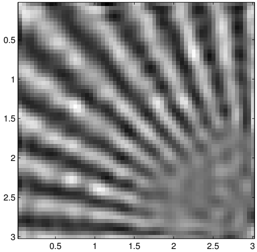
(a) 100 speckle patterns



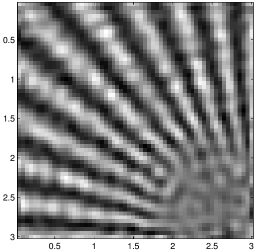
(b) 300 speckle patterns

Simulations

200 Illuminations



(a) Patch size 2x2



(b) Patch size 10x10

Conclusion

- Introduce the marginal approach to image reconstruction using speckle patterns
- Demonstrate the super-resolution capacity of marginal approach and its patch version
- Future work
  - A more efficient numerical optimization algorithm
  - 3D image reconstruction problem will be explored

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