

Lab Report

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1 Introduction

This report is about naive matrix multiplication and heat conduction kernels analysis using CUDA C/C++ language. It is an architecture developed by NVIDIA that uses the computing power of multiple GPU (Compute Unified Device Architecture) cores to offload CPU usage and thus increase device performance. The design of the CUDA architecture assumes full scalability of programs, it means an executable program written today is to run without any changes on increasingly powerful graphics processors in the future. When programming with CUDA technology, the GPU is seen as a computational device capable of executing a large number of threads in parallel. The GPU then works as a coprocessor for the main CPU.

In other words, the program is divided into separate applications that each thread executes at the same time. The program executable by these coprocessors is called the kernel. the following examples will present the use of this vases architecture with an analysis of how this process works.

2 Experimental Part

2.1 Matrix Multiplication

Let's start with an example of matrix multiplication. We call this object a table of numbers, in the sciences it is convenient to use this representation, because it simplifies the model for us, and causes less effort in various calculations.

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1k} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2k} \\ \vdots & & & \ddots & \vdots \\ a_{(n-1)1} & \cdots & \cdots & \cdots & a_{(n-1)k} \\ a_{n1} & a_{n2} & \cdots & \cdots & a_{nk} \end{bmatrix}}_{A_{nk}(t)} \cdot \underbrace{\begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1m} \\ b_{21} & b_{22} & b_{23} & \cdots & b_{2m} \\ \vdots & & & \ddots & \vdots \\ b_{(k-1)1} & \cdots & \cdots & \cdots & b_{(k-1)m} \\ b_{k1} & b_{k2} & \cdots & \cdots & b_{km} \end{bmatrix}}_{B_{km}(t)} = \underbrace{\begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1m} \\ c_{21} & c_{22} & c_{23} & \cdots & c_{2m} \\ \vdots & & & \ddots & \vdots \\ c_{(n-1)1} & \cdots & \cdots & \cdots & c_{(n-1)m} \\ c_{n1} & c_{n2} & \cdots & \cdots & c_{nm} \end{bmatrix}}_{C_{nm}(t)}$$

where each component is given a formula:

$$c_{ij} = \sum_{l=1}^k a_{il}b_{lj}.$$

The number of multiplications of each such term is k, and these k terms must be added to each element of this new matrix. Taking into account the resulting size of the $n \times m$ matrix, these operations are quite a lot for one core, let alone for someone who would like to count them manually on a piece of paper... It just takes time to do this, as you can rightly see, unnecessarily, because each element is counted independently of the others. And this is where NVIDIA comes in with a brilliant approach to breaking this process down into some kind of co-processes called kernels. Our task is divided into parts performed by individual threads.

Let's look at the code of matrix multiplication in the CUDA architecture, where we will divide the multiplication of the elements of each row and column into individual threads.

```

1 #include <stdio.h>
2
3 #define N 64
4
5 __global__ void matrixMulGPU( int * a, int * b, int * c )
6 {
7     int val = 0;
8
9     int row = blockIdx.x * blockDim.x + threadIdx.x;
10    int col = blockIdx.y * blockDim.y + threadIdx.y;
11
12    if (row < N && col < N)
13    {
14        for ( int k = 0; k < N; ++k )
15            val += a[row * N + k] * b[k * N + col];
16        c[row * N + col] = val;
17    }
18 }
19
20 void matrixMulCPU( int * a, int * b, int * c )
21 {
22     int val = 0;
23
24     for( int row = 0; row < N; ++row )
25         for( int col = 0; col < N; ++col )
26         {
27             val = 0;
28             for ( int k = 0; k < N; ++k )
29                 val += a[row * N + k] * b[k * N + col];
30             c[row * N + col] = val;
31         }
32 }
33
34 int main()
35 {
36     int *a, *b, *c_cpu, *c_gpu;
37
38     int size = N * N * sizeof( int ); // Number of bytes of an N x N matrix
39
40     // Allocate memory
41     cudaMallocManaged ( &a, size );
42     cudaMallocManaged ( &b, size );
43     cudaMallocManaged ( &c_cpu, size );
44     cudaMallocManaged ( &c_gpu, size );
45
46     // Initialize memory
47     for( int row = 0; row < N; ++row )
48         for( int col = 0; col < N; ++col )
49         {
50             a[row*N + col] = row;
51             b[row*N + col] = col+2;
52             c_cpu[row*N + col] = 0;
53             c_gpu[row*N + col] = 0;
54         }
55
56     dim3 threads_per_block (16, 16, 1); // A 16 x 16 block threads
57     dim3 number_of_blocks ((N / threads_per_block.x) + 1, (N / threads_per_block.y) + 1, 1);
58
59     matrixMulGPU <<< number_of_blocks, threads_per_block >>> ( a, b, c_gpu );
60
61     cudaDeviceSynchronize(); // Wait for the GPU to finish before proceeding
62
63     // Call the CPU version to check our work
64     matrixMulCPU( a, b, c_cpu );
65
66     // Compare the two answers to make sure they are equal
67     bool error = false;
68     for( int row = 0; row < N && !error; ++row )
69         for( int col = 0; col < N && !error; ++col )
70             if (c_cpu[row * N + col] != c_gpu[row * N + col])
71             {
72                 printf("FOUND ERROR at c[%d][%d]\n", row, col);
73                 error = true;
74                 break;
75             }
76     if (!error)
77         printf("Success!\n");
78
79     // Free all our allocated memory
80     cudaFree(a); cudaFree(b);
81     cudaFree( c_cpu ); cudaFree( c_gpu );
82 }

```

Fig. 1: CUDA C/C++ code for matrix multiplication.

The above code is written in the syntax typical for CUDA, it means it contains some kind of idioms characteristic for this language, it means expressions not found in C/C++ itself, although we support a version based on these languages. It is characteristic here, among other things, to define the functions themselves as global (we declare them with the `__global__` prefix), so that they are visible to the GPU. Undoubtedly, the biggest advantage, not complete, but partial replacement of loops in the code by spreading the work to other machine units (GPU threads), thus freeing you from the information overload, and thus their processing by the CPU. However, we must remember about the appropriate Malloc of memory for the variables processed by the GPU and the `cudaDeviceSynchronize()` function, which will ensure that the GPU will finish its work before giving us the results of the operation. We also need to determine how much computing power we engage in the execution of the process, saving the number of threads, the number of functions performed on the program at the same time and blocks (We will determine them by dividing the data by the number of our functions, read threads, so that the task is distributed as much as possible even way). It is obvious that by increasing the computing power we reduce the time needed to complete the task, but as we will see, we cannot reduce the effective time of the process to zero. As we can see in line 9, we omit the instance related to loops iterating over the matrix at the expense of the definition of another quantity that will help us navigate through the objects of the matrix. Because it so happens that an object such as a matrix does not exist in the computer's memory as we see it, the first thread of the second row is not at all close to the first thread, but in the first row, it is away from it exactly by the number of threads in the row in which it is located. In this approach, iterating over the matrix becomes simple, because if we are interested in a specific thread, it is enough to check in which block it is located, and then add the differences between the general position and the number of the block times its size. Finally, of course, we also need to clean up that busy memory to free ourselves from memory leaks.

I base the analysis of programs on changing parameters, such as the number of executable threads, the program execution time changes. I'm seeing this as a result of using `!nsys profile -stats=true ./[filename]`. Each time the program is called, it happens at a different time, so if I notice a gross error involving a drastic increase in execution time compared to a program using fewer threads, I repeat the call. In this way, I receive a series of measurements for the case of a problem with matrices and heat conduction, which I adjust to the process duration curve depending on the number of threads.

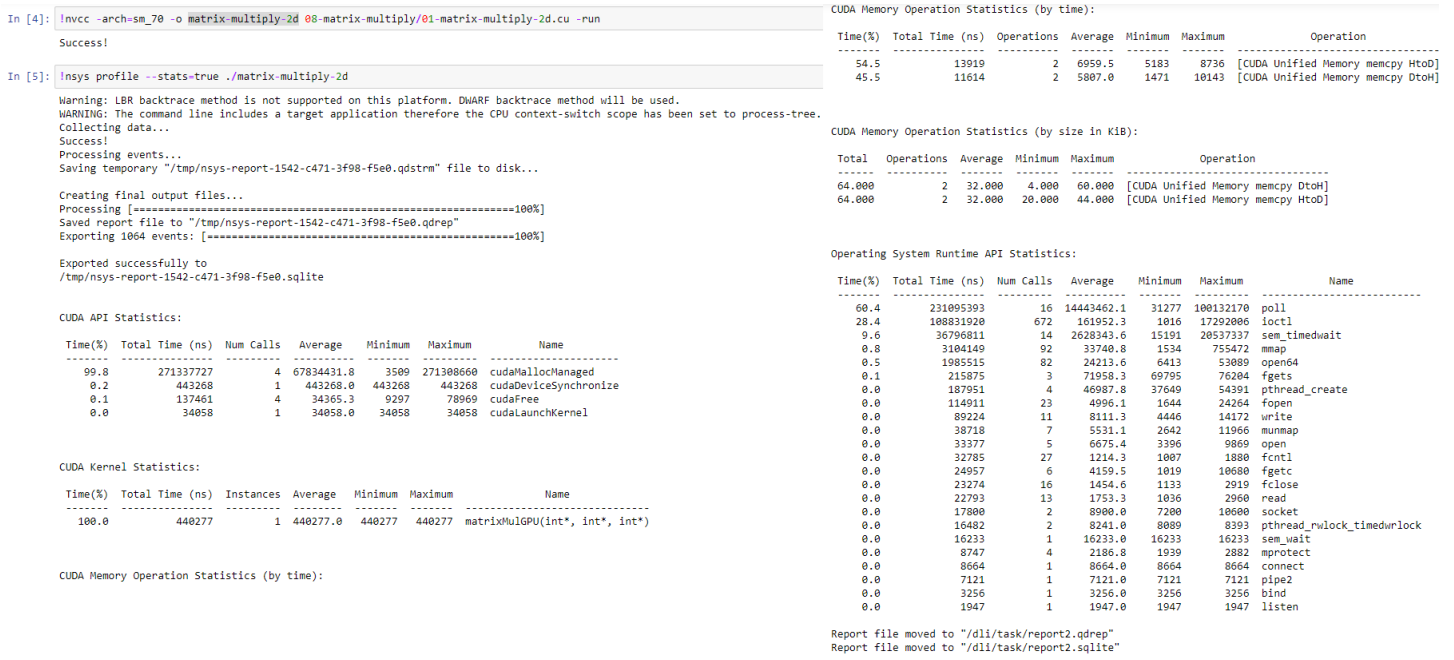


Fig. 2: Program statistics from Figure 1.

In Figure 2, apart from the general parameters of the executed program, we can see a section called "CUDA Kernel Statistics", this is where the information on how long the process took for a single thread is located

Tab. 1: Kernel time execution table depending on the number of threads for matrix multiplication.

Threads_x	Kernel Time [ms]						Mean [ms]	Mean Deviation [ms]
1	0,57	0,53	0,50	0,54	0,53	0,42	0,51	0,03
2	0,51	0,52	0,50	0,51	0,43	0,42	0,48	0,02
3	0,56	0,52	0,53	0,50	0,51	0,42	0,51	0,02
4	0,55	0,54	0,50	0,50	0,49	0,42	0,50	0,02
5	0,51	0,55	0,50	0,48	0,51	0,45	0,50	0,02
6	0,55	0,49	0,50	0,45	0,45	0,43	0,48	0,02
7	0,48	0,50	0,49	0,46	0,46	0,46	0,48	0,01
8	0,54	0,51	0,52	0,48	0,47	0,42	0,49	0,02
9	0,43	0,49	0,50	0,49	0,45	0,44	0,47	0,02
10	0,39	0,49	0,53	0,49	0,42	0,40	0,45	0,03
11	0,50	0,50	0,51	0,46	0,46	0,43	0,47	0,02
12	0,51	0,52	0,48	0,47	0,46	0,43	0,48	0,02
13	0,49	0,48	0,45	0,45	0,43	0,42	0,45	0,02
14	0,53	0,47	0,51	0,45	0,43	0,43	0,47	0,02
15	0,52	0,48	0,48	0,46	0,46	0,48	0,48	0,01
16	0,50	0,48	0,46	0,43	0,41	0,44	0,46	0,02
Threads_y	1	2	4	8	16	32		

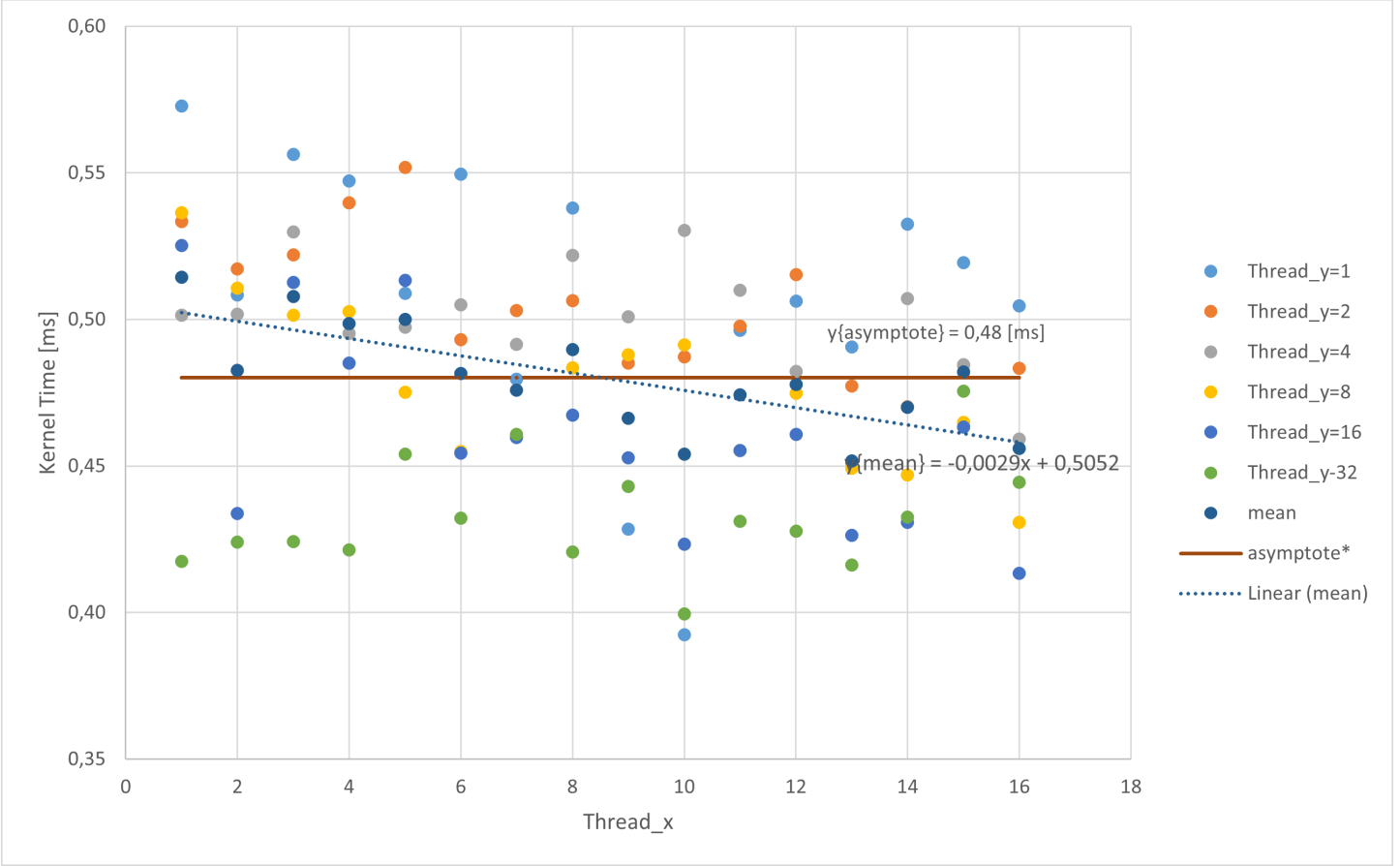


Fig. 3: Matrix plot for data from tab 1. *Actually it is not an asymptote as we know it, but a value around which the kernel execution time oscillates.

Based on the measurement data obtained with each program call, I determine the average time for a given Thread_x between different Thread_y (these are the values on the vertical line Thread_x = n; n = 1,2,...,16). fitting the line to the data, we get a decreasing function, but it changes slightly with the change in the number of threads. theoretically, we can determine its point of intersection with the x axis. We get this for Thread_x=173, inserting this value into the body of the code for any Thread_y (preferably to the one closest to the mean value, i.e. reading Thread_y=4 from the graph) we get about 0.47 [ms], so it is not a descending line, but a constant around which the data oscillates. if I take the average of the averages from the 1 table and use the uncertainty transfer law, I get the asymptote that the thread execution time tends to with a deviation $\sigma_y = 0,2[\text{ms}]$. This means that the time of the executing kernel will saturate for a certain value, regardless of the number of blocks or threads used.

2.2 Heat Conduction

The formula to represent 2D heat conduction is given as follows:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right),$$

where T is temperature, t is time, α is the thermal diffusivity, and x and y are points in a grid. To solve this problem, one possible finite difference approximation is:

$$\frac{\Delta T}{\Delta t} = \alpha \left(\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} \right),$$

where ΔT is the temperature change over time Δt and i,j are indices in a grid. The implementation of this problem into accelerated CUDA code is below.

```

1 #include <stdio.h>
2 #include <math.h>
3
4 // Simple define to index into a 1D array from 2D space
5 #define I2D(num, c, r) ((r)*(num)+(c))
6
7 __global__
8 void step_kernel_mod(int ni, int nj, float fact, float* temp_in, float* temp_out)
9 {
10     int i00, im10, ip10, i0m1, i0p1;
11     float d2tdx2, d2tdy2;
12
13     int j = blockIdx.x * blockDim.x + threadIdx.x;
14     int i = blockIdx.y * blockDim.y + threadIdx.y;
15
16     // loop over all points in domain (except boundary)
17     if (j > 0 && i > 0 && j < nj-1 && i < ni-1) {
18         // find indices into linear memory
19         // for central point and neighbours
20         i00 = I2D(ni, i, j);
21         im10 = I2D(ni, i-1, j);
22         ip10 = I2D(ni, i+1, j);
23         i0m1 = I2D(ni, i, j-1);
24         i0p1 = I2D(ni, i, j+1);
25
26         // evaluate derivatives
27         d2tdx2 = temp_in[im10]-2*temp_in[i00]+temp_in[ip10];
28         d2tdy2 = temp_in[i0m1]-2*temp_in[i00]+temp_in[i0p1];
29
30         // update temperatures
31         temp_out[i00] = temp_in[i00]+fact*(d2tdx2 + d2tdy2);
32     }
33 }
34
35 void step_kernel_ref(int ni, int nj, float fact, float* temp_in, float* temp_out)
36 {
37     int i00, im10, ip10, i0m1, i0p1;
38     float d2tdx2, d2tdy2;
39
40
41     // loop over all points in domain (except boundary)
42     for ( int j=1; j < nj-1; j++ ) {
43         for ( int i=1; i < ni-1; i++ ) {
44             // find indices into linear memory
45             // for central point and neighbours
46             i00 = I2D(ni, i, j);
47             im10 = I2D(ni, i-1, j);
48             ip10 = I2D(ni, i+1, j);
49             i0m1 = I2D(ni, i, j-1);
50             i0p1 = I2D(ni, i, j+1);
51
52             // evaluate derivatives
53             d2tdx2 = temp_in[im10]-2*temp_in[i00]+temp_in[ip10];
54             d2tdy2 = temp_in[i0m1]-2*temp_in[i00]+temp_in[i0p1];
55
56             // update temperatures
57             temp_out[i00] = temp_in[i00]+fact*(d2tdx2 + d2tdy2);
58         }
59     }
60 }
61
62 int main()
63 {
64     int istep;
65     int nstep = 200; // number of time steps
66
67     // Specify our 2D dimensions
68     const int ni = 200;
69     const int nj = 100;
70     float tfac = 8.418e-5; // thermal diffusivity of silver
71
72     float *temp1_ref, *temp2_ref, *temp1, *temp2, *temp_tmp;
73
74     const int size = ni * nj * sizeof(float);
75
76     temp1_ref = (float*)malloc(size);
77     temp2_ref = (float*)malloc(size);
78     cudaMallocManaged(&temp1, size);
79     cudaMallocManaged(&temp2, size);
80
81     // Initialize with random data
82     for( int i = 0; i < ni*nj; ++i ) {
83         temp1_ref[i] = temp2_ref[i] = temp1[i] = temp2[i] = (float)rand()/(float)(RAND_MAX/100.0f);
84     }
85
86     // Execute the CPU-only reference version
87
88     for (istep=0; istep < nstep; istep++) {
89         step_kernel_ref(ni, nj, tfac, temp1_ref, temp2_ref);
90         // swap the temperature pointers
91         temp_tmp = temp1_ref;
92         temp1_ref = temp2_ref;
93         temp2_ref = temp_tmp;
94     }
95     dim3 tblocks(32, 16, 1);
96     dim3 grid((nj/tblocks.x)+1, (ni/tblocks.y)+1, 1);
97     cudaError_t ierrSync, ierrAsync;
98
99     // Execute the modified version using same data
100     for (istep=0; istep < nstep; istep++) {
101         step_kernel_mod<<< grid, tblocks >>>(ni, nj, tfac, temp1, temp2);
102
103         ierrSync = cudaGetLastError();
104         ierrAsync = cudaDeviceSynchronize(); // Wait for the GPU to finish
105         if (ierrSync != cudaSuccess) { printf("Sync error: %s\n", cudaGetErrorString(ierrSync)); }
106         if (ierrAsync != cudaSuccess) { printf("Async error: %s\n", cudaGetErrorString(ierrAsync)); }
107
108         // swap the temperature pointers
109         temp_tmp = temp1;
110         temp1 = temp2;
111         temp2 = temp_tmp;
112     }
113
114     float maxError = 0;
115     // Output should always be stored in the temp1 and temp1_ref at this point
116     for( int i = 0; i < ni*nj; ++i ) {
117         if (abs(temp1[i]-temp1_ref[i]) > maxError) { maxError = abs(temp1[i]-temp1_ref[i]); }
118     }
119
120     // Check and see if our maxError is greater than an error bound
121     if (maxError > 0.0005f)
122         printf("Problem! The Max Error of %.5f is NOT within acceptable bounds.\n", maxError);
123     else
124         printf("The Max Error of %.5f is within acceptable bounds.\n", maxError);
125     free( temp1_ref );
126     free( temp2_ref );
127     cudaFree( temp1 );
128     cudaFree( temp2 );
129     return 0;
130 }

```

Fig. 4: Code in CUDA C/C++ architecture for heat conduction .

As a result of using the nsys function, we receive program execution statistics, in particular, we will be interested in the program execution time.

```

In [7]: Invc -arch-sm_70 -o heat-conduction 09-heat/01-heat-conduction.cu --run
The Max Error of 0.00001 is within acceptable bounds.

CUDA Memory Operation Statistics (by size in KiB):
Total Operations Average Minimum Maximum Operation
-----
256.000 9 28.444 4.000 100.000 [CUDA Unified Memory memcpy HtoD]
128.000 4 32.000 4.000 60.000 [CUDA Unified Memory memcpy DtoH]

In [8]: Insys profile --stats=true ./heat-conduction
Warning: LBR backtrace method is not supported on this platform. DWARF backtrace method will be used.
WARNING: The command line includes a target application therefore the CPU context-switch scope has been set to process
Collecting data...
The Max Error of 0.00001 is within acceptable bounds.
Processing events...
Saving temporary "/tmp/nsys-report-bicc-7e9e-4997-29e9.qdstrm" file to disk...

Creating final output files...
Processing [=====100%]
Saved report file to "/tmp/nsys-report-bicc-7e9e-4997-29e9.qdrep"
Exporting 1873 events: [=====100%]

Exported successfully to
/tmp/nsys-report-bicc-7e9e-4997-29e9.sqlite

CUDA API Statistics:
Time(%) Total Time (ns) Num Calls Average Minimum Maximum Name
-----
97.9 258616954 2 129388477.0 30782 258586172 cudaMallocManaged
1.7 4420831 200 22100.2 6143 609684 cudaDeviceSynchronize
0.4 1106907 200 5534.5 3756 60906 cudaLaunchKernel
0.1 148330 2 74165.0 53010 95320 cudaFree

CUDA Kernel Statistics:
Time(%) Total Time (ns) Instances Average Minimum Maximum Name
-----
100.0 3894439 200 19472.2 16415 610546 step_kernel_mod(int, int, float, float*, float*)

CUDA Memory Operation Statistics (by time):
Time(%) Total Time (ns) Operations Average Minimum Maximum Operation
-----
71.0 57080 9 6342.2 2335 18079 [CUDA Unified Memory memcpy HtoD]
29.0 23261 4 5815.3 1439 10176 [CUDA Unified Memory memcpy DtoH]

Operating System Runtime API Statistics:
Time(%) Total Time (ns) Num Calls Average Minimum Maximum Name
-----
71.3 370335382 21 17635018.2 32603 100130704 poll
18.5 96258472 672 143241.8 1012 17291780 ioctl
9.1 47383979 17 2787292.9 23254 20540284 sem_timedwait
0.6 2997415 92 32580.6 1445 753061 mmap
0.3 1592705 82 19423.2 4744 45103 open64
0.0 214050 3 71350.0 68733 75067 fgetc
0.0 171686 4 42921.5 32872 51181 pthread_create
0.0 111155 23 4832.8 1800 25351 fopen
0.0 83060 11 7550.9 4471 11696 write
0.0 35665 5 7133.0 3878 9827 open
0.0 34665 8 4333.1 1444 8991 munmap
0.0 32290 28 1153.2 1007 1615 fcntl
0.0 27745 13 2134.2 1403 3320 read
0.0 24800 6 4148.3 1059 10746 fgetc
0.0 22650 16 1415.6 1046 2279 fclose
0.0 17364 2 8682.0 7459 9905 socket
0.0 14587 1 14587.0 14587 14587 pthread_rwlock_timedwrlock
0.0 8515 4 2128.8 1726 2544 mprotect
0.0 8387 1 8387.0 8387 8387 pipe2
0.0 8247 1 8247.0 8247 8247 connect
0.0 2601 1 2601.0 2601 2601 bind
0.0 1761 1 1761.0 1761 1761 listen

Report file moved to "/dli/task/report4.qdrep"
Report file moved to "/dli/task/report4.sqlite"

```

Fig. 5: Program statistics from Figure 4.

Tab. 2: Kernel time execution table depending on the number of threads for heat conduction.

Threads_x	Kernel Time [ms]							Mean [ms]	Mean Deviation [ms]
1	13,89	7,97	4,76	3,22	2,52	1,75	1,65	5,11	1,69
2	7,83	4,55	3,02	2,19	1,86	1,71	1,84	3,28	0,85
3	5,73	3,65	2,67	2,00	1,71	1,64	1,84	2,75	0,57
4	4,66	2,98	2,63	1,97	1,61	1,63	1,58	2,44	0,43
5	3,98	3,12	2,40	1,83	1,82	1,73	1,69	2,37	0,34
6	3,53	2,77	2,36	2,08	2,01	1,67	1,80	2,32	0,25
7	3,20	2,70	2,10	1,71	2,05	1,84	1,89	2,21	0,21
8	2,95	2,59	1,90	1,86	2,12	1,84	1,82	2,15	0,17
9	3,10	2,24	2,20	1,92	2,10	1,90	1,88	2,19	0,17
10	2,65	2,36	2,33	1,96	2,18	1,89	2,21	2,22	0,1
11	3,04	2,18	2,37	2,19	2,19	1,92	2,06	2,28	0,14
12	2,80	2,25	2,44	2,40	2,21	2,02	2,18	2,33	0,1
13	2,97	2,11	2,36	2,05	2,26	2,16	2,29	2,31	0,12
14	3,02	2,43	2,38	2,15	2,04	2,18	2,42	2,37	0,13
15	2,99	2,23	2,07	2,13	2,18	2,19	2,54	2,33	0,13
16	2,82	2,36	2,46	2,51	2,29	2,27	2,34	2,44	0,08
Threads_y	1	2	4	8	16	32	64		

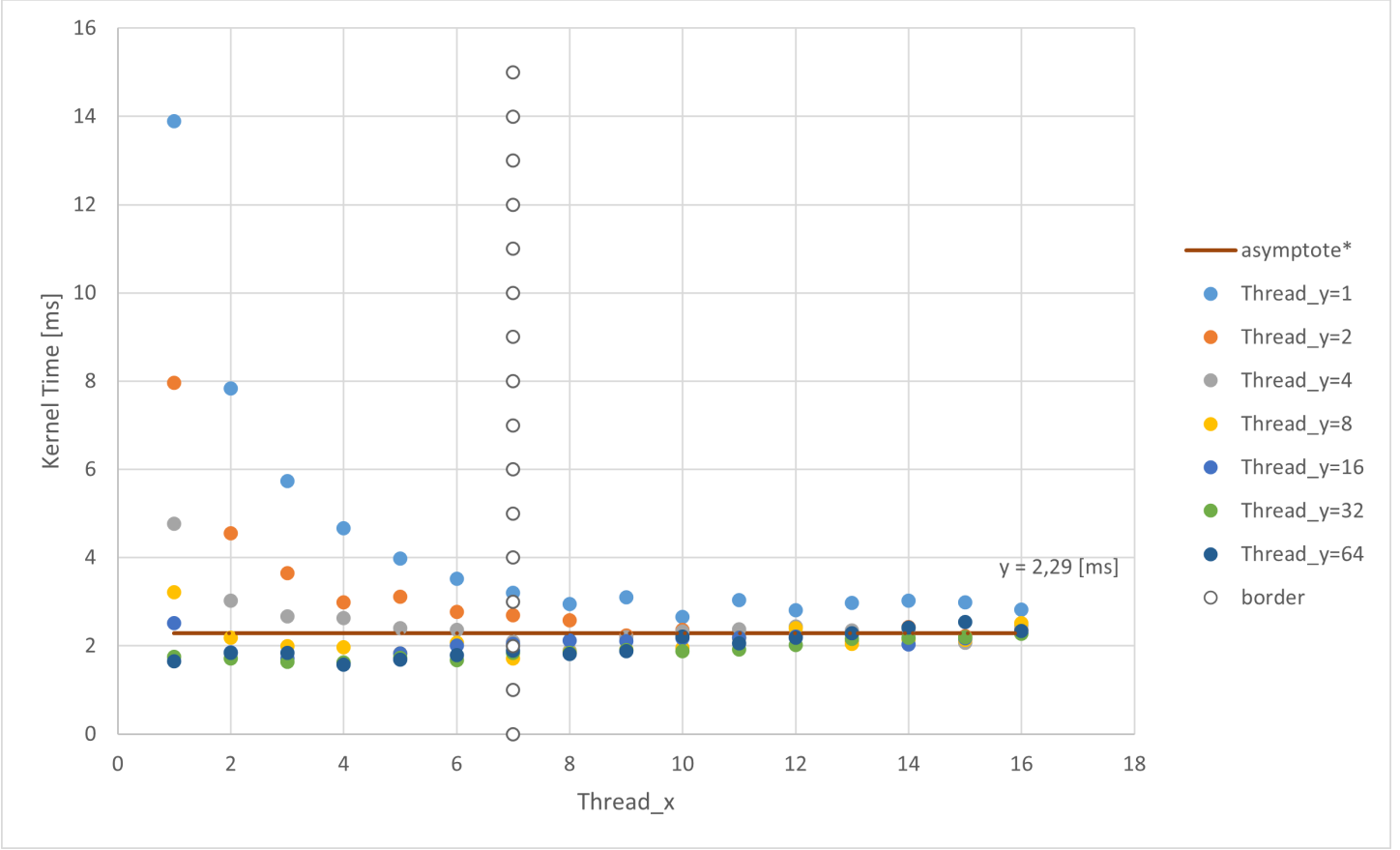


Fig. 6: Heat conduction plot for data from tab 2. *Actually it is not an asymptote as we know it, but a value around which the kernel execution time oscillates, in this case, for the average around which the data oscillates, I took the values from Thread number 7 because below 7 they diverge.

The case of heat conduction is so much simpler that you can see at first glance how the values of thread execution times for this process drastically decrease to a certain constant value. The graph of the dependence of these times on the number of threads becomes constant already for $\text{Thread_x} = 7$, so to determine this value, I will take the average values between the points, similarly as in the case of matrix multiplication, with the difference that I will omit the points on the Thread_x axis from 1 to 6. So, taking the average of the average distances between thread execution times for a given Thread_x , it obtains a value of 2.29 [ms], with an deviation of 0.2 [ms], also obtained by the uncertainty transfer law.

3 Summary

In this report, I presented the process implemented by the CUDA architecture on the examples of matrix multiplication with a very large dimension and heat conduction , approximating the given partial differential equation by very small temperature equally small spatial increments. in which this process will be resolved. As a result of a more detailed analysis of the graph of the process duration versus the number of threads, they come to the conclusion that the execution time reaches a certain constant value, which it does not fully achieve due to errors, but the measurement points remain close to the curve despite the increasing number of threads. To sum up, although we will not be able to reduce the execution time of the program, you will underestimate any value of time, it is a fair price for the incredibly fast access to results compared to an analogous program, but operating in the architecture of only one process.