

1 Overview

In this lecture we discussed the definition of complex systems, basic rules of modelling and population dynamics.

2 Ethnic groups

As United States Census showed people of different ethnicities tend to live in the neighbourhood of people of similar background (unfortunately I haven't been able to find same datasets prof used in class. The one linked is best I found). This behaviour was first described in paper by Schelling[1]. This paper shows how simple changes made in micro scale can show very sharp results in macro scale. Computer Science Department of Harding University prepared very good explanation and visualisation of Schelling model¹.

3 Complex System

Complex Systems as is don't have a precise definition. Rather they are defined by a set of properties. Those rules include:

- Being composed of many parts
- Having simple rule in micro scale
- Being hard to predict in macro scale
- Buzzword for complex systems - emergence (novelty)

Example of the complex system are the Storks of Oldenburg described by Robert Matthews[2] which shows that there is a correlation between amount of breeding storks and newborn babies in European countries. That does not of course mean that storks bring babies, as correlation does not imply causation.

¹For anyone interested original paper by Schelling can be found eg. here

4 Population dynamics and logistic growth model

Quoting Wikipedia: "Population dynamics is the branch of life sciences that studies the size and age composition of populations as dynamical systems, and the biological and environmental processes driving them". From very beginning of population dynamics, there were tries to use mathematical language to describe it. The first one to do it was Pierre François Verhulst who suggested an equation (later corrected and popularized by Pearl and Reed) in such form as shown in 1

$$\frac{dN}{dt} = rN(1 - \frac{N}{K}) \quad (1)$$

where N stands for number of Individuals, and K is presumed equilibrium. This is a formula for a logistic curve.

In discrete form this equation goes like this $\frac{n_{t+1}-n_t}{\delta t} = rn_t(1 - n_t)$ which for delta getting smaller to 0, gives us a differential equation from definition, guiding us to the 1.

References

- [1] Thomas C. Schelling Dynamic models of segregation. *The Journal of Mathematical Sociology*, 1:2, 143-186, 1971.
- [2] Robert Matthews Storks Deliver Babies (p=0.008) *Teaching Statistics*, Vol. 22, No. 2, 2000.