

Kinetic Energy — 04.03, 2019

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1 Overview

In the last lecture we discussed simple models in statistical physics such as models of atoms and spins and mentioned liquid crystals.

In this lecture we talked about Kinetic theory of gases and thermodynamics.

2 Kinetic theory of gases

There are about 10^{27} atoms in the classroom. They are all in constant movement.

2.1 Pressure

Let's have an area A and apply force F to it. This is called pressure and we denote it as $p = \frac{F}{A}$

2.2 Kinetic equation of state for ideal gas

Note: We assume no forces between molecules.

Now, let's have a small area. Molecules move, so they are bound to hit this area. Let's consider molecules only in a certain cuboid in time dt . This model is shown on graphic 1. We will look only on movement in x axis for now. We can express volume of our cuboid as $dV = dA * v_x * dt$. Now, let's consider what happens when a molecule hits that area. Its mass doesn't change (of course), but its velocity changes direction. Assuming all collisions are elastic (which we will) its value doesn't change. This is change of momentum, which is expressed as $\vec{p} = m\vec{v}$.

Momentum in unit of volume can be expressed in terms of density, ie. $\rho * v_x$. This means we can substitute and see that total momentum in unit of volume is equal to

$$dP_{tot} = \rho * v_x * dV = \rho * v_x^2 dA * dt \quad (1)$$

as dV , ie. "infinitely small volume" in this model can be expressed as $dV = dA * v_x * dt$ which is base times height of our model geometrically speaking. This gives change of momentum equal to

$$dP = 2dP_{tot} \quad (2)$$

Now, we can express ρ as number of particles divided by the volume we describe. Because they go in two directions (we look now only on the movement in x axis) to, and away from the plane, we take half of those particles and also take their mass into account. $\rho = \frac{mN}{2V}$.

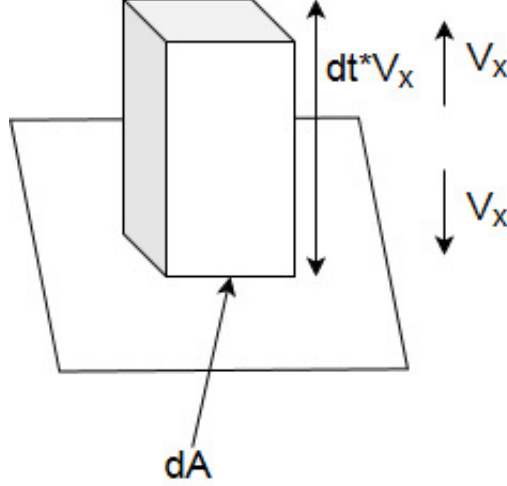


Figure 1: Sample model

Substituting, we get

$$dP = 2 * \frac{mN}{2V} * v_x^2 dA * dt \quad (3)$$

From Newton, we know that force is equal to change of momentum in time

$$F = \frac{dP}{dt} = \frac{Nm}{V} * v_x^2 dA \implies p = \frac{dF}{dA} = \frac{mN}{V} * v_x^2 \quad (4)$$

This model has one fault. It assumes all velocities are equal, when in fact they are gaussian. Generalized, we can take average of them, but because $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$ and "full" velocity consists of all three of those, we can say $\langle v^2 \rangle = 3 * \langle v_x^2 \rangle$. In such case momentum is equal to $p = \frac{mN}{V} * \frac{\langle v^2 \rangle}{3}$, which after transformation gives us equation of state for ideal gas, ie.

$$pV = \frac{1}{3} * Nm \langle v^2 \rangle \quad (5)$$

2.3 Temperature

We define temperature as follows

$$T = \frac{2}{3k_B} \frac{m \langle v^2 \rangle}{2} \quad (6)$$

This is absolute temperature measured in Kelvin. The second part is average kinetic energy. Expressing this equation in terms of velocity gives $\langle v^2 \rangle = T 3k_B \frac{1}{m}$, which after transforming and simplifying gives us

$$pV = Nk_B T \quad (7)$$

which is our equation of state. k_B meaning Boltzman constant.

We can write this equation in form $f(T,p,N)=0$, or alternatively we can use Avogadro number to express the number of moles ($n = \frac{N}{N_o}$). Then $pV = n * N_o * k_B * T$, substituting R, we get familiar from school equation

$$pV = nRT \quad (8)$$

3 Thermodynamics

3.1 First law of thermodynamics

We have a system in connection with external world (eg. a room). We'll not discuss change of amount of particles here.

Now, internal energy is the energy of moving particles. It exists even when there's no mechanical energy in a system. We will denote this energy as $dU = \delta Q - \delta W$. Let's remind ourselves that in thermodynamics work is connected to expansion of a system.

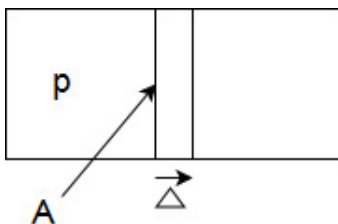


Figure 2: Expansion of a system

Work can be expressed here as $W = p * A * \vec{\Delta}$ which means $dW = pdV$. With this we can say

$$dU = \delta Q - pdV \quad (9)$$

3.2 Entropy

We will now discuss Boltzman entropy. Let's have Ω microscopic states of macroscopic system. They are all equally probable. Boltzman entropy can be expressed as

$$S_B = k_B \ln \Omega \quad (10)$$

Let's consider a system and introduce an arbitrary parameter.¹ This parameter will be (in general, for n particles) equal to $\Omega^{(n)} = (\frac{V}{V_o})^{(n)}$, which will give us slightly changed formula for entropy

$$S_B = k_B \ln \left(\frac{V}{V_o} \right)^{(n)} \quad (11)$$

¹I didn't note too much this part. Can anybody explain what this parameter does?