Introduction to Complex Systems Summer 2019

Logistic map and q-Potts Model — 01.04, 2019

mgr inż. Bartłomiej Nowak *Scribe: Krzysztof Agieńczuk*

1 Overview

This was a guest lecture by mgr inż. **Bartłomiej Nowak**

In the last lecture we talked about Power Laws.

In this lecture we covered: q-Potts standard model and quick reminder of logistic map.

2 Logistic map

Logistic map is an equation of form

$$x_{n+1} = a(1 - x_n)x_n \tag{1}$$

Fixed point is a for which value of next point is equal to value in the previous point. It conveys to calculating

$$x = a(1 - x)x \tag{2}$$

Stability of such point is can be checked by calculating the derivative of map in a fixed point and checking whether it's absolute value is smaller than 1

$$|f'(x*)| < 1 \tag{3}$$

Fixed points tend to go to 1 point. Sometimes we obtain 2,3,4-cycle which alternate between 2,3,4 points (it is possible to get cycle between more points). Three cycles are "hidden" in 4 cycles.

Checking stability of cycles is similar to checking stability of normal map. We just plug one function into the other.

$$\left|\tfrac{\partial}{\partial x*}(f^n(x*))\right| < 1 \tag{4}$$

where f^n is $f(f(f...))$ made n times.

3 Standard q-Potts model

Hamiltonian is used to describe energy in our system.

$$H = -J\sum_{i,j}\delta\sigma_i\sigma_j \tag{5}$$

where δ is Kronecker delta and σ

$$\sigma = exp(\frac{i2\pi S_k}{q}) \quad S_k = 1, 2, ..., q \tag{6}$$

i,j means we we move only to our neighbours. We can use both von Neuman and Moore definitions of neighbourhood.

For q=2 q-Potts model simplifies to Issing model. For q less than 4 hysteresis exists in a system