Faculty of Economic and Management

Gdańsk University of Technology

KREMÓWKI 

ASSIGNMENT-01

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Course: Decision Analysis

Problem 1 – Continuous Linear Problem

1. Problem Statement

A local bakery specializes in producing five types of baked goods: bread loaves, pastries, cakes, kremówki (cream cakes), and eklerki (eclairs). The bakery operates with limited resources in terms of labor hours, oven time and ingredient (flour) availability. The goal is to determine the optimal production quantities for each product to maximize profit while adhering to these constraints.

x1	Number of bread loaves							
x2	Number of pastries							
x3	Number of cakes							
x4	Number of kremówki							
x5	Number of eklerki							
	Product	x1	x2	x3	x4	x5		
	Profit per unit	5,5	7	15	5,2	3		
	Labour hours	0,6	0,7	3	0,9	0,85		
	Oven time (hours)	1,1	1,9	4	1,1	0,95		
	Ingredients	4	3	10	2	2		
	How much to produce?	0	0	0	0	0	>20	<150
	Profit =	0		LHS	RHS			
				0	420	labour hours	<	
				0	700	oven time	<	
				0	1300	ingredients	<	
				0	450	sum of produ	>	

New Decision Variables:

- x1: Number of bread loaves produced.
- x2: Number of pastries produced.
- x3: Number of cakes produced.
- x4: Number of kremówki (cream cakes) produced.
- x5: Number of eklerki (eclairs) produced.

Profit Contributions:

- Bread profit = $5,5 per loaf.
- Pastry profit = $7 per piece.

- Cake profit = $15 per cake.
- Kremówki profit = $5,2 per piece.
- Eklerki profit = $3 per piece.

2. Constraints

1. Labor Hours (60 hours limit):

0,6*x1 + 0,7*x2 + 3*x3 + 0,9*x4 + 0,85*x5 <= 420

2. Oven Time (30 hours limit):

1,1*x1 + 1,9*x2 + 4*x3 + 1,1*x4 + 0,95*x5 <= 700

3. Ingredients (70 units limit):

4*x1 + 3*x2 + 10*x3 + 2*x4 + 2*x5 <= 1300

4. Non-Negativity:

x1, x2, x3, x4, x5 >= 0

5. Sum of products:

x1+x2+x3+x4+x5 >= 450

6. Each product at least 20 units:

x1 >=20, x2 >= 20, x3 >=20, x4 >=20, x5 >=20

7. Each product at most 150 units:

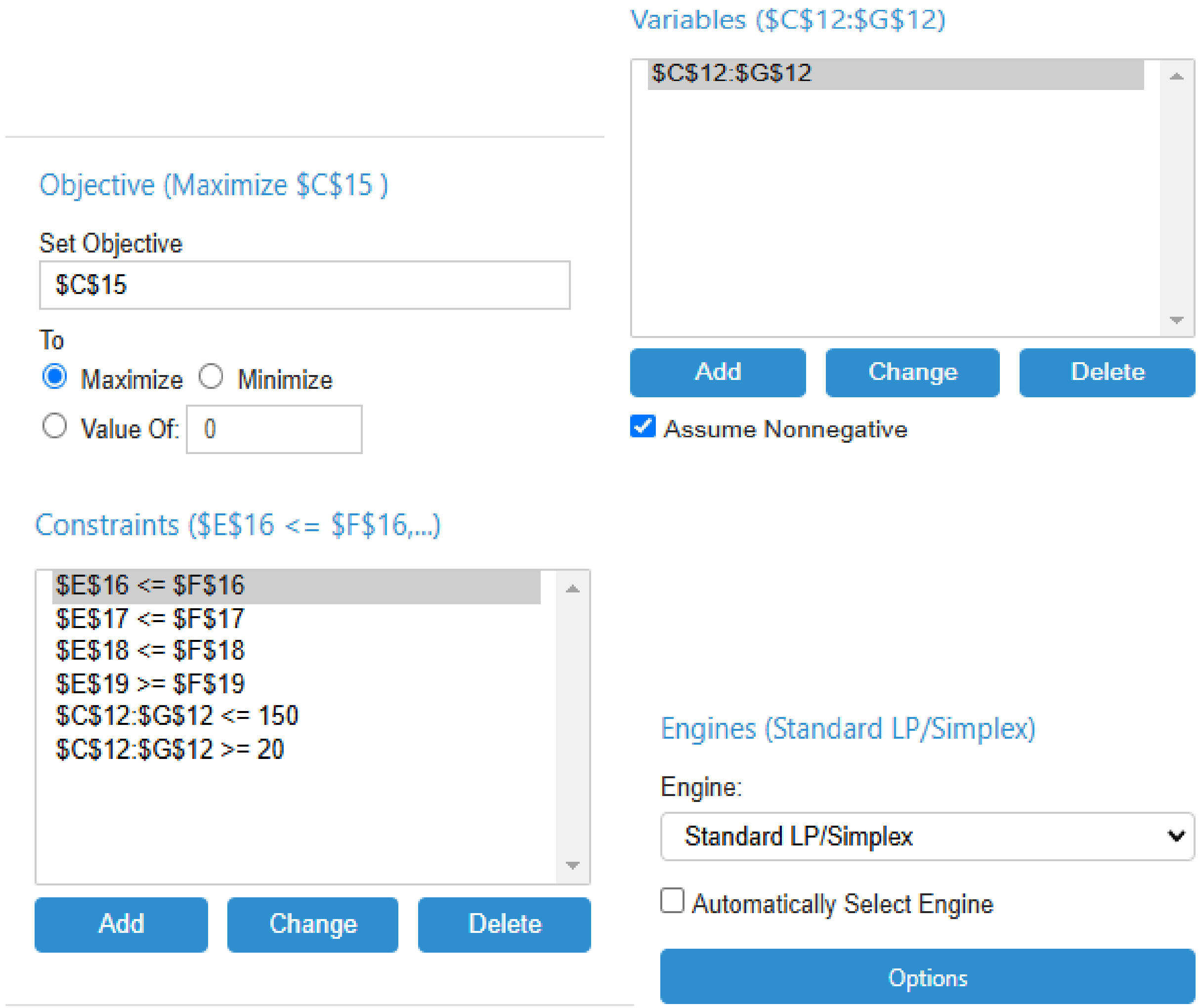
x1 <= 150, x2 <= 150, x3 <= 150, x4 <= 150, x5 <= 150

3. Mathematical Formulation

Objective Function:

Maximize Z (Profit) = 5,5*x1 + 7*x2 + 15*x3 + 5,2*x4 + 3*x5

4. Solver Configuration



5. Additional Analysis

Solution:

Based on Simplex method solution to our problem is as following, we should produce this many units:

Bread loaves: 20 Pastries: 150 Cakes: 25,4 Kremówki: 150 Eklerki: 108

x1	Number of bread loaves							
x2	Number of pastries							
x3	Number of cakes							
x4	Number of kremówki							
x5	Number of eklerki							
	Product	x1	x2	x3	x4	x5		
	Profit per unit	5,5	7	15	5,2	3		
	Labour hours	0,6	0,7	3	0,9	0,85		
	Oven time (hours)	1,1	1,9	4	1,1	0,95		
	Ingredients	4	3	10	2	2		
	How much to produce?	20	150	25,4	150	108	>20	<150
	Profit =	2645		LHS	RHS			
				420	420	labour hours	<	
				676,2	700	oven time	<	
				1300	1300	ingredients	<	
				453,4	450	sum of prod	>	

Answer report:

Objective Cell (Max)						
	Cell	Original Val	Final Value			
	Sheet1'!C	0	2645			
Decision Variable Cells						
	Cell	Original Val	Final Value			
	C12	0	20			
	D12	0	150			
	E12	0	25,4			
	F12	0	150			
	G12	0	108			
Constraints						
	Cell	Original Val	Final Value	Lower Boun	Upper Boun	Slack
	E16	0	420	-1E+30	420	0
	E17	0	676,2	-1E+30	700	23,8
	E18	0	1300	-1E+30	1300	0
	E19	0	453,4	450	1E+30	-3,4

Sensitivity report:

Objective Cell (Max)						
	Cell	Original Valı	Final Value			
	Sheet1'!C'	0	2645			
Decision Variable Cells						
		Final	Reduced	Maximum	Minimum	
	Cell	Value	Cost	Objective Co	Objective Coefficient	
	C12	20	-0,5	6	-1E+30	
	D12	150	2,5	1E+30	4,5	
	E12	25,4	0	15,0000001	14,43181807	
	F12	150	2,2	1E+30	3	
	G12	108	0	3,20833338	2,999999981	
Constraints						
		Final	Shadow	Constraint	Allowable	Allowable
	Cell	Value	Price	R.H.Side	Increase	Decrease
	E16	420	0	420	6,75	1,0625
	E17	676,2	0	700	1E+30	23,8
	E18	1300	1,5	1300	3,953488372	15,8823529
	E19	453,4	0	450	3,4	1E+30

- The problem was solved using **Excel Solver** with the Simplex LP method. The setup included 5 decision variables (production levels) and 6 constraints (resource availability).
- The solution took less than 5 seconds to compute, showing the efficiency of the Simplex method for this relatively small problem size.

Constraints and Slack Values:

- Constraint E16 (value = 420) is binding, as the final value equals the upper bound, indicating that this constraint limits the solution and cannot be relaxed further. Changes to this constraint may help increase profit even more.
- Constraint E18 (value = 1300) is also binding, suggesting it is a critical factor in the solution. Changes to this constraint may help increase profit even more.
- Constraints E17 (value = 676.2) and E19 (value = 453.4) are **non-binding**:
  - E17 has a slack of 23.8, meaning the resource could be increased by this amount without affecting the optimal solution.
  - E19 has a slack of -3.4, meaning

Key Findings:

- The **shadow price** of 1.5 for the ingredient constraint indicates that increasing the available ingredient units by 1 would increase the objective value (profit) by 1.5 units.
- The **allowable increase** of 3.95 units for the ingredient constraint shows how much flexibility is available before the constraint becomes non-binding.
- Other constraints (E16, E17, E19) have zero shadow prices, meaning they are not limiting the solution and could accommodate changes without affecting the result.

Interpretation of Reduced Costs:

- Variables D12 and F12 have positive reduced costs (2.5 and 2.2), suggesting that slight increases in their objective coefficients would improve the total profit.
- Variables C12, E12, G12 have zero or negative reduced costs, indicating they are optimally contributing to the objective.

Applicability:

- This type of linear programming model could represent a real-world problem like optimizing production in a small factory or bakery. It demonstrates how resources (e.g., labor, ingredients, oven time) can be allocated effectively to maximize profit.

Problem 2 – Binary Linear Problem

1. Problem Statement

A bakery plans to invest in various areas to improve its business operations. The available investment options include marketing, purchasing new equipment and technology (ovens), employee training, minor renovations, developing new product lines, and expanding the franchise model. The bakery needs to determine which investment areas to prioritize to maximize the return on investment (ROI) while staying within the allocated budget and adhering to strategic business constraints.

New Decision Variables:

- x1: Whether to invest in marketing (1 = yes, 0 = no).
- x2: Whether to invest in new ovens and technology (1 = yes, 0 = no).
- x3: Whether to invest in employee training (1 = yes, 0 = no).

- x4: Whether to invest in minor renovations (1 = yes, 0 = no).
- x5: Whether to invest in developing new product lines (1 = yes, 0 = no).
- x6: Whether to invest in expanding the franchise model (1 = yes, 0 = no).

Each variable is binary ($x_i \in \{0, 1\}$).

Profit Contributions:

The estimated ROI (in thousands of dollars) from each investment is as follows:

- Marketing: 120
- New ovens and technology: 200
- Employee training: 150
- Minor renovations: 80
- Developing new product lines: 180
- Expanding the franchise model: 250

2. Constraints

2.1 Budget Constraint

The total cost of investments cannot exceed the allocated budget of $450,000. The costs of each investment area (in thousands of dollars) are:

- Marketing: 50
- New ovens and technology: 150
- Employee training: 100
- Minor renovations: 50
- Developing new product lines: 120
- Expanding the franchise model: 180

Constraint:

$50x1 + 150x2 + 100x3 + 50x4 + 120x5 + 180x6 \leq 450$

2.2 **Strategic Constraints**

- At least one investment must target employee development:

x3 = 1

- At least two investments must focus on operational growth (ovens, product lines, or franchise model):

x2 + x5 + x6 ≥ 2

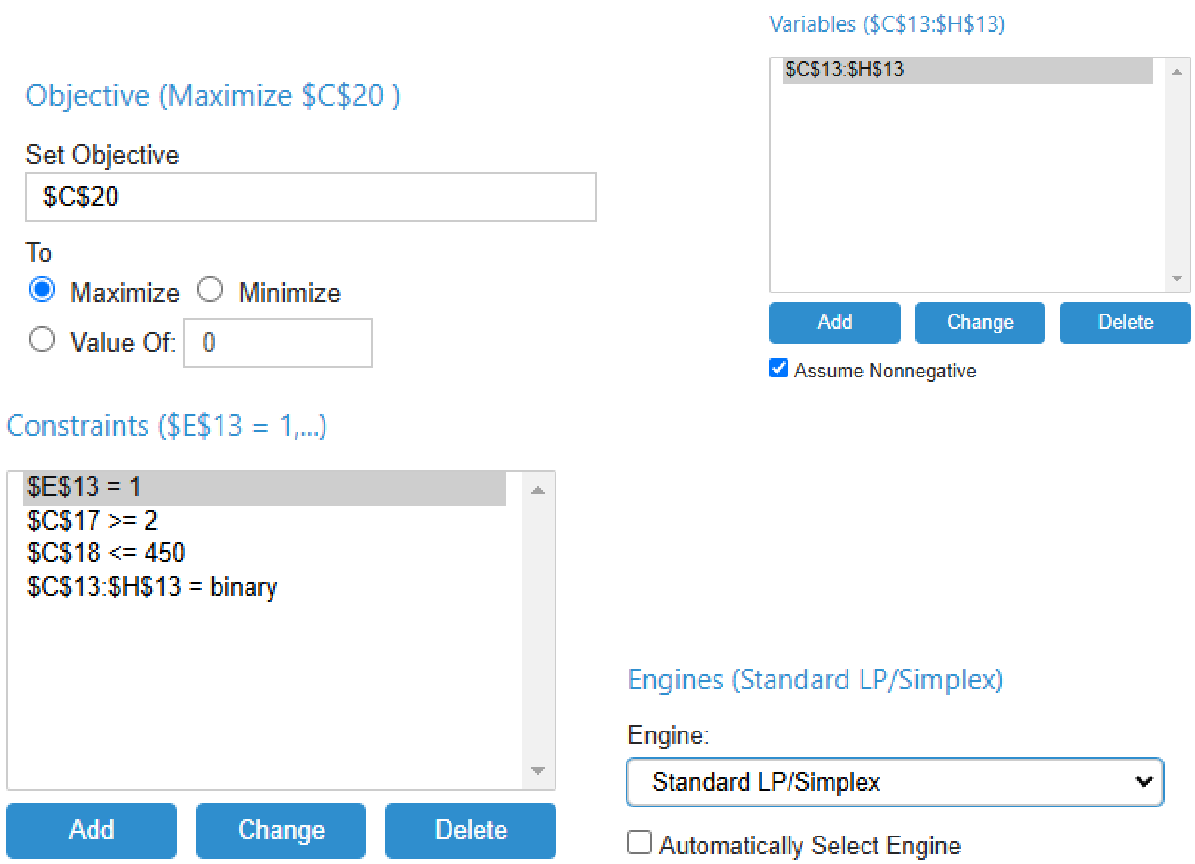
3. Mathematical Formulation

Objective Function:

Maximize the total ROI(profit):

Maximize Z = 120x1 + 200x2 + 150x3 + 80x4 + 180x5 + 250x6

4. Solver Configuration



5. Additional Analysis

Decision Variables	
x1	Whether to invest in marketing (1 = yes, 0 = no)
x2	Whether to invest in new ovens and technology (1 = yes, 0 = no)
x3	Whether to invest in employee training (1 = yes, 0 = no)
x4	Whether to invest in minor renovations (1 = yes, 0 = no)
x5	Whether to invest in developing new product lines (1 = yes, 0 = no)
x6	Whether to invest in expanding the franchise model (1 = yes, 0 = no)

Efficiency Contributions	
Investment	Roi (in thousands of dollars)
Marketing (x1)	120
New ovens and technology (x2)	200
Employee training(x3)	150
Minor renovations(x4)	80
Developing new product lines(x5)	180
Expanding the franchise model(x6)	250

Budget Constraint	
Investment	Cost (in thousands of dollars)
Marketing (x1)	50
New ovens and technology (x2)	150
Employee training(x3)	100
Minor renovations(x4)	50
Developing new product lines(x5)	120
Expanding the franchise model(x6)	180

Budget (in thousands of dollars)	450

Constraints

$x3 = 1$

$x2 + x5 + x6 \geq 2$

$50x1 + 150x2 + 100x3 + 50x4 + 120x5 + 180x6 \leq 450$

$x1, x2, x3, x4, x5, x6 \in \{0, 1\}$

Solution:

Investment	x1	x2	x3	x4	x5	x6
(yes - 1, no - 0)	1	0	1	0	1	1

Sum of x2,x5,x6	2
Money spend	450
Income	700
Profit	250

Answer raport:

Objective Cell (Max)

	Cell	Original Valu	Final Value
	Second proble	0	250

Decision Variable Cells

	Cell	Original Valu	Final Value
	C14	0	1
	D14	0	0
	E14	0	1
	F14	0	0
	G14	0	1
	H14	0	1

Constraints

	Cell	Original Valu	Final Value	Lower Bound	Upper Bound	Slack
	C18	0	2	2	1E+30	0
	C19	0	450	-1E+30	450	0

Key Findings:

- Objective Value: The optimal profit achieved is 250.
- Decision Variables: The selected investments are:
  - x1 (Marketing) = 1
  - x4 (Minor Renovations) = 1
  - x6 (Expanding Franchise Model) = 1
  - x3 (Employee Training) = 1 (per constraint).

Investments x2 (New Ovens and Technology) and x5 (Developing New Product Lines) remain at 0.

- Resource Utilization:
  - The budget constraint has a slack of 0, showing that all available funds (450) have been fully allocated.
  - The condition $x2 + x5 + x6 \geq 2$ has no slack, meeting the lower bound exactly.

Applicability:

This approach can be applied to scenarios like project portfolio optimization, where companies need to decide which projects to fund under a limited budget to maximize overall profitability.