Lab Assignments #4
FIR filter design

What to do?

- Design FIR filters using Matlab
- Plot results (label axis and titles)
- Create a short report including plots and comments

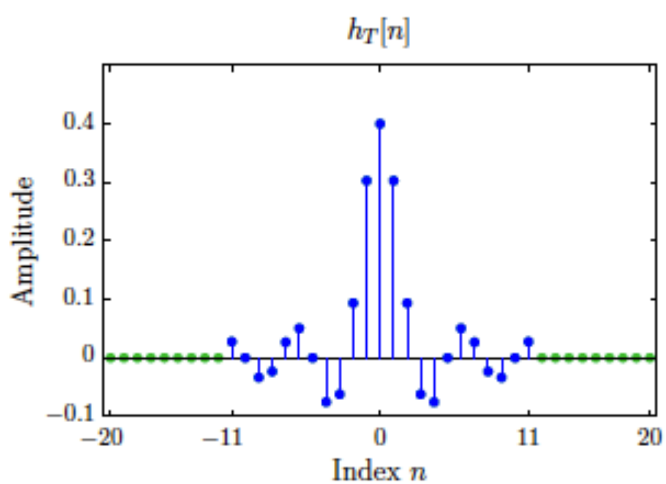
Assignment #4.1

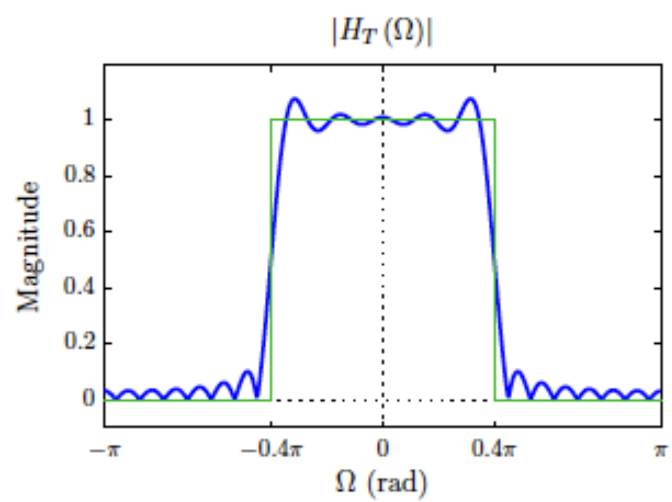
FIR filter design using Fourier series

- The truncated impulse response has $2M+1$ samples for $n = -M, \ldots, M$.
- Truncation of the ideal impulse response causes the spectrum of the filter to deviate from the ideal spectrum.

The system function for the resulting filter is

$$H_T(\Omega) = \sum_{n=-\infty}^{\infty} h_T[n]\, e^{-j\Omega n} = \sum_{n=-M}^{M} h_d[n]\, e^{-j\Omega n}$$





Assignment #4.1 cont

FIR filter design using Fourier series

Fourier series design

Using the Fourier series method, design a length-15 FIR lowpass filter to approximate an ideal lowpass filter with $\Omega_c = 0.3\pi$ rad.

Solution: The impulse response of the ideal lowpass filter is

$$h_d[n] = \frac{0.3\pi}{\pi}\,\mathrm{sinc}\left(\frac{0.3\pi n}{\pi}\right) = 0.3\,\mathrm{sinc}\,(0.3n)$$

Since $N = 2M + 1 = 15$ we have $M = 7$. The truncated impulse response is

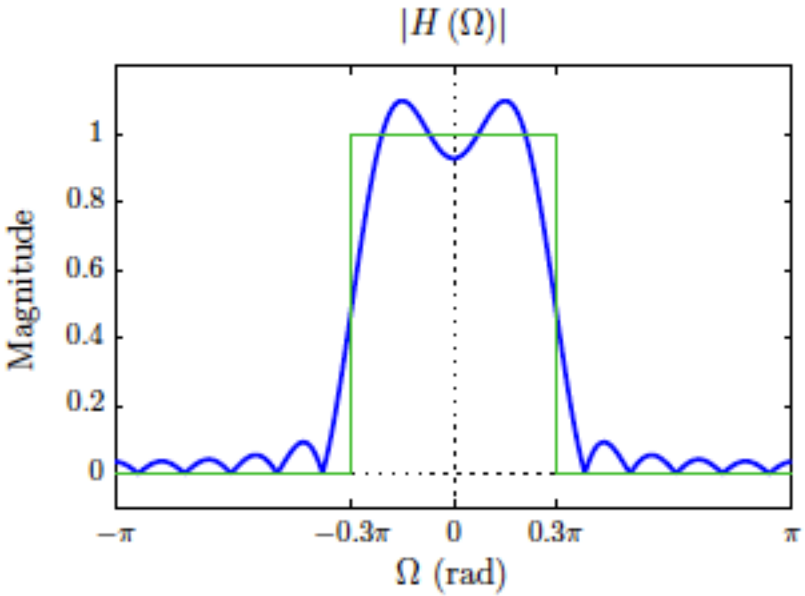
$$h_T[n] = \begin{cases} 0.3\,\mathrm{sinc}\,(0.3n)\ , & -7,\ldots,7 \\ 0\ , & \text{otherwise} \end{cases}$$

The impulse response of the FIR filter is

$$\begin{aligned} h[n] &= h_T[n-7] = 0.3\,\mathrm{sinc}\,(0.3\,(n-7))\ , \qquad n = 0,\ldots,14 \\ &= \{\underset{\substack{\uparrow \\ n=0}}{0.014},\ -0.031,\ -0.064,\ -0.047,\ 0.033,\ 0.151,\ 0.258,\ 0.3,\ 0.258,\ 0.151, \\ &\qquad 0.033,\ -0.047,\ -0.064,\ -0.031,\ 0.014\,\} \end{aligned}$$

Assignment #4.1 cont

- Calculate and plot h[n] using Matlab
- Calculate H[Ω] using fft in Matlab
- Plot H[Ω] using Matlab

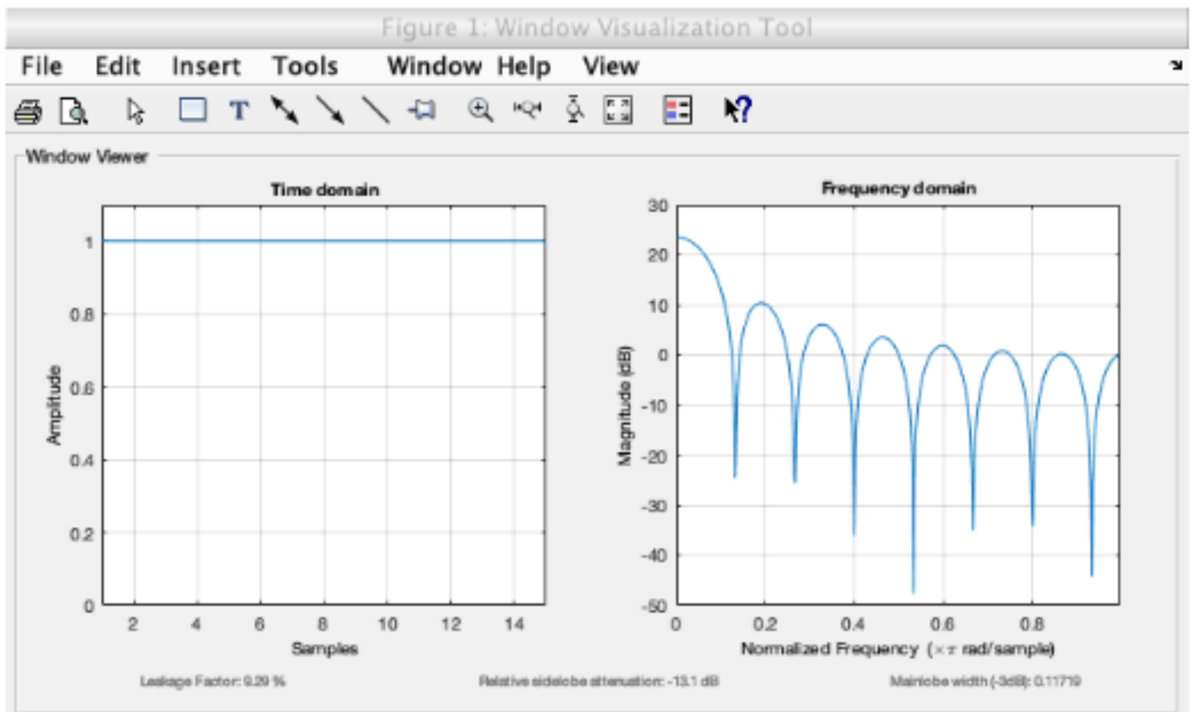


Assignment #4.1 cont

Note – You designed this filer using **rectangular window**

You can calculate this window using Matlab

```
L = 15;
wvtool(rectwin(L))
```



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Assignment #4.2

Fourier series design using window functions

Design a length-15 FIR lowpass filter to approximate an ideal lowpass filter with $\Omega_c = 0.3\pi$ rad. Use Hamming and Blackman windows for two separate designs.

Solution:

$$h_T[n] = 0.3\ \mathrm{sinc}\,(0.3n)\ w[n]\ , \qquad n = -7, \ldots, 7$$

$$h[n] = h_T[n-7] = 0.3\ \mathrm{sinc}\,(0.3\,(n-7))\ w[n-7]\ , \qquad n = 0, \ldots, 14$$

Using Hamming window:

$$h[n] = \{\underset{\substack{\uparrow\\ n=0}}{0.0011}, -0.0039, -0.0161, -0.0205, 0.0211, 0.1251, 0.2458, 0.3, 0.2458,$$
$$0.1251, 0.0211, -0.0205, -0.0161, -0.0039, 0.0011\}$$

Using Blackman window:

$$h[n] = \{\underset{\substack{\uparrow\\ n=0}}{0.0000}, -0.0006, -0.0058, -0.0111, 0.0151, 0.1081, 0.2370, 0.3, 0.2370,$$
$$0.1081, 0.0151, -0.0111, -0.0058, -0.0006, 0.0000\}$$

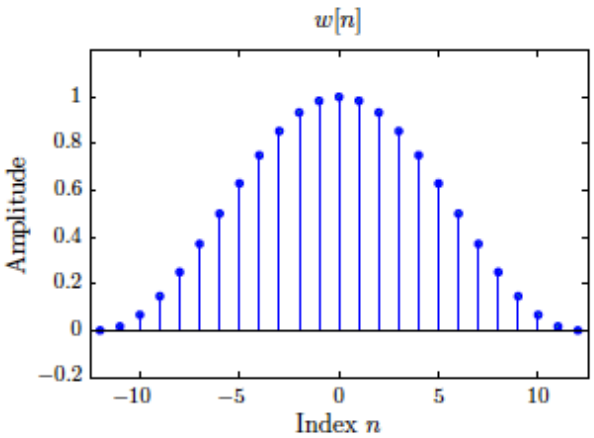
Assignment #4.2

Window functions

Hanning window:

$$w[n] = 0.5 - 0.5\cos\left(\frac{\pi(n+M)}{M}\right),$$

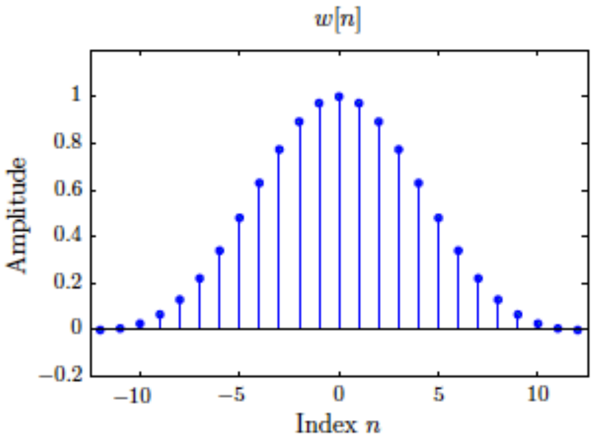
$$-M \leq n \leq M$$



Blackman window:

$$w[n] = 0.42 - 0.5\cos\left(\frac{\pi(n+M)}{M}\right) + 0.08\cos\left(\frac{2\pi(n+M)}{M}\right),$$

$$-M \leq n \leq M$$



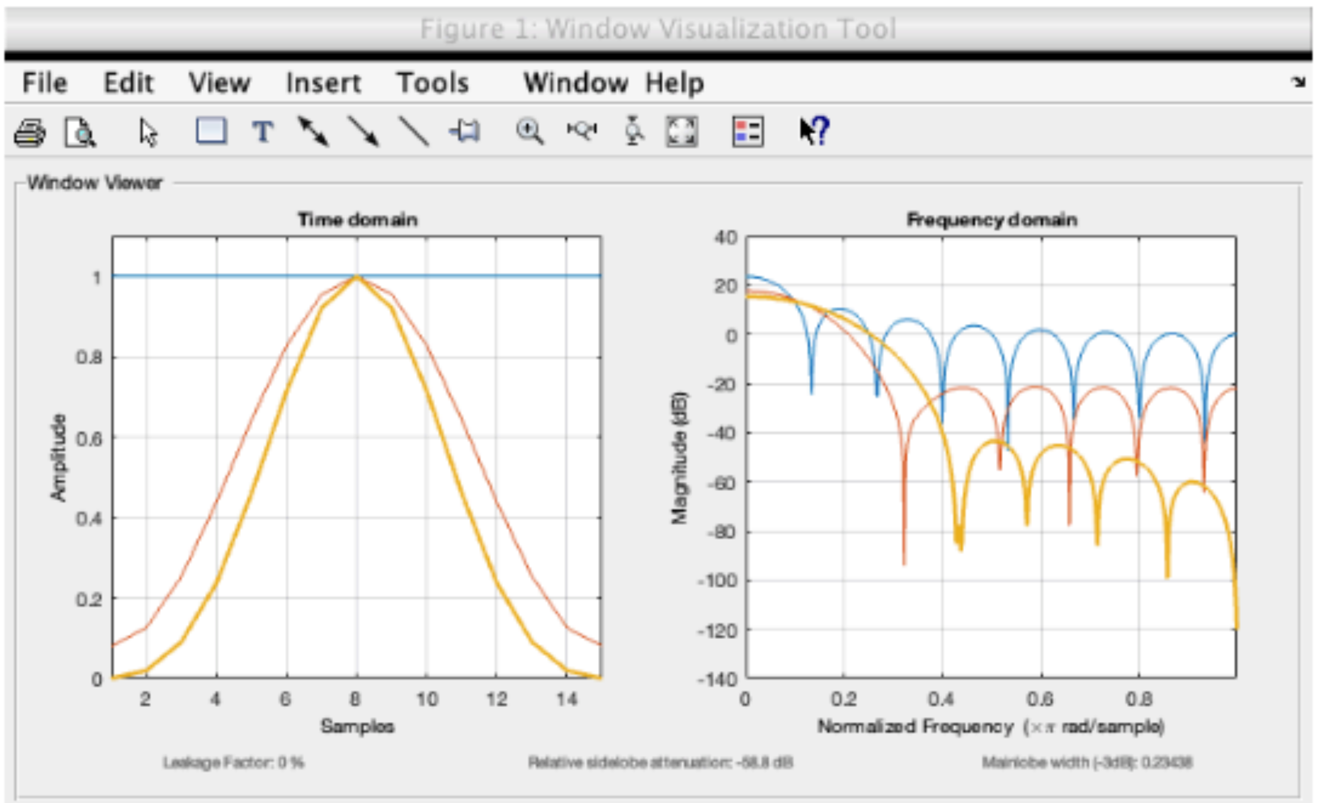
- Implement the window eqauations in Matlab and calcuate the w[n] coefficients of both windows, length 15.
- Print the w[n] coefficients
- Did you get the same the window coefficients as those given above?

Assignment #4.2

Window functions

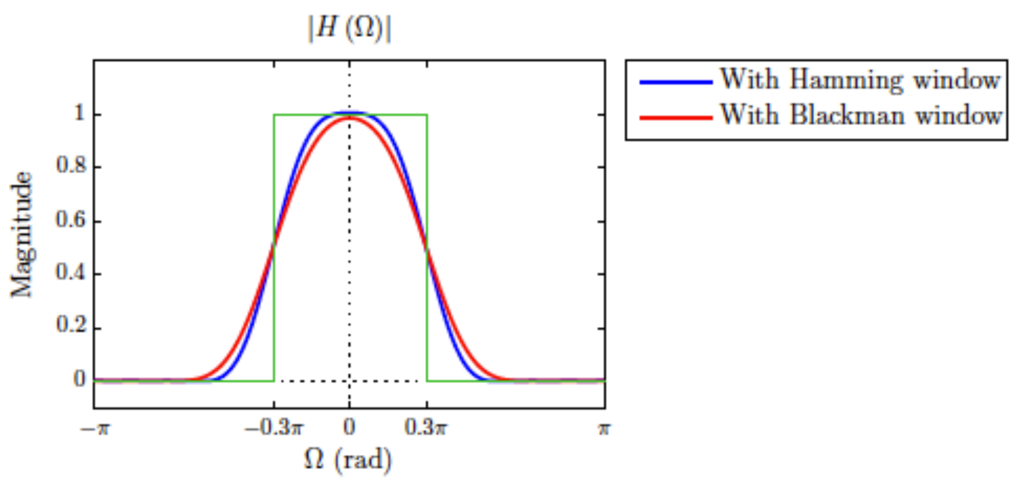
Calcuate and plot both windows with length 15 using Matlab
Hint: you may use matlab window functions

```
L=15;
wvtool(rectwin(L),hamming(L),blackman(L))
```



Assignment #4.2 cont

- Calculate and plot h[n] truncted with:
  1. Rectangular window
  2. Hanning window
  3. Blackman window
- Calculate the respective H[Ω] using fft
- Plot obtained results
  Compare amplitude responses of the designed filers, comment differences



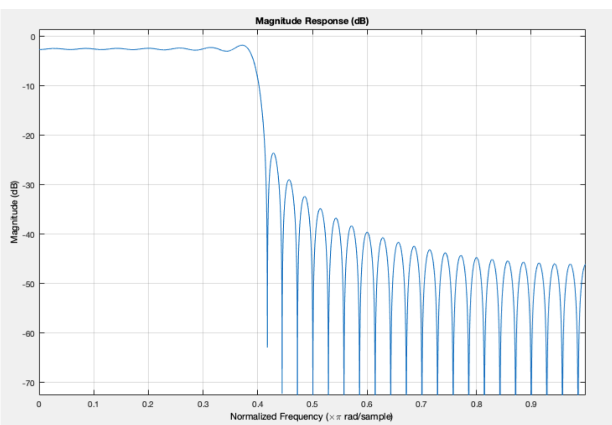
Assignment #4.3

You can design the FIR filters using Matlab's FIR Filter Design functions

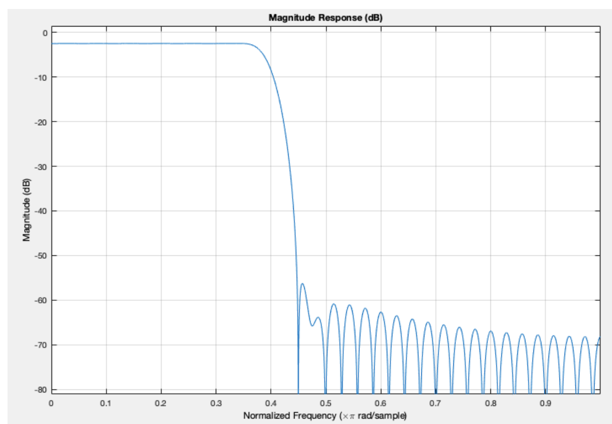
Using the Fourier series method, design length-71 FIR LP filters to approximate ideal lowpass filters with $\Omega_C = 0.3\pi$ rad using different windows.

```
%  FIR filter design
%
%%
b = 0.3*sinc(0.3*(-35:35));
%b = b.*hamming(71)';
%b = b.*blackman(71)';
fvtool(b,1)
```

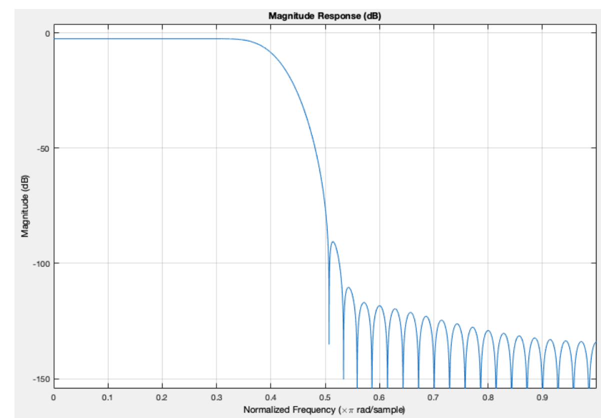
Rectangular window



Hamming window



Blackman window



Compare amplitude responses of the designed filters, what can you gain and loose using soft windows? What you gained increasing filter length from 15 to 71?

Assignment #4.3 cont

Read Matlab summary

FIR vs. IIR Filters

Digital filters with finite-duration impulse response (all-zero, or FIR filters) have both advantages and disadvantages compared to infinite-duration impulse response (IIR) filters.

FIR filters have the following primary advantages:
•They can have exactly linear phase.
•They are always stable.
•The design methods are generally linear.
•They can be realized efficiently in hardware.
•The filter startup transients have finite duration.

The primary disadvantage of FIR filters is that they often require a much higher filter order than IIR filters to achieve a given level of performance. Correspondingly, the delay of these filters is often much greater than for an equal performance IIR filter.