Bayes decisions and Model evaluation

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Model evaluation for classification

Assess how good our model is on a held–out evaluation (test) set

We start considering binary classification problems

A possible solution is to compute the accuracy of the model, or, equivalently, the error rate, defined as

$$accuracy = \frac{\#\ of\ correctly\ classified\ samples}{\#\ of\ samples}$$

$$error\ rate = \frac{\#\ of\ incorrectly\ classified\ samples}{\#\ of\ samples} = 1 - accuracy$$

Model evaluation for classification

Accuracy can be misleading if the classes are not balanced

Let's consider, for example, rain prediction in arid climates. Over one year, the model makes the following predictions:

	Rain	Clear
Prediction: Rain	15 days	30 days
Prediction: Clear	20 days	300 days

$$accuracy = \frac{300 + 15}{365} \approx 86\%$$

Model evaluation for classification

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$$accuracy = \frac{300 + 15}{365} \approx 86\%$$

86% looks like a good accuracy ... But a model that always predicts Clear would achieve an accuracy of $\approx 90\%$!

Model evaluation for classification

Let's analyze the outcomes table

	Rain	Clear
Prediction: Rain	15 days	30 days
Prediction: Clear	20 days	300 days

This table is also called confusion matrix. In general:

	Class $\mathcal{H}_F$	Class $\mathcal{H}_T$
Prediction $\mathcal{H}_F$	True Negative	False Negative
Prediction $\mathcal{H}_T$	False Positive	True Positive

Model evaluation for classification

We can compute different accuracy measures

- False negative rate FNR (false rejection / miss rate): $\frac{FN}{FN+TP}$
- False positive rate FPR (false acceptance): $\frac{FP}{FP+TN}$
- True positive rate TPR (recall, sensitivity): $\frac{TP}{FN+TP} = 1 - \text{FNR}$
- True negative rate TNR (specificity): $\frac{TN}{FP+TN} = 1 - \text{FPR}$
- ...

We can also compute a *weighted* accuracy

$$acc = \alpha FPR + (1 - \alpha) FNR$$

The weight α measures how important are different kind of errors (we shall see this in more detail later)

Model evaluation for classification

Different kind of errors may have different impact on applications

Systems providing only hard decisions do not allow for trade-offs between different error types

Rather than labels, often decision functions output scores

- Generative models: log–likelihood ratios

$$s = \log \frac{f(x|\mathcal{H}_T)}{f(x|\mathcal{H}_F)}$$

- Discriminative models: posterior log–likelihood ratios

$$s = \log \frac{P(\mathcal{H}_T|x)}{P(\mathcal{H}_F|x)}$$

- Non–probabilistic models: score (e.g. SVM)

$$s = \boldsymbol{w}^T \boldsymbol{x}$$

Model evaluation for classification

A higher score means we should favor class $\mathcal{H}_T$

Class assignment is performed by comparing scores to a threshold t

$$s \geq t \longrightarrow \mathcal{H}_T$$

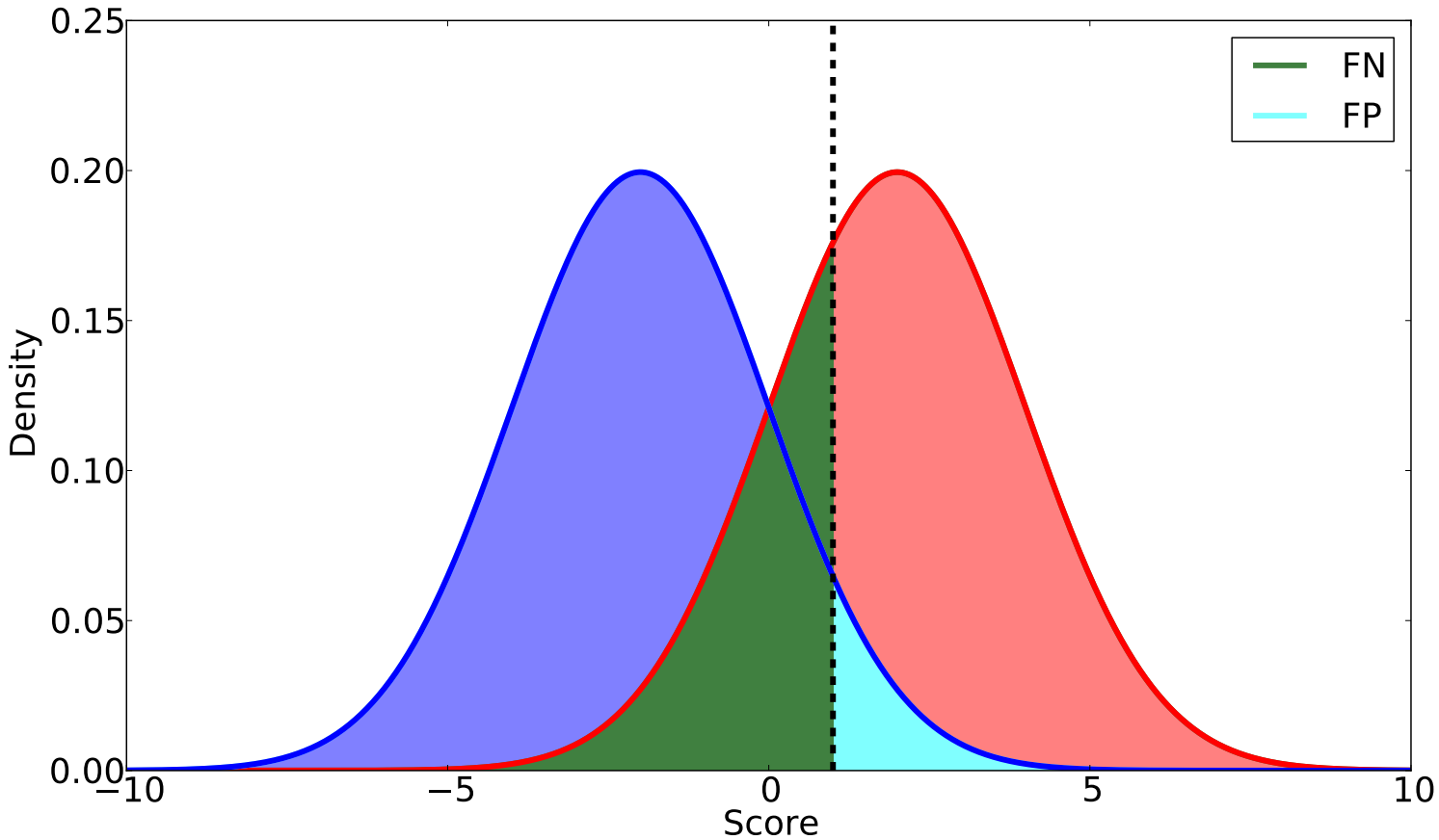
$$s < t \longrightarrow \mathcal{H}_F$$

Different thresholds correspond to different error rates

Thresholds are related to class priors and error costs

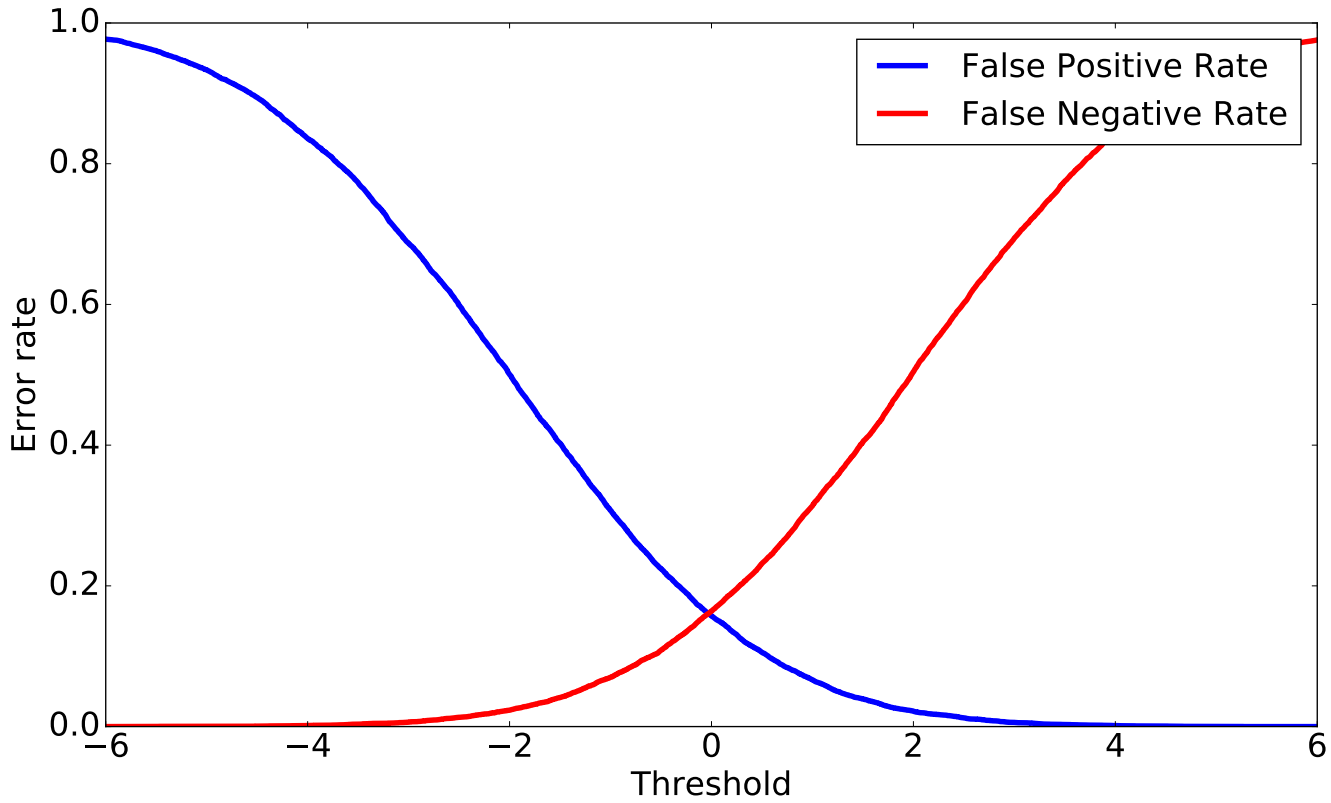
Model evaluation for classification

Score thresholding:



Model evaluation for classification

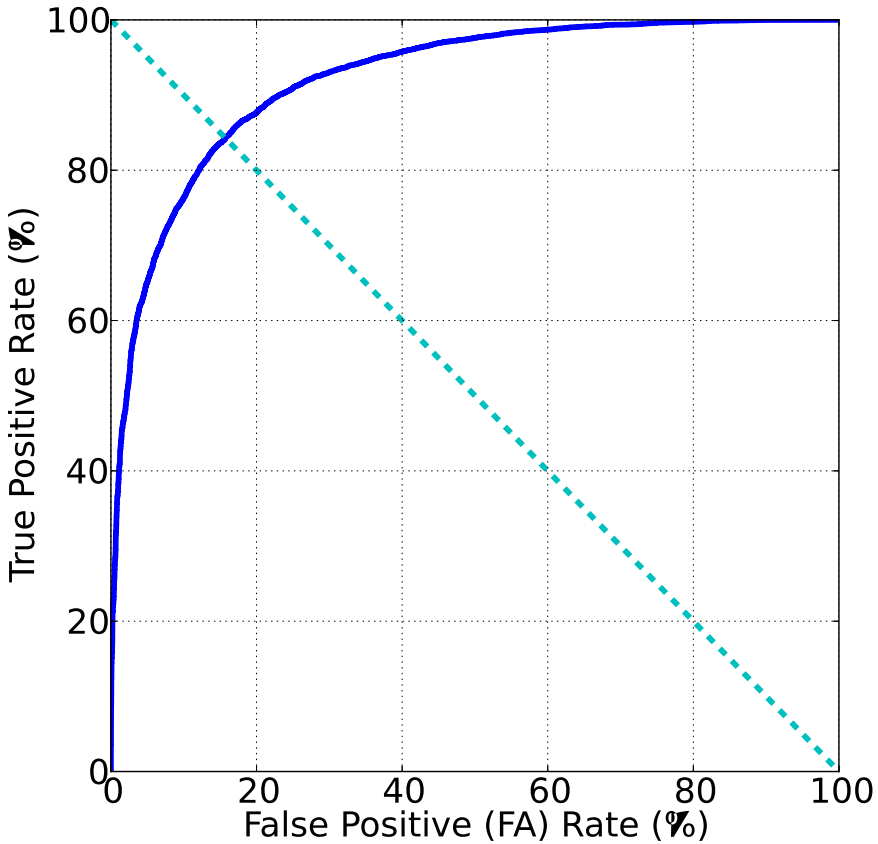
We can visualize the performance of the classifier for different thresholds by plotting the error rates as a function of the threshold



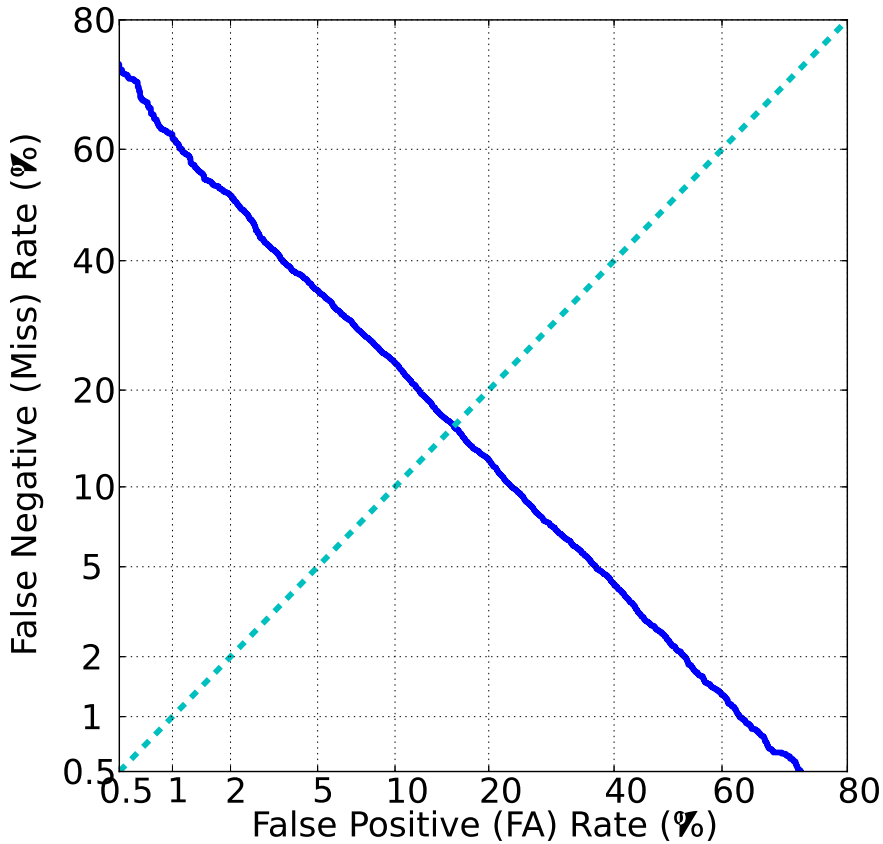
Equal Error Rate (EER): The error rate for which $FPR = FNR$

Model evaluation for classification

- Receiver Operating Characteristic (ROC) curve



- Detection Error Trade-off (DET) curve



Bayes decisions

The goal of the classifier is to allow us to choose an action a to perform among a set of actions $\mathcal{A}$

Example: accepting vs rejecting a sample

Example: assign label a to the sample

We can associate to each action a cost $C(a|k)$ that we have to pay when we choose action a and the sample belongs to class k

In the following we consider the set of actions corresponding to labeling a sample with label a

Bayes decisions

$C(a|k)$ represents the cost of labeling the sample as belonging to class a when it actually belongs to class k

We do not know k, however we have a classifier $\mathcal{R}$ that allows us computing class posterior probabilities $P(C = k|x)$ for sample x

We can thus compute the expected cost of action a when the posterior probability for each class is $P(C = k|x, \mathcal{R})$

$$\mathcal{C}_{x,\mathcal{R}}(a) = \mathbb{E}[C(a|k)|x, \mathcal{R}] = \sum_{k=1}^{K} C(a|k)P(C = k|x, \mathcal{R})$$

It measures the cost that we expect to pay given our knowledge of the class distribution $P(C = k|x, \mathcal{R})$

Bayes decisions

The Bayes decision consists in choosing the action $a^*(x, \mathcal{R})$ that minimizes the expected cost: $a^*(x, \mathcal{R}) = \arg\min_a \mathcal{C}_{x,\mathcal{R}}(a)$

It represents the action that will result in the lower expected cost, according to the recognizer beliefs

Different recognizers may have different posterior beliefs, and thus provide different decisions

Bayes decisions

For example, let's consider we have a 3-class problem, with cost matrix and priors given by

$$\boldsymbol{C} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{\pi} = \begin{bmatrix} 0.3 \\ 0.4 \\ 0.3 \end{bmatrix}$$

For a test sample $\boldsymbol{x}_t$, we have computed the posterior class probabilities (using the prior $\boldsymbol{\pi}$)

$$\boldsymbol{q}_t = \begin{bmatrix} P(C=1|\boldsymbol{x}_t, \mathcal{R}) \\ P(C=2|\boldsymbol{x}_t, \mathcal{R}) \\ P(C=3|\boldsymbol{x}_t, \mathcal{R}) \end{bmatrix} = \begin{bmatrix} 0.40 \\ 0.25 \\ 0.35 \end{bmatrix}$$

Bayes decisions

The expected cost of actions “Predict k” are

$$C_{\boldsymbol{x}_t,\mathcal{R}}(1) = 0 \times 0.40 + 1 \times 0.25 + 2 \times 0.35 = 0.95$$
$$C_{\boldsymbol{x}_t,\mathcal{R}}(2) = 1 \times 0.40 + 0 \times 0.25 + 1 \times 0.35 = 0.75$$
$$C_{\boldsymbol{x}_t,\mathcal{R}}(3) = 2 \times 0.40 + 1 \times 0.25 + 0 \times 0.35 = 1.05$$

or, in matrix form:

$$\begin{bmatrix} C_{\boldsymbol{x}_t,\mathcal{R}}(1) \\ C_{\boldsymbol{x}_t,\mathcal{R}}(2) \\ C_{\boldsymbol{x}_t,\mathcal{R}}(3) \end{bmatrix} = \boldsymbol{C}\boldsymbol{q}_t$$

The optimal decision would therefore to assign label 2, even though it has the lowest posterior probability, since the expected cost due to mis-calssifications would be lower.

Bayes decisions

Let's now consider again a binary problem

We have four costs:

	Class $\mathcal{H}_F$	Class $\mathcal{H}_T$
Prediction $\mathcal{H}_F$	$C(\mathcal{H}_F\|\mathcal{H}_F)$	$C(\mathcal{H}_F\|\mathcal{H}_T)$
Prediction $\mathcal{H}_T$	$C(\mathcal{H}_T\|\mathcal{H}_F)$	$C(\mathcal{H}_T\|\mathcal{H}_T)$

Without loss of generality we assume

$$C(\mathcal{H}_T|\mathcal{H}_T) = 0\ , \quad (\mathcal{H}_F|\mathcal{H}_F) = 0$$

i.e. correct decisions have no cost.

We also assume $C(\mathcal{H}_F|\mathcal{H}_T) \geq 0$ and $C(\mathcal{H}_T|\mathcal{H}_F) \geq 0$

Bayes decisions

The costs reflect the costs of the two different kind of errors:

	Class $\mathcal{H}_F$	Class $\mathcal{H}_T$
Prediction $\mathcal{H}_F$	0	$C(\mathcal{H}_F \vert \mathcal{H}_T) = C_{fn}$
Prediction $\mathcal{H}_T$	$C(\mathcal{H}_T \vert \mathcal{H}_F) = C_{fp}$	0

C_{fn} is the cost of false negative errors, C_{fp} is the cost of false positive errors

Bayes decisions

The expected Bayes cost for action $\mathcal{H}_T$ (i.e. for predicting $\mathcal{H}_T$) is

$$\mathcal{C}_{x,\mathcal{R}}(\mathcal{H}_T) = C_{fp}P(\mathcal{H}_F|x,\mathcal{R}) + 0 \cdot P(\mathcal{H}_T|x,\mathcal{R}) = C_{fp}P(\mathcal{H}_F|x,\mathcal{R})$$

whereas the cost for action $\mathcal{H}_F$ (i.e. for predicting $\mathcal{H}_F$) is

$$\mathcal{C}_{x,\mathcal{R}}(\mathcal{H}_F) = C_{fn}P(\mathcal{H}_T|x,\mathcal{R}) + 0 \cdot P(\mathcal{H}_F|x,\mathcal{R}) = C_{fn}P(\mathcal{H}_T|x,\mathcal{R})$$

The optimal decision is the labeling that has lowest cost

Bayes decisions

For binary problems, the optimal decision can be expressed as

$$a^*(x,\mathcal{R}) = \begin{cases} \mathcal{H}_T & \text{if } C_{fp}P(\mathcal{H}_F|x,\mathcal{R}) < C_{fn}P(\mathcal{H}_T|x,\mathcal{R}) \\ \mathcal{H}_F & \text{if } C_{fp}P(\mathcal{H}_F|x,\mathcal{R}) > C_{fn}P(\mathcal{H}_T|x,\mathcal{R}) \end{cases}$$

and we can choose any action when the two costs are equal.

Alternatively, we can express the optimal decision (up to tie-breaking) as

$$a^*(x,\mathcal{R}) = \begin{cases} \mathcal{H}_T & \text{if } r(x) > 0 \\ \mathcal{H}_F & \text{if } r(x) < 0 \end{cases}$$

where

$$r(x) = \log \frac{C_{fn}P(\mathcal{H}_T|x,\mathcal{R})}{C_{fp}P(\mathcal{H}_F|x,\mathcal{R})}$$

Bayes decisions

If $\mathcal{R}$ is a generative model for x, then we can express r in terms of costs, prior probabilities and conditional likelihoods as

$$r(x) = \log \frac{\pi_T C_{fn}}{(1-\pi_T)C_{fp}} \cdot \frac{f_{X|\mathcal{H},\mathcal{R}}(x|\mathcal{H}_T)}{f_{X|\mathcal{H},\mathcal{R}}(x|\mathcal{H}_F)}$$

where $\pi_T = P(\mathcal{H} = \mathcal{H}_T)$ is the prior probability for class $\mathcal{H}_T$.

The decision rule thus becomes

$$r(x) \lessgtr 0 \iff \log \frac{f_{X|\mathcal{H},\mathcal{R}}(x|\mathcal{H}_T)}{f_{X|\mathcal{H},\mathcal{R}}(x|\mathcal{H}_F)} \lessgtr -\log \frac{\pi_T C_{fn}}{(1-\pi_T)C_{fp}}$$

Bayes decisions

The triplet (π_T, C_{fn}, C_{fp}) represents the working point of an application for a binary classification task.

We can show that the triplet is actually redundant, in the sense that we can build equivalent applications $(\pi'_T, C'_{fn}, C'_{fp})$ which have the same decision rule as the original application, but different costs and priors.

For example, the application $(\tilde{\pi}, 1, 1)$ with

$$\tilde{\pi} = \frac{\pi_T C_{fn}}{\pi_T C_{fn} + (1 - \pi_T) C_{fp}}$$

is equivalent to the application (C_{fn}, C_{fp}, π_T). Indeed, we have

$$\frac{\tilde{\pi}}{1 - \tilde{\pi}} = \frac{\frac{\pi_T C_{fn}}{\pi_T C_{fn} + (1 - \pi_T) C_{fp}}}{1 - \frac{\pi_T C_{fn}}{\pi_T C_{fn} + (1 - \pi_T) C_{fp}}} = \frac{\pi_T C_{fn}}{(1 - \pi_T) C_{fp}}$$

Bayes decisions

We can interpret $\tilde{\pi}$ as an effective prior: if the class prior for $\mathcal{H}_T$ was $\tilde{\pi}$ and we assumed uniform costs, we would obtain the same decisions as for our original application

Similarly, we can devise an equivalent application where the effective prior is uniform $\tilde{\pi} = \frac{1}{2}$, and the application prior π_T absorbed in "effective" classification costs (we won't prove it here)

Bayes decisions

We have, up to now, considered how to perform decisions for a sample x

We now return to the problem of evaluating the goodness of our decisions

Remember that our decisions are taken using the posterior class distribution defined by the classifier $\mathcal{R}$

We can compute the cost of the Bayes decisions taken with the recognizer $\mathcal{R}$:

$$\mathcal{C}^*(x, \mathcal{R}|c) = C(a^*(x, \mathcal{R})|c)$$

If actions correspond to class labeling, it's the cost of predicting the minimum-expected-cost label a^* when the actual label is c.

Empirical Bayes Risk

We would like to know how well the classifier performs on some data.

We can formalize this as computing an expectation of the Bayes cost of optimal Bayes decisions made with $\mathcal{R}$ for the evaluation population.

We define the posterior Bayes risk $\mathcal{B}$ as the expected value of the Bayes cost of Bayes decisions made by $\mathcal{R}$ over evaluation data sampled from $X, C|\mathcal{E}$:

$$\mathcal{B} = \mathbb{E}_{X,C|\mathcal{E}}\left[\mathcal{C}^*(x, \mathcal{R}|c)\right] = \sum_{c=1}^{K} \pi_c \int \mathcal{C}^*(x, \mathcal{R}|c) f_{X|C,\mathcal{E}}(x|c) dx$$

Empirical Bayes Risk

The distribution $X|C, \mathcal{E}$ is the conditional distribution of the evaluation population

$\mathcal{E}$ represents the evaluator

Note that the distribution reflects the knowledge of the evaluator $\mathcal{E}$, not the knowledge of the recognizer $\mathcal{R}$

The evaluator $\mathcal{E}$ is measuring how good are the decisions made by the recognizer $\mathcal{R}$ for his own task (data sampled from $X|C, \mathcal{E}$)

Empirical Bayes Risk

In general, we won't have access to $f_{X|C,\mathcal{E}}(x|c)$.

However, if we have at our disposal a set of labeled evaluation samples $(x_1, c_1) \ldots (x_N, c_N)$, then we can approximate the expectations by averaging the cost over the samples

Indeed, if samples x_i are generated by $X|C, \mathcal{E}$, as the number of samples per class becomes large, it's possible to show that

$$\int \mathcal{C}^*(x, \mathcal{R}|c) f_{X|C,\mathcal{E}}(x|c) dx \approx \frac{1}{N_c} \sum_{i|c_i=c} \mathcal{C}^*(x_i, \mathcal{R}|c)$$

i.e. the integral can be approximated by the average cost computed over samples of each class

Empirical Bayes Risk

We can finally define the empirical Bayes risk as

$$\mathcal{B}_{emp} = \sum_{c=1}^{K} \frac{\pi_c}{N_c} \sum_{i|c_i=c} \mathcal{C}^*(x_i, \mathcal{R}|c)$$

The risk measures the costs of our decisions over the evaluation samples.

We can use $\mathcal{B}_{emp}$ to compare recognizers.

A recognizer that has lower cost will provide more accurate answers

Empirical Bayes Risk

The empirical Bayes risk can be computed from the confusion matrix and the matrix of costs

For example, let's consider again the 3-class problem, with cost matrix and priors given by

$$\boldsymbol{C} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{\pi} = \begin{bmatrix} 0.3 \\ 0.4 \\ 0.3 \end{bmatrix}$$

For all test samples, we computed the Bayes decisions. We can then compute the confusion matrix, Let's assume that we obtain

$$\boldsymbol{M} = \begin{bmatrix} 205 & 111 & 56 \\ 145 & 199 & 121 \\ 50 & 92 & 225 \end{bmatrix}$$

Empirical Bayes Risk

We can compute, for each class, the term

$$\frac{\pi_c}{N_c} \sum_{i|c_i=c} \mathcal{C}^*(x_i, \mathcal{R}|c)$$

For samples that belong to class 1, we have

$$\pi_1 = 0.3 \ , \quad N_1 = 205 + 145 + 50 = 400 \ .$$

For samples that are correctly classified (205) the cost is 0; for samples that are classified as class 2 (145) the cost is 1; for samples that are classified as class 3 (50) the cost is 2. Thus

$$\frac{\pi_1}{N_1} \sum_{i|c_i=1} \mathcal{C}^*(x_i, \mathcal{R}|c) = \frac{0.3}{400} (0 \times 205 + 1 \times 145 + 2 \times 50) = 0.18375$$

Empirical Bayes Risk

Similarly,

$$\frac{\pi_2}{N_2}\sum_{i|c_i=2}\mathcal{C}^*(x_i,\mathcal{R}|c) = \frac{0.4}{402}\left(1\times 111 + 0\times 199 + 1\times 92\right) \approx 0.20199$$

$$\frac{\pi_3}{N_3}\sum_{i|c_i=3}\mathcal{C}^*(x_i,\mathcal{R}|c) = \frac{0.3}{402}\left(2\times 56 + 1\times 121 + 0\times 225\right) \approx 0.17388$$

The empirical Bayes risk is

$$\mathcal{B}_{emp} \approx 0.18375 + 0.20199 + 0.17388 = 0.55962$$

Empirical Bayes Risk

Let's now consider again the binary problem

	Class $\mathcal{H}_F$	Class $\mathcal{H}_T$
Prediction $\mathcal{H}_F$	0	$C(\mathcal{H}_F\|\mathcal{H}_T) = C_{fn}$
Prediction $\mathcal{H}_T$	$C(\mathcal{H}_T\|\mathcal{H}_F) = C_{fp}$	0

We have seen that we can compute predicted labels by comparing the log-likelihood ratio to a threshold that depends on (π_T, C_{fn}, C_{fp})

Empirical Bayes Risk

Let c_i^* be the predicted label for sample x_i. The empirical Bayes risk is

$$\begin{aligned}
\mathcal{B}_{emp} &= \frac{\pi_T}{N_T} \sum_{i|c_i=\mathcal{H}_T} \mathcal{C}^*(x_i, \mathcal{R}|\mathcal{H}_T) + \frac{1-\pi_T}{N_F} \sum_{i|c_i=\mathcal{H}_F} \mathcal{C}^*(x_i, \mathcal{R}|\mathcal{H}_F) \\
&= \pi_T \frac{\sum_{i|c_i=\mathcal{H}_T} C_{fn} \mathbb{I}[h_i^* = \mathcal{H}_F]}{N_T} + (1-\pi_T) \frac{\sum_{i|c_i=\mathcal{H}_F} C_{fp} \mathbb{I}[h_i^* = \mathcal{H}_T]}{N_F} \\
&= \pi_T \frac{\sum_{i|c_i=\mathcal{H}_T, c_i^*=\mathcal{H}_F} C_{fn}}{N_T} + (1-\pi_T) \frac{\sum_{i|c_i=\mathcal{H}_F, c_i^*=\mathcal{H}_T} C_{fp}}{N_F} \\
&= \pi_T C_{fn} \cdot FNR + (1-\pi_T) C_{fp} \cdot FPR \\
&= \pi_T C_{fn} P_{fn} + (1-\pi_T) C_{fp} P_{fp}
\end{aligned}$$

where $P_{fn} = FNR$ is the false negative rate (false rejection rate) and $P_{fp} = FPR$ is the false positive rate (false acceptance rate)

$\mathcal{B}_{emp}$ is also called (un-normalized) Detection Cost Function (DCF)

Model evaluation for classification

Detection Cost Functions:

- Define the costs of different kind of errors (C_{fn}, C_{fp})
- Define the class prior probability (π_T, $\pi_F = 1 - \pi_T$)
- Evaluate by computing empirical Bayes risk

$$DCF_u(C_{fn}, C_{fp}, \pi_T) = \pi_T C_{fn} P_{fn} + (1 - \pi_T) C_{fp} P_{fp}$$

- P_{fn} and P_{fp} are the false negative and false positive rates, and depend on the selected threshold t

Model evaluation for classification

C_{fn}, C_{fp} and π_T depend only on the application

A dummy system that always accepts a test segment ($c_t^* = \mathcal{H}_T$):

$$P_{fp} = 1, P_{fn} = 0 \implies DCF_u = (1 - \pi_T) C_{fp}$$

A dummy system that always rejects a test segment ($c_t^* = \mathcal{H}_F$):

$$P_{fp} = 0, P_{fn} = 1 \implies DCF_u = \pi_T C_{fn}$$

Normalized DCF: we compare the system DCF w.r.t. the best dummy system

$$DCF(\pi_T, C_{fn}, C_{fp}) = \frac{DCF_u(\pi_T, C_{fn}, C_{fp})}{\min(\pi_T C_{fn}, (1 - \pi_T) C_{fp})}$$

Model evaluation for classification

Normalized DCF is invariant to scaling

We can thus re-scale the un-normalized DCF by $\frac{1}{\pi_T C_{fn} + (1-\pi_T) C_{fp}}$

Let $\tilde{\pi} = \frac{\pi_T C_{fn}}{\pi_T C_{fn} + (1-\pi_T) C_{fp}}$, so that $1 - \tilde{\pi} = \frac{(1-\pi_T) C_{fp}}{\pi_T C_{fn} + (1-\pi_T) C_{fp}}$

The un-normalized DCF becomes

$$DCF_u(\tilde{\pi}) = \tilde{\pi} P_{fn} + (1 - \tilde{\pi}) P_{fp}$$

whereas the corresponding normalized DCF has the same value

In terms of normalized DCF, the applications (π_T, C_{fp}, C_{fn}) and $(\tilde{\pi}, 1, 1)$ are again equivalent

Model evaluation for classification

We can observe that the error rate we defined at the beginning as

$$e = \frac{\#\ of\ incorrectly\ classified\ samples}{\#\ of\ samples} = 1 - accuracy$$

corresponds to

$$e = \frac{N_T P_{fn} + N_F P_{fp}}{N} = \frac{N_T}{N} P_{fn} + \frac{N_F}{N} P_{fp}$$

i.e., up to a scaling factor, to the DCF of an application $(\frac{N_T}{N}, 1, 1)$

The weighted error rate

$$e = \frac{1}{2}(P_{fn} + P_{fp})$$

corresponds to the application $(\frac{1}{2}, 1, 1)$

Model evaluation for classification

For systems producing well-calibrated log–likelihood ratios

$$s = \log \frac{f_{X|C}(x|\mathcal{H}_T)}{f_{X|C}(x|\mathcal{H}_F)}$$

the optimal threshold is given by

$$t = -\log \frac{\tilde{\pi}}{1-\tilde{\pi}}$$

Indeed, since $\tilde{\pi}$ is the effective prior probability of $\mathcal{H}_T$, the posterior log–likelihood ratio is

$$\log \frac{P(\mathcal{H}_T|x)}{P(\mathcal{H}_F|x)} = \log \frac{f_{X|C}(x|\mathcal{H}_T)}{f_{X|C}(x|\mathcal{H}_F)} + \log \frac{\tilde{\pi}}{1-\tilde{\pi}}$$

Model evaluation for classification

LLRs allow disentangling the classifier from the application

In general, systems often do not produce well–calibrated LLRs

- Non-probabilistic scores (e.g. SVM)
- Mis-match between train and test populations
- Non-accurate model assumptions

In these cases, we say that scores are mis-calibrated

The theoretical threshold $-\log \frac{\tilde{\pi}}{1-\tilde{\pi}}$ is not optimal anymore

Model evaluation for classification

For a given application, we can measure the additional cost due to the use of mis-calibrated scores

We can define the minimum cost DCF_{min} corresponding to the use of the optimal threshold for the evaluation set

We consider varying the threshold t to obtain all possible combinations of P_{fn} and P_{fp} for the evaluation set

We select the threshold corresponding to the lowest DCF

Model evaluation for classification

The corresponding value DCF_{min} is the cost we would pay if we knew before-hand the optimal threshold for the evaluation

We can think of this value as a measure of the quality of the classifier

We can also compute the actual DCF obtained using the threshold corresponding to the effective prior $\tilde{\pi}$

The difference between the actual and minimum DCF represents the loss due to score mis-calibration

Model evaluation for classification

We can also compare different systems over different applications through Bayes error plots

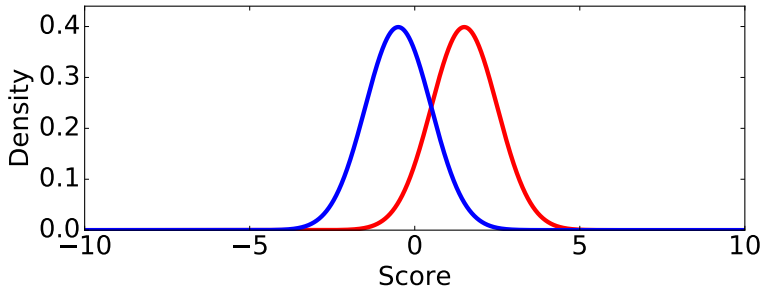
These plots can be used to report actual and / or minimum DCF for different applications

A binary application is parametrized by a single value $\tilde{\pi}$

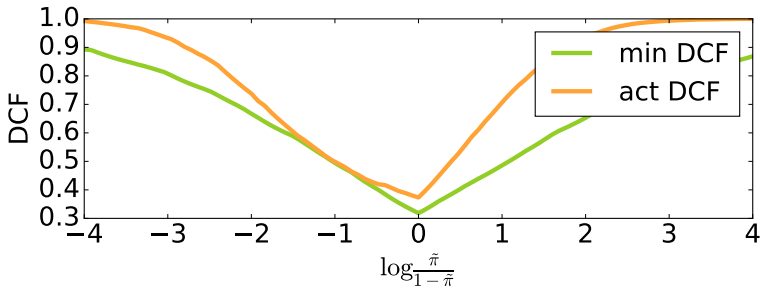
We can thus plot the DCF as a function of prior log-odds $\log \frac{\tilde{\pi}}{1-\tilde{\pi}}$, i.e. the negative of the Bayes optimal threshold.

Model evaluation for classification

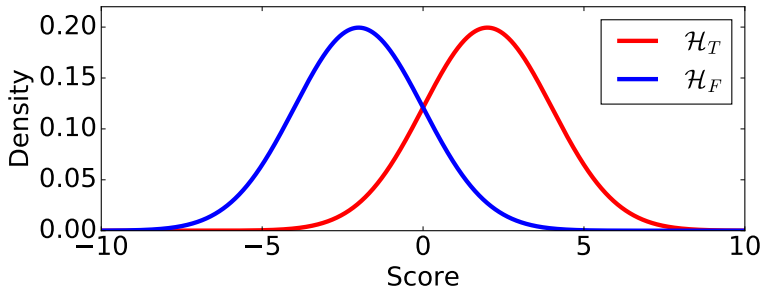
Non calibrated scores



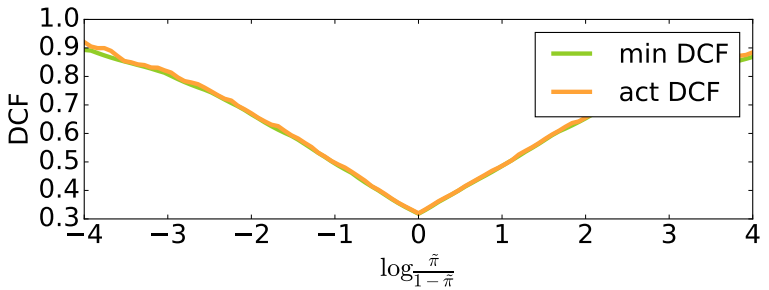
Bayes error plot



Calibrated scores



Bayes error plot



Model evaluation for classification

To reduce mis-calibration costs we can adopt different calibration strategies

We can use a validation set to find a (close-to) optimal threshold for a given application

More general approaches look for functions that transform the classifier scores s into well-calibrated LLRs, possibly in a way that is as much as possible independent from the target application

These usually also employ a validation set (calibration set in this case) to estimate the mapping between the classifier scores and well-calibrated scores

Model evaluation for classification

Score calibration approaches:

- Isotonic regression
- Prior-weighted logistic regression
- Generative score models (e.g. Gaussian score models)

Model evaluation for classification

Evaluation of multiclass tasks is more complex

As we have seen, we can build confusion matrices

We can also compute the empirical Bayes risk for multiclass problems

We can compute a normalized detection cost, obtained by scaling the Bayes risk by the cost of the best dummy system — in this case, we have K dummy systems, each of them predicting a different class k regardless of the sample

For the multiclass problem, it's more difficult to separate the costs due to mis-calibration from those due to poor discriminant capabilities of the classifier