Zestaw 4

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3N

Zbudowano wielomian interpolacyjny oparty na nastepujacych danych:

```
In[1]:= x₀ = -1.2300;
       y₀ = 1.5129;
       x₁ = -1.1900;
       y₁ = 1.4161;
       x₂ = -0.7400;
       y₂ = 0.5476;
       x₃ = 0.1100;
       y₃ = 0.0121;
       x₄ = 2.5600;
       y₄ = 6.5536;
```

```
In[11]:= XY = {{-1.2300, 1.15129}, {-1.1900, 1.14161},
           {-0.7400, 0.5476}, {0.1100, 0.0121}, {2.5600, 6.5536}};
```

```
In[12]:= Lagrange[XY_, rozm_] :=
           Module[{n = rozm - 1, wyn, suma = 0, resz, X, Y},
            X = Transpose[XY]₍₁₎;
            Y = Transpose[XY]₍₂₎;
            Do[wyn = 1;
             Do[resz = Which[j == k, 1,

               j ≠ k, (x - X₍j+1₎)/(X₍k+1₎ - X₍j+1₎)];

              wyn = wyn * resz; , {j, 0, n, 1}];
             suma = suma + Y₍k+1₎ * wyn; , {k, 0, n, 1}];

             Return[Print["Wielomian interpolacyjny: "];
              Expand[suma]]; ];
```

```
In[13]:= Lagrange[XY, 5]
```

```
Wielomian interpolacyjny:
```

$$Out[13]= -0.09908 + 0.700368\, x + 2.77272\, x^2 + 0.498601\, x^3 - 0.504699\, x^4$$

4N

```
In[14]:= f[x_] := 1/(1 + 5 x²);
```

```
In[15]:= X = Table[x, {x, -1, 1, 1/32}];
```

Wartosci funkcji w punktach x odpowiednio od -1 do 1 z krokiem $\frac{1}{32}$ wynosz„:

In[16]:= **Y = Map[f, X]**

Out[16]= $\left\{\frac{1}{6}, \frac{1024}{5829}, \frac{256}{1381}, \frac{1024}{5229}, \frac{64}{309}, \frac{1024}{4669}, \frac{256}{1101}, \frac{1024}{4149}, \frac{16}{61}, \frac{1024}{3669}, \frac{256}{861}, \frac{1024}{3229}, \frac{64}{189}, \right.$
$\frac{1024}{2829}, \frac{256}{661}, \frac{1024}{2469}, \frac{4}{9}, \frac{1024}{2149}, \frac{256}{501}, \frac{1024}{1869}, \frac{64}{109}, \frac{1024}{1629}, \frac{256}{381}, \frac{1024}{1429}, \frac{16}{21}, \frac{1024}{1269},$
$\frac{256}{301}, \frac{1024}{1149}, \frac{64}{69}, \frac{1024}{1069}, \frac{256}{261}, \frac{1024}{1029}, 1, \frac{1024}{1029}, \frac{256}{261}, \frac{1024}{1069}, \frac{64}{69}, \frac{1024}{1149}, \frac{256}{301},$
$\frac{1024}{1269}, \frac{16}{21}, \frac{1024}{1429}, \frac{256}{381}, \frac{1024}{1629}, \frac{64}{109}, \frac{1024}{1869}, \frac{256}{501}, \frac{1024}{2149}, \frac{4}{9}, \frac{1024}{2469}, \frac{256}{661}, \frac{1024}{2829},$
$\left. \frac{64}{189}, \frac{1024}{3229}, \frac{256}{861}, \frac{1024}{3669}, \frac{16}{61}, \frac{1024}{4149}, \frac{256}{1101}, \frac{1024}{4669}, \frac{64}{309}, \frac{1024}{5229}, \frac{256}{1381}, \frac{1024}{5829}, \frac{1}{6} \right\}$

In[17]:= **XY = Transpose[Distribute[{X, Y}]];**

In[18]:= **Lagrange[XY_] := Module$\Big[$ {j, k, n, X, Y},**

    $X_{k\_}$:= Transpose[XY]$_{[\![1,k+1]\!]}$;

    $Y_{k\_}$:= Transpose[XY]$_{[\![2,k+1]\!]}$;

    n = Length[XY] - 1;

    Return$\Big[\sum_{k=0}^{n} Y_k \left(\prod_{j=0}^{k-1} \frac{x - X_j}{X_k - X_j} \right) \left(\prod_{j=k+1}^{n} \frac{x - X_j}{X_k - X_j} \right) \Big]$;$\Big]$;

In[19]:= **For$\Big[$n = 2, n ≤ 15, n++,**

    x1 = -1;

    x2 = 1;

    XXY = N$\Big[$Table$\Big[\Big\{ x1 + \frac{x2 - x1}{n} k, f\Big[x1 + \frac{x2 - x1}{n} k \Big] \Big\}, \{k, 0, n\}\Big]\Big]$;

    Cdot = ListPlot[XY, PlotStyle → {Red, PointSize[0.02]}];

    P[x_] = Lagrange[XY];

    graph1 = Plot[f[x], {x, -2, 2}, PlotStyle → Green];

    graph2 = Plot[P[x], {x, -1, 1}, PlotStyle → Blue];

    b = Show[graph1, graph2, Cdot] $\Big]$;

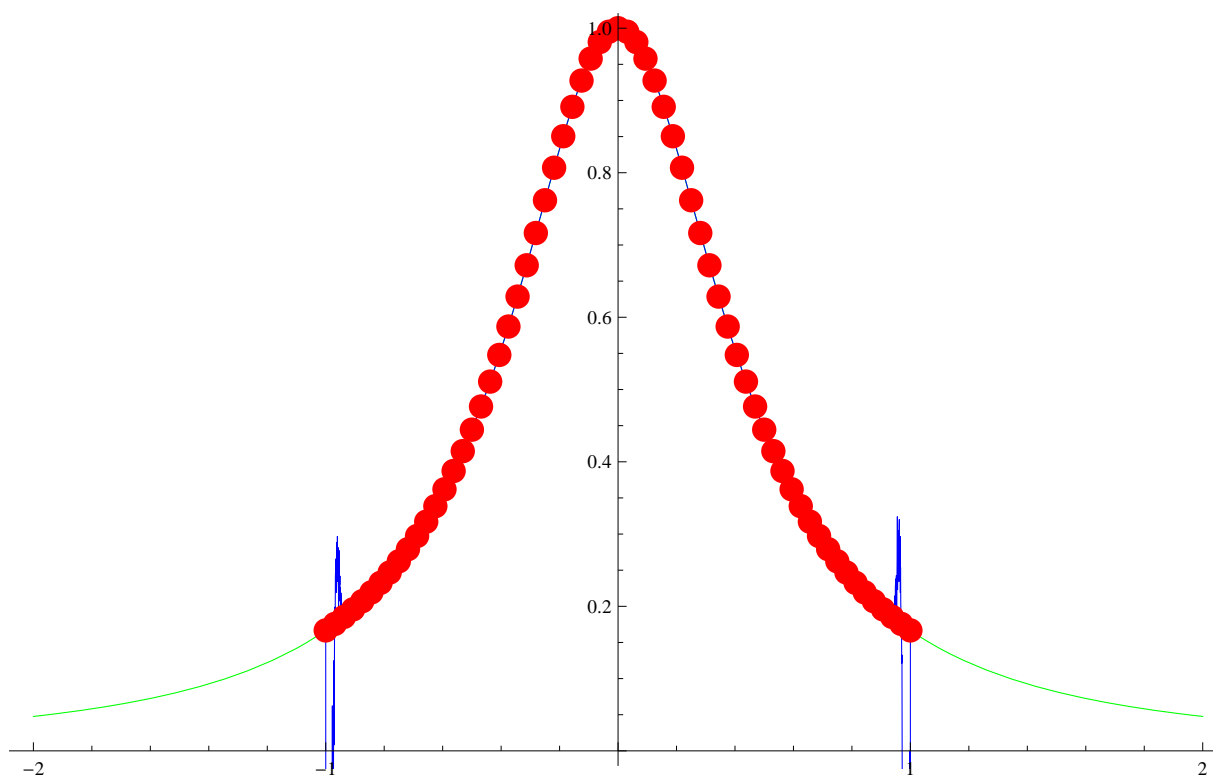Wykres   wielomianu:

kolor niebieski - wykres wielomianu,

kolor zielony - wykres funkcji,

kolor czerwony - wezly i wartosci funkcji.

In[20]:= **b**

Out[20]=

In[21]:=

5N

In[22]:=

Skonstruowano funckje, jak w zadaniu 4N:

In[23]:= $\mathbf{f[x\_] := \dfrac{1}{1 + 5\,x^2}};$

In[24]:= $\mathbf{X = Table\left[x, \left\{x, -1, 1, \dfrac{1}{32}\right\}\right]};$

In[25]:= $\mathbf{Y = Map[f, X]};$

In[26]:= $\mathbf{XY = N[Transpose[Distribute[\{X, Y\}]]]};$

```mathematica
In[27]:= SplajnNat[XY0_] := Module[{XY = XY0},
    D = Module[{k}, n = Length[XY] - 1; X = Transpose[XY][[1]];
      Y = Transpose[XY][[2]]; h = d = Table[0, {n}]; m = Table[0, {n + 1}];
      a = b = c = v = Table[0, {n - 1}]; s = Table[0, {n}, {4}];
      h[[1]] = X[[2]] - X[[1]];
      d[[1]] = (Y[[2]] - Y[[1]])/h[[1]];
      For[k = 2, k ≤ n, k++,
        h[[k]] = X[[k+1]] - X[[k]];
        d[[k]] = (Y[[k+1]] - Y[[k]])/h[[k]];
        a[[k-1]] = h[[k]];
        b[[k-1]] = 2 (h[[k-1]] + h[[k]]);
        c[[k-1]] = h[[k]];
        v[[k-1]] = 6 (d[[k]] - d[[k-1]])]; ];
    TrD := Module[{k, t},
      m[[1]] = 0;
      m[[n+1]] = 0;
      For[k = 2, k ≤ n - 1, k++,
        t = a[[k-1]]/b[[k-1]];
        b[[k]] = b[[k]] - t c[[k-1]];
        v[[k]] = v[[k]] - t v[[k-1]]; ];
      m[[n]] = v[[n-1]]/b[[n-1]];
      For[k = n - 2, 1 ≤ k, k--,
        m[[k+1]] = (v[[k]] - c[[k]] m[[k+2]])/b[[k]]; ]; ];
    Pol := Module[{k},
      For[k = 1, k ≤ n, k++,
        s[[k,1]] = Y[[k]];
        s[[k,2]] = d[[k]] - 1/6 h[[k]] (2 m[[k]] + m[[k+1]]);
        s[[k,3]] = m[[k]]/2;
        s[[k,4]] = (m[[k+1]] - m[[k]])/(6 h[[k]]); ]; ];
    CS[t_] := Module[{j},
      For[j = 1, j ≤ n, j++,
        If[X[[j]] ≤ t && t < X[[j+1]], k = j]; ];
      If[t < X[[1]], k = 1];
      If[X[[n+1]] ≤ t, k = n];
      w = t - X[[k]];
      Return[((s[[k,4]] w + s[[k,3]]) w + s[[k,2]]) w + s[[k,1]]]; ];

    D;
    TrD;
    Pol; ]
```
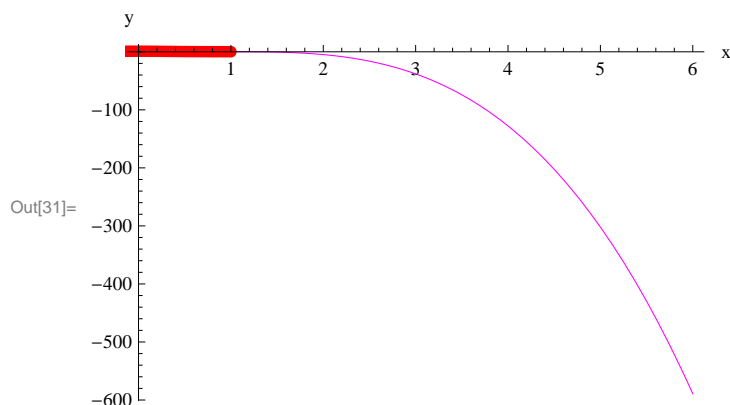
```
In[28]:=  SplajnNat[XY];
          dots = ListPlot[XY, PlotStyle → {Red, PointSize[0.02]}, DisplayFunction → Identity];
          gr = Plot[CS[x], {x, 0, 6}, PlotStyle → {Magenta}, DisplayFunction → Identity];
          Show[gr, dots, AxesLabel → {"x", "y    "}]
          Print["Splajn y = ", Sᵢ, "[x]"];
          Print["Punkty = ", XY];
          Print["Wspó‡czynniki: "]
          MatrixForm[Chop[s]]
```

Set::wrsym : Symbol D is Protected. ≫


```
Out[31]=
```

Splajn y = $S_i[x]$

Punkty = {{-1., 0.166667}, {-0.96875, 0.175673}, {-0.9375, 0.185373}, {-0.90625, 0.195831},
  {-0.875, 0.20712}, {-0.84375, 0.219319}, {-0.8125, 0.232516}, {-0.78125, 0.246806},
  {-0.75, 0.262295}, {-0.71875, 0.279095}, {-0.6875, 0.297329}, {-0.65625, 0.317126},
  {-0.625, 0.338624}, {-0.59375, 0.361965}, {-0.5625, 0.387292}, {-0.53125, 0.414743},
  {-0.5, 0.444444}, {-0.46875, 0.476501}, {-0.4375, 0.510978}, {-0.40625, 0.547887},
  {-0.375, 0.587156}, {-0.34375, 0.628607}, {-0.3125, 0.671916}, {-0.28125, 0.716585},
  {-0.25, 0.761905}, {-0.21875, 0.806935}, {-0.1875, 0.850498}, {-0.15625, 0.89121},
  {-0.125, 0.927536}, {-0.09375, 0.957905}, {-0.0625, 0.980843}, {-0.03125, 0.995141},
  {0., 1.}, {0.03125, 0.995141}, {0.0625, 0.980843}, {0.09375, 0.957905}, {0.125, 0.927536},
  {0.15625, 0.89121}, {0.1875, 0.850498}, {0.21875, 0.806935}, {0.25, 0.761905},
  {0.28125, 0.716585}, {0.3125, 0.671916}, {0.34375, 0.628607}, {0.375, 0.587156},
  {0.40625, 0.547887}, {0.4375, 0.510978}, {0.46875, 0.476501}, {0.5, 0.444444},
  {0.53125, 0.414743}, {0.5625, 0.387292}, {0.59375, 0.361965}, {0.625, 0.338624},
  {0.65625, 0.317126}, {0.6875, 0.297329}, {0.71875, 0.279095}, {0.75, 0.262295},
  {0.78125, 0.246806}, {0.8125, 0.232516}, {0.84375, 0.219319}, {0.875, 0.20712},
  {0.90625, 0.195831}, {0.9375, 0.185373}, {0.96875, 0.175673}, {1., 0.166667}}

Wspó‡czynniki:

```
Out[35]//MatrixForm=
        ⎛ 0.166667   0.28362       0        4.70397  ⎞
        ⎜ 0.175673   0.297402   0.440997   -0.815858 ⎟
        ⎜ 0.185373   0.322574   0.36451    0.708839  ⎟
        ⎝ 0.195831   0.347432   0.430964   0.349951  ⎠
```

0.20712	0.375393	0.463772	0.499656
0.219319	0.405842	0.510614	0.516631
0.232516	0.439269	0.559049	0.572019
0.246806	0.475886	0.612675	0.618753
0.262295	0.515991	0.670684	0.667513
0.279095	0.559864	0.733263	0.712503
0.297329	0.60778	0.80006	0.750977
0.317126	0.659984	0.870464	0.777536
0.338624	0.716666	0.943358	0.785085
0.361965	0.777926	1.01696	0.763891
0.387292	0.843724	1.08857	0.701095
0.414743	0.913814	1.1543	0.580113
0.444444	0.987657	1.20869	0.380277
0.476501	1.06431	1.24434	0.0769452
0.510978	1.14231	1.25155	− 0.357489
0.547887	1.21949	1.21804	− 0.95104
0.587156	1.29283	1.12888	− 1.72728
0.628607	1.35832	0.966946	− 2.69741
0.671916	1.41085	0.714063	− 3.84901
0.716585	1.44421	0.353218	− 5.13206
0.761905	1.45125	− 0.127913	− 6.44525
0.806935	1.42437	− 0.732155	− 7.62808
0.850498	1.35626	− 1.44729	− 8.46701
0.89121	1.241	− 2.24107	− 8.72375
0.927536	1.07538	− 3.05892	− 8.18887
0.957905	0.860202	− 3.82663	− 6.75171
0.980843	0.601257	− 4.4596	− 4.46351
0.995141	0.309456	− 4.87805	− 1.5624
1.	0	− 5.02453	1.5624
0.995141	− 0.309456	− 4.87805	4.46351
0.980843	− 0.601257	− 4.4596	6.75171
0.957905	− 0.860202	− 3.82663	8.18887
0.927536	− 1.07538	− 3.05892	8.72375
0.89121	− 1.241	− 2.24107	8.46701
0.850498	− 1.35626	− 1.44729	7.62808
0.806935	− 1.42437	− 0.732155	6.44525
0.761905	− 1.45125	− 0.127913	5.13206
0.716585	− 1.44421	0.353218	3.84901
0.671916	− 1.41085	0.714063	2.69741
0.628607	− 1.35832	0.966946	1.72728
0.587156	− 1.29283	1.12888	0.95104
0.547887	− 1.21949	1.21804	0.357489
0.510978	− 1.14231	1.25155	− 0.0769452
0.476501	− 1.06431	1.24434	− 0.380277
0.444444	− 0.987657	1.20869	− 0.580113
0.414743	− 0.913814	1.1543	− 0.701095
0.387292	− 0.843724	1.08857	− 0.763891
0.361965	− 0.777926	1.01696	− 0.785085
0.338624	− 0.716666	0.943358	− 0.777536

$$
\begin{pmatrix}
0.317126 & -0.659984 & 0.870464 & -0.750977 \\
0.297329 & -0.60778 & 0.80006 & -0.712503 \\
0.279095 & -0.559864 & 0.733263 & -0.667513 \\
0.262295 & -0.515991 & 0.670684 & -0.618753 \\
0.246806 & -0.475886 & 0.612675 & -0.572019 \\
0.232516 & -0.439269 & 0.559049 & -0.516631 \\
0.219319 & -0.405842 & 0.510614 & -0.499656 \\
0.20712 & -0.375393 & 0.463772 & -0.349951 \\
0.195831 & -0.347432 & 0.430964 & -0.708839 \\
0.185373 & -0.322574 & 0.36451 & 0.815858 \\
0.175673 & -0.297402 & 0.440997 & -4.70397
\end{pmatrix}
$$