Zestaw 3

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3N

Przy uzyciu metody potegowej znaleziono 2 najwieksze na modul wartosci wlasne.

In[1]:= $A = \begin{pmatrix} \frac{19}{12} & \frac{13}{12} & \frac{5}{6} & \frac{5}{6} & \frac{13}{12} & \frac{-17}{12} \\ \frac{13}{12} & \frac{13}{12} & \frac{5}{6} & \frac{5}{6} & \frac{-11}{12} & \frac{13}{12} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{13}{12} & \frac{-11}{12} & \frac{5}{6} & \frac{5}{6} & \frac{13}{12} & \frac{13}{12} \\ \frac{-17}{12} & \frac{13}{12} & \frac{5}{6} & \frac{5}{6} & \frac{13}{12} & \frac{19}{12} \end{pmatrix}$;

```
In[2]:= X01 = {1, 1, 1, 1, 1, 1}; X02 = {-1, 1, 1, 1, 1, 1};
```

```
In[3]:= Potegowa[A0_, V0_, precyzja_, iter_] :=
  Module[{A = N[A0], c1, i, er, l, l0, X, k, X0 = N[V0], Y},
   Normalizacja[W_] := W/Sqrt[W.W];
   maxsize[W_] := Module[{rozm, w = W},
     If[Abs[w[[-1]]] ≥ Abs[w[[1]]],
      rozm = w[[-1]],
      rozm = w[[1]]];
     Return[rozm];];
   k = 1;
   While[k ≤ 2,
    If[k == 2,
     X0 = N[X02]];
    l0 = 0;
    i = 0;
    While[i ≤ iter,
     i++;
     Y = A.X0;
     l = maxsize[Y];
     X = 1/l Y;
     Print["λ"_k, "=", NumberForm[l, 6]];
     Print["X"_k, "=", MatrixForm[X]];
     er = Max[{Abs[l - l0], Normalizacja[X - X0]}];

     If[er < precyzja, Break;];
     X0 = X;
     l0 = l; k++];];];
```

Wynik:

```
In[4]:= Potegowa[A, X01, 0.00001, 0]
```

λ_1=4.

$$X_1=\begin{pmatrix}1.\\1.\\1.\\1.\\1.\\1.\end{pmatrix}$$

λ_2=6.83333

$$X_2=\begin{pmatrix}0.121951\\0.268293\\0.341463\\0.341463\\0.268293\\1.\end{pmatrix}$$

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Sprowadzic macierz do postaci trojdiagonalnej i znalezc jej wszytskie wartosci wlasne.

In[5]:=
$$A=\begin{pmatrix}\frac{19}{12}&\frac{13}{12}&\frac{5}{6}&\frac{5}{6}&\frac{13}{12}&\frac{-17}{12}\\\frac{13}{12}&\frac{13}{12}&\frac{5}{6}&\frac{5}{6}&\frac{-11}{12}&\frac{13}{12}\\\frac{5}{6}&\frac{5}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{5}{6}\\\frac{5}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{5}{6}&\frac{5}{6}\\\frac{13}{12}&\frac{-11}{12}&\frac{5}{6}&\frac{5}{6}&\frac{13}{12}&\frac{13}{12}\\\frac{-17}{12}&\frac{13}{12}&\frac{5}{6}&\frac{5}{6}&\frac{13}{12}&\frac{19}{12}\end{pmatrix};$$

Do sprowadzenia macierzy do postaci trojdiagonalnej uzyto metody Householdera.

```
In[6]:= Householder[A0_] :=
  Module[{k},
   A1 = N[A0];
   n = Length[A1];
   Q1 = Table[0, {i, n}];
   V1 = Table[0, {i, n}];
   W1 = Table[0, {i, n}];
   For[k = 1, k ≤ n - 2, k++,
    (*1*)
    Module[{i, j, sum},
      sum = 0;
      For[j = k + 1, j ≤ n, j++,
```

```
    sum = sum + (A1[[j,k]])^2;];
   s0 = Sqrt[sum];
   If[A1[[k+1,k]] < 0, s0 = -s0; ];
   r0 = Sqrt[2 (A1[[k+1,k]] + s0) s0];
   For[j = 1, j ≤ k, j++, W1[[j]] = 0; ];
   W1[[k+1]] = (A1[[k+1,k]] + s0)/r0;
   For[j = k + 2, j ≤ n, j++,
    W1[[j]] = A1[[j,k]]/r0; ];
   (*2*)
   For[j = 1, j ≤ k, j++, V1[[j]] = 0; ];
   For[i = k + 1, i ≤ n, i++,
    sum = 0;
    For[j = k + 1, j ≤ n, j++,
     sum = sum + A1[[i,j]] W1[[j]]; ];
    V1[[i]] = sum; ];
   (*3*)
   c = 0;
   For[j = k + 1, j ≤ n, j++,
    c = c + W1[[j]] V1[[j]]; ];
   For[j = 1, j ≤ k, j++,
    Q1[[j]] = 0; ];
   For[j = k + 1, j ≤ n, j++,
    Q1[[j]] = V1[[j]] - c W1[[j]]; ];
   (*4*)
   For[j = k + 2, j ≤ n, j++,
    A1[[j,k]] = 0;
    A1[[k,j]] = 0; ];
   A1[[k+1,k]] = - s0;
   A1[[k,k+1]] = - s0;
   For[j = k, j ≤ n, j++,
    A1[[j,j]] = A1[[j,j]] - 4 Q1[[j]] W1[[j]]; ];
   For[i = k + 1, i ≤ n, i++,
    For[j = i + 1, j ≤ n, j++,
      A1[[i,j]] = A1[[i,j]] - 2 W1[[i]] Q1[[j]] - 2 Q1[[i]] W1[[j]];
      A1[[j,i]] = A1[[i,j]]; ]; ]; ]; ];
  Print["Forma trojdiagonalna macierzy: "];
  Return[N[A1] // MatrixForm] ];
```

In[7]:= `B = Householder[A]`

Forma trojdiagonalna macierzy:

Out[7]//MatrixForm=

$$\begin{pmatrix} 1.58333 & -2.39647 & 0. & 0. & 0. & 0. \\ -2.39647 & -0.0125957 & 0.934759 & 0. & 0. & 0. \\ 0. & 0.934759 & 2.36902 & -2.07886 & 0. & 0. \\ 0. & 0. & -2.07886 & 0.060241 & -1.26585\times10^{-15} & 0. \\ 0. & 0. & 0. & -1.26585\times10^{-15} & 1.27901 & -0.448514 \\ 0. & 0. & 0. & 0. & -0.448514 & 1.72099 \end{pmatrix}$$

In[8]:=
$$B = \begin{pmatrix} 1.5833333333333333` & -2.396467307424734` & 0.` & 0.` \\ -2.396467307424734` & -0.01259572752922189` & 0.9347592788434194` & 0.` \\ 0.` & 0.9347592788434194` & 2.3690214303404695` & -2.07886320644073 \\ 0.` & 0.` & -2.0788632064407335` & 0.060240963855419 \\ 0.` & 0.` & 0.` & -1.2658490090568385 \\ 0.` & 0.` & 0.` & 0.` \end{pmatrix}$$

Nastepnie przeksztalcono macierz w macierz diagonalna przy uzyciu metody QR:

In[9]:=
```
QR[A_, m0_] :=
  Module[ {A0 = N[A], A1, i, m = m0},
   Print[ Chop[A0, 5.0 × 10^-6] // MatrixForm ]
    i = 1;
   Do[
     {Q0, R0} = QRDecomposition[A0];
     A1 = R0.Transpose[Q0];
     If [i == m, Print["Macierz diagonalna z wartosciami wlasnymi na przekatnej: "];
      Print[ MatrixForm[Chop[A1, 5.0 × 10^-6]]];, MatrixForm[Chop[A1, 5.0 × 10^-6]];

      A0 = A1] , {i, 1, m}] Return[0];];
```

In[10]:=
```
QR[B, 73]
```

$$\begin{pmatrix} 1.58333 & -2.39647 & 0 & 0 & 0 & 0 \\ -2.39647 & -0.0125957 & 0.934759 & 0 & 0 & 0 \\ 0 & 0.934759 & 2.36902 & -2.07886 & 0 & 0 \\ 0 & 0 & -2.07886 & 0.060241 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.27901 & -0.448514 \\ 0 & 0 & 0 & 0 & -0.448514 & 1.72099 \end{pmatrix}$$

Macierz diagonalna z wartosciami wlasnymi na przekatnej:

$$\begin{pmatrix} 4. & 0 & 0 & 0 & 0 & 0 \\ 0 & 3. & 0 & 0 & 0 & 0 \\ 0 & 0 & -2. & 0 & 0 & 0 \\ 0 & 0 & 0 & 2. & 0 & 0 \\ 0 & 0 & 0 & 0 & -1. & 0 \\ 0 & 0 & 0 & 0 & 0 & 1. \end{pmatrix}$$

Dla sprawdzenia polioczno wartosci wbudowana funkcja Mathematici, ktora dzia‡a na podstawie algortmu Laczosa.

In[11]:=
```
Eigenvalues[B] // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix} 4. \\ 3. \\ -2. \\ 2. \\ -1. \\ 1. \end{pmatrix}$$

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Macierz podana w cwiczeniu hermitowska nie byla, wiec wstawiono "minus" przy i w prawym gornym rogu macierzy.

Skorzystano z rozłozenia macierzy na dwie macierze: H=A+iB, gdzie H=$\begin{pmatrix} A & -B \\ B & A \end{pmatrix}$.

In[12]:=

$$H = \begin{pmatrix} 0 & 1 & 0 & -i \\ 1 & 0 & -i & 0 \\ 0 & i & 0 & 1 \\ i & 0 & 1 & 0 \end{pmatrix};$$

In[13]:=

```
rozm = 4; (*rozmiar macierzy*)
```

In[14]:=
```
RzeczywSymetr[H_, rozm_] := Module[
  {l1, k, j, n = rozm, W = H, A = Array[0 &, {rozm, rozm}], B = Array[0 &, {rozm, rozm}],
   Hrz = Array[0 &, {2 * rozm, 2 * rozm}], G = Array[0 &, {2 * rozm, 2 * rozm}]},
  Do[
   Do[
     If [ Im[W[[k, j]]] != 0,
       B[[k, j]] = W[[k, j]];
       B[[k, j]] = 1/i B[[k, j]];,
       A[[k, j]] = W[[k, j]];];
     , {j, 1, n}];
   , {k, 1, n}];

  Print["A=", A // MatrixForm];
  Print["B=", B // MatrixForm];
  Print["-B=", -B // MatrixForm]

   Do[Do[
      Hrz[[k, j]] = A[[k, j]];
      Hrz[[k, j+n]] = -B[[k, j]];
      Hrz[[k+n, j]] = B[[k, j]];
      Hrz[[k+n, j+n]] = A[[k, j]];
       , {j, 1, n}];
    , {k, 1, n}];
  Print["Macierz rzeczywista symetryczna: "];
  Print["Hrz= ", Hrz // MatrixForm];
  Print["Wartosci wlasne: ", Eigenvalues[Hrz] // MatrixForm];
  Print[
   "Do kazdej wartosci wlasnej sa dwa wektory wlasne, bo macierz H nxn ma n wektorow
     wlasnych, ale macierz H roz‡ozona na dwie macierze ma rozmiar
     2nx2n wiŒc ma 2n wektorow wlasnych. To pierwszy zestaw: "];
  Print[G = Eigenvectors[Hrz] // MatrixForm]
   Return[G];
 ];
```

In[15]:=
```
RzeczywSymetr[H, 4]
```

$$A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$B=\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$-B=\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

Macierz rzeczywista symetryczna:

$$H_{rz}=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Wartosci wlasne: $\begin{pmatrix} -2 \\ -2 \\ 2 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Do kazdej wartosci wlasnej sa dwa wektory wlasne, bo macierz H nxn ma n wektorow wlasnych, ale macierz H roz‡ozona na dwie macierze ma rozmiar 2nx2n wiŒc ma 2n wektorow wlasnych. To pierwszy zestaw:

$$\begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[16]:= **G** = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$;

```
In[17]:= X = Array[0 &, {4, 8}];
         Y = Array[0 &, {4, 8}];
```

```
In[19]:= Do[Do[
           X[[k,j]] = G[[k,j]];
           Y[[k,j]] = G[[k+rozm,j]];
           , {j, 1, 8}];
         , {k, 1, rozm}];
         Do[Do[
           G[[k,j]] = Y[[k,j]];
           G[[k+rozm,j]] = -X[[k,j]];
           , {j, 1, 8}];
         , {k, 1, rozm}];
         Print["To drugi zestaw wektorow wlasnych: ", G // MatrixForm];
```

To drugi zestaw wektorow wlasnych: $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

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Znalezc przyblizony wektor wlasny do wartosci wlasnej λ=0.38197

In[22]:= $A = \begin{pmatrix} 2 & -1 & 0 & 0 & 1 \\ -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 \\ 1 & 0 & 0 & -1 & 2 \end{pmatrix}$;

```
l = 0.38197;
```

```
In[24]:= Func[A_, l_] := Module[{x, b, B, Al = A, n = Length[A]},
           b = Array[0 &, {n}];
           b[[1]] = 1;
           Normalizacja[W_] := Sqrt[Abs[W[[1]]]^2 + Abs[W[[2]]]^2 + Abs[W[[3]]]^2];
```

$B = l * \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$;

```
           Do[
            Al[[i,i]] = Al[[i,i]] - B[[i,i]];
            , {i, 1, n, 1}];
           Do[x = LinearSolve[Al, b] ;
            b = x / Normalizacja[x];,
            {i, 1, n}
           ];
           Print["Przyblizony wektor wlasny:"];
           Return[ b // MatrixForm];
          ];
```

In[25]:=
```
Func[A, l] // MatrixForm
```

```
Przyblizony wektor wlasny:
```

Out[25]//MatrixForm=

$$\begin{pmatrix} -0.850651 \\ -0.525731 \\ 0. \\ 0.525731 \\ 0.850651 \end{pmatrix}$$