

Enhancing 3D reconstruction with Android Sensor Fusion data



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Abstract

The main subject of this thesis addresses the possibility of enhancing the existing 3D reconstruction algorithms with additional Android sensor fusion data. The primary fundamentals behind the reconstruction techniques and the problems related to them are explained in order to introduce the reader to these topics. Related works are discussed in order to explain the strengths and weaknesses of the state-of-art reconstruction approaches. The proposed enhancements using initial rotation in the standard 8-point and 5-point algorithms are explained in detail. Both of them allow for focusing on correcting rotation matrix error instead of calculating the rotation itself. This, in turn, serves for reducing ambiguity of the decomposition of essential matrix to the relative rotation and translation of the cameras. The primary idea behind the implementation of the 3-point algorithm for translation is also described. The most important aspects of the implemented "*SensorEnhancedImageCamera*" Android application and "*Enhanced3DReconstructor*" were discussed. The three proposed initial pair reconstruction methods were evaluated, which showed that each algorithm is able to improve certain aspects of the 3D reconstruction. Different reconstruction strategies were evaluated. As a result it was discovered that the enhanced structure computation can be faster and more accurate than the standard methodology. Using initial rotation and translation pose estimation results in significantly faster convergence and better error reduction with Bundle Adjustment. This thesis covers only some notions of the Structure from Motion, therefore plans for future development were established.

Zusammenfassung

Das Hauptthema dieser Arbeit befasst sich mit der Möglichkeit der Verbesserung der vorhandenen 3D-Rekonstruktionsalgorithmen mit zusätzlichen Android Sensorfusion Daten. Die wichtigsten Grundlagen hinter den Rekonstruktionstechniken und die damit verbundenen Probleme sind, um den Leser zu diesen Themen vorstellen erläutert. Ähnliche Arbeiten sind erforderlich, um die Festigkeiten und Schwächen der Standard-Technik Rekonstruktion Anstze erklären diskutiert. Die vorgeschlagenen Verbesserungen mit Anfangsrotation in den Standard 8-Punkte und 5-Punkte-Algorithmen werden im Detail erklärt. Beide von ihnen können zur Fokussierung auf die Rotationsmatrix Fehler Korrektur anstelle der Berechnung des Rotation selbst. Dies wiederum serviert zur Reduzierung der Mehrdeutigkeit des Essential-Matrix Zersetzung zu der relativen Rotation und Translation den Kameras. Die Hauptidee hinter der Umsetzung der 3-Punkt-Algorithmus für die Translation ist ebenfalls beschrieben. Die wichtigsten Aspekte die implementierten *SSensorVerbesserteBildkamera*“ Android-Anwendung und ”*Verbesserte3DReconstructor*“ wurden diskutiert. Die drei vorgeschlagenen ersten Paar Rekonstruktionsmethoden wurden bewertet, die zeigten, dass jeder Algorithmus kann bestimmte aspects der 3D-Rekonstruktion zu verbessern. Verschiedene Rekonstruktionsstrategien wurden bewertet. Als Ergebnis wurde festgestellt, dass die verbesserte Struktur Berechnung kann schneller und genauer als die Standardmethode sein. Mit anfnglichen Rotation und Translation Posenschtzung resultiert in signifikant schneller Konvergenz und besseres Fehlerreduktion mit Bündelausgleichung. Diese Diplomarbeit umfasst nur einige Vorstellungen von der Struktur aus Bewegung, daher plant für die zukünftige Entwicklung entstanden.

To ...

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I would like to acknowledge the thousands of individuals who have coded for the LaTeX project for free. It is due to their efforts that we can generate professionally typeset PDFs now.

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Acronyms

BA Bundle Adjustment. ix, 3

Base line The distance between the left and the right camera in a stereo pair.. ix

Epipolar lines TODO. ix

Essential Matrix TODO. ix

Extrasinc??? camera parameters TODO. ix

FoV Field of View. ix

Intersinc camera parameters TODO. ix

Pitch Rotation around the y axis.. ix

Pose Estimation TODO. ix

Quaternion TODO. ix

RANSAC Bundle Adjustment. ix

Roll Rotation around the x axis.. ix

Rotation Matrix TODO. ix

SIFT TODO. ix

SVD Singular Value Decomposition. ix

Yaw Rotation around the z axis.. ix

Chapter 1

Introduction

Mobile and wearable devices are becoming more and more popular. Modern smart-phones despite having extremely good camera's also use advanced sensor's, like accelerometers, gyroscope, magnetometer, barometer etc.. There is also a big need and growing market of Augmented Reality (AR) and Virtual Reality(VR). That is why image analysis and recognition, as well as 3-D reconstruction techniques are really hot topic. Unfortunately algorithms that support these techniques are very time and memory consuming, that is why it is really hard to run them on mobile devices, which have many limitations in terms of CPU speed and RAM memory capacity.

1.1 Purpose of this thesis

Author of this document will present the reader with an overview of the idea of 3D reconstruction techniques. This thesis also inculdes brief description of related research conducted in this area. After short analysis of efficiency, accuracy and common problems of few chosen algorithms this thesis will propose their enhacement using data acquired from Android sensor fusion. Two application was created in order to verify and test proposed algorithms and reconstruction strategies. Both Android and Desktop applications will be discussed in important implementation asspects. This thesis also explains the reader meaning of testing results. At the end conclusions and encountered errors are discussed. Finally futur plans for conducted research is presented.

1. INTRODUCTION

1.2 Scope

The author researched, how fusion of accelerometer, gyroscope and magnetometer can be used in order to improve fundamental, essential matrix calculations and also relative pose estimation. Unfortunately raw data sensors are really noisy and therefore not reliable enough to rely solely on them to enhance reconstruction. Heuristic measurement of translation used for enhancement using linear-acceleration was also conceived and implemented by author. In this research introduced are initial rotation enhanced 8-point and 5-point reconstruction algorithms. Also alternative 3-point algorithm for translation estimation is also discussed and implemented. This thesis deals only with initial pair images reconstruction and relative pose estimation for Structure from Motion computation. It is not researched, what are the best methodology for corresponding feature computation, but other possibilities of point matching are discussed. Finally it is researched, how improved versions of standard 8-point and 5-point algorithms influence convergence speed and error reduction of Bundle Adjustment.

1.3 Initial assumptions

The general process of 3-D reconstruction is quite broad, that's why the author of the thesis focus only on certain aspects of this topic. That is why author did not write all algorithms from the scratch, but built his versions on top of OpenCV library and open-source project called "Relative Pose Estimation" created by TODO . In terms of sensor fusion, currently state of art approach can be found on Android platform. 3D reconstruction process will be processed in separate desktop application.

1.4 Thesis Outline

In **Chapter 2** fundamental technology behind 3D reconstruction and Structure from Motion was discussed. Also sensors used to perform sensor fusion on Android platform were briefly introduced.

In **Chapter 3** brief overview of related research was made. Most interesting discoveries, which lead to conception of proposed methods were also described.

In **Chapter 4** theoretical concept of proposed 8-point and 5-point rotation enhanced algorithms were presented. Also alternative approach to 3-point translation estimation

1.4 Thesis Outline

was made. Finally author proposed few most promising Structure from motion reconstruction strategies.

In **Chapter 5** most important implementation aspects of rotation and translation measurements were discussed. Also enhancements modifications to standard 8-point and 5-point algorithms were discussed. Also 3-point translation estimation approach and relative pose enhancements implementation was discussed.

In **Chapter 6** evaluation of proposed initial pair reconstruction methods and strategies was presented and discussed.

In **Chapter 7** conclusions observed during master thesis project period, as well as encountered problems were discussed. Thesis ends with further development plans establishment.

Chapter 2

Fundamentals

This chapter explains the basic theory behind 3D reconstruction and Structure from Motion. All information referred to herein can be found in (?). It also includes a brief overview of sensor fusion performed with the use of accelerometer, gyroscope and magnetometer data.

2.1 3-D reconstruction in general

There are many possibilities of performing reconstruction, starting from two-view reconstruction, multiple-view reconstruction and ending with the use of stereo calibrated cameras like the ones used in Kinect[reference]. Reconstruction can also be performed with a single hand-held camera either from a video or a sequence of images. As few as two pictures taken from different angles of a single object are sufficient to perform 3D model generation. Reconstruction process consists of the following steps:

1. **Image Acquisition**, where images are acquired
2. **Feature extraction and corresponding points matching**, where distinctive features are extracted from the images and compared
3. **Fundamental & Essential Matrices**, where matrices meeting the requirements of basic epipolar geometry are calculated
4. **Camera parameters estimation**, where external and internal camera parameters are estimated
5. **Triangulation**, where camera projection matrices are composed and used in order to calculate 3D cloud points

2.1.1 Feature extraction and corresponding points matching

Usually, each image used in reconstruction has to be analysed in order for the distinctive features to be found. Afterwards all features in images are compared in order to find corresponding matches (2.1). There are multiple features detectors and extractors available for use http://en.wikipedia.org/wiki/Feature_detection_%28computer_vision%29. Some of them are more suitable for edge detection, while others are best used for corner or blob detection. One of the most popular and robust feature detection method is scale-invariant feature transform (SIFT) http://en.wikipedia.org/wiki/Scale-invariant_feature_transform. The use of these descriptors allows for detecting local features in images and describing them with metrics including scale, rotation and translation invariant.

2.1.2 Fundamental & Essential Matrix estimations

Once proper matches are found, it can be proven that there exists Fundamental matrix F for which the following equation is satisfied:

$$x'^T * F * x = 0 \quad (2.1)$$

where x and x' are uncalibrated notions of points correspondence (?) TODO chapter. It is known that solutions of this equation are highly sensitive to the occurrence of outliers. Usually, to make fundamental matrix estimations more accurate some outlier removing algorithms need to be used. One of the most robust approaches includes the use of of RANdom SAmple Consensus (Bundle Adjustment (RANSAC)) <http://en.wikipedia.org/wiki/RANSAC>. The example of sample fitting can be seen in 2.2 Its basic idea relies on choosing a random subset from among all matches, solving a problem of reduced dataset and establishing how many points from the original set satisfy the equation. The use of F matrix allows for calculating epipolar lines for each point (2.3). These lines cross the exactly same points in both images and can be used for dense feature matching since matches need to be searched for exclusively in the surroundings of these lines. When enough inliers are found, points that do not satisfy the equation can be removed from further processing. Once internal camera parameters K are known, the image points found can be calibrated and expressed in

2. FUNDAMENTALS

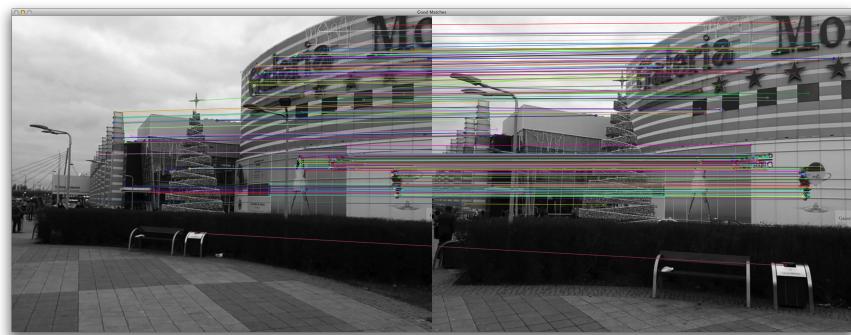


Figure 2.1

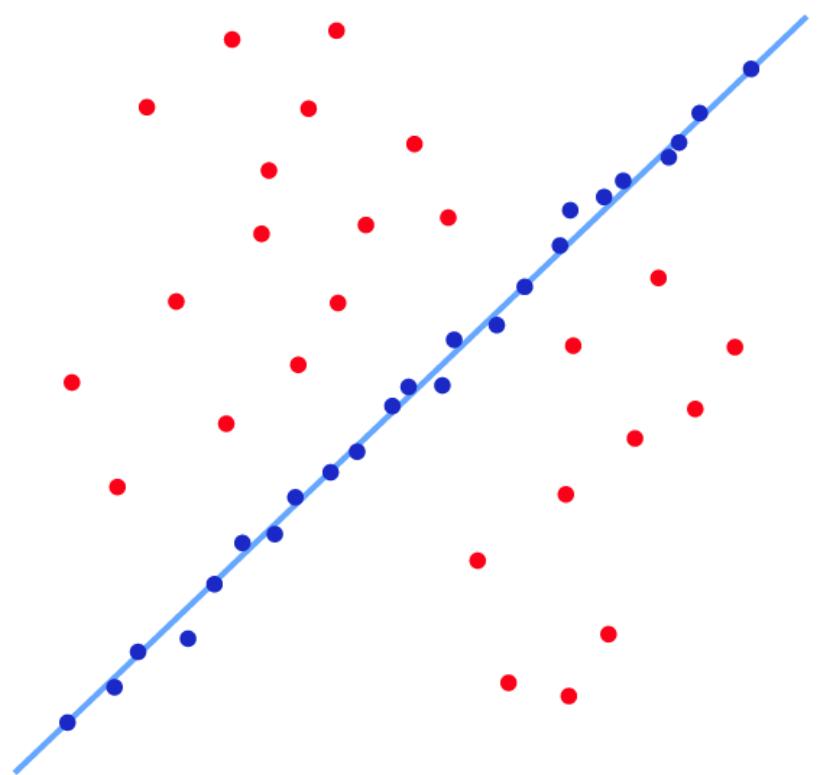


Figure 2.2: RANSAC fitting for 2D image TODO reference

2.1 3-D reconstruction in general

camera reference position system. Such calibrated points satisfy the following essential matrix E equation:

$$x_c'^T * E * x_c = 0 \quad (2.2)$$

which is very similar to a fundamental equation 2.1. This results in:

$$E = K^T * F * K \quad (2.3)$$

with K being the internal camera parameters. This equation 2.3 is important in terms of decomposition of F matrix to relative rotation and translation.

2.1.3 Camera parameters estimations

Internal camera parameters are expressed by the following matrix:

$$\begin{bmatrix} \alpha_x & x_0 \\ \alpha_y & y_0 \\ & 1 \end{bmatrix} \quad (2.4)$$

where $x = fmx$ and $y = fmy$ represent the focal length of the camera expressed in pixel dimensions in the x and y direction respectively. Similarly, $x_0 = (x_0, y_0)$ is the principal point of pixel dimensions. These parameters need to be calculated only once for each camera model. Cameras can be calibrated with special reference boards of the known dimensions and characteristics. For the proper 3D reconstruction it is essential to properly estimate external camera parameters, such as rotation (orientation angles of the camera) and global position of the camera. It is often the case in 3D reconstruction that these parameters are not known. However, it is shown in Chapter 9 of Multiple View Geometry in Computer Vision ((?)), how essential matrix can be decomposed using Singular Value Decomposition Singular Value Decomposition (SVD) to relative camera positioning system of two projections:

$$P1 = K * [I \mid 0] \quad (2.5)$$

, the second one is equal to

$$P2 = K * [RDiff \mid tDiff] \quad (2.6)$$

Unfortunately, there are four possible solutions for such a decomposition and it is not always possible to identify the correct one.

2. FUNDAMENTALS

2.1.4 Points Triangulation

Once internal and external (global or relative) camera parameters are calculated, the triangulation can be performed in order to acquire an up-to-affine reconstruction model (??). The only element that cannot be determined in such a relative case is the scale. This process is described in detail in [TODO hartley chapter 10].

2.2 Structure from Motion

The term "Structure from Motion (SfM)" refers to the reconstruction performed from the consecutive sequences of a moving camera. It is a popular research topic and the two main approaches, namely the Pose Estimation and Homography Estimation, can be used for the purposes of reconstructing a 3D model of an object.

2.2.1 3D Pose Estimation

Assuming that some of the 3D cloud points are already known, the matches between 2D features in a new image and 3D point cloud positions can be established. Such 3D-2D matches can be used to estimate the camera position. This allows for reconstructing new 3D points and merging them smoothly into a functional model. Unfortunately, this process is also highly sensitive to the occurrence of outliers, therefore adequate measures have to be undertaken to reduce their influence. One of the main advantages of this method is its speed. On the other hand, its effectiveness relies strongly on the existing 3D cloud quality.

2.2.2 Homography estimation

A new image can be reconstructed with a previous one in a standard way in order to receive an up-to-scale 3D model. Afterwards, a newly acquired model can be merged into an existing one using homography estimation between the corresponding 3D points. Such a strategy is slower than the previous one, but it is not influenced by the quality of the existing 3D model. Although it is highly sensitive to outliers as well, if used properly it can produce more new 3D points.

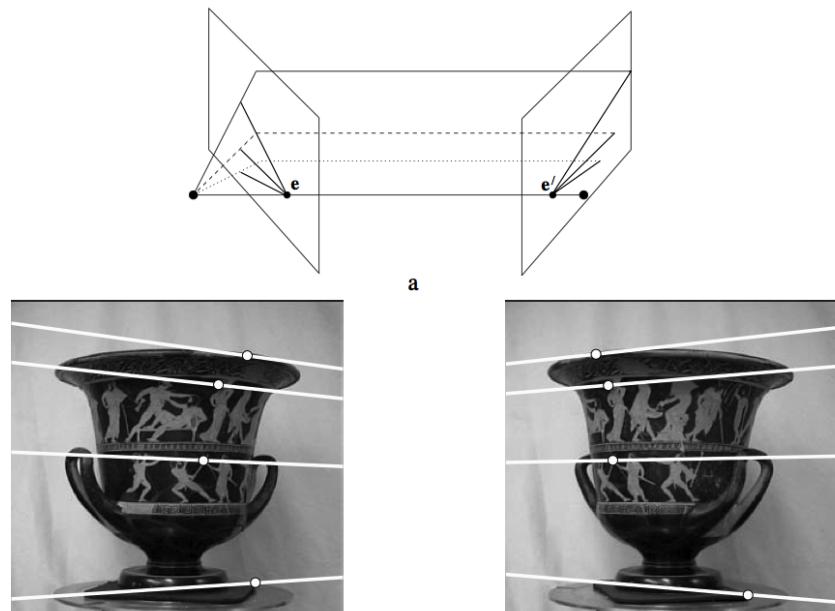


Figure 2.3: Epipolar lines found in an image of a vase TODO reference

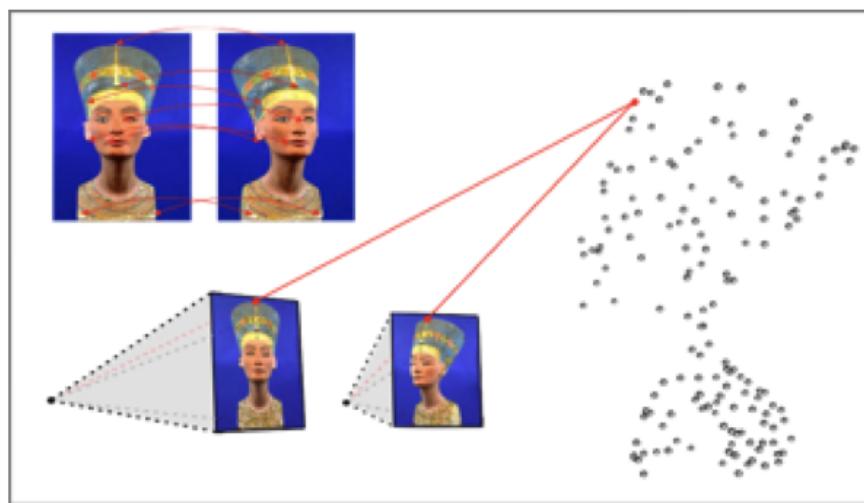


Figure 2.4

2. FUNDAMENTALS

2.2.3 Structure Adjustment

Special refinement methods can be used to compensate for an error of mismatched points propagating through images. They include, among others, Bundle Adjustment Bundle Adjustment (BA) http://en.wikipedia.org/wiki/Bundle_adjustment. The algorithm, using the information concerning the corresponding matches between multiple sets of images, iteratively modifies either both the camera external parameters and 3D points positions or one of them. The main disadvantage of the method is its execution time. It is too time-consuming to be used in real-time applications. The basic idea of BA is expressed in figure 2.5.

2.3 Mobile Sensors overview

There are many sensors available in the nowadays smartphones, such as accelerometer, gyroscope, magnetometer, barometer, GPS, etc. All of them have their advantages and disadvantages, and thus the errors of some might be compensated for with the strengths of the others in order to, for example, accurately compute camera rotation angles.

2.3.1 Accelerometer

An accelerometer is a device that measures acceleration along 3 axes of the device [reference]. Generally, an accelerometer allows for measuring total acceleration by sensing what force is applied to its micro strings. An accelerometer, which lies on a flat surface perpendicular to the Earth's surface will indicate approximately 1G upwards. This gravitation vector can be used to calculate the relative camera rotation, but it is difficult to define to which direction the gravity vector points when the device is moving in not a linear manner. The gravity vector is obtained from the accelerometer data and tracked during unexpected movements with the use of gyroscope sensor.

2.3.2 Gyroscope

A gyroscope is a device for measuring or maintaining orientation, based on the principles of conservation of angular momentum [reference]. A standard gyroscope consists of a spinning wheel mounted on two gimbal rings, which allows it to rotate in all three axes. The spinning wheel will resist changes in orientation, due to an effect of the

conservation of angular momentum. A conventional gyroscope measures orientation, in contrast to MEMS (Micro Electro-Mechanical System) types, which measure angular rate, and are therefore called rate-gyros [reference]. MEMS gyroscopes contain vibrating elements to measure the Coriolis effect. In the end the angular velocity can be calculated in each axis. It is important to note that whereas the accelerometer and the magnetometer measure acceleration and angle relative to the Earth, gyroscope measures angular velocity relative to the body.

2.3.3 Magnetometer

A magnetometer is an instrument used to measure the strength and/or direction of the magnetic field in the surrounding area of the instrument. Its main idea works in the same manner as the conventional compass. With some magentometers magnetic field can be measured in three directions, relative to the spatial orientation of the device [reference]. Using magnetometers allows for estimating relative position in geomagnetic north position system.

2.3.4 Sensor Fusion

Sensor fusion is the process of combining sensory data derived from disparate sources in a way that the resulting information is to some extent more valuable than it would be possible should these sources be used individually. The term "more valuable" in this case may mean "more accurate", "more complete" or "more dependable", or refer to the result of an emerging view, such as stereoscopic vision (calculation of depth information by combining two-dimensional images from two cameras at slightly different viewpoints) [reference]. One of the strategies available is Kalman filtering (2.6) This approach is used in sensor fusion available in Android API, where the gravity vector calculation is enhanced with the accelerometer and gyroscope data. The gravity vector can be decomposed in order to estimate the camera's rotation angles. Magnetometer can be used optionally in order to acquire rotation angles referenced to the Earth's coordinate system. However, magnetometer is sensitive to changes in the electromagnetic fields, therefore its measurement can be noisy in confined spaces filled with electronic devices. Subtracting gravity from the actual acceleration measurements allows for estimating linear acceleration. The use of linear acceleration makes it possible to measure the relative change of the device's translation. However, it requires a double integration

2. FUNDAMENTALS

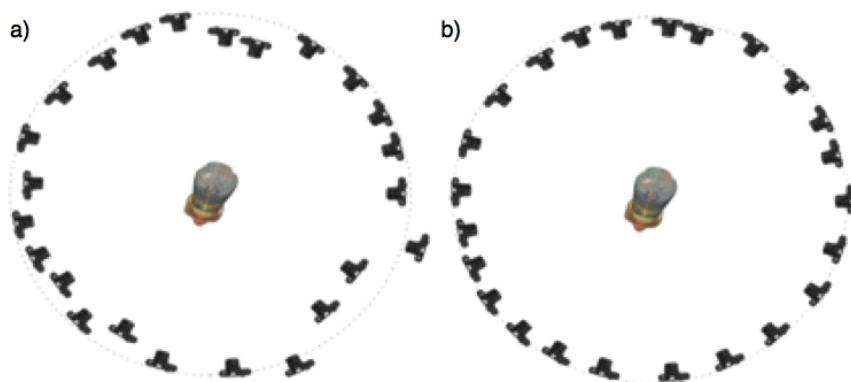


Figure 2.5: TODO reference

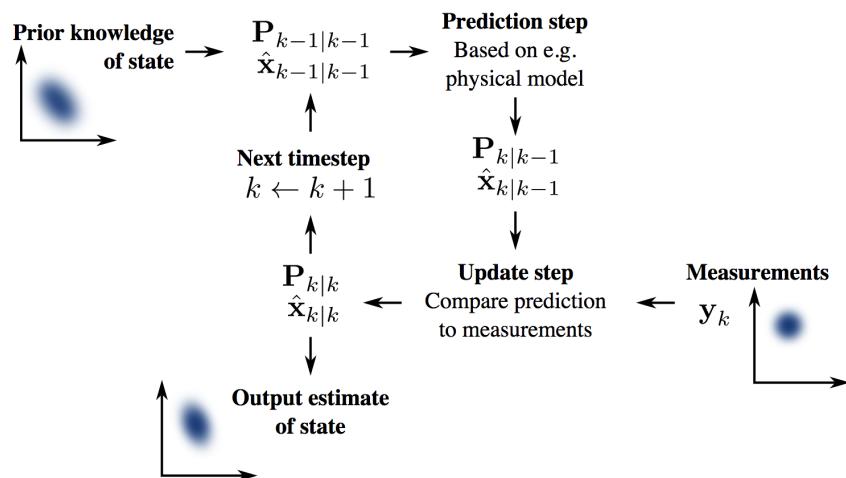


Figure 2.6

2.3 Mobile Sensors overview

over time, which results in increasing the translation's noise. It is important to note that calculating rotation is significantly more accurate than estimating the translation.

Chapter 3

Related Work

There are many approaches to the reconstruction problems, starting with the raw one-by-one pixel analysis for density matching, through high level abstraction of objects recognition and extraction and ending with the light and shadows-based reconstruction (?(?)?). However this thesis does not focus on high-level abstraction reconstruction, but on refining relative poses estimation steps. As described in the theoretical part, in order to estimate the first two relative camera positions, an essential matrix must be decomposed. Since the basic epipolar geometry equation discovery, many scientist have introduced various ways to solve this linear problems, wchich differed primarily in terms of the accuracy and speed. One of the first was the 8-point algorithm (?), which can be used to compute a fundamental matrix. This can be done without any prior knowledge of the scene or camera external and internal parameters. Fundamental matrix has to be converted to essential matrix in order for the relative camera positions to be found and that is why it is necessary to calculate internal camera parameters. Next, the 5-point algorithm approaches were introduced (?)(?). They used internal camera parameters for the proper essential matrix estimation. In his paper M. David Nister showed that the 5-point algorithm outperforms almost all similar algorithms in terms of accuracy and speed. Only the 8-point algorithm can be regarded as equally efficient. One of the state-of-art real-time and robust approaches is the iterative 5-point algorithm created by Vincent Lui and Tom Drummond(?).

Most of the algorithms are very sensitive to the occurence of outliers. One of the most common approaches to use to tackle this problem is RANSAC modelling(?).

A research group from Technische Universität Berlin made an interesting comparison and evaluation of the methods discovered by 2008 (?). It was discovered that estimation of camera rotation is much more stable than translation calculation. Furthermore, there are a lot of ambiguities regarding the choice of the correct solution of epipolar geometry equation.

In certain situations, where external camera parameters as rotation and translation can be measured, more accurate algorithms were proposed. In 2011 D. Scaramuzza from Zurich proposed a 1-point algorithm (?), which shows how to describe and use a model of a camera mounted on a car to enhance 3D reconstruction. The 4-point algorithm introduced in 2013 using information of rotation angle in certain axis from additional sensor, as shown in paper (?), can outperform even the 5-point algorithms. Lately the scientists have been creating more complex models to estimate relative stereometry, e.g. a team from Zurich proposed a way to enhance reconstruction with additional 6DOF sensor (?).

There are also non standard approaches like (?), where it is shown how to estimate the relative pose from three lines with two of the lines being parallel and orthogonal to the third one. Very accurate estimations can also be obtained for special camera model cases, when there is no rotation(?). All these references show that enhanced models often help to achieve more accurate or faster solutions to standard epipolar geometry problems.

Accuracy and speed constitute significant elements of the process of creating the systems capable of augmenting our reality. One of the first successful tracking and mapping reconstruction-based system was proposed by a research team from Oxford University (?). They showed how two simultaneously working threads can be used to both create a model of the environment being scanned and use this knowledge to apply graphical effects to objects presented on a video stream. Some of these concepts have already been applied to robotic vision. The authors showed efficiency of the proposed system for a robot walking in cluttered indoor workspaces(?). There were also approaches of porting reconstruction strategies to mobile devices(?)(?)(?)(?).

The major conclusions to be drawn include the fact that rotation estimation is more stable than translation estimation and that the reconstruction accuracy is improved by a better described model of camera movement. Some of the researches drafted many

3. RELATED WORK

scientific papers regarding rotation-improved solution space searching (?) (?). There are even some approaches involving the use of additional IMU accelerometer for the reconstruction process enhancement (?).

Chapter 4

Concept

As indicated in a similar research it is very attractive to use additional data to enhance reconstruction and reduce ambiguity in finding correct solution for 3D reconstruction. Additional camera or model information help to implement faster, more stable and robust algorithms. This thesis will show how the prior knowledge of rotation or translation acquired via mobile sensor fusion can be used to enhance process of 3D reconstruction from series of images. When it comes to relying on hand-held smartphones, collected sensor data are very noisy. This thesis shows how even noisy information can be used in reconstruction processes. Initially, only the camera rotation estimation was supposed to be used under the scope of this thesis. However, the first trials of reconstruction proved to be not sufficient enough and additional rotation error matrix estimations were developed. From the analysis of theory and related works it seems that both epipolar equations and pose estimation techniques can be improved by additional rotation and translation information data. Author of this thesis proposes the use of an environment, where a user can decide what type of strategy to use.

4.1 Requirements

The proposed methodology needs an input in the form of a series of images with additional information about the position of the camera - euclidian rotation and, optionally, translation. The use of smartphone is not necessary; any camera with sensor-fusioned accelerometer and gyroscope (magnetometer is optional and the use thereof, as discussed in [TODO], has both advantages and disadvantages), capable of performing the

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rotation and translation estimation, can be used. The internal camera parameters need to be calculated before the reconstruction process commences. Additional sensor data need not be fully accurate. Noisy external camera parameters can still be successfully used in order to enhance the reconstruction process.

4.2 Enhancing epipolar geometry equations with initial rotation matrix

The initial pair reconstruction step is of utmost importance and needs to be accurate in order to let the other images calculate relative position based on the initially reconstructed 3D cloud points. Analysing the standard fundamental geometry equation and the relative camera based system ($P = [I|0]$, $P' = [R|t]$) results in the following:

$$x_r^T * K^{-T} * [T]_x * R * K^{-1} * x = 0 \quad (4.1)$$

It can also be noted that:

$$[T]_x = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \text{ where } T = [t_x, t_y, t_z] \quad (4.2)$$

As already discussed, rotation can be distorted with noise. This can be written down as:

$$R = R_{error} * R_{init} \quad (4.3)$$

where R_{init} is initial rotation matrix constructed from measured angles and R_{error} is rotation error matrix. Looking at this from a different point of view 4.3 one can interpret it as the multiplication of the two rotation matrices: One estimated, but close to local optimum initial rotation matrix and the second one responsible for correction of noise error. Instead of on the relative rotation matrix calculation, the primary idea of the algorithm proposed in this thesis is based on the entire rotation matrix calculation, which can be acquired from the essential matrix SVD decomposition, where only the rotation R_{error} can be estimated. Eventually, 4.1 can be rewritten as:

$$x_r^T * K^{-T} * [T]_x * R_{error} * R_{init} * K^{-1} * x = 0 \quad (4.4)$$

Having:

$$\begin{aligned} h_r^T &= x_r^T * K^{-T} \\ h &= R_{init} * K^{-1} * x \\ G &= [T]_x * R_{error} \end{aligned} \quad (4.5)$$

4.2 Enhancing epipolar geometry equations with initial rotation matrix

With such notation one can notice that:

$$h'^T * G * h = 0 \quad (4.6)$$

which resembles the already known fundamental([??eq:fundamntalEquation](#)) and essential equations (2.2). Naturally, h' and h both are expressed in homogenous coordinates. From the analysis it follows that G has 6DOF: 3 due to an unknown translation and another 3 due to an unknown correction angles (created by rotation error matrix decomposition). Theoretically, such matrix can be resolved for instance by both 5 and 8-point algorithms. Therefore the standard fundamental and essential equation solvers can be used in order to retrieve both $[T]_x$ and R_{error} . The finally estimated R_{error} and calculated R_{init} has to be multiplied in order for the new rotation estimation of R (4.3) to be retrieved. Pursuant to Appendix 6 "Multiple View Geometry in Computer Vision"(A6.9.1 ([?](#))), the use of Rodrigues parametrization for small angles (and noise in initial rotation matrices estimations can be expresed by small angles) the rotation matrix, and thus R_{error} , approximately equals to:

$$R_{error} \cong \begin{bmatrix} 1 & -w_z & w_y \\ w_z & 1 & -w_x \\ -w_y & w_x & 1 \end{bmatrix} \quad (4.7)$$

Such a criterion with a special matrix design can be used when decomposing G to resolve some ambiguity in choosing the proper solution. The standard four solution ambiguity with two possible rotations and translations can be reduced to two possible translation calculations. The concept described constitutes a basis ofthe implemented, enhanced 8-point and 5-point algorithms.

4.2.1 Alternative 3-point algorithm for finading a translation

Following the 4.6 it can be stated that:

$$x'^T * K^{-T} * \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} * R * K^{-1} * x = 0 \quad (4.8)$$

Having:

$$\begin{aligned} h'^T &= x'^T * K^{-T} \\ h &= K^{-1} * x \end{aligned} \quad (4.9)$$

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and

$$\begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} * \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} * \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = 0 \quad (4.10)$$

and multiplying it one receives what follows:

$$h_1 * h_2 * t_z - h_1 * h_3 * t_y - h_2 * h_1 * t_z + h_2 * h_3 * t_x + h_3 * h_1 * t_y - h_3 * h_2 * t_x = 0 \quad (4.11)$$

what can be grouped:

$$t_x * (h_2 * h_3 - h_3 * h_2) + t_y * (h_3 * h_1 - h_1 * h_3) + t_z * (h_1 * h_2 - h_2 * h_1) = 0 \quad (4.12)$$

rewritten as:

$$\begin{bmatrix} t_x & t_y & t_z \end{bmatrix} * \begin{bmatrix} (h_2 * h_3 - h_3 * h_2) \\ (h_3 * h_1 - h_1 * h_3) \\ (h_1 * h_2 - h_2 * h_1) \end{bmatrix} = 0 \quad (4.13)$$

and solved for instance with SVD with only three points. This is a truly fast way of estimating translation for the reconstructed images. However, in such situation the overall accuracy strictly depends on the precise measurements of camera orientation.

4.3 Pose estimation

One of the main goal was to develop a more accurate and faster version of the reconstruction algorithm. That is why the relative pose estimation techniques were used to enrich models with additional points. In general, this approach produces less outliers than the homography merging techniques. This also allows to maintain the scale of the reconstructed images. For any point within the image, which has a corresponding 3D point the following condition is met:

$$x = P * X \quad (4.14)$$

where x is image point expressed in homogenous coordinates (x , y , 1) and X homogenous 3D point (X , Y , Z , 1). Projection matrix of the camera design is as follows:

$$P = K * [R | t] \quad (4.15)$$

4.3.1 Rotation enhancements

Applying the similar approach to initial pair reconstruction enhancements, it can be noted that:

$$\begin{aligned} x &= K * [R_{init} + dR | t] * X \\ x &= K * [R_{init} | 0] * X + K * [dR | t] * X \\ x - K * [R_{init} | 0] &= K * [dR | t] * X \end{aligned} \quad (4.16)$$

Substituting $x_m = x - K * [R_{init} | 0]$ the following is received:

$$x_m = K * [dR | t] * X \quad (4.17)$$

This can be solved using a standard PNP calculating algorithms. Rotation can be used as an initial solution for the pose estimation in order to focus solely on the rotation error and translation estimation.

4.3.2 Rotation & translation enhancements

Similar to 4.16:

$$\begin{aligned} x &= K * [R_{init} + dR | T_{init} + dt] * X \\ x &= K * [R_{init} | T_{init}] * X + K * [dR | dt] * X \\ x - K * [R_{init} | T_{init}] &= K * [dR | dt] * X \end{aligned} \quad (4.18)$$

end in the end by substituting $x_n = x - K * [R_{init} | T_{init}]$ the following is received:

$$x_n = K * [dR | dt] * X \quad (4.19)$$

This can also be solved using a standard PNP[TODO reference] calculation algorithm. Rotation and translation can be used as an initial solution for the pose estimation in order to focus solely on the rotation error and translation error estimation.

4.4 Known rotations & translations

Where accurate rotations and translations of cameras are known, no aditional pose calculations are needed. Such a situation is interesting, since everything needed for corresponding points triangulation is already known. However, mobile sensor data measurements are very noisy. In that case Bundle Adjustment can be used in order to refine the measured calculated camera positions and the final 3D cloud reconstruction.

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4.5 Reconstruction process strategy

Finally, all methods described herein can be combined in different reconstruction strategies. For the initialization of 3D cloud point the following strategies can be used:

1. **Known rotations and translations**, which theoretically allows for an up-to-metrical reconstruction
2. **Known rotations**, where translation is estimated with an alternative 3-point algorithm
3. **Noisy rotations**, where enhanced 8-point fundamental or 5-point essential algorithms are used in order to calculate rotation error and relative translation
4. **Unknown external camera parameters**, where standard 8-point fundamental or 5-point essential algorithms are used in order to calculate rotation and relative translation

For the pose estimation the following methodologies can be used:

1. **Known rotation and translations**, where no additional calculations are required and an up-to-metrical model can be acquired
2. **Initial rotation**, where relative translation needs to be calculated
3. **Noisy rotation and translation**, where both rotation and translation errors need to be calculated
4. **Unknown external camera parameters**, where standard pose estimation has to be used

Chapter 5

Implementation

This chapter describes the chosen implementation environment. It also describes an Android application which was created in order to acquire image sequences with additional sensor data. The structure of both Android and Desktop projects is explained and essential implementation details of the proposed algorithms and strategies are discussed herein.

5.1 Choosing Environment

Currently, Android has one of the best APIs which allows programmers to acquire sensor fused data of rotation and linear acceleration. In order to avoid coding anew all epipolar geometry algorithms, C++ version of OpenCV library was used. Image acquiring process was separated from model reconstruction. This allowed to save a lot of time in debugging which is highly difficult in native C++ development on Android. In order to enrich images with sensor data an Android app was created and a standalone C++ desktop app to perform processing and evaluation.

5.2 Project Structure

Source codes of both applications can be found on an attached CD and in the Github repository at the following web address: TODO <https://github.com/KrzysztofWrobel/MasterThesisSource.git>. In general, the project was split into two subprojects: Android and Desktop. In addition, some of the sample datasets were added to the dataset folder in order to perform evaluations similar to the ones described in the next chapter.

5. IMPLEMENTATION

5.3 ”Sensor Enhanced Images Camera” - Android Gradle based project

In order to capture images and associate them with sensor data, a custom photo capture app called ”Sensor Enhanced Images Camera” was created. Using this application, the camera orientation angles can be tracked in real-time in reference to the Earth’s coordinate system. Also linear acceleration, current velocity and relative translation from the location of the last picture taken can be tracked. Upon the photo capture the current camera rotation and relative translation are stored in a custom JSON file. A corresponding image is saved along with this JSON file in folder, which is created upon every app start-up. It allows for taking different dataset captures without them being mixed up.

5.3.1 Installation

In order to compile and distribute Android application, a Gradle based built system was used. This is currently a recommended way to maintain Android based projects [TODO ref. google]. At present this app supports only devices with Android 4.0 and above. In order to compile this project it is recommended to install Android Studio, which already contains the needed SDK and also has a built-in Gradle support. More information about configuration, compilation and installation steps can be found in README.md in the main catalog of the attached source code.

5.3.2 User Interface

As mentioned earlier the app allows to track all necessary external camera parameters in real-time. This is how the first version of user interface looked:

5.3 "Sensor Enhanced Images Camera" - Android Gradle based project

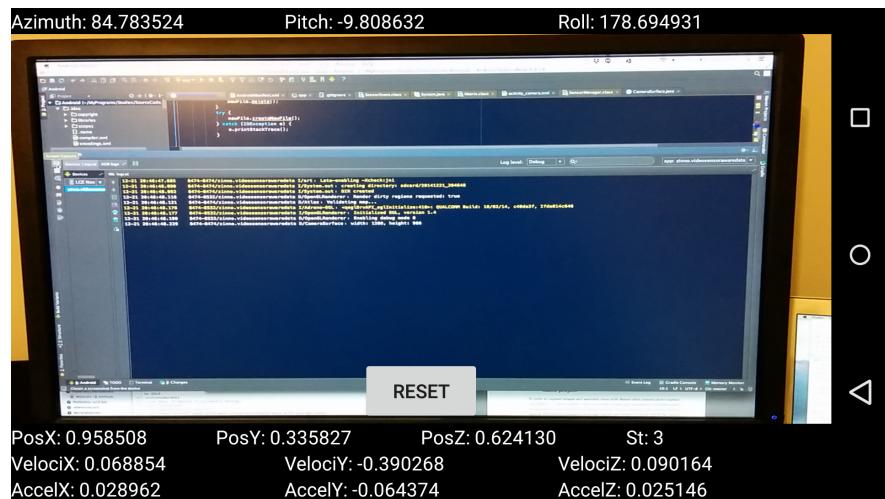


Figure 5.1

5. IMPLEMENTATION

In order to capture a photo, a user only has to click on the screen center. No additional configuration is required. To use the so acquired datasets in another program, the smartphone has to be connected to the computer. Folders are created with time suffixes in the main catalog of an internal SD card.

5.3.3 Important Implementation Aspects

There were two important aspects of an Android application implementation:

1. Camera rotation calculation
2. Heuristic movement estimation

5.3.3.1 Rotation calculation

Android SDK already has a built-in API, which can be used to access sensor fused data. In particular, for measuring the rotation, the *Sensor.TYPE_ROTATION_VECTOR* was used. It returns 9-degree quaternion in reference to geomagnetic north pole. In order to decompose it to euler angles a helper method was used:

```
private float[] euclidianAnglesFromQuaternion(float[] quaternion) {  
    ...  
    //Converts quaternion to 4x4 rotation matrix  
    SensorManager.getRotationMatrixFromVector(rotMat, quaternion);  
    ...  
    //Extracts euclidian Angles in radians  
    SensorManager.getOrientation(rotMat, orientation);  
    ...  
    return orientation;  
}
```

5.3.3.2 Custom heuristics for move estimation

Using the currently available Android sensor data it is difficult to estimate the relative translation of the device. Linear acceleration measurements, which can be accessed with *Sensor.TYPE_LINEAR_ACCELERATION* constant, are very noisy. Such noisy data with double integration over time results in quite big errors after some time. First integration is required to acquire current velocity change. Second integration over velocity is required to calculate position change in particular moment. However, it

5.3 "Sensor Enhanced Images Camera" - Android Gradle based project

can be improved using human-walking model description with the following heuristical constraints over calculated velocity:

1. When new linear acceleration sensor data is ready, first apply lowPass filtering in order to reduce some high frequency noise.
2. Change sensor datda vector from local camera coordinates to global reference system (multiply with inverted rotation matrix)
3. Decide depending on current state, if deviceMovmentState has changed:
 - (a) If device was previously IDLE and incoming Acceleration value is bigger than $0.5 \frac{m}{s^2}$ change device state to MOVING and reset current velocity to $0 \frac{m}{s}$
 - (b) If device was previously MOVING and incoming Acceleration value is smaller than $0.1 \frac{m}{s^2}$ change device state to IDLE
4. Only if device is currently moving:
 - (a) Update device velocity
 - (b) Check current speed with walk constraint (People walk with avarage speed of $1.5 \frac{m}{s}$) and eventually scale it down to maximum value
 - (c) Update device postion

A code snippet of the described movement estimation heuristics was derived from Android application and can be found in the following listing 5.1. Maximum walking speed can be adjusted, but by default it should be set to around $1.5 \frac{m}{s}$ TODO http://en.wikipedia.org/wiki/Preferred_walking_speed. The most important factor of the accurate reconstruction are correleations between movements in X,Y and Z axes. Noise in sensors is equally distributed on each axis, so it estimated translation should keep these movements' correlations properly.

5.3.3.3 Custom Sensor Data File format

As mentiond earlier each photo sensor information is stored in a separe file. By default the following information is stored for each captured image:

- **ID**, which indicates order of the photos
- **Path**, which relates the path to a corresponding image file

5. IMPLEMENTATION

```
1  public void onSensorChanged(SensorEvent event) {
2      ...
3      } else if (event.sensor == mLinearAcceleration) {
4          ...
5              //Filtering out some noise
6              newGlobalAcceleration = lowPass(newGlobalAcceleration,
7                  currentGlobalAcceleration);
8              //Switch linear acceleration from phone local coordinates to
9                  global coordinates
10             Matrix.multiplyMV(newGlobalAcceleration, 0,
11                 invertedRotationMatrix.clone(), 0, newGlobalAcceleration,
12                 0);
13
14             double distance = getLength(currentGlobalAcceleration);
15             long currentTimeMillis = System.currentTimeMillis();
16             //Decide state of the device. Distance is in m/s^2
17             if (distance > 0.5 && deviceState == State.IDLE) {
18                 if (currentTimeMillis - movingEndTime > 300) {
19                     deviceState = State.MOVING;
20                     currentGlobalVelocity = {0,0,0};
21                 }
22             }
23
24             if (deviceState == State.MOVING) {
25                 //Update current device velocity
26                 currentGlobalVelocity += currentGlobalAcceleration * dT;
27
28                 double velocity = getLength(currentGlobalVelocity);
29                 //Check and adjust current velocity. People walk with
30                     avarage speed 1.5\frac{m}{s}
31                 if (velocity > WALKING_MAX_VELOCITY) {
32                     currentGlobalVelocity /= velocity /
33                         WALKING_MAX_VELOCITY;
34                 }
35
36                 //Update device relative position, s = v0 * t + a * t
37                     ^2/2;
38                 currentRelativePosition += currentGlobalVelocity * dT +
39                     currentGlobalAcceleration * dT * dT / 2;
40             }
41             ...
42 }
```

Listing 5.1: Snippet from Android source code position estimation heuristic

5.3 "Sensor Enhanced Images Camera" - Android Gradle based project

- Euler Angles - pitch, roll, azimuth

- Relative position coordinate

All these pieces of information are stored as a list and when the user has completed

taking pictures, entire information set is saved into JSON file named sensor.txt. A

sample file can be found in the Additional Materials[TODO].

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5.4 "Enhanced 3D Reconstructor" - OSX CMake-based project

In order to evaluate algorithms there was a need to prepare a comfortable project environment which would allow for numerical and visual comparison of multiple datasets. CMake was chosen in order to simplify building and compilation process. The entire code architecture was developed and tested on Apple MacbookAir with OSX 10.9 Mavericks. In order to compile this project the following elements need to be installed first:

- CMake 2.8
- OpenCV 2.4.10
- Point Cloud Library (PCL) 1.7
- Boost 1.55
- cvsba (<http://www.uco.es/investiga/grupos/ava/node/39>) [TODO reference move]

5.4.1 User Interface

There are two targets defined in CMake file:

1. **Test efficiency**, which was used to evaluate pair reconstruction methods, draw epipolar lines in images and calculate samson error distances
2. **Test reconstruction**, which was used to evaluate proposed initialization pair reconstruction methods with different pose estimation methods

5.4.1.1 Test Efficiency

There is no graphical interface for reconstruction parameters configuration. All parameters needed are configured from the command-line interface. The following parameters have to be configured for this purpose:

1. **Enhanced Photo Data folder path**
2. **Initial SIFT features set size**

The calculated execution times and execution time are printed to the console output.

5.4 "Enhanced 3D Reconstructor" - OSX CMake-based project

5.4.1.2 Test reconstruction

There is no graphical interface for reconstruction parameters configuration either. All necessary parameters have to be configured from the command-line interface. The following variables need to be configured:

- 1. Enhanced Photo Data folder path**
- 2. Initial SIFT features set size**
- 3. Init pair reconstruct method**
 - Standard 8-point algorithm
 - Enhanced 8-point algorithm
 - Standard 5-point algorithm
 - Enhanced 5-point algorithm
 - Alternative 3-point translation estimation
 - None (use existing rotations and translations informations)
- 4. Pose estimation method:**
 - Standard OpenCV Pose Estimation
 - Rotation enhanced Pose Estimation
 - Rotation and translation enhanced Pose Estimation
 - None (use existing rotations and translations)
- 5. Whether drop outliers or not**
- 6. Whether use Bundla Adjustment or not**

The output received by the user is a reconstructed model in a file with *.asc extension. The reconstructed model is also visible in PCL Visualizer integrated into the code. The user can navigate through the model with mouse and 'F' key, which centers the camera view on a selected point.

5.4.2 Important Implementation Aspects

Most of the necessary algorithms were already implemented in OpenCV. In addition an opensource project with 5-point algorithm implementation was used. Its source code can be found here: [TODO reference].

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5.4.2.1 Rotation matrix generation

To parse Android generated sensor data file Boost JsonParser was used. As mentioned earlier, Android saves decomposed Euler angles. In order to properly multiply these angles to acquire a proper rotation matrix the following code inspired on MathWorld Wolfram [TODO <http://mathworld.wolfram.com/EulerAngles.html>]

```
Mat getRotation3DMatrix(double pitch, double azimuth, double roll) {
    Mat D = (Mat_<T>(3, 3) <<
        cos(roll), -sin(roll), 0,
        sin(roll), cos(roll), 0,
        0, 0, 1);

    Mat C = (Mat_<T>(3, 3) <<
        cos(azimuth), 0, -sin(azimuth),
        0, 1, 0,
        sin(azimuth), 0, cos(azimuth));
    Mat B = (Mat_<T>(3, 3) <<
        1, 0, 0,
        0, cos(pitch), -sin(pitch),
        0, sin(pitch), cos(pitch));
    //Important
    return B * C * D;
}
```

5.4.2.2 Enhancing epipolar equations

As could have been observed in ??, in order to enhance initial pair reconstruction algorithms same as the standard ones can be used, but with specially conditioned point matching. The standard 8-point algorithm is already available in OpenCV library. The implemented enhanced 8-point version can be found in 'Multiview.cpp' file, but its main difference is that it uses sets of specially modified input points, which are conditioned and the first one is additionally rotated with Initial Rotation Matrix. In OpenCV it can be done with:

```
...
undistortPoints(points1Exp, points1Exp, K, distCoeffs, rotDiffGlobal
);
undistortPoints(points2Exp, points2Exp, K, distCoeffs);
...
```

5.4 "Enhanced 3D Reconstructor" - OSX CMake-based project

where rotDiffGlobal is the relative rotation matrix between two cameras. The estimated fundamental matrix needs to be transformed to essential matrix for further decomposition. In order to choose proper matrix decomposition the following code is used:

```
void chooseProperMatrixFromEnhanced(Mat &dRx, Mat &dR1x, Mat &TdRExp,
    Mat &dR, Mat &T) {
    dR = dRx;
    if (decideProperMatrix(dRx, 0.05)) {
        dR = constraintMatrix(dRx);
    } else if (decideProperMatrix(dR1x, 0.05)) {
        dR = constraintMatrix(dR1x);
    } else if (decideProperMatrix(-dRx, 0.05)) {
        dR = constraintMatrix(-dRx);
    } else if (decideProperMatrix(-dR1x, 0.05)) {
        dR = constraintMatrix(-dR1x);
    }

    Mat skewT = TdRExp * dR.inv();
    cout << "skewT" << skewT << endl;

    Mat tdecx = Mat(3,1, CV_64FC1);
    tdecx.at<double>(0) = (skewT.at<double>(2,1) - skewT.at<double>(1,2))
        /2;
    tdecx.at<double>(1) = (skewT.at<double>(0,2) - skewT.at<double>(2,0))
        /2;
    tdecx.at<double>(2) = (skewT.at<double>(1,0) - skewT.at<double>(0,1))
        /2;
    T = tdecx;
}

bool decideProperMatrix(Mat dRot, double tolerance){
    double a00 = abs(dRot.at<double>(0,0) - 1);
    double a11 = abs(dRot.at<double>(1,1) - 1);
    double a22 = abs(dRot.at<double>(2,2) - 1);
    if((a00 + a11 + a22)/3 < tolerance) {
        return true;
    } else {
        return false;
    }
}
```

5. IMPLEMENTATION

These lines help to decide on a properly constrained rotation error matrix 4.7 and calculate relative translation between cameras.

5.4.2.3 Alternative 3-point translation estimation

When it is known or assumed that the acquired rotation data is accurate, the translation can be calculated with the 3-point algorithm. The ones proposed in 4.11 and 4.13 were implemented. Similarly to other epipolar estimations methods implemented in OpenCV, this one also has the ability to properly filter outliers with the use of RANSAC algorithm is used. The following listing contains this implementation:

```
...
for (iterationNumber = 0; iterationNumber < niters; iterationNumber
++) {

    getSubsety(prev_points_raw, next_points_raw, point1s, point2s,
    300, modelPoints);
    Mat t = findTranslation(point1s, point2s, rotDiffGlobal, Kinv);
    Mat F1 = constructFundamentalMatrix(rotDiffGlobal, t, Kinv);

    int goodCount = findInliersy(prev_points_raw, next_points_raw,
        F1, errors, statuses, reprojThreshold);
    if (goodCount > maxGoodCount) {
        swap(statuses, goodStatuses);
        FEnhanced = F1;
        tEnhanced = t / t.at<double>(2);
        maxGoodCount = goodCount;
        niters = cvRANSACUpdateNumIters(confidence,
            (double) (count - maxGoodCount) / count, modelPoints
            , niters);
    }

}
...

Mat findTranslation(std::vector<cv::Point2d> &points1, std::vector<
cv::Point2d> &points2, Mat &rotDiff, Mat &Kinv) {

    Mat hg1 = Mat::zeros(points1.size(), 3, CV_64FC1);
    Mat hg2 = Mat::zeros(points2.size(), 3, CV_64FC1);
    for (int i = 0; i < points1.size(); i++) {
        hg1.at<double>(i, 0) = points1[i].x;
```

5.4 "Enhanced 3D Reconstructor" - OSX CMake-based project

```
hg1.at<double>(i, 1) = points1[i].y;
hg1.at<double>(i, 2) = 1;
hg2.at<double>(i, 0) = points2[i].x;
hg2.at<double>(i, 1) = points2[i].y;
hg2.at<double>(i, 2) = 1;
}

hg1 = hg1 * (rotDiff * Kinv).t();
hg2 = hg2 * (Kinv).t();

Mat A = Mat::zeros(hg1.rows, 3, CV_64FC1);
for (int i = 0; i < hg1.rows; i++) {
    A.at<double>(i, 0) = (hg2.at<double>(i, 2) * hg1.at<double>(i,
        1) - hg2.at<double>(i, 1) * hg1.at<double>(i, 2));
    A.at<double>(i, 1) = (hg2.at<double>(i, 0) * hg1.at<double>(i,
        2) - hg2.at<double>(i, 2) * hg1.at<double>(i, 0));
    A.at<double>(i, 2) = (hg2.at<double>(i, 1) * hg1.at<double>(i,
        0) - hg2.at<double>(i, 0) * hg1.at<double>(i, 1));
}

SVD svd1(A);
Mat tCalc = svd1.vt.row(2);

//Translation between cameras estimated and Fundamental Matrix from
//that as well
Mat T = (tCalc.t());

return T;
```

5.4.2.4 Enhancing pose estimation

OpenCV already has the rotation and translation enhanced pose estimation implemented. The only thing that has to be done is performing the conversion of euclidian rotation matrix to its Rodrigues representation. The initial rotation and translation need to be passed as input variables. Pose estimation methods implementations are as follows:

```
Mutiview::FindPoseEstimation(
    cv::Mat &rvec,
    cv::Mat &t,
    cv::Mat &R,
    cv::Mat &K,
    cv::Mat &distCoeffs,
```

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```
    std::vector<cv::Point3d> ppcloud,
    std::vector<cv::Point2d> imgPoints,
    vector<int> inliers)
Mutiview::FindPoseEstimationEnhanced(
    cv::Mat &rvec,
    cv::Mat &t,
    cv::Mat &R,
    cv::Mat &RInit,
    cv::Mat &TInit,
    cv::Mat &K,
    cv::Mat &distCoeffs,
    std::vector<cv::Point3d> ppcloud,
    std::vector<cv::Point2d> imgPoints,
    vector<int> inliers)
```

can be found in 'Mutiview.cpp' file.

Chapter 6

Evaluation

All implemented algorithms were tested in terms of speed, accuracy and effectiveness. As regards the speed test, it included the measurement of the reconstruction execution time. In the accuracy testing Sampson Error measurement was used (this variable measures the distance between all points and their corresponding epipolar lines in an image). Effectiveness was tested with the use of the visual comparison of reconstructed 3D cloud points.

6.1 Acquiring datasets

For the purpose of analysis the following datasets were captured with the implemented "Sensor Enhanced Images Camera" application and Nexus 5 camera:

1. Warsaw University of technology main building(4 images 1024x768pixels)
2. Advertisement Pole ???
3. Warsaw Business School Gate and Entrance ??
4. Warsaw Shopping Center Back???
5. Warsaw Shopping Center Front???

Since most of the algorithms proposed in the (TODOreference to Concept) section require internal camera parameters to be known, the camera used in the course of conducting this study was calibrated and its parameters were stored in "*out_camera_data.yml*" file. All of these datasets can be found on attached CD or in Github repository.

6. EVALUATION

6.2 Test Environment

All tests were performed on MacBook Air with 1.7GHz dual-core Intel Core i7 processor and 8GB 1600MHz DDR3 RAM using "Enhanced 3D Reconstructor" implemented as described in the Implementation chapter. Numerical tests which allowed for measuring total errors and execution time were run on the "Warsaw University of Technology" dataset. Initial pair reconstruction ability of each method proposed was measured as well as various reconstruction strategies. Finally, for each dataset the most effective methods were used to reconstruct sparse models.

6.3 Testing initial pair reconstruction methods

The key question to be answered at this point was whether the proposed sensor enhancement gave better results than the standard algorithms. The following methods were tested: TODO methods should be already described in Concept and implementation

1. **Standard 8-point** - based on [] the fundamental matrix decomposition, implemented in OpenCV
2. **Enhanced 8-point** - the proposed camera rotation enhanced version of the above 8-point algorithm
3. **Alternative 3-point** - the proposed 3-point algorithm for translation estimation.
4. **Known rotations and translations** - calculation from known cameras rotations and translations
5. **Standard essential 5-point** - based on [] the essential matrix decomposition, implemented in []
6. **Enhanced essential 5-point** - similar to 2. rotation-enhanced version of the above 5-point algorithm

6.3.1 Accuracy - Epipolar lines correspondence

In the case of initial pair images one of the most important factors include the epipolar constraint. With the use of a properly estimated fundamental matrix, drawing corresponding epipolar lines in both images is possible. Furthermore, points lying on two matching epipolar lines in different images can be easily matched. In other words, epipolar lines cross exactly the same points in both images. The more accurate they

6.3 Testing initial pair reconstruction methods

are the more corresponding pairs can be found, e.g. for the purposes of performing dense reconstruction. Sampson Error is one of the metrics that can be used in order to estimate the accuracy of epipolars lines; the smaller the error's value the more accurate the lines. Its primary function is to measure the total distance between all points and their corresponding epipolar lines. However, each of the proposed methods has different outliers removal capabilities, therefore in order to compare their efficiency,

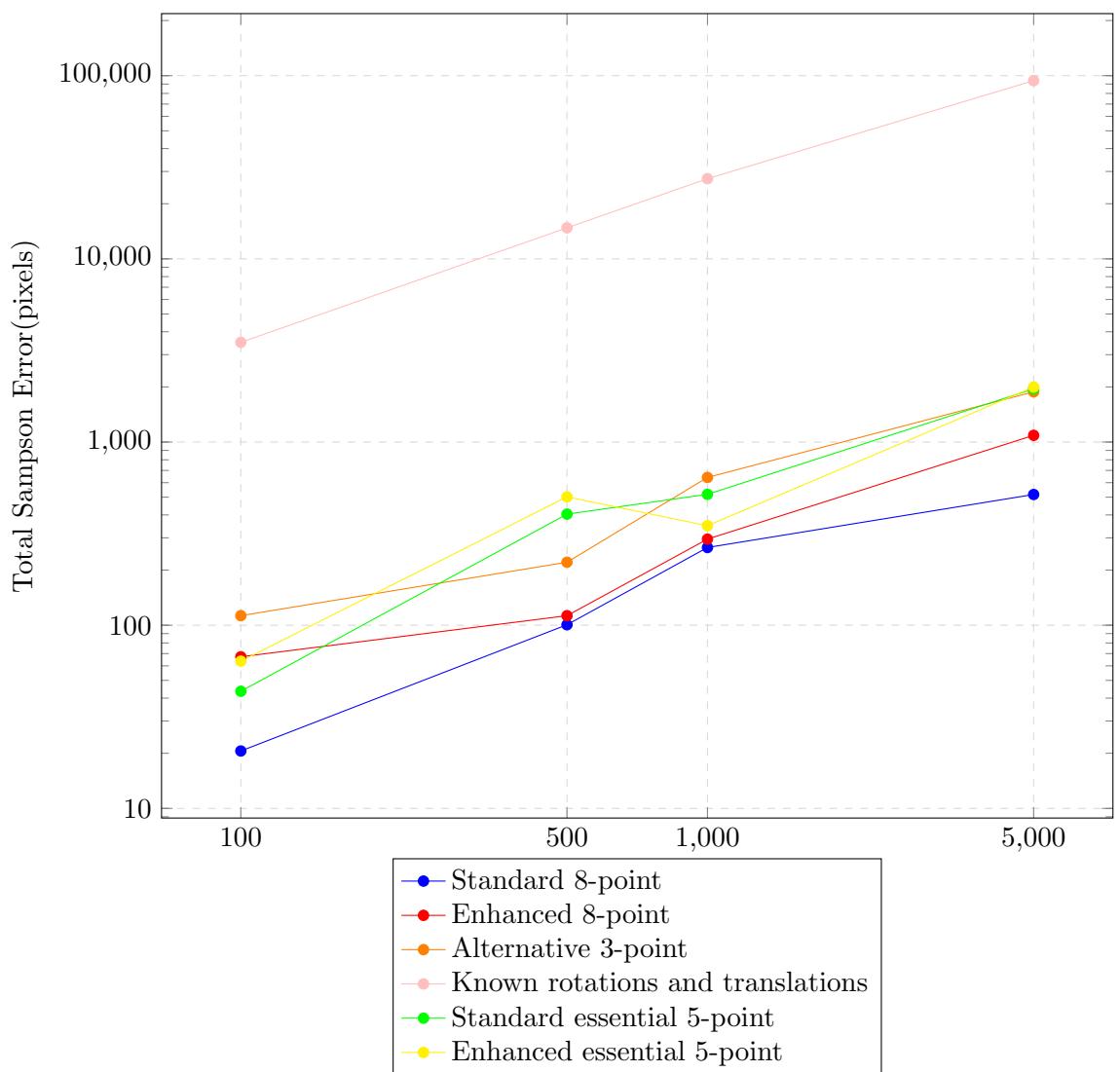


Figure 6.1: Chart showing the sum of Sampson Errors in a picture(1024x768pixels) per various initial Sift features sets sizes

6. EVALUATION

the average of Samson Error per a corresponding pair was calculated. The following chart 6.1 shows the total Sampson Errors for different sets of initial SIFT features. The major observation made based on these results is that most of the algorithms remove outliers properly. As could have been expected the only one that is not efficient in this regard is the method involving drawing epipolar lines from heuristically estimated movement and noisy rotation, which produces significantly larger error than the other algorithms. Analysing the per pair Sampson Error results 6.2 it can be noted that the proposed sensor enhanced methods did not improve either the 8-point or 5-point algorithms, but the 3-point algorithm developed for the purposes of this thesis proved to be much faster than the 8-point algorithm and also quite accurate. To present the discussed errors, estimated epipolar lines for 300 initial SIFT features set were drawn on the figures 6.3 - 6.4. It can be seen that both the standard 8-point algorithm and the proposed rotation-enhanced version return very good results in terms of epipolar lines accuracy. The alternative 3-point algorithm gives slightly worse results due to the uncompensated mobile sensors' noise. In this particular dataset the essential 5-point method was inefficient in producing proper essential matrix estimation, contrary to the proposed sensor enhanced 5-point algorithm, which found satisfactory correspondency in epipolar lines.

To verify whether the epipolar lines calculation is influenced by the number of SIFT features, the same process was applied to 1000 SIFT features set(6.5 - 6.6). In general, the proposed enhanced algorithms are not better than standard versions in terms of accuracy of epipolar lines estimation. It is mostly because during calculation some of the noise in sensor data is propagated on epipolar geometry estimation. However, the enhanced versions are always close to optimal solutions, while the standard versions can often fail in terms of fundamental matrix estimation.

6.3 Testing initial pair reconstruction methods

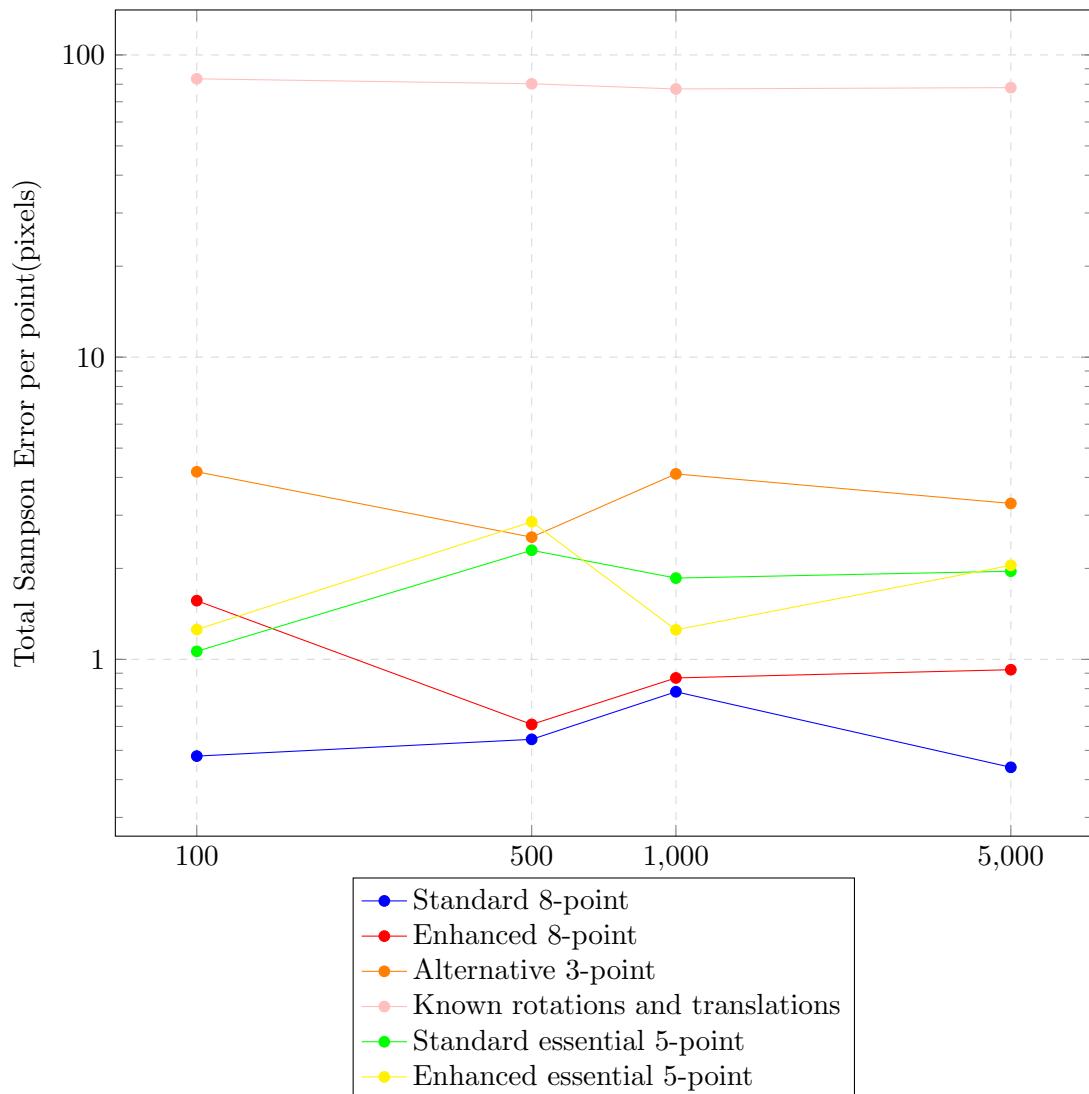


Figure 6.2: Chart showing per point Sampson Error in a picture(1024x768pixels) per various initial Sift features sets sizes

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Figure 6.3: The results of drawing estimated epipolar lines for the Warsaw University Dataset with 300 Sift points. 1) Standard Fundamental 8-point algorithm (upper pair), 2) Rotation Enhanced Fundamental 8-point algorithm (middle pair), 3) Alternative 3-point algorithm (bottom pair)

6.3 Testing initial pair reconstruction methods



Figure 6.4: The results of drawing estimated epipolar lines for the Warsaw University Dataset with 300 Sift points. 1) Fundamental matrix created from rotation and translation (upper pair), 2) Standard Essential matrix 5-point algorithm (middle pair), 3) Rotation Enhanced Essential 5-point algorithm (bottom pair)

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Figure 6.5: The results of drawing estimated epipolar lines for the Warsaw University Dataset with 1000 Sift points. 1) Standard Fundamental 8-point algorithm (upper pair), 2) Rotation Enhanced Fundamental 8-point algorithm (middle pair), 3) Alternative 3-point algorithm (bottom pair)

6.3 Testing initial pair reconstruction methods



Figure 6.6: The results of drawing estimated epipolar lines for the Warsaw University Dataset with 1000 Sift points. 1) Fundamental matrix created from rotation and translation (upper pair), 2) Standard Essential matrix 5-point algorithm (middle pair), 3) Rotation Enhanced Essential 5-point algorithm (bottom pair)

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6.3.2 Time comparison

The chart 6.7 below shows the avaraged execution time for 100 estimation attempts. Execution time of the 8-point algorithms are very similar, which is understandable,

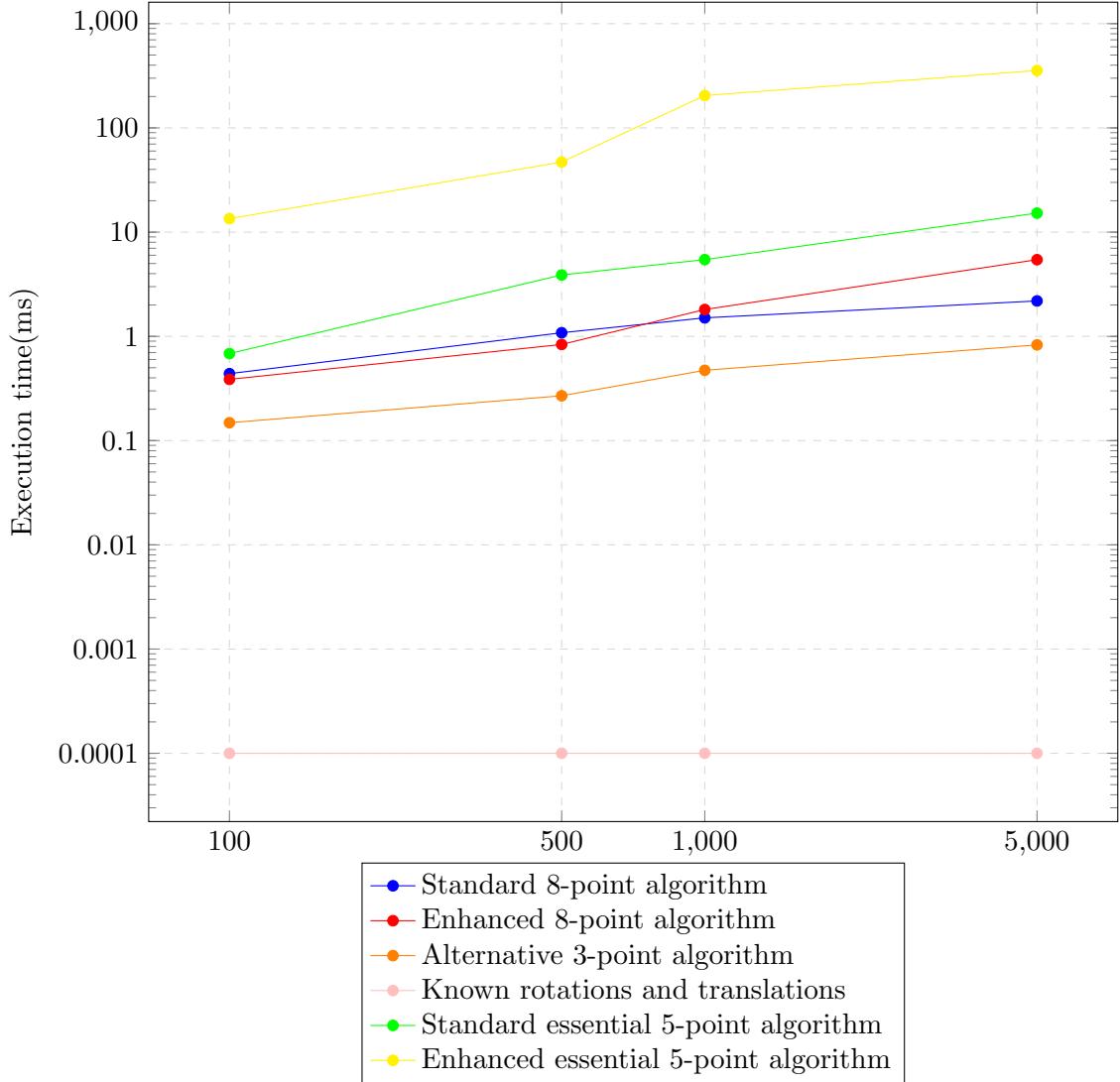


Figure 6.7: Execution time of the proposed algorithms(1024x768pixels) per initial SIFT feature set size

since their implementation is nearly identical and they differ only in the characteristics of the analysed dataset. It can be seen, however, that execution time values of the 5-point epipolar estimations differ significantly depending on the version. In this

situation either finding optimal solution with initial rotation is much more difficult or implementation of these methods is significantly different in terms of memory allocation. Execution time of the 3-point algorithm is few times faster than the one of the 8-point algorithms. The reconstruction with the sole use of the known rotations and translations has a hundred times shorter execution time than the standard algorithms, but at the same time it does not produce properly correlated epipolar lines.

6.4 Testing reconstruction strategies

As was shown in 6.3 not every initial reconstruction method gives good results. Only the most effective techniques were used to prepare a number of completely different strategies:

1. Standard 8-point + OpenCV Pose Estimation
2. Enhanced 8-point + OpenCV Pose Estimation
3. Enhanced 8-point + Initial Rotation and Translation OpenCV Pose Estimation
4. Alternative 3-point + OpenCV Pose Estimation
5. Alternative 3-point + Initial Rotation and Translation OpenCV Pose Estimation

The first method gives the basic reference to standard 3D reconstruction strategy. The second one was chosen for its consistency in 3D reconstruction. It has never failed to produce solution close to optimal. The third one was prepared in order to determine how the enhanced pose estimation influences the final reconstruction outcomes. The strategies number four and five were used to see if the models can be produced faster without impacting the final accuracy too heavily. Finally, the sixth method was proposed in order to verify whether the entire reconstruction process can be performed using the sensor data only.

6.4.1 Accuracy

Accuracy of the 3D reconstruction was measured using Bundle Adjustment algorithm from SBA library[ref]. It allows for calculating the error based on the projective constraint both before and after Bundle Adjustment. The tests performed with different strategies for the "Warsaw University" dataset are presented on 6.8. The significant differences in the initial error can be explained by different numbers of reconstructed

6. EVALUATION

points after initial phase and inconsistent and unknown scale of the finally reconstructed models. The enhanced initial pair reconstruction and pose estimation methods have bigger impact on Bundle Adjustment error reduction.

6.4 Testing reconstruction strategies

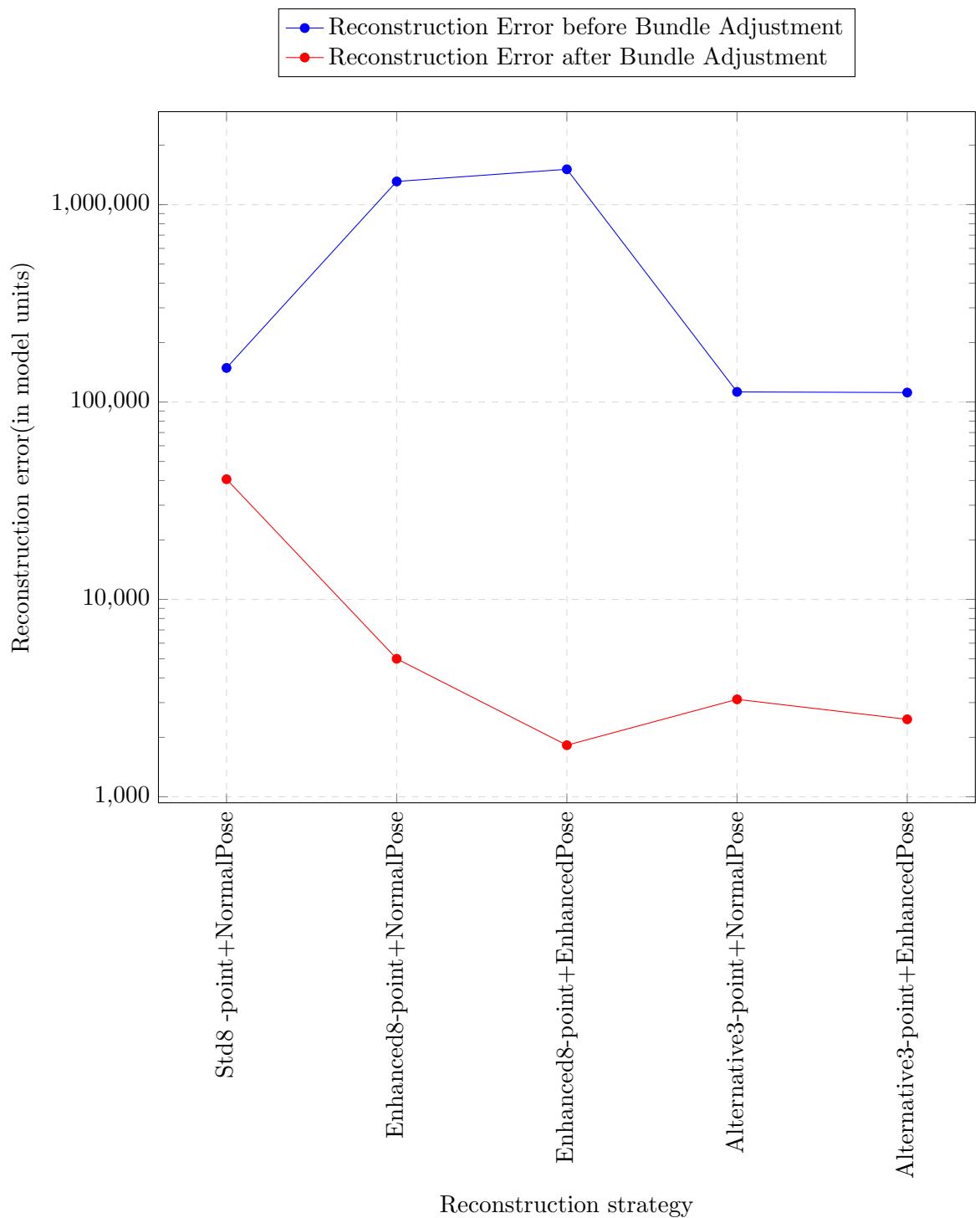


Figure 6.8: Influence of Bundle Adjustment on the models produced with different reconstruction strategies

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In general, Bundle Adjustmet process allows to rearrange and modify 3D points positions and change external parameters of estimated cameras. However, in the case of the enhanced algorithms the estimated camera positions are already very close to their optimal orientations. This allows Bundle Adjustment method to focus mainly on 3D points modifications. It can be observed that the enhanced pose estimation results in further reduction of BA error when compared to the standard strategies.

6.4.2 Execution time

The chart 6.10 presentes the execution time without Bundle Adjustment. Comparing it to the execution times in 6.7 one can notice that most of the time is allocated for SIFT correspondences matching, which constitutes a bottleneck in the proposed reconstruction methodology [reference to concept pose esitmation choosing]. Furthermore, the reconstruction time was also measured with Bundle Adjustment process at the end(6.11). It was found that Bundle Adjustment works significantly better with enhanced initial pair reconstruction and pose estimation. In that case the cloud points are better organised than in the standard reconstruction, which results in faster convergence of BA. Sample difference between the cloud points before and after BA can be seen in 6.9

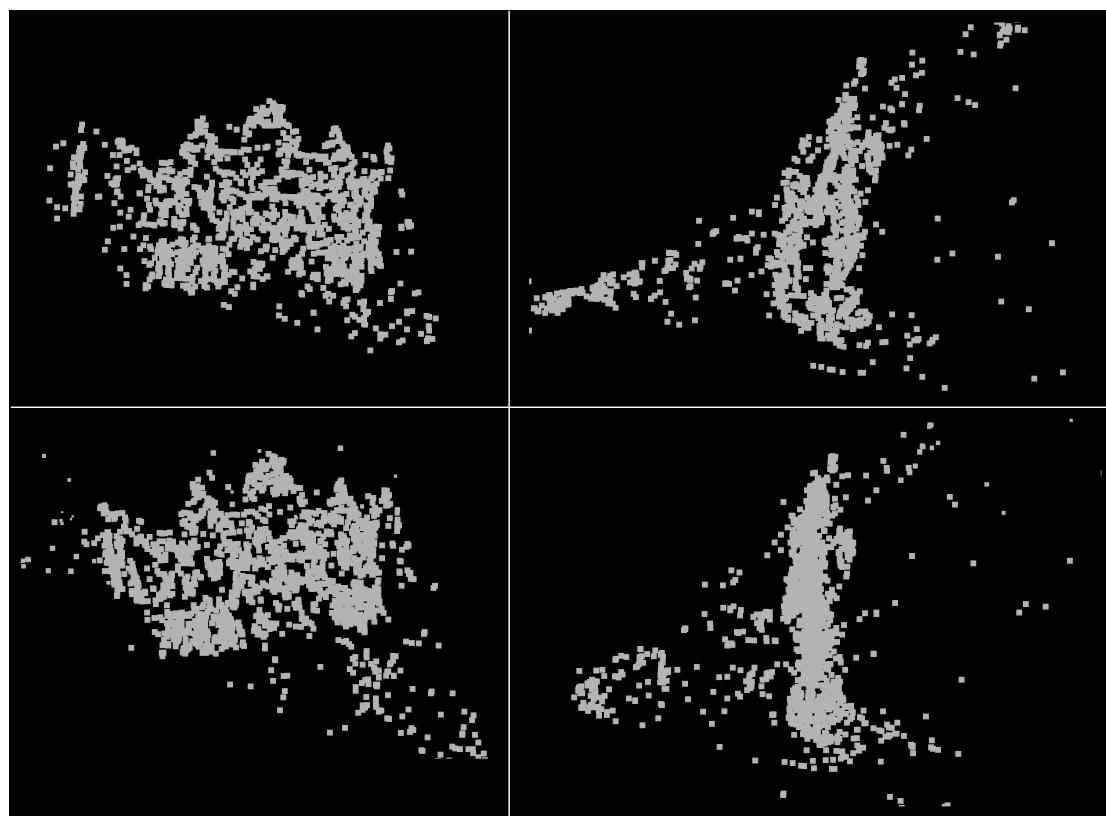


Figure 6.9: 3D point clouds before Bundle Adjustment (upper) and after (bottom) for enhanced 8-point with enhanced pose estimation. Warsaw University of Technology dataset with 1000 SIFT corresponding features (left - front, right - side)

6. EVALUATION

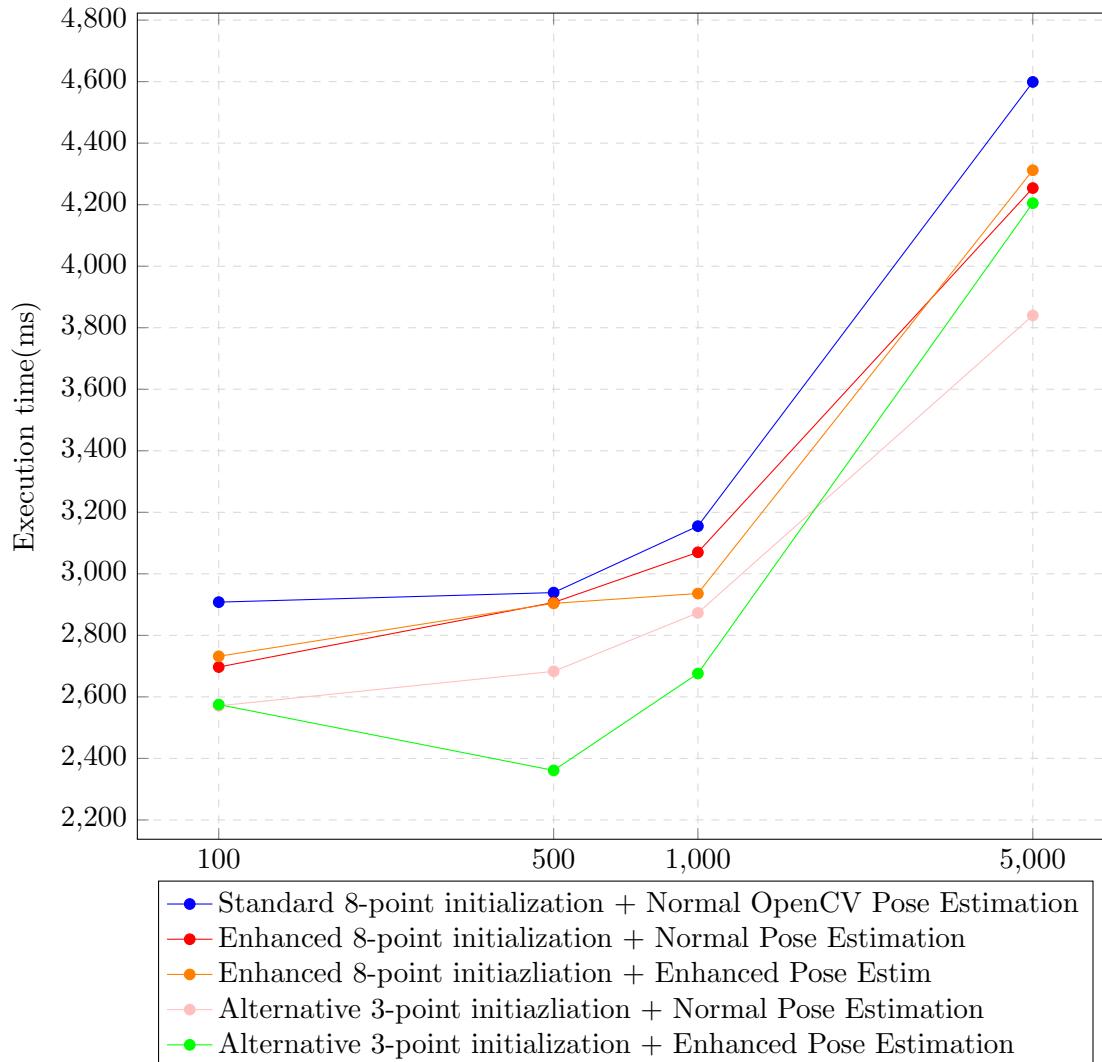


Figure 6.10: Total reconstruction execution time (4 images with resolution 1024x768pixels) per SIFT features set size

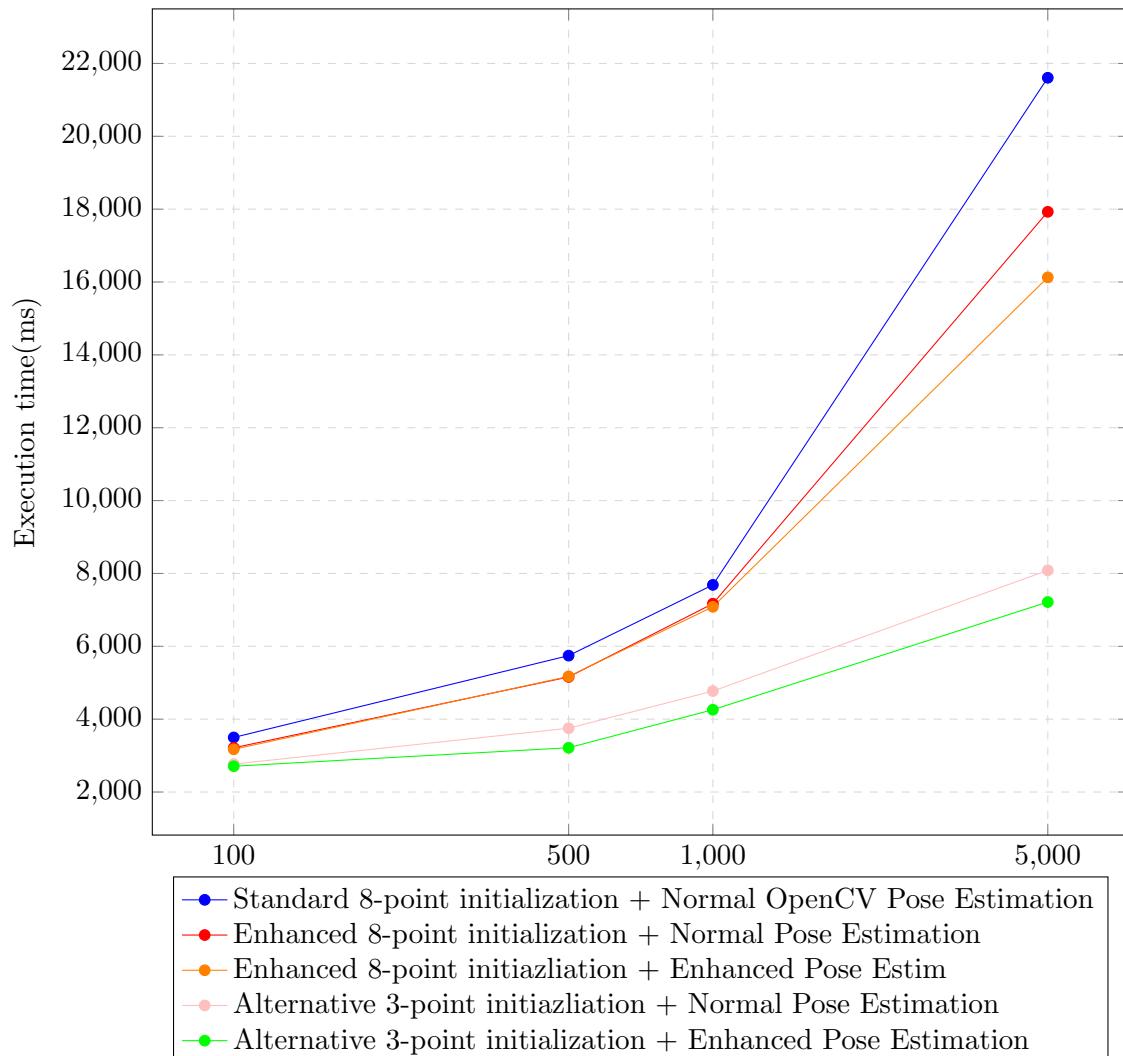


Figure 6.11: Total execution time of reconstruction with Bundle Adjustment (4 images with resolution 1024x768pixels) per SIFT features set size

6. EVALUATION

6.5 Effectivness

The numerical measuerments used do not always correspond to the proper 3D model reconstructions. The following pages present the reconstruction effects of the proposed strategies. Figure 6.12 shows the reconstructed effects for different initialization pair methods and 4000 SIFT features set. It can be observed that both standard and enhanced 8-point methods produces very good results, which differ only in terms of final reconstruction scale. Alternative 3-point algorithm produces slighlly worse and distorted models due to uncompenstaed noise in the rotations of the camera used. A model reconstructed from the known rotations and translations is very distorted, but nonetheless stil recognizable. It could prove useful should a very fast reconstruction be needed.

The matter is slightly different when 400 SIFT feature points are used. [reference] In the first case all algorithms were able to find solutions close to optimal. However, in the second attempt the traingulation test, which is used to identify proper deconmposition in standard 8-point approach, failed and produced an unrecognizable model (6.14). It can be seen that pose estimations enhancments result primarily in the reduction of outliers (6.15).

Figure 6.16 shows that reconstruction from the known rotations and translations produces many outliers.

More reconstructed models can be found in [TODO Materials]

6.5 Effectivness

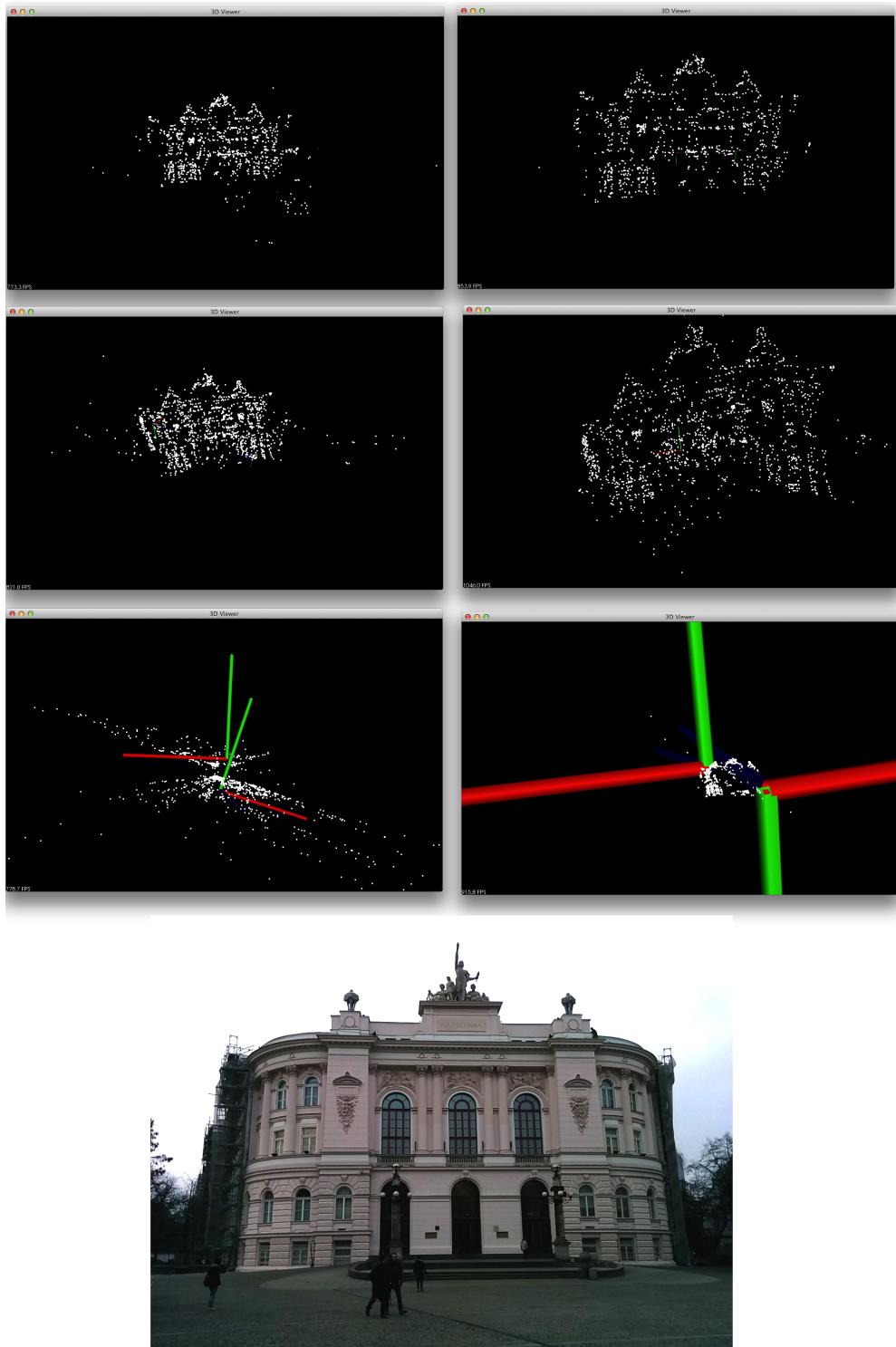


Figure 6.12: Reconstructed models for the proposed initial reconstruction methods and 4000 SIFT features. From upper left to bottom right: 1) standard 8-point, 2) enhanced 8-point, 3) alternative 3-point, 4) known rotations and translations, 5) standard 5-point, 6) enhanced 5-point

6. EVALUATION

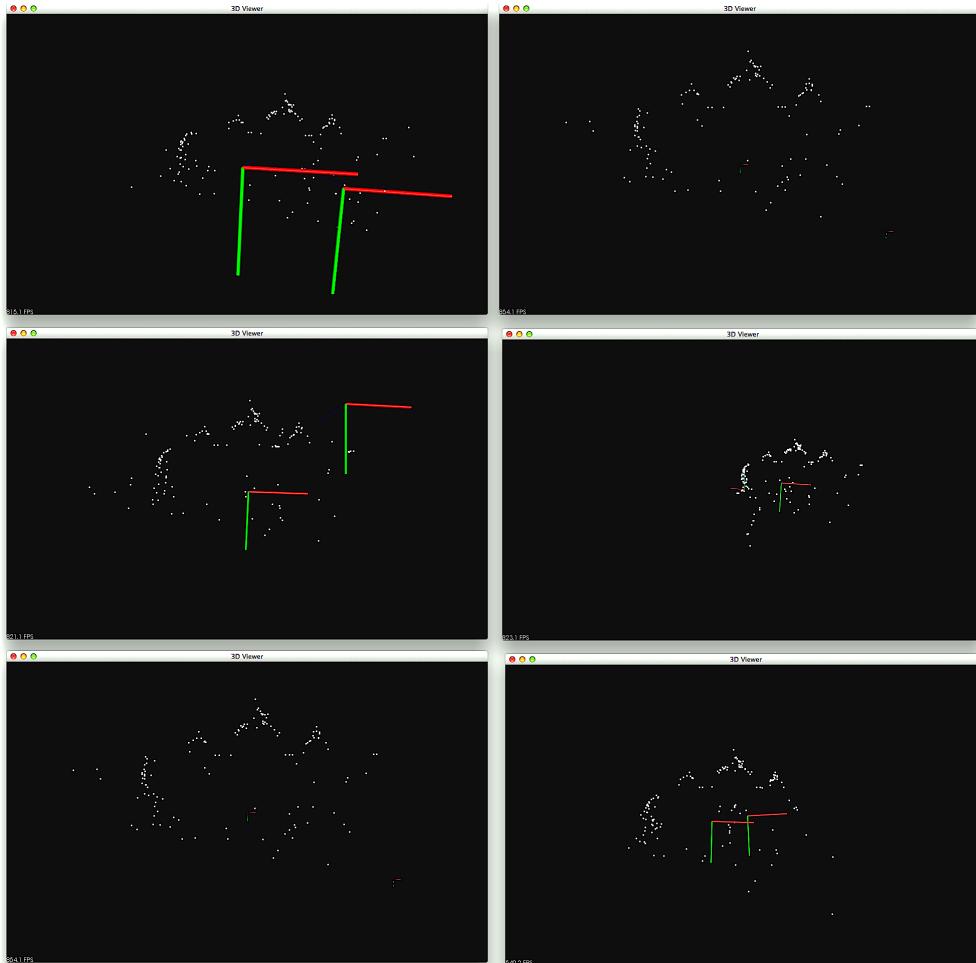


Figure 6.13: Reconstructed models for the proposed initial reconstruction methods and 400 SIFT features. From upper left to bottom right: 1) standard 8-point, 2) enhanced 8-point, 3) alternative 3-point, 4) known rotations and translations, 5) standard 5-point, 6) enhanced 5-point

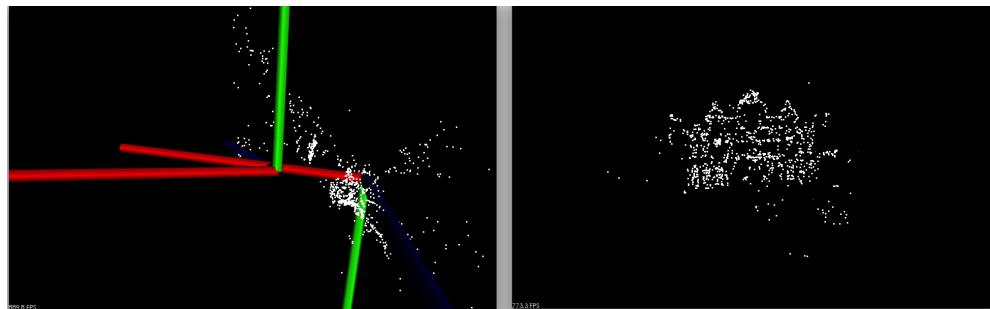


Figure 6.14: Fail test case of Standard 8-point triangulation(left) in comparison to fortunate reconstruction(right)

6.5 Effectivness



Figure 6.15: Pose estimation methods comparison (Views from front and side). Left:
Normal Pose Estimation, right: Enhanced Rotation and Translation Pose Estimation
. Less outliers appear in the reconstruction if enhancment is applied.

6. EVALUATION



Figure 6.16: Reconstruction results from known translations and rotations from different angles. The upper one shows the front face of building, the others present views from side angles. In the reconstructed model many outliers are present.

Chapter 7

Conclusion

This chapter describes the work performed in the course of investigating the master thesis subject, discusses the problems encountered and proposes ideas for the future development.

7.1 Summary

The idea of enhancing standard 3D reconstruction algorithms with Android sensor fusion data was conceptualised, implemented and verified in several different versions. At the beginning, only the alternative 3-point version was planned to be implemented for the purposes of this thesis. However, it did not produce results of enough accuracy. After few iterations of implementation and testing of the 3-point algorithm it was concluded that it is not enough to base the reconstruction solely on the rotation data from Android sensor fusion. That was when the enhanced 8-point and 5-point versions were designed. In addition, the author proposed different reconstruction strategies, which can be used depending on accuracy and speed trade-offs preferred. They were built on personal observations of the reconstructions performed and inspired by related works in the field. In order to acquire datasets for algorithm, an evaluation Android application named "Sensor Enhanced Images Camera" was developed. Upon capturing the picture it automatically stores the current device's rotation information and the proposed heuristically estimated translation. To evaluate the proposed methods and collected datasets a desktop application named "Enhanced 3D reconstructor" was implemented. It can be used in two different modes: 1) Efficiency testing - for comparing Samson

7. CONCLUSION

Error and visual estimation of calculated epipolar lines. The proposed enhancements to the standard 8-point algorithm allow to unambiguously calculate the proper rotation and translation. Application of the enhanced 5-point algorithm resulted in better accuracy than in the case of the standard 5-point algorithm in terms of Sampson Error. Execution time of the enhanced reconstruction methods is generally longer than of the standard ones. The use of initial rotation and translation estimation in pose estimation results in a greater reconstruction accuracy, particularly in terms of outliers removal and Bundle Adjustment convergence speed. In general, applying Bundle Adjustment of sensor enhanced reconstruction results in greater error reduction and shorter execution time in comparison to the refining standard ones. The major problem, a bottleneck of some sort, of the proposed reconstruction process was the time needed for matching corresponding items. The study showed that the use of the sensor data alone is not enough to create accurate 3D models. However, it is possible to find a recognizable model among the reconstructed 3D cloud of points. The only problem is how to distinguish a recognizable model from the one reconstructed from uncorrect 3D points. It turned out that estimating rotation error matrix (R_{error}) is quite accurate and useful for that purpose and the proposed Rodrigues decomposition and its rounding, such as diagonal of R_{error} consisting of a diagonal identity matrix, returns better accuracy of 3D reconstruction and unambiguously defines the proper camera decomposition. Heuristical movement estimation is not entirely accurate and does not have significant impact on the reconstruction process. Finally, the proposed 3-point algorithm for translation estimation allows for faster and quite accurate recreation of the structure.

7.2 Dissemination

So far no one has used the implemented applications. Nonetheless the Android app can be useful for further 3D reconstruction research and it is planned to be published to Google Play Store once most needed improvements are made and it is properly tested. "Enhanced 3D reconstructor" has been already published to GitHub as open-source project distributed on LGPL license [reference].

7.3 Problems Encountered

The majority of the problems were related to bugs which appeared during implementation of the proposed algorithms and adaptation of the third party libraries. Android API allows a user to obtain rotation quaternion and offers a way to decompose it, but it does not explain how it is calculated. Decomposition of rotation matrix to euclidian angles and their composition needs to be done in the same order. Some tests were conducted in order to establish its proper rotation matrix composition. All of these problems were successfully resolved. It turned out that using the pose estimation instead of homography merging was not the optimal solution. Relaying on the pose estimation produced very small amount of points and sometimes reconstruction was forced stopped after analysing merely a few images.

7.4 Future work

Firstly, it would be useful to establish how the homography merging approach would influence the accuracy of 3D reconstruction. Secondly, other correspondence matching approaches should be tested. An optical flow estimation using video sequences could constitute one example thereof. This would both allow for a very quick relative pose estimation and could be used for dense model reconstruction afterwards. All of the libraries used are available or can be ported to Android, therefore it might be valuable to determine whether it is possible to achieve a real-time tracking and mapping (similar to (?)).

Chapter 8

Materials & methods

```
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26 ]
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8. MATERIALS & METHODS

100 SIFT Features	Total Sampson Error	Sampson Error per Point	Points left	Execution time(ms)
8-point OpenCV	20.5793	0.478588	43	0.4387
Alternative 3-point	112.749	4.17588	27	0.1484
8-point enhanced	67.2559	1.56409	43	0.3867
Known rot and trans	3501.23	83.3625	42	0.0001
Essential 5-point	43.5866	1.06309	41	0.684
5-point enhanced	1863.66	45.4552	41	13.4643

Table 8.1: Efficiency table of proposed methods for 100 SIFT features in Warsaw University of technology dataset. Columns: Total Sampson Error, Average Sampson error per point, Amount of points left after outliers removal, Execution time

500 SIFT features	Total Sampson Error	Sampson Error per Point	Points left & Execution time(ms)
8-point OpenCV	100.584	0.543697	185 1.0833
Alternative 3-point	220.722	2.53704	87 0.2692
8-point enhanced	112.7	0.609189	185 0.8362
Known rot and trans	14770.7	80.2756	184 0.0001
Essential 5-point	404.098	2.29601	176 3.8827
5-point enhanced	501.987	2.8522	176 47.0683

Table 8.2: Efficiency table of proposed methods for 500 SIFT features in Warsaw University of technology dataset. Columns: Total Sampson Error, Average Sampson error per point, Amount of points left after outliers removal, Execution time

1000 SIFT features	Total Sampson Error	Sampson Error per Point	Points left	Execution time(ms)
8-point OpenCV	265.637	0.781287	340	1.5055
Alternative 3-point	640.895	4.1083	156	0.4725
8-point enhanced	295.152	0.868093	340	1.8099
Known rot and trans	27394.4	77.1673	355	0.0001
Essential 5-point	518.293	1.85768	279	5.4482
5-point enhanced	349.393	1.2523	279	204.2998

Table 8.3: Efficiency table of proposed methods for 1000 SIFT features in Warsaw University of technology dataset. Columns: Total Sampson Error, Average Sampson error per point, Amount of points left after outliers removal, Execution time

5000 SIFT features	Total Sampson Error	Sampson Error per Point	Points left	Execution time(ms)
8-point OpenCV	517.189	0.439413	1177	2.187
Alternative 3-point	1879.98	3.28094	573	0.8286
8-point enhanced	1087.78	0.924199	1177	5.4395
Known rot and trans	93951.1	77.9677	1205	0.0001
Essential 5-point	1949.53	1.95736	996	15.2223
5-point enhanced	19966.6	20.0468	996	355.464

Table 8.4: Efficiency table of proposed methods for 5000 SIFT features in Warsaw University of technology dataset. Columns: Total Sampson Error, Average Sampson error per point, Amount of points left after outliers removal, Execution time

References

Declaration

I herewith declare that I have produced this paper without the prohibited assistance of third parties and without making use of aids other than those specified; notions taken over directly or indirectly from other sources have been identified as such. This paper has not previously been presented in identical or similar form to any other German or foreign examination board. The thesis work was conducted from XXX to YYY under the supervision of PI at ZZZ.

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