

# Class 2a: Review of concepts in Probability and Statistics

Business Forecasting

# Summary

- In the last class:
  - We discussed the organization of the course
  - We overviewed forecasting methods
  - We learned about methods of qualitative forecasting
  - *Reference:* Forecasting Methods and Applications, chapter 1
- This set of classes:
  - We will start learning about exploratory analysis preparing the forecast
  - We will learn about various **data types**
  - We will learn how to **summarize data graphically**
  - We will learn how to **summarize data with summary statistics**
  - We will learn about **comparisons and associations**
  - *Reference:* Forecasting Methods and Applications, chapter 2.1-2.4

# Scenario

- Nowadays, many online pharmacies appeared which write prescriptions and make drugs subscriptions
  - Example in Mexico: *Choiz*
- At the same time, a new wave of very effective anti-diabetes drugs appeared which help to lose weight
  - Example: *Ozempik*
- You are consulting a business which wants to provide subscription services for these drugs in Mexico
- Your boss asks you to do exploratory market research for potential sales forecast

# Parameters vs Statistics

- You need to know how many people in Mexico have diabetes

## Parameter

- Call  $\mu_d$  the proportion of Mexican population which has diabetes
  - Usually the parameter is an **unknown** number **describing the whole population**
  - You want to learn what it is
  - In our example,  $\mu_d$  is a parameter that you want to learn
  - More generally, parameter describes an aspect of the entire population

## Statistic

- But you don't have data on the whole population. At best you can get a sample from a survey
  - So you will try to estimate this parameter with sample
  - Statistic is a **guess of the parameter** which can be **calculated from the sample**
  - You will calculate a statistic  $\hat{\mu}_p$  which is the proportion of diabetics in the sample

# Parameters vs Statistics

- What is population, sample, parameter and statistic in the following examples?
- You want to know the probability that a user who got a match on tinder will go out on a date with that person. You survey 1000 users and ask them about each match they got if they went on a date. You then calculate the share of dates which ended up in a match for these users.
- You want to know what whether starbucks baristas are faster than Cielito Querido baristas. You go to 10 starbucks and 10 Cielito Querido and measure the time it takes to make a coffee. You then calculate the average time it takes to make a coffee in each of these chains.
- You want to know the average age of people who go to the gym. You go to a gym and ask 100 people about their age. You then calculate the average age of these people.
- You want to know the variance of internet speed during in Mexico City. You visit 500 households and calculate the variance of their internet speed.



# Types of Data

# Longitudinal Data

- Observations are collected for the same subject (entity) over a period of time
- Same as time series data
- Example: Tracking a company's annual revenue and number of employees over several years

## Longitudinal Data Example

Show  entries

Year	Revenue	Employees
2018	50000	50
2019	52000	55
2020	55000	60
2021	58000	65
2022	60000	70

Showing 1 to 5 of 5 entries

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# Cross-Sectional Data

- Observations are collected at a single point in time
- Example: A survey of customers' satisfaction with a product and likelihood of repurchase at a certain point in time

## Cross-Sectional Data Example

Show  entries

Customer_ID	Satisfaction_Score	Repurchase_Likelihood
1	7	Likely
2	8	Unlikely
3	5	Likely
4	9	Likely

Showing 1 to 4 of 4 entries

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# Panel Data

- Combines both longitudinal and cross-sectional data
- Observations are collected for multiple subjects over multiple points in time
- Example: Tracking the annual revenue and number of employees of several companies over a few years

## Panel Data Example

Show  entries

Year	Company	Revenue	Employees
2018	A	50000	50
2018	B	52000	55
2018	C	55000	60
2019	A	58000	65
2019	B	60000	70

Showing 1 to 5 of 15 entries

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# Q1

Show  entries

Month	Cryptocurrency	Market_Cap
Jan	Bitcoin	60000
Jan	Ethereum	40000
Jan	Dogecoin	10000
Feb	Bitcoin	62000
Feb	Ethereum	41000

Showing 1 to 5 of 36 entries

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## Panel data

- Multiple time observation per subject (currency) and multiple subjects

# Q2

Show  entries

Country	Population_Millions	GDP_Billions	Internet_Users_Millions
United States	331	21433	246
China	1439	15308	904
India	1380	3160	560
Brazil	213	1848	126
Russia	145	1690	116

Showing 1 to 5 of 5 entries

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## Cross-sectional data

- Single (time) observation per subject (user), many subjects

# Q3

Show  entries

Year	Electric_Car_Sales
2020	20000
2021	30000
2022	40000
2023	50000
2024	60000

Showing 1 to 5 of 5 entries

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## Longitudinal data

- Multiple (time) observations of a single subject

# Primary vs Secondary Data

- **Primary data** is original data collected **directly from the source** for a **specific research purpose**.
  - Experimental data (if used by team designing the experiment)
  - Survey data (if used by survey team designing the survey)
  - It is customized for a particular research objective
- **Secondary data** is data that has **already been collected by someone else** for a different purpose but **can be used for a new research question or analysis**
  - National statistics - death certificates
  - National surveys reused by researchers
  - Data from medical records
  - Data on stock market

# What kind of data is it?

- Surveys: A company conducts a customer satisfaction survey to gather feedback from its customers regarding their products and services.
- Sales Reports: A business can analyze past sales data from previous years to identify trends and make strategic decisions.
- Interviews: A researcher interviews individuals to understand their opinions on a particular topic, such as political preferences or healthcare choices.
- Census Data: Government census data can be used by researchers to study demographic trends or population characteristics in a specific region
- Observations: An ecologist observes the behavior of a particular species in its natural habitat to gather data for a research project.
- Social Media Data: Companies can analyze social media posts and user engagement data from platforms like Twitter or Facebook to gain insights into customer preferences and sentiment.
- Experiments: Scientists conduct laboratory experiments to test a specific hypothesis and collect data directly from the experiments.
- Academic Journals: Researchers can review published studies and articles to gather data and insights related to their research topic.

# Variable Types



# Variable Types

We have two general types: **Categorical** and **Numerical** variables

## Categorical Variables

- Variables that can be divided into one or more groups or categories.
  - **Ordinal:** These variables can be logically ordered or ranked.
    - *Variable:* Customer Satisfaction Survey Results
    - *Example:* Very Unsatisfied, Unsatisfied, Neutral, Satisfied, Very Satisfied
  - **Nominal:** These variables cannot be ordered or ranked.
    - *Variable:* Social Media Platforms Used
    - *Example:* Facebook, Instagram, Twitter, LinkedIn, TikTok, Snapchat

# Numerical Variables

- Variables that hold numeric value and ordering is possible
  - **Discrete:** These variables can only take certain values
    - *Example:* Number of App Downloads from App Store
    - *Example:* Number of children you have
    - *Example:* Size of coke products: 0.33L, 0.5L, 1L, 2.25L



# Numerical Variables

- Variables that hold numeric value and ordering is possible
  - **Discrete:** These variables can only take certain values
    - *Example:* Number of App Downloads from App Store
    - *Example:* Number of children you have
    - *Example:* Size of coke products
- **Continuous:** These variables can take any value within a range
  - *Example:* Time spent on a Webpage
  - *Example:* Exchange rate between MXN and USD
- What's the main difference between ordinal and discrete?
  - We could say 1=Very unsatisfied, 2=Unsatisfied
  - But we cannot say that very unsatisfied has half of satisfaction of person who is just unsatisfied!
  - We can order, but these numbers don't have meaning in terms of distance between them

# Mexican Health Survey

Representative sample of the Mexican population  $n=37858$

Show  entries

age	gender	weight	location_type	diabetes	Mother_diabetes	Difficulty_walking
51	Male	77.4657	Urban	0	1	A lot of difficulty
41	Female	80.0499	Urban	0	0	A lot of difficulty
44	Male	87.1874	Urban	0	1	No difficulty
68	Female	54.9827	Urban	0	0	No difficulty
52	Female	34.3283	Urban	0	0	A lot of difficulty

Showing 1 to 5 of 37,858 entries

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- *Age*: Numerical, Discrete
- *Gender*: Categorical, Nominal
- *Weight*: Numerical, Continuous
- *Location\_type*: Categorical, Nominal
- *Diabetes*: Categorical, Nominal
- *Mother\_diabetes*: Categorical, Nominal
- *Difficulty\_walking*: Categorical, Ordinal

# Summarizing Data

Graphical summaries

# Categorical variables

## Frequency Tables

**Frequency table:** present the absolute frequencies (counts) and relative frequencies (shares) of each category.

- Relative frequency of category  $i$ :  $p_i = \frac{n_i}{N}$ 
  - $n_i$  is count of category  $i$
  - $N$  is total count in the sample

Show  entries

Location

Category	n_i	p_i
Rural	9899	0.261
Urban	27959	0.739
Total	37858	1

Showing 1 to 3 of 3 entries

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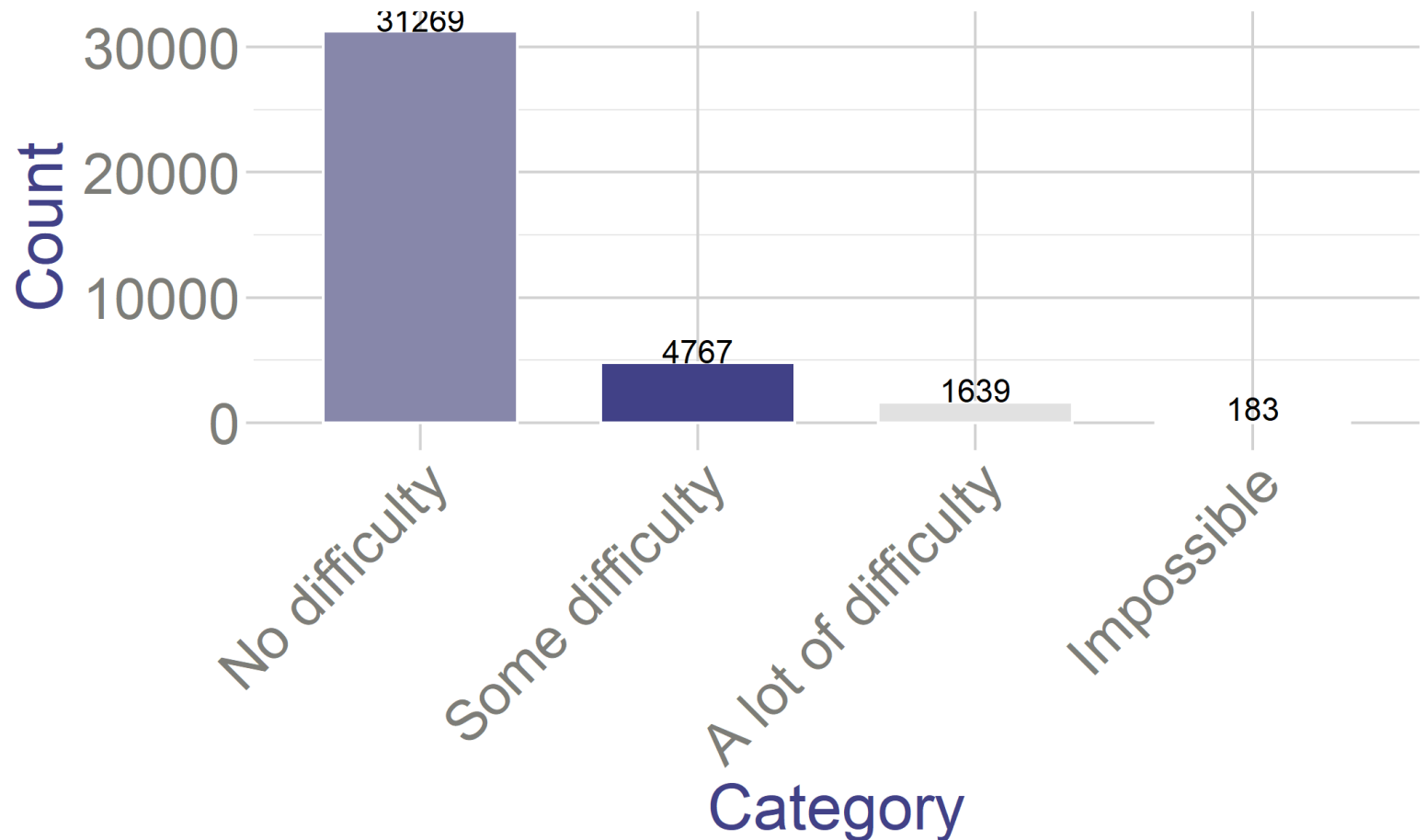
Show  entries

Difficulty Waking

Category	n_i	p_i
A lot of difficulty	1639	0.043
Impossible	183	0.005
No difficulty	31269	0.826
Some difficulty	4767	0.126
Total	37858	1

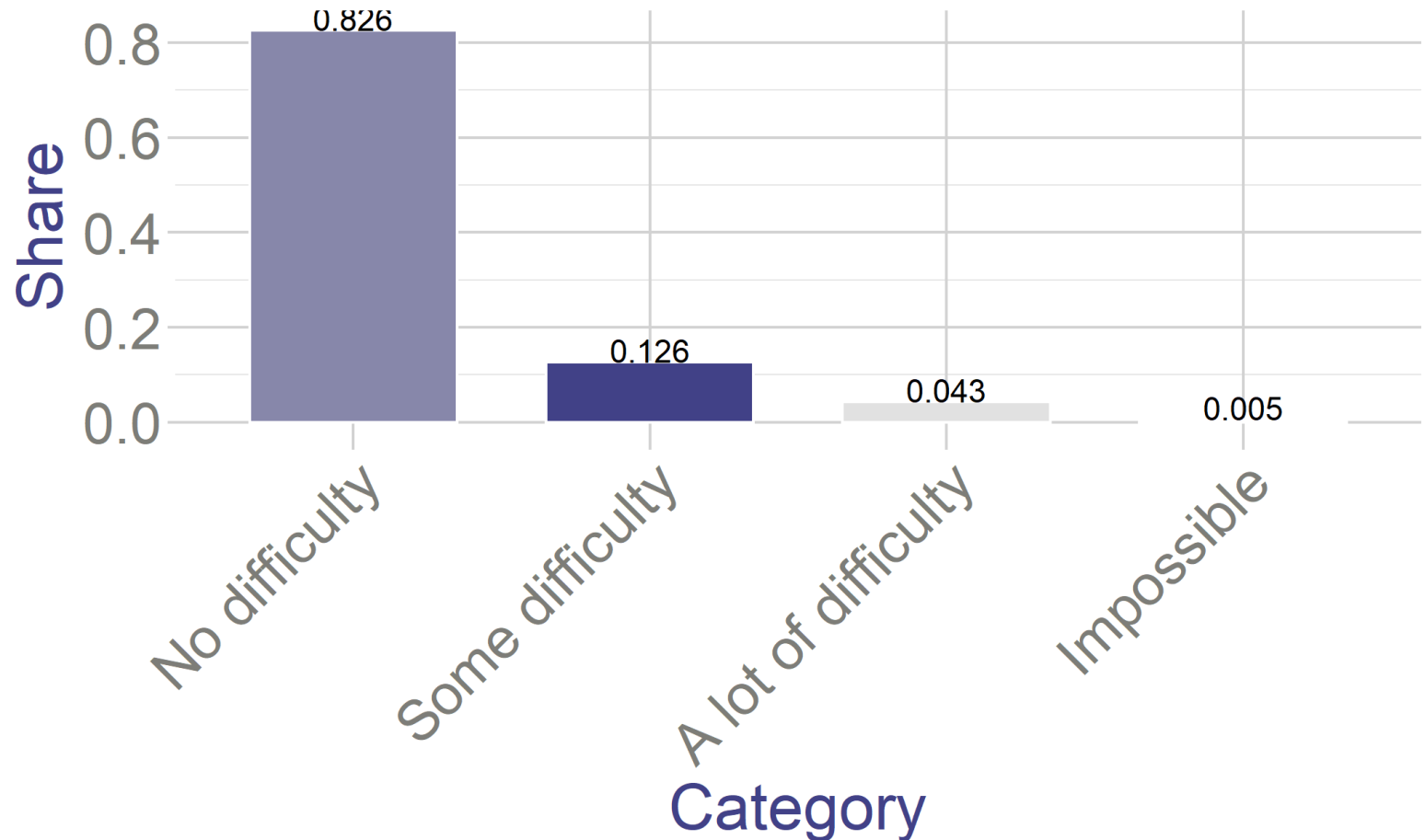
# Bar Charts

**Bar charts** visually represents the frequency count of each category



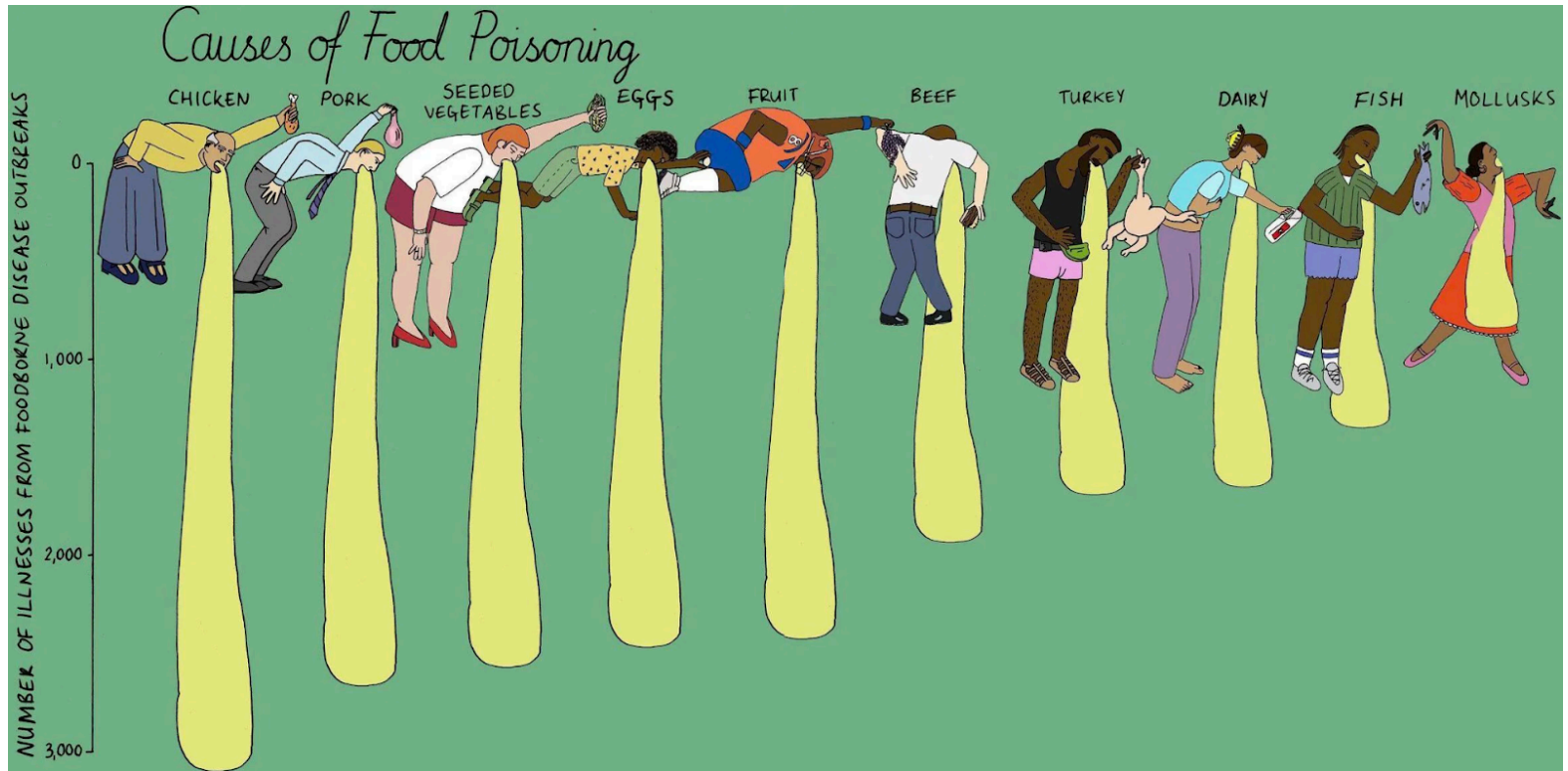
# Bar Charts

**Bar charts** visually represents the frequency count of each category



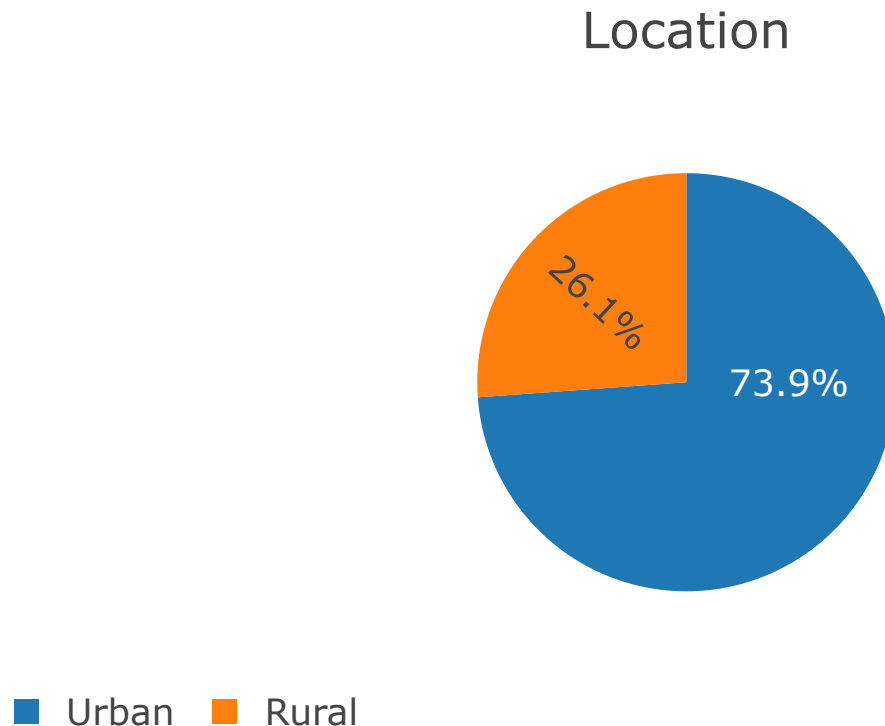


# More Creative Bar Chart



# Pie Charts

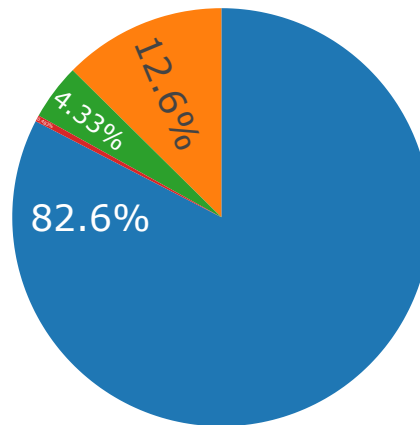
**Pie chart:** Each slice is proportional to the category's frequency



# Pie Charts

**Pie chart:** (Angle of) Each slice is proportional to the category's frequency

Difficulty Walking





■ No difficulty ■ Some difficulty ■ A lot of difficulty ■ Impossible

## My favorite pie chart

NETFLIX



 Time spent looking for movie

 Time spent watching movie

# Treemaps

**Treemap:** each group is represented by a rectangle, which area is proportional to its value.

## Data

Show  entries

Firm	Revenue	Industry
Apple	274515	Tech
Microsoft	143015	Tech
Johnson & Johnson	82483	Health
JPMorgan Chase	142422	Finance
Alphabet	182527	Tech
Pfizer	51907	Health
Bank of America	85205	Finance
Intel	77956	Tech

Showing 1 to 8 of 20 entries

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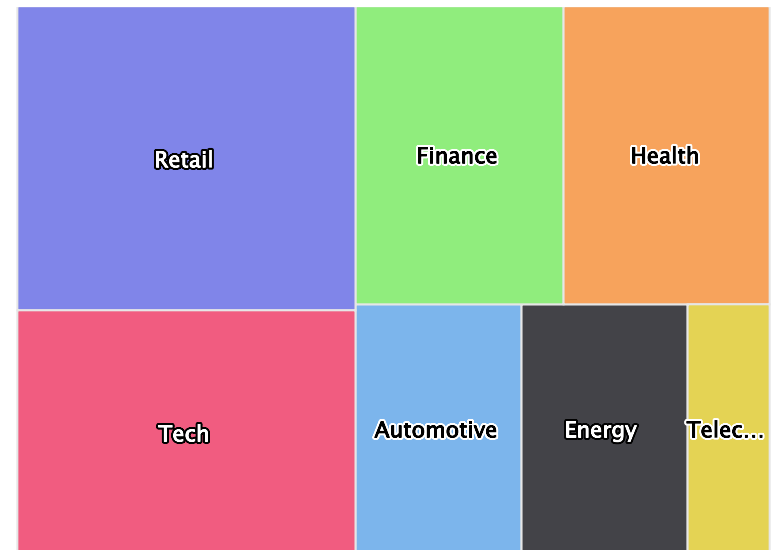
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## Treemap

Treemap of Industry Composition



# Numerical variables: Discrete

**Dotplot:** present one dot for each observation. Stacks observation of similar value

- Clearly see the distribution and the outliers
- Useless for larger data

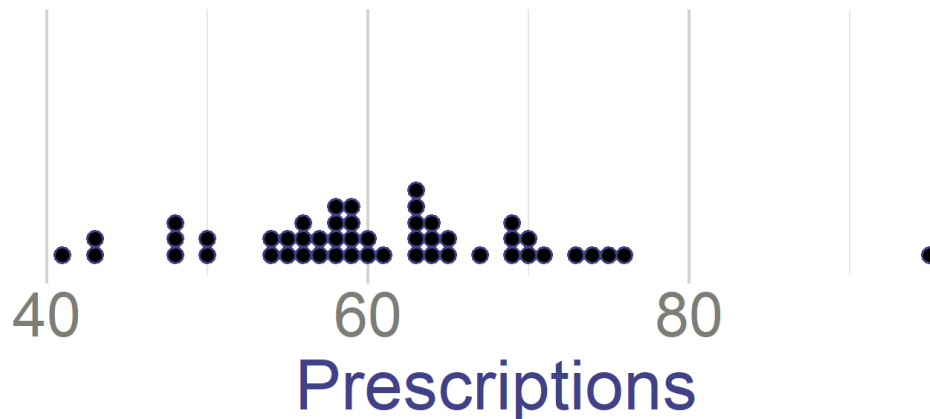
Show  entries

Number of prescriptions per physician

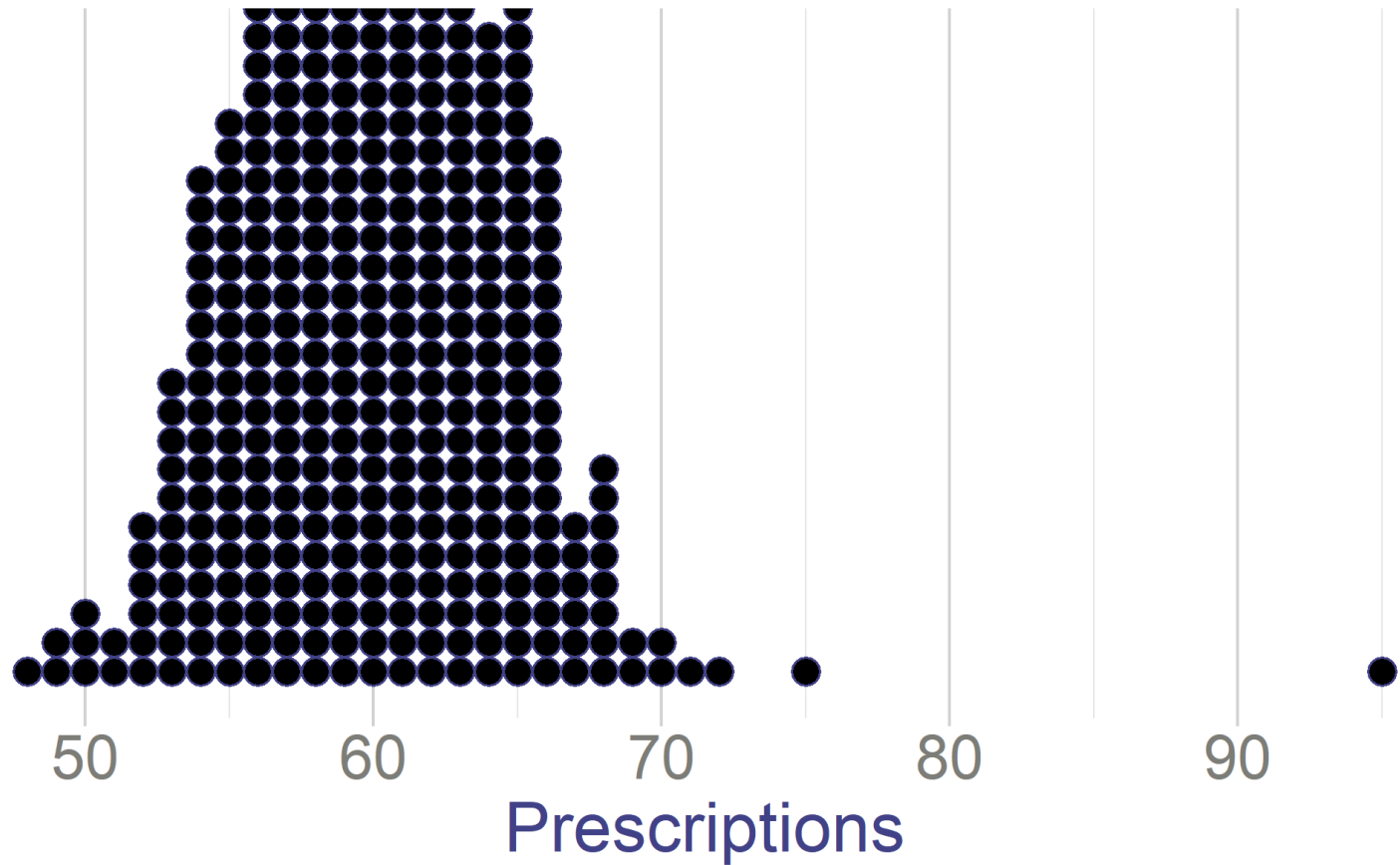
Physician	Prescriptions
Dr.1	70
Dr.2	56

Showing 1 to 2 of 50 entries

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## Numerical variables: Discrete



# Frequency Distribution

Suppose we survey people age 30-50 how many partners they had in their life.

- What's the distribution of partners?
- Calculate relative frequencies
- Show them on a bar graph

## Data

Show  entries

Number_of_partners	n_i	p_i
0	5	0.033
1	9	0.06
2	13	0.087
3	22	0.147
4	27	0.18
5	19	0.127

Showing 1 to 6 of 22 entries

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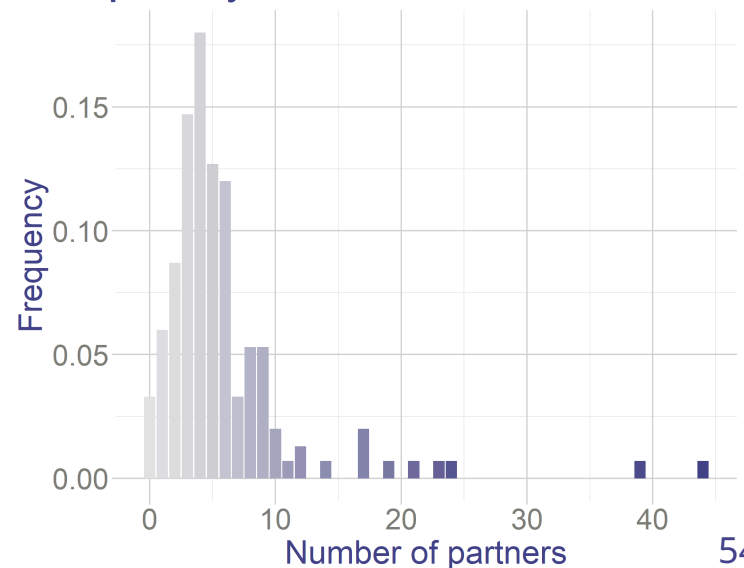
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## Distribution

### Frequency of Number of Partners

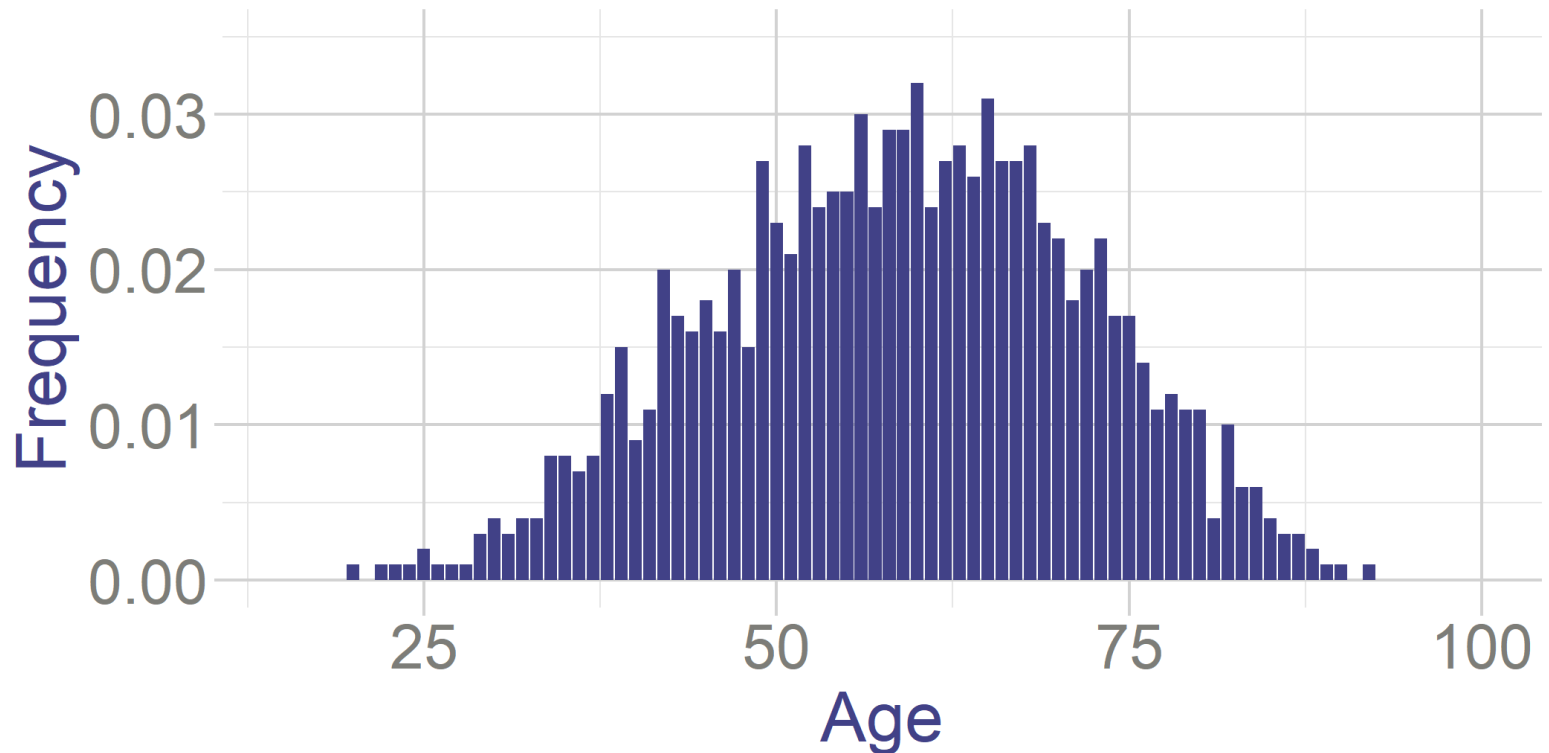




# Frequency Distribution

We can also show frequency of age of people who have diabetes from our data

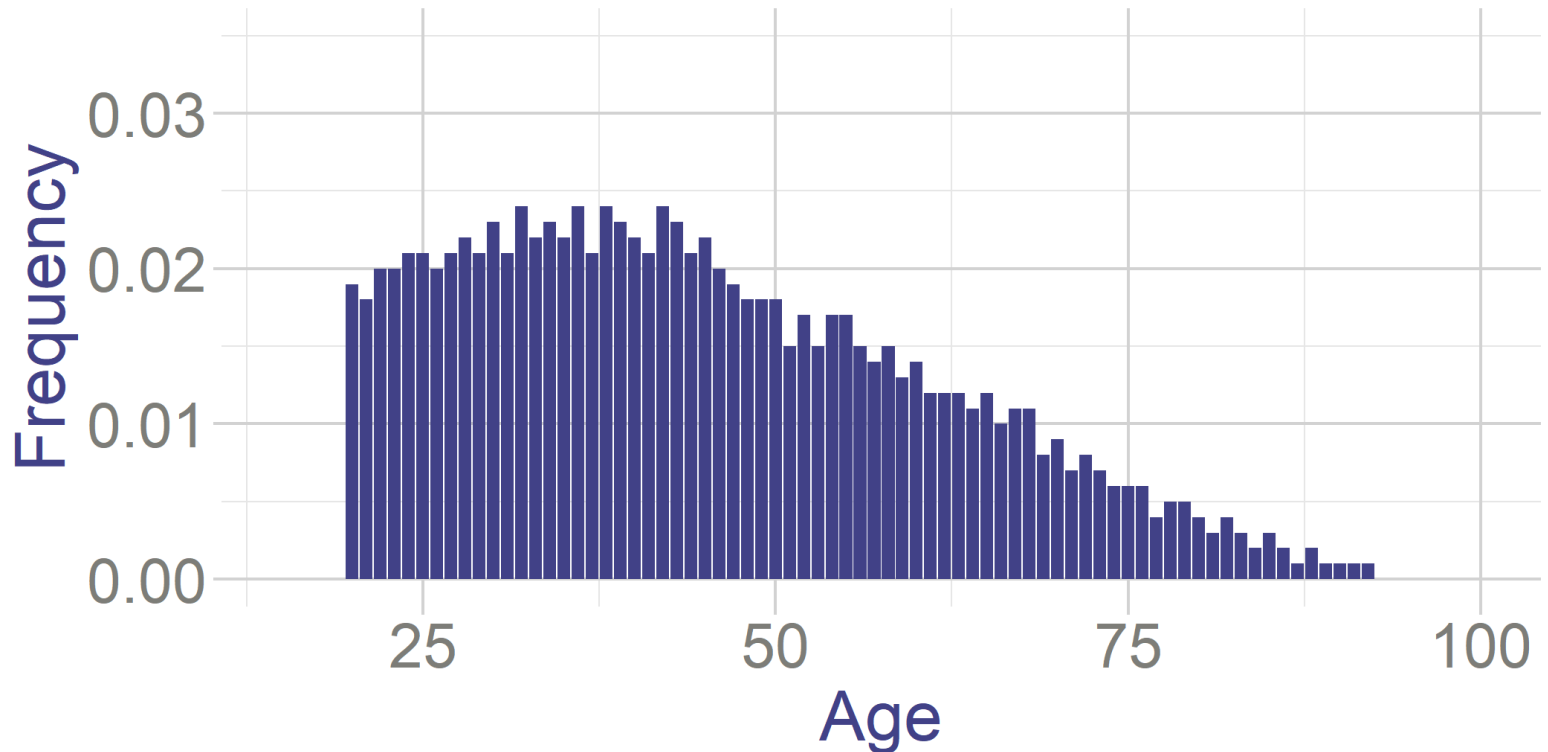
## Frequency of Age



# Frequency Distribution

Compare it to the age distribution in the adult population (20+)

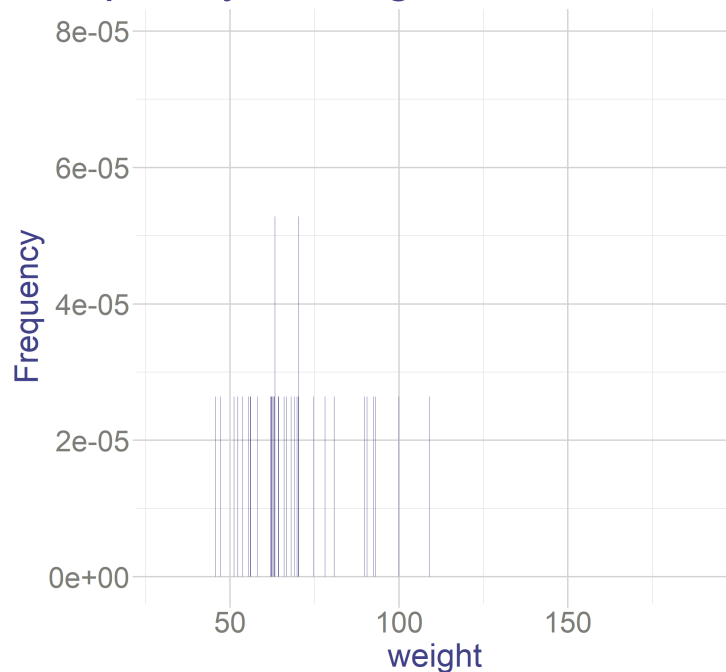
## Frequency of Age



# Numerical Variables: Continuous

- What about continuous values? Why can't we do the same?

## Frequency of weight



Show  entries

weight	n_i	p_i
30.3745	1	0.0000264
30.4593	1	0.0000264
30.5235	1	0.0000264
30.6135	1	0.0000264
30.7581	1	0.0000264
30.9106	1	0.0000264

Showing 1 to 6 of 36,297 entries

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- Most values never repeat, so they have very low relative frequency

# Histograms

**Solution:** Group similar values together

- Construct intervals and show how many observations are in a given interval

## Process

1. Decide how many intervals
2. And how wide they are
3. Then calculate the absolute and relative frequencies of each interval
4. Plot it with bars

## My approach

- I want  $k$  (example  $k=5$ ) equal intervals
- Divide the range of the data into  $k$  equal intervals
  - *Range* is max-min of the data

```
# Calculate max and min
max_value <- max(Health_data$weight)
min_value <- min(Health_data$weight)

# Calculate the difference
range <- max_value - min_value
```

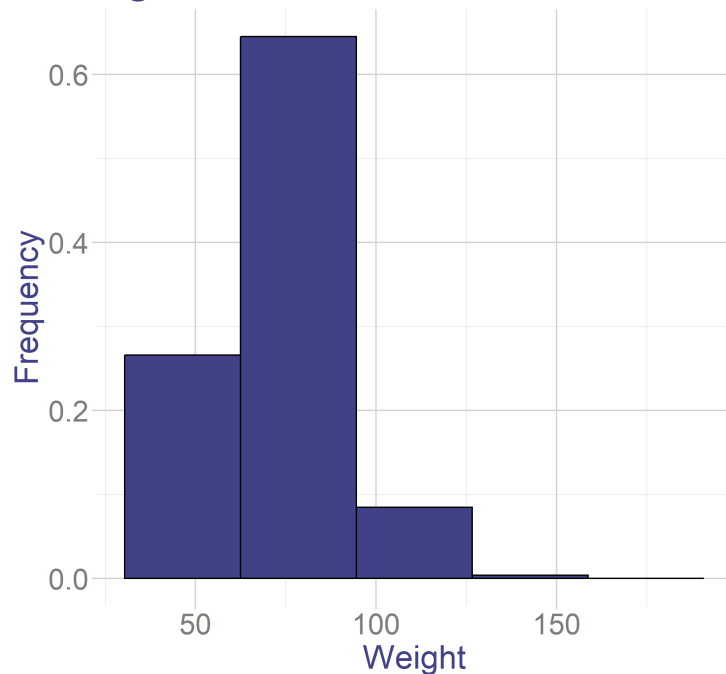
```
## [1] "Range= 190.8078 - 30.3745 = 160.4333"
```

- With 5 intervals, each will be 32kg wide
- The first one starts at the minimum value (30.3745)
- The last one ends at the maximum value (190.8078)
- Calculate how many observations I have in each interval and what's the relative frequency

# Histograms

- Midpoint represents middle of the interval - center of the bar
- $P_i$  is cumulative frequency: share of observations in this or smaller interval
  - Example:  $P_{(62.46-94.55)} = 0.911$
  - Interpretation: 91.1% of people have weight lower than 94.55kg

## Histogram with 5 Classes



Show 6 entries

Interval	Midpoint	n_i	p_i	P_i
30.37 - 62.46	46.42	10068	0.2659411	0.2659411
62.46 - 94.55	78.5	24430	0.6453061	0.9112472
94.55 - 126.63	110.59	3206	0.0846849	0.9959321
126.63 - 158.72	142.68	143	0.0037773	0.9997094
158.72 - 190.81	174.76	11	0.0002906	1

Showing 1 to 5 of 5 entries

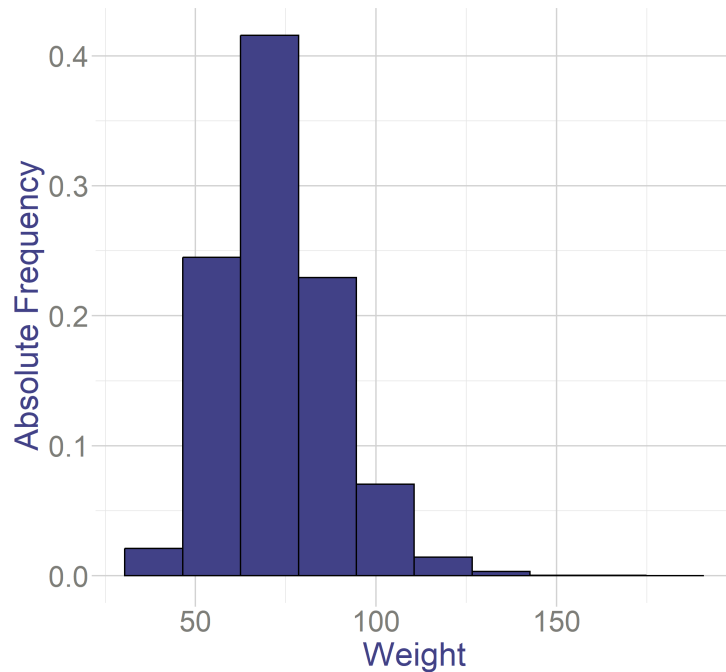
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# Histogram with 10 Classes

Now, let's increase the number of classes to 10.



Show 6 entries

Interval	Midpoint	n_i	p_i	P_i
30.37 - 46.42	38.4	796	0.0210259	0.0210259
46.42 - 62.46	54.44	9272	0.2449152	0.2659411
62.46 - 78.5	70.48	15742	0.415817	0.6817581
78.5 - 94.55	86.53	8688	0.2294891	0.9112472
94.55 - 110.59	102.57	2661	0.070289	0.9815362
110.59 - 126.63	118.61	545	0.0143959	0.9959321

Showing 1 to 6 of 10 entries

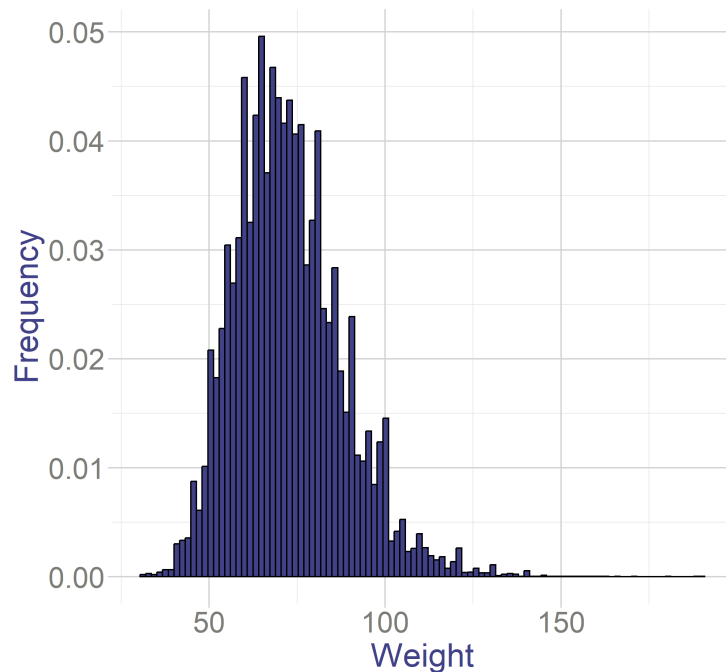
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# Histogram with 100 Classes



Show  entries

Interval	Midpoint	n_i	p_i	P_i
30.37 - 31.98	31.18	8	0.0002113	0.0002113
31.98 - 33.58	32.78	11	0.0002906	0.0005019
33.58 - 35.19	34.38	7	0.0001849	0.0006868
35.19 - 36.79	35.99	16	0.0004226	0.0011094
36.79 - 38.4	37.59	24	0.0006339	0.0017433
38.4 - 40	39.2	24	0.0006339	0.0023772

Showing 1 to 6 of 100 entries

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- Helps to see the distribution and outliers
- Is more always better?
- With smaller intervals, histogram tends to the **probability density function**



# Probability Density Function (PDF)

## Definition

- **Probability Density Function (pdf)** describes the probability distribution of a continuous random variable.
- It **does not** give probability at a given value (this is always 0 for continuous variable)
- It shows in which intervals that variable the most often appears
- It is used to calculate the probability of the random variable being in a given interval
- Area under it always adds up to 1

## Example

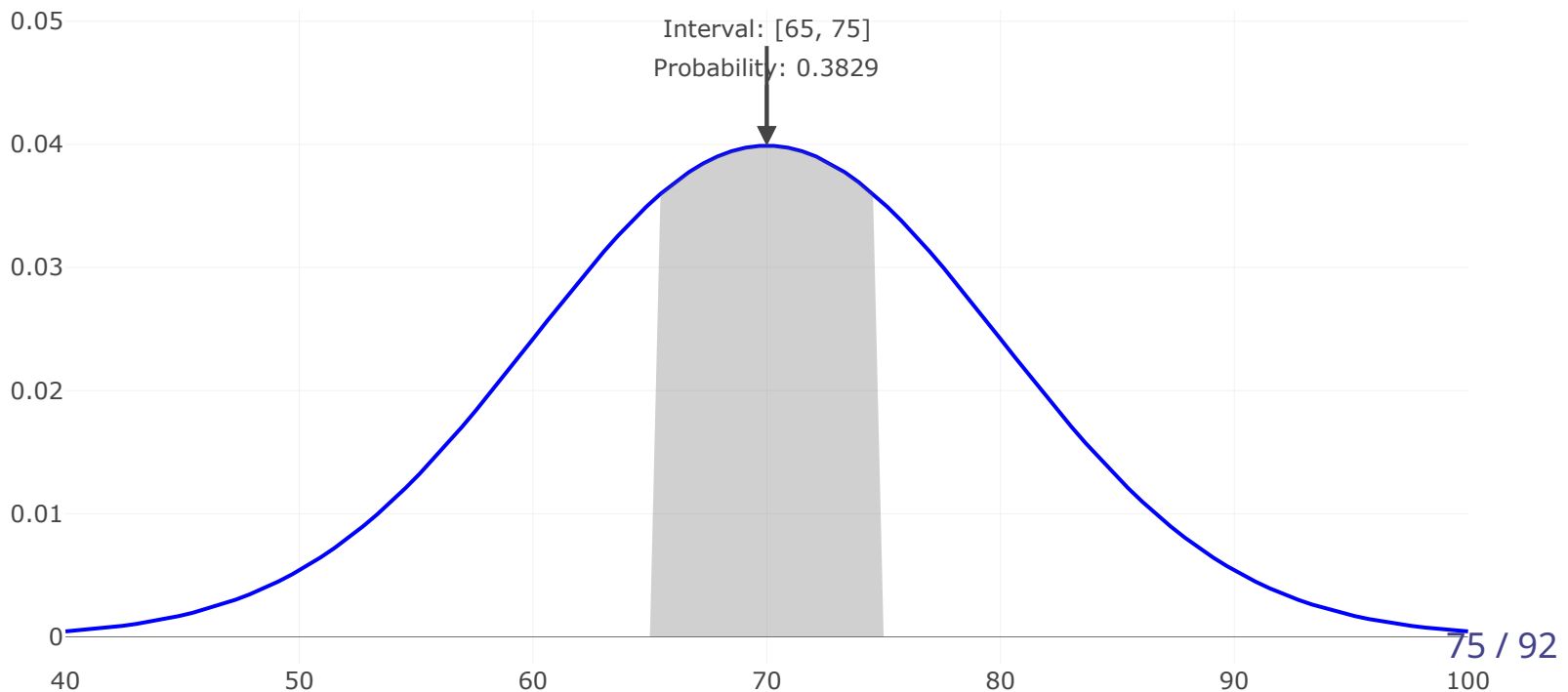
We have a random variable  $X$  representing the weight of adults in Mexican population. The PDF of  $X$  helps to describe the likelihood of finding a person of a specific weight within a range (e.g., between 58kg and 60kg).

# How They Work

To calculate the probability of  $X$  falling within a specific range  $[a, b]$ , you need to integrate the PDF from  $a$  to  $b$ :

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

The total area under the PDF curve is equal to 1

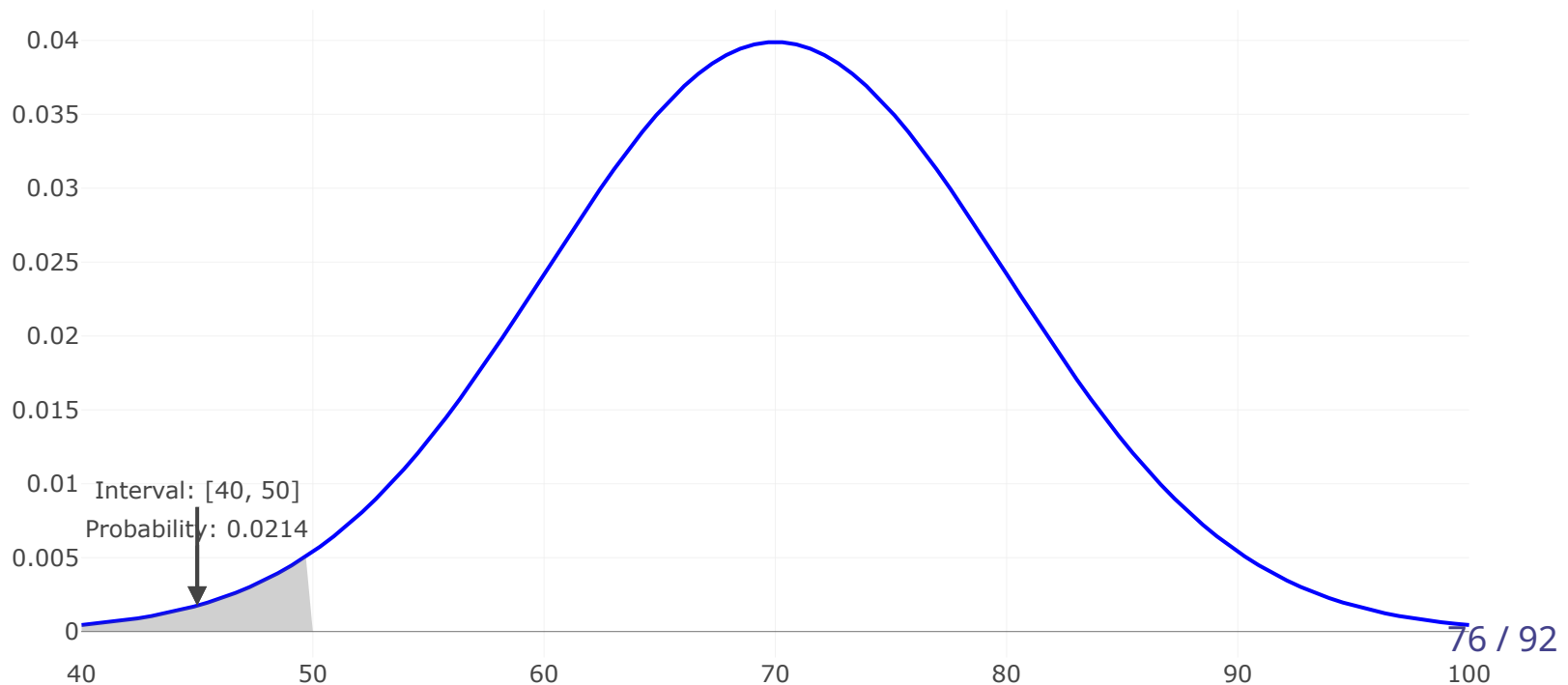


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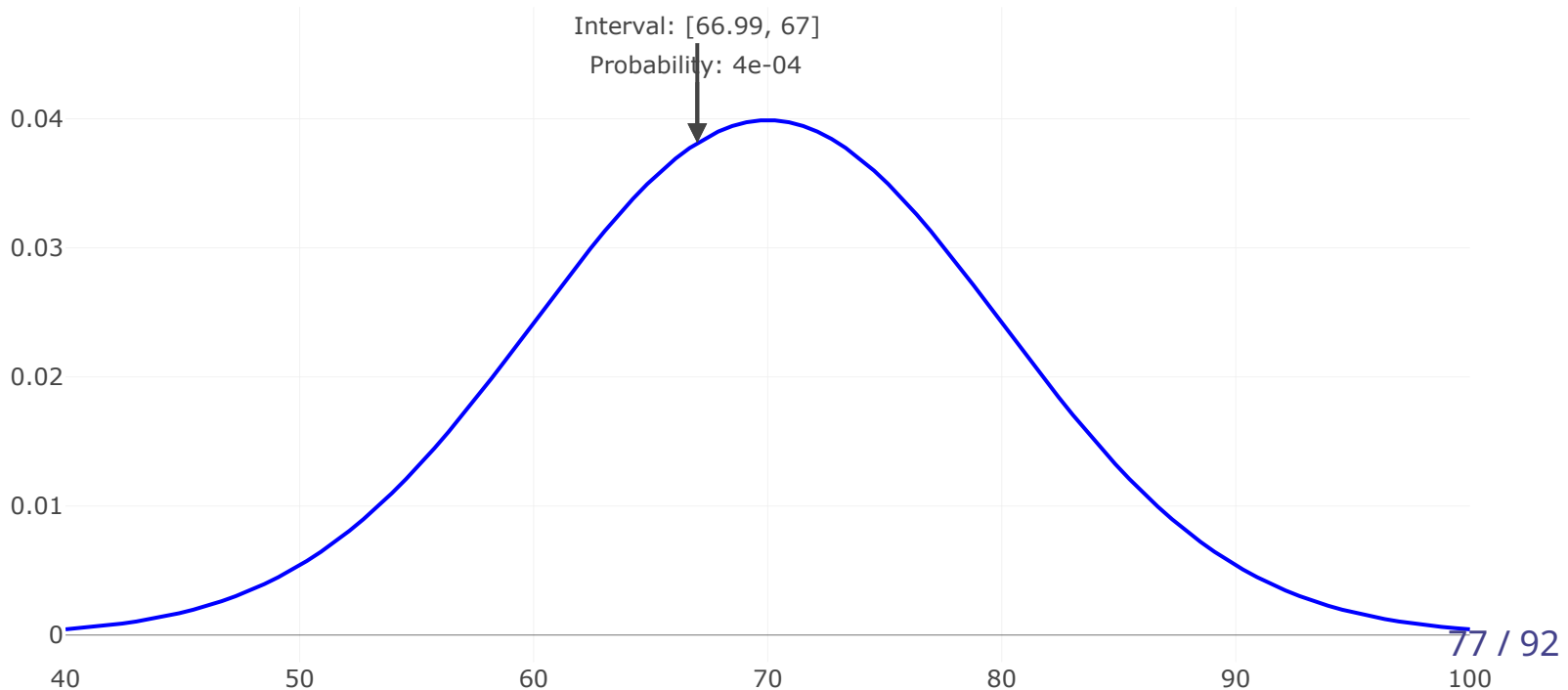


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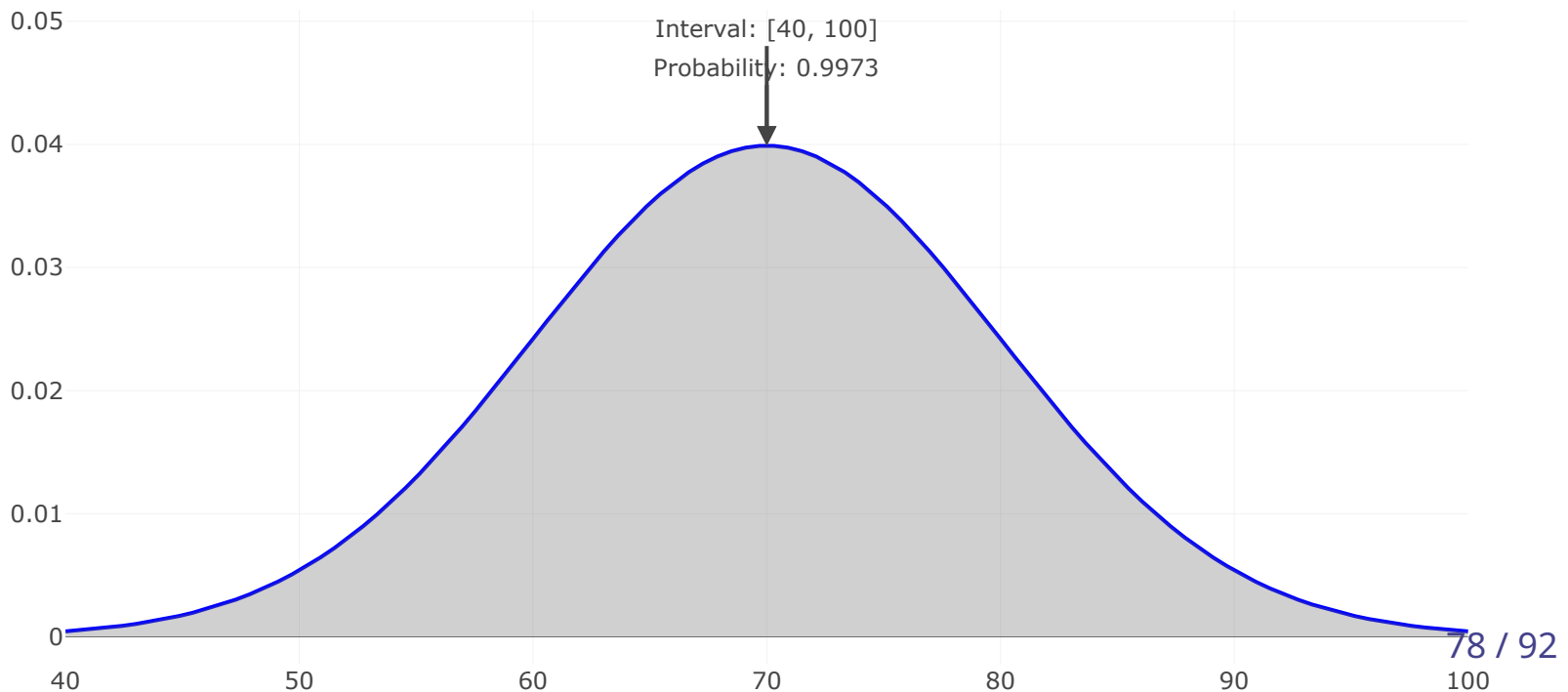


# How They Work

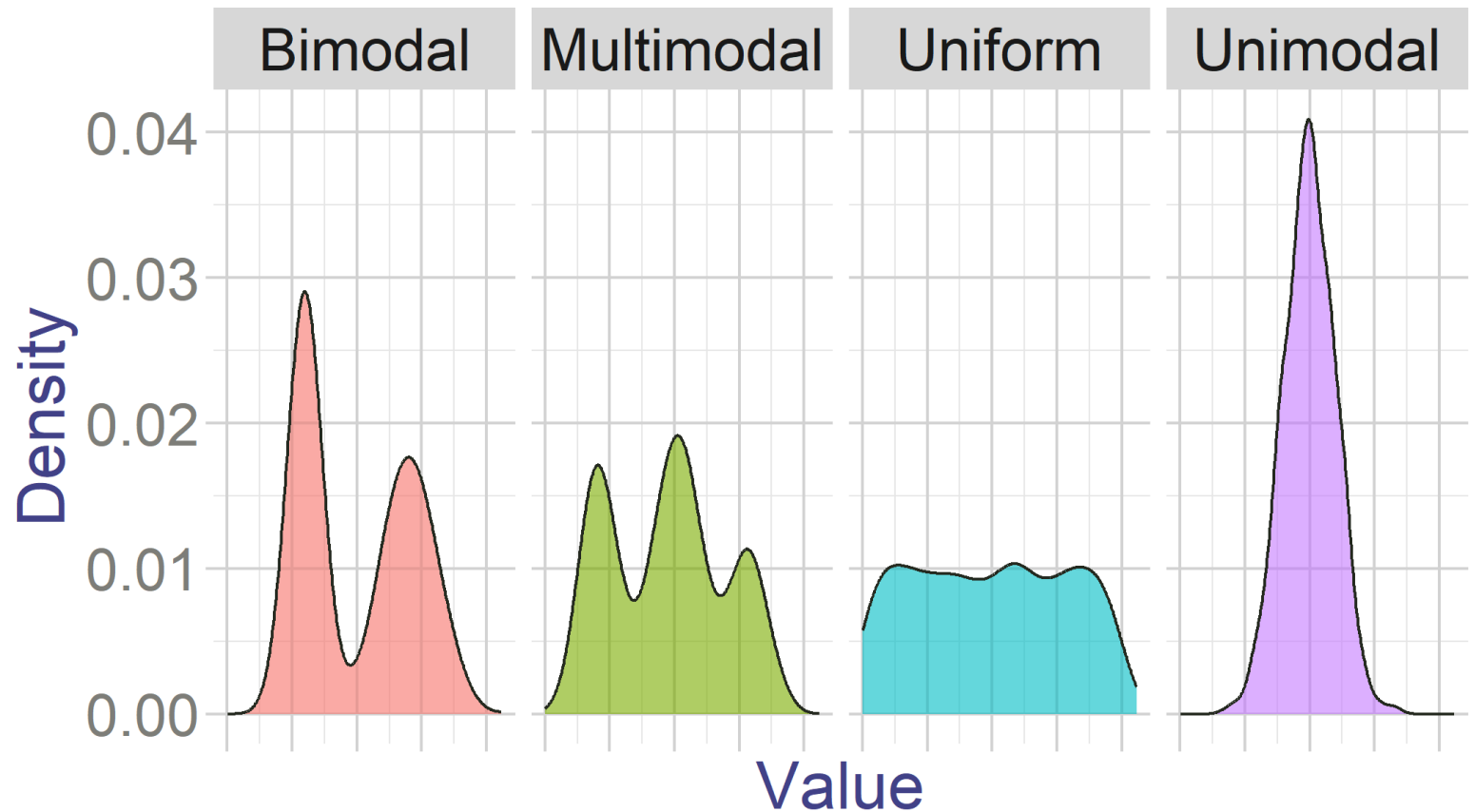
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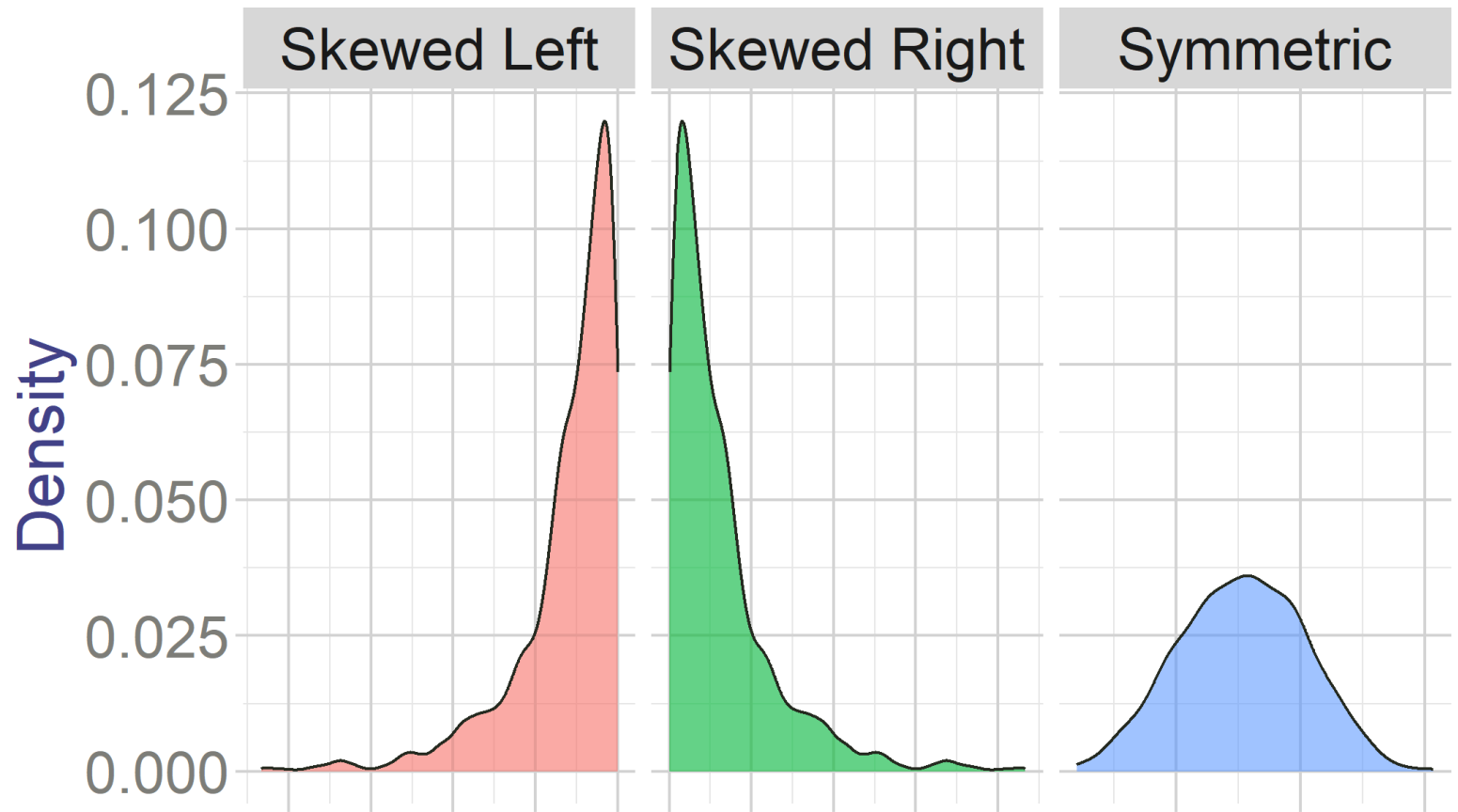
# Distribution Shapes: Modality



# Which is uniformly distributed

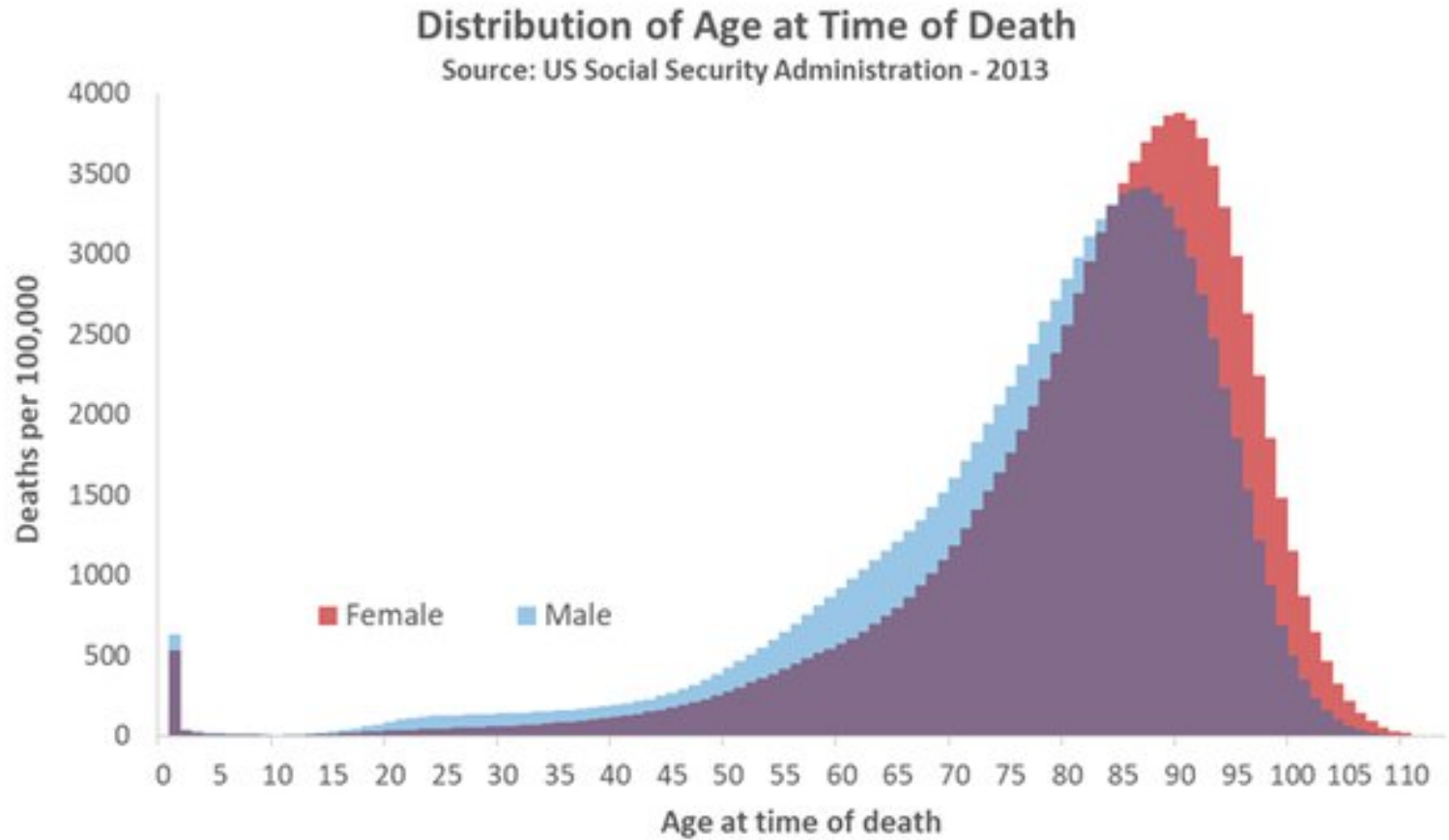
1. weights of adult females
2. salaries of a random sample of people from CDMX
3. House prices in CDMX
4. birthdays of classmates (day of the month)

# Distribution Shapes: Skewness





# Age at death



**We want to know how many people  
weight more than 100kg**

# Cumulative Distribution Function (CDF)

The **Cumulative Distribution Function** (CDF) gives the probability that a random variable  $X$  will take on a value less than or equal to a specific value.

For a continuous random variable  $X$  with PDF  $f(x)$ , the CDF  $F(x)$  is defined as:

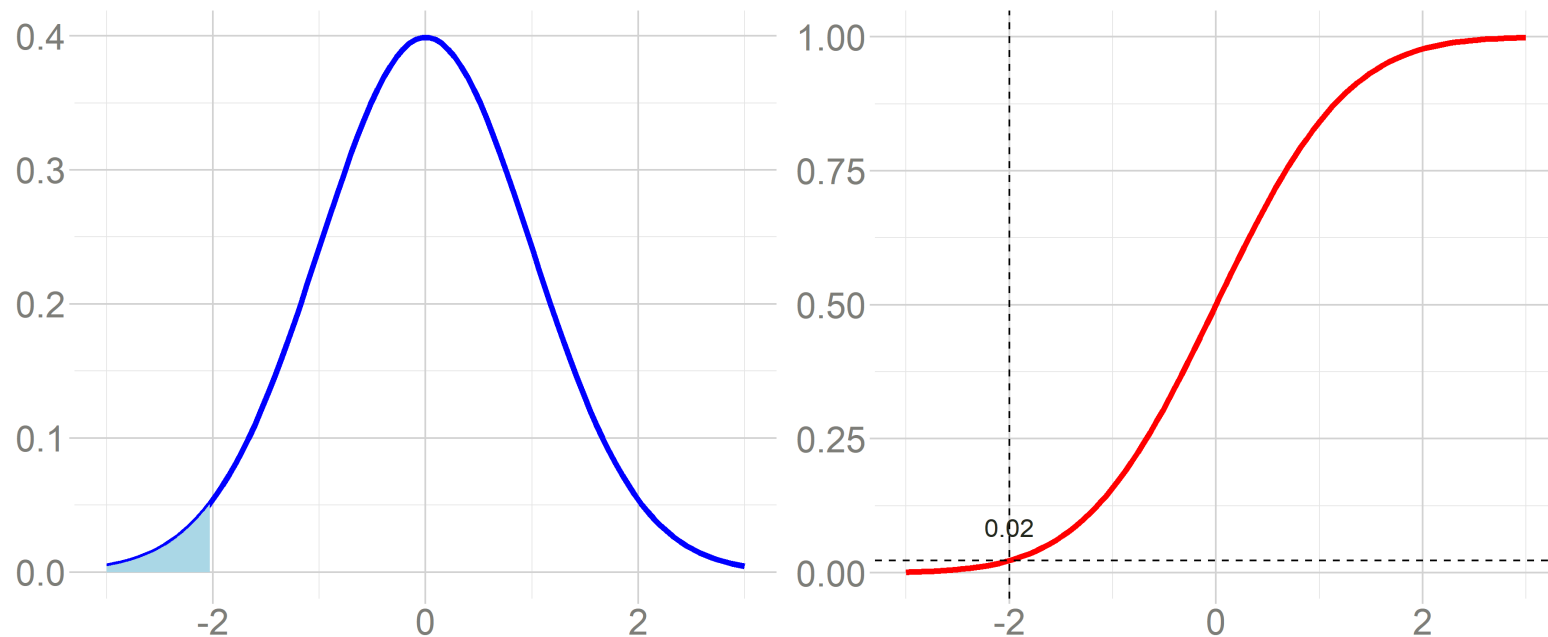
$$F(x) = \int_{-\infty}^x f(t) dt = P(X \leq x)$$

Characteristics:

- The CDF starts (for minus infinity) at 0 (minimum)
- It approaches 1 as  $x$  approaches infinity (maximum)
- It is non decreasing
- It is right continuous

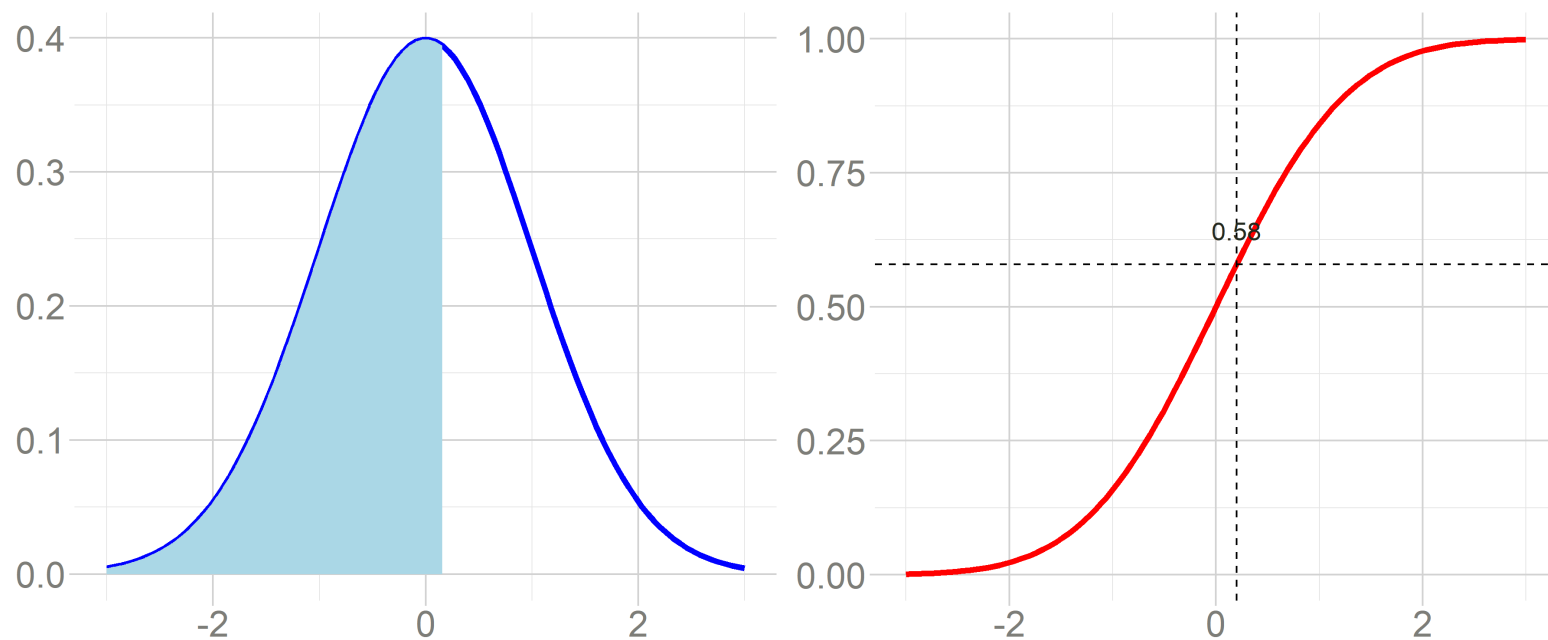
# Example 1: Standard Normal

$$F(-2) = \int_{-\infty}^{-2} f(t) dt = P(X \leq -2) = 0.02$$



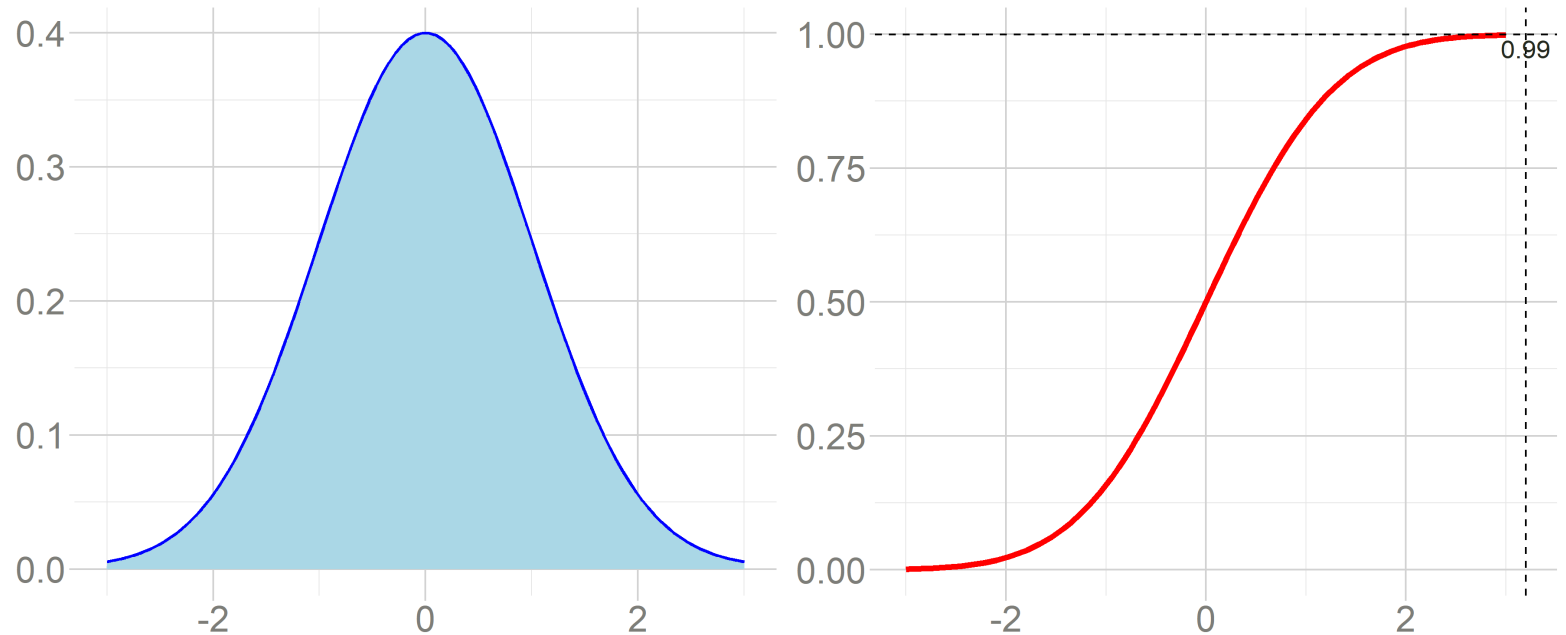
## Example 2: Standard Normal

$$F(0.2) = \int_{-\infty}^{0.2} f(t) dt = P(X \leq 0.2) = 0.58$$



# Example 3: Standard Normal

$$F(3.2) = \int_{-\infty}^{3.2} f(t) dt = P(X \leq 3.2) = 0.99$$

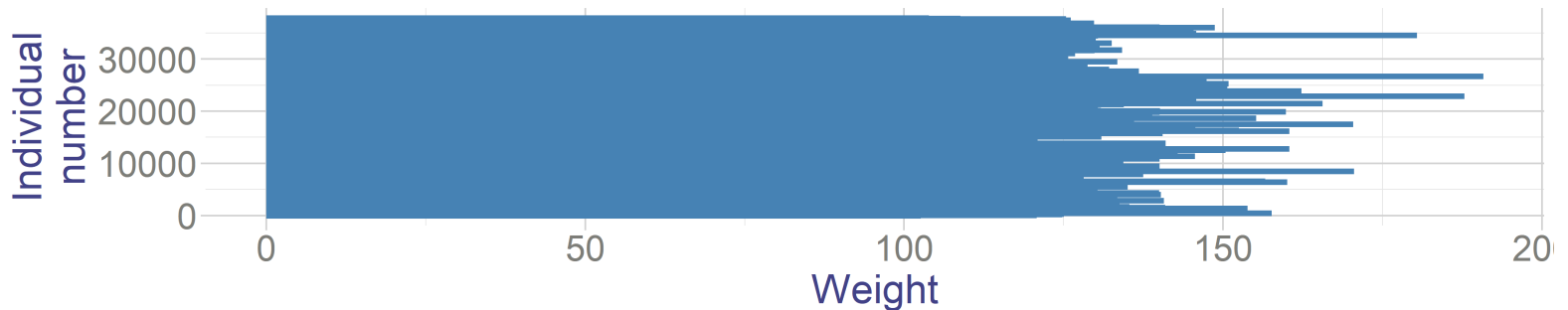


# Empirical CDF

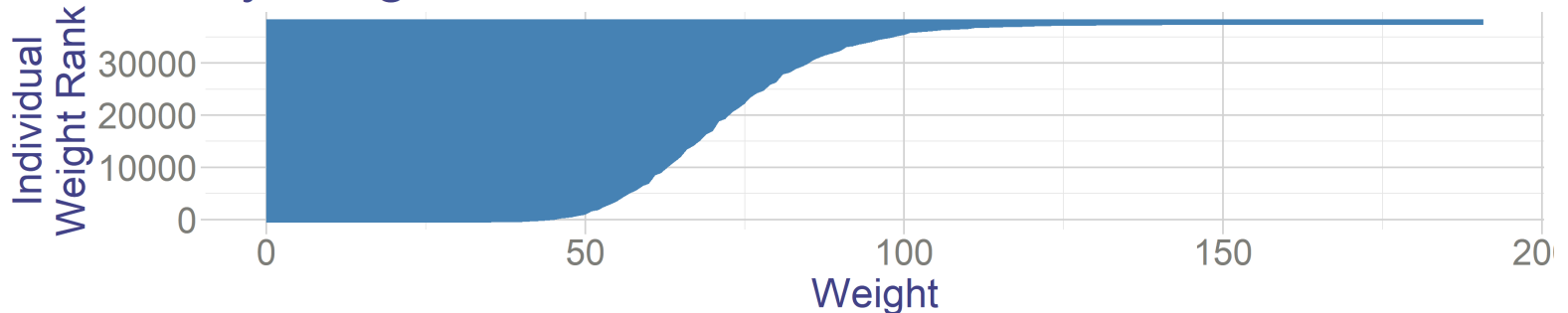
We can do similar thing with our weight data.

Intuition on how it comes up:

## Individual's weight



## Sorted by weight

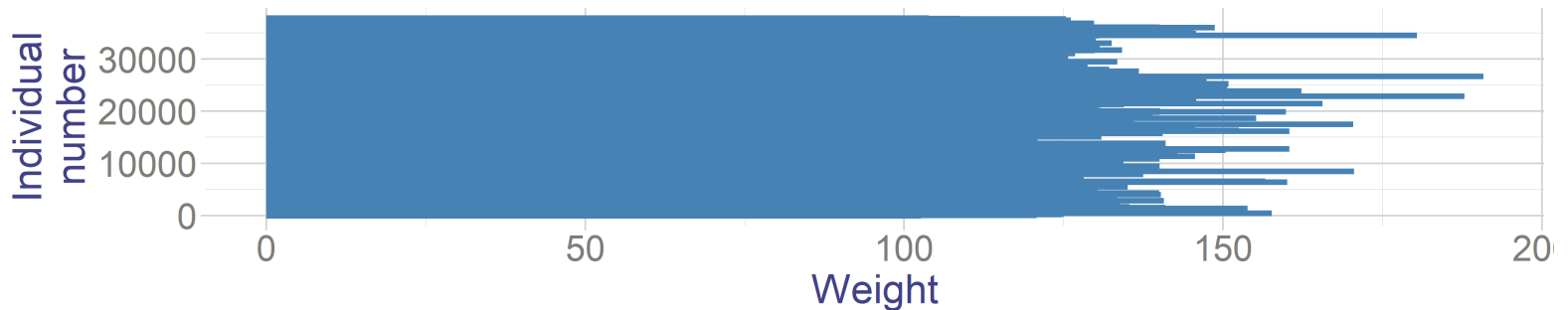


# Empirical CDF

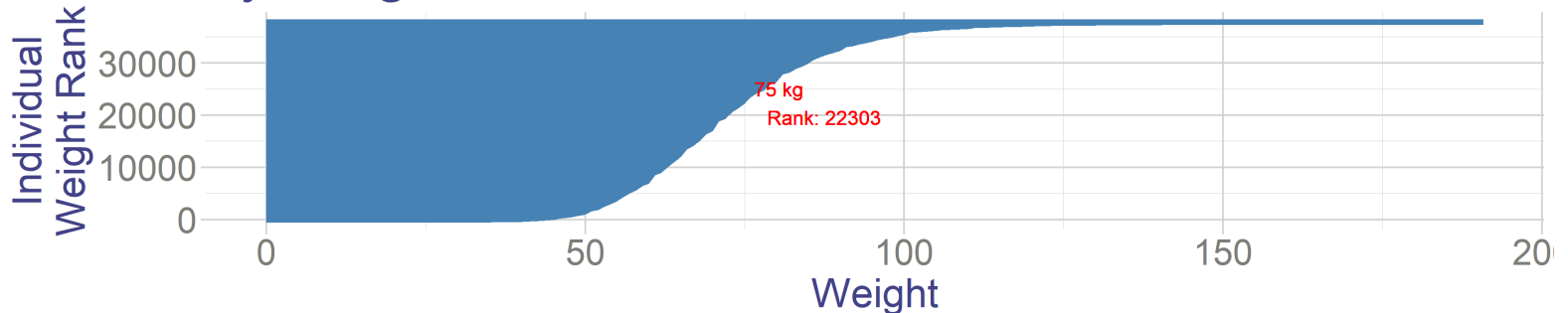
We can do similar thing with our weight data.

Intuition on how it comes up:

## Individual's weight



## Sorted by weight





# Empirical CDF

$$ECDF(x) = \frac{\sum I(w_i \leq x)}{N} = \frac{\text{Number of people with weight lower than } x}{N}$$

- $I(w_i < x) = 1$  if weight of person  $i$  is lower than  $x$  (*Indicator Function*)
- $N$  is total number of people (*Sample Size*)
- Share of people with weight lower than  $x$

- So how do we calculate share of people with weight > 100kg?  
 $P(\text{weight} > 100) = 1 - P(\text{weight} \leq 100) = 1 - ECDF(100)$

# Exercises:

- Review Exercises:
  - PDF 1: 1,2,3,4,5,6,7
  - PDF 2: 14,15,16,17

