Class 4b: Simple Linear Regression diagnostics

Business Forecasting

Roadmap

This class

- Testing assumptions behind residuals
  - Linearity
  - Constant Variance
  - Uncorrelated Residuals
  - Normality

Let's revisit the assumptions behind linear model:

1. Model is linear in the parameter and with additive error term
2. $E(u_i) = 0$
3. $E(u_i|x) = 0$
4. $Var(u_i) = \sigma^2$
5. $cov(u_i, u_j) = 0$

Additional assumption needed for hypothesis testing and confidence intervals:

1. $u_i \sim N(0, \sigma)$

Assumptions

Visualizing residuals, we can test:

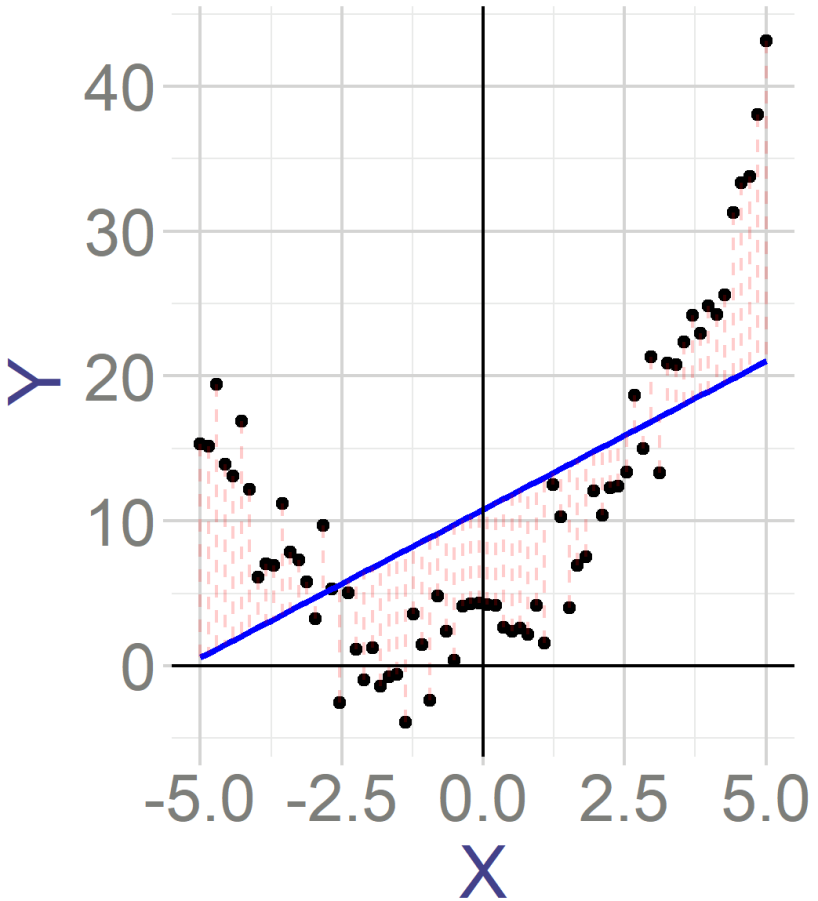
- Linearity
- Constant Variance (homoskedasticity)
- Uncorrelated errors

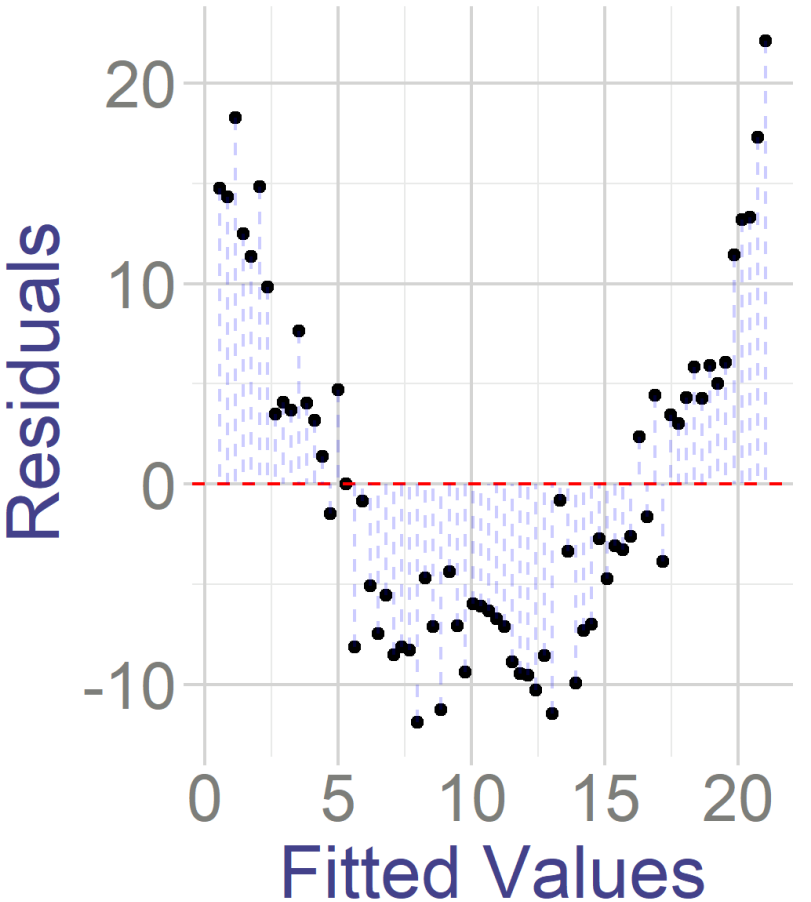
With vizuale and numerical tests we can analyze:

- Normality

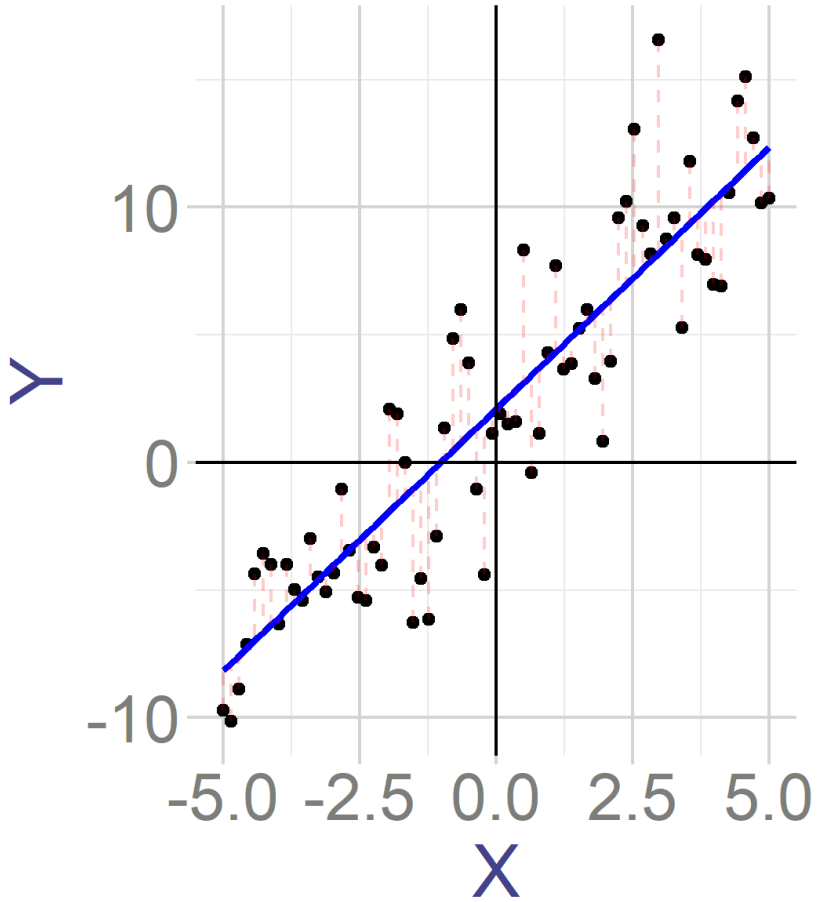
Linearity diagnostic

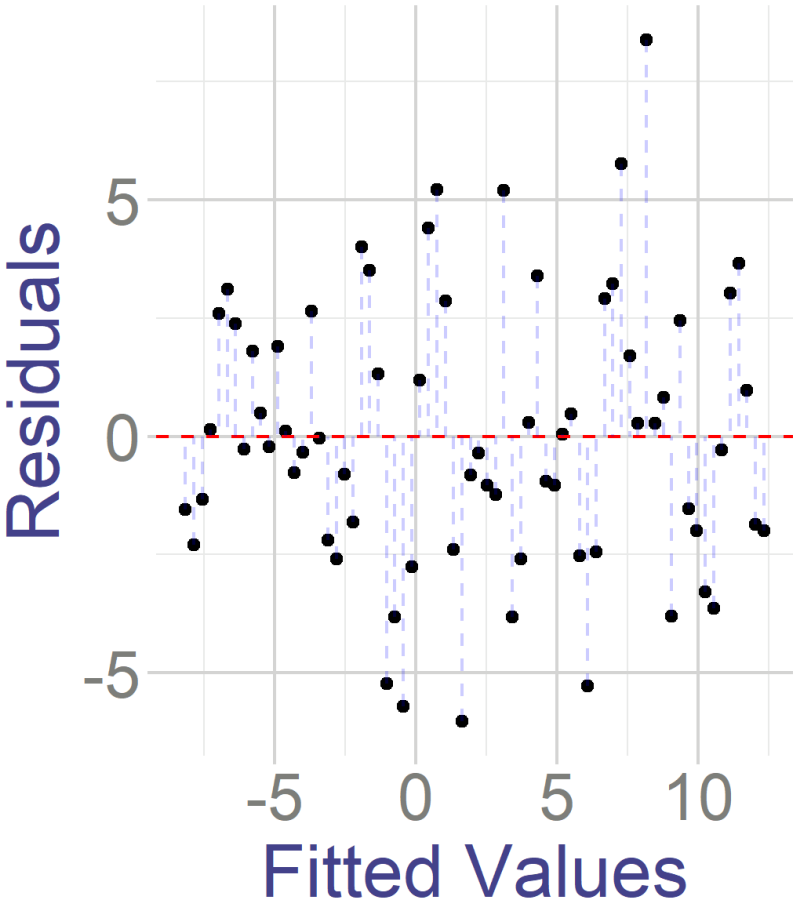
1. Plot residuals against fitted values of y $\hat{y}$
2. Check if residuals have systematic non-linear pattern





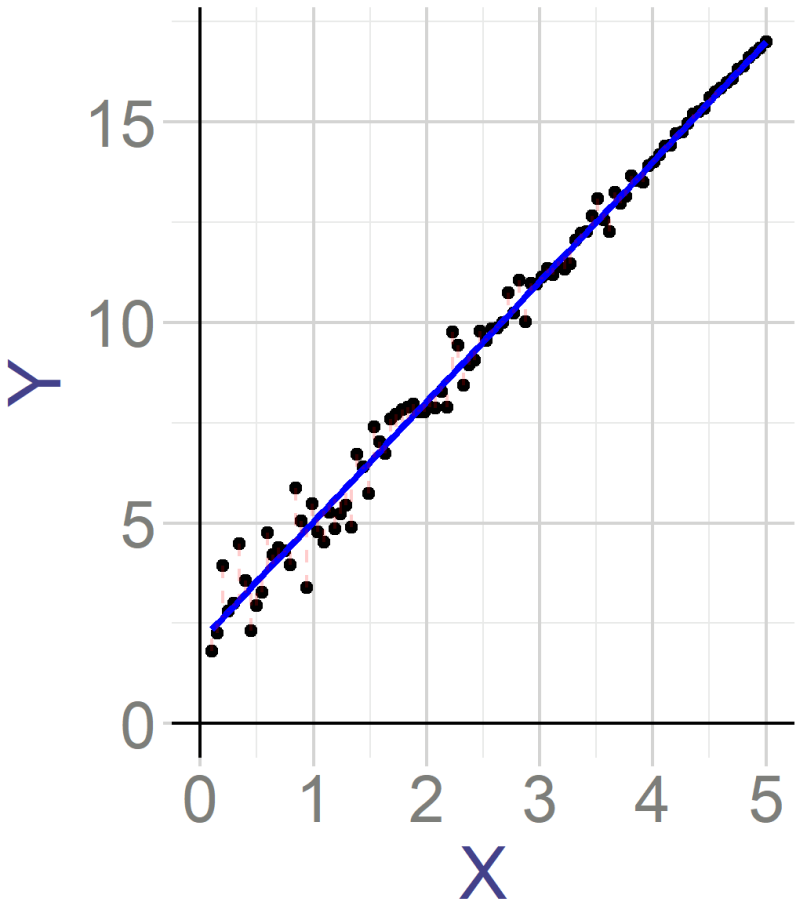
Linearity diagnostic

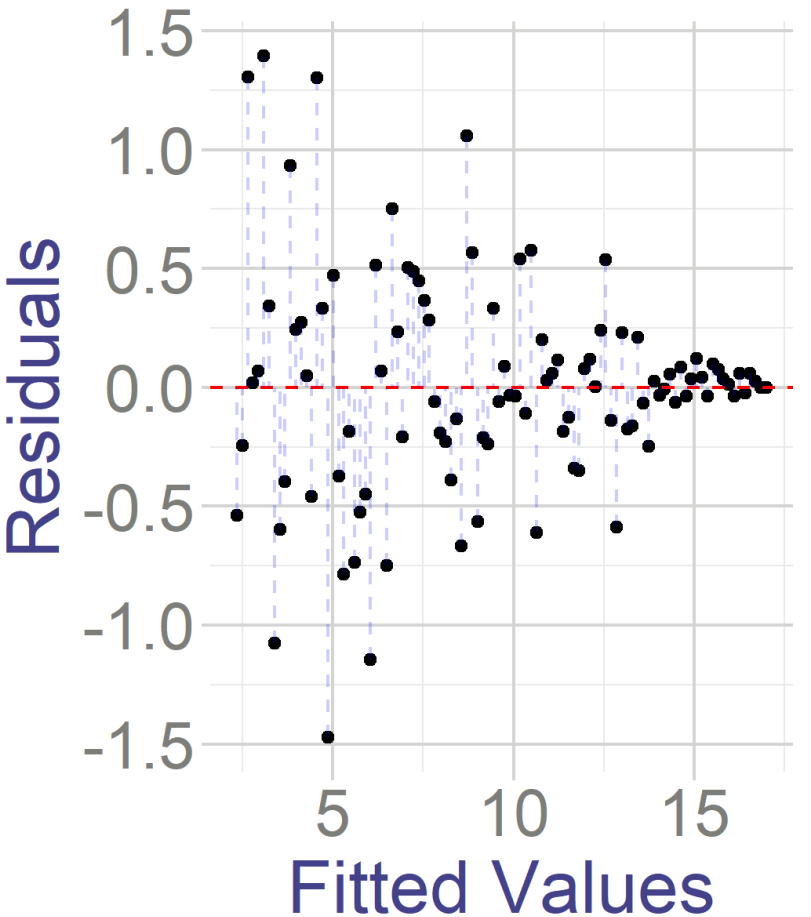




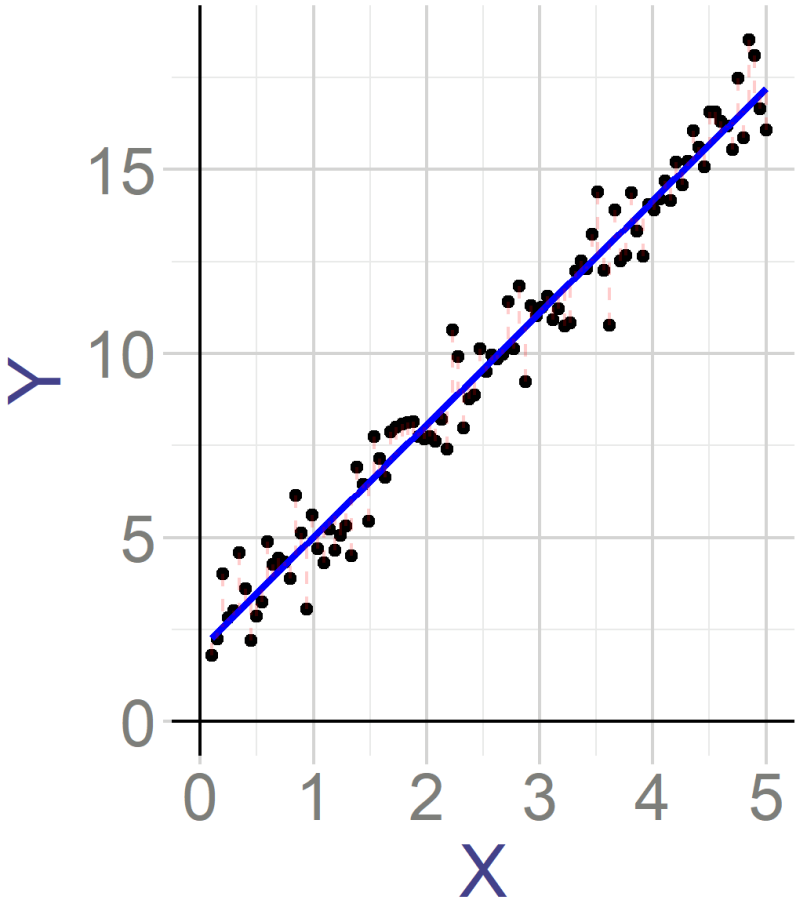
Constant variance (homoskedasticity) diagnostic

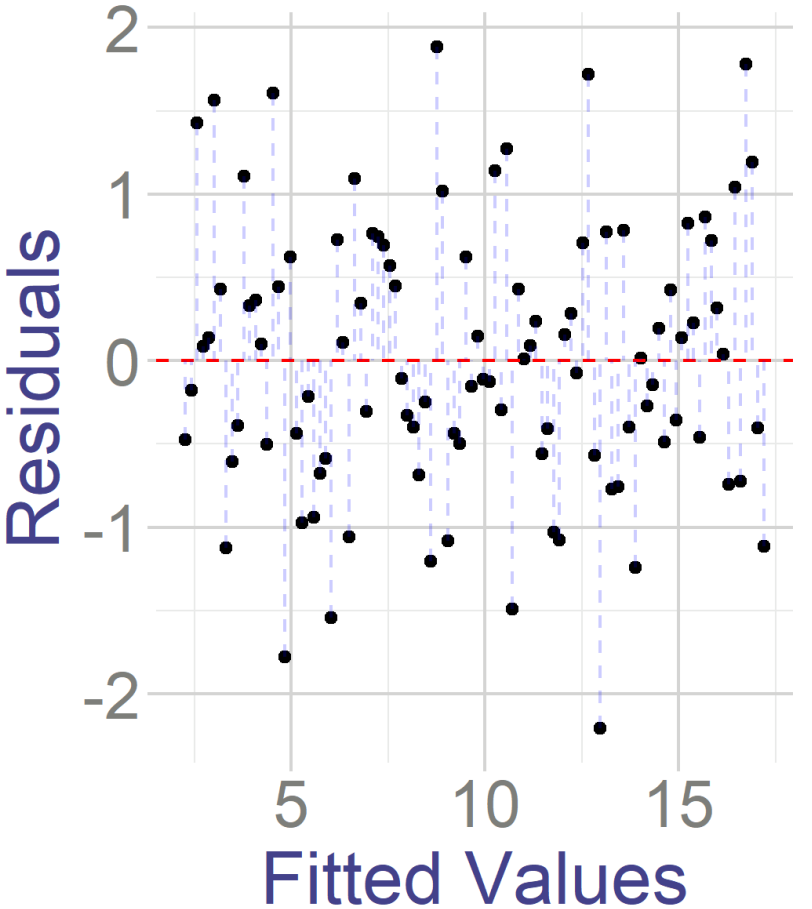
1. Plot residuals against fitted values of y $\hat{y}$
2. Check if variance changes as $\hat{y}$ changes (heteroskedasticity)



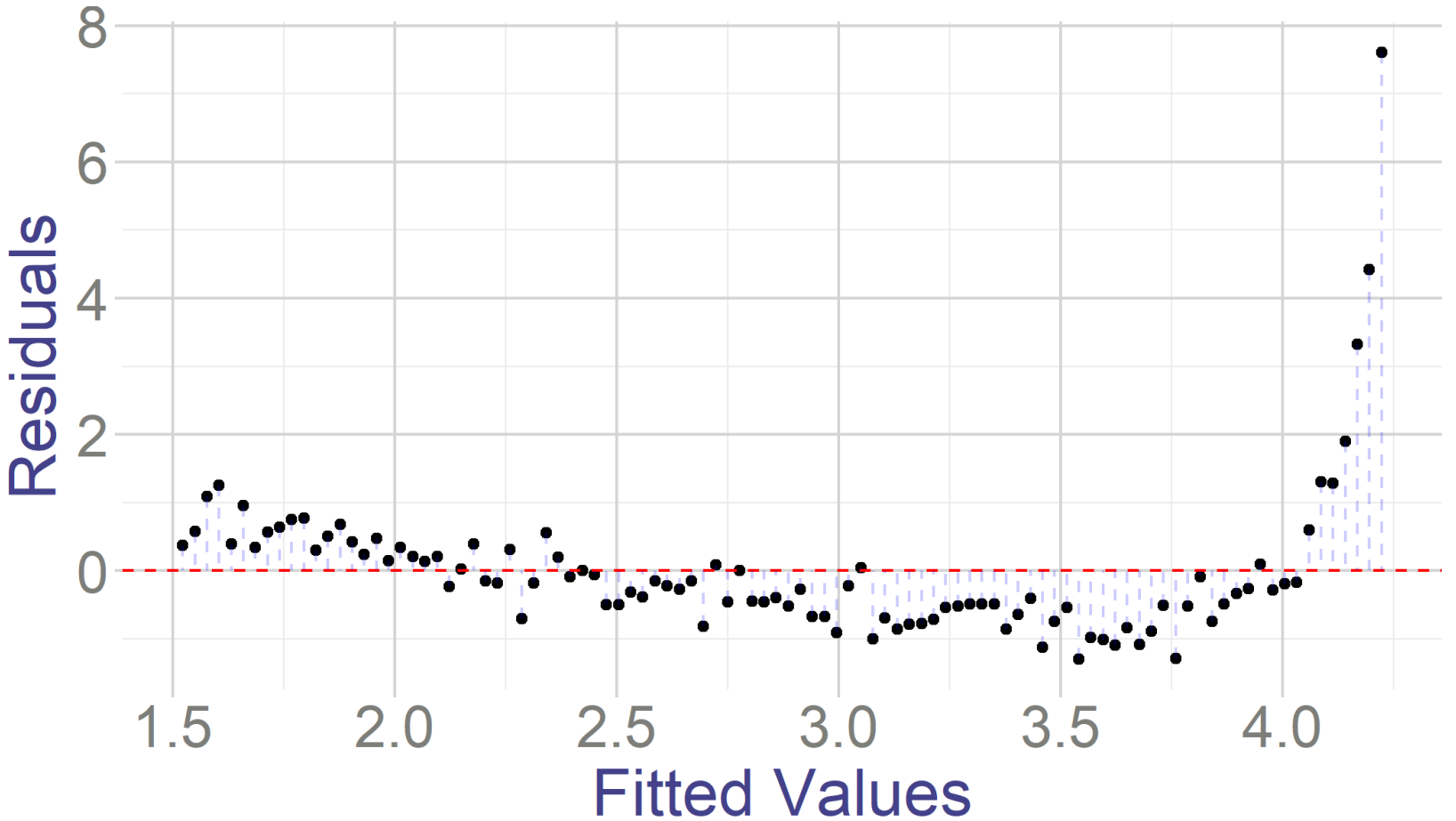


Constant variance (homoskedasticity) diagnostic

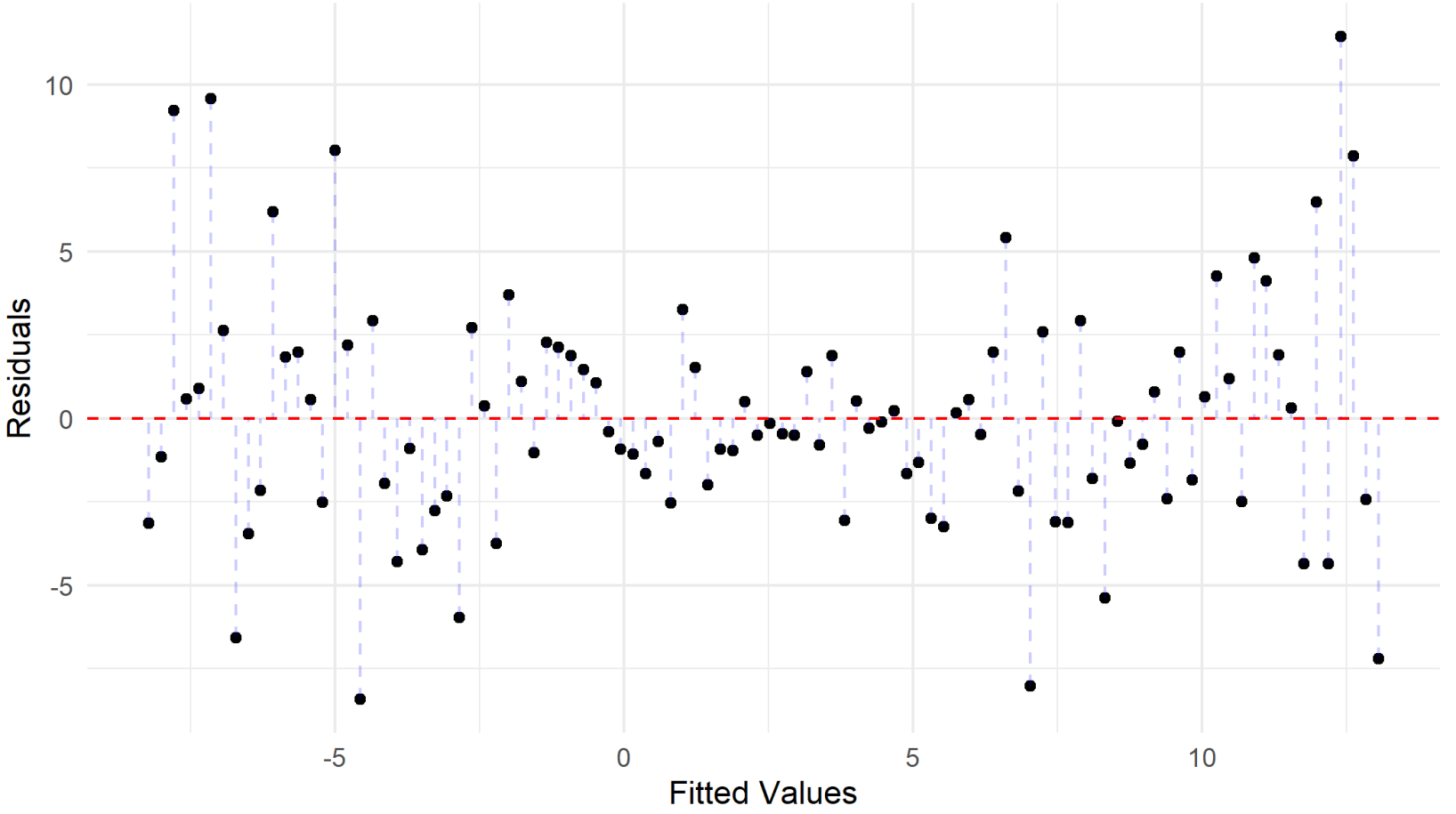




What goes wrong here?

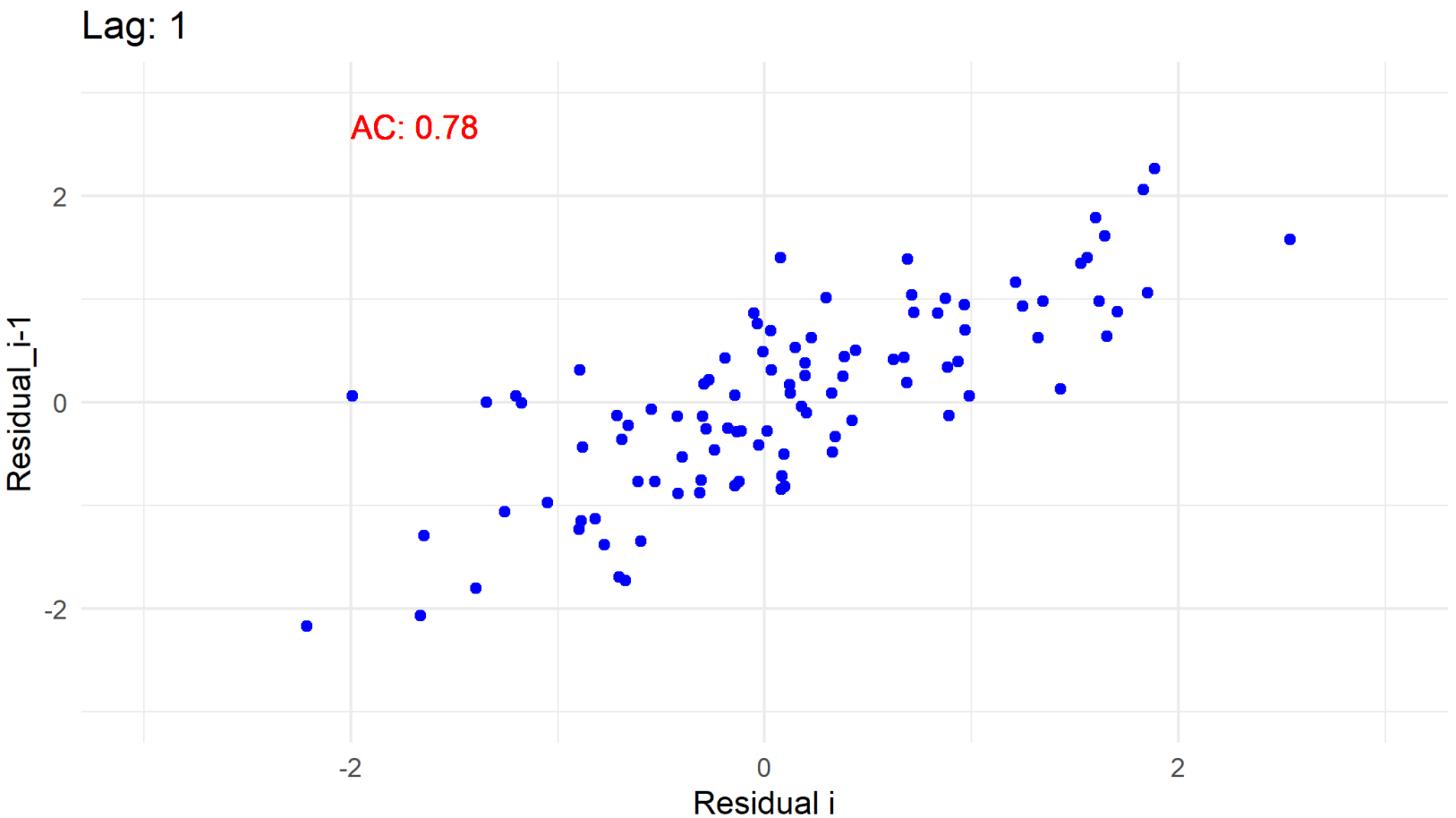


What goes wrong here?



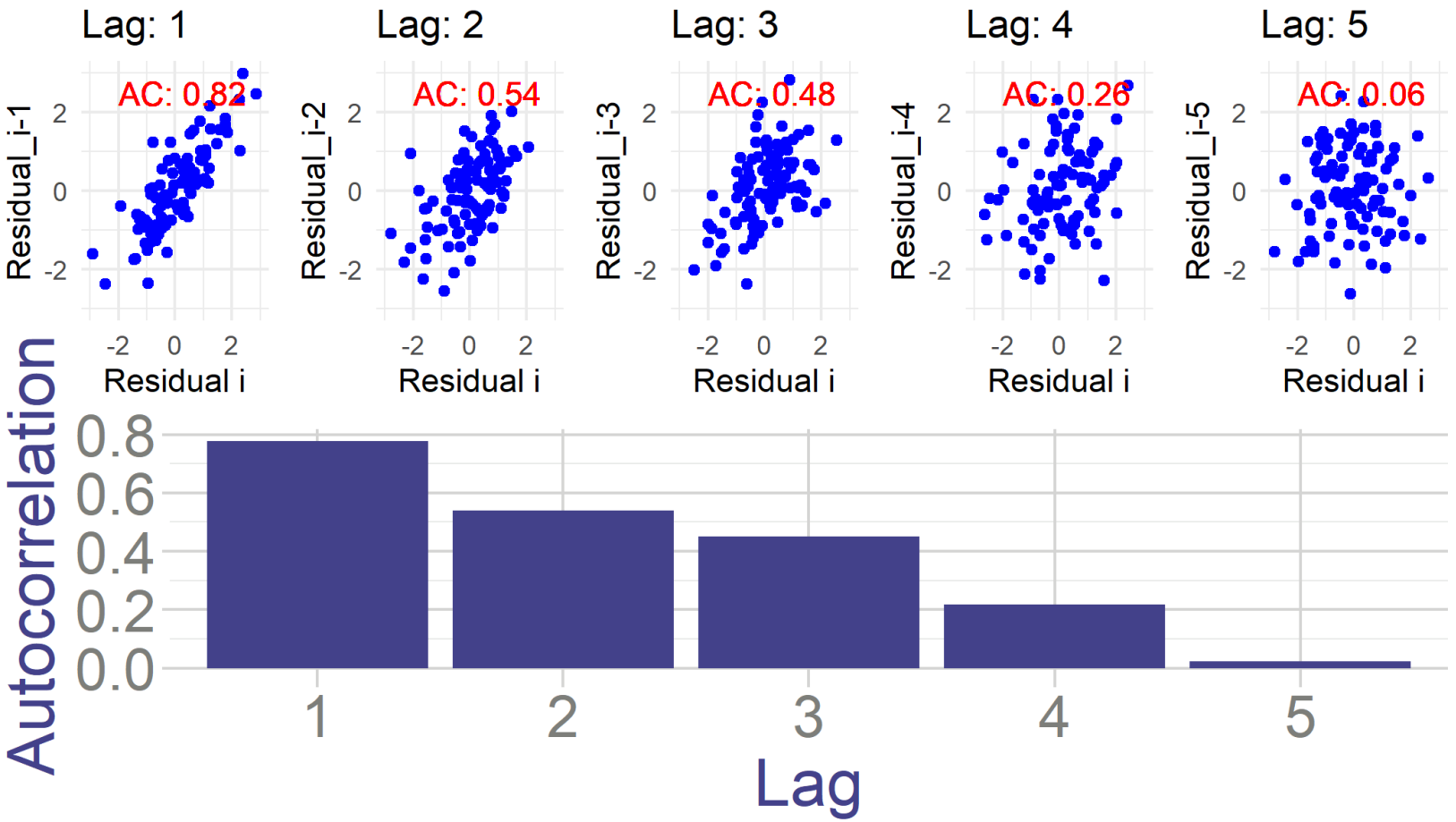
Correlation of error terms

1. Plot fitted residuals vs their value in previous observation e_i vs e_{i-1}
2. So e_2 vs e_1, e_5 vs e_4 etc



Correlation of error terms

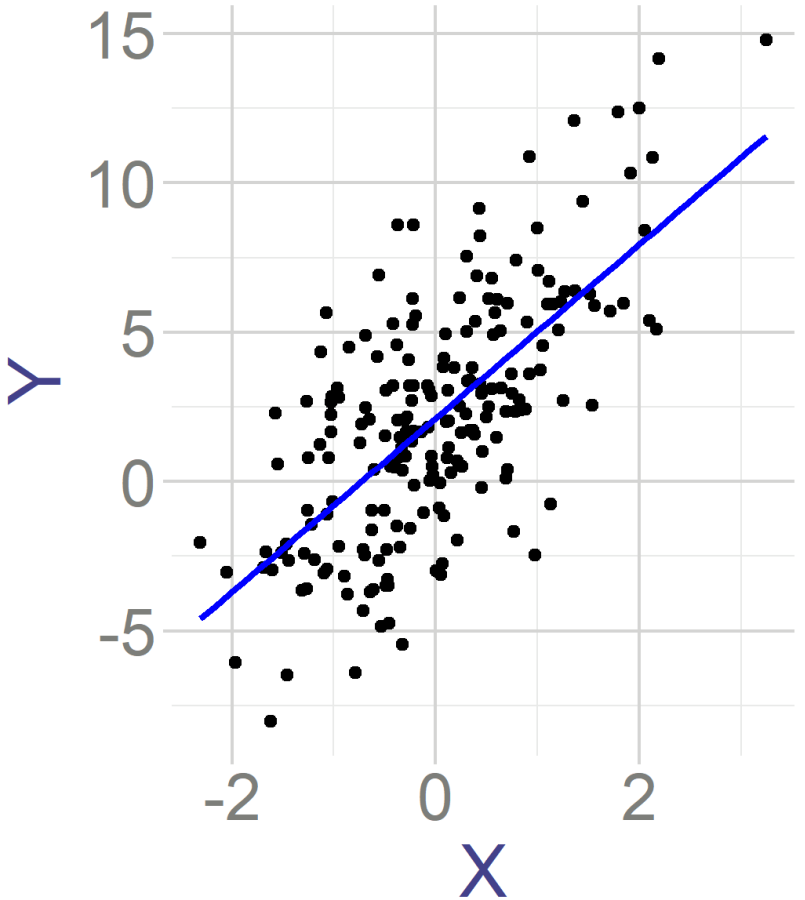
1. We can also calculate the correlation with other lags
   - Example: e_i vs e_{i-3}
   - Note down correlation at each lag: $\rho(e_i, e_{i-1})$, $\rho(e_i, e_{i-2})$, $\rho(e_i, e_{i-3})$

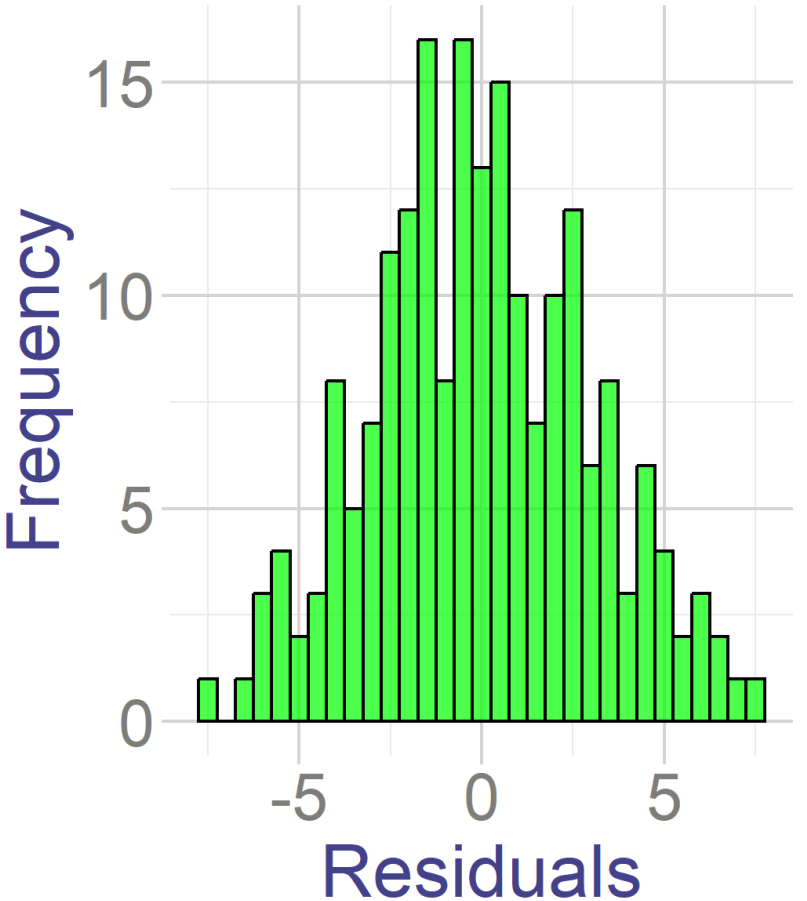


Normality

We can start by looking at histograms

- Some are obvious

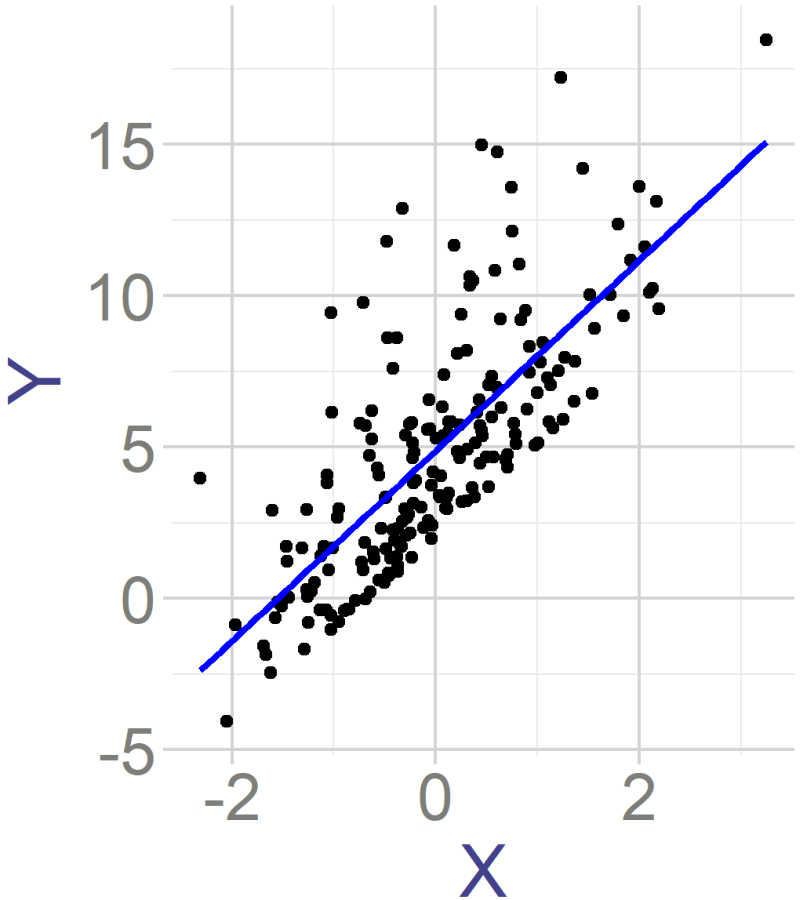


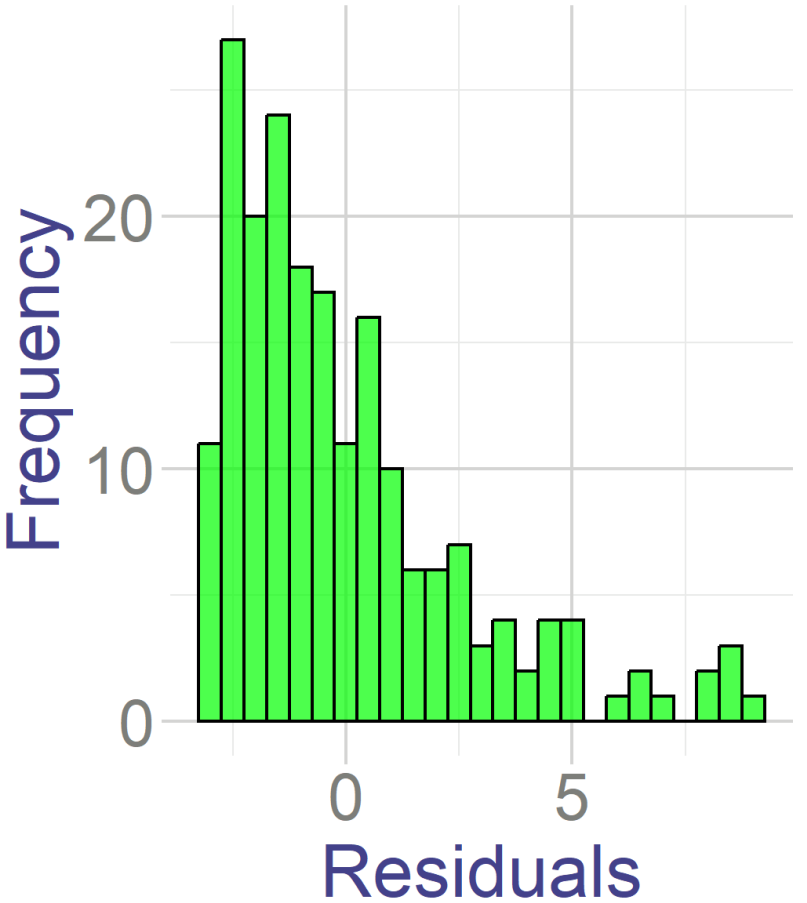


Normality

We can start by looking at histograms

- Some are obvious

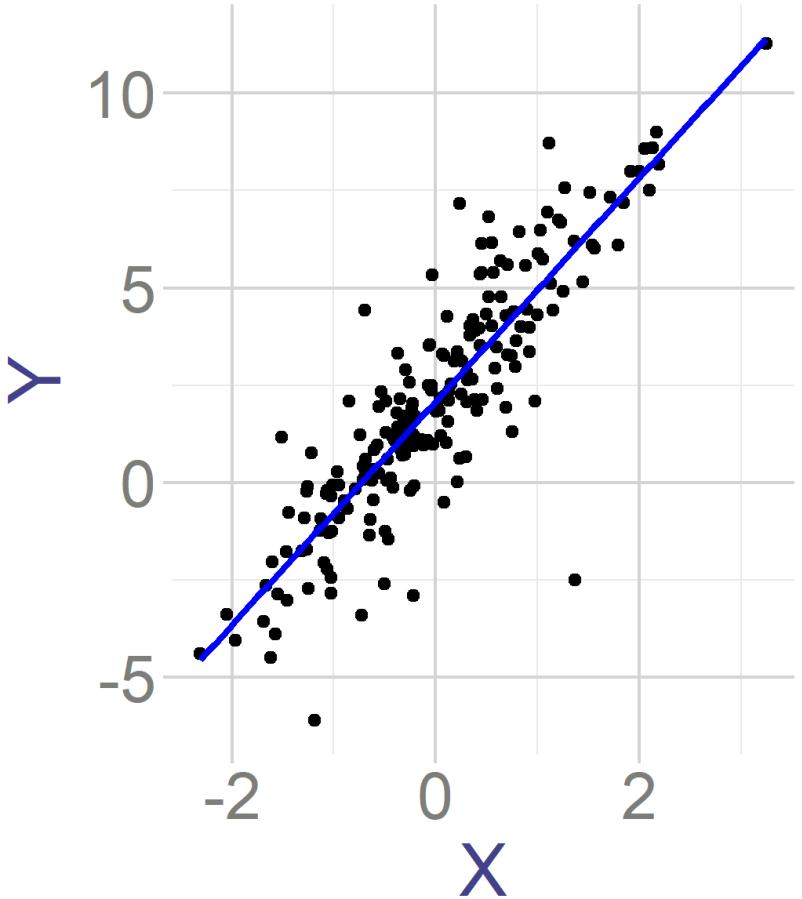


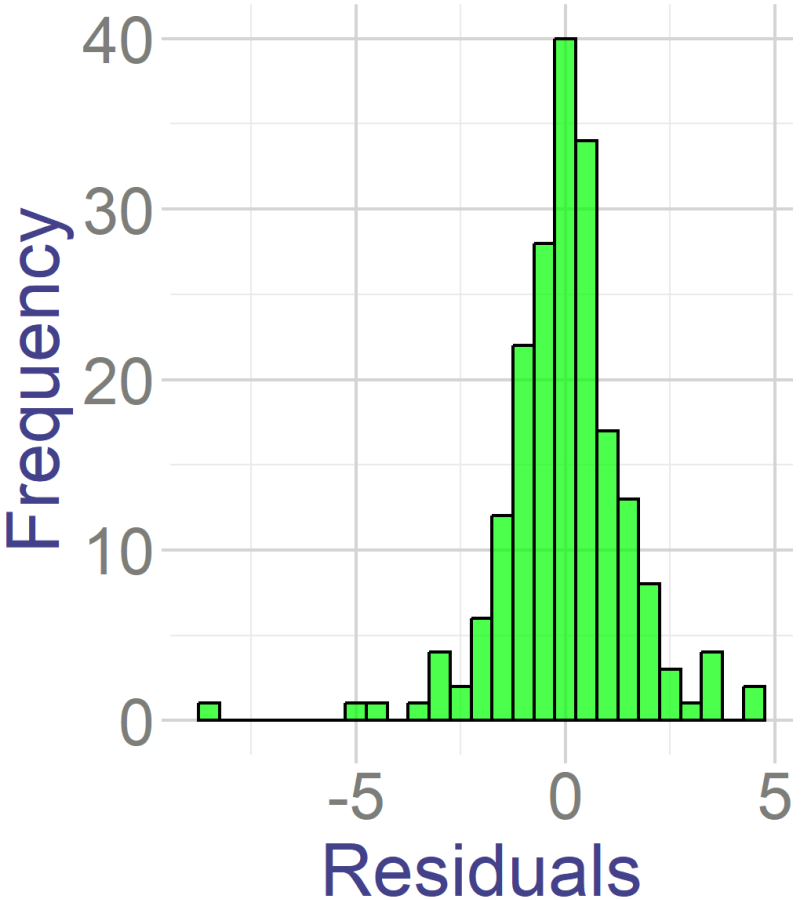


Normality

We can start by looking at histograms

- Some are **NOT** obvious
- Then we need a different test





Normality

Q-Q Plot: Quantlie-Quantile Plot

- Comparing quantiles of our data to what they should be if they come from normal distribution
- Procedure:
  - Standardize residuals
  - Sort them
  - Check which quantile they represent
  - This is equivalent to checking cumulative probability: $CP_i = \frac{Index}{n+1}$
  - Where index is the number of observation if we sort if from smallest to largest
    - 1 is the smallest, 2 is the second smallest, ..., n is the largest
  - In other words, what share of data is smaller than this observation
  - Compare them to quantiles from standard normal
  - If our distribution is normal, the quantiles should be similar

Q-Q plot

Show 10 entries

Residuals	Standardized_Residuals	Index	Cumulative_Probability	Quantile_Normal
-1.681	-0.713	23	0.228	-0.745
-0.691	-0.351	39	0.386	-0.29
4.676	1.609	95	0.941	1.563
0.212	-0.022	51	0.505	0.013
0.388	0.043	54	0.535	0.088
5.145	1.78	96	0.95	1.645
1.383	0.406	69	0.683	0.476
-3.795	-1.485	5.5	0.054	-1.607
-2.061	-0.852	19	0.188	-0.885
-1.337	-0.587	29	0.287	-0.562

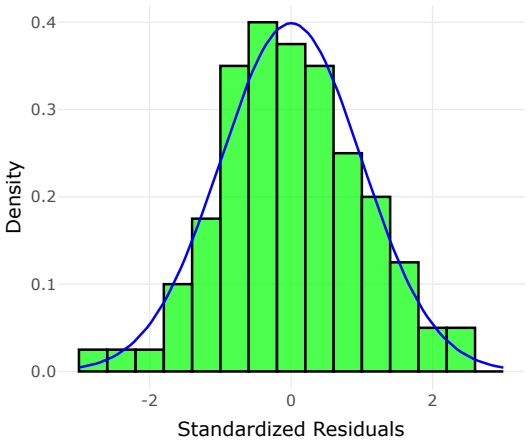
Showing 1 to 10 of 100 entries

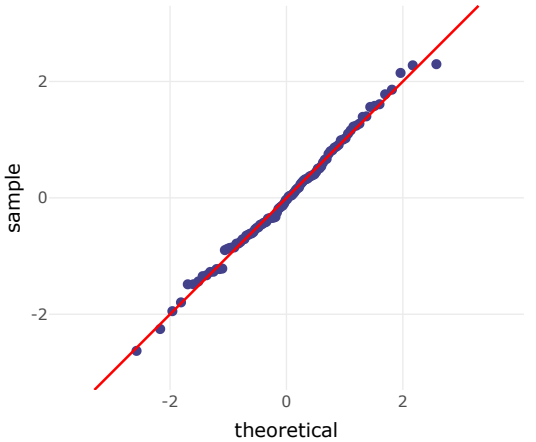
Previous 1 2 3 4 5 ... 10 Next

Q-Q plot

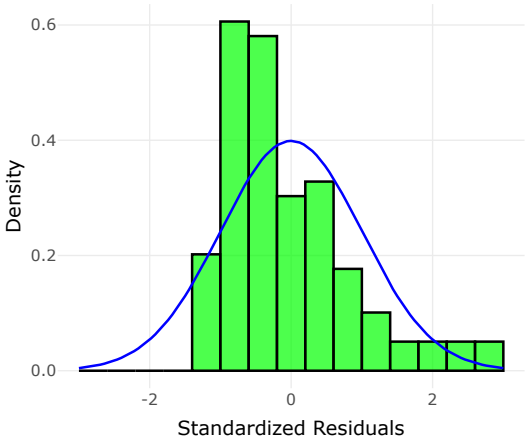
- Next, we plot the sample quantiles vs the standard normal quantiles
- If they are similar, they should be on the straight line

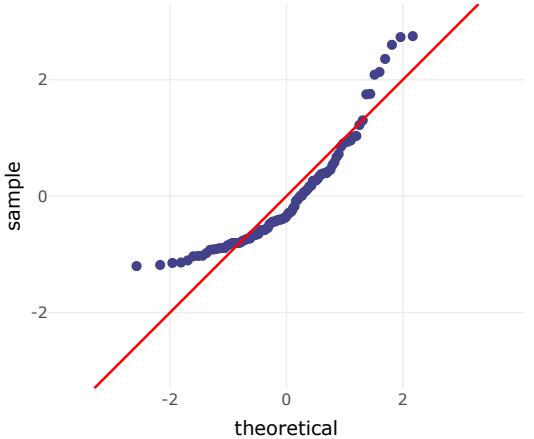
Q-Q plot



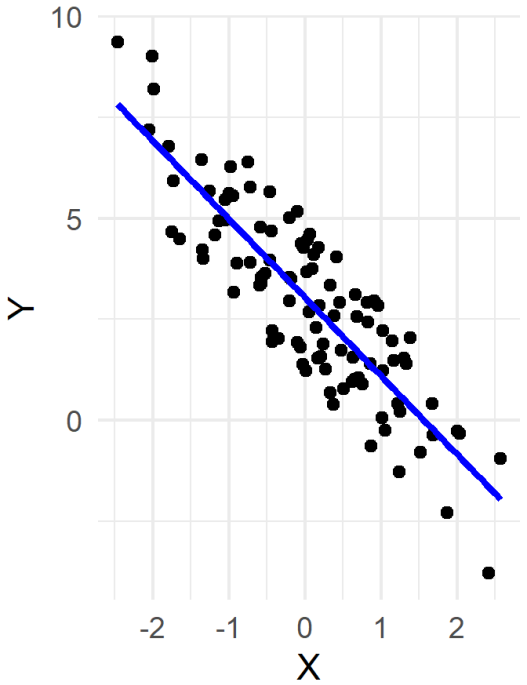


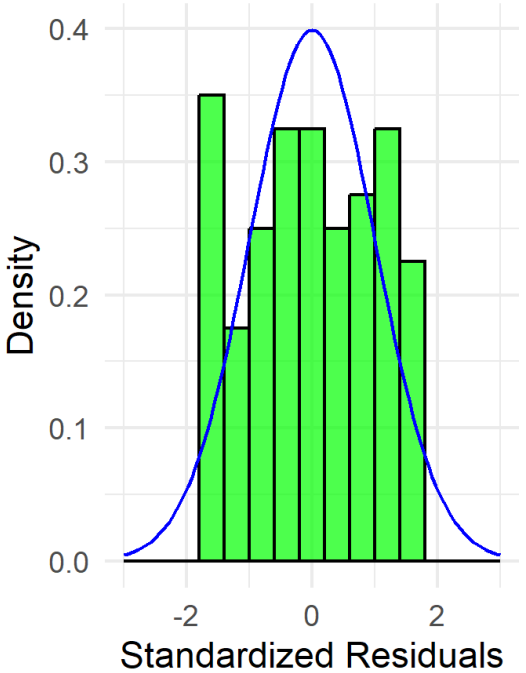
Q-Q plot

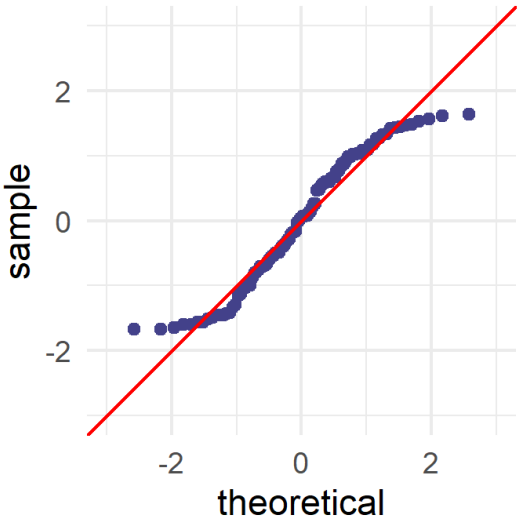

0.6
0.4
0.2
0.0
Density
-2
0
2
Standardized Residuals


2
0
-2
sample
-2
0
2
theoretical

Q-Q plot


10
5
0
Y
-2
-1
0
1
2
X


0.4
0.3
0.2
0.1
0.0
Density
-2
0
2
Standardized Residuals


2
0
-2
sample
-2
0
2
theoretical

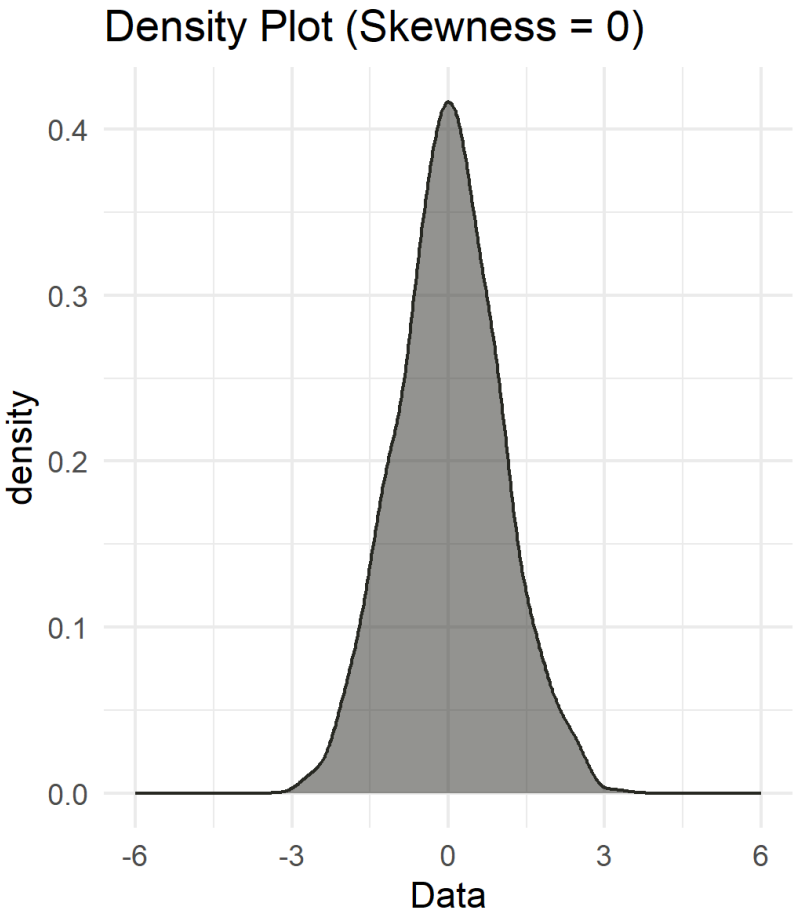
Jarque-Bera Test

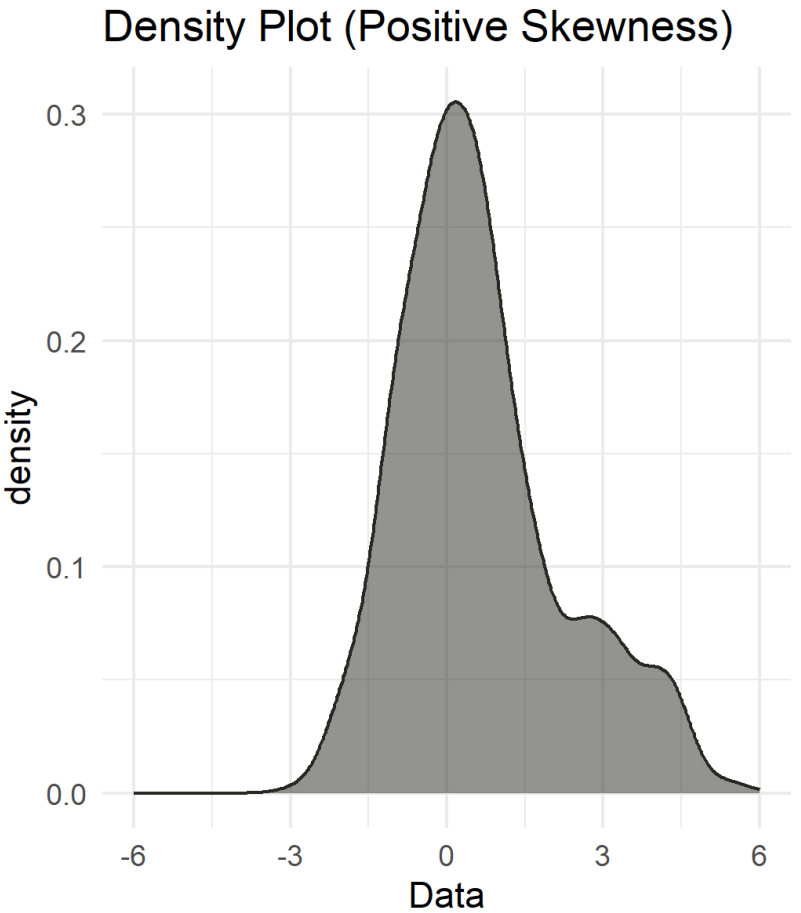
- We can also use the **Jarque-Bera Test** to see whether sample comes from a normal distribution

Intuition

- Skeweness of normal distribution is 0 (it's symmetric)

$$S = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{\frac{3}{2}}}$$


Density Plot (Skewness = 0)
density
0.0
0.1
0.2
0.3
0.4
-6
-3
0
3
6
Data


Density Plot (Positive Skewness)
density
0.0
0.1
0.2
0.3
-6
-3
0
3
6
Data

Jarque-Bera Test

- We can also use the **Jarque-Bera Test** to see whether sample comes from a normal distribution

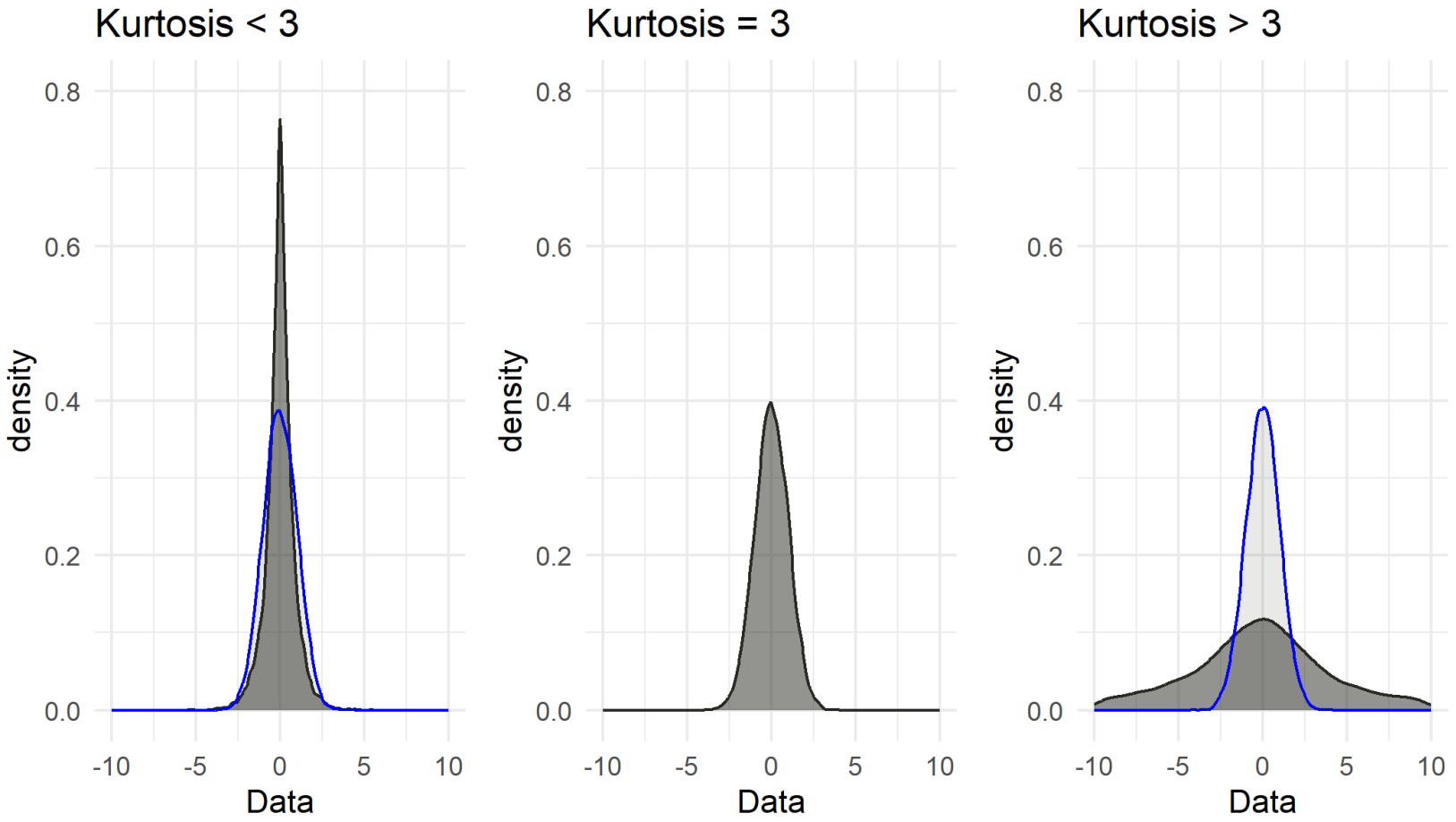
Intuition

- Skeweness of normal distribution is 0 (it's symmetric)

$$S = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{\frac{3}{2}}}$$

- Kurtosis of normal distribution is 3
- Excess kurtosis is 0

$$EK = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2} - 3$$


Kurtosis < 3
Kurtosis = 3
Kurtosis > 3
density
0.0
0.2
0.4
0.6
0.8
-10
-5
0
5
10
Data

Jarque-Bera Test

- We can also use the **Jarque-Bera Test** to see whether sample comes from a normal distribution

Intuition

- Skeweness of normal distribution is 0 (it's symmetric)

$$S = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{\frac{3}{2}}}$$

- Kurtosis of normal distribution is 3
- Excess kurtosis is 0

$$EK = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2} - 3$$

- We will test, whether our sample has (more or less)
  - Skewenss of 0
  - Kurtosis of 3

Jarque-Bera Test

The test statistic for our test is:

$$JB = \frac{n}{6}\left(\frac{S^2}{2} + \frac{EK^2}{4}\right)$$

- Its value will be high if:
  - Skeweness deviates significantly from 0
  - Kurtosis deviates significantly from 3
- If the data really comes from normal (that's our null hypothesis), then:

$$JB \sim \chi_2$$

- It follows the Chi-squared distribution with 2 degrees of freedom.

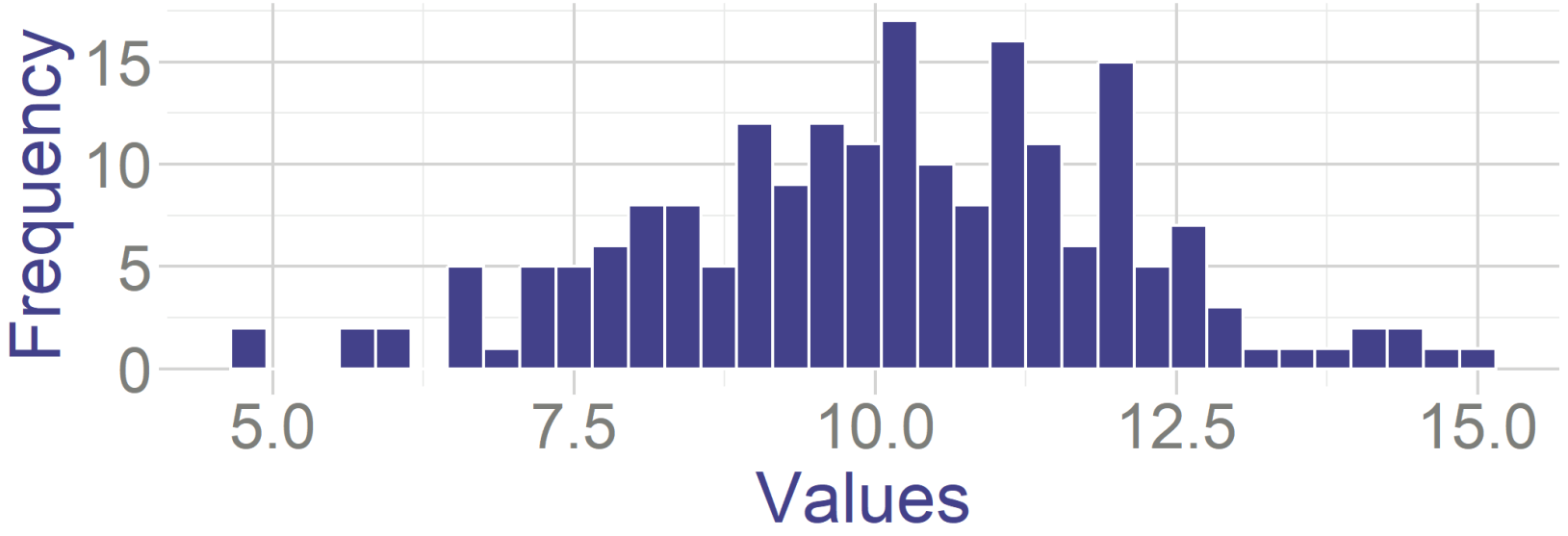
Jarque-Bera Test

So in our usual testing setting:

- H_0 : e_i comes from normal (JB is small)
- H_A : e_i Does not come from normal (JB is large)
- It's a one sided test, so we reject at α if

$$JB_{test} > \chi_{1-\alpha,2}$$

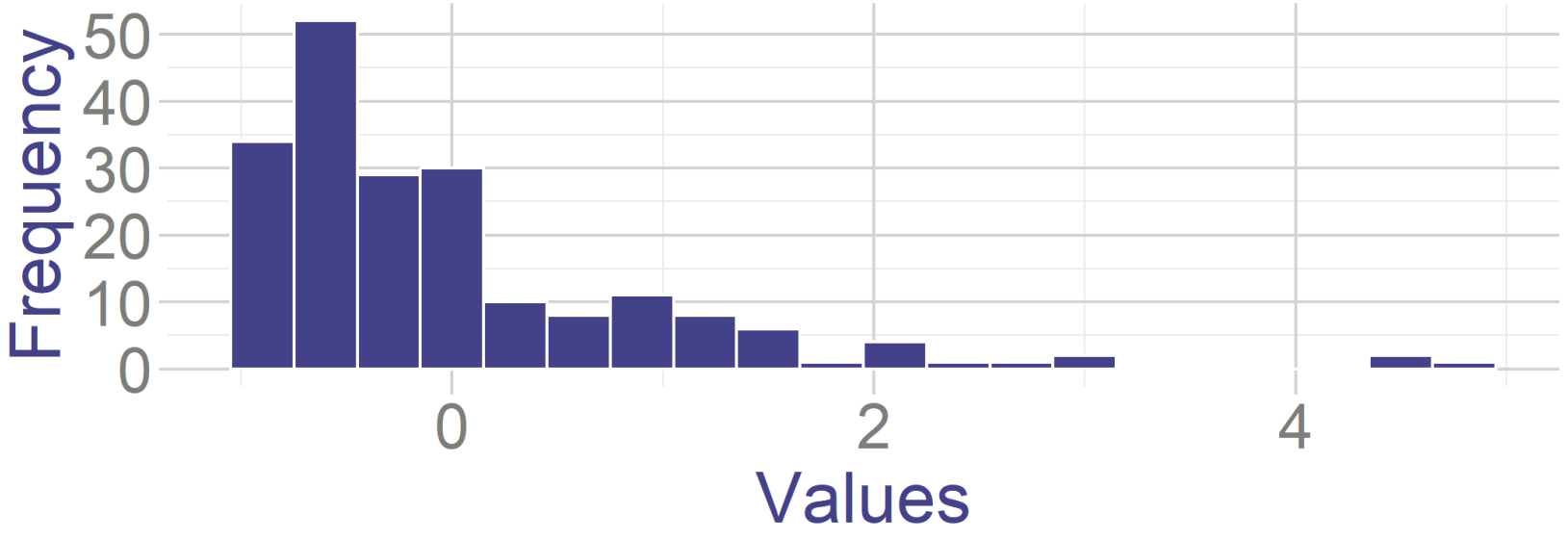
Jarque-Bera Test




```
library(tseries)
jarque.bera.test(sample_data)
```

```
##
##     Jarque Bera Test
##
## data:  sample_data
## X-squared = 0.96619, df = 2, p-value = 0.6169
```

Jarque-Bera Test




```
library(tseries)
jarque.bera.test(sample_data)
```

```
##
##     Jarque Bera Test
##
## data:  sample_data
## X-squared = 420.19, df = 2, p-value < 2.2e-16
```