Class 2d: Review of concepts in Probability and Statistics

Business Forecasting

Summarizing Data

Comparisions and Associations

Comparisions

- Descriptive and visual comparisons
- NOT declaring statistically significant differences, just eyeballing
- That's coming next

Comparing categorical variables

Does mother's diabetes predict diabetes?

- We have two categorical variables
- We can use frequency table to see how diabetes is distributed among groups with healthy vs diabetic mother

	No Diabetes	Has Diabetes
Mother No Diabetes	25270	2427
Mother Has Diabetes	8283	1721

Comparing categorical variables

Does mother's diabetes predict diabetes?

- Are relative frequencies more helpful?
- Share of each subgroup within the sample

	No Diabetes	Has Diabetes	Total
Mother No Diabetes	0.67	0.06	0.73
Mother Has Diabetes	0.22	0.05	0.27
Total	0.89	0.11	1.00

- Can we compare numbers in the *Has Diabetes* column?
- **Marginal frequencies** are total probabilities by group

Table of frequency

- We want to compare whether someone with diabetic mother is more likely to have diabetes than someone with healthy mother
- So we want to see whether:

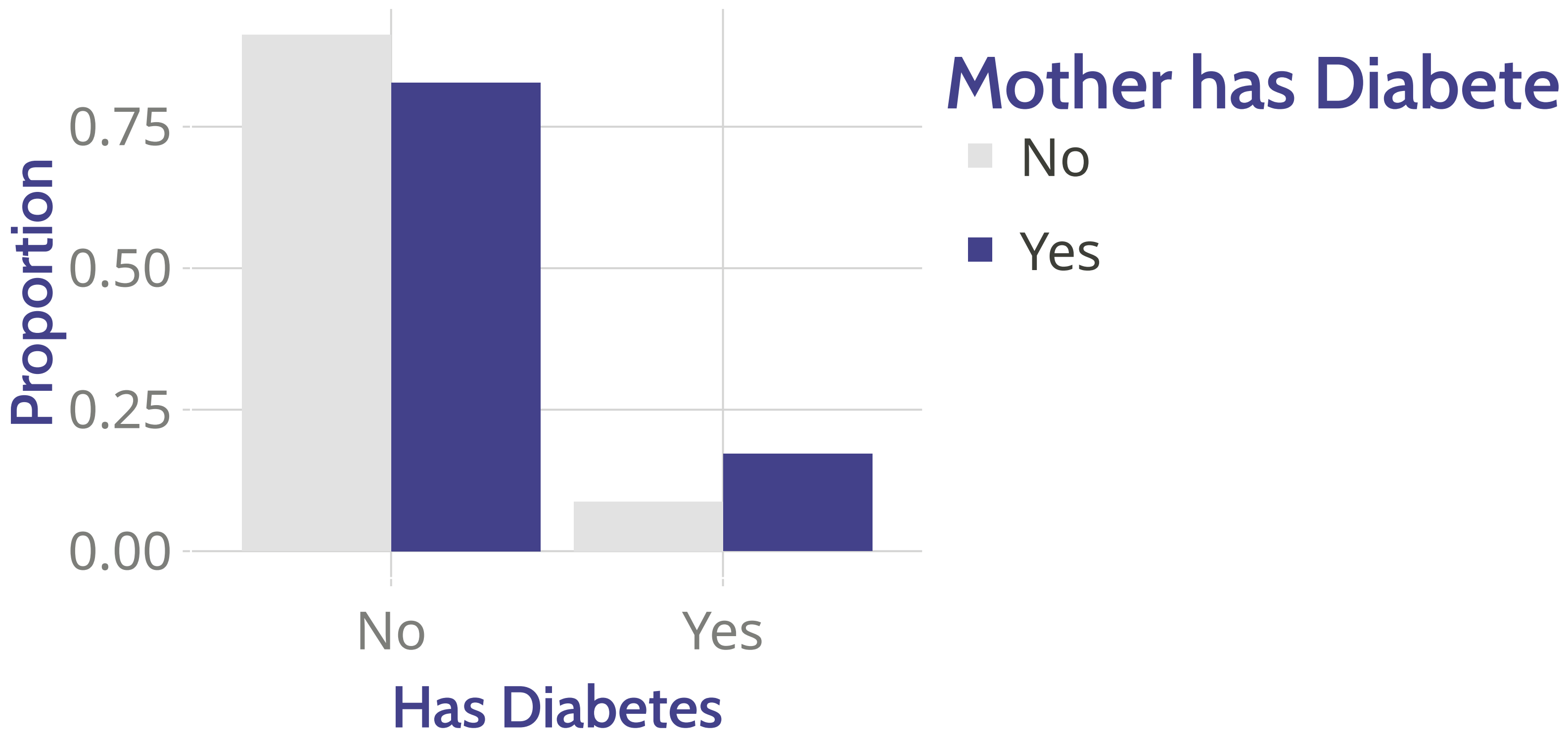
$$P(Diabetes_i = 1|\text{Mother has diabetes}_i = 1) > P(Diabetes_i = 1|\text{Mother has diabetes}_i = 0)$$

- We want to look at the **relative conditional frequencies**
- They are usually in **contingency tables**
  - Share with diabetes within group of people whose mothers have diabetes
  - Share with diabetes within group of people whose mothers are healthy

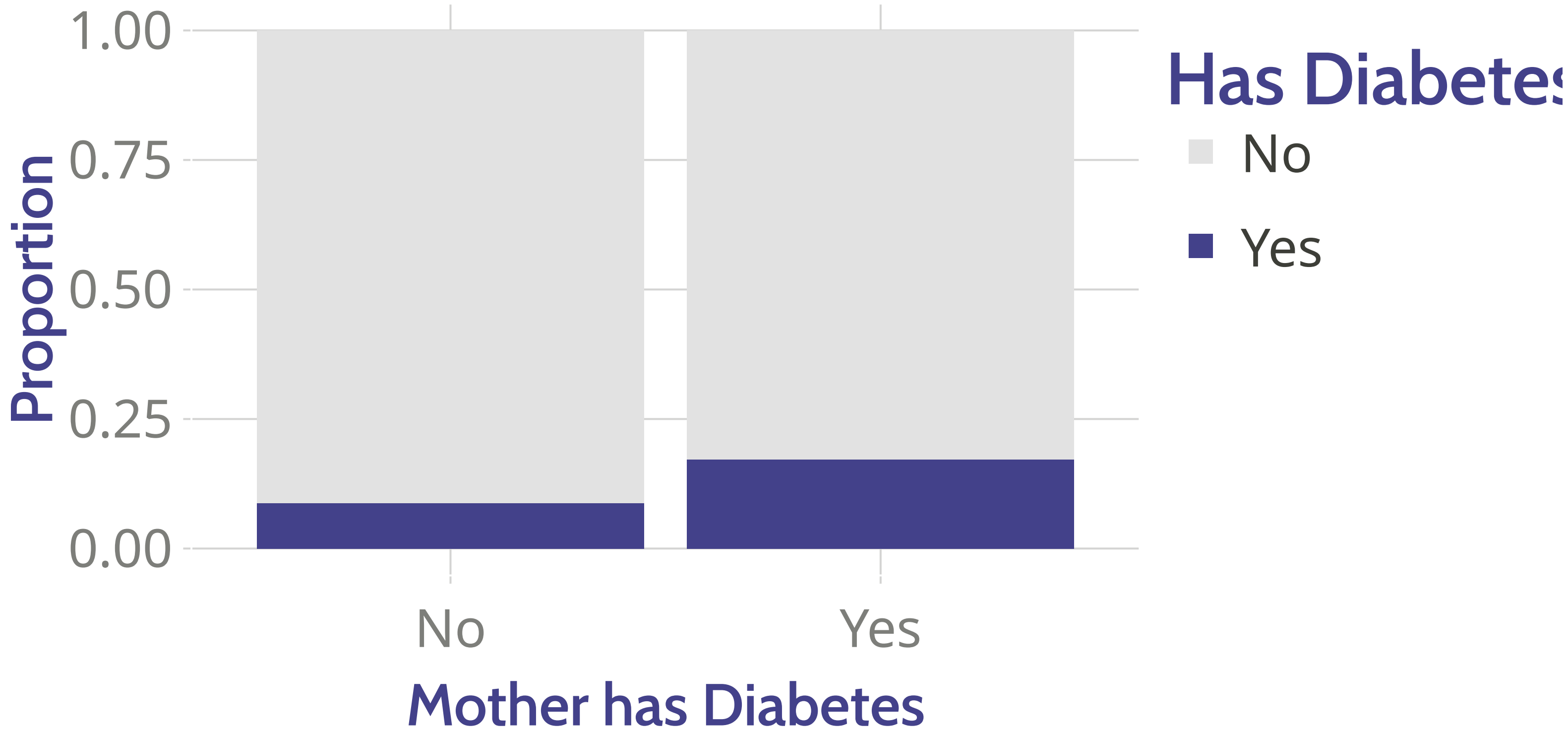
	No Diabetes	Has Diabetes
Mother No Diabetes	0.91	0.09
Mother Has Diabetes	0.83	0.17

- What about marginal frequencies here?
  - Row sums should add up to 1
    - $P(Diabetes_i = 1|\text{Mother has diabetes}_i = 1) + P(Diabetes_i = 0|\text{Mother has diabetes}_i = 1)$
  - Column sums are meaningless
    - $P(Diabetes_i = 1|\text{Mother has diabetes}_i = 1) + P(Diabetes_i = 1|\text{Mother has diabetes}_i = 0)$

- We can visualize it on a barplot



- Or better on a **stacked barplot**

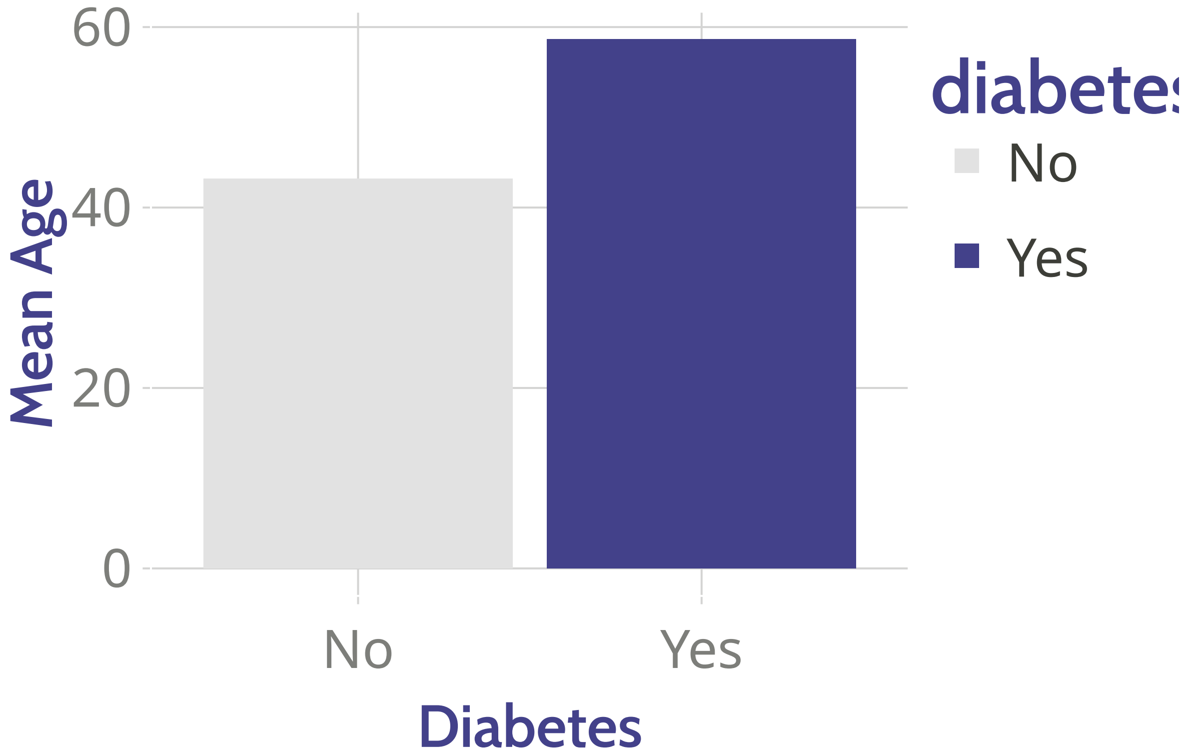


- *Stacked barplot* clearly shows the distribution of diabetes within each group
- Does it mean that having diabetic mother **causes** higher change of having diebetes?

Comparing quantitative variables

Discrete variables

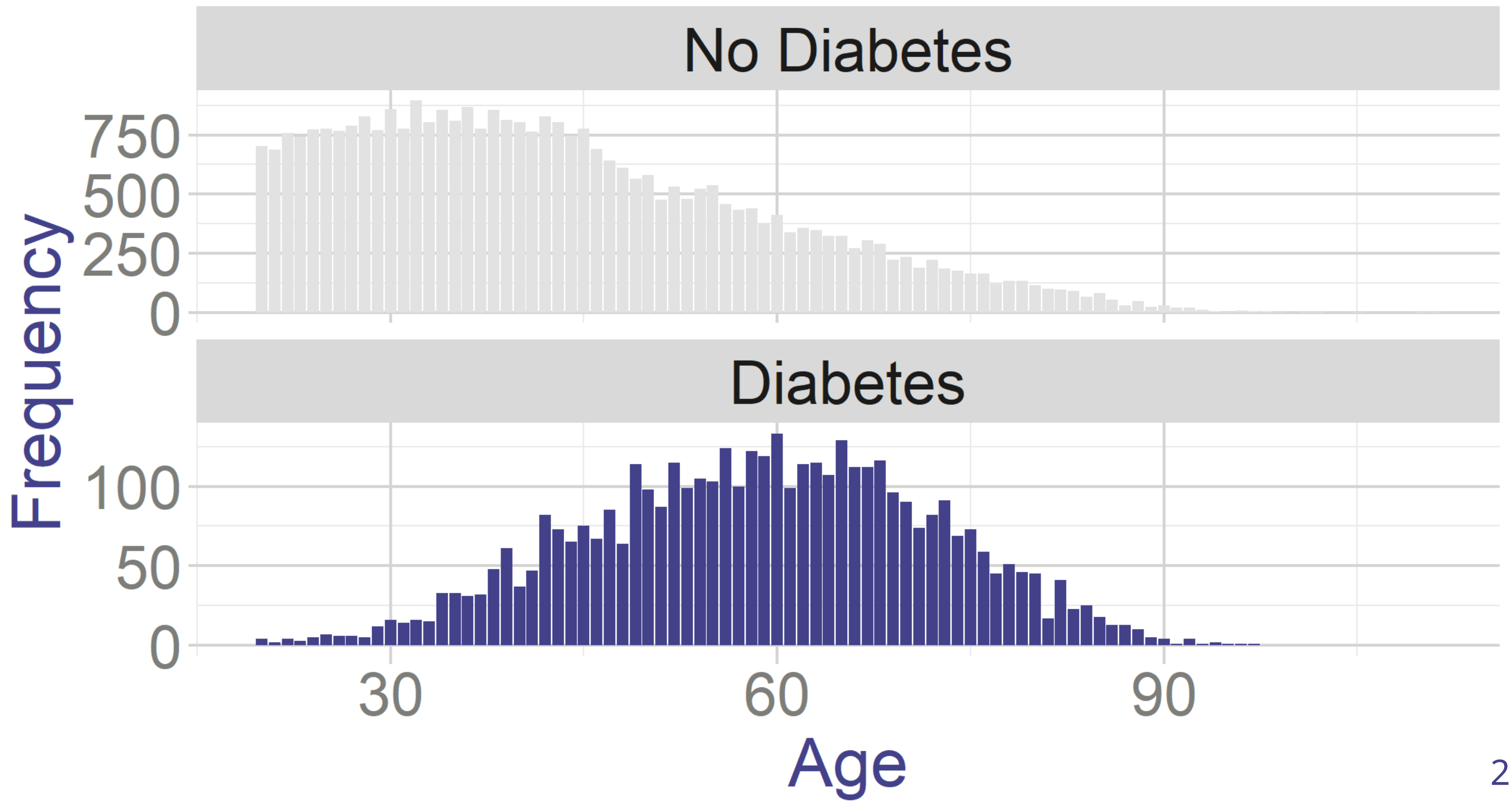
- For quantitative variables we can compare some summary statistics
  - *Example* means in two subpopulations
  - Are people older with diabetes older than people without it?



Comparing quantitative variables

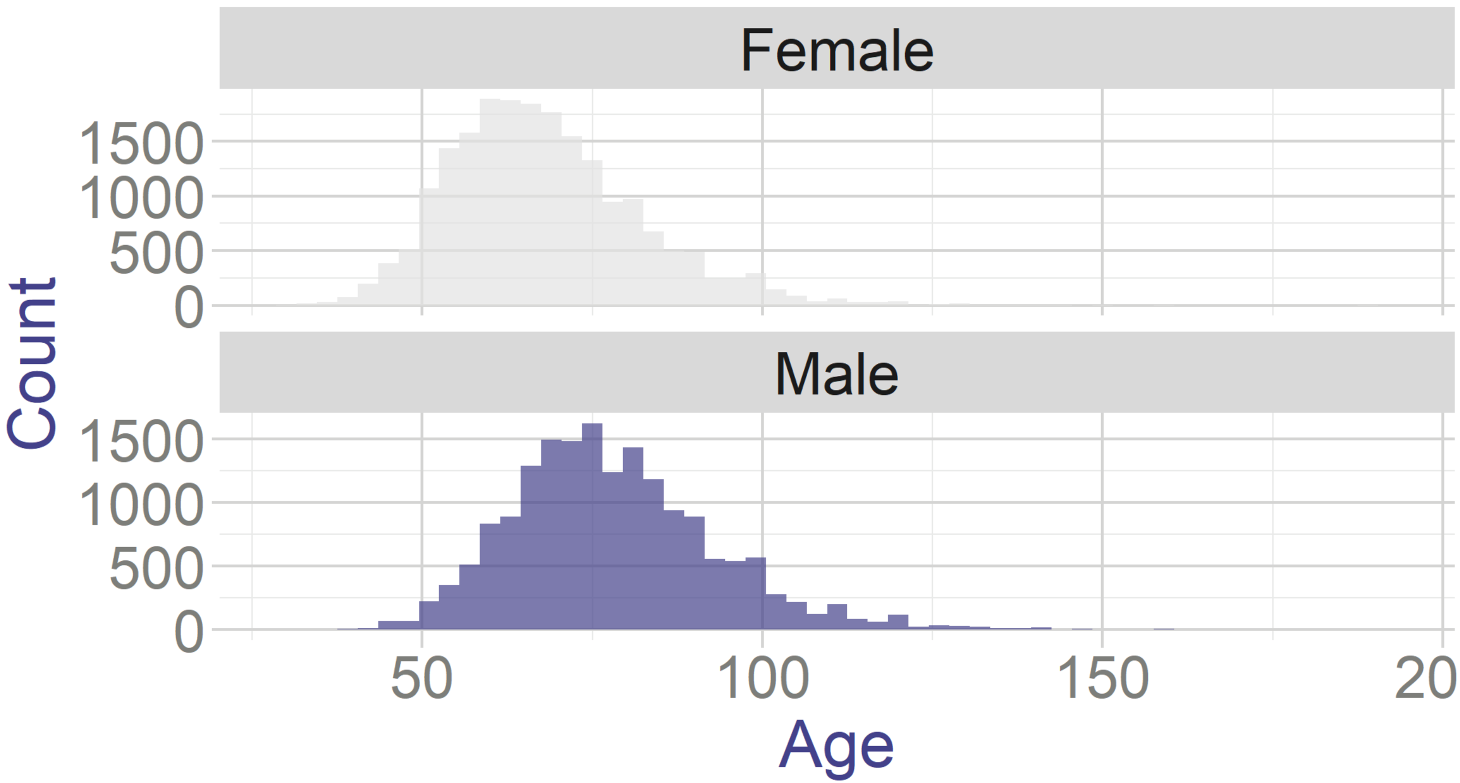
Discrete variables

- Or we can do Box and Whiskers plots as before
- Or we can compare the whole distributions of frequencies



Continous variables

- For continuous variables we can use the same methods (except frequency distribution)
- Instead, we can compare densities or histograms
- Are men heavier than women?



Association

- Likely people would subscribe to the website to lose weight
- But do these people have resources?
- What is the relationship between Body Mass Index (BMI) and Income?
- More generally, how to measure association between two quantitative variables
- Association between qualitative variables is measured with contingency tables

Associations

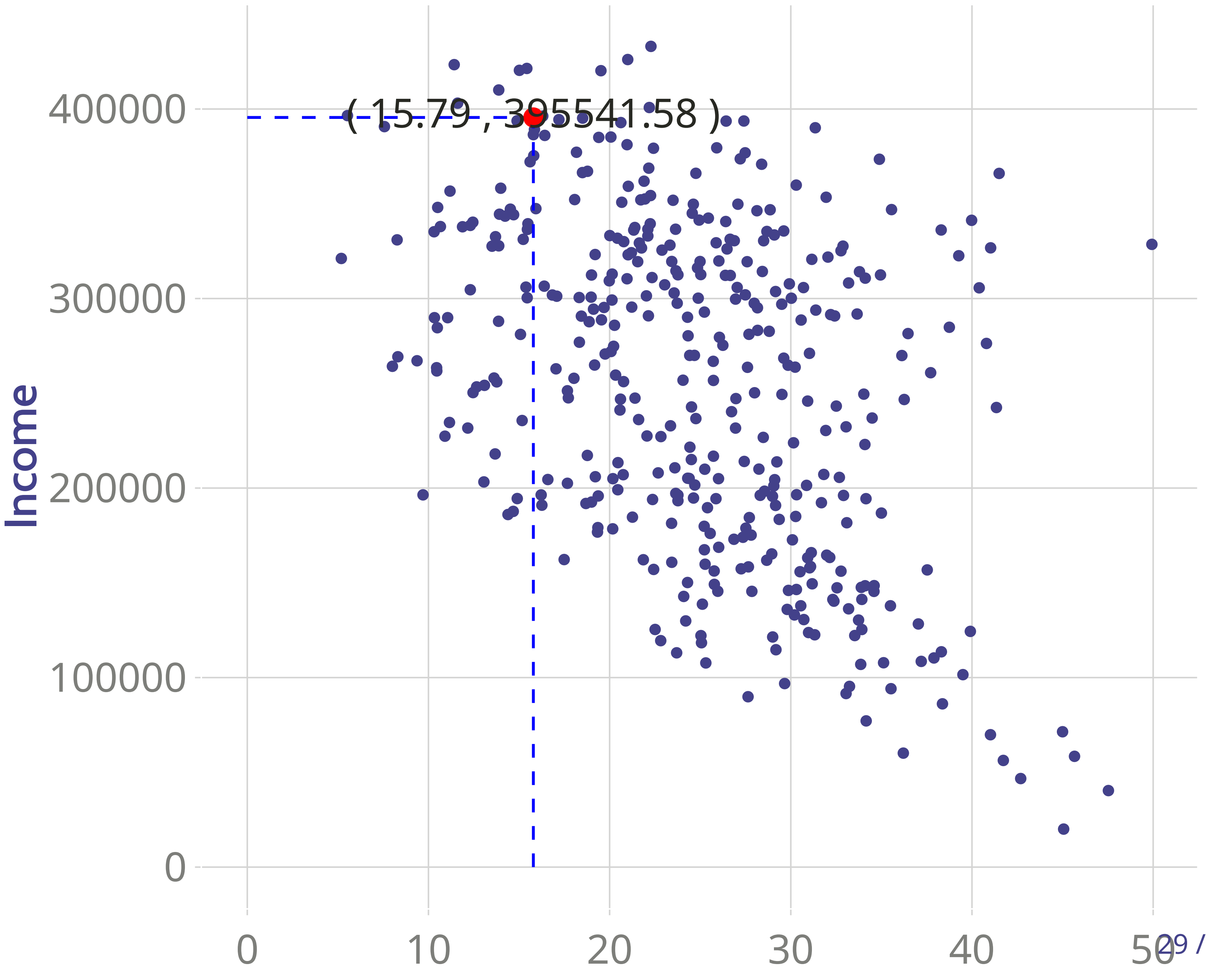
- Suppose we surveyed people from Guadalajara and CDMX about their BMI, education and income.
- Scatter plots show associations between two quantitative variables
  - We put variables of interest (*example*: Y and X) on the axis
  - We place observation on the cartesian plane using their values of variable X and Y: $\{(x_1, y_1), (x_2, y_2)..\}$
- In our case:
  - X axis is BMI
  - Y axis is Income
  - An individual i is placed on these axis based on $(BMI_i, Income_i)$

Show 4 entries

City	BMI	Education	Income
Mexico City	19.52	17.5	420224.44
Mexico City	22.16	15.3	368793.49
Mexico City	36.47	11.3	281512.52
Mexico City	24.56	13.4	344991.58

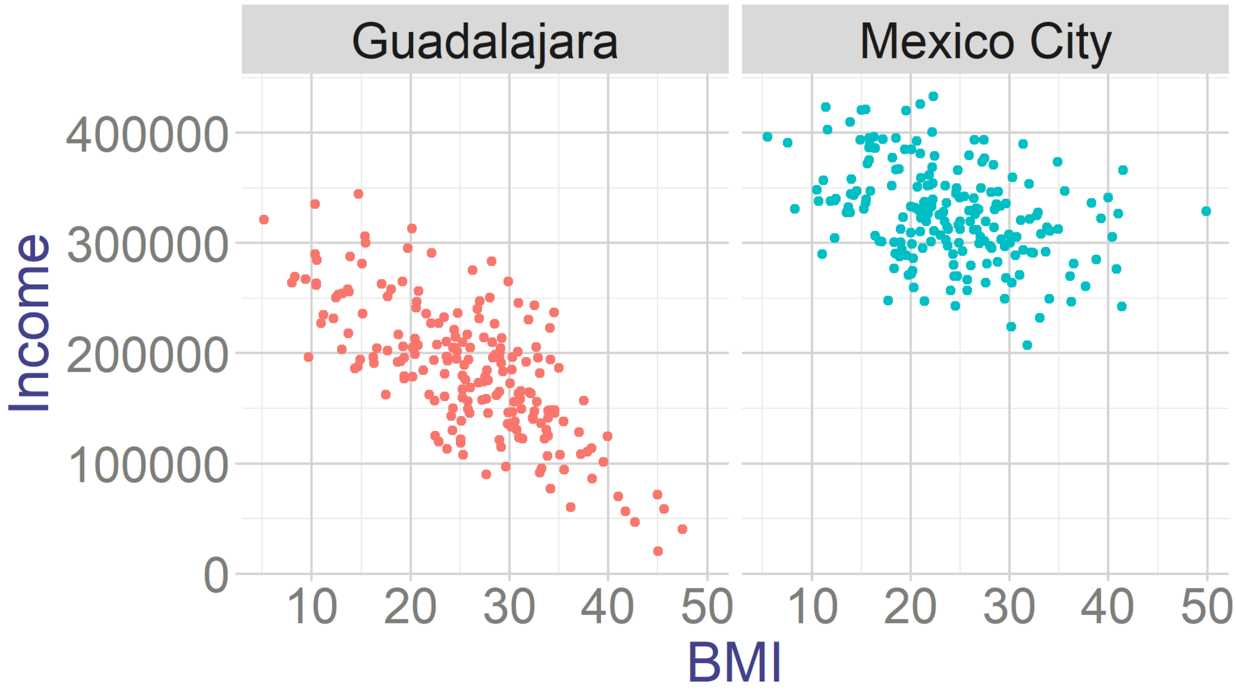
Showing 1 to 4 of 400 entries

Previous 1 2 3 4 5 ... 100 Next


(15.79 , 395541.58)
Income
400000
300000
200000
100000
0
0
10
20
30
40
50

Assocations

- Would you say that the relationship is stronger in Guadalajara or in Mexico City?



- How to measure the strength of the relationship?

Associations

Covariance

- **Covariance** measures the strength of the relationship between two variables.

$$\text{Cov}(X, Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_X)(y_i - \mu_Y)$$

And it's sample equivalent is:

$$\hat{\text{Cov}}(X, Y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

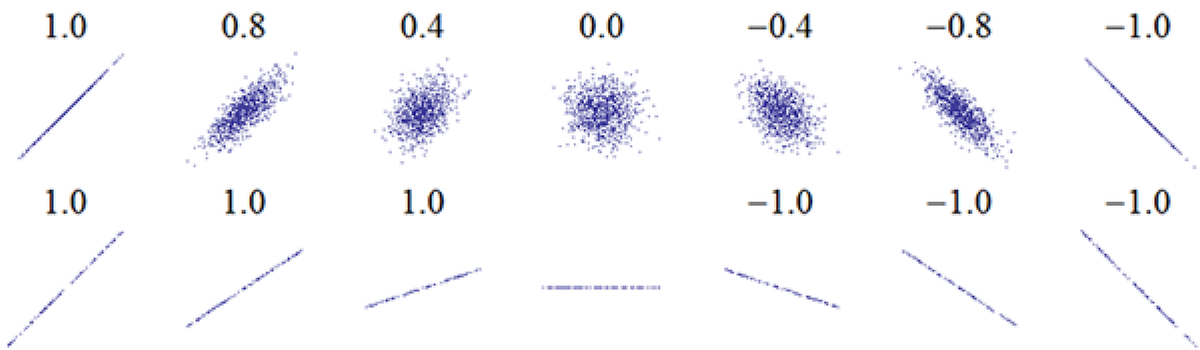
- Covariance whether the two variables move together

- Covariance increases when:
  - The relationship is stronger
  - The deviations of variables are larger

We use the Correlation coefficient to quantify the strength and direction of a relationship between two variables. $e.g.$, think about height and weight, or hours of sleep and irritability.

● The Pearson product-moment correlation coefficient is scale free and it ranges between -1 and 1.
● It is typically denoted by r, for sample data or by ρ (the greek symbol Rho), to indicate the population value.
● You have probably examined XY scatterplots to visualize this type of bivariate relationship, and have begun to evaluate the 2 dimensional attributes of the scattercloud to gain a sense of direction and strength of the relationship.

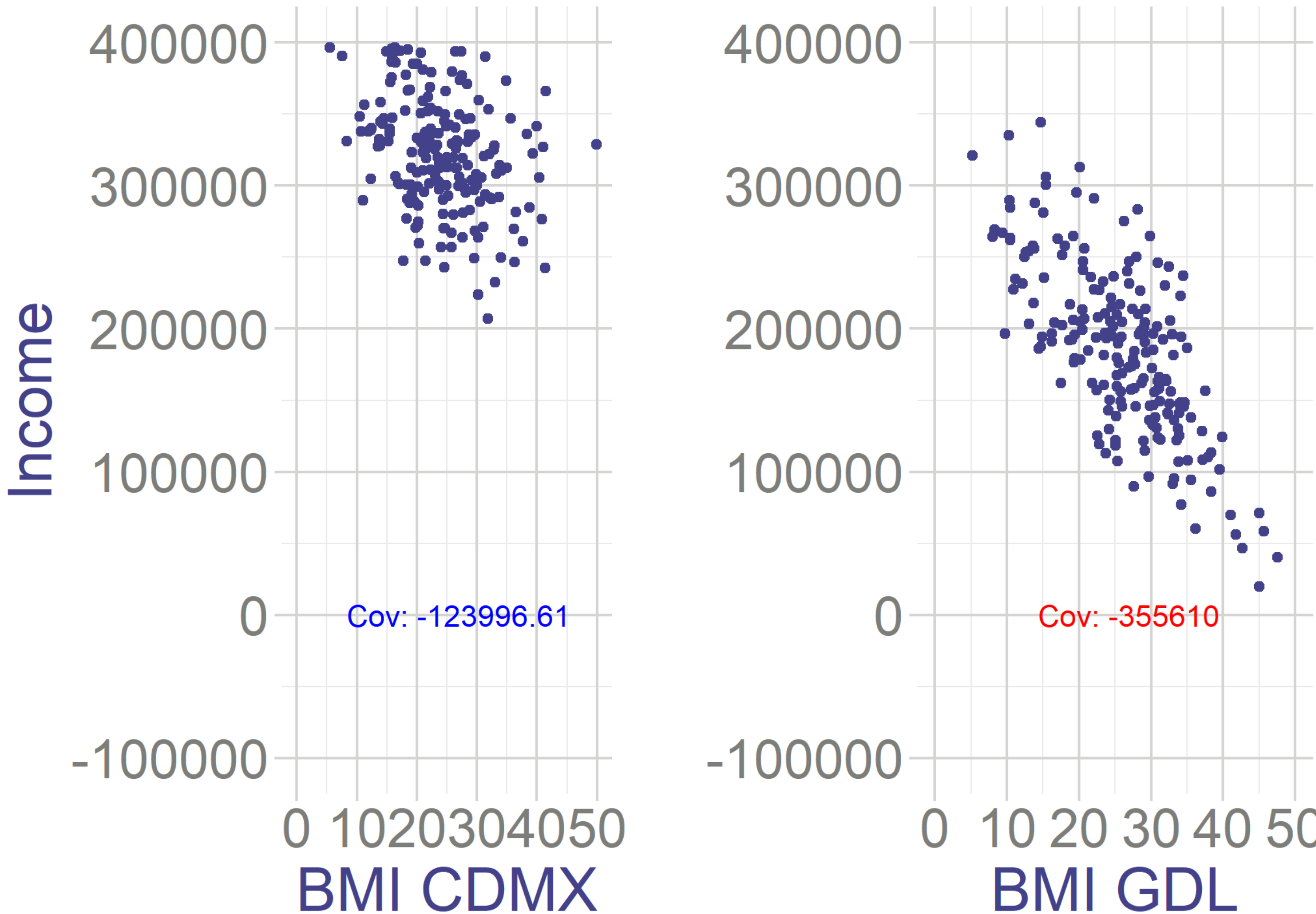
● Often, introductory textbooks show a figure like the following which depicts a series of XY scatterplots reflecting correlation patterns of differing size and sign. This one is the Wikipedia illustration.



● A correlation of -1 means that the X and Y variables have a perfect negative relationship and the data points fit a straight line with a negative slope.
● Similarly, a correlation of +1 means that X and Y have a perfect positive relationship and fall on a line with positive slope.

Source: https://shiny.rit.albany.edu/stat/rectangles/

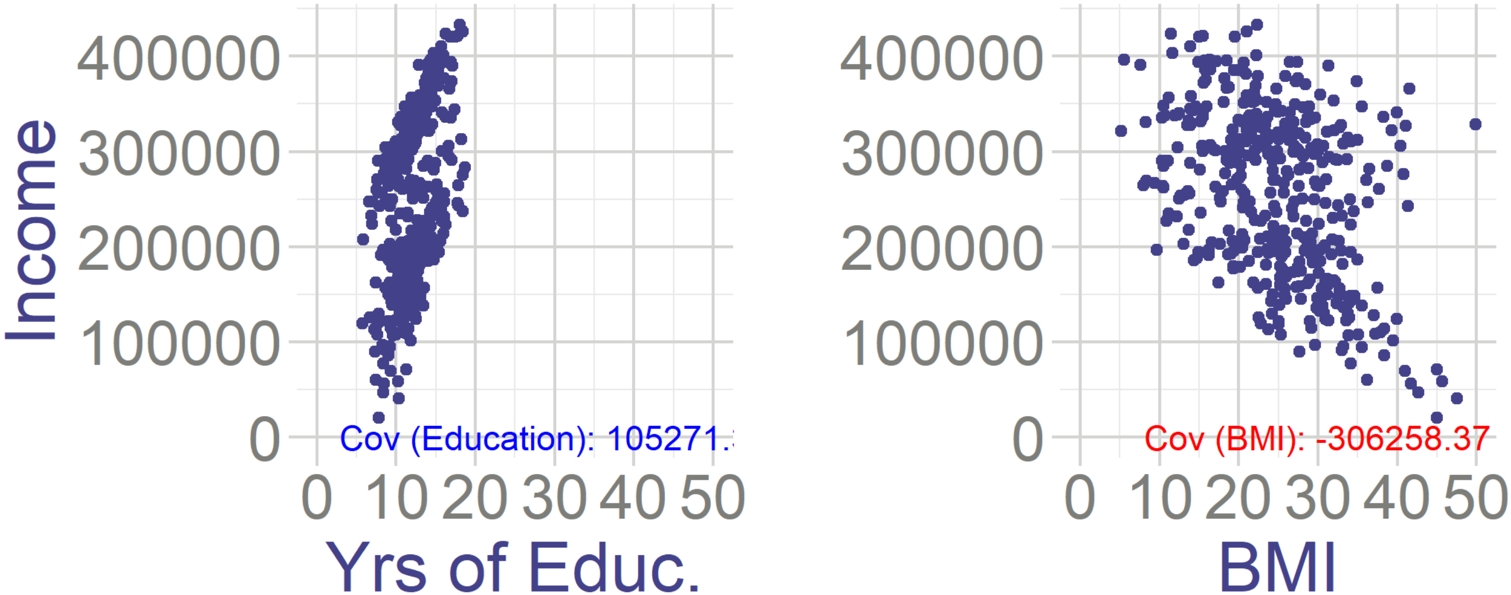
Covariance



Income
400000
300000
200000
100000
0
-100000
Cov: -123996.61
0 1020304050
BMI CDMX
400000
300000
200000
100000
0
-100000
Cov: -355610
0 10 20 30 40 50
BMI GDL

Covariance

- What has stronger relationship with Income: BMI or Years of Education?



- BMI has larger covariance
- But we can't compare covariances of different variables
- Covariance depends on the scales (or units) of the variable
- All else equal, larger standard deviation implies larger covariance
  - The squares are just bigger

Correlation

- **Correlation measures** the strength of a linear relationship between two variables.
- It ranges between -1 and 1

Population Correlation coefficient:

$$\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$$

Sample Correlation coefficient:

$$\hat{\rho}(X,Y) = \frac{\hat{\mathrm{Cov}}(X,Y)}{s_X \cdot s_Y}$$

Where $s_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

Correlation

- Correlation is preferred over covariance because it's **scale-independent** and easier to interpret.
- Suppose that instead of measuring income (Y variable) in MXN , we measure it in Dollars.
  - Z income in dollars $Z = \frac{Y}{16}$
  - Is $Cov(X, Z) = Cov(X, Y)$?

$$
\begin{aligned}
cov(X, Z) &= \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_X)(z_i - \mu_Z) \\
&= \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_X)(\frac{y_i}{16} - \frac{\mu_Y}{16}) \\
&= \frac{1}{16} \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_X)(y_i - \mu_Y) \\
&\neq cov(X, Y)
\end{aligned}
$$

Correlation

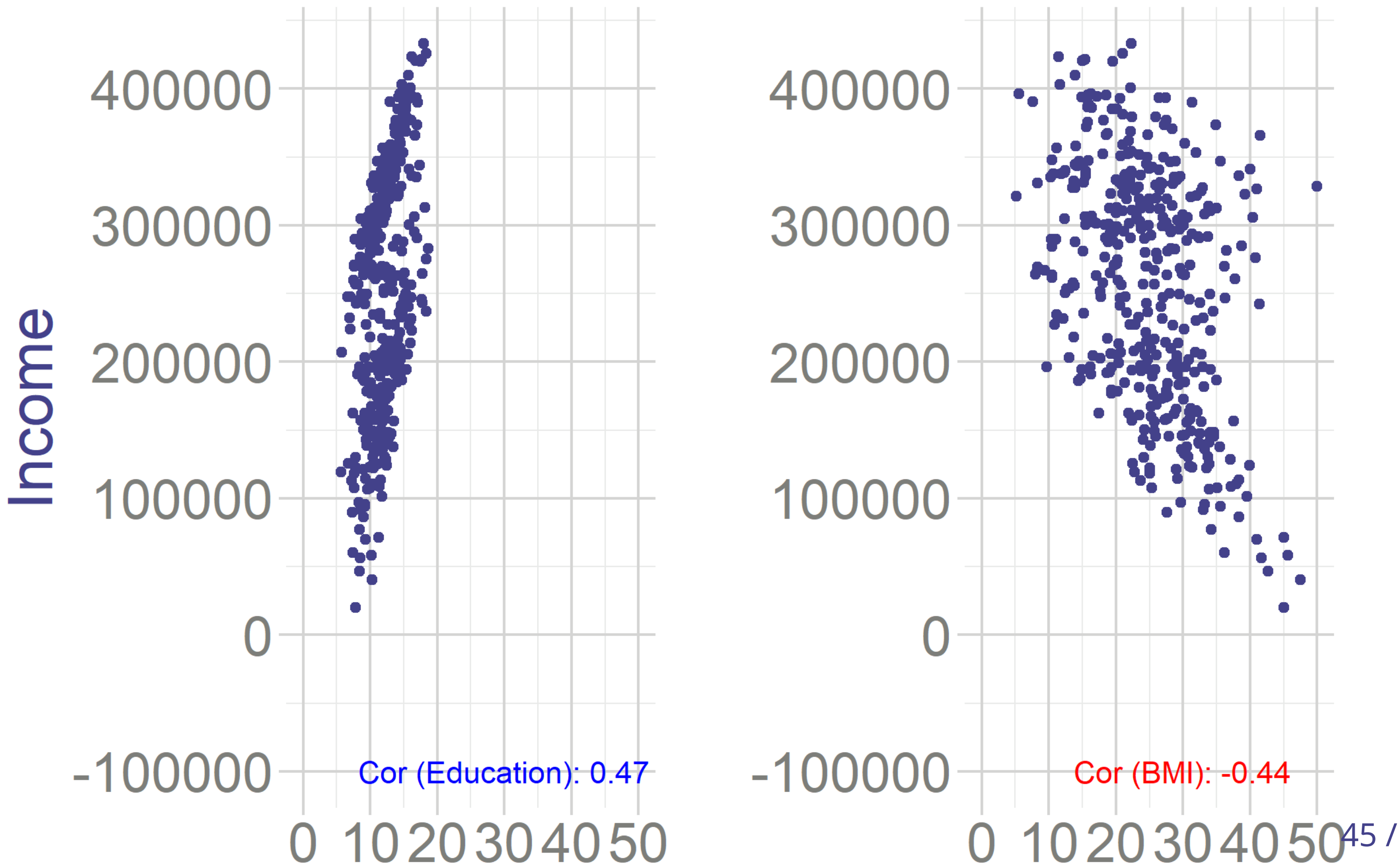
- Correlation is preferred over covariance because it's **scale-independent** and easier to interpret.

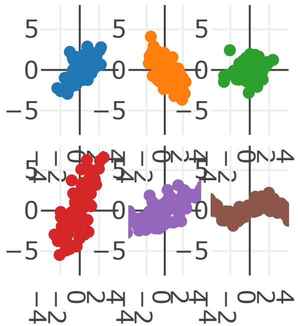
- Suppose that instead of measuring income (Y variable) in MXN , we measure it in Dollars.
    - Z income in dollars $Z = \frac{Y}{16}$
    - Is $\rho(X, Z) = \rho(X, Y)$?

$$\begin{aligned}
\rho(X, Z) &= \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_X)(z_i - \mu_Z))}{\sqrt{\sum_{i=1}^{N}(x_i - \mu_X)^2} \cdot \sqrt{\sum_{i=1}^{N}(z_i - \mu_Z)^2}} \\
&= \frac{\frac{1}{N}\sum_{i=1}^{N}\sum_{i=1}^{N}(x_i - \mu_X)(\frac{y_i}{16} - \frac{\mu_Y}{16})}{\sqrt{\sum_{i=1}^{N}(x_i - \mu_X)^2} \cdot \sqrt{\sum_{i=1}^{N}(\frac{y_i}{16} - \frac{\mu_Y}{16})^2}} \\
&= \frac{\frac{1}{16}\frac{1}{N}\sum_{i=1}^{N}\sum_{i=1}^{N}(x_i - \mu_X)(y_i - \mu_Y)}{\frac{1}{16}\sqrt{\sum_{i=1}^{N}(x_i - \mu_X)^2} \cdot \sqrt{\sum_{i=1}^{N}(y_i - \mu_Y)^2}} \\
&= \rho(X, Y)
\end{aligned}$$

Correlation

- Correlation with education is actually stronger



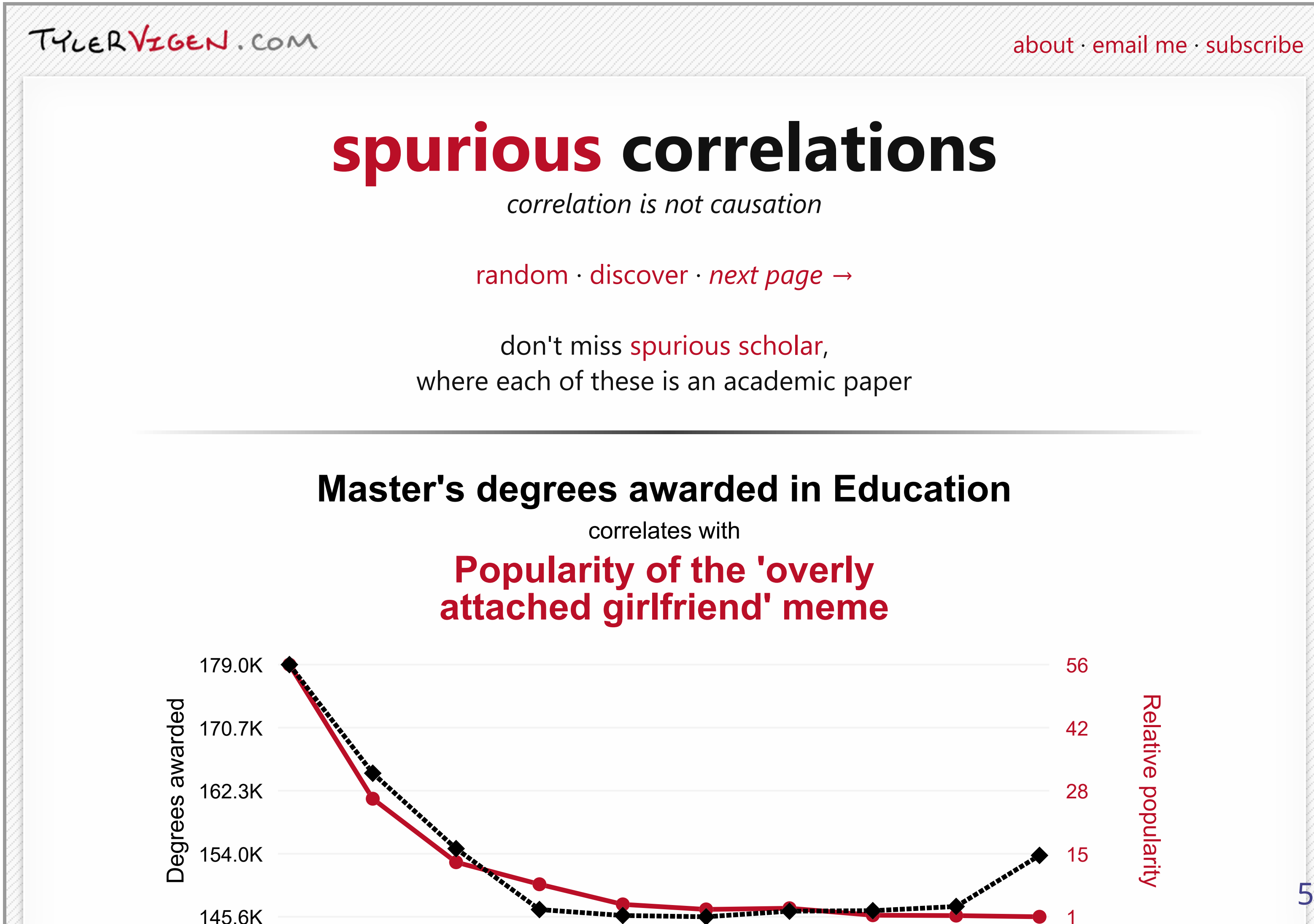


Correlation

1. Correlation is a value between -1 and 1: $-1 \leq \rho(X, Y) \leq 1$.
2. Perfect positive correlation: $\rho = 1$. Perfect negative correlation: $\rho = -1$.
3. No linear correlation: $\rho = 0$, but this doesn't imply independence.
4. Correlation measures **linear** relationships; nonlinear relationships might not be accurately captured.
5. Correlation doesn't imply causation; a relationship could be coincidental.

Causality vs Correlation

Causality vs Correlation



Causality vs Correlation

- Less obvious examples
- You look at historical data from some media campaign
- You notice that people who were more exposed to ads were less likely to buy that product
- What can you conclude?
- Are people who were exposed to ads similar to people who were not?
- Maybe they were targeted in the first place because they are less likely to buy and you want to change it?

Causality vs Correlation

- Less obvious examples
- Education usually correlates with Income (correlation)
- Does it mean that if decide to get a degree, you will earn more? (causality)