

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

A. 0.3875
A. 0.2676
B. 0.5
C. 0.6987

Solution:

Here the normal distribution $\mu = 45$

Standard deviation $\sigma = 8.0$.

Let X is the amount of time to take complete the repair on a customer's car.

To finish the work in one hour we have $X \leq 50$. Then we have to find $P(X \leq 50)$

Z value

$$Z = (X - \mu) / \sigma = (X - 45) / 8$$

Then,

$$\begin{aligned} P(X \leq 50) &= P(Z \leq (50 - 45) / 8) \\ &= P(Z \leq 0.625) \end{aligned}$$

Probability that the service manager will not meet his demand will be = $100\% - 73.4\% = 26.6\%$ or 0.2676

Option B is the answer.

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.

A. More employees at the processing center are older than 44 than between 38 and 44.
Ans: True

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
Ans: True
3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans: We know that $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ are two independent random variables then ,

$$X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$$

$$X - Y \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$$

Let us consider $Z = aX + bY$, where X and Y are as defined above, that is Z is linear combination of X and Y , then $Z \sim N(a\mu_1 + b\mu_2, a^2\sigma_1^2 + b^2\sigma_2^2)$.

Therefore,

$$2X_1 \sim N(2\mu, 4\sigma^2)$$

$$\text{Consider } X_1 + X_2 \sim N(\mu + \mu, \sigma^2 + \sigma^2) \sim N(2\mu, 2\sigma^2)$$

$$2X_1 - (X_1 + X_2) \sim N(4\mu, 6\sigma^2)$$

4. Let $X \sim N(100, 20^2)$. Find two values, a and b , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9

B. 80.2, 119.8

C. 22, 78

D. 48.5, 151.5

E. 90.1, 109.9

Ans: D

Explanation:

Given, $P(a < X < b) = 0.99$, mean = 100, Standard deviation = 20

The probability area under the curve = 0.99

The probability outside the a and b area = $1 - 0.99 = 0.01$

Then, we have to find the probability towards the area in left and right tails

Let the probability towards left from $a = -0.01/2 = -0.005$

The probability towards right from $b = +0.01/2 = +0.005$

Using python we have to find Z value (Standard Normal Variable)

Z value is given as `stats.norm.ppf(pval)`

For Probability 0.005 the Z Value is -2.576

By using Z value we can calculate X value i.e

$$Z = (X - \mu) / \sigma \Rightarrow X = Z \sigma + \mu$$

$$X = Z(-0.005)(20) + 100 = -(-2.57)(20) + 100 = 151.5$$

$$X = Z(0.005)(20) + 100 = (-2.57)(20) + 100 = 48.5$$

Therefore two values, a and b , symmetric about the mean for the given standard normal distribution are 48.5, 151.5

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
- A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: 99million rupees to 980.5million rupees

B. Specify the 5th percentile of profit (in Rupees) for the company

Ans: 169.907million rupees

C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Profit 2