Principles of Programming Languages

Assignment 4

1. Which statement is true:

a. $\{f: [T1 \rightarrow T2], g: [T1 \rightarrow T2], a: T1\} \vdash \big(f(g\ a)\big): T2$ – false, a is of type T1, g receives T1 and returns T2, f takes as input T1 and not T2.

b. $\{x: T1, y: T2, f: [T2 \rightarrow T1]\} \vdash (f\ y): T1$ – true, f receives 1 argument of type T2 (y): and returns T1.

c. $\{f: [T1 \rightarrow T2]\} \vdash \big(lambda\ (x)\ (f\ x)\big): [T1 \rightarrow T2]$ –false, there is no information on the left side about x, could be the wrong type. The $lambda$ exp returns T2 and not $[T1 \rightarrow T2]$.

d. $\{f: [T1 * T2 \rightarrow T3]\} \vdash \big(lambda\ (x)\ (f\ x\ 100)\big): [T1 \rightarrow T3]$ – false, there is no information on the left side about x, could be the wrong type. T2 could be a number, but the $lambda$ exp returns T3 and not $[T1 \rightarrow T3]$.

2. Perform type inference algorithm for the expression:

a. $\big((lambda\ (x1)\ (+\ x1\ 1))\ 4\ \big)$

1. Rename bound vars: $\big((lambda\ (x1)\ (+\ x1\ 1))\ 4\ \big) \rightarrow$ $\big((lambda\ (x)\ (+\ x\ 1))\ 4\ \big)$.

2. Assign type vars for every sub expression:

Expression	Variable
$\big((lambda\ (x)\ (+\ x\ 1))\ 4\ \big)$	$T0$
$(lambda\ (x)\ (+\ x\ 1))$	$T1$
$(+\ x\ 1)$	$T2$
$+$	T_+
x	Tx
1	$Tnum1$
4	$Tnum4$

3. Construct type equations:

Expression	Equation
$\big((lambda\ (x)\ (+\ x\ 1))\ 4\ \big)$	$T1 = [Tnum4 \rightarrow T0]$
$(lambda\ (x)\ (+\ x\ 1))$	$T1 = [Tx \rightarrow T2]$
$(+\ x\ 1)$	$T_+ = [Tx * Tnum1 \rightarrow T2]$
$+$	$T_+ = [Number * Number \rightarrow Number]$
1	$Tnum1 = Number$
4	$Tnum4 = Number$

4. Solve the equations:

Equation	Substitution
$T1 = [Tnum4 \rightarrow T0]$	$\{\square\}$
$T1 = [Tx \rightarrow T2]$	
$T_+ = [Tx * Tnum1 \rightarrow T2]$	
$T_+ = [Number * Number \rightarrow Number]$	
$Tnum1 = Number$	
$Tnum4 = Number$	

Step 1: $T1 = [Tnum4 \rightarrow T0] \circ Substituotion =$
$(T1 = [Tnum4 \rightarrow T0]) \rightarrow$
$Substituotion = Substituotion \circ (T1 = [Tnum4 \rightarrow T0])$.

Equation	Substitution
$T1 = [Tx \rightarrow T2]$	$\{T1 = [Tnum4 \rightarrow T0]\}$
$T_+ = [Tx * Tnum1 \rightarrow T2]$	
$T_+ = [Number * Number \rightarrow Number]$	
$Tnum1 = Number$	
$Tnum4 = Number$	

Step 2: $T1 = [Tx \rightarrow T2] = [Tnum4 \rightarrow T0]$ both sides are composite that needs to be split.

Equation	Substitution
$T_+ = [Tx * Tnum1 \rightarrow T2]$	$\{T1 = [Tnum4 \rightarrow T0]\}$
$T_+ = [Number * Number \rightarrow Number]$	
$Tnum1 = Number$	
$Tnum4 = Number$	
$Tx = Tnum4$	
$T2 = T0$	

Step 3: $T_+ = [Tx * Tnum1 \rightarrow T2] \circ Substituotion = (T_+ = [Tx * Tnum1 \rightarrow T2]) \rightarrow Substituotion = Substituotion \circ (T_+ = [Tx * Tnum1 \rightarrow T2])$.

Equation	Substitution
$T_+ = [Number * Number \rightarrow Number]$	$\{ T1 = [Tnum4 \rightarrow T0], T_+ = [Tx * Tnum1 \rightarrow T2] \}$
$Tnum1 = Number$	
$Tnum4 = Number$	
$Tx = Tnum4$	
$T2 = T0$	

Step 4: $T_+ = [Number * Number \rightarrow Number] = [Tx * Tnum1 \rightarrow T2]$ split.

Equation	Substitution
$Tnum1 = Number$	$\begin{Bmatrix} T1 = [Tnum4 \to T0], \\ T_+ = [Tx * Tnum1 \to T2] \end{Bmatrix}$
$Tnum4 = Number$	
$Tx = Tnum4$	
$T2 = T0$	
$Tx = Number$	
$T2 = Number$	

Step 5: substituted $Tnum1$.

Equation	Substitution
$Tnum4 = Number$	$\begin{Bmatrix} T1 = [Tnum4 \to T0], \\ T_+ = [Tx * Number \to T2], \\ Tnum1 = Number \end{Bmatrix}$
$Tx = Tnum4$	
$T2 = T0$	
$Tx = Number$	
$T2 = Number$	

Step 6: substituted $Tnum4$.

Equation	Substitution
$Tx = Tnum4$	$\begin{Bmatrix} T1 = [Number \to T0], \\ T_+ = [Tx * Number \to T2] \\ Tnum1 = Number, \\ Tnum4 = Number \end{Bmatrix}$
$T2 = T0$	
$Tx = Number$	
$T2 = Number$	

Step 7: first substituted Tx to $Number$ $(Tnum4 = Number)$, then substituted each equation that has Tx with $Number$.

Equation	Substitution
$T2 = T0$	$\left\{\begin{array}{c} T1 = [Number \to T0], \\ T_+ = [Tx * Number \to T2] \\ Tnum1 = Number, \\ Tnum4 = Number, \\ Tx = Number \end{array}\right\}$
$Tx = Number$	
$T2 = Number$	

Step 8: substituted $T2$ with $T0$.

Equation	Substitution
$Tx = Number$	$\left\{\begin{array}{c} T1 = [Number \to T0], \\ T_+ = [Tx * Number \to T0] \\ Tnum1 = Number, \\ Tnum4 = Number, \\ Tx = Number, \\ T2 = T0 \end{array}\right\}$
$T2 = Number$	

Step 9: substituted Tx with $Number$, we will get $Number = Number$ and delete it from the left side.

Equation	Substitution
$T2 = Number$	$\left\{\begin{array}{c} T1 = [Number \to T0], \\ T_+ = [Tx * Number \to T0] \\ Tnum1 = Number, \\ Tnum4 = Number, \\ Tx = Number, \\ T2 = T0 \end{array}\right\}$

Step 10: first substituted $T2$ with $T0$, then we will get $T0 = Number$ and substituted each equation that has $T0$ with $Number$.

Equation	Substitution
	$\left\{\begin{array}{c} T1 = [Number \rightarrow Number], \\ T_{+} = [Tx * Number \rightarrow Number] \\ Tnum1 = Number, \\ Tnum4 = Number, \\ Tx = Number, \\ T2 = T0, \\ T0 = Number \end{array}\right\}$

The inference succeeds the type of $T0$ is $Number$.

b. $\left(\left(lambda\ (f1\ x1)\ (f1\ x1\ 1)\right)\ 4\ +\right)$

1. Rename bound vars: $\left(\left(lambda\ (f1\ x1)\ (f1\ x1\ 1)\right)\ 4\ +\right) \rightarrow$ $\left(\left(lambda\ (f\ x)\ (f\ x\ 1)\right)\ 4\ +\right)$.
2. Assign type vars for every sub expression:

Expression	Variable
$\left(\left(lambda\ (f\ x)\ (f\ x\ 1)\right)\ 4\ +\right)$	T_0
$\left(lambda\ (f\ x)\ (f\ x\ 1)\right)$	T_1
$(f\ x\ 1)$	T_2
f	T_f
x	T_x
1	T_{num1}
4	T_{num4}
+	T_+

3. Construct type equations:

Expression	Equations
$\left(\left(lambda\ (f\ x)\ (f\ x\ 1)\right)\ 4\ +\right)$	$T_1 = [T_{num4} * T_+ * T_{num1} \rightarrow T_0]$
$\left(lambda\ (f\ x)\ (f\ x\ 1)\right)$	$T_1 = [T_f * T_x * T_{num1} \rightarrow T_2]$
$(f\ x\ 1)$	$T_f = [T_x * T_{num1} \rightarrow T_2]$
1	$T_{num1} = Number$
4	$T_{num4} = Number$
+	$T_+ = [Number * Number \rightarrow Number]$

4. Solve the equations:

Equation	Substitution
$T_1 = [T_{num4} * T_+ * T_{num1} \rightarrow T_0]$	$\{\square\}$
$T_1 = [T_f * T_x * T_{num1} \rightarrow T_2]$	
$T_f = [T_x * T_{num1} \rightarrow T_2]$	
$T_{num1} = Number$	
$T_{num4} = Number$	
$T_+ = [Number * Number \rightarrow Number]$	

Step 1: $T1 = [T_{num4} * T_+ * T_{num1} \rightarrow T_0] \circ Substituotion =$
$(T1 = [T_{num4} * T_+ * T_{num1} \rightarrow T_0]) \rightarrow$
$Substituotion = Substituotion \circ$
$(T1 = [T_{num4} * T_+ * T_{num1} \rightarrow T_0])$.

Equation	Substitution
$T_1 = [T_f * T_x * T_{num1} \rightarrow T_2]$	$\{T_1 = [T_{num4} * T_+ * T_{num1} \rightarrow T_0]\}$
$T_f = [T_x * T_{num1} \rightarrow T_2]$	
$T_{num1} = Number$	
$T_{num4} = Number$	
$T_+ = [Number * Number \rightarrow Number]$	

Step 2: $T1 = [T_{num4} * T_+ * T_{num1} \rightarrow T_0] = [T_f * T_x \rightarrow T_2]$ both sides are composite that needs to be split.

Equation	Substitution
$T_f = [T_x * T_{num1} \rightarrow T_2]$	$\{T_1 = [T_{num4} * T_+ * T_{num1} \rightarrow T_0]\}$
$T_{num1} = Number$	
$T_{num4} = Number$	
$T_+ = [Number * Number \rightarrow Number]$	
$T_f = T_{num4}$	
$T_x = T_+$	
$T_{num1} = T_{num1}$	

Step 3: $T_f = [T_x * T_{num1} \rightarrow T_2] \circ Substituotion =$
$(T_f = [T_x * T_{num1} \rightarrow T_2]) \rightarrow$
$Substituotion = Substituotion \circ (T_f = [T_x * T_{num1} \rightarrow T_2])$.

Equation	Substitution
$T_{num1} = Number$	$\{T_1 = [T_{num4} * T_+ * T_{num1} \rightarrow T_0], T_f = [T_x * T_{num1} \rightarrow T_2]\}$
$T_{num4} = Number$	
$T_+ = [Number * Number \rightarrow Number]$	
$T_f = T_{num4}$	
$T_x = T_+$	
$T_{num1} = T_{num1}$	

Step 4: substituted $Tnum1$.

Equation	Substitution
$T_{num4} = Number$	$\begin{Bmatrix} T_1 = [T_{num4} * T_+ * Number \rightarrow T_0], \\ T_f = [T_x * Number \rightarrow T_2], \\ T_{num1} = Number \end{Bmatrix}$
$T_+ = [Number * Number \rightarrow Number]$	
$T_f = T_{num4}$	
$T_x = T_+$	
$T_{num1} = T_{num1}$	

Step 5: substituted $Tnum4$.

Equation	Substitution
$T_+ = [Number * Number \rightarrow Number]$	$\begin{Bmatrix} T_1 = [Number * T_+ * Number \rightarrow T_0], \\ T_f = [T_x * Number \rightarrow T_2], \\ T_{num1} = Number, \\ T_{num4} = Number \end{Bmatrix}$
$T_f = T_{num4}$	
$T_x = T_+$	
$T_{num1} = T_{num1}$	

Step 6: substituted T_+.

Equation	Substitution
$T_f = T_{num4}$	$\begin{Bmatrix} T_1 = \begin{bmatrix} Number * [Number * Number \rightarrow Number] * Number \\ \rightarrow T_0 \end{bmatrix}, \\ T_f = [T_x * Number \rightarrow T_2], \\ T_{num1} = Number, \\ T_{num4} = Number, \\ T_+ = [Number * Number \rightarrow Number] \end{Bmatrix}$
$T_x = T_+$	
$T_{num1} = T_{num1}$	

Step 7: $T_f = T_{num4} = [T_x * Number \rightarrow T_2]$, FAILED, $Number$ cannot be a composite expression.

$$(f\ x\ 1) = (4 + 1) \rightarrow not\ a\ procedure$$