Mathematics problems

11 $\frac{x^{50}}{x^3 x^4} \cdot \frac{x^3}{x^2} = x^{(50+3-3-2-2)} = \underline{x^{46}}$

12 $8^2 \cdot 4^x \cdot 2^x = 8^7 \mid 8^2$

$2^{3x} \cdot 2^x = 8^2$

$2^{3x} = 2^{32}$

$\underline{x = 2}$

13 $\frac{x}{y} = 3 \Rightarrow \frac{y}{x} = \frac{1}{3}$ $x^{-4} \cdot y^4 = \left(\frac{y}{x}\right)^4 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$

14 $\frac{\sqrt{4^5}}{16^2} = \sqrt{\frac{2^{10}}{2^{12}}} = \sqrt{2^2} = \underline{2}$

15 a) $x + (y+z) = (y+x) + z$ true

b) $y(x+z) = xy + zy$ true

c) $x^{y+z} = x^z + x^y$ false

d) $\frac{y^z}{y^x} = x^{y-z}$ false

16 $\ln(x) \geq e$

$x \geq e^e$ $\quad x \in [e^e, \infty)$

2.1 $f(x) = a + bx$

$x = 0 \Rightarrow a = 32$

$x = 100$ $f(x) = 212$

$\begin{cases} a + 100b = 212 \\ a = 32 \end{cases}$

$100b = 180$

$b = 1,8$

$f(x) = 32 + 1,8x$

$32 + 1,8x = x$

$0,8x = -32$

$x = -40$

2.2 $f(x) = 3x - 12$

$f(y) = 0$

$3y - 12 = 0$

$3y = 12$

$y = 4$

2.3 $9^{x^2-6x+2} = 81$

$x^2-6x+2 = 2$

$x^2 = 6x$

$x = 0$ or $x = 6$

2.4 GDP growth +3% triple

$1{,}03^x = 3$

$\ln 1{,}03^x = \ln 3$

$x \ln 1{,}03 = \ln 3$

$x = \frac{\ln 3}{\ln 1{,}03} \approx 37{,}16 \Rightarrow 38$ years

2.5 $\log_\pi \left(\frac{1}{\pi^5}\right) = \log_\pi (\pi)^{-5} = -5 \log_\pi \pi = -5$

3. 3.1 $\sum_{i=0}^{\infty} \left(\frac{1}{5^i} + 0{,}3^i\right) = \sum_{i=0}^{\infty} \left(\frac{1}{5^i}\right) + \sum_{i=0}^{\infty} 0{,}3^i =$

$\left(\frac{1}{5}\right)^0 + \frac{1}{1-\frac{1}{5}} + 0.3^0 + \frac{0.3}{1-0.3} = 1 + \frac{5}{4} + 1 + \frac{3}{7} = 2 + \frac{1}{4} + \frac{3}{7} = 2 + \frac{19}{28} =$

$= \frac{75}{28}$

3.2. $\lim_{x \to 5} \frac{x^2-25}{x-5} = \lim_{x \to 5^-} (x+5) = 10$

3.3 $f(x) = x^3 - 4$ at $(-2; -12)$

$f'(x) = 3x^2$ $f'(x) = 3 \cdot 4 = 12$

$f''(x) = 6x$

$f''(-2) = -12$ concave

3.4. $f(x) = \frac{x^5+3}{x^2-1} =$

$f'(x) = \frac{(x^5+3)'(x^2-1) - (x^5+3)(x^2-1)'}{(x^2-1)^2} = \frac{5x^4(x^2-1) - 2x(x^5+3)}{(x^2-1)^2} =$

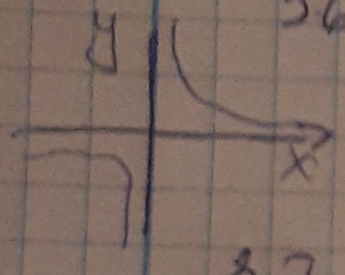
$= \frac{3x^6 - 5x^4 - 6}{(x^2-1)^2} = \frac{3x^6 - 5x^4 - 6}{(x^2-1)^2}$

35. $f(x) = x^3 + 3$

$f'(x) = 9x^4$

$f''(x) = 72x^7$

36. $f(x) = \frac{1}{x}$ continuous at 0?



No $\lim_{x \to 0} \frac{1}{x}$ there is no solution as you can't divide by zero

function is not continuous at 0

3.7

$f(x) = 4x^3 - 12x$

$f'(x) = 12x^2 - 12 \Rightarrow x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x = \pm 1$ - inflection points

$f'(x) = 0$ inflection points

$f''(x) = 24x$

$f''(-1) = -24$ local maximum at $x = -1$

$f''(1) = 24$ - local minimum at $x = 1$

3.8 $f(x,y) = x^3 - y^2$

$f(2,3) = 2^3 - 3^2 = 8 - 9 = -1$

3.9 $f(x,y) = \ln(x - 3y) \Rightarrow x - 3y > 0$

3.10 $\frac{\partial}{\partial x}\left(x^5 y^7 + \frac{x^2}{y^3}\right) = 5x^4 y^7 + 2x \cdot \frac{1}{y^3}$

3.11 $f(x,y) = \sqrt{xy} - x - y$

$\frac{\partial f(x,y)}{\partial x} = \sqrt{y} \cdot (x^{\frac{1}{2}})' - 1 = \frac{\sqrt{y}}{2\sqrt{x}} - 1 = \frac{1}{2}\sqrt{\frac{y}{x}} - 1$

$\frac{\partial f(x,y)}{\partial y} = \frac{1}{2}\sqrt{\frac{x}{y}} - 1$

$\frac{1}{2}\sqrt{\frac{y}{x}} - 1 = 0$

$\sqrt{\frac{x}{y}} = 2$

$\sqrt{\frac{y}{x}} = \sqrt{\frac{x}{y}} \Rightarrow x = y$

3.2 $\max x^2y^2$ st $2x+y=9$

$L = f(x,y) - \lambda g(x,y)$ $\quad L = x^2y^2 - \lambda g(x,y)$

$\frac{\partial L}{\partial x} = 2xy^2 - 2\lambda = 0 \Rightarrow 2xy^2 = 2\lambda \Rightarrow xy^2 = \lambda$

$\frac{\partial L}{\partial y} = 2yx^2 - \lambda = 0 \Rightarrow 2yx^2 = \lambda$

$xy^2 = 2x^2y \quad | : xy$

$y = 2x$

$2x + 2x = 9$

$4x = 9$

$x = \frac{9}{4} \Rightarrow y = \frac{9}{2}$

$f\left(\frac{9}{4}, \frac{9}{2}\right) = \left(\frac{9}{4}\right)^2 \cdot \left(\frac{9}{2}\right)^2 = \frac{9^2 \cdot 9^2}{2^6} = \frac{81^2}{2^6} = \frac{3^8}{2^6} = \frac{6561}{64} = 102,515625$

4.1 $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix}$

$BA = \begin{bmatrix} 2\cdot1+2\cdot0+7\cdot1 & 5\cdot1+0+6 \\ 2\cdot9+2\cdot1+7\cdot5 & 5\cdot9+1+5\cdot6 \end{bmatrix} = \begin{bmatrix} 9 & 11 \\ 55 & 76 \end{bmatrix}$

4.2 $A = \begin{bmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 5\cdot8+3\cdot2 & 5\cdot4+3\cdot1 & 5\cdot0+3\cdot2 \\ 0\cdot8+1\cdot2 & 0\cdot4+1\cdot1 & 0+1\cdot2 \\ 1\cdot8+2\cdot2 & 1\cdot4+2\cdot1 & 1\cdot0+8\cdot2 \end{bmatrix} = \begin{bmatrix} 46 & 23 & 6 \\ 2 & 1 & 2 \\ 12 & 6 & 2 \end{bmatrix}$

5.1 $\Omega = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

5.2 $p = 0,001$ $\quad p_1 = 0,999$

$0,98$ $\quad 1 - 99,7\% = 0,003$

$P = \frac{0,001 \cdot 0,98}{0,001 \cdot 0,98 + 0,999 \cdot 0,003} = \frac{0,00098}{0,00098 + 0,002997} = \frac{0,00098}{0,003977} = 24,64\%$

53 $E(x) = np = 20 \cdot \frac{1}{6} = \frac{20}{6} = 3\frac{2}{3}$