Задача 1.

$f(x) = \frac{1}{x_2 - x_1}$ - функция плотности равномерного распеределения

$E(x) = \frac{x_1 + x_2}{2}$

$$\int_{x_1}^{x_2} x^2 f(x)\, dx - (E(X))^2 = \frac{1}{x_2 - x_1} \cdot \int_{x_1}^{x_2} x^2\, dx - (\frac{x_1 + x_2}{2})^2 = \frac{1}{x_2 - x_1} \cdot \frac{x^3}{3}\Big|_{x_1}^{x_2} - (\frac{x_1 + x_2}{2})^2 =$$

$$D(x) = \frac{x^3}{3(x_2 - x_1)}\Big|_{x_1}^{x_2} - (\frac{x_1 + x_2}{2})^2 = \frac{x_2^3}{3(x_2 - x_1)} - \frac{x_1^3}{3(x_2 - x_1)} - (\frac{x_1 + x_2}{2})^2 = \frac{x_2^3 - x_1^3}{3(x_2 - x_1)} - (\frac{x_1 + x_2}{2})^2 =$$

$$= \frac{(x_2 - x_1)(x_2^2 + x_2 x_1 + x_1^2)}{3(x_2 - x_1)} - (\frac{x_1 + x_2}{2})^2 = \frac{(x_2^2 + x_2 x_1 + x_1^2)}{3} - (\frac{x_1 + x_2}{2})^2 = \frac{x_2^2 + x_2 x_1 + x_1^2}{3} -$$

$$- \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} = \frac{4(x_2^2 + x_2 x_1 + x_1^2) - 3(x_1^2 + 2x_1 x_2 + x_2^2)}{12} = \frac{4x_2^2 + 4x_2 x_1 + 4x_1^2 - 3x_1^2 - 6x_1 x_2 - 3x_2^2}{12} =$$

$$= \frac{x_2^2 - 2x_1 x_2 + x_1^2}{12} = \frac{(x_2 - x_1)^2}{12}$$

Задача 2.

1. $p(y) = 0, \quad -\infty < y < 0$

$$F(y) = \int_{-\infty}^{0} 0\, dy = 0$$

$p(y) = y, \quad 0 \leqslant y \leqslant 1$

$$F(y) = \int_{-\infty}^{0} 0\, dy + \int_{0}^{y} y\, dy = 0 + \frac{y^2}{2}\Big|_0^y = \frac{y^2}{2}$$

$p(y) = 1, \quad 1 < y \leqslant 1.5$

$$F(y) = \int_{-\infty}^{0} 0\, dy + \int_{0}^{1} y\, dy + \int_{1}^{y} 1\, dy = \frac{y^2}{2}\Big|_0^1 + y\Big|_1^y = 0 + \frac{1}{2} + y - 1 = y - \frac{1}{2}$$

$p(y) = 0, \quad 1.5 < y \leqslant +\infty$

$$F(y) = \int_{-\infty}^{0} 0\,dy + \int_{0}^{1} y\,dy + \int_{1}^{1.5} 1\,dy + \int_{1.5}^{+\infty} 0\,dy = 0 + \frac{1}{2} + (1.5-1) + 0 = \frac{1}{2} + \frac{1}{2} = 1$$

$$F(y) = \begin{cases} 0, \text{ при } -\infty < y < 0 \\ \frac{y^2}{2}, \text{ при } 0 \leqslant y \leqslant 1 \\ y - \frac{1}{2}, \text{ при } 1 < y \leqslant 1.5 \\ 1, \text{ при } 1.5 < y \leqslant +\infty \end{cases}$$

2. $F(3) = 1$

3. $P(0.7 \leqslant Y \leqslant 1.1) = F(1.1) - F(0.7) = 1.1 - 0.5 - \frac{0.7^2}{2} = 0.355$

Задача 3.

1. $\displaystyle\int_a^b p(y)\,dy = \int_0^\pi a\cdot sin(y)\,dy = a\cdot\int_0^\pi sin(y)\,dy = a\cdot(-cos(y))\Big|_0^\pi = a\cdot(-cos(\pi)-(-cos(0)) = 2a$

$$\int_a^b p(y)\,dy = 1 \Leftrightarrow \int_0^\pi a \cdot sin(y)\,dy = 1 \Rightarrow a = \frac{1}{2}$$

2. $p(y) = \frac{1}{2}sin(y), \quad 0 \leqslant y \leqslant \pi$

$$F(y) = \int_0^y \frac{1}{2}sin(y)\,dy = -\frac{cos(y)}{2}\Bigg|_0^y = -\frac{cos(y)+1}{2}$$

$p(y) = 0, \quad \pi < y < 2\pi$

$$F(y) = \int_0^\pi \frac{1}{2}sin(y)\,dy + \int_\pi^y 0\,dy = 1$$

$$F(y) = \begin{cases} -\frac{cos(y)-1}{2}, \text{ при } 0 \leqslant y \leqslant \pi \\ 1, \text{ при } \pi < y < 2\pi \end{cases}$$

3. $P(Y \leqslant \frac{\pi}{6}) = F(\frac{\pi}{6}) = -\frac{cos(\frac{\pi}{6})-1}{2} = -\frac{\sqrt{3}-2}{4} \approx 0.0669$

4. $E(Y) = \int_0^{\pi} y \cdot f(y)\, dy = \int_0^{\pi} y \cdot \frac{1}{2} \cdot sin(y)\, dy = \frac{1}{2} \int_0^{\pi} y \cdot sin(y)\, dy = \frac{1}{2} \cdot (y(-cos(y)))-$

$- \int_0^{\pi} -cos(y)\, dy = \frac{1}{2} \cdot (y(-cos(y)) + sin(y)) = \left. \frac{-y \cdot cos(y) + sin(y)}{2} \right|_0^{\pi} = \frac{-\pi \cdot cos(\pi) + sin(\pi)}{2} -$

$- \frac{0 \cdot cos(0) + sin(0)}{2} = \frac{-\pi \cdot (-1)}{2} = \frac{\pi}{2} \approx 1.57$