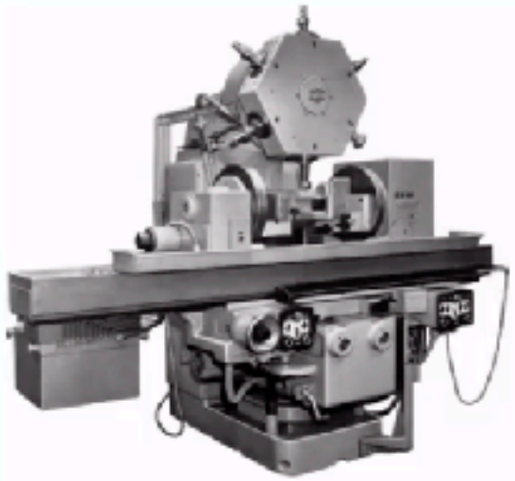
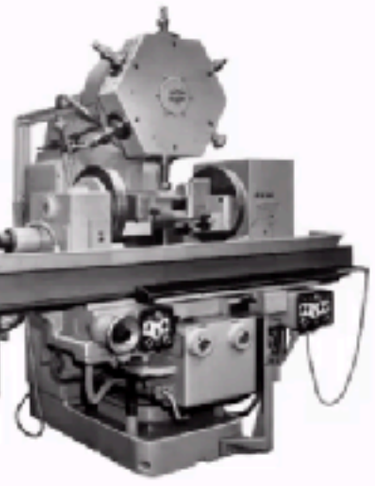
Bayes' Theorem

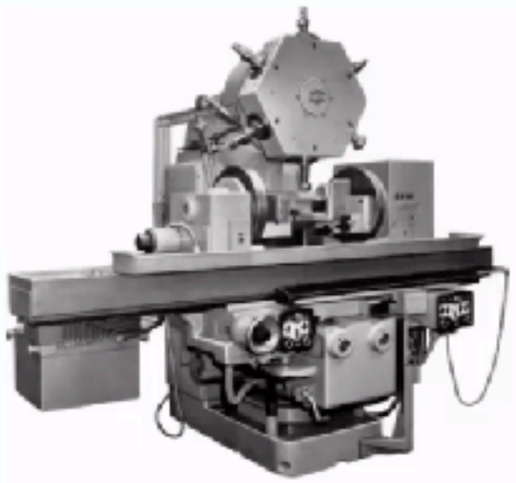
Bayes Theorem

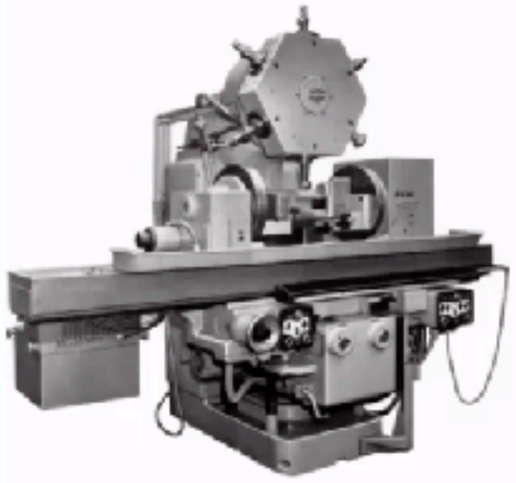
Bayes Theorem



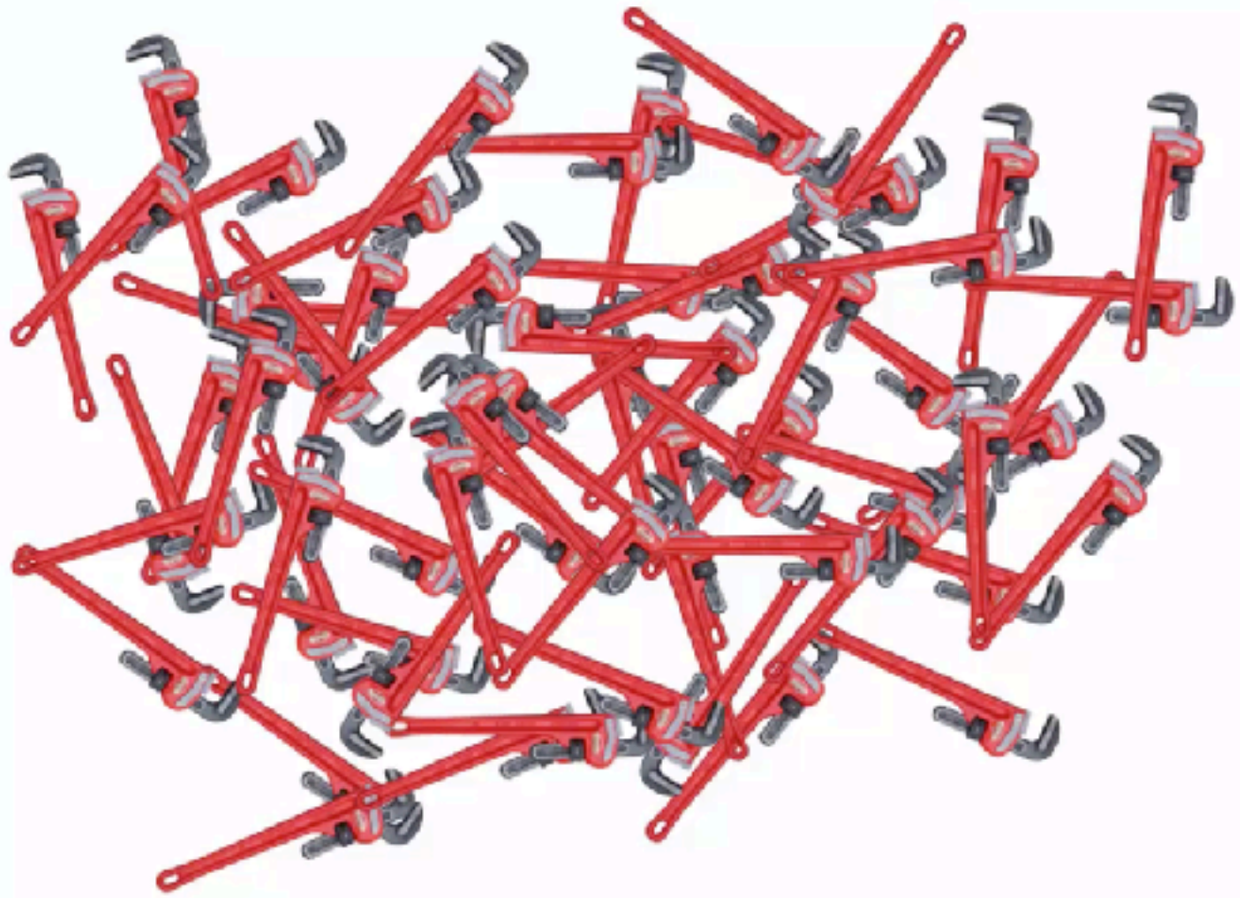


Bayes Theorem

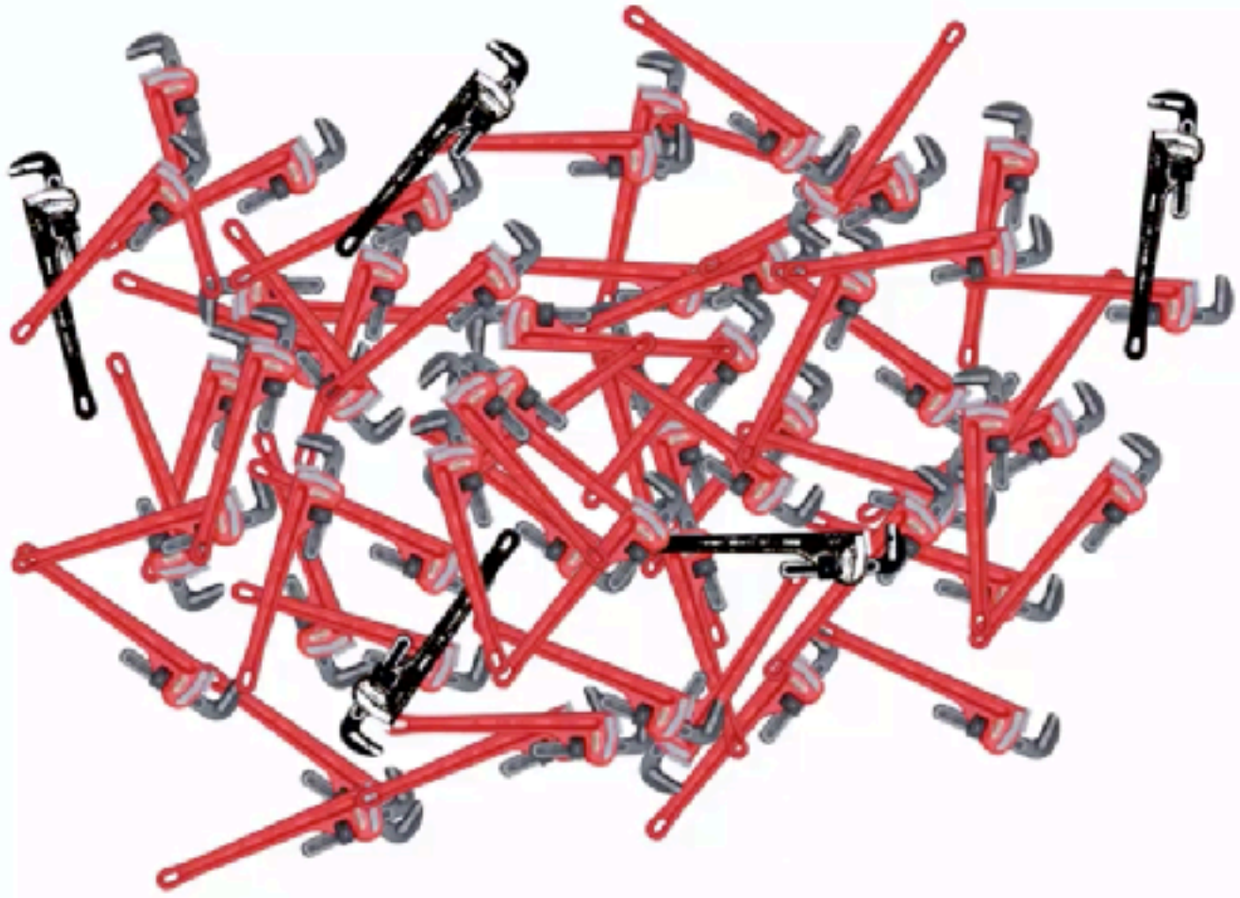




Bayes Theorem

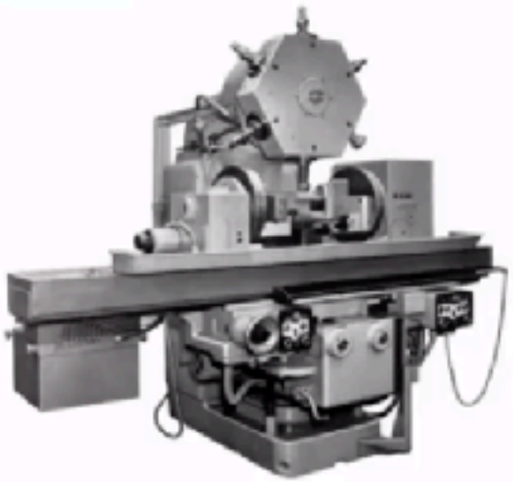


Bayes Theorem



Bayes Theorem

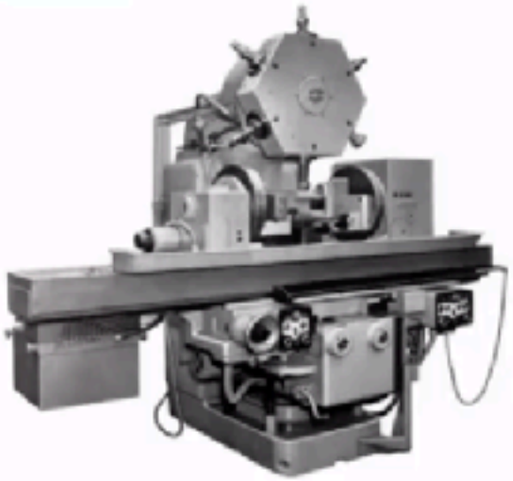
What's the probability?



m2

Bayes Theorem

What's the probability?



m2

Bayes Theorem

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

Bayes Theorem

Mach1: 30 wrenches / hr
Mach2: 20 wrenches / hr

Bayes Theorem

Mach1: 30 wrenches / hr
Mach2: 20 wrenches / hr

Out of all produced parts:
We can SEE that 1% are defective

Out of all defective parts:
We can SEE that 50% came from mach1
And 50% came from mach2

Bayes Theorem

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Question:
What is the probability that a part produced by mach2 is defective = ?

Bayes Theorem

Mach1: 30 wrenches / hr
Mach2: 20 wrenches / hr

-> P(Mach1) = 30/50 = 0.6
-> P(Mach2) = 20/50 = 0.4

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We can SEE that 1% are defective

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Bayes Theorem

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Mach2: 20 wrenches / hr

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-> P(Mach2) = 20/50 = 0.4

Out of all produced parts:
We can SEE that 1% are defective

-> P(Defect) = 1%

Out of all defective parts:
We can SEE that 50% came from mach1
And 50% came from mach2

-> P(Mach1 | Defect) = 50%

Question:
What is the probability that a part produced by mach2 is defective = ?

Bayes Theorem

Mach1: 30 wrenches / hr
Mach2: 20 wrenches / hr

-> P(Mach1) = 30/50 = 0.6
-> P(Mach2) = 20/50 = 0.4

Out of all produced parts:
We can SEE that 1% are defective

-> P(Defect) = 1%

Out of all defective parts:
We can SEE that 50% came from mach1
And 50% came from mach2

-> P(Mach1 | Defect) = 50%
-> P(Mach2 | Defect) = 50%

Question:
What is the probability that a part produced by mach2 is defective = ?

-> P(Defect | Mach2) = ?

Bayes Theorem

Mach1: 30 wrenches / hr
Mach2: 20 wrenches / hr

~~-> P(Mach1) = 30/50 = 0.6~~
-> P(Mach2) = 20/50 = 0.4

Out of all produced parts:
We can SEE that 1% are defective

-> P(Defect) = 1%

Out of all defective parts:
We can SEE that 50% came from mach1
And 50% came from mach2

~~-> P(Mach1 | Defect) = 50%~~
-> P(Mach2 | Defect) = 50%

Question:
What is the probability that a part produced by mach2 is defective = ?

-> P(Defect | Mach2) = ?

Bayes Theorem

Mach1: 30 wrenches / hr
Mach2: 20 wrenches / hr
Out of all produced parts:
We can SEE that 1% are defective
Out of all defective parts:
We can SEE that 50% came from mach1
And 50% came from mach2
Question:
What is the probability that a part produced by mach2 is defective = ?

-> P(Mach2) = 20/50 = 0.4
-> P(Defect) = 1%
-> P(Mach2 | Defect) = 50%
-> P(Defect | Mach2) = ?

$$P(\text{Defect} \mid \text{Mach2}) = \frac{P(\text{Mach2} \mid \text{Defect}) * P(\text{Defect})}{P(\text{Mach2})}$$

Bayes Theorem

Mach1: 30 wrenches / hr
Mach2: 20 wrenches / hr
Out of all produced parts:
We can SEE that 1% are defective
Out of all defective parts:
We can SEE that 50% came from mach1
And 50% came from mach2
Question:
What is the probability that a part produced by mach2 is defective = ?

-> P(Mach2) = 20/50 = 0.4
-> P(Defect) = 1%
-> P(Mach2 | Defect) = 50%
-> P(Defect | Mach2) = ?

$$P(\text{Defect} \mid \text{Mach2}) = \frac{0.5 \quad 0.01}{0.4}$$

Bayes Theorem

Mach1: 30 wrenches / hr
Mach2: 20 wrenches / hr
Out of all produced parts:
We can SEE that 1% are defective
Out of all defective parts:
We can SEE that 50% came from mach1
And 50% came from mach2
Question:
What is the probability that a part produced by mach2 is defective = ?

-> P(Mach2) = 20/50 = 0.4
-> P(Defect) = 1%
-> P(Mach2 | Defect) = 50%
-> P(Defect | Mach2) = ?

$$P(\text{Defect} \mid \text{Mach2}) = \frac{0.5 * 0.01}{0.4} = 0.0125$$

It's intuitive!

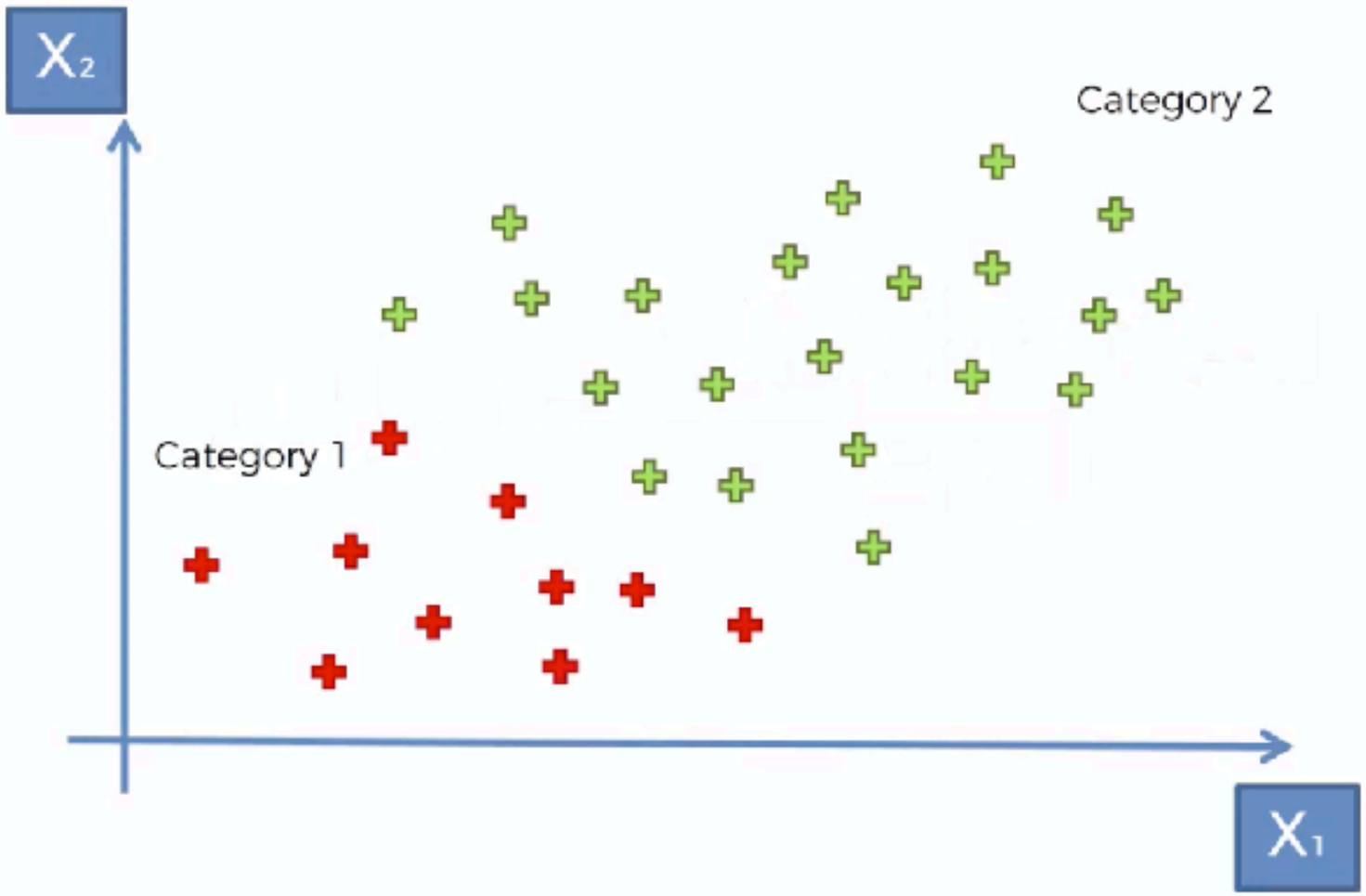
$$P(\text{Defect} \mid \text{Mach2}) = \frac{P(\text{Mach2} \mid \text{Defect}) * P(\text{Defect})}{P(\text{Mach2})} = 1.25\%$$

Let's look at an example:

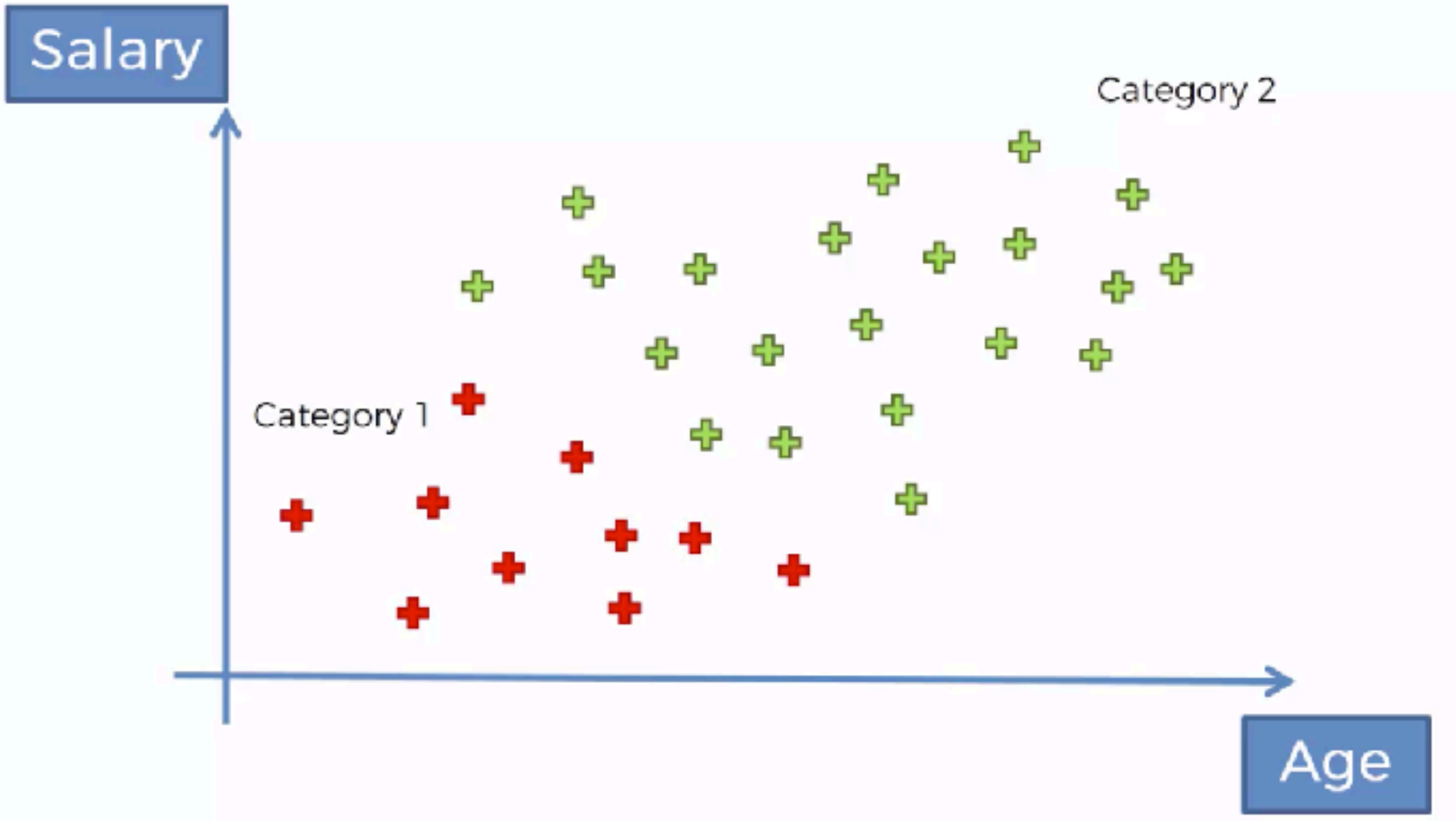
- 1000 wrenches
- 400 came from Mach2
- 1% have a defect = 10
- of them 50% came from Mach2 = 5
- % defective parts from Mach2 = 5/400 = 1.25%

Naïve Bayes Classifier Intuition

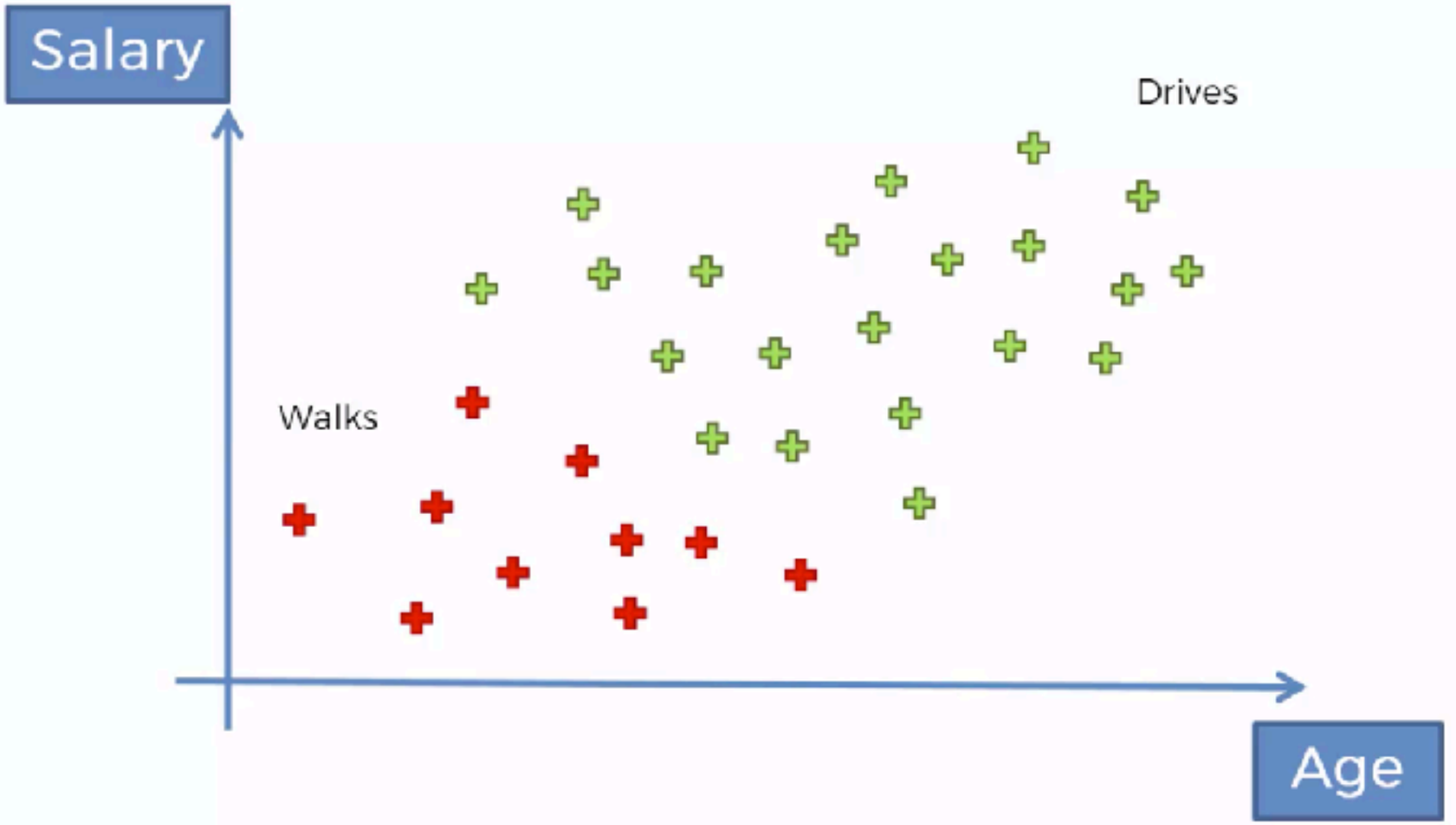
Naïve Bayes



Naïve Bayes



Naïve Bayes



Plan of Attack

Step 1

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

Step 1

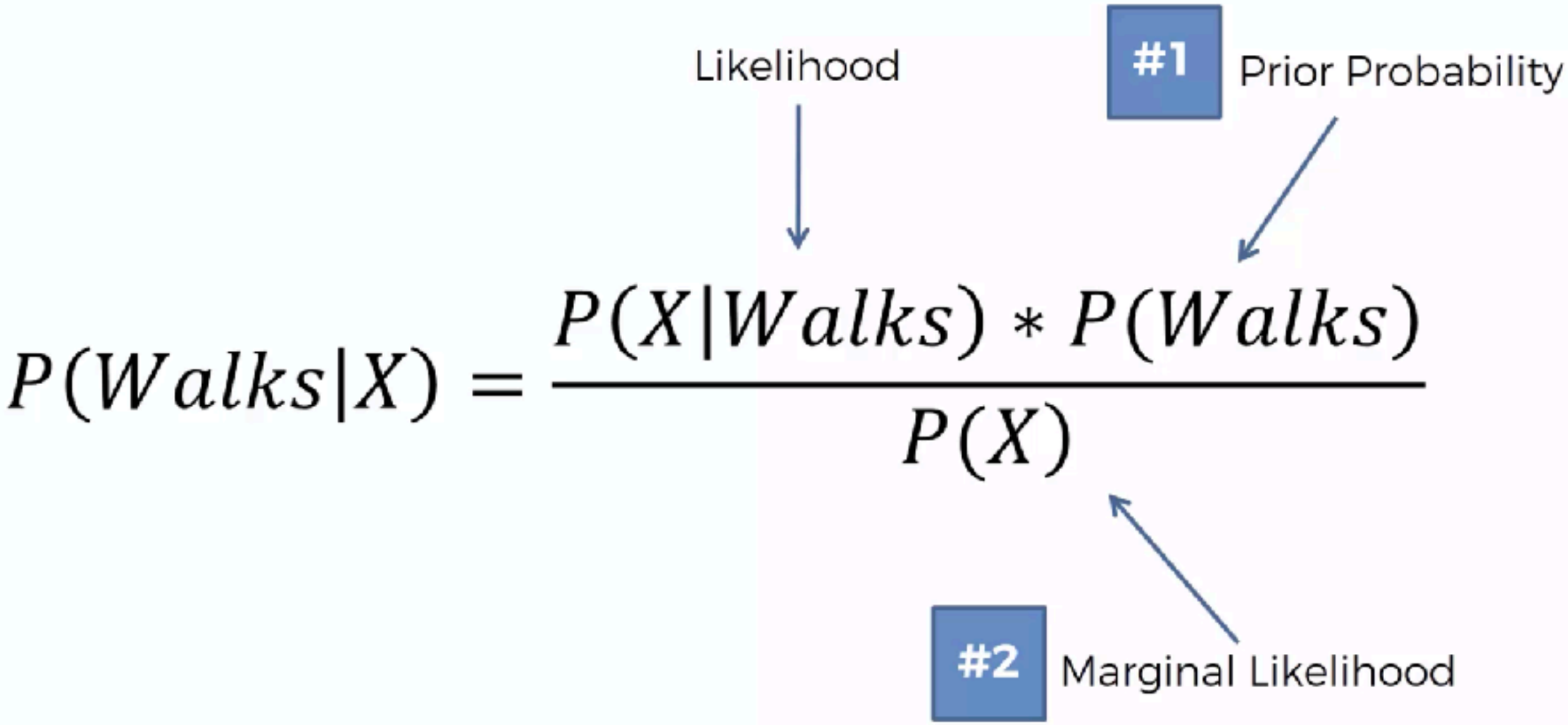
$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

Step 1

#1 Prior Probability

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

Step 1


Likelihood
#1
Prior Probability
$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$
#2
Marginal Likelihood

Step 1

#4 Posterior Probability

#3 Likelihood

#1 Prior Probability

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

#2 Marginal Likelihood

Step 2

#4 Posterior Probability

#3 Likelihood

#1 Prior Probability

$$P(Drives|X) = \frac{P(X|Drives) * P(Drives)}{P(X)}$$

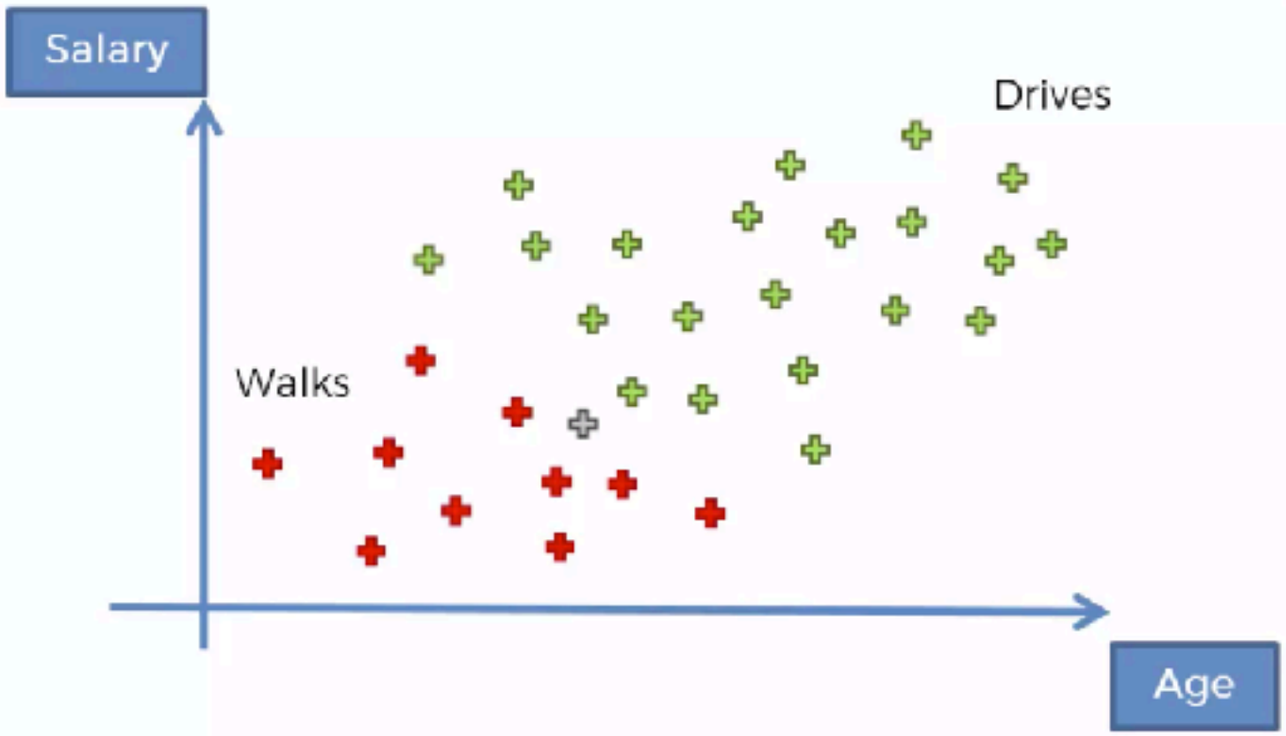
#2 Marginal Likelihood

Step 3

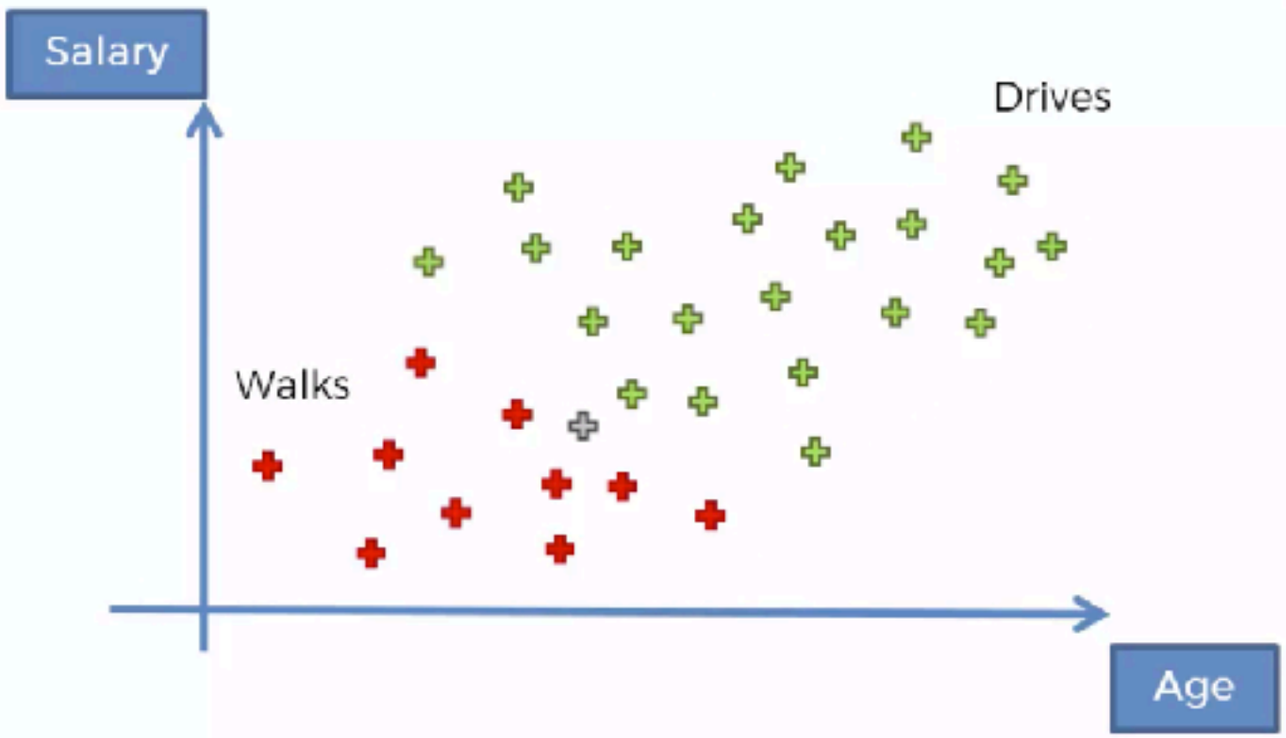
$$P(Walks|X) \; v.s. \; P(Drives|X)$$

Naïve Bayes: Step 1

#1. P(Walks)



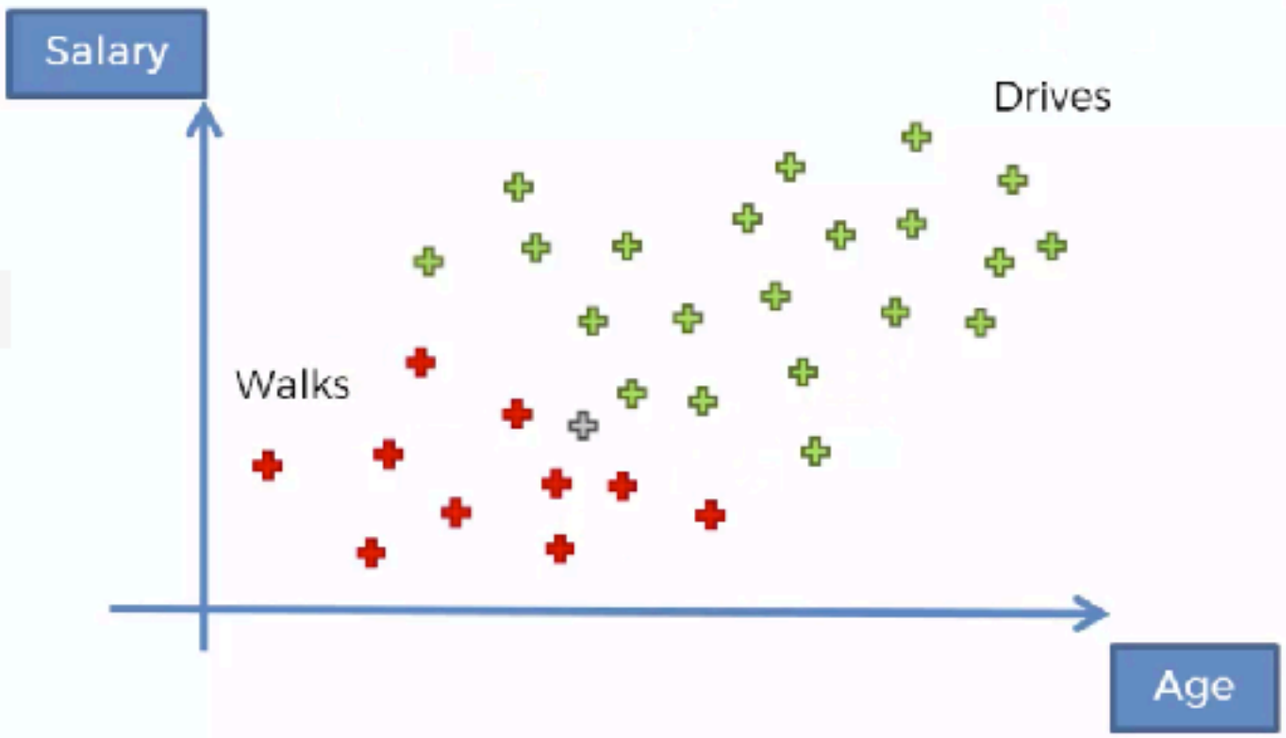
Naïve Bayes: Step 1



#1. P(Walks)

$$P(Walks) = \frac{Number\ of\ Walkers}{Total\ Observations}$$

Naïve Bayes: Step 1



#1. P(Walks)

$$P(Walks) = \frac{Number\ of\ Walkers}{Total\ Observations}$$

$$P(Walks) = \frac{10}{30}$$

Naïve Bayes: Step 1

#4 Posterior Probability

#3 Likelihood

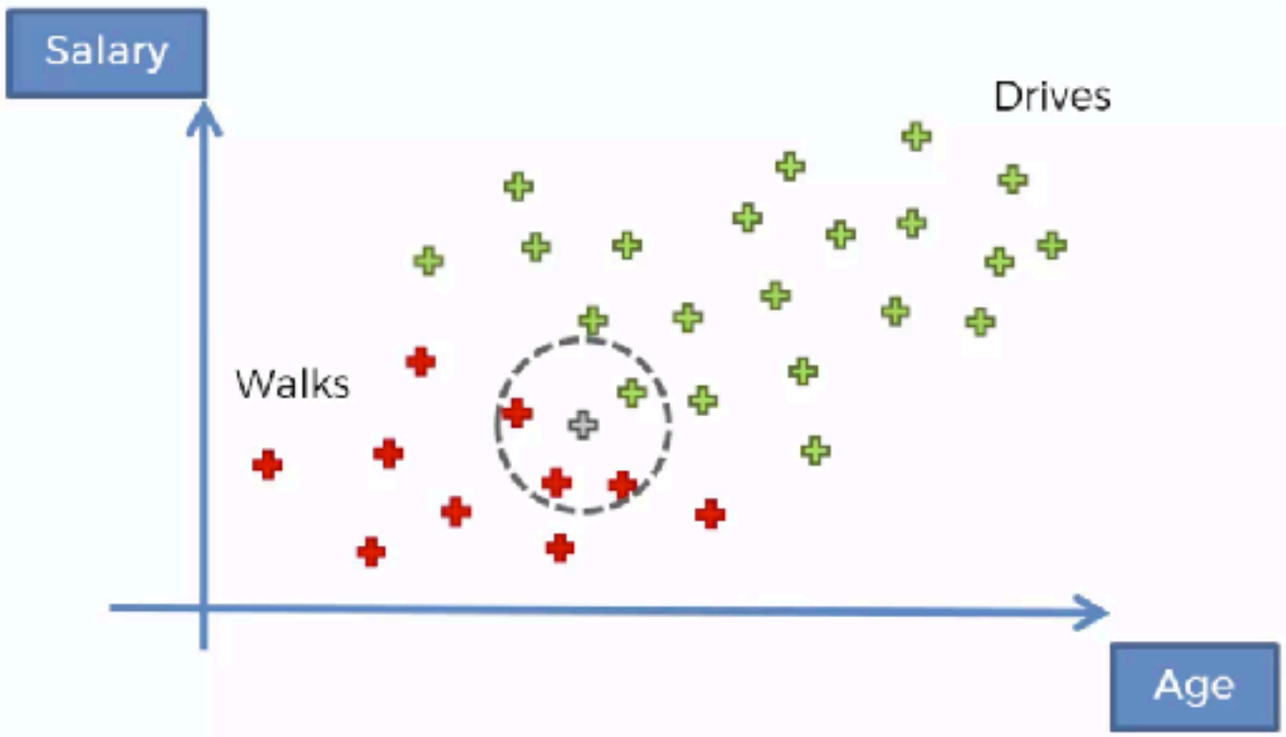
#1 Prior Probability

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

#2 Marginal Likelihood

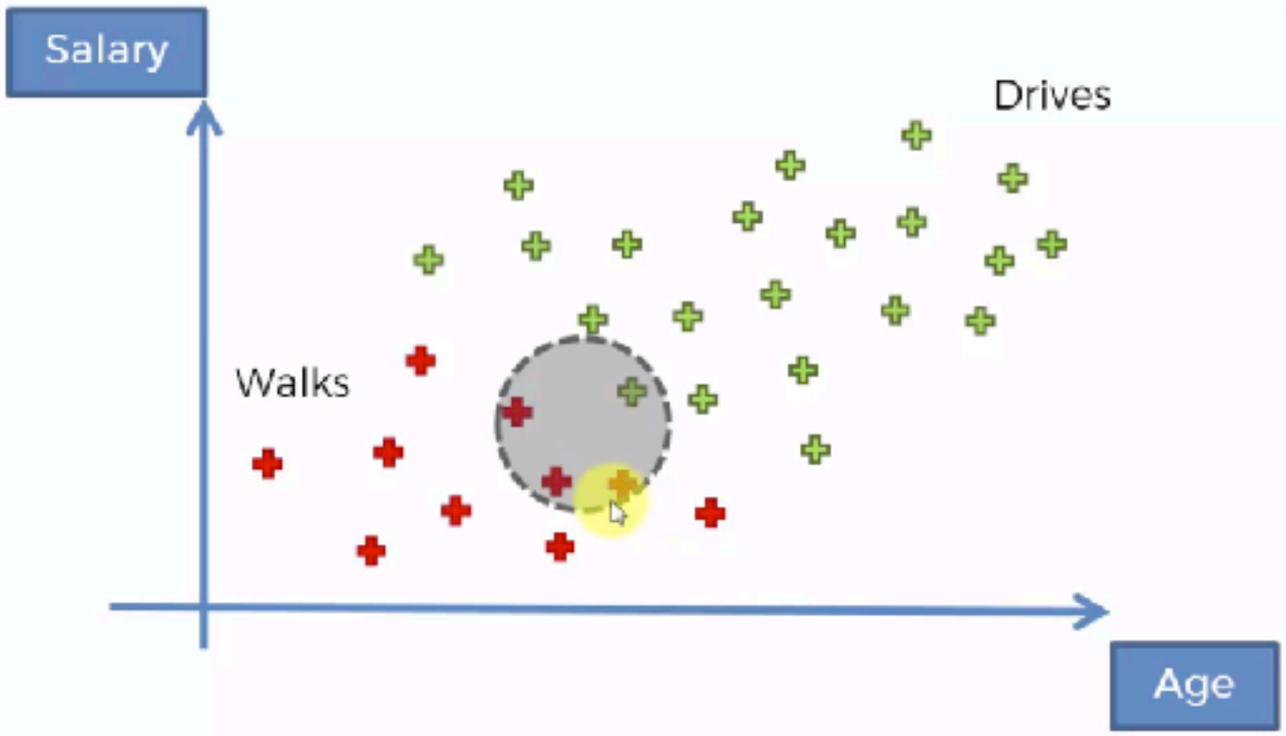
Naïve Bayes: Step 1

#2. P(X)

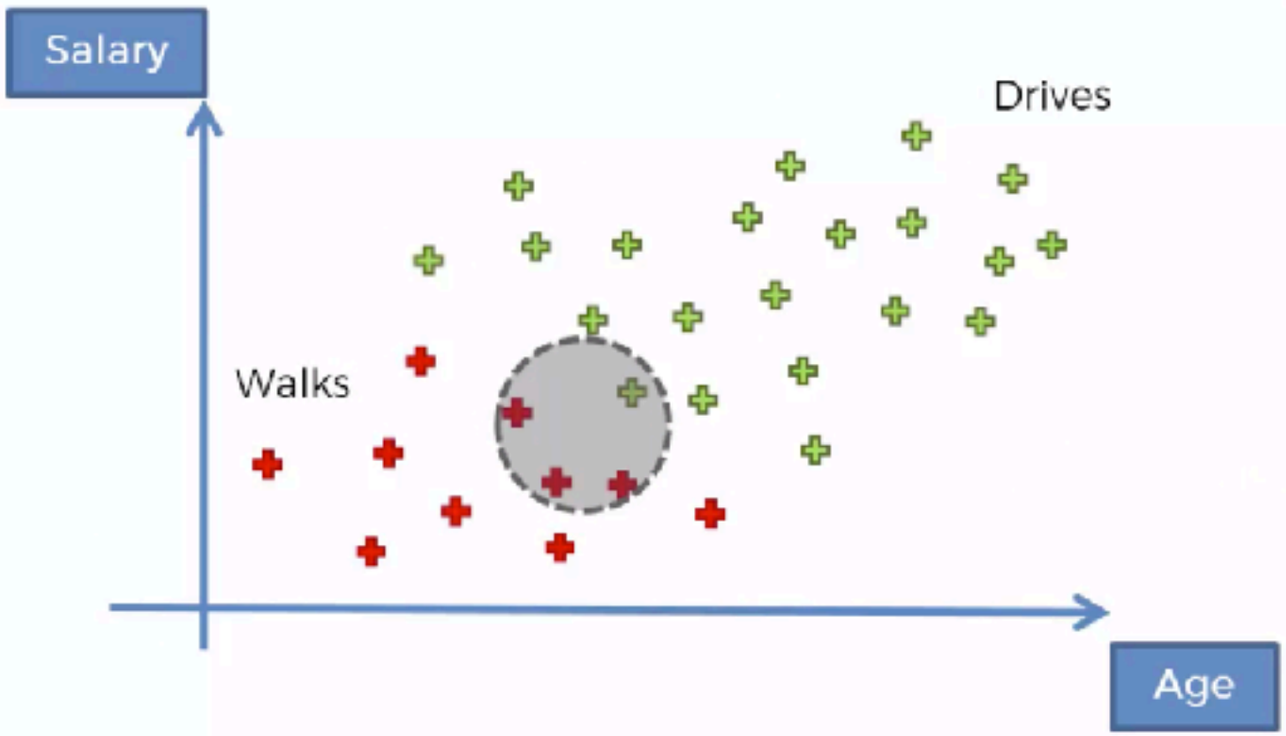


Naïve Bayes: Step 1

#2. P(X)



Naïve Bayes: Step 1



#2. P(X)

$$P(X) = \frac{Number\ of\ Similar\ Observations}{Total\ Observations}$$

$$P(X) = \frac{4}{30}$$

Naïve Bayes: Step 1

#4 Posterior Probability

#3 Likelihood

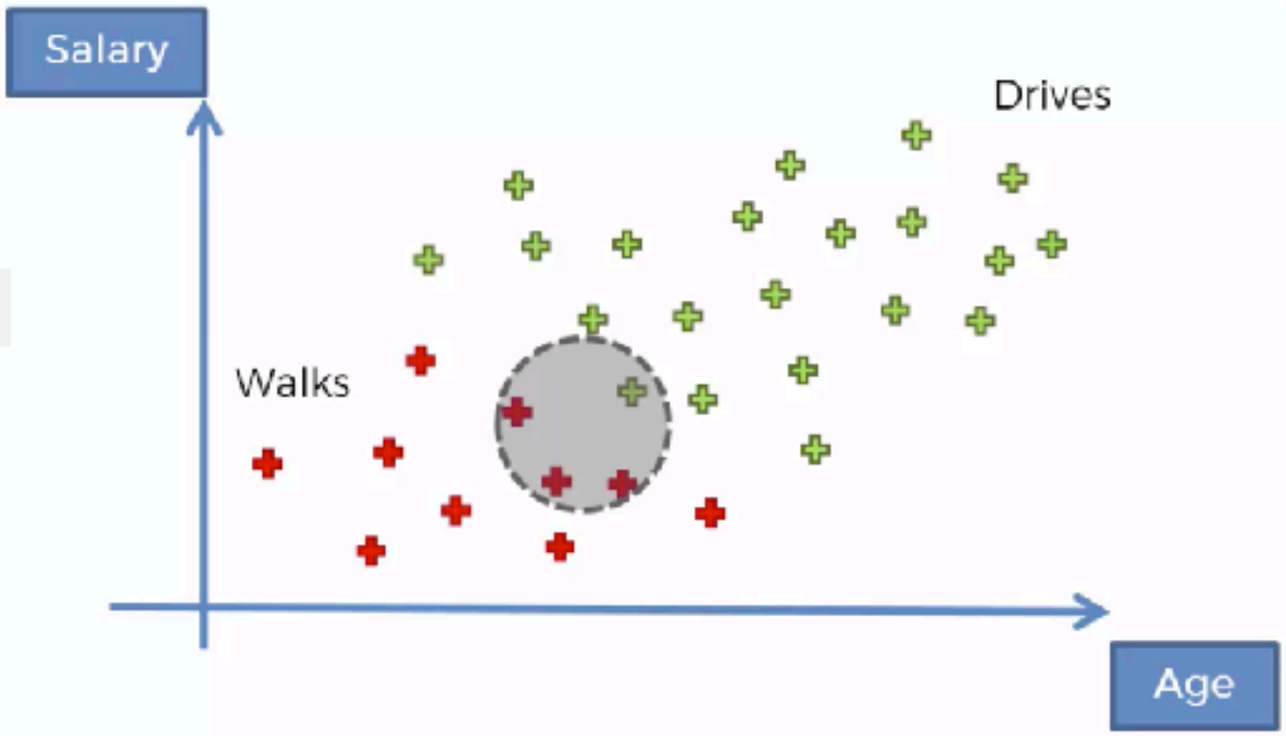
#1 Prior Probability

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

#2 Marginal Likelihood

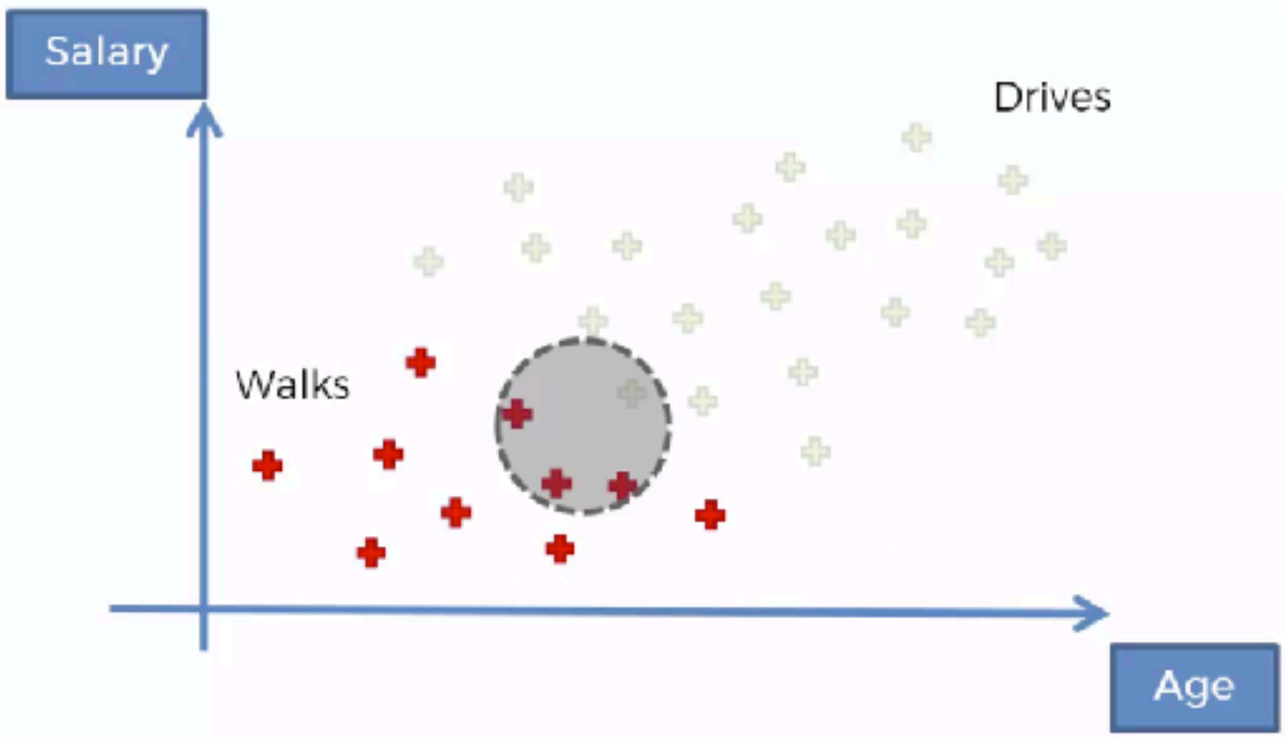
Naïve Bayes: Step 1

#3. P(X|Walks)

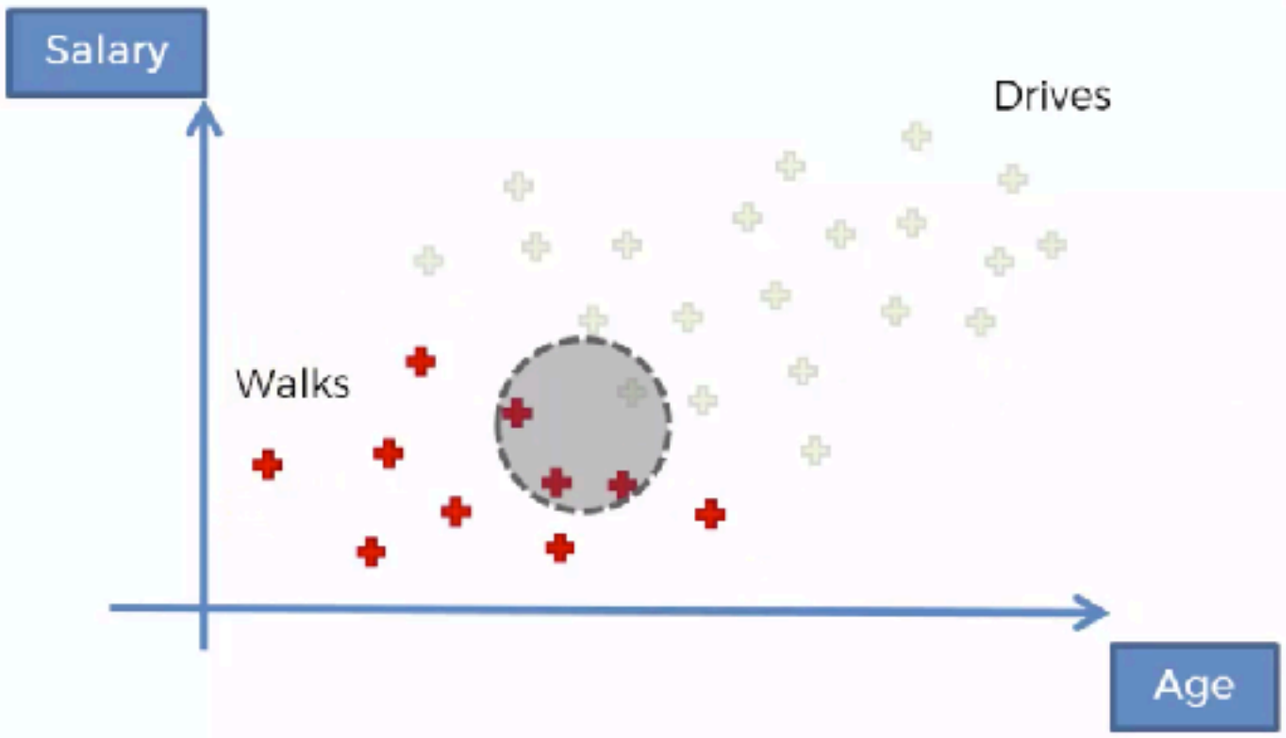


Naïve Bayes: Step 1

#3. P(X|Walks)



Naïve Bayes: Step 1



#3. P(X|Walks)

$$P(X|Walks) = \frac{Number\ of\ Similar\ Observations\ Among\ those\ who\ Walk}{Total\ number\ of\ Walkers}$$

$$P(X|Walks) = \frac{3}{10}$$

Naïve Bayes: Step 1

#4 Posterior Probability

#3 Likelihood

#1 Prior Probability

$$P(Walks|X) = \frac{\frac{3}{10} * \frac{10}{30}}{4} = 0.75$$

#2 Marginal Likelihood

Step 2

#4 Posterior Probability

#3 Likelihood

#1 Prior Probability

$$P(Drives|X) = \frac{P(X|Drives) * P(Drives)}{P(X)}$$

#2 Marginal Likelihood

Naïve Bayes: Step 2

#4 Posterior Probability

#3 Likelihood

#1 Prior Probability

$$P(Drives|X) = \frac{\frac{1}{20} * \frac{20}{30}}{\frac{4}{30}} = 0.25$$

#2 Marginal Likelihood

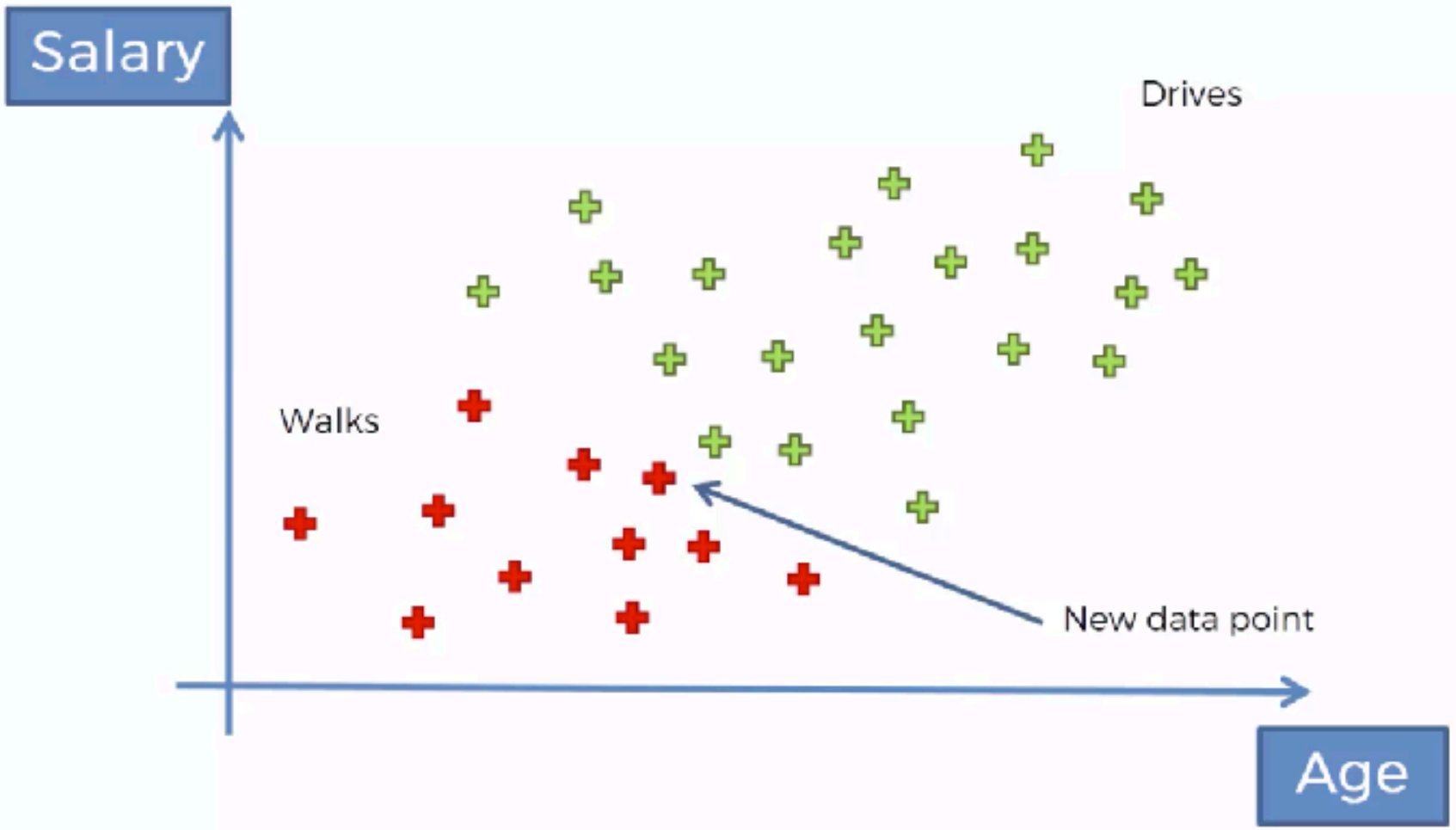
Step 3

$$P(Walks|X) \; v.s. \; P(Drives|X)$$

Step 3

$$P(Walks|X) > P(Drives|X)$$

Naïve Bayes



Naïve Bayes

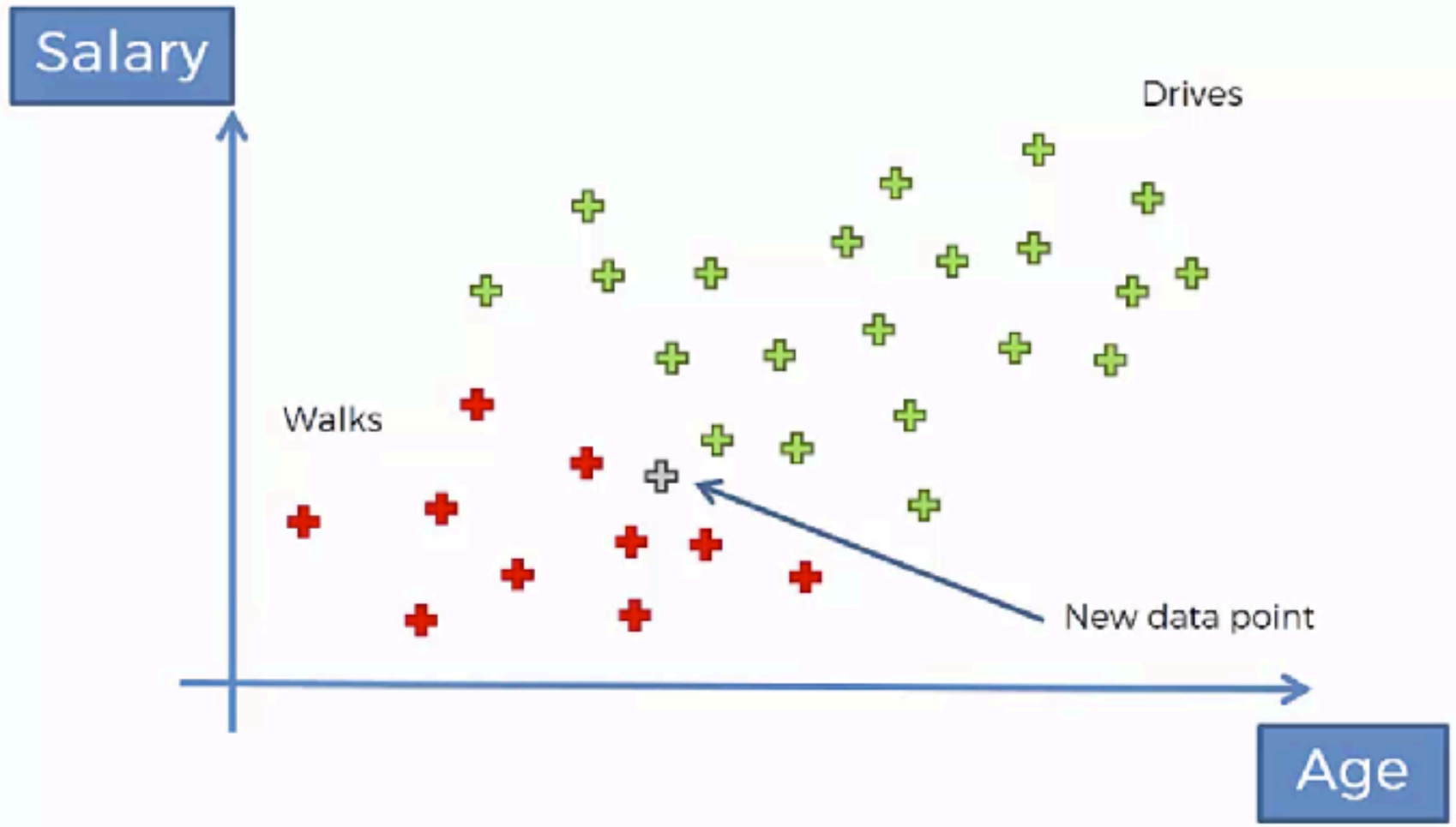
Q: Why "Naïve"?

Naïve Bayes

Q: Why "Naïve"?

A: Independence assumption

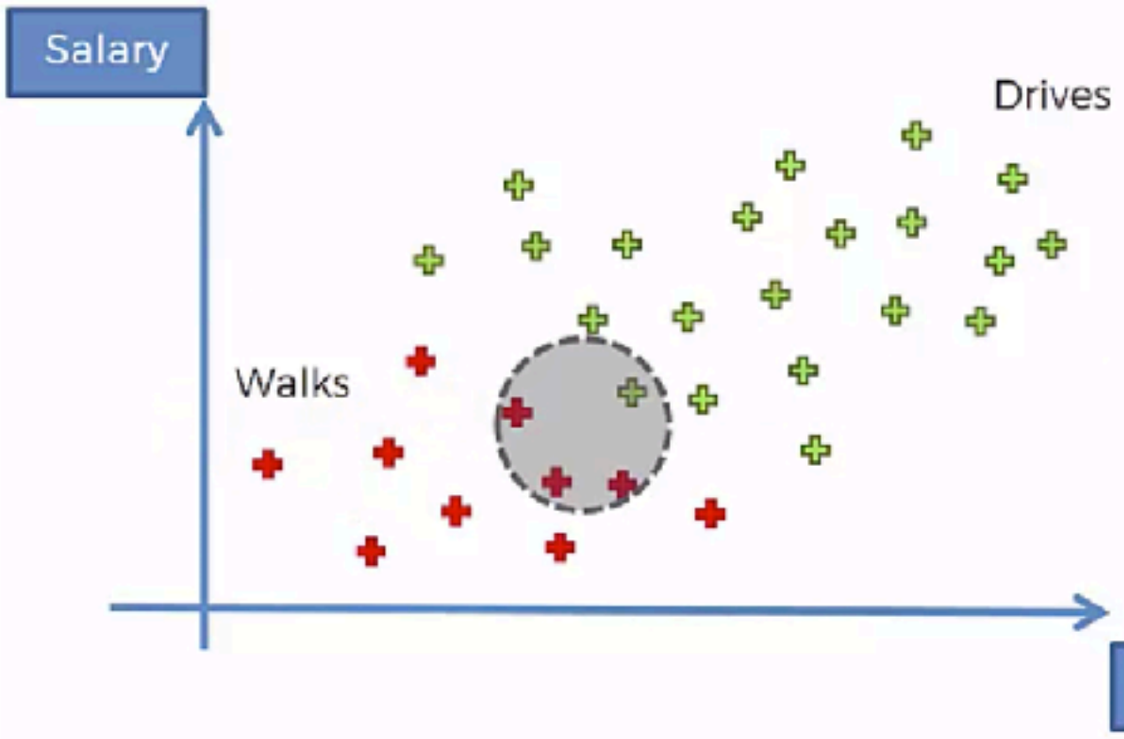
Naïve Bayes



Naïve Bayes

$$P(X)$$

Naïve Bayes: Step 2



#2. P(X)

$$P(X) = \frac{Number\ of\ Similar\ Observations}{Total\ Observations}$$

$$P(X) = \frac{4}{30}$$

NOTE: Same both times

Step 1

#4 Posterior Probability

#3 Likelihood

#1 Prior Probability

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

#2 Marginal Likelihood

Step 2

#4 Posterior Probability

#3 Likelihood

#1 Prior Probability

$$P(Drives|X) = \frac{P(X|Drives) * P(Drives)}{P(X)}$$

#2 Marginal Likelihood

Step 3

$$\frac{P(X|Walks) * P(Walks)}{P(X)} \quad v.s. \quad \frac{P(X|Drives) * P(Drives)}{P(X)}$$

Step 3

$$\frac{P(X|Walks) * P(Walks)}{\cancel{P(X)}} \quad v.s. \quad \frac{P(X|Drives) * P(Drives)}{\cancel{P(X)}}$$