

Bayes' Theorem

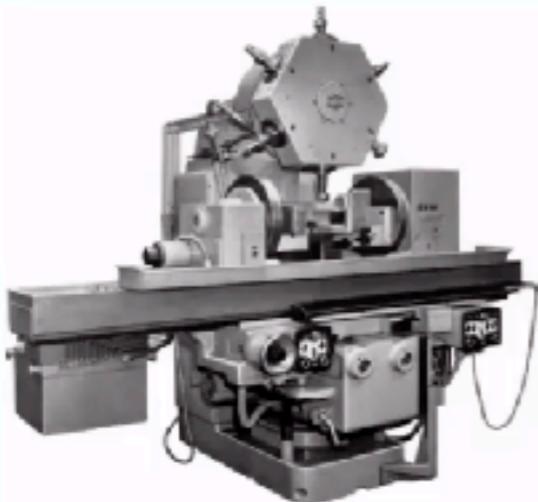
Bayes Theorem



Bayes Theorem



Bayes Theorem



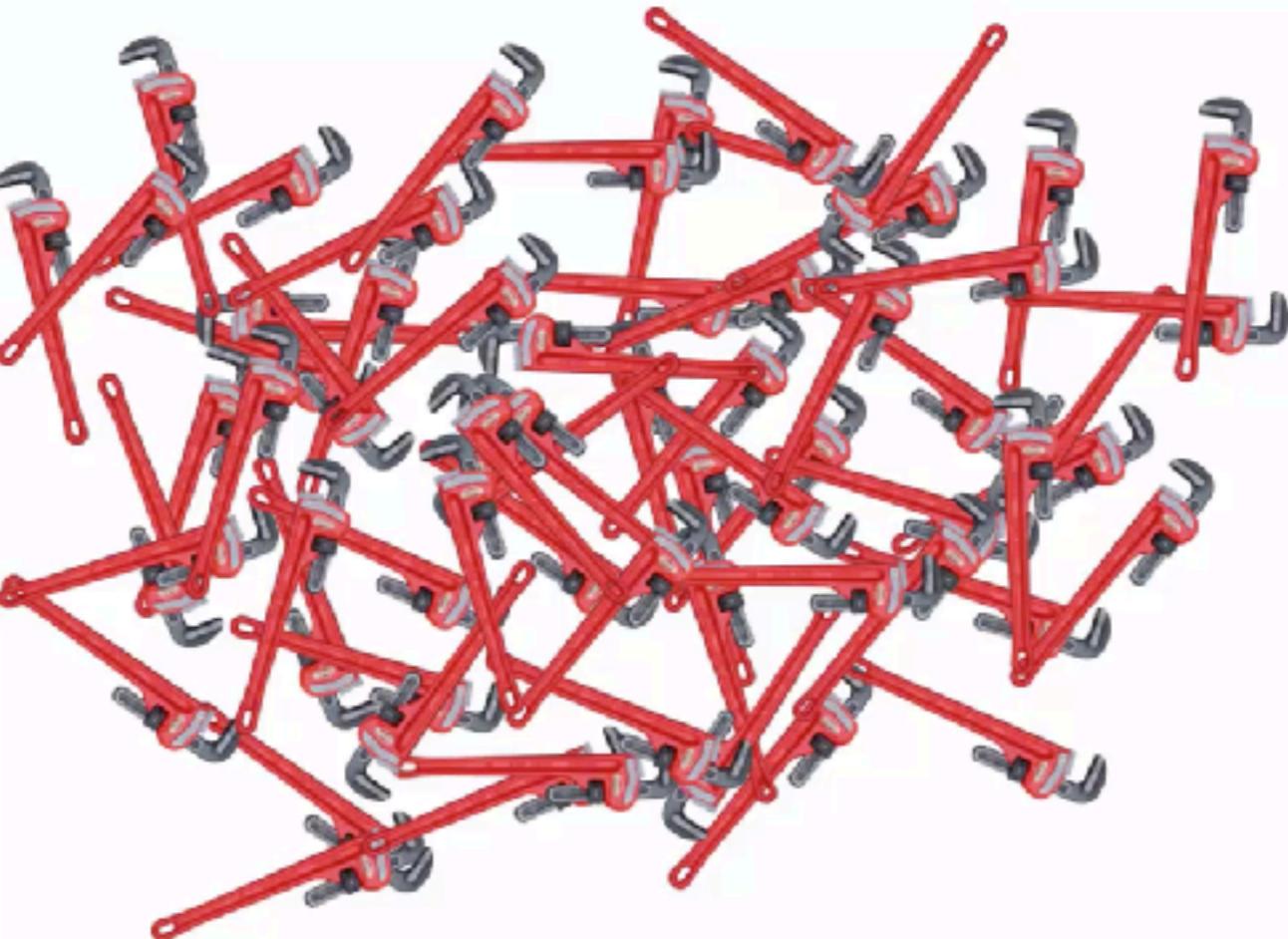
m1 m1



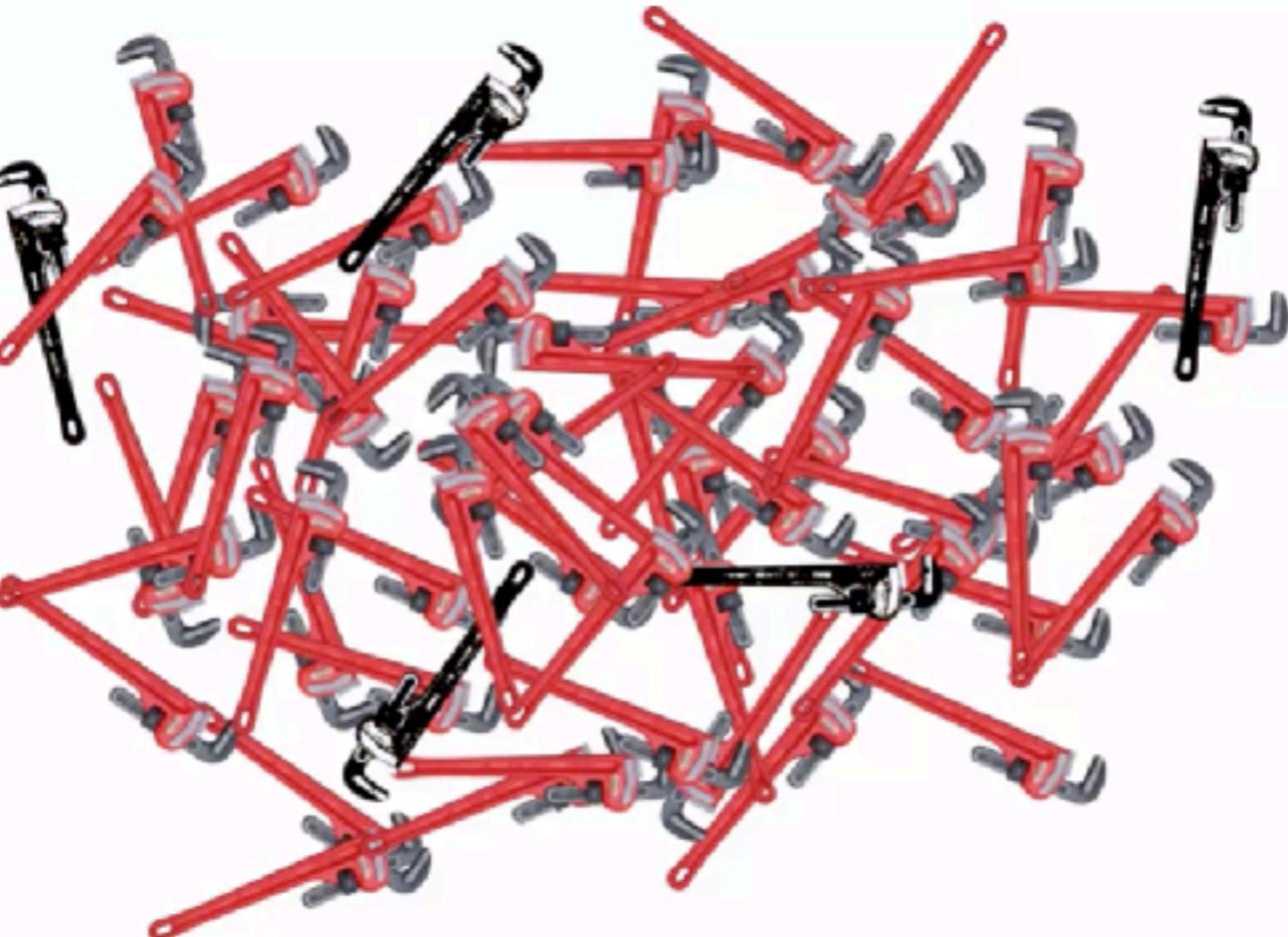
m2 m2



Bayes Theorem



Bayes Theorem



Bayes Theorem

What's the probability?

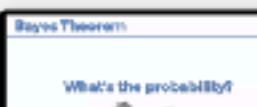


m2



Bayes Theorem

What's the probability?



Bayes Theorem

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

Out of all produced parts:

We can SEE that 1% are defective

Out of all defective parts:

We can SEE that 50% came from mach1

And 50% came from mach2

Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

Out of all produced parts:

We can SEE that 1% are defective

Out of all defective parts:

We can SEE that 50% came from mach1

And 50% came from mach2

Question:

What is the probability that a part
produced by mach2 is defective = ?

Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

$$\rightarrow P(\text{Mach1}) = 30/50 = 0.6$$

$$\rightarrow P(\text{Mach2}) = 20/50 = 0.4$$

Out of all produced parts:

We can SEE that 1% are defective

Out of all defective parts:

We can SEE that 50% came from mach1

And 50% came from mach2

Question:

What is the probability that a part
produced by mach2 is defective = ?

Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

$$\rightarrow P(\text{Mach1}) = 30/50 = 0.6$$

$$\rightarrow P(\text{Mach2}) = 20/50 = 0.4$$

Out of all produced parts:

We can SEE that 1% are defective

$$\rightarrow P(\text{Defect}) = 1\%$$

Out of all defective parts:

We can SEE that 50% came from mach1

And 50% came from mach2

$$\rightarrow P(\text{Mach1} \mid \text{Defect}) = 50\%$$

Question:

What is the probability that a part
produced by mach2 is defective = ?



Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

$$\rightarrow P(\text{Mach1}) = 30/50 = 0.6$$

$$\rightarrow P(\text{Mach2}) = 20/50 = 0.4$$

Out of all produced parts:

We can SEE that 1% are defective

$$\rightarrow P(\text{Defect}) = 1\%$$

Out of all defective parts:

We can SEE that 50% came from mach1

And 50% came from mach2

$$\rightarrow P(\text{Mach1} \mid \text{Defect}) = 50\%$$

$$\rightarrow P(\text{Mach2} \mid \text{Defect}) = 50\%$$

Question:

What is the probability that a part
produced by mach2 is defective = ?

$$\rightarrow P(\text{Defect} \mid \text{Mach2}) = ?$$

Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

$$\cancel{\rightarrow P(\text{Mach1}) = 30/50 = 0.6}$$

$$\rightarrow P(\text{Mach2}) = 20/50 = 0.4$$

Out of all produced parts:

We can SEE that 1% are defective

$$\rightarrow P(\text{Defect}) = 1\%$$

Out of all defective parts:

We can SEE that 50% came from mach1

And 50% came from mach2

$$\cancel{\rightarrow P(\text{Mach1} \mid \text{Defect}) = 50\%}$$

$$\rightarrow P(\text{Mach2} \mid \text{Defect}) = 50\%$$

Question:

What is the probability that a part
produced by mach2 is defective = ?

$$\rightarrow P(\text{Defect} \mid \text{Mach2}) = ?$$

Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

Out of all produced parts:

We can SEE that 1% are defective

Out of all defective parts:

We can SEE that 50% came from mach1

And 50% came from mach2

Question:

What is the probability that a part
produced by mach2 is defective = ?

$$\rightarrow P(\text{Mach2}) = 20/50 = 0.4$$

$$\rightarrow P(\text{Defect}) = 1\%$$

$$\rightarrow P(\text{Mach2} \mid \text{Defect}) = 50\%$$

$$\rightarrow P(\text{Defect} \mid \text{Mach2}) = ?$$

$$P(\text{Defect} \mid \text{Mach2}) = \frac{P(\text{Mach2} \mid \text{Defect}) * P(\text{Defect})}{P(\text{Mach2})}$$

Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

Out of all produced parts:

We can SEE that 1% are defective

Out of all defective parts:

We can SEE that 50% came from mach1

And 50% came from mach2

Question:

What is the probability that a part
produced by mach2 is defective = ?

$$\rightarrow P(\text{Mach2}) = 20/50 = 0.4$$

$$\rightarrow P(\text{Defect}) = 1\%$$

$$\rightarrow P(\text{Mach2} \mid \text{Defect}) = 50\%$$

$$\rightarrow P(\text{Defect} \mid \text{Mach2}) = ?$$

P(Defect | Mach2) =  0.01

0.5 0.01
 0.4

Bayes Theorem

Mach1: 30 wrenches / hr

Mach2: 20 wrenches / hr

Out of all produced parts:

We can SEE that 1% are defective

Out of all defective parts:

We can SEE that 50% came from mach1

And 50% came from mach2

Question:

What is the probability that a part
produced by mach2 is defective = ?

$$\rightarrow P(\text{Mach2}) = 20/50 = 0.4$$

$$\rightarrow P(\text{Defect}) = 1\%$$

$$\rightarrow P(\text{Mach2} \mid \text{Defect}) = 50\%$$

$$\rightarrow P(\text{Defect} \mid \text{Mach2}) = ?$$

$$P(\text{Defect} \mid \text{Mach2}) = \frac{0.5 * 0.01}{0.4} = 0.0125$$

It's intuitive!

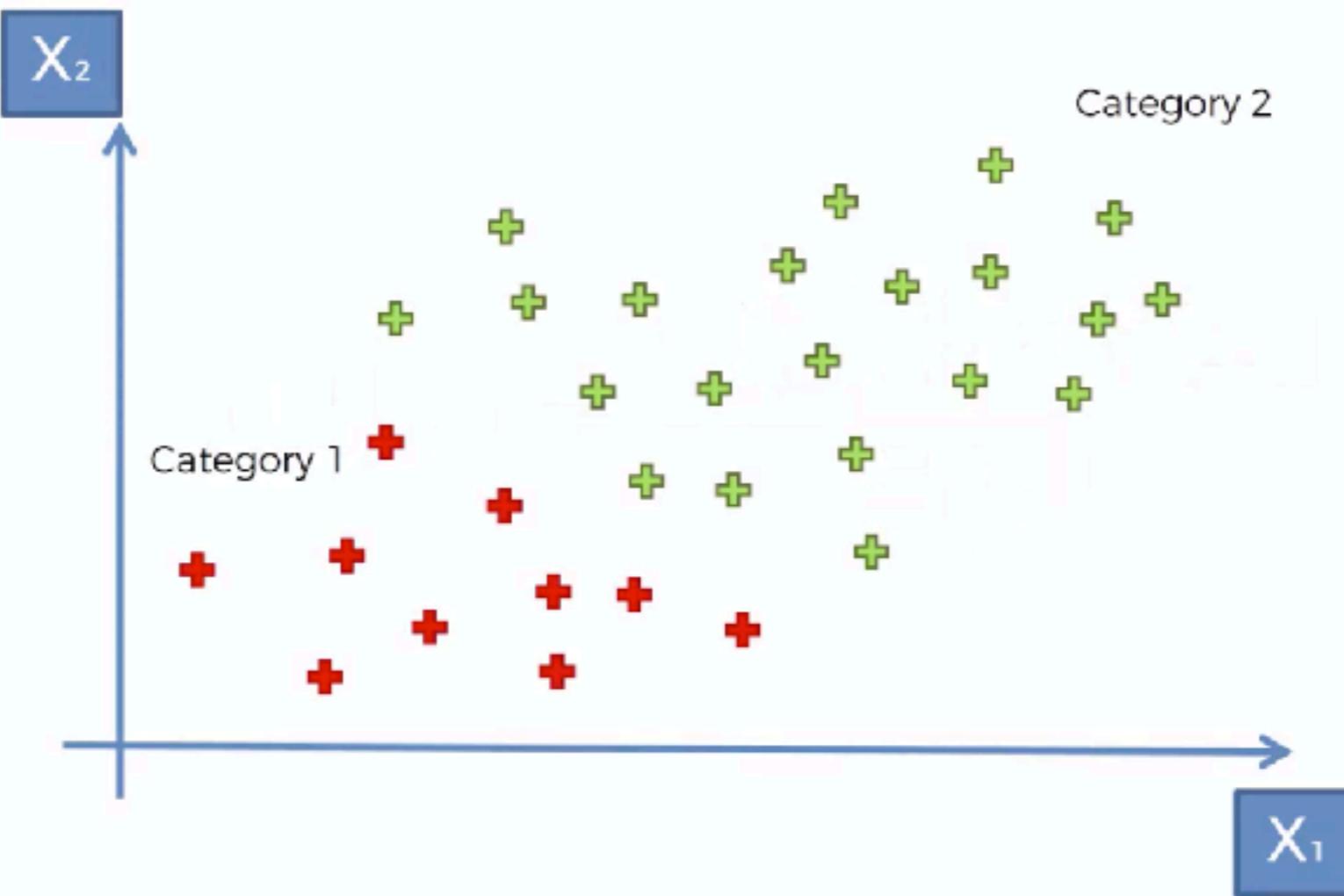
$$P(\text{Defect} \mid \text{Mach2}) = \frac{P(\text{Mach2} \mid \text{Defect}) * P(\text{Defect})}{P(\text{Mach2})} = 1.25\%$$

Let's look at an example:

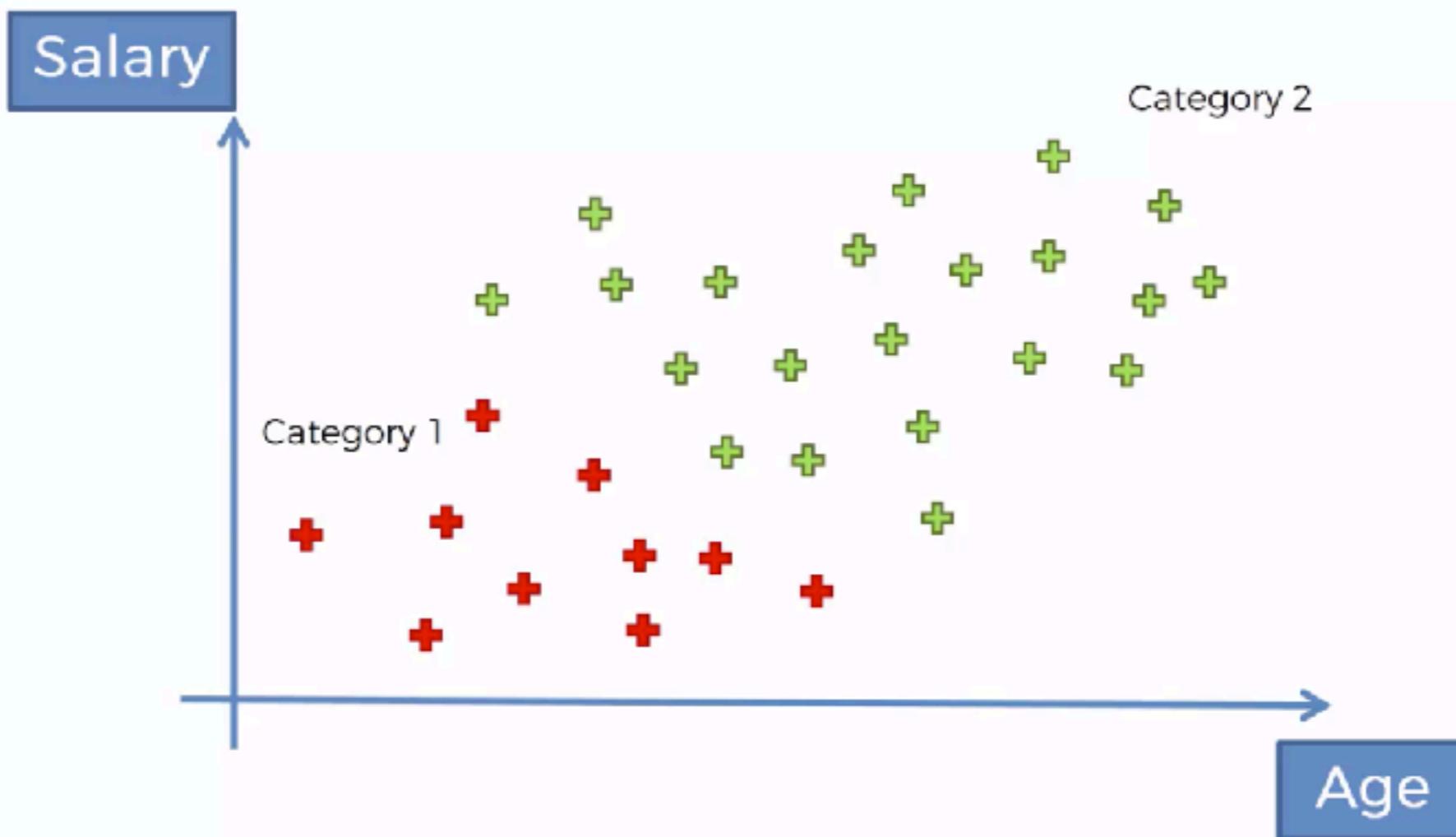
- 1000 wrenches
- 400 came from Mach2
- 1% have a defect = 10
- of them 50% came from Mach2 = 5
- % defective parts from Mach2 = 5/400 = 1.25%

Naïve Bayes Classifier Intuition

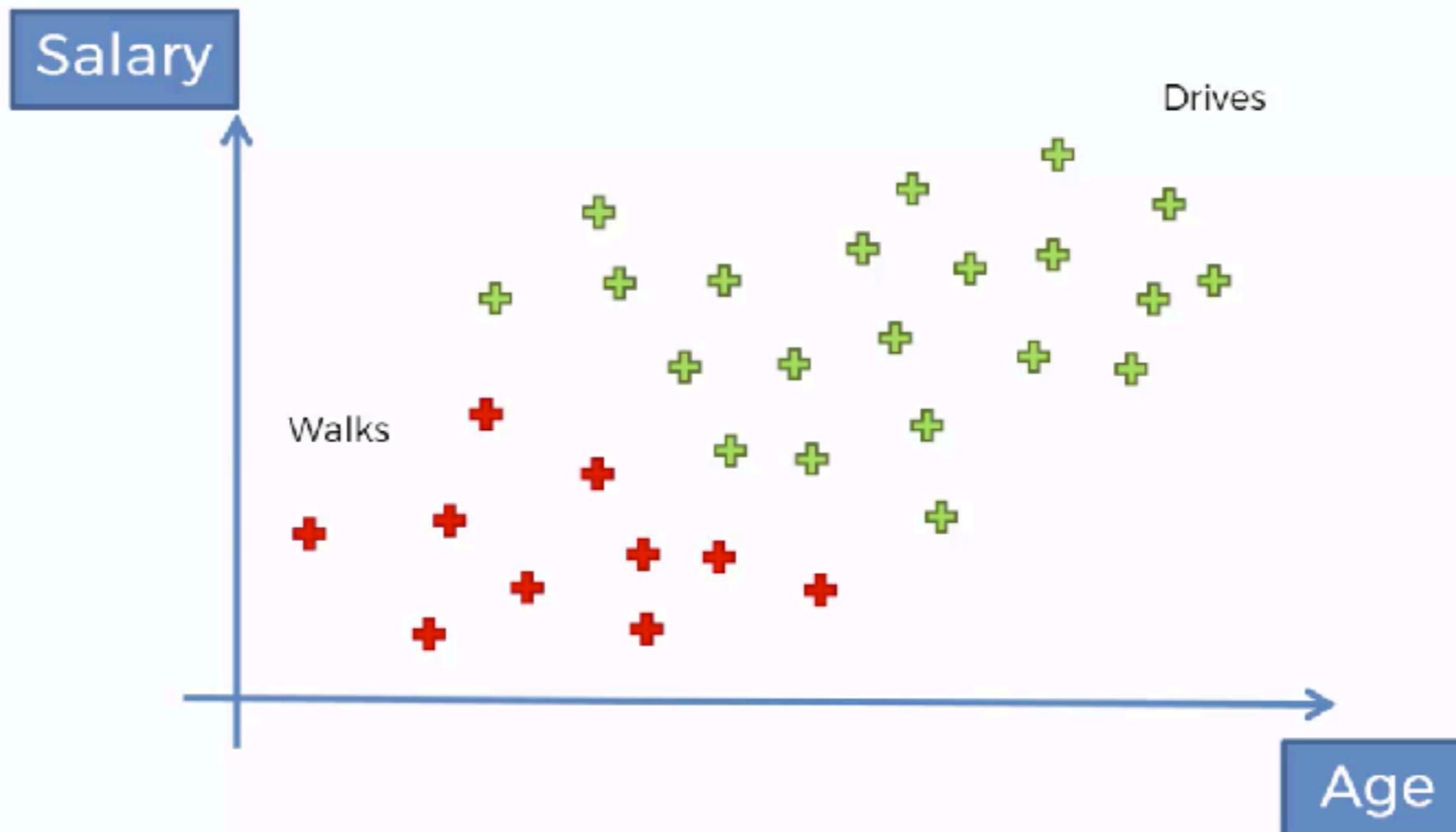
Naïve Bayes



Naïve Bayes



Naïve Bayes



Naïve Bayes

Plan of Attack

Step 1

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

Step 1

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

Step 1

#1

Prior Probability

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

Step 1

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

Likelihood

#1 Prior Probability

#2 Marginal Likelihood

Step 1

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

#4

Posterior Probability

#3

Likelihood

#1

Prior Probability

#2

Marginal Likelihood

Step 2

$$P(Drives|X) = \frac{P(X|Drives) * P(Drives)}{P(X)}$$

Diagram illustrating the components of the Bayes' Theorem formula:

- #4 Posterior Probability
- #3 Likelihood
- #1 Prior Probability
- #2 Marginal Likelihood

Arrows point from the labels to their corresponding terms in the formula:

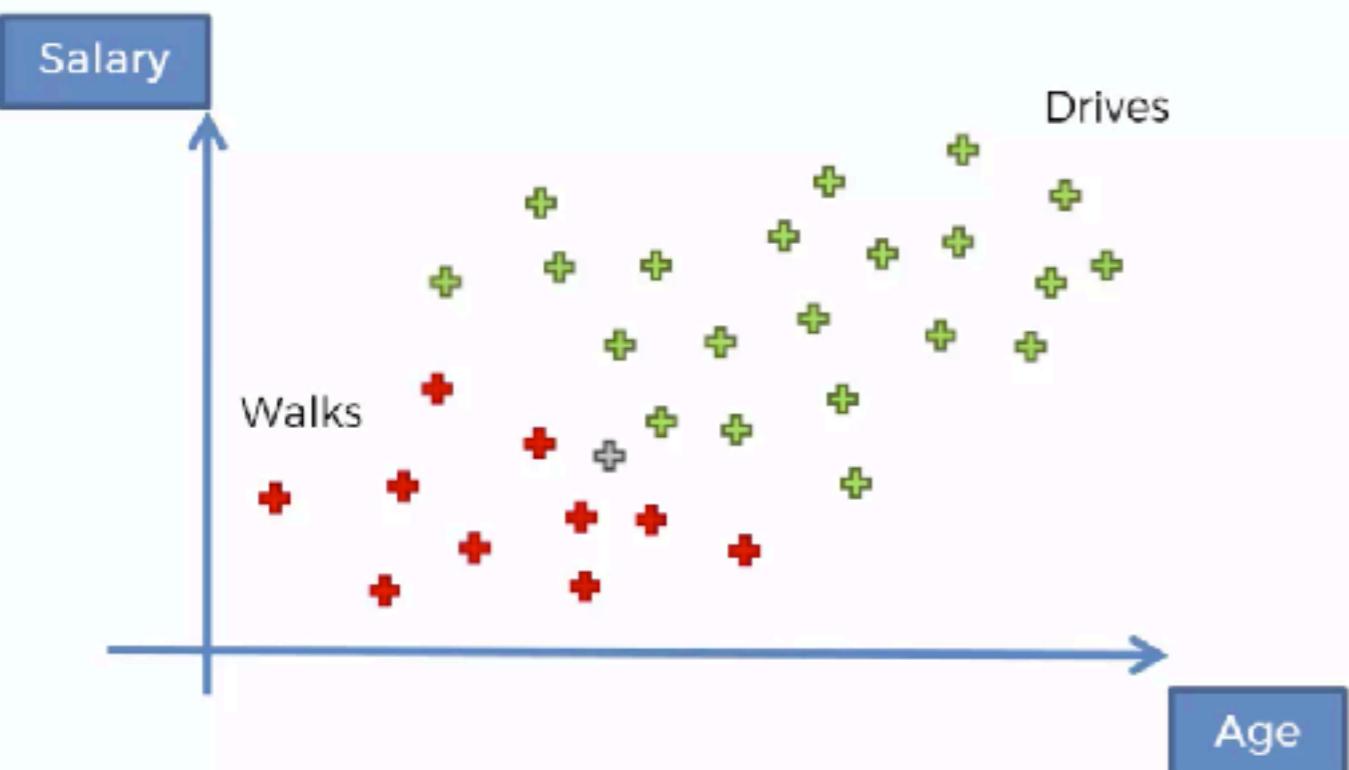
- #4 points to $P(Drives|X)$
- #3 points to $P(X|Drives)$
- #1 points to $P(Drives)$
- #2 points to $P(X)$

Step 3

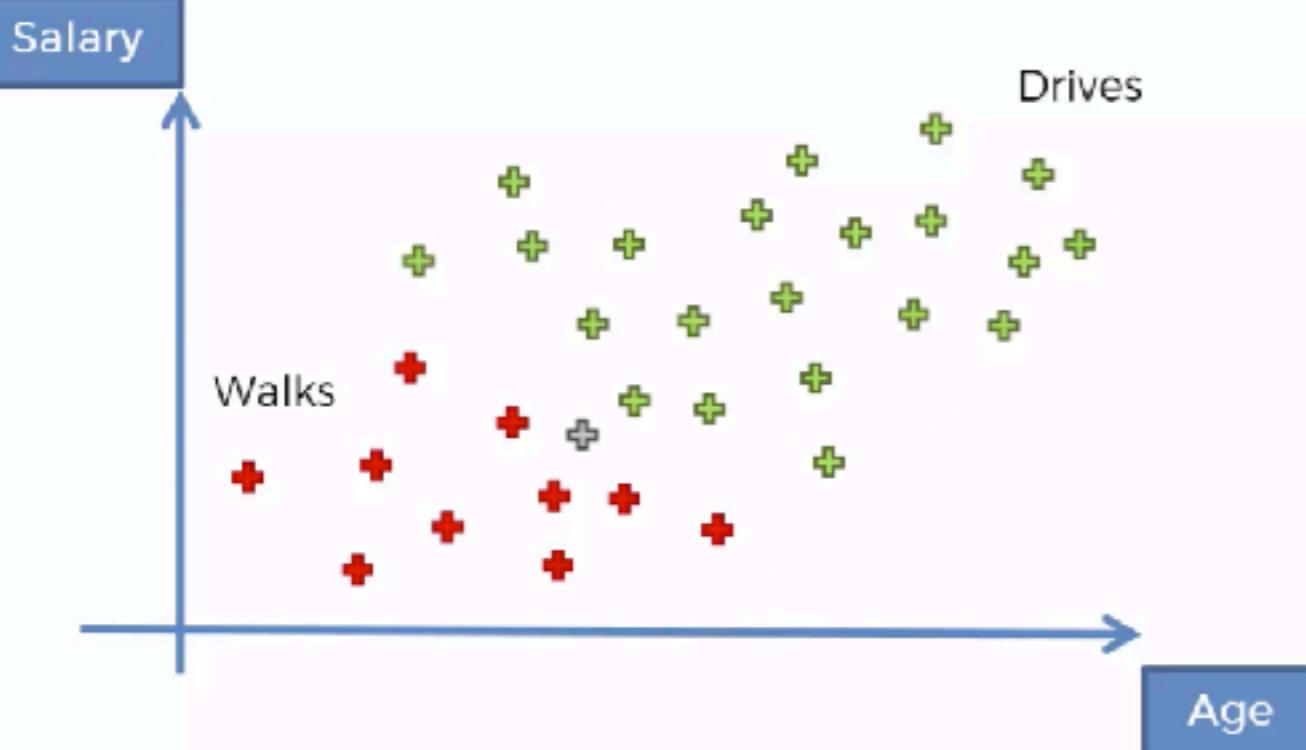
$P(\text{Walks}|X)$ v.s. $P(\text{Drives}|X)$

Naïve Bayes: Step 1

#1. $P(\text{Walks})$



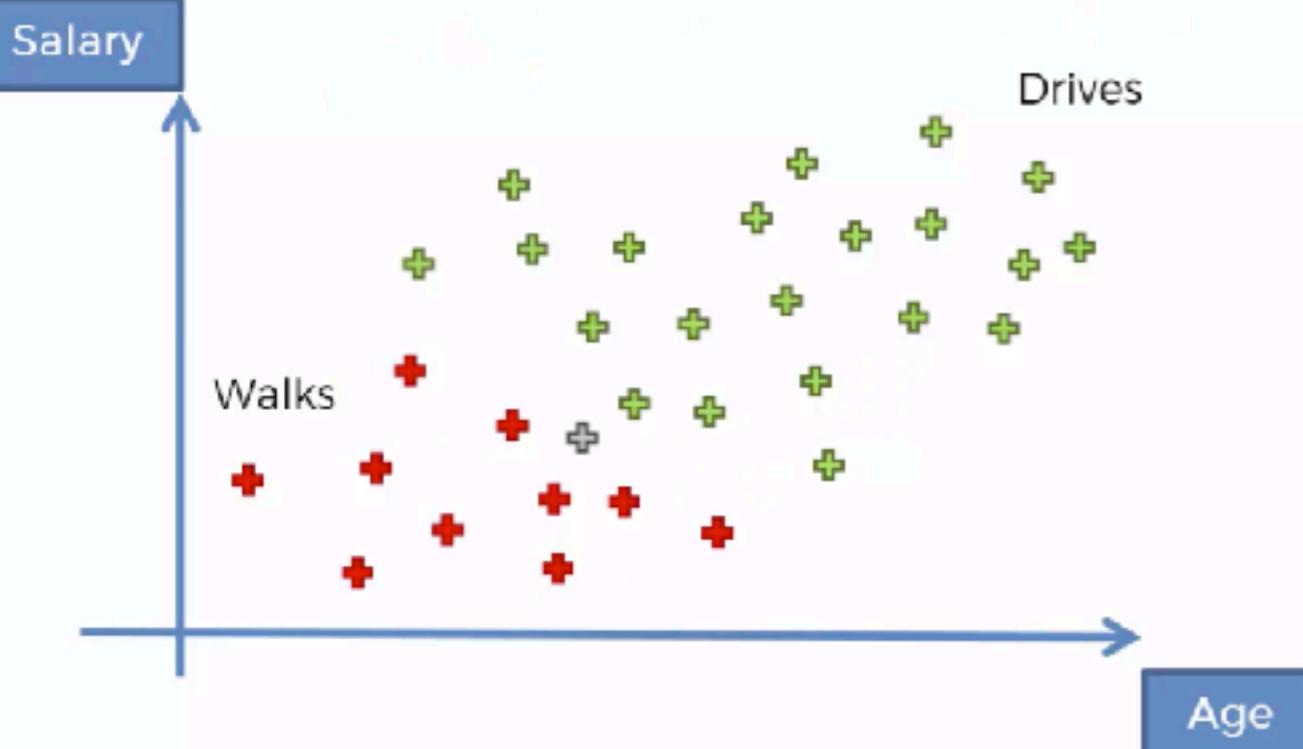
Naïve Bayes: Step 1



#1. $P(\text{Walks})$

$$P(\text{Walks}) = \frac{\text{Number of Walkers}}{\text{Total Observations}}$$

Naïve Bayes: Step 1



#1. $P(\text{Walks})$

$$P(\text{Walks}) = \frac{\text{Number of Walkers}}{\text{Total Observations}}$$

$$P(\text{Walks}) = \frac{10}{30}$$

Naïve Bayes: Step 1

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

#4

Posterior Probability

#3

Likelihood

#1



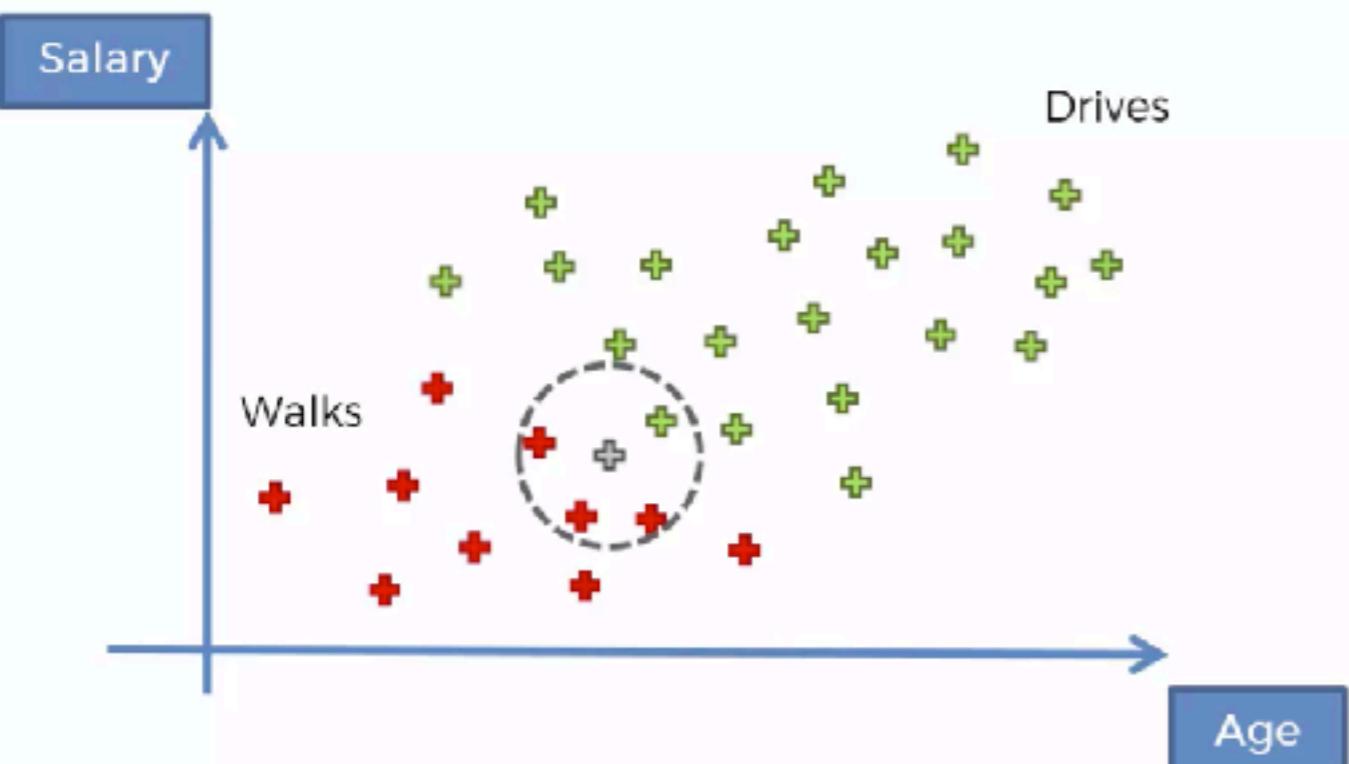
Prior Probability

#2

Marginal Likelihood

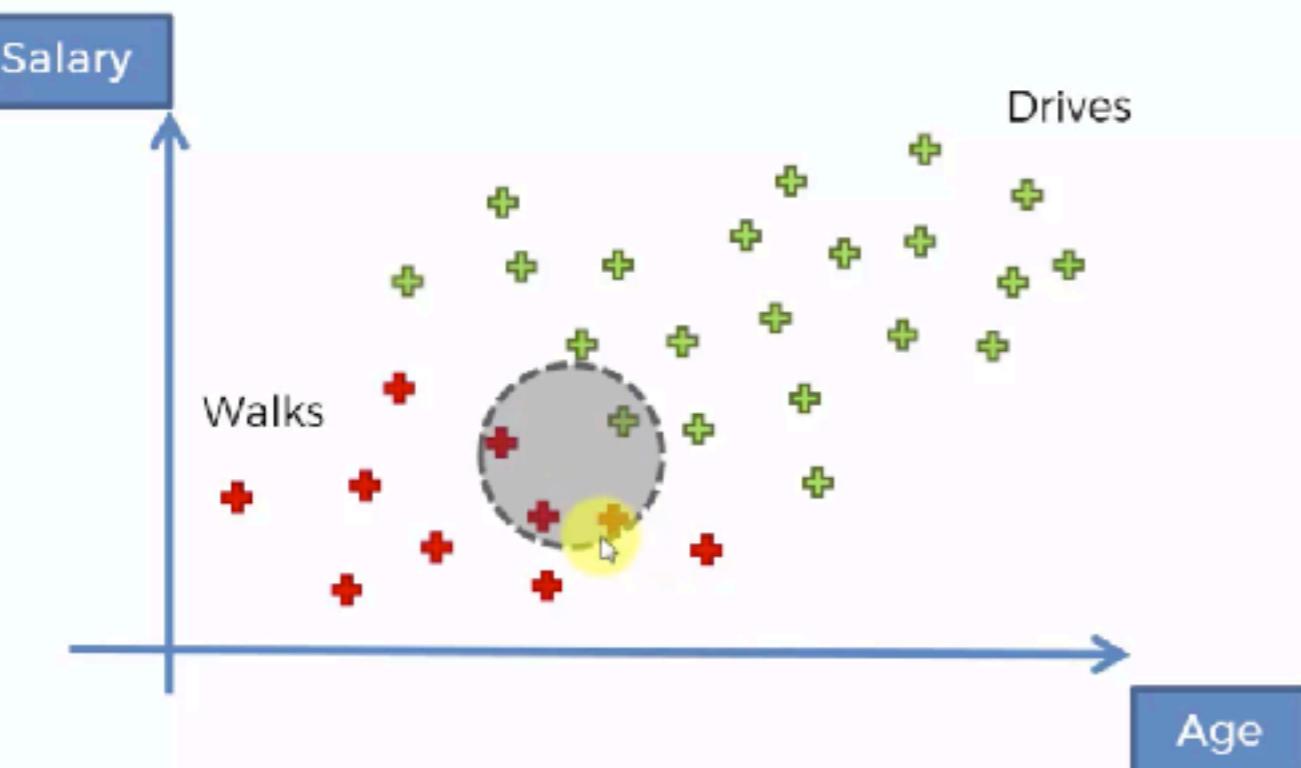
Naïve Bayes: Step 1

#2. $P(X)$

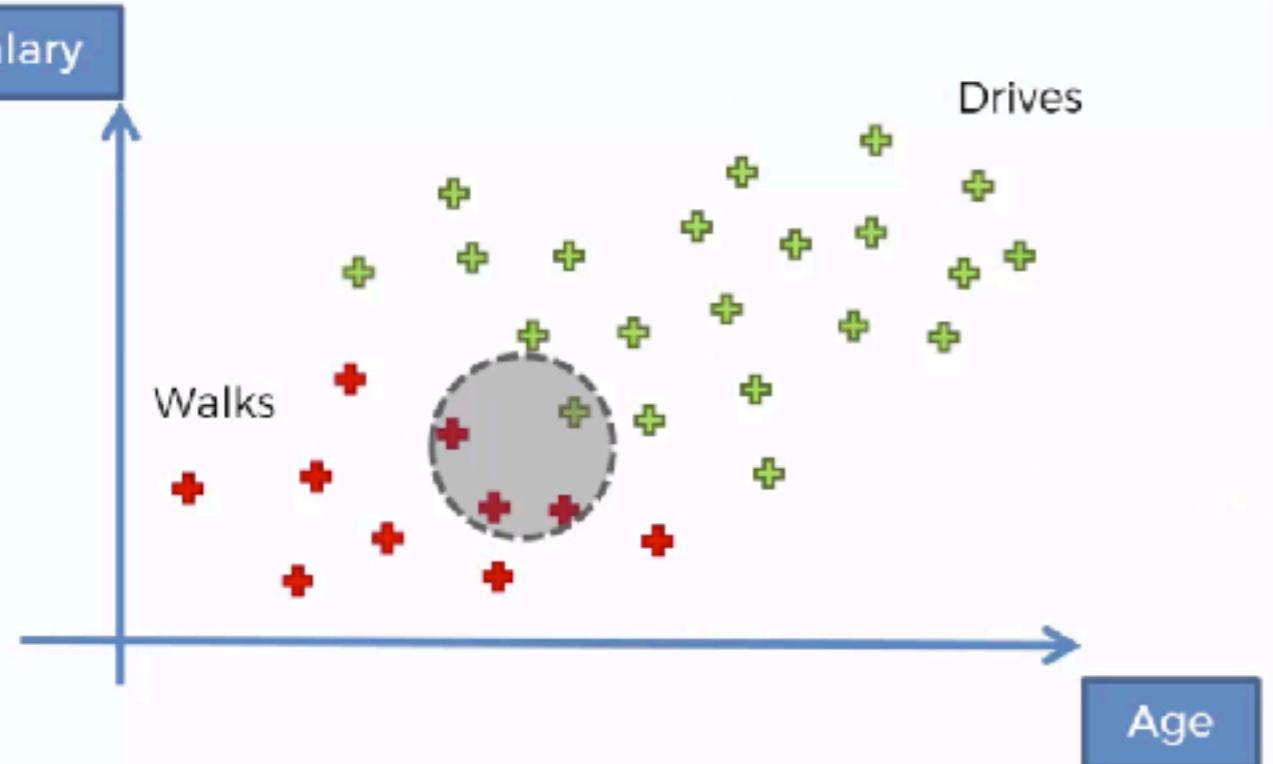


Naïve Bayes: Step 1

#2. $P(X)$



Naïve Bayes: Step 1



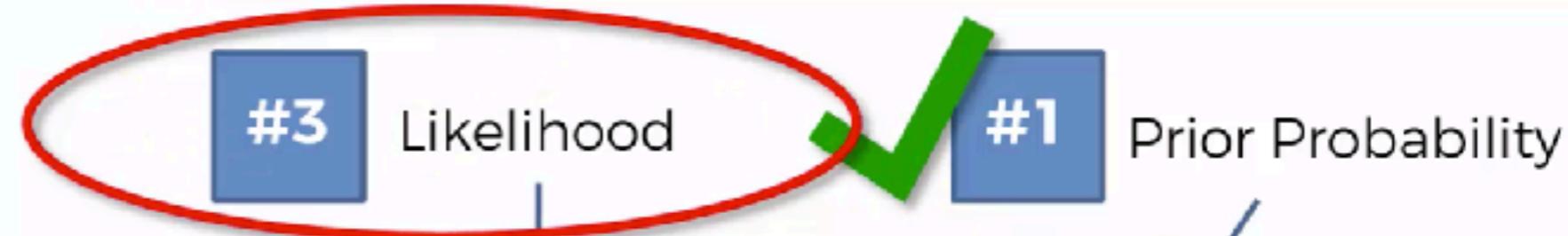
#2. $P(X)$

$$P(X) = \frac{\text{Number of Similar Observations}}{\text{Total Observations}}$$

$$P(X) = \frac{4}{30}$$

Naïve Bayes: Step 1

#4 Posterior Probability

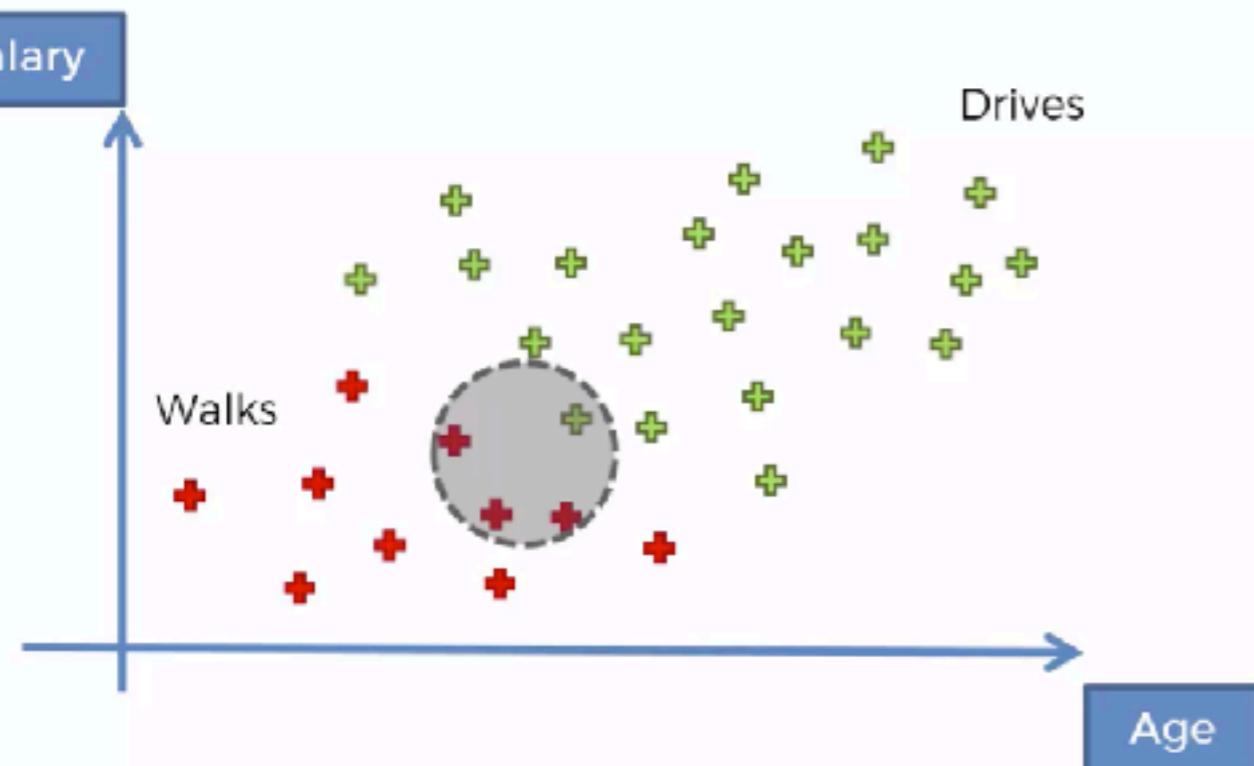


$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

#2 Marginal Likelihood

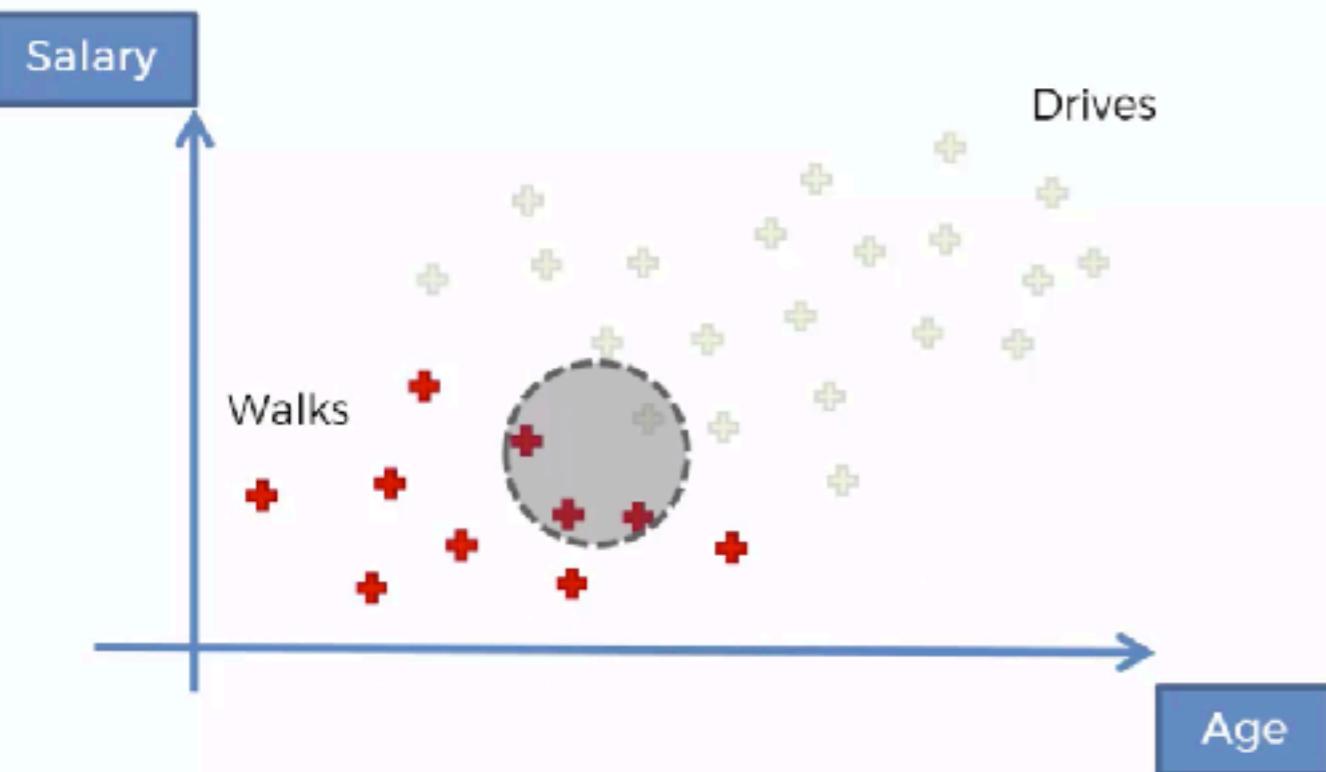
Naïve Bayes: Step 1

#3. $P(X|Walks)$

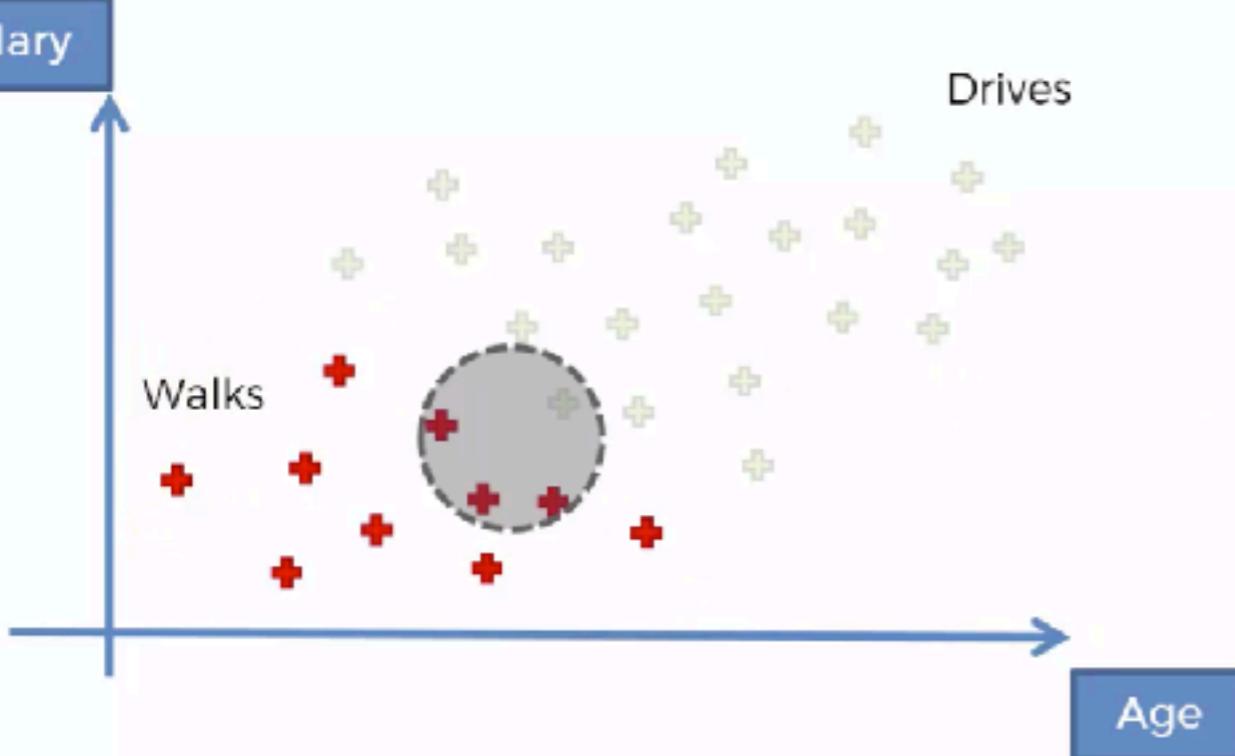


Naïve Bayes: Step 1

#3. $P(X|Walks)$



Naïve Bayes: Step 1



#3. $P(X|Walks)$

*Number of Similar Observations
Among those who Walk*

$$P(X|Walks) = \frac{\text{Number of Similar Observations}}{\text{Total number of Walkers}}$$

$$P(X|Walks) = \frac{3}{10}$$

Naïve Bayes: Step 1

#4

Posterior Probability

 #3

Likelihood

 #1

Prior Probability

$$P(Walks|X) = \frac{\frac{3}{10} * \frac{10}{30}}{\frac{4}{30}} = 0.75$$

#2

Marginal Likelihood

Step 2

#4

Posterior Probability

#3

Likelihood

#1

Prior Probability

$$P(Drives|X) = \frac{P(X|Drives) * P(Drives)}{P(X)}$$

#2

Marginal Likelihood

Naïve Bayes: Step 2

#4

Posterior Probability



#3

Likelihood



#1

Prior Probability

$$P(\text{Drives}|X) = \frac{\frac{1}{20} * \frac{20}{30}}{\frac{4}{30}} = 0.25$$



#2

Marginal Likelihood



Step 3

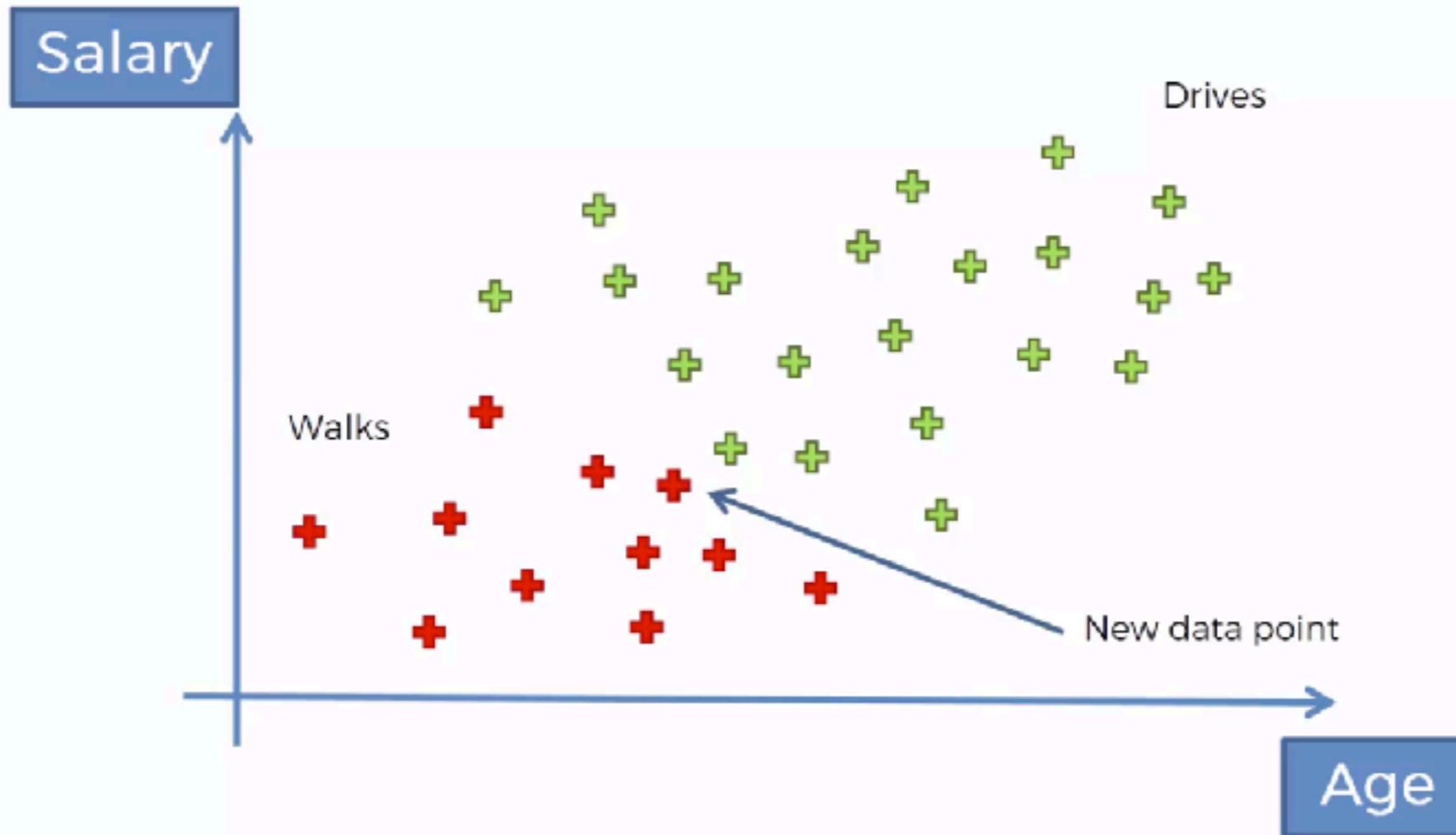
$P(\text{Walks}|X)$ v. s. $P(\text{Drives}|X)$



Step 3

$$P(\text{Walks}|X) > P(\text{Drives}|X)$$

Naïve Bayes



Naïve Bayes

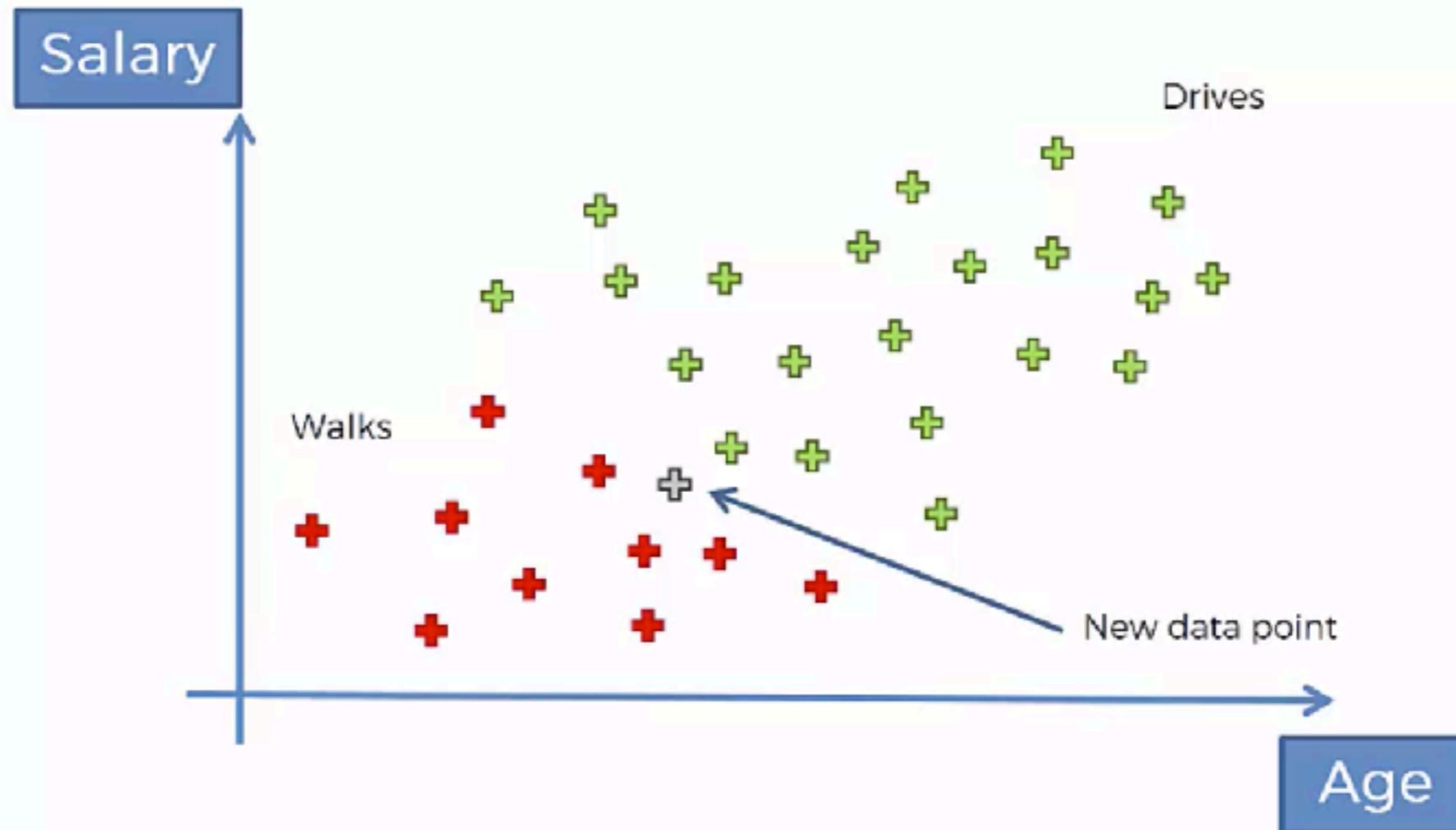
Q: Why “Naïve”?

Naïve Bayes

Q: Why “Naïve”?

A: Independence assumption

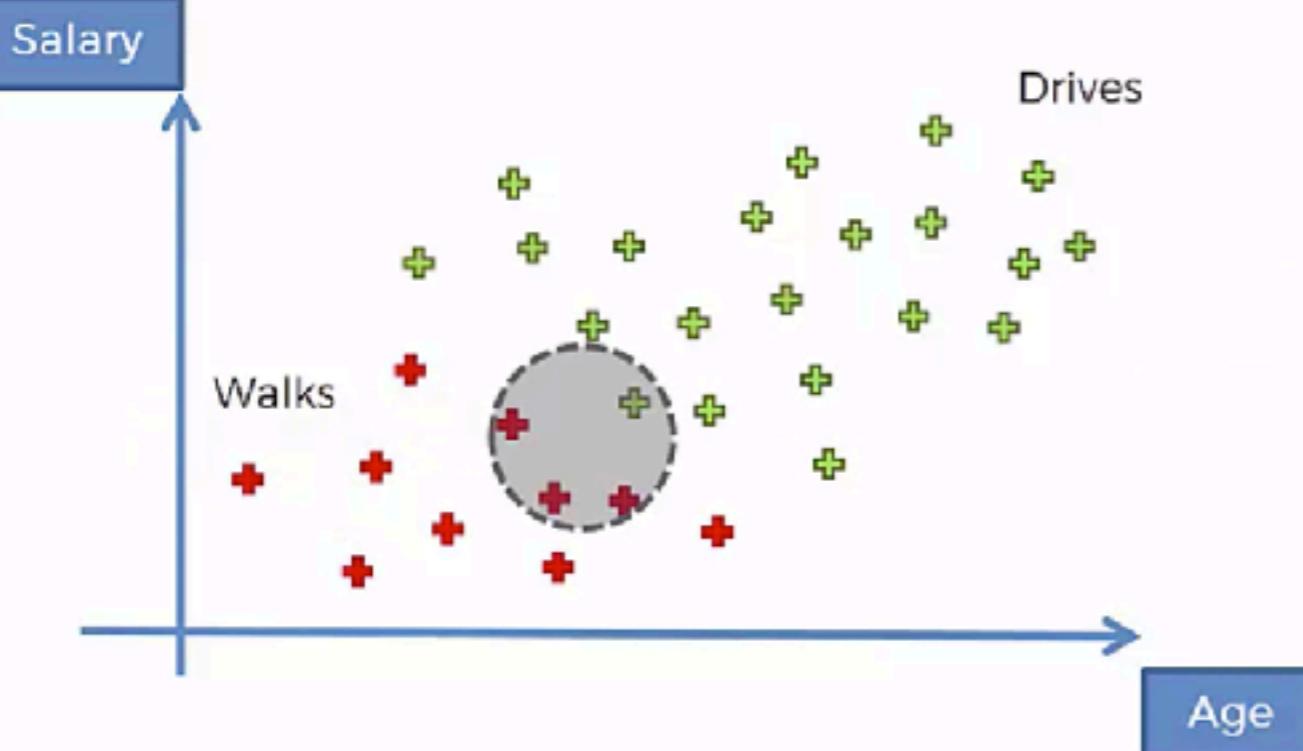
Naïve Bayes



Naïve Bayes

P(X)

Naïve Bayes: Step 2



#2. $P(X)$

$$P(X) = \frac{\text{Number of Similar Observations}}{\text{Total Observations}}$$

$$P(X) = \frac{4}{30}$$

NOTE: Same both times

Step 1

$$P(Walks|X) = \frac{P(X|Walks) * P(Walks)}{P(X)}$$

#4

Posterior Probability

#3

Likelihood

#1

Prior Probability

#2

Marginal Likelihood

Step 2

$$P(Drives|X) = \frac{P(X|Drives) * P(Drives)}{P(X)}$$

#4 Posterior Probability

#3 Likelihood

#1 Prior Probability

#2 Marginal Likelihood

The diagram illustrates the components of Bayes' Theorem. At the top, four numbered boxes are arranged horizontally: #4 Posterior Probability (blue), #3 Likelihood (blue), #1 Prior Probability (blue), and #2 Marginal Likelihood (blue). Arrows point from each of these boxes to their respective terms in the formula below. A yellow circle with a cursor icon is positioned near the left side of the equation.

Step 3

$$\frac{P(X|Walks) * P(Walks)}{P(X)} \text{ v.s. } \frac{P(X|Drives) * P(Drives)}{P(X)}$$

↳

Step 3

$$\frac{P(X|Walks) * P(Walks)}{\cancel{P(X)}} \quad v.s. \quad \frac{P(X|Drives) * P(Drives)}{\cancel{P(X)}}$$