K. Kethem

6) Fourier transform of gaussian

$g(t) = e^{-at^2}$

$F_t[g(t)](k) = \int_{-\alpha}^{\alpha} e^{-at^2} e^{-i2\pi kt} dt$

$= \int_{-\alpha}^{\alpha} e^{-at^2} [\cos(2\pi kt) - i\sin(2\pi kt)] dt$

$\Rightarrow \int_{-\alpha}^{\alpha} e^{-at^2} \cos(2\pi kt) dt$

$-i\int_{-\alpha}^{\alpha} e^{-at^2} \sin 2\pi kt \, dt$

$\Rightarrow \int_{-\alpha}^{\alpha} e^{-at^2} \cos(2\pi kt) dt - 0$

Second part is odd so integrating it gives 0.

$\Rightarrow \int_{-\alpha}^{\alpha} e^{-at^2} \cos(2\pi kt) dt$

Apparently this gives $\sqrt{\frac{\pi}{a}} \cdot e^{-\pi^2 k^2/a}$

according to Abramowitz & Stegun (1972, P. 302 equation 7.4.6)

$F_t[e^{-at^2}]k = \sqrt{\frac{\pi}{a}} e^{-\pi^2 k^2/a}$

this is gaussian.