2019 IEEE 14th International Conference on Industrial and Information Systems (ICIIS), 18-20 Dec., Peradeniya, Sri Lanka

A Queuing Model for Outpatient Department to Reduce Unnecessary Waiting Times

W.M.N.B. Weerakoon
Department of Computing & Information Systems,
Faculty of Applied Sciences,
Sabaragamuwa University of Sri Lanka,
Belihuloya, Sri Lanka.
wmnbweerakoon@std.appsc.sab.ac.lk

S. Vasanthapriyan
Department of Computing & Information Systems,
Faculty of Applied Sciences,
Sabaragamuwa University of Sri Lanka,
Belihuloya, Sri Lanka.
priyan@appsc.sab.ac.lk

U.A.P. Ishanka
Department of Computing & Information Systems,
Faculty of Applied Sciences,
Sabaragamuwa University of Sri Lanka,
Belihuloya, Sri Lanka.
ishankauap@appsc.sab.ac.lk

***Abstract*— In most of the government health care centers, the patients feel like helpless because of long waiting queues. The reason for overcrowding is the failure of effective management of patient queues. Unnecessary queue waits not only boost patient frustration with their disease but also waste precious time of the patient. It may not always be a better solution to solve this problem by increasing the number of employees. Because the increasing number of employees may be costly than the total waiting cost of patients. The current mean waiting time is analyzed by using queuing theory. Two fundamental parameters of queuing theory analysis are arrival rate (λ) and service rate (μ) and they were used to analyze not only current mean waiting time but also to predict mean waiting times by the increasing number of servers. Finally, a new system has been proposed to reduce mean waiting times in outpatient department (OPD). The arrival times and service times of each patient for each service point (registration, consultation and pharmacy) were gathered straight over a week. The tea breaks and lunchtimes were not considered while calculating service rates. The present queue system adapts Tandem queuing theory and the queue discipline is first come first serve discipline. The analytical results show that the mean waiting times of OPD system can be reduced in a considerable amount from the proposed system. Further, this rate is achieved through theoretical analysis and a simulated patient management system is suggested as the administration can be used to decrease waiting times in queues.**

Keywords*— *Queuing theory, Patients management system, Tandem queues

I. Introduction

Heath is wealth. Sri Lankan government provides healthcare free for all citizens. There is an extensive network of healthcare centers located in countrywide.[1] The ineffective patient queue management leads overcrowding of patients in OPDs and this is one of major challenging problems in OPDs. Unnecessary queue waits increase not only patient frustration with their diseases but also waste precious time for the patient.[2] [3] The queuing theory is the mathematics of waiting lines. This research used queuing theory to analyze and find current waiting times of the patients in OPD and predict waiting times by the increase of servers at each service point.

The researchers have been identified a hospital outpatient department for this study, which always busy and overcrowding. The OPD has one registration counter, three doctors' consultation rooms and two pharmacy counters. The OPD consultations start at 8.00 a.m. to noon and 2.00 p.m. to 4.00 p.m. on weekdays and 8.00 a.m. to noon on Saturdays. Patients arrive before starts the consultation and waiting in queues to be served. However, all the doctors are not going to serve the patients parallel. There is always one doctor and one pharmacy counter available to serve patients. All three doctors are available and two pharmacy counters are open at clinic days. One doctor is available and one pharmacy counter is open for OPD patients even in clinic days while others are serving clinic patients. A large number of patients can be seen waiting in a queue to be served at the period of morning to noon. Therefore this study focus on waiting queues at registration counter, consultation and pharmacy counter.

The arrival of patients' and their flow in OPD illustrates in figure 1.

Figure 1. Flow chart of OPD

Patients arrive at OPD and waiting in waiting queue to get register and after registration, patients are waiting in waiting queue to be served by a doctor and after doctor's consultation, patients are waiting in a waiting queue to have drugs from pharmacy and then leave the OPD. This study mainly focuses on normal OPD patients' process and their waiting time analysis.

In the past, many professionals have been selected to effectively analyze patient waiting time and manage patients.[4] The results of these investigations cannot be implemented directly. Because for their evaluation, the alternatives are specific to the hospital as they have chosen to study. The kinds of patients vary from hospital to hospital and the hospital facilities are also vary. Therefore doing proper research is significance to give an appropriate solution to reduce long waiting times in OPD queues.

This research's main goal is to enhance the quality of life by decreasing unnecessary waiting in hospital queues and effectively handling patients.

Typically, patients in the OPD queue have to wait for a long time to register, meet with a doctor, and to have drugs from a pharmacy. Following research questions are going to cover from this study.

- When are particularly and frequently waiting lines happening?
- What are the hospital queuing bottlenecks?

- What are the present hospital queuing procedures?
- What are the latest methods of reducing waiting times?
- How can the impact of long queues be mitigated?

A. Related work

A similar research has performed a methodology which gather data by observations and the distribution patterns of data were identified by using Arena Input Analyzer. Then performed an experimental analysis. Finally, a best schedule rule for the hospital has been suggested. [5]

A predictive model has been proposed after a study to detect the delay in bank teller queues. They have tested four models for this study. One using queuing theory and others using data mining algorithms. The results indicate that Grandlent boost machine algorithm is most efficient than others to predict waiting time overflow on bank teller queues. [6]

An ambulatory treatment system. The system having models like single service queuing model (M/M/1) and multi service queuing model (M/M/C), built up the queuing optimization model. Zhuzhou hospital has been used to perform actual study. Analyze the actual data of the outpatient **treatment, use** χ^2 to conduct the expectation analysis of the arrival rate and the service rate and design the full static inspection. The work improved the use of hospital resources and offered hospital management data and guidance. [7]

II. Method

Doing a systematic literature review is essential to identify the results of previous researches and background, research gaps and limitations too.[8] Then it is necessary to collect data to apply Queuing theory. Finally, finds an optimal solution to reduce unnecessary waiting times in OPD queues and propose a solution based on queuing theory.

A. Queuing Theory

Queuing theory is the mathematics of waiting lines and used to predict and evaluate the system performance. [9] Patients who want to obtain medical services always arrive randomly and require instant services at that moment. If the services are operating at the peak level when patients arrive to OPD, they have to wait in line to be served. According to the characteristics of medical services, we cannot say exact times when patients arrive at OPD and the time which takes to serve a patient. Therefore, the goal of queuing theory analyze is to **find economic equilibrium between patients' wasted** time in OPD queue and service cost. Some useful symbols, notations and equations were given below.

Pn : The probability of exactly n customers in the queue systems.

Ls : The number of the jobs in the queue system in equilibrium state.

Lq : The number of the jobs in queuing in equilibrium state.

Ws : The mean waiting time for each individual job in the queuing system.

Wq : The mean waiting time for each individual job in the queue.

λ : Average arrival rate (e.g., number of arrivals per hour).

μ : Average service rate per server (e.g., 1/average service time).

c : Number of servers.

ρ : The utilization rate for the service facility ($\rho = \lambda / \mu c$).

Arrival rate (λ) and Service rate (μ) are being used to find mean waiting time in the queue (Wq) and mean waiting time in the system (Ws).[10] The probability distribution of interarrival time and service time is assumed by almost all queuing models as in exponential distribution, and the number of patients arriving per unit of time follows the distribution of Poisson. The queue discipline is first come first serve. A single-queue single-server system with a single source of traffic and an infinite storage capability (M/M/1) queuing model and a single-queue multi-server (M/M/C) queuing model calculations done for three station Tandem queue systems. [11]

1) Data collection

The patients' **arrival times** to the OPD, the registration start times, registration end times, consultation start times, consultation end times, drug issuing start times and drug issuing end times were directly observed. Data were gathered for a one week, Monday to Friday. The total number of 289 **patients' data** were observed. Patient arrivals listed in table 1.

Table I. Number of patients' arrival per day

Day	Number of Patients
Monday	75
Tuesday	67
Wednesday	71
Thursday	87
Friday	76
Average	75.2

The researchers extracted from the observed data that the **average number of 75 patients' come into OPD daily. Some** patients are arriving at OPD before 8.00 a.m. and also registration starts before 8.00 a.m. normally, the hospital staffs have half an hour tea break and two hour lunch break.

2) Arrival rate (λ) calculation

One of two different ways of arrival rate calculation is, get the inverse value of average interarrival time. The other one is to take average number of arrivals.[12] The first mentioned method has been used to calculate arrival rate for this study. To take arrival rate per hour, the value was multiplied by 60.

The hospital staffs usually take two hour lunch break from 12.00 noon to 2.00 p.m. and between 12.00 noon to 1.30 p.m. time period and did **not show any patients' arrivals.** Therefore, this time period does not consider as interarrival time for arrival rate calculations. Calculated arrival rates are presented in table 2.

Table II. Arrival rate per hour

	Monday	Tuesday	Wednesday	Thursday	Friday
Average interarrival time (minutes)	4.90	5.24	5.19	4.67	4.58
Arrival rate (λ)	12.22	11.45	11.54	12.86	13.10

3) Service rate (μ) calculation

Get the inverse value of average service time and multiplied by 60, then it gives service rate per hour. There are three service stations. The registration, consultation and drug issuing. The service rates needs to be calculated for each service point. There is one server at each service station. Estimated service rates for each service stations are presented in table 3.

Table III. Service rate per hour

Service rate (μ) per hour	Monday	Tuesday	Wednesday	Thursday	Friday
Registration (μ_1)	60	59.11	59.16	47.02	53.02
Consultation (μ_2)	16.67	15.36	15.78	15.91	16.89
Drug issuing (μ_3)	16.85	15.82	15.71	17.28	15.61

4) Calculate the number of patients in the system (Ls) and Number of patients' in queue (Lq)

The total number of each service stations number of patients, is the number of patients in the entire system.

The patients' are waiting in queue to be served at each service point. The total number of patients' who waits in waiting queue at each service point, is the number of patients' in the queue for the entire system.

The studying system is a three station Tandem queue system and each service point has one server. Therefore, M/M/1 queue model calculations were used.

Following equation (1) used to find the number of patients' in each service station of system. The total number of patients in the system will be used to calculate mean waiting time in the system.

$$Ls = \lambda / (\mu - \lambda) \tag{1}$$

If number of patients in registration is Ls1, number of patients in a consultation is Ls2 and number of patients in pharmacy is Ls3. Then, the number of patients in the system, Ls:

$$Ls = Ls1 + Ls2 + Ls3 \tag{2}$$

is given by the above equation (2). The following table 4 presents the calculated results of number of patients in each service station and number of patients in the entire system.

Table IV. Number of patients in system

	Ls_1	Ls_2	Ls_3	Ls
Monday	0.2559	2.7551	2.6435	5.6546
Tuesday	0.2402	2.8571	2.6185	5.7159
Wednesday	0.2424	2.7272	2.7653	5.7350
Thursday	0.3762	4.2051	2.9038	7.4852
Friday	0.3282	3.4615	5.2142	9.0040

The following equation (3) used to calculate number of patients' in the queue at each service point. The total number of patients in the queue will be used to calculate mean waiting time in the queue.

$$Lq = \lambda 2 / (\mu (\mu - \lambda)) \tag{3}$$

If number of patients in registration queue is Lq1, number of patients in consultation queue is Lq2 and number of patients in pharmacy queue is Lq3. Then, number of patients in entire system queue, Lq:

$$Lq = Lq1 + Lq2 + Lq3 \tag{4}$$

is given by the above equation (4). The following table 5 presents the calculated results of number of patients' in each service station queue and number of patients' in the entire system queue.

Table V. Number of patients' in system queue

	Lq_1	Lq_2	Lq_3	Lq
Monday	0.0521	2.0214	1.9180	3.9915
Tuesday	0.0465	2.1164	1.8949	4.0578
Wednesday	0.0473	1.9955	2.0308	4.0737
Thursday	0.1028	3.3972	2.1600	5.6601
Friday	0.0811	2.6856	4.3752	7.1419

5) Calculate the mean waiting time in system (Ws) and mean waiting time in queue (Wq)

Number of patients in the system divide by arrival rate equals to mean waiting time in system. Number of patients in the queue divided by arrival rate equals to mean waiting time in queue. The equations to calculate mean waiting in system (Ws) (5) and mean waiting time in queue (Wq) (6) are;

$$Ws = Ls / \lambda \tag{5}$$

$$Wq = Lq / \lambda \tag{6}$$

The following table 6 presents the mean waiting time in system and mean waiting time in queue in hours (h).

Table VI. Mean waiting times

	Ws (h)	Wq (h)
Monday	0.4624	0.3264
Tuesday	0.4990	0.3543
Wednesday	0.4967	0.3528
Thursday	0.5821	0.4402
Friday	0.6871	0.5450

The next stage is to calculate mean waiting time in the system and mean waiting time in queue by raising servers to discover the most appropriate amount of servers for each service station to minimize waiting times.

M/M/C queue model calculations needed when a service station has two or more servers. The equations for number of **patients' in the system (7), number of patients in the queue (8),** mean waiting in system (9) and mean waiting time in queue (10) were given below. The total number of patients in the system and total number of patients in the queue will be used to calculate mean waiting times.

$$Ls = Lq + \lambda / \mu \quad (7)$$

$$Lq = [(\lambda/\mu)^{c} \rho / C! (1-\rho)^{2}] P_0 \quad (8)$$

$$Ws = Wq + 1/\mu \quad (9)$$

$$Wq = Lq/\lambda \quad (10)$$

The calculated results were presented in following table 7 and table 8.

Table VII. Mean waiting time in system and queue according to the number of doctors

Number of doctors (servers)	Server utilization %	Mean waiting time in system	Mean waiting time in queues	Day
1	37.37	0.225	0.165	Monday
1	74.074	0.249	0.185	Tuesday
1	73.171	0.236	0.173	Wednesday
1	80.788	0.327	0.264	Thursday
1	77.586	0.264	0.205	Friday
2	36.367	0.069	0.009	Monday
2	37.035	0.075	0.01	Tuesday
2	36.583	0.073	0.01	Wednesday
2	40.395	0.075	0.012	Thursday
2	38.792	0.07	0.01	Friday
3	24.451	0.061	0.001	Monday
3	24.69	0.066	0.001	Tuesday
3	24.38	0.065	0.001	Wednesday
3	26.93	0.064	0.002	Thursday
3	25.861	0.06	0.001	Friday
4	18.338	0.06	0	Monday
4	18.518	0.065	0	Tuesday
4	18.291	0.064	0	Wednesday
4	20.198	0.063	0	Thursday
4	19.396	0.059	0	Friday

Table VIII. Mean waiting time in system and queue according to the number of dispensers

Number of dispensers (servers)	Server utilization %	Mean waiting time in system	Mean waiting time in queues	Day
1	73.367	0.225	0.165	Monday
1	72.362	0.229	0.165	Tuesday
1	73.44	0.24	0.176	Wednesday
1	69.758	0.191	0.133	Thursday
1	83.908	0.398	0.334	Friday
2	36.683	0.069	0.009	Monday
2	36.181	0.073	0.01	Tuesday
2	36.72	0.074	0.01	Wednesday
2	34.879	0.06	0.008	Thursday
2	41.954	0.078	0.014	Friday
3	24.456	0.061	0.001	Monday
3	24.121	0.064	0.001	Tuesday
3	24.48	0.065	0.001	Wednesday
3	23.253	0.059	0.001	Thursday
3	27.969	0.066	0.002	Friday
4	18.342	0.06	0	Monday
4	18.09	0.063	0	Tuesday
4	18.36	0.064	0	Wednesday
4	17.44	0.058	0	Thursday
4	20.977	0.064	0	Friday

The next step is to compare the outcomes of the mean waiting times calculated according to the number of servers at each service station. Compare all the possibilities and then propose an ideal solution.

III. Results

According to the results presented in table 3, the researchers identified that OPD system bottleneck is in consultation. Because, the service rate is very low in consultation compare to the registration service rate. Therefore, one registration counter is enough to serve the **patients' in registration** service station.

When increasing the servers in consultation service station, mean waiting time is going to zero while the number of servers are four. The following figure 2 presents the analyzed results for mean waiting time in system and mean waiting time in queue when increasing number of doctors.

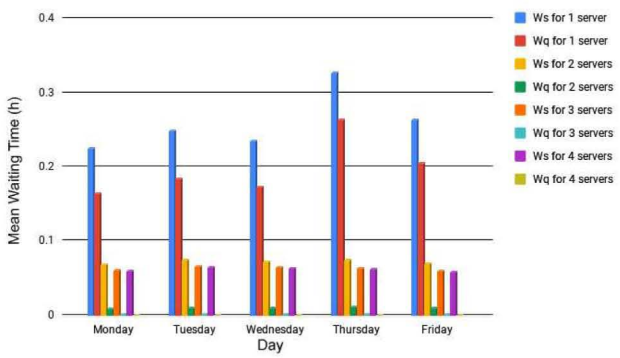

Figure 2. Mean waiting time in system and mean waiting time in queue compared to number of doctors in each day.

By assigning four doctors, patients can be served without leaving any patients waiting in queue. But from further studying the graph, it can recognize that the mean waiting times in significant amount do not decrease after raising physicians more than two. So that the researchers recommend two doctors for the system. **However, raising doctors' more** than two decreases the mean waiting time in queues, but raising doctors more than two is also increasing the total cost to doctors. In general understanding, considering the cost of decreased waiting in queue may be very small compared to doctors ' costs. As a reasonable number of doctors, the researcher recommends two and one registration counter.

Following figure 3 clearly shows that the mean waiting time in pharmacy queue is reduced to zero when there are four dispensers. Here it can also be recognized that the mean waiting times do not decrease from a significant amount after two dispensers. Therefore, the viable quantity of dispensers is two for this system.

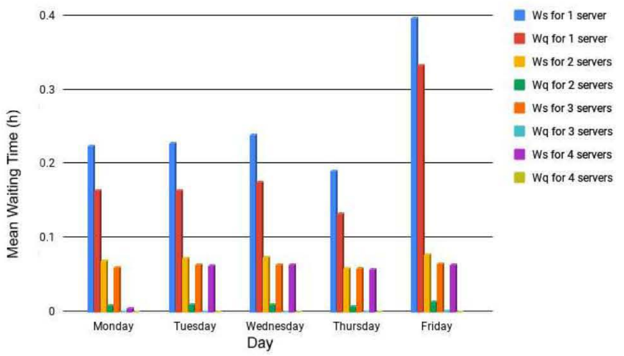

Figure 3. Mean waiting time in system and queue compared to the number of dispensers in each day.

The all above calculations were done considering that the other service stations have only one sever when increasing the servers of one service station. The researchers identified that the suitable registration counters are one and two doctors. When studying server utilization for two doctors and two dispensers, it demonstrates that the two values are tightly the same (table 7 and table 8). If the two service points work at the same speed, no long waiting queue will propagate. However, the following figure 4 shows the mean waiting time in the system and mean waiting time in queue by increasing the number of dispensers while one registration counter and two doctors are on duty.

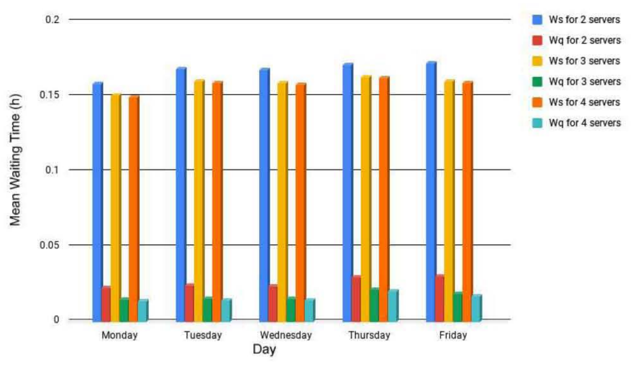

Figure 4. Mean waiting times compared to increase the number of dispensers while two doctors and one registration counter works in each day.

There is no significant reduction in waiting time from the growing number of two to three and three to four dispensers. The reduced mean waiting time is a tiny number of seconds. Considering all these outcomes, one registration counter, two doctors and two dispensers can be suggested.

The findings show that the mean waiting time from the suggested system can be reduced to 65.69 % - 74.90 %. The mean waiting time of the suggested system queue can be reduced to 93.08 % - 94.43 %. (table 9)

Table IX. Waiting times reduce rates

Day	Percentage of reduced waiting time in system %	Percentage of reduced waiting time in system queue %
Monday	65.69	93.08
Tuesday	66.21	93.25
Wednesday	66.23	93.28
Thursday	70.55	93.28
Friday	74.90	94.43

These findings are obtained from the evaluation of the queues and a new system has been suggested according to these outcomes.

IV. Discussion and Conclusion

The ineffective patient queue management leads overcrowding of patients in OPDs. Queuing theory is the mathematics of waiting lines. For this research, the patients arrival times and service times at each station for each patient were directly observed and analyzed using queuing theory. Current waiting times in the system and queue were calculated. Bottleneck of the system identified according to the results. Then, queuing analysis has been performed by increasing number of servers at each service point. Then the optimal number of servers were identified according to compared results of waiting times which resulted by queuing analysis. Finally, a new system has been suggested for the hospital administration to reduce waiting times of the patients in OPD.

For this research, the researcher directly noted information such as patient arrival times and service times. Previous researches have taken this information from some kind of EMR (Electronic medical record) scheme, doctors ' roster or patient registration document.[13] Also, three station Tandem queue analysis has been performed for this research while other researches perform for two stations.

A. Limitations

Theoretically, the suggested scheme demonstrates that it decreases mean waiting times to a significant level. In order to demonstrate it in practice, it requires to be implemented in OPD. Implementing the suggested system in OPD is a time-consuming method for the hospitals. Therefore, the study **can't** compare the theoretical outcomes with real outcomes.

From comparing the cost of service and cost of waiting times for the current system and proposed system, can identify the economic impact from the proposed systems. But, it is very hard to introduce cost variables to calculate service cost and waiting cost in OPD.

B. *Future Work*

Introducing user friendly patient flow simulator for health care management and integrating a patient pre-scheduling and different queuing disciplines with multiple server concepts are future directions of the current study.

ACKNOWLEDGMENT

I express my great appreciation to the Department of Computing & Information Systems, Faculty of Applied Sciences, Sabaragamuwa University of Sri Lanka for providing opportunities, guidance, advice, enormous encouragement, endless experimental recommendations, spending a lot of precious time and endless assist in the achievement of this study.

REFERENCES

[1] R. S. Jayasekara and T. Schultz, "Health status, trends, and issues in Sri Lanka," *Nurs. Heal. Sci.*, vol. 9, no. 3, pp. 228–233, 2007.

[2] C. Siriwardhana and K. Wickramage, "Conflict, forced displacement and health in Sri Lanka: A review of the research landscape," *Confl. Health*, vol. 8, no. 1, pp. 1–9, 2014.

[3] M. Bahadori, E. Teymourzadeh, R. Ravangard, and M. Raadabadi, "Factors affecting the overcrowding in outpatient healthcare," *J. Educ. Health Promot.*, vol. 6, no. 1, p. 21, 2017.

[4] P. A. D. Dilrukshi, H. D. I. M. Nirmanamali, G. H. J. Lanel, and M. A. S. C. Samarakoon, "A Strategy to Reduce the Waiting Time at the Outpatient Department of the National Hospital in Sri Lanka," *Int. J. Sci. Res. Publ.*, vol. 6, no. 2, pp. 281–2250, 2016.

[5] R. Geometry and G. Analysis, "A Simulation Approach for Reduced Outpatient Waiting Time," pp. 4–9.

[6] R. C. and G. R. Roberto Mour˜ao, Ricardo Carvalho, "Predicting Waiting Time Overflow on Bank Teller Queue.pdf." 2017.

[7] X. Shan, L. Jing, L. Zhifeng, Q. Dongjun, and T. Ying, "The Study and Application of Intelligent Queuing in Outpatient Department," pp. 0–4, 2013.

[8] C. Kitchenham, "Guidelines for performing Systematic Literature Reviews in Software Engineering," 2007.

[9] C. M. H. Donald gross, John F. Shortle, James M.Thopson, "fundamental of queuing theory," *A JOHN WILEY SONS, INC.*, vol. 4th editio, 2008.

[10] G. Agrawal, "A queuing model for health centre," *ICEIE 2010 - 2010 Int. Conf. Electron. Inf. Eng. Proc.*, vol. 1, no. Iceie, pp. 542–547, 2010.

[11] P. R. Savariappan, P. Chandrasekhar, V. Mani, and S. Thobias, "Statistical Analysis for a Three Service Point Tandem Queue with Blocking," *Sri Lankan J. Appl. Stat.*, vol. 13, no. 0, p. 1, 2013.

[12] K. W. Cho, S. M. Kim, Y. M. Chae, and Y. U. Song, "Application of queueing theory to the analysis of changes in outpatients' waiting times in hospitals introducing EMR," *Healthc. Inform. Res.*, vol. 23, no. 1, pp. 35–42, 2017.

[13] C. Report, D. Hospital, and S. Lanka, "" eHospital - Dompe " project – the story of the transformation of a district hospital in Sri Lanka," pp. 40–43, 2013.