NATIONAL INSTITUTE OF TECHNOLOGY, HAMIRPUR

FOUNDATION ENGINEERING-II (CED-313)
ASSIGNMENT-II

SUBMITTED BY:
SHUBHAM AGARWAL
191074
Semester-V, 3rd Year
CIVIL ENGINEERING, NITH

SUBMISSION DATE:
18/08/2021

SUBMITTED TO:
PROF. R.K. SHARMA
CIVIL ENGINEERING DEPT.
NIT HAMIRPUR (H.P)

CHAPTER NAME:
EARTH PRESSURE

Questions and Answers:

Q 1. (a) Derive an expression for the active earth pressure behind a retaining wall due to a
cohesionless backfill using Rankine's theory for the following conditions:
(i) Dry or moist backfill without surcharge
Rankine assumed that the soil element is subjected to only two types of stresses:

i. Vertical stress (σ_z) due to the weight of the soil
ii. Lateral earth pressure (p_a).

Assuming the back of the wall as smooth and vertical, Rankine considered that the active earth pressure (p_a) acts horizontally for a backfill with a horizontal surface [Fig. 15.7(a)]. In the active case, the vertical stress is more than the horizontal stress. Since both the stresses are considered as principal stresses – Major principal stress, $\sigma_1 = \sigma_z = \gamma h$ and minor principal stress, $\sigma_3 = p_a$
Figure 15.6 shows the Mohr's circle of stresses and the failure envelope for the active case. When the soil element reaches the state of plastic equilibrium with sufficient movement of the wall away from the backfill, the Mohr's circle of stresses touches the Coulomb's failure envelope, as shown in Fig. 15.6.

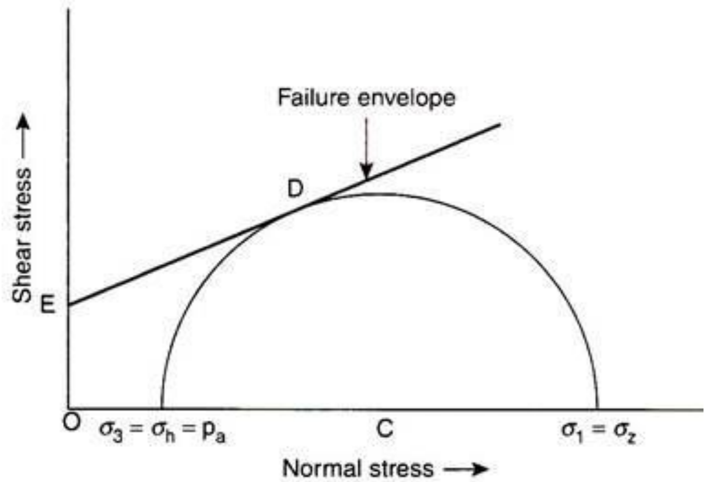


Figure 15.6 Mohr's circle in active case showing principal stresses.

It is known that the principal stresses are related to the shear parameters of the backfill material by the Bell's equation as follows –

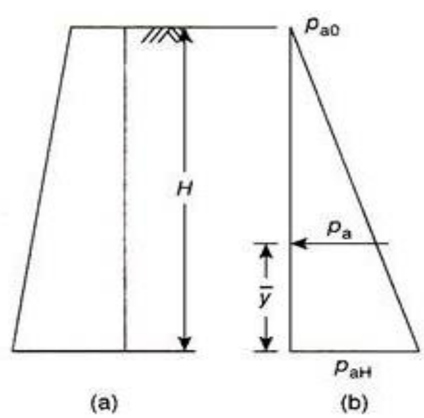
$\sigma_1 = \sigma_3 \tan 2\alpha + 2c \tan \alpha$ …(15.7)
Considering a dry cohesionless backfill, we have c = 0, hence –
$\sigma_1 = \sigma_z = \gamma h$ (in active case) and $\sigma_3 = p_a$

$$\gamma h = p_a \tan^2 \alpha + 0 \quad \Rightarrow \quad p_a = \frac{1}{\tan^2 \alpha} \gamma h \quad \Rightarrow \quad p_a = K_a \gamma h$$

Substituting these values in Eq. (15.7), we have –

$$K_a = \frac{1}{\tan^2 \alpha} = \cot^2 \alpha = \frac{(1 - \sin\phi)}{(1 + \sin\phi)}$$

where K_a is the Rankine's coefficient of active earth pressure and is given by –



Active earth pressure for a dry cohesionless backfill: (a) Retaining wall and (b) active earth pressure diagram.

Equation (15.8) indicates that the active earth pressure is zero at the top surface of the backfill (h = 0) and increases linearly with depth below the surface. The distribution of active earth pressure is shown in Fig. 15.7(b).

The total or resultant active earth pressure exerted on the wall is obtained by computing the area of the pressure diagram.
Total active earth pressure = Area of the pressure diagram
that is,

$$P_a = A = \frac{1}{2} \times p_{aH} \times H = \frac{1}{2} \times K_a \gamma H \times H \quad \Rightarrow \quad P_a = \frac{K_a \gamma H^2}{2}$$

(ii) Submerged backfill

Fully Submerged Cohesionless Backfill:

Figure 15.10(a) shows a retaining wall with a fully submerged backfill, with the groundwater table at the surface of the backfill. The principle of determination of active earth pressure is to multiply the effective vertical stress with the lateral pressure coefficient (Ka) and then add the hydrostatic pressure due to water table, if any. This is because the hydrostatic pressure is equal in all directions as per Pascal's law, and hence, the lateral pressure coefficient (Ka) should not be applied to the hydrostatic pressure.

The active earth pressure at any depth h below the surface of the backfill, as per Rankine's theory, is given by
$p_a = K_a \gamma' h + \gamma_w h$ ...(15.16)
where Ka, the Rankine's coefficient of active earth pressure, is –
$K = (1 - \sin \phi)/(1 + \sin \phi)$
Here γ' is the submerged density of backfill material and γw the density of water is 9.81 $kN/m^3 = 1\ t/m^3 = 1\ g/cc$. The active earth pressure at the base of the wall is –
$P_aH = K_a \gamma' H + \gamma_w H$
Figure 15.10(b) shows the active earth pressure distribution program. Total or resultant active earth pressure exerted on the wall is obtained by computing the area of the pressure diagram. That is –

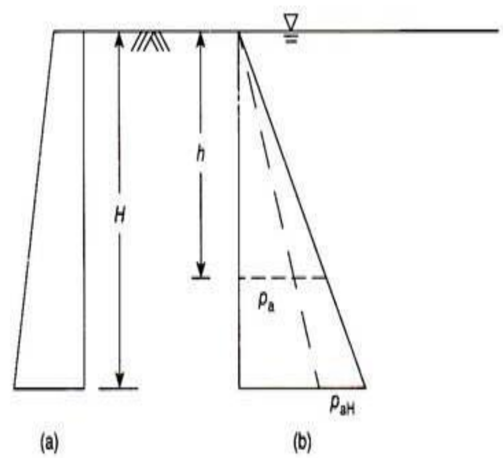


Figure 15.10 Rankine's active earth pressure for a fully submerged cohesionless backfill: (a) Retaining wall and (b) active earth pressure diagram.

Total active earth pressure = Area of the pressure diagram

or $$P_a = A = \frac{1}{2} \times p_{aH} \times H = \frac{1}{2} \times (K_a \gamma' H + \gamma_w H) \times H \quad \Rightarrow \quad P_a = \frac{K_a \gamma' H^2}{2} + \frac{\gamma_w H^2}{2}$$

Total active earth pressure acts horizontally through the centroid of the pressure diagram.
The vertical distance of total active earth pressure above the base of the wall = y
For a triangular pressure distribution, we know that y=H/3 above the base of the wall.

Partially Submerged Cohesionless Backfill:

Figure 15.12(a) shows a retaining wall of height H, with a partially submerged backfill, with the groundwater table at a depth h_1 below the surface of the backfill. The soil above the water table may be either partially or fully saturated. The bulk density of the soil is to be used for computation of vertical stress for soil above the water table.

The active earth pressure at depth h_1 below the surface of the backfill is given by –

$p_{a1} = K_a \gamma h_1$ ...(15.18)

where

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

where γ is the bulk density of the backfill material above the water table, γ' the submerged density of the backfill material, and γ_w the density of water is 9.81 kN/m^3 = 1 t/m^3= 1 g/cc.

The active earth pressure at the base of the wall is given by –

$P_{a2} = K_a\gamma h_1+K_a\gamma' h_2+ \gamma_w h_2$ ...(15.19)

Figure 15.12(b) shows the active earth pressure diagram. Due to the use of submerged density, the slope of the pressure diagram ($K_a\gamma'$) decreases below the water table (dotted line) as compared with that ($K_a\gamma$) above the water table. As the water pressure is added, the slope of the active pressure diagram ($K_a\gamma'$ + γ_w) is more than that above the water table (solid line). The total or resultant active earth pressure exerted on the wall is obtained by computing the area of the pressure diagram. That is –

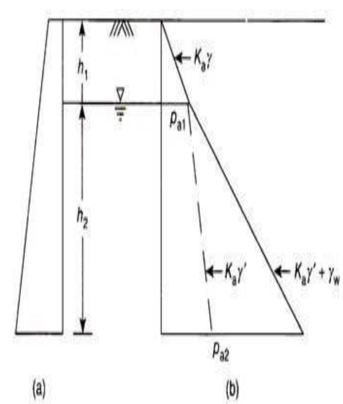


Figure 15.12 (a) Retaining wall with partially submerged backfill and (b) active earth pressure diagram.

Total active earth pressure = Area of the pressure diagram

or $\quad P_a = A = A_1 + A_2 + A_3 \quad \Rightarrow \quad P_a = \left[\frac{1}{2}\times p_{a1}\times h_1\right]+[p_{a1}\times h_2]+\left[\frac{1}{2}\times(p_{a2}-p_{a1})\times h_2\right]$

Total active earth pressure acts horizontally through the centroid of the pressure diagram. Vertical distance of total active earth pressure above the base of the wall is $\bar{y}$. The distance of the centroid can be computed from the principles of mechanics using –

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{\Sigma A_i y_i}{P_a}$$

where $\bar{y}$ is the distance of line of action of P_a above the base of the wall, A_1, A_2, A_3 are the areas of segments 1, 2, and 3 of the pressure diagram as shown in Fig. 15.12(b), and y_1, y_2, y_3 the distances of the centroid of segments 1, 2, and 3 from the base of the wall.

(iii) Backfill with uniform surcharge

Figure 15.9(a) shows a retaining wall with a horizontal backfill subjected to additional pressure (surcharge) of intensity q (kN/m^2) on the backfill surface. The surcharge applied at the top may be assumed to be uniform throughout the depth of the wall. The simple principle for the determination of active earth pressure at any level in Rankine's theory is to multiply the vertical stress at that depth with the Rankine's coefficient of active earth pressure. Vertical stress at any depth below the top of the backfill –

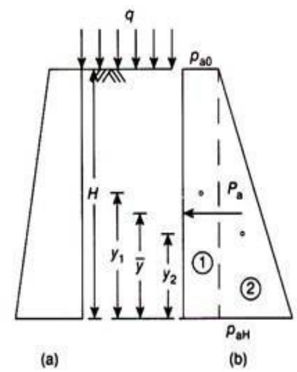


Figure 15.9 Active earth pressure for a cohesionless backfill subjected to surcharge: (a) Retaining wall and (b) active earth pressure diagram.

$\sigma_v = \gamma h + q$ …(15.12)

Hence, active earth pressure at any depth is given by –

$p_a = K_a \sigma_v = K_a(\gamma h + q) \Rightarrow p_a = K_a\gamma h + K_a q$ …(15.13)

When h = 0, active earth pressure at the top of the backfill is given by –

$p_{a0} = K_a \times \gamma \times 0 + K_a q = K_a q$

When h = H, active earth pressure at the bottom of the wall is given by –

$P_aH = K_a \times \gamma \times H + K_a q = K_a\gamma H + K_a q$

Thus, for a backfill subjected to a surcharge q at the top, the active earth pressure distribution is trapezoidal, as shown in Fig. 15.9(b), with intensity p_{a0} at top and p_aH at bottom.

Total active earth pressure is obtained by computing the area of the pressure diagram –

$P_a = (K_a q) \times H + 1/2 \times (K_a\gamma H) \times H \Rightarrow P_a = K_a qH + K_a\gamma H^2/2$ …(15.14)

Total active earth pressure acts horizontally through the centroid of the pressure diagram. From the principles of mechanics, the distance of the centroid above the base of the wall is given by –

$y = \Sigma aiyi/\Sigma Ai$ …(15.15)

where Ai is the area of each part of the pressure diagram, that is, A_1 and A_2 and y_i the distance of the centroid of each part of the pressure diagram above the base of the wall, that is, y_1 and y_2.

(iv) **Backfill with sloping surface.**

Figure 15.20(a) shows a retaining wall with a cohesionless backfill having its surface sloping at an angle β with the horizontal.

Consider a soil element of width b, along the slope, at any depth h below the surface of the backfill.

Rankine considered that the soil element is subjected to two stresses:

1. Vertical stress, σv, due to self-weight of the soil, acting vertically downward on the inclined planes AB and CD of the soil element.
2. Active earth pressure, pa, acting parallel to the surface of the backfill on the vertical planes BC and AD.

The two stresses are called conjugate stresses because the direction of each stress is parallel to the plane on which the other stress is acting. As shear stress also acts on plane AB of the soil element, the vertical stress is not a principal stress. Similarly, lateral pressure is also not a principal stress. Volume of the soil above the element per unit length will be –

$V = h \times b \cos \beta \times 1 = bh \cos \beta$

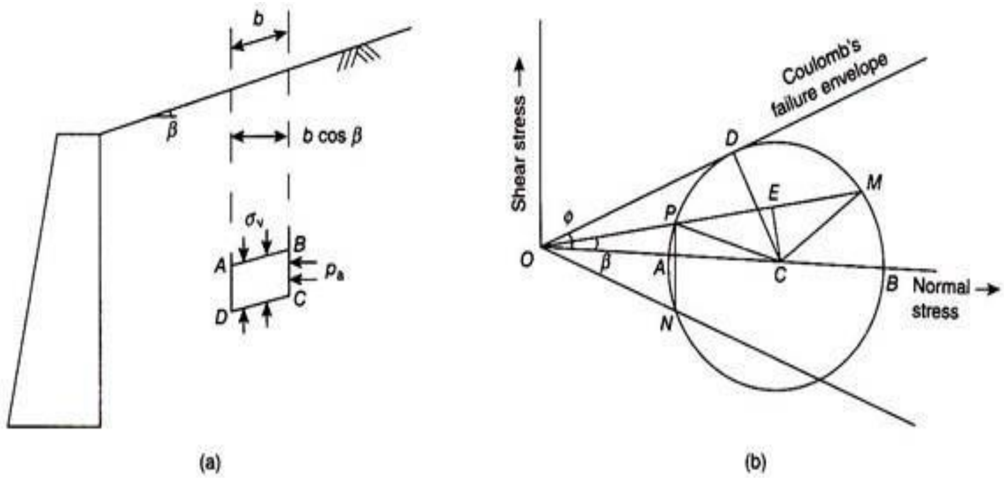


Figure 15.20 Rankine's active earth pressure for a retaining wall with a cohesionless backfill with a sloping surface: (a) Retaining wall with a sloping backfill and (b) active earth pressure determination from the Mohr's circle.

Weight of the soil above the element will be –

$W = \gamma \times V = \gamma \times (bh \cos \beta)$

Resisting area of the soil element will be –

$A = b \times 1 = b$

Vertical stress on the soil element will be –

$\sigma_v = W/A = \gamma \times (bh \cos \beta)/b = \gamma h \cos \beta$

Figure 15.20(b) shows the Mohr's circle of stresses for the soil element. Point M represents plane AB, and hence, OM gives the vertical stress. In fact, the vertical stress is the resultant of the normal stress and the shear stress acting on plane AB. When the backfill is in plastic equilibrium, the Mohr's circle passes through point M and will be tangential to the Coulomb's failure envelope. To find pole P on the Mohr's circle, a line is drawn from point M parallel to plane AB (on which it is acting) to intersect the Mohr's circle at point P, as shown in Fig. 15.20(b).

Now from point P, a line is drawn parallel to plane AD (on which p_a is acting) to intersect the Mohr's circle at point N. Point N represents plane AD on which the active earth pressure p_a is acting. Hence, ON gives the value of p_a.

$$\frac{P_a}{\sin(\alpha-\phi)} = \frac{W}{\sin(180-\alpha+\phi+\theta-\delta)} \quad \Rightarrow \quad P_a = \frac{W\sin(\alpha-\phi)}{\sin(180-\alpha+\phi+\theta-\delta)}$$

$$P_a = \frac{W\sin(\alpha-\phi)}{\sin(\alpha-\phi-\theta+\delta)} \tag{15.85}$$

$$W = \text{Area of } \Delta\text{ABC} \times 1 \times \gamma \quad \Rightarrow \quad W = \frac{1}{2} \times \text{BC} \times \text{AD} \times \gamma \tag{15.86}$$

In ΔABC

$$\frac{\text{BC}}{\sin A} = \frac{\text{AB}}{\sin C} \quad \Rightarrow \quad \angle A = 90+\beta-(90-\theta) = 90+\beta-90+\theta = \beta+\theta$$

Therefore,

$$\text{BC} = \frac{\text{AB}\sin A}{\sin C} = \text{AB}\frac{\sin(\beta+\theta)}{\sin(\alpha-\beta)}$$

In ΔABD

$$\frac{\text{AD}}{\sin B} = \frac{\text{AB}}{\sin 90} \quad \Rightarrow \quad \angle B = 180-(\theta+\alpha)$$

Therefore,

$$\text{AD} = \text{AB}\frac{\sin[180-(\theta+\alpha)]}{\sin 90} = \text{AB}\sin[180-(\theta+\alpha)] \quad \Rightarrow \quad \text{AD} = \text{AB}\sin(\theta+\alpha)$$

Substituting the values of BC and AD in Eq. (15.86), we get

$$W = \frac{1}{2} \times \text{AB}\frac{\sin(\beta+\theta)}{\sin(\alpha-\beta)} \times \text{AB}\sin(\theta+\alpha) \times \gamma \quad \Rightarrow \quad W = \frac{1}{2} \times \text{AB}^2 \frac{\sin(\beta+\theta)}{\sin(\alpha-\beta)} \times \sin(\theta+\alpha) \times \gamma$$

In ΔABE

$$\sin\theta = \frac{H}{\text{AB}} \quad \Rightarrow \quad \text{AB} = \frac{H}{\sin\theta}$$

$$W = \frac{\gamma H^2}{2\sin^2\theta}\frac{\sin(\beta+\theta)}{\sin(\alpha-\beta)} \times \sin(\theta+\alpha)$$

Substituting the value of W in Eq. (15.85), we get

$$P_a = \frac{\gamma H^2}{2} \times \frac{\sin(\beta+\theta) \times \sin(\theta+\alpha)}{\sin^2\theta \times \sin(\alpha-\beta)} \times \frac{\sin(\alpha-\phi)}{\sin(\alpha-\phi-\theta+\delta)} \tag{15.87}$$

It should be noted that active earth pressure is assumed to act at an angle P with the horizontal, parallel to the backfill surface.
If β = 0 is substituted in Eq. (15.30)
$K_a = (1 - \sin\phi)(1 + \sin\phi)$
which is the same as Eq. (15.9) for a cohesionless backfill with a horizontal surface.
Equation (15.29) indicates that active earth pressure is zero at the top surface

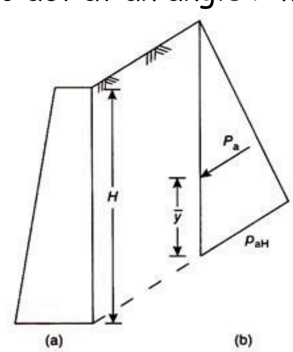


Figure 15.21 (a) Retaining wall with a sloping backfill and (b) active earth pressure diagram.

of the backfill (h = 0) and increases linearly with depth below the surface. Active earth pressure at the base of the wall –

$P_aH = K_a\gamma H$

The distribution of active earth pressure is shown in Fig. 15.21(b) for the wall with inclined backfill shown in Figs. 15.20 and 15.21(a).

The total or resultant active earth pressure exerted on the wall is obtained by computing the area of the pressure diagram.

Total active earth pressure = Area of the pressure diagram that is -

$$P_a = A = \frac{1}{2} \times p_{aH} \times H = \frac{1}{2} \times K_a \gamma H \times H = \frac{K_a \gamma H^2}{2}$$

(b) Explain active and passive earth pressures. Discuss how will you determine active earth pressure of a cohesive soil?

Sol. Active earth pressure:

When the wall moves away from the backfill, there is a decrease in the pressure on the wall and this decrease continues until a minimum value has reached after which their is no reduction in the pressure and the value will become constant. This kind of pressure is known as active earth pressure.

Passive earth pressure:

When the wall moves towards the back fill, there is an increase in the pressure on the wall and this increase continues until a maximum value has reached after which their is no increase in the pressure and the value will become constant. This kind of pressure is known as passive earth pressure.

This means that when the wall is about to slip due to lateral thrust from the backfill, a resistive force is applied by the soil in front of the wall

Active Earth pressure of cohesive soils

• Cohesive soil is a partially self supporting soil and exerts a smaller pressure on a retaining wall than a cohesionless soil with the same angle of friction.

• Rankine’s theory is for cohesionless soils only

• In cohesive soils, the failure envelope has a cohesion intercept whereas that for cohesionless soils is zero. Plastic equilibrium equation

We know that the principal stresses are related to soil properties by Bell’s equation, which is as follows –

$\sigma_1 = \sigma_3$ tan2α + 2c tan α …(15.35)

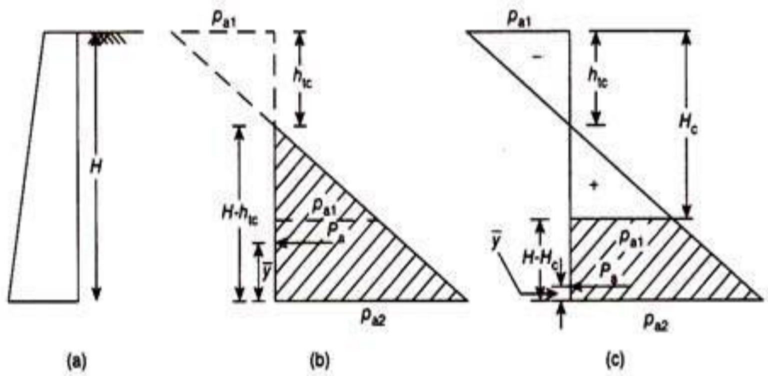


Figure 15.24 Rankine's active earth pressure for a cohesive backfill: (a) Retaining wall, (b) after formation of the tension crack, and (c) before formation of the tension crack.

Consider a soil element at any depth h below the surface of the backfill, as shown in Fig. 15.24(a). In the active case, major principal stress –

$\sigma_1 = \sigma_v = \gamma_h$

Minor principal stress –

$\sigma_3 = Pa$

Substituting these in Eq. (15.35), we have –

$\sigma_v = p_a \tan^2\alpha + 2c \tan\alpha = \gamma_h \Rightarrow p_a \tan^2\alpha = \gamma_h - 2c \tan\alpha$

$$p_a = \frac{\gamma h}{\tan^2\alpha} - \frac{2c\tan\alpha}{\tan^2\alpha} = \frac{\gamma h}{\tan^2\alpha} - \frac{2c}{\tan\alpha}$$

$$p_a = K_a\gamma h - 2c\sqrt{K_a} \tag{15.36}$$

where K_a is the Rankine's coefficient of active earth pressure, hence,

$$K_a = \frac{1}{\tan^2\alpha} = \frac{(1-\sin\phi)}{(1+\sin\phi)} = \cot^2\alpha$$

When $h = 0$

$$p_a = -2c\sqrt{K_a} = -2c\ \cot\alpha \tag{15.37}$$

When $h = H$

$$p_a = K_a\gamma H - 2c\sqrt{K_a} \tag{15.38}$$

Thus, active earth pressure is negative at the top of the wall and increases linearly with the increase in depth. As the soil is weak in tension, tension cracks will develop in the negative active earth pressure zone of the backfill. The depth of a tension crack can be obtained by substituting $p_a = 0$ in Eq. (15.38) –

$$p_a = K_a \gamma h_{tc} - 2c\sqrt{K_a} = 0 \qquad \Rightarrow \qquad h_{tc} = \frac{2c}{\gamma K_a} \times \sqrt{K_a}$$

$$h_{tc} = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2c}{\gamma}\tan\alpha \tag{15.39}$$

$$\alpha = 45 + \frac{\phi}{2}$$

If the soil is able to withstand the negative active earth pressure, the negative pressure over the depth h_{tc} is balanced by a positive pressure over the same depth below. Hence, the resultant active earth pressure is zero over the depth $H_c = 2h_{tc}$, known as critical height. Thus, excavations in cohesive soils can stand with vertical sides without any lateral support over the critical height, provided no tension crack is developed in the negative pressure zone. Critical height –

$$H_c = 2h_{tc} = \frac{4c}{\gamma\sqrt{K_a}} = \frac{4c}{\gamma}\tan\alpha \tag{15.40}$$

The total active earth pressure can be obtained by computing the area of the pressure diagram.

Q 2. (a) Describe Coulomb's wedge theory for determining the earth pressure behind a retaining wall.

Sol. Coulomb's theory states that the total lateral earth pressure is equal to the reaction exerted by the retaining wall when the wedge of soil tends to slide and acts at an angle 8 with the normal to the back of the wall.

Coulomb's Theory for Active Earth Pressure for Cohesionless Backfill:

As per Coulomb's theory, a wedge of soil above a failure plane moves outward and downward in the active case when the wall moves away from the backfill due to lateral earth pressure. Figure 15.42(a) shows a retaining wall of height H with a cohesionless backfill, with its surface inclined at an angle β with the horizontal. The back of the wall is inclined at an angle θ with the horizontal. Consider the failure plane BC at an inclination of α with the horizontal. The wedge of soil ABC tends to slide outward and downward away from the rest of the backfill. The wall resists the movement of the wedge and exerts a reaction P_a, inclined at an angle Δ with the normal to the wall, where Δ is the angle of wall friction. The magnitude of total active earth pressure is equal to P_a.

The total active earth pressure is determined in Coulomb's theory by considering the equilibrium of the wedge of soil ABC.

The forces acting on the wedge are:

i. Weight (W) of the soil in the wedge of soil ABC, acting vertically downward.

ii. The reaction (P_a) on the contact surface AB of the wall with the backfill, acting at an angle Δ with the normal to the back of the wall. As the wedge moves outward and downward, P_a acts inward and upward, opposing the movement of the wedge.

iii. The reaction (R) on the trial failure plane BC, which is the contact surface of the wedge with the rest of the backfill. The reaction R acts at an angle ϕ with the normal to the surface BC. This reaction acts upward and outward, opposing the movement of the wedge.
From, Fig. 15.42(a), angle of P_a with the vertical –
$\Psi = \theta - \Delta$ ....(15.82)
Similarly, from Fig. 15.42(a), angle of R with the vertical –

$\lambda = \alpha - \phi$ ...(15.83)
A trial value of α is assumed and the force diagram is constructed. Figure 15.42(b) shows the force diagram abc. A line ab is drawn parallel to the line of action of W, with the length ab equal to W to some scale. Now, a line bc′ is drawn parallel to the line of action of Pa, that is, at an angle $(\theta - \Delta)$ with ab. Another line ac″ is drawn parallel to the line of action of R, that is, at an angle $(\alpha - \phi)$ with ab. The two lines bc′ and ac″ intersect at point c, which completes the force diagram abc. The length of the line be gives the value of P_a to the scale of the force diagram for the assumed trial value of α.

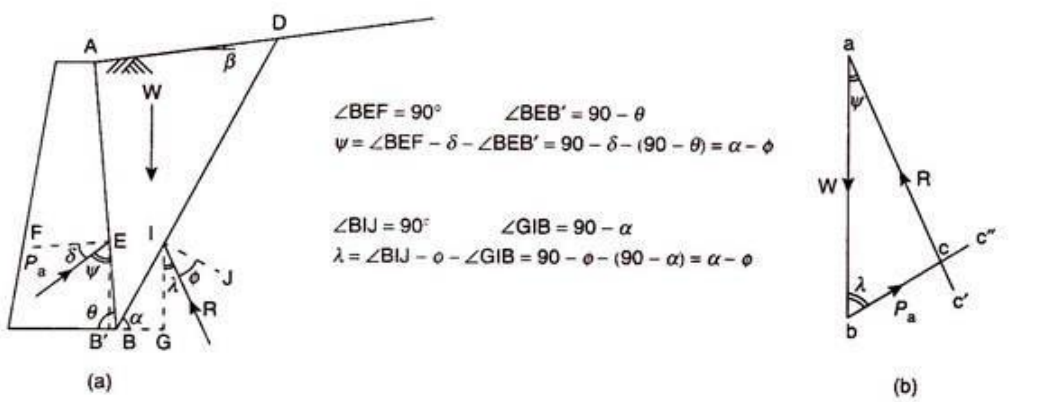


Figure 15.42 Coulomb's wedge theory: (a) Retaining wall with a trial slip surface and (b) force diagram.

The procedure is repeated for other failure planes, taking different trial values of α, and the corresponding values of P_a are determined. The maximum value of P_a, among the trial values, is taken as the active earth pressure. The corresponding trial failure plane is taken as the critical failure plane.
The active earth pressure acts along the same line of action as P_a, but opposite in direction. To determine the point of application of P_a, a line is drawn from the centroid of the wedge of soil ABC parallel to the critical failure plane to intersect the back of the wall at point P, which is the approximate point of application of P_a.

Coulomb's theory assumes that the failure surface is a plane surface, the actual surface being a curved surface. In the active case, however, the error involved in the estimation of P_a with a plane failure surface is found to be small. The angle of wall friction may be determined by conducting a shear box test with the wall material in the bottom half and the backfill material in the upper half of the shear box. The value of Δ is found to be in the range of $\phi/3 - 2\phi/3$.

Expression for Coulomb's Active Earth Pressure:

Referring to the force diagram shown in Fig. 15.42(b) and applying Lami's theorem –
$P_a/\sin(\alpha - \phi) = W/\sin C$ …(15.84)
In Δabc

$$\angle C = 180 - (\angle A + \angle B) = 180 - [(\alpha - \phi) + (\theta - \delta)] \quad \Rightarrow \quad \angle C = 180 - \alpha + \phi + \theta - \delta$$

Substituting this value of angle C in Eq. (15.84), we get –

$$\frac{P_a}{\sin(\alpha - \phi)} = \frac{W}{\sin(180 - \alpha + \phi + \theta - \delta)} \quad \Rightarrow \quad P_a = \frac{W \sin(\alpha - \phi)}{\sin(180 - \alpha + \phi + \theta - \delta)}$$

$$P_a = \frac{W \sin(\alpha - \phi)}{\sin(\alpha - \phi - \theta + \delta)} \tag{15.85}$$

$$W = \text{Area of } \Delta ABC \times 1 \times \gamma \quad \Rightarrow \quad W = \frac{1}{2} \times BC \times AD \times \gamma \tag{15.86}$$

In ΔABC

$$\frac{BC}{\sin A} = \frac{AB}{\sin C} \quad \Rightarrow \quad \angle A = 90 + \beta - (90 - \theta) = 90 + \beta - 90 + \theta = \beta + \theta$$

Therefore,

$$BC = \frac{AB \sin A}{\sin C} = AB \frac{\sin(\beta + \theta)}{\sin(\alpha - \beta)}$$

In ΔABD

$$\frac{AD}{\sin B} = \frac{AB}{\sin 90} \quad \Rightarrow \quad \angle B = 180 - (\theta + \alpha)$$

Therefore,

$$AD = AB \frac{\sin[180 - (\theta + \alpha)]}{\sin 90} = AB \sin[180 - (\theta + \alpha)] \quad \Rightarrow \quad AD = AB \sin(\theta + \alpha)$$

Substituting the values of BC and AD in Eq. (15.86), we get

$$W = \frac{1}{2} \times AB \frac{\sin(\beta + \theta)}{\sin(\alpha - \beta)} \times AB \sin(\theta + \alpha) \times \gamma \quad \Rightarrow \quad W = \frac{1}{2} \times AB^2 \frac{\sin(\beta + \theta)}{\sin(\alpha - \beta)} \times \sin(\theta + \alpha) \times \gamma$$

In ΔABE

$$\sin\theta = \frac{H}{AB} \quad \Rightarrow \quad AB = \frac{H}{\sin\theta}$$

$$W = \frac{\gamma H^2}{2\sin^2\theta} \frac{\sin(\beta + \theta)}{\sin(\alpha - \beta)} \times \sin(\theta + \alpha)$$

Substituting the value of W in Eq. (15.85), we get

$$P_a = \frac{\gamma H^2}{2} \times \frac{\sin(\beta + \theta) \times \sin(\theta + \alpha)}{\sin^2\theta \times \sin(\alpha - \beta)} \times \frac{\sin(\alpha - \phi)}{\sin(\alpha - \phi - \theta + \delta)} \tag{15.87}$$

The condition for maximum active earth pressure is obtained by differentiating Eq. (15.87) with respect to α and equating to zero –

$dP_a/d\alpha = 0$

The final expression for active earth pressure is given by –
$P_p = K_a (\gamma H2/2)$ …(15.88)
where

$$K_a = \frac{\sin^2(\theta+\phi)}{\sin^2\theta\sin(\theta-\delta)\left[1+\sqrt{\{\sin(\phi+\delta)\sin(\phi-\beta)\}/\{\sin(\theta-\delta)\sin(\theta+\beta)\}}\right]^2} \quad (15.89)$$

In Eqs. (15.88) and (15.89), H is the vertical height of the wall; θ the angle of back of the wall with horizontal; ϕ the angle of shearing resistance; Δ the angle of wall friction of the back of the wall; β the angle of backfill surface with horizontal; and γ the density of backfill material.

(b) A vertical excavation was made in a clay deposit having unit weight of 20 kN/m 3 . It caved in after the depth of excavation reached 4 meters. Calculate the value of Cohesion, assuming f to be zero. If the same clay is used as a backfill against a retaining wall upto a height of 8 m, calculate (i) total active earth pressure (ii) total passive earth pressure. Assume that the wall yields far enough to allow Rankine deformation conditions to establish.

Sol.

Given, $r = 20\frac{KN}{m^3},$

and as $H_c = \frac{4C}{r}.tan\alpha$

we know here, C=r then, C= $20\frac{KN}{m^3},$

1. $P_a = \int_{low}^{H}(K_a rH - 2C\sqrt{K_a}$

   $= \int_2^8 (20h-40) = (10\,h^2 - 40)_2^8$

   $= \frac{20}{2}(64-4) - 40(6)$

   =360 Kn/m2

2. $P_p = \int_0^H \left(K_f rH - 2C\sqrt{K_p}\right)dx$

   = 20*64/2 + 2X20X8

   = 640 + 320

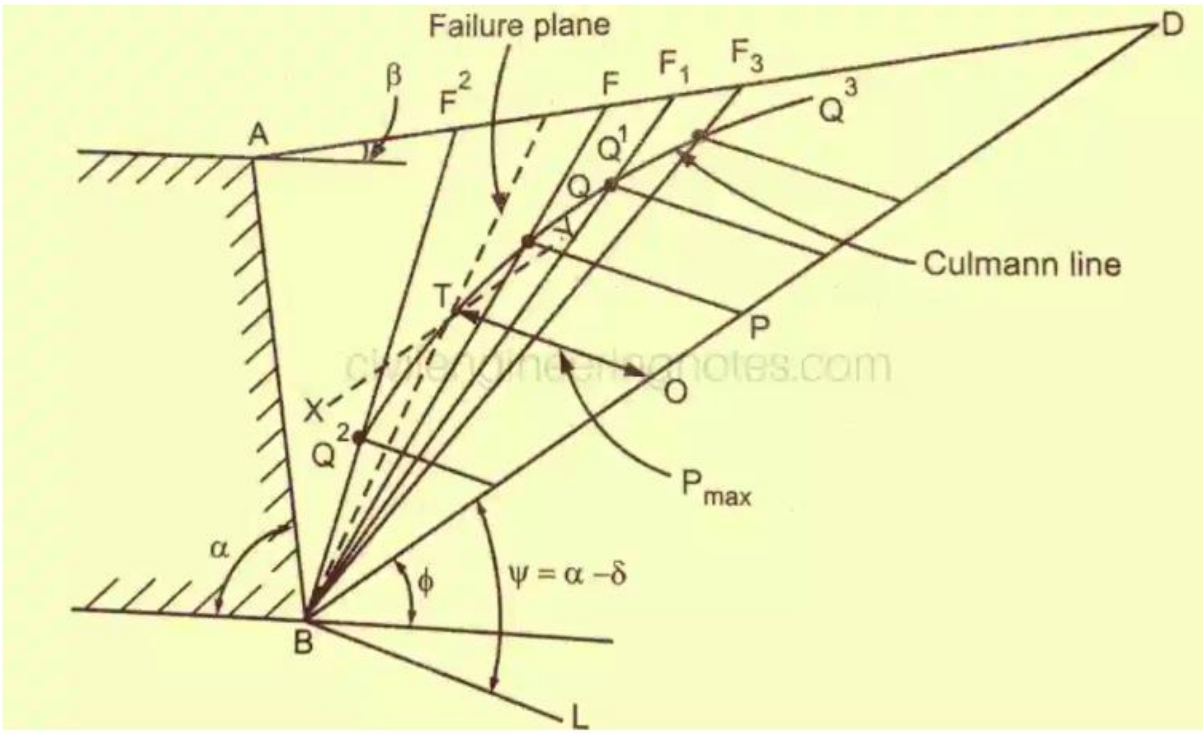
   960 KN/m2

Q 3. (a) Discuss how will you estimate the active earth pressure on a retaining wall by the Culmann's graphical method.

Sol. According to Coulomb's wedge theory, Culmann's method allows us to graphically calculate the magnitude of the earth pressure and locate the most dangerous rupture surface.

This method has more general application than Poncelet's and is, in fact, a simplified version of the more general trial wedge method. It is suitable for any form of ground surface, various forms of surcharge loads, and layered backfill with different unit weights for different layers.

The measures or steps in the procedure of culmann's method are as follows :



Culmann's Graphical Method

Procedure of Culmann's Method

- To scale, draw the retaining wall **AB**.
- **BD** is the **φ**-line.
- A line **BL** is drawn at an angle of 0 with line **BD**, resulting in **Ψ= α – δ**.
- The weight **(W)** of the failure wedge **ABF (γ. ABF)** is calculated using a failure surface **BF** which is totally assumed.
- **BP = W** is obtained by plotting the weight **(W)** along **BD**.
- To intersect the failure surfaces **BF** and **Q**, a line **PQ** is drawn from point **P** parallel to **BL**.
- The magnitude of P_a needed to maintain equilibrium for the presumed failure plane is represented by the length **PQ**.

The process is replicated with several different failure planes **BF_2**, **BF_1**, **BF_3**, etc. As a result, the points **Q^2, Q^1, Q^3** and so on are obtained.

- A smooth curve is drawn joining points **Q^2, Q, Q^1, Q^3** etc. This curve is called Culmann's Line.
- A line **XY** (dotted) tangential to the culmann line and parallel to **BD** is drawn. The point of tangency is designated by the letter **T**.

- Drawing a line **TO** from **T** on **BD** and parallel to **BL** measures the magnitude of largest value of $\mathbf{P_a}$. It's the same as the coulomb's pressure ($\mathbf{P_a}$).
- The failure plane actually passes through point **T (dotted)**.

(b) A retaining wall 5 m high has a vertical back and supports cohesive backfill whose surface is level with the top of the wall. The properties of the backfill are:
Angle of friction f = zero, unit weight γ = 18 kN/m 3 and cohesion c = 20 kN/m 2 . Determine the magnitude and point of application of the active earth pressure per metre length of the wall considering the effect of development of tension cracks.
Sol.

$h_{tc} = \frac{2C}{r} tan\alpha = \frac{2X120}{18} X1$ = 2.22 m

And,

$P_{aH} = K_a rH - 2C\sqrt{K_a}$

And, $K_a = \frac{1-Sin\varphi}{1+Sin\varphi}$

and we know that here Ka = 1

and also $P_a H$= 1X18X5 – 2X 20

= 50 KN/ m^2

So ,

Total Pressure = ½*2.78*50

= 69.5 KN/m^2

and application of part of pressure is

H/S = 278/3

= 0.93 from base.

Q 4. Explain how will you design a gravity retaining wall.
Solu. Gravity retaining walls derive their stability by self-weight.
The main design criteria are as follows:
i.To prevent overturning of the wall about toe.
ii.To prevent sliding of the wall at its base.

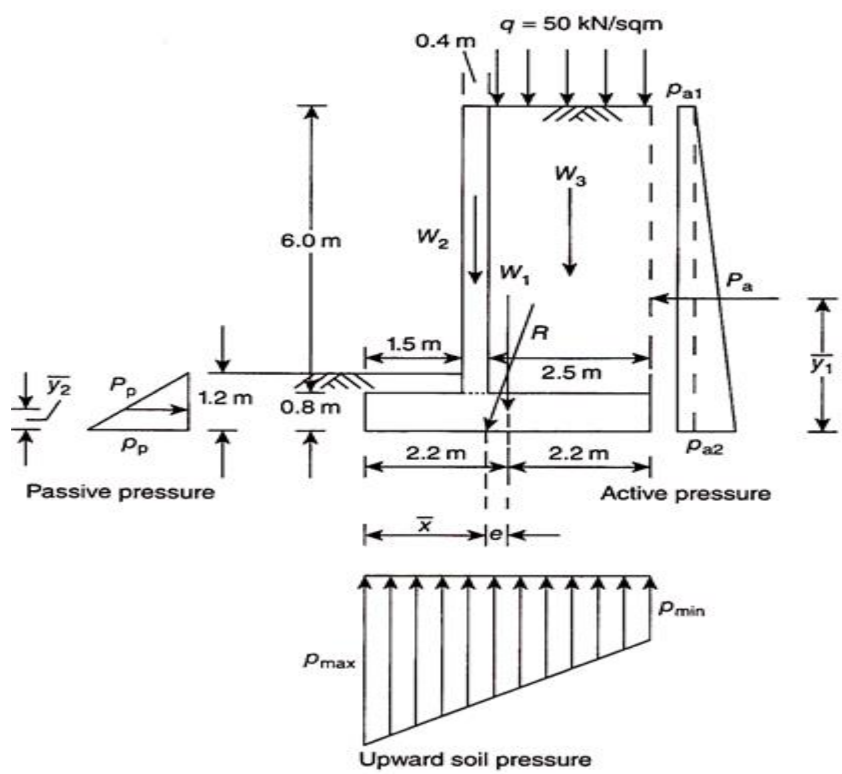


Figure 16.7 Stability analysis of cantilever retaining wall.

iii.To prevent tension anywhere in the base soil.
iv. To prevent bearing failure of the base soil.
From Figure, it is evident that the various forces/pressures acting on the wall are:
i. Active earth pressure of intensity p_{a1} and p_{a2} at the top and bottom of the wall, respectively.
ii. Passive earth pressure of intensity p_p at the bottom of the wall on the front side of the wall.
iii. Weight of the wall, consisting of W_1 and W_2.
iv. Weight of the backfill above the heel slab (W_3).
v. Surcharge pressure of intensity q acting on the top of the backfill.
vi. Upward soil reaction of intensity p_{max} below the toe and p_{min} below the heel of the wall.
The design principles of gravity retaining walls are discussed below:

1. Stability against Overturning:

The wall must be safe against overturning. Overturning is caused by the horizontal component of the resultant active earth pressure, Pa, in the form of an overturning moment, ΣM_o, about the toe of the wall. It is resisted by downward forces, W_1, W_2, W_3, and W_4 (due to surcharge) and the resultant passive earth pressure (P_p), in the form of a resisting moment, ΣM_R, about the toe of the wall.
Stability analysis is carried out by considering all forces per unit length (say 1 m) of the wall. P_a and P_p are calculated using Rankine's theory or Coulomb's theory. The weight of the wall is obtained as the product of the corresponding cross-sectional area (W_1 or W_2) and the density of the wall material. The weight of soil over the heel slab is calculated as the product of the cross-sectional area of the soil above the toe slab and the density of the backfill material.

$$\sum M_o = P_{aH} \times \overline{y}_1 \qquad (16.1)$$

The overturning moment, Mo, about the toe is computed by the following equation –
where $\overline{y_1}$ is the distance of the line of action P_a above the base of the wall.
i. The resisting moment, MR, about the toe is computed by –

$$\sum M_R = W_1 \times x_1 + W_2 \times x_2 + W_3 \times x_3 + W_4 \times x_4 + P_{pH} \times \overline{y}_2 \qquad (16.2)$$

where, x_1, x_2, x_3 and x_4 are the horizontal distances of W_1, W_2, W_3, and W_4, respectively, from the toe of the wall and $\overline{y_2}$is the distance of the line of action P_p above the base of the wall.
ii. The factor of safety against overturning is defined by –

$$F_o = \frac{\sum M_R}{\sum M_o} \qquad (16.3)$$

The factor of safety against overturning should be generally about 2-3.

2.Stability against Sliding:

The wall tends to slide away from the backfill due to net horizontal later pressure acting on the wall. This is resisted by the frictional force between the base of the wall and the underlying soil.
i. Horizontal component of active earth pressure acting on the wall is given by –
P_aH= P_a cosa ...(16.4)
where P_a is the total active earth pressure acting at an angle a with horizontal.

ii. Frictional force at the base of the wall is given by –
FH = $\mu\Sigma$Fy …(16.5)
iii. The factor of safety against sliding is defined by –

$$F_s = \frac{P_{pH} + \mu \sum F_y}{P_{aH}} \qquad (16.6)$$

where μ is the coefficient of friction between the base of the slab and the underlying soil = tanϕ.
The factor of safety against sliding should not be less than 1.5. Usually, the condition for stability against overturning is automatically satisfied if the condition for stability against sliding is fulfilled.

3. Stability against Tension:
The soil reaction pressure at the base of the wall will be minimum at the heel and can be computed using –

$$p_{min} = \frac{\sum F_y}{b}\left[1 - \frac{6e}{b}\right] \qquad (16.7)$$

where Σf_Y is the algebraic sum of vertical components of all forces acting on the wall, obtained from Eq. (16.8), b the width of the base slab, and e the eccentricity of the resultant force from the center of the width of the base slab, obtained from Eq. (16.9) –

$$\sum F_y = W_1 + W_2 + W_3 + W_4 + P_{ay} \qquad (16.8)$$

$$e = \frac{b}{2} - \bar{x} \qquad (16.9)$$

Tension should not be allowed to develop at the heel, and hence, p_{min} should be equal to or more than zero. In other words, the eccentricity of the resultant force –
e ≤ b/6 …(16.10)
or the resultant should lie within the middle third of the base.

4. Stability against Bearing Failure:

$$p_{max} = \frac{\sum F_y}{b}\left(1 + \frac{6e}{b}\right) \qquad (16.11)$$

The maximum soil reaction pressure occurs at the toe of the wall and can be determined by –
The factor of safety against bearing failure is determined using –

$$F_b = \frac{q_u}{p_{max}} \qquad (16.12)$$

The factor of safety against bearing failure should not be less than 2.5.

Q 5. A 6 metres high vertical wall supports saturated, cohesive backfill with horizontal surface. The top 3m of the backfill weighs 18 kN/m 3 and has an apparent cohesion of 18 kN/m 2 . The bulk unit weight and apparent cohesion of the bottom 3 m of the backfill are respectively 20 kN/m 3 and 24 kN/m 2 . Find the likely depth of tension crack behind the wall? If the tension cracks develop, what will be the total active pressure? Draw pressure distribution diagram and determine point of application of the resultant pressure.

Sol.

Topsoil layer AB,

$Ka_1 = (1 - Sin\varphi_1)/ (1 + Sin\varphi_1) = 1;$ $\varphi_1 = 0$

$Pa_1 = Ka_1r_1z - 2c(Ka_1)^{-1/2}$

At point A; $Pa_1 = -2 \times 18 \times 1 = -36\ KN/m^2$

At point B; $Pa_2 = -(18 \times 3) - (2 \times 18) = 18\ KN/m^2$

Depth of tension crack,

$Z_0 = ac_1/(r_1(Ka_1)^{-1/2})$

Thrust contribution, $Pa_1 = \frac{1}{2} \times 18(3 - 2) \times 1 = 9\ KN/m$

Line of action of $Pa_1 = 3 + ((3 - 2)/3) = 3.33m$ from base

Bottom soil layer BC,

$Ka_2 = (1 - Sin\varphi_2)/(1 + Sin\varphi_2) = 1;$

$q = r_1H_1 - 18 \times 3 = 54\ KN/m$

Thrust contribution, $Ka_2r_1H_1H_2 = 54 \times 3 = 162\ KN/m$

Line of action = 3/2 = 1.5m

Effect of soil layer or grains of soil mass:

$Pa_3 = Ka_2r_2 - 2c(Ka_2)^{-1/2}$

$(Pa_3)_{2:3} = 1 \times 20 \times 3 - 2 \times 24 = 12\ KN/m$

Now, depth of tension crack in layer BC,

$h_{tc} = (2c/r)\tan a$

$h_{tc} = (2 \times 24)/20 = 2.4m$

Thrust = $\frac{1}{2} \times 12\ (3 - 2.4)*1$

= $6 \times 0.6 \times 1 = 3.6\ KN/m$

Line of action of $Pa_3 = 0.6/3 = 0.2$ m from base

Total thrust per unit length of the wall,

$Pa = Pa_1 + Pa_2 + Pa_3$

$Pa = 9 + 162 + 3.6$

$Pa = 174.6\ KN/m$

Line of action of thrust,

$= (9 \times 3.33 + 162 \times 1.5 + 3.6 \times 6.2)/174.6$

= 1.5659 m from the base