

1.3 | Unit Conversion

Learning Objectives

By the end of this section, you will be able to:

- Use conversion factors to express the value of a given quantity in different units.

It is often necessary to convert from one unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you may need to convert units of feet or meters to miles.

Let's consider a simple example of how to convert units. Suppose we want to convert 80 m to kilometers. The first thing to do is to list the units you have and the units to which you want to convert. In this case, we have units in *meters* and we want to convert to *kilometers*. Next, we need to determine a conversion factor relating meters to kilometers. A **conversion factor** is a ratio that expresses how many of one unit are equal to another unit. For example, there are 12 in. in 1 ft, 1609 m in 1 mi, 100 cm in 1 m, 60 s in 1 min, and so on. Refer to **Appendix B** for a more complete list of conversion factors. In this case, we know that there are 1000 m in 1 km. Now we can set up our unit conversion. We write the units we have and then multiply them by the conversion factor so the units cancel out, as shown:

$$80 \cancel{\text{ m}} \times \frac{1 \text{ km}}{1000 \cancel{\text{ m}}} = 0.080 \text{ km}.$$

Note that the unwanted meter unit cancels, leaving only the desired kilometer unit. You can use this method to convert between any type of unit. Now, the conversion of 80 m to kilometers is simply the use of a metric prefix, as we saw in the preceding section, so we can get the same answer just as easily by noting that

$$80 \text{ m} = 8.0 \times 10^1 \text{ m} = 8.0 \times 10^{-2} \text{ km} = 0.080 \text{ km},$$

since “kilo-” means 10^3 (see **Table 1.2**) and $1 = -2 + 3$. However, using conversion factors is handy when converting between units that are not metric or when converting between derived units, as the following examples illustrate.

Example 1.2

Converting Nonmetric Units to Metric

The distance from the university to home is 10 mi and it usually takes 20 min to drive this distance. Calculate the average speed in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units, then we can get the average speed into the desired units by picking the correct conversion factors and multiplying by them. The correct conversion factors are those that cancel the unwanted units and leave the desired units in their place. In this case, we want to convert miles to meters, so we need to know the fact that there are 1609 m in 1 mi. We also want to convert minutes to seconds, so we use the conversion of 60 s in 1 min.

Solution

1. Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now. Average speed and other motion concepts are covered in later chapters.) In equation form,

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}}.$$

2. Substitute the given values for distance and time:

$$\text{Average speed} = \frac{10 \text{ mi}}{20 \text{ min}} = 0.50 \frac{\text{mi}}{\text{min}}.$$

3. Convert miles per minute to meters per second by multiplying by the conversion factor that cancels miles and leave meters, and also by the conversion factor that cancels minutes and leave seconds:

$$0.50 \frac{\cancel{\text{mile}}}{\cancel{\text{min}}} \times \frac{1609 \text{ m}}{1 \cancel{\text{ mile}}} \times \frac{1 \cancel{\text{ min}}}{60 \text{ s}} = \frac{(0.50)(1609)}{60} \text{ m/s} = 13 \text{ m/s}.$$

Significance

Check the answer in the following ways:

1. Be sure the units in the unit conversion cancel correctly. If the unit conversion factor was written upside down, the units do not cancel correctly in the equation. We see the “miles” in the numerator in 0.50 mi/min cancels the “mile” in the denominator in the first conversion factor. Also, the “min” in the denominator in 0.50 mi/min cancels the “min” in the numerator in the second conversion factor.
2. Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of meters per second and, after the cancellations, the only units left are a meter (m) in the numerator and a second (s) in the denominator, so we have indeed obtained these units.



1.2 Check Your Understanding Light travels about 9 Pm in a year. Given that a year is about 3×10^7 s, what is the speed of light in meters per second?

Example 1.3

Converting between Metric Units

The density of iron is 7.86 g/cm^3 under standard conditions. Convert this to kg/m^3 .

Strategy

We need to convert grams to kilograms and cubic centimeters to cubic meters. The conversion factors we need are $1 \text{ kg} = 10^3 \text{ g}$ and $1 \text{ cm} = 10^{-2} \text{ m}$. However, we are dealing with cubic centimeters ($\text{cm}^3 = \text{cm} \times \text{cm} \times \text{cm}$), so we have to use the second conversion factor three times (that is, we need to cube it). The idea is still to multiply by the conversion factors in such a way that they cancel the units we want to get rid of and introduce the units we want to keep.

Solution

$$7.86 \frac{\cancel{\text{g}}}{\cancel{\text{cm}}^3} \times \frac{\text{kg}}{10^3 \cancel{\text{g}}} \times \left(\frac{\cancel{\text{cm}}}{10^{-2} \text{ m}} \right)^3 = \frac{7.86}{(10^3)(10^{-6})} \text{ kg/m}^3 = 7.86 \times 10^3 \text{ kg/m}^3$$

Significance

Remember, it's always important to check the answer.

1. Be sure to cancel the units in the unit conversion correctly. We see that the gram (“g”) in the numerator in 7.86 g/cm^3 cancels the “g” in the denominator in the first conversion factor. Also, the three factors of “cm” in the denominator in 7.86 g/cm^3 cancel with the three factors of “cm” in the numerator that we get by cubing the second conversion factor.
2. Check that the units of the final answer are the desired units. The problem asked for us to convert to kilograms per cubic meter. After the cancellations just described, we see the only units we have left are “kg” in the numerator and three factors of “m” in the denominator (that is, one factor of “m” cubed, or “m³”). Therefore, the units on the final answer are correct.



1.3 Check Your Understanding We know from **Figure 1.4** that the diameter of Earth is on the order of 10^7 m, so the order of magnitude of its surface area is 10^{14} m^2 . What is that in square kilometers (that is, km^2)? (Try doing this both by converting 10^7 m to km and then squaring it and then by converting 10^{14} m^2 directly to square kilometers. You should get the same answer both ways.)

Unit conversions may not seem very interesting, but not doing them can be costly. One famous example of this situation was