

1.6 | Significant Figures

Learning Objectives

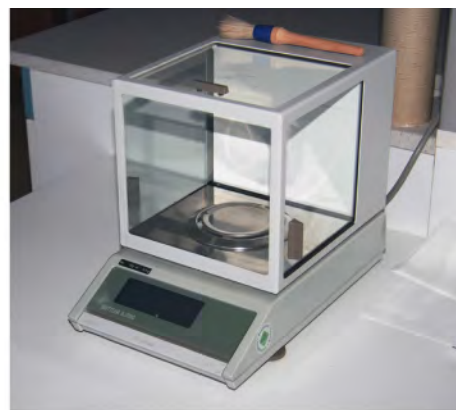
By the end of this section, you will be able to:

- Determine the correct number of significant figures for the result of a computation.
- Describe the relationship between the concepts of accuracy, precision, uncertainty, and discrepancy.
- Calculate the percent uncertainty of a measurement, given its value and its uncertainty.
- Determine the uncertainty of the result of a computation involving quantities with given uncertainties.

Figure 1.11 shows two instruments used to measure the mass of an object. The digital scale has mostly replaced the double-pan balance in physics labs because it gives more accurate and precise measurements. But what exactly do we mean by *accurate* and *precise*? Aren't they the same thing? In this section we examine in detail the process of making and reporting a measurement.



(a)



(b)

Figure 1.11 (a) A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The “known masses” are typically metal cylinders of standard mass such as 1 g, 10 g, and 100 g. (b) Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. A mechanical balance may read only the mass of an object to the nearest tenth of a gram, but many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit a: modification of work by Serge Melki; credit b: modification of work by Karel Jakubec)

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the accepted reference value for that measurement. For example, let's say we want to measure the length of standard printer paper. The packaging in which we purchased the paper states that it is 11.0 in. long. We then measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the reference value of 11.0 in. In contrast, if we had obtained a measurement of 12 in., our measurement would not be very accurate. Notice that the concept of accuracy requires that an accepted reference value be given.

The **precision** of measurements refers to how close the agreement is between repeated independent measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements is to determine the range, or difference, between the lowest and the highest measured values. In this case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by, at most, 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9 in., 11.1 in., and 11.9 in., then the measurements would not be very precise because there would be significant

variation from one measurement to another. Notice that the concept of precision depends only on the actual measurements acquired and does not depend on an accepted reference value.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let's consider an example of a GPS attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target and think of each GPS attempt to locate the restaurant as a black dot. In **Figure 1.12(a)**, we see the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low-precision, high-accuracy measuring system. However, in **Figure 1.12(b)**, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high-precision, low-accuracy measuring system.

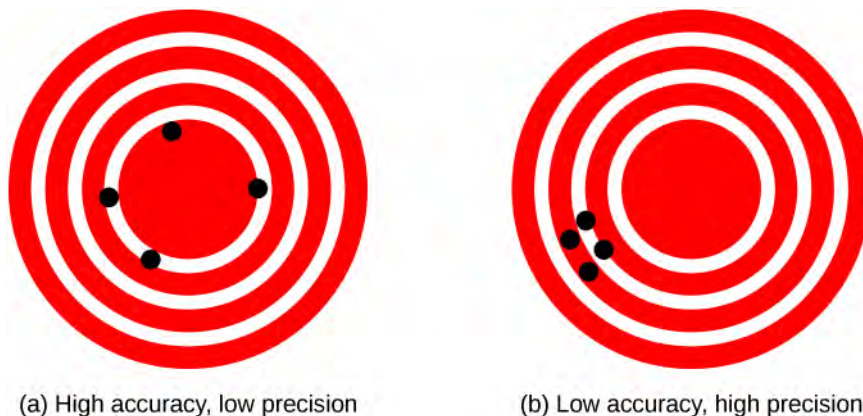


Figure 1.12 A GPS attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. (a) The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (b) The dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit a and credit b: modification of works by "DarkEvil"/Wikimedia Commons)

Accuracy, Precision, Uncertainty, and Discrepancy

The precision of a measuring system is related to the **uncertainty** in the measurements whereas the accuracy is related to the **discrepancy** from the accepted reference value. Uncertainty is a quantitative measure of how much your measured values deviate from one another. There are many different methods of calculating uncertainty, each of which is appropriate to different situations. Some examples include taking the range (that is, the biggest less the smallest) or finding the standard deviation of the measurements. Discrepancy (or "measurement error") is the difference between the measured value and a given standard or expected value. If the measurements are not very precise, then the uncertainty of the values is high. If the measurements are not very accurate, then the discrepancy of the values is high.

Recall our example of measuring paper length; we obtained measurements of 11.1 in., 11.2 in., and 10.9 in., and the accepted value was 11.0 in. We might average the three measurements to say our best guess is 11.1 in.; in this case, our discrepancy is $11.1 - 11.0 = 0.1$ in., which provides a quantitative measure of accuracy. We might calculate the uncertainty in our best guess by using the range of our measured values: 0.3 in. Then we would say the length of the paper is 11.1 in. plus or minus 0.3 in. The uncertainty in a measurement, A , is often denoted as δA (read "delta A "), so the measurement result would be recorded as $A \pm \delta A$. Returning to our paper example, the measured length of the paper could be expressed as 11.1 ± 0.3 in. Since the discrepancy of 0.1 in. is less than the uncertainty of 0.3 in., we might say the measured value agrees with the accepted reference value to within experimental uncertainty.

Some factors that contribute to uncertainty in a measurement include the following:

- Limitations of the measuring device
- The skill of the person taking the measurement
- Irregularities in the object being measured
- Any other factors that affect the outcome (highly dependent on the situation)

In our example, such factors contributing to the uncertainty could be the smallest division on the ruler is 1/16 in., the person using the ruler has bad eyesight, the ruler is worn down on one end, or one side of the paper is slightly longer than the other.