Example 2.14

The Unit Vector of Direction

If the velocity vector of the military convoy in Example 2.8 is $\vec{\mathbf{v}} = (4.000\,\hat{\mathbf{i}} + 3.000\,\hat{\mathbf{j}} + 0.100\,\hat{\mathbf{k}})\text{km/h}$, what is the unit vector of its direction of motion?

Strategy

The unit vector of the convoy's direction of motion is the unit vector $\hat{\mathbf{v}}$ that is parallel to the velocity vector. The unit vector is obtained by dividing a vector by its magnitude, in accordance with Equation 2.26.

Solution

The magnitude of the vector $\vec{\mathbf{v}}$ is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{4.000^2 + 3.000^2 + 0.100^2}\text{km/h} = 5.001\text{km/h}.$$

To obtain the unit vector $\hat{\mathbf{v}}$, divide $\vec{\mathbf{v}}$ by its magnitude:

$$\begin{aligned}\hat{\mathbf{v}} &= \frac{\vec{\mathbf{v}}}{v} = \frac{(4.000\,\hat{\mathbf{i}} + 3.000\,\hat{\mathbf{j}} + 0.100\,\hat{\mathbf{k}})\text{km/h}}{5.001\text{km/h}} \\ &= \frac{(4.000\,\hat{\mathbf{i}} + 3.000\,\hat{\mathbf{j}} + 0.100\,\hat{\mathbf{k}})}{5.001} \\ &= \frac{4.000}{5.001}\hat{\mathbf{i}} + \frac{3.000}{5.001}\hat{\mathbf{j}} + \frac{0.100}{5.001}\hat{\mathbf{k}} \\ &= (79.98\,\hat{\mathbf{i}} + 59.99\,\hat{\mathbf{j}} + 2.00\,\hat{\mathbf{k}}) \times 10^{-2}.\end{aligned}$$

Significance

Note that when using the analytical method with a calculator, it is advisable to carry out your calculations to at least three decimal places and then round off the final answer to the required number of significant figures, which is the way we performed calculations in this example. If you round off your partial answer too early, you risk your final answer having a huge numerical error, and it may be far off from the exact answer or from a value measured in an experiment.

2.10 **Check Your Understanding** Verify that vector $\hat{\mathbf{v}}$ obtained in Example 2.14 is indeed a unit vector by computing its magnitude. If the convoy in Example 2.8 was moving across a desert flatland—that is, if the third component of its velocity was zero—what is the unit vector of its direction of motion? Which geographic direction does it represent?

2.4 | Products of Vectors

Learning Objectives

By the end of this section, you will be able to:

- Explain the difference between the scalar product and the vector product of two vectors.
- Determine the scalar product of two vectors.
- Determine the vector product of two vectors.
- Describe how the products of vectors are used in physics.

A vector can be multiplied by another vector but may not be divided by another vector. There are two kinds of products of vectors used broadly in physics and engineering. One kind of multiplication is a *scalar multiplication of two vectors*.

Taking a scalar product of two vectors results in a number (a scalar), as its name indicates. Scalar products are used to define work and energy relations. For example, the work that a force (a vector) performs on an object while causing its displacement (a vector) is defined as a scalar product of the force vector with the displacement vector. A quite different kind of multiplication is a *vector multiplication of vectors*. Taking a vector product of two vectors returns as a result a vector, as its name suggests. Vector products are used to define other derived vector quantities. For example, in describing rotations, a vector quantity called *torque* is defined as a vector product of an applied force (a vector) and its distance from pivot to force (a vector). It is important to distinguish between these two kinds of vector multiplications because the scalar product is a scalar quantity and a vector product is a vector quantity.

The Scalar Product of Two Vectors (the Dot Product)

Scalar multiplication of two vectors yields a scalar product.

Scalar Product (Dot Product)

The **scalar product** $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is a number defined by the equation

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\varphi, \tag{2.27}$$

where φ is the angle between the vectors (shown in **Figure 2.27**). The scalar product is also called the **dot product** because of the dot notation that indicates it.

In the definition of the dot product, the direction of angle φ does not matter, and φ can be measured from either of the two vectors to the other because $\cos\varphi = \cos(-\varphi) = \cos(2\pi - \varphi)$. The dot product is a negative number when $90° < \varphi \le 180°$ and is a positive number when $0° \le \varphi < 90°$. Moreover, the dot product of two parallel vectors is $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos 0° = AB$, and the dot product of two antiparallel vectors is $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos 180° = -AB$. The scalar product of two *orthogonal vectors* vanishes: $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos 90° = 0$. The scalar product of a vector with itself is the square of its magnitude:

$$\vec{\mathbf{A}}^{\,2} \equiv \vec{\mathbf{A}} \cdot \vec{\mathbf{A}} = AA\cos 0° = A^2. \tag{2.28}$$

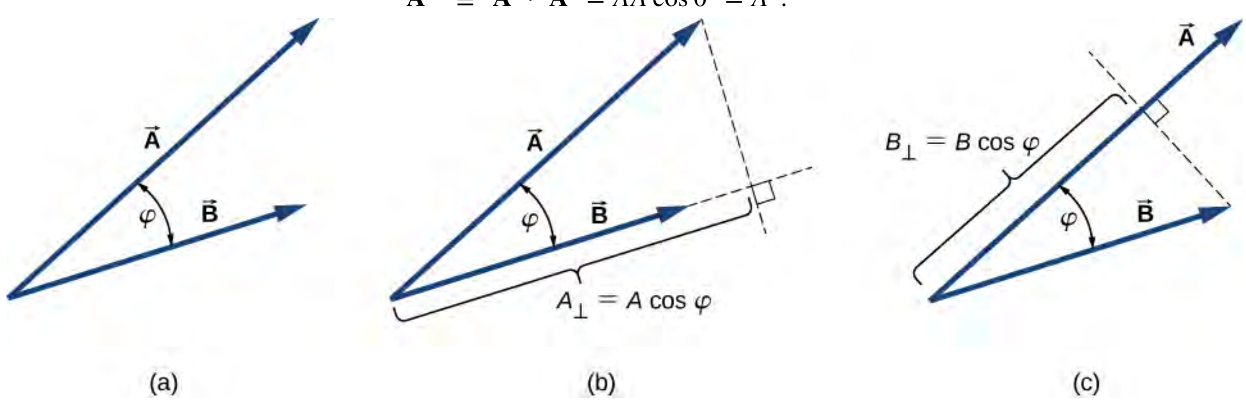

Figure 2.27 The scalar product of two vectors. (a) The angle between the two vectors. (b) The orthogonal projection $A_\perp$ of vector $\vec{\mathbf{A}}$ onto the direction of vector $\vec{\mathbf{B}}$. (c) The orthogonal projection $B_\perp$ of vector $\vec{\mathbf{B}}$ onto the direction of vector $\vec{\mathbf{A}}$.

Example 2.15

The Scalar Product

For the vectors shown in **Figure 2.13**, find the scalar product $\vec{\mathbf{A}} \cdot \vec{\mathbf{F}}$.