Assigment - 2

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Q1

Given

- $\rightarrow$ V is set of all Real func.
- $\rightarrow$ $f(g(x)) = f(x) + g(x)$
- $\rightarrow$ $\vec{O} = g(x) = x$
- $\rightarrow$ 2 Rules which are Broken in order for V to be a vetor space

Soln

Property 1

Commutivity

$$f(g(x)) = f+g$$

When $g(x) = x+1$

$f(x) = x^2$

$$f(g(x)) = (x+1)^2$$

$$g(f(x)) = x^2+1$$

$$g(f(x)) \neq f(g(x))$$

$\therefore$ Commutativity fails

Property 2

Distributivity

$$cf(cg(x)) \neq cf(g(x))$$

and When

$g(x) = x+1$

$f(x) = x^2$

$$c(f+g) = c \cdot f(g(x)) = c\,(x+1)^2$$

$$cf + cg = cf(c\,g(x))$$

$$= c\,(cx + c)^2$$

$$c(f+g) \neq cf + cg$$

Q2 *Given*

$$x = (x_1, x_2, x_3, x_4) \; \forall R^4$$

to find

Specific Vectors x with Dims

a) 0 b) 1 c) 3 d) 4

a) $\dim(S) = 0$ this means that all Permutations are a zero vector

$\therefore$

$$x = (0, 0, 0, 0)$$

b) $\dim(S) = 1$
ie, all permutations are a Scalar Multiple of each other
$\therefore x = (1, 1, 1, 1)$

c) $\dim(S) = 3$

Every Non Zero Vector
$x = (x_1, x_2, x_3, x_4)$
Where the Dot Product with $(1, 1, 1, 1)$ is 0.

One such is
$x = (1, 1, -1, -1)$

d) the Standard basis of $\mathbb{R}^4$. ie $(1, 0, 0, 0)$
Since any Vector Space is a Subspace of itself.

Q3 Given

- V_1, V_2, V_3, V_4 is a Basis of $\mathbb{R}^4$.
- if W is a Subspace then Some Subset of V is a Basis for W

Soln

all Subsets of
$V = \{ V_1, V_2, V_3, V_4 \}$
are Either the Standard basis of $\mathbb{R}^4$ or it's Linear Combination
So

$V_1 = \{ 1 \quad 0 \quad 0 \quad 0 \}$
$V_2 = \{ 0 \quad 1 \quad 0 \quad 0 \}$
$V_3 = \{ 0 \quad 0 \quad 1 \quad 0 \}$
$V_4 = \{ 0 \quad 0 \quad 0 \quad 1 \}$

Consider

$W = \{ a \ b \quad c \ d \}$
Where $a = b$ and $c = d$.
and all it's permutations.

$\Rightarrow$ No Subset of V can Span

W as Each Element of V has only 1 Non Zero Element and Each Element of Basis of W have 2 Non Zero Element

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Q.5 Given

$\Rightarrow$ Non Empty Subset of Inner Product Space V.

$$\Rightarrow S^{\perp} = \{ v \in V \mid \langle v, s \rangle = 0 \ \forall s \in S \}$$

To find

$S^{\perp}$ when

a) $S = \{(1, 2, -2), (1, -1, 3)\}$ in $V = \mathbb{R}^3$ wrt Usual Inner Product.

b) $S = \{1+x, x^2\}$ in $V = P_2(\mathbb{R})$ wrt

Inner Product

$$\langle p(x), q(x) \rangle = \int_{-1}^{1} p(x) q(x)\, dx$$

Soln Let $V = (x, y, z)$ in $\mathbb{R}^3$

a) $S = \{(1, 2, -2), (1, -1, 3)\}$

$$S^{\perp} = \begin{cases} \langle v, (1, 2, -2) \rangle \\ \langle v, (1, -1, 3) \rangle \end{cases}$$

$$S^{\perp} = \begin{cases} x + 2y - 2z = 0 \\ x - y + 3z = 0 \end{cases}$$

from 1 : $x = -2y + 2z$

Substituting into 2

$$(-2y + 2z) - y + 3z = 0$$

$$-3y + 5z = 0$$

$$y = 5/3\, z$$

$$\therefore x = \frac{-10}{3} z + 2z$$

$$x = \frac{-4}{3} z$$

$$\therefore \quad v = z\left(\frac{-4}{3}, \frac{5}{3}, 1\right)$$

$$S^{\perp} = \operatorname{Span}\left\{\left(\frac{-4}{3}, \frac{5}{3}, 1\right)\right\}$$

b) $S = \{1+x, x^2\}$ in $P_2(\mathbb{R}^2)$

$$\langle p, q \rangle = \int_{-1}^{1} pq \, dx$$

$$p = a + bx + cx^2$$

$$S^{\perp} = \begin{cases} \langle p, 1+x \rangle \\ \langle p, x^2 \rangle \end{cases}$$

1) $\langle p, 1+x \rangle =$

$$\int^{1}$$

$$= \int_{-1}^{1} (a+bx+cx^2)(1+x)$$

$$= \int_{-1}^{1} a + (a+b)x + (b+c)x^2 + cx^3$$

$$S_1^{\perp} = 2a + (b+c)\frac{2}{3} = 0$$

$\underline{\underline{2}}$

$$\int_{-1}^{1} (a+bx+cx^2)\ (x^2)$$

$$S_2^{\perp} = a\frac{2}{3} + c\frac{2}{5} = 0$$

$$S^{\perp} = \begin{cases} 2a+(b+c)2/3 = 0 & -\ 1 \\ a\frac{2}{3} + c\frac{2}{5} = 0 & -\ 2 \end{cases}$$

$$S^{\perp} = \begin{cases} 3a + b + c = 0 \\ 5a + 3c = 0 \end{cases}$$

$$c = \frac{-5a}{3}$$

$$b = \frac{-4}{3}a$$

$$P = a\left(1, -\frac{4}{3}, -\frac{5}{3}\right)$$

$$S^{\perp} = \text{Span}\left\{\left(1, -\frac{4}{3}, -\frac{5}{3}\right)\right\}$$

Q6

Given

$$I_3 = I_2 + I_1$$

on Left loop

$$-50V + 5I_1 + 20I_3 = 0$$

on Right loop

$$-30V - 20I_3 - 10I_2 = 0$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 5 & 0 & 20 & -50 \\ 0 & 10 & 20 & -30 \end{bmatrix}$$

$$R_3 = R_3 - R_2$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 0 & 20 & -50 \\ 0 & 10 & 0 & 20 \end{bmatrix}$$

$$10 I_2 = 20V$$
$$I_2 = 2V$$
$$I_3 = I_2 + I_1$$

$$5 I_1 + 20 I_3 = 50V$$
$$5 I_1 + 2(2V + I_1) = 5 V$$
$$5 I_1 + 4V + 2 I_1 = 5V$$
$$7 I_1 + 4v = 5V$$
$$I_1 = 1/7 V$$
$$I_2 = 2V$$
$$I_3 = \frac{15}{7} V$$

Q7 Given

→ A is a 2×2 Sym. Mtrx.

→ $\lambda = \{3, -2\}$

To find :-

U, Σ, V^T

U is the set of Eigen values

$\therefore U = [e_1, e_2]$

$$\Sigma = \begin{bmatrix} |\lambda_1| & 0 \\ 0 & |\lambda_2| \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$= V^T = U^T$

$\because$ left and Right Singular vector are the same. $U = V$

Q8

if $A = 4A$

$4A = 4U \Sigma V^T$

$= U(4\Sigma)V^T \Rightarrow \Sigma' = \begin{bmatrix} 12 & 0 \\ 0 & 8 \end{bmatrix}$

$\because$ A is Symmetric

$A^T = A^{-1}$

SVD of A^T is same as of A

$$\Sigma = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

c) SVD of A^{-1}

if A is Invertible then

$U \Sigma V^T \Rightarrow A^{-1} = V \Sigma^{-1} U^T$

$\therefore$ Singular Values of

A^{-1} are $1/3$ and $1/2$

SVD of $A^{-1} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix}$

$V^T = U^T$