Logical AI

Btech CSE 3rd Semester

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Introduction

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Logical AI refers to the application of **formal logic** to represent knowledge and reasoning in artificial intelligence.

The goal is to enable machines to reason and make decisions based on logical representations of knowledge.

Propositional Logic: (also known as **Boolean Logic**) deals with statements that are either **true** or **false**. These statements, called **propositions**, are combined using logical operators.

Operators in Propositional Logic:

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AND (∧): True if both propositions are true.

OR (∨): True if at least one proposition is true.

NOT (¬): Inverts the truth value of a proposition.

I**MPLIES (→)**: True if the first proposition implies the second.

EQUIVALENT (↔): True if both propositions are either both true or both false.

AND (∧) Operator (Conjunction)

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- AND returns true if both propositions are true.
- Expression: $P \land Q$
- Example: Meaning: “It is raining AND the ground is wet.”

Truth Table:

P	Q	PQ
True	True	True
True	False	False
False	True	False
False	False	False

OR (∨) Operator (Disjunction)

- OR returns true if at least one of the propositions is true.
- Expression: $P \vee Q$
- Example: Meaning: “It is raining OR the ground is wet.”

Truth Table:

P	Q	PQ
True	True	True
True	False	True
False	True	True
False	False	False

IMPLIES (→) Operator (Implication)

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IMPLIES is true if the first proposition implies the second. This means P → Q is true in all cases except when P is true and Q is false.

Expression: P → Q

Meaning: “If it is raining, then the ground is wet.”

Truth Table:

P	Q	PQ
True	True	True
True	False	False
False	True	True
False	False	True

EQUIVALENT (↔) Operator (Biconditional)

- EQUIVALENT is true if both propositions are either true or both false.
- Expression: $P \leftrightarrow Q$
- Meaning: “It is raining IF AND ONLY IF the ground is wet.”

Truth Table:

P	Q	PQ
True	True	True
True	False	False
False	True	False
False	False	True

Predicate Logic

Predicate Logic (also called First-Order Logic or FOL) extends propositional logic by dealing with objects and their relationships. It allows for more expressive reasoning by using quantifiers and variables to make statements about collections of objects

Components of Predicate Logic:

- Predicates: Functions that represent relationships or properties (e.g., $P(x)$: “x is a person”).

Quantifiers:

- **Universal Quantifier ($\forall$): Means “for all.”**
- **Existential Quantifier ($\exists$): Means “there exists.”**
- **Variables: Represent objects (e.g., x, y).**

Universal Quantifier (∀)

- The universal quantifier expresses that something is true for all elements in a domain.

Expression: ∀x \ P(x)

Meaning: “For all x, P(x) is true.”

Example: P(x) : “x is mortal.”

The expression ∀x \ P(x) means “All x are mortal.”

Another example:

∀x \ (P(x) → Q(x)) : “For all x, if x is a human, then x is mortal.”

Existential Quantifier (∃)

The existential quantifier expresses that there is at least one element in the domain for which the predicate is true.

Expression: $\exists x \setminus P(x)$

Meaning: "There exists an x such that P(x) is true."

Example:

P(x) : "x is a doctor."The expression $\exists x \setminus P(x)$ means "There exists at least one x who is a doctor."

Another example:

$\exists x \setminus (P(x) \wedge Q(x))$: "There exists at least one x such that x is a person AND x is happy."

Example for Combining Operators in Predicate Logic

Statement:

"All humans are mortal, and there exists a human named Socrates who is mortal."

In Predicate Logic:

H(x) : "x is a human."

M(x) : "x is mortal."

s : Socrates.

The statement can be written as:

1. **Universal Statement: $\forall x\,(H(x) \rightarrow M(x))$ (For all x, if x is human, then x is mortal).**
2. **Existential Statement: $H(s) \wedge M(s)$ (Socrates is human and Socrates is mortal)**