Q. 1 A company wants to compare the **average salaries** of male and female employees. A random sample of **20 male and 20 female employees** is taken, and their salaries are recorded.

Question:

At a 5% significance level, is there a significant difference in the average salaries of male and female employees?

Hint: t test as n<30

Q. 2 A smartphone company claims that the average battery life of its latest model is **12 hours**. A consumer group tests **40 randomly selected phones** and finds an average battery life of **11.5 hours** with a standard deviation of **1.8 hours**.

Question:

At a 5% significance level, can we conclude that the true mean battery life is different from 12 hours?

Hint: z test as n>=30

Q.3 A supermarket believes that **customers are equally likely to shop on any day of the week.** A sample of **700 customers** shows the following distribution:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Customers	90	85	100	120	110	105	90

Question:

At a 5% significance level, can we conclude that customer shopping preferences are **not equally distributed** across the days of the week?

Q.4 A retailer wants to determine whether a **customer's preferred payment method** is related to their **shopping preference** (Online or In-Store). A survey of 300 customers provides the following data:

Payment Method	Online	In-Store	Total
Credit Card	50	70	120

UPI	80	50	130
Cash	40	10	50
Total	170	130	300

The retailer wants to test whether **payment method and shopping preference are independent** at a **5% significance level**.

Q.5 Let X and Y be discrete random variables with the following joint probability mass function (PMF):

$$P(X = x, Y = y) = \begin{cases} \frac{1}{10}, & (x, y) \in \{(1,1), (1,2), (2,1), (2,3), (3,2)\} \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the marginal PMFs $P_X(x)$ and $P_Y(y)$.
(b) Find $P(Y = 2|X = 1)$.
(c) Are X and Y independent? Justify your answer.

Q.6 Suppose the joint probability density function (PDF) of X and Y is given by:

$$f(x, y) = \begin{cases} 6(1 - y), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the marginal density functions $f_X(x)$ and $f_Y(y)$.
(b) Find $P(X > 0.5, Y < 0.5)$.
(c) Are X and Y independent? Explain.

Q.7 Given a joint PDF:

$$f(x, y) = c(x + y), \quad 0 \leq x \leq 1, 0 \leq y \leq 1$$

(a) Find the value of c that makes it a valid probability distribution.
(b) Compute $E[X]$ and $E[Y]$.
(c) Find $E[XY]$.
(d) Compute $\text{Cov}(X, Y)$.
(e) Are X and Y uncorrelated?

Q.8 Suppose X and Y are independent uniform random variables on [0,1], and we define:

$$Z = X + Y, \quad W = X - Y$$

(a) Find the joint distribution of Z and W.

(b) Find $P(Z > 1.5)$.

(c) Compute $E[Z]$ and $E[W]$.

Q.9

Let X and Y follow a bivariate normal distribution with means $\mu_X = 2$, $\mu_Y = 3$, standard deviations $\sigma_X = 1$, $\sigma_Y = 2$, and correlation coefficient $\rho = 0.5$.

(a) Write down the joint PDF of X and Y.

(b) Find $E[X + Y]$.

(c) Find $\mathrm{Var}(X + Y)$.

Q.10 and Q.11

Let X and Y be two random variables with $E[X] = 4$, $E[Y] = 5$, $\mathrm{Var}(X) = 9$, $\mathrm{Var}(Y) = 16$, and $\mathrm{Cov}(X, Y) = 6$.

(a) Compute the correlation coefficient $\rho(X, Y)$.

(b) Interpret the result in terms of the strength and direction of the linear relationship between X and Y.

Suppose X and Y have a covariance of $\mathrm{Cov}(X, Y) = -10$. If $a = 2$ and $b = -3$, find $\mathrm{Cov}(aX + bY, X)$ using the property:

$$\mathrm{Cov}(aX + bY, Z) = a\mathrm{Cov}(X, Z) + b\mathrm{Cov}(Y, Z)$$

Q. 12 Consider two random variables X and Y with joint PMF given in Table

- Find P(X=0,Y≤1)
- Find the marginal PMFs of X and Y
- Find P(Y=1|X=0)
- Are X and Y independent?

	$Y=0$	$Y=1$	$Y=2$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
$X=1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

Q. 13

Let X and Y be two independent $N(0,1)$ random variables. Define:

$$Z = 2X + 3Y^2$$

$$W = X + Y$$

Find $\mathrm{Cov}(Z, W)$.

Q. 14 A box contains **5 red**, **4 blue**, and **3 green** balls. Two balls are drawn at random one after another **without replacement**.

What is the probability that the second ball drawn is blue, given that the first ball drawn was red?

Q. 15 A factory produces **60%** of its items from **Machine A** and **40%** from **Machine B**.

The defect rate is **3%** for Machine A and **5%** for Machine B.

If a randomly selected item is found to be defective, what is the probability that it was produced by **Machine B**?

Q. 16 Let X and Y be two discrete random variables with joint probability distribution given by:

X	**Y**	**P(X,Y)**
0	0	0.25

0	1	0.25
1	0	0.25
1	1	0.25

Are X and Y independent? Justify your answer using the definition of independence.