Probability: Understanding the Odds

Probability is a branch of mathematics that deals with the likelihood of events occurring. It helps us understand the chances of something happening, from coin tosses to complex scientific experiments.

NC

Probability Basics

Definition

Probability is a measure of the likelihood of an event to occur. Many events cannot be predicted with total certainty. We can predict only the chance of an event to occur i.e., how likely they are going to happen, using it.

Probability can range from 0 to 1, where 0 means the event to be an impossible one and 1 indicates a certain event. Probability for Class 10 is an important topic for the students which explains all the basic concepts of this topic.

The probability of all the events in a sample space adds up to 1.

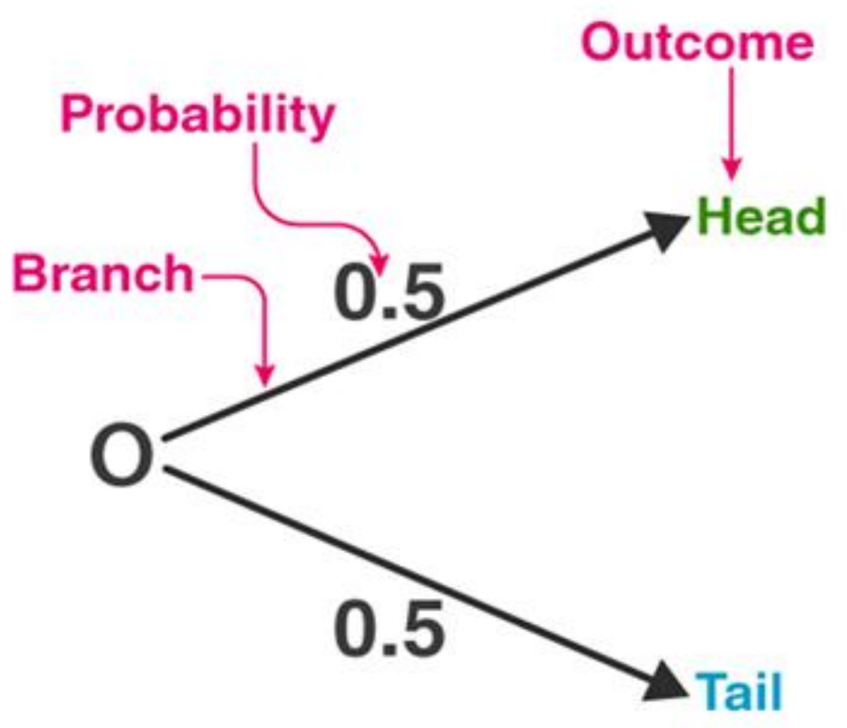
For example, when we toss a coin, either we get Head OR Tail, only two possible outcomes are possible (H, T).

But when two coins are tossed then there will be four possible outcomes, i.e. {(H, H), (H, T), (T, H), (T, T)}.

Formula

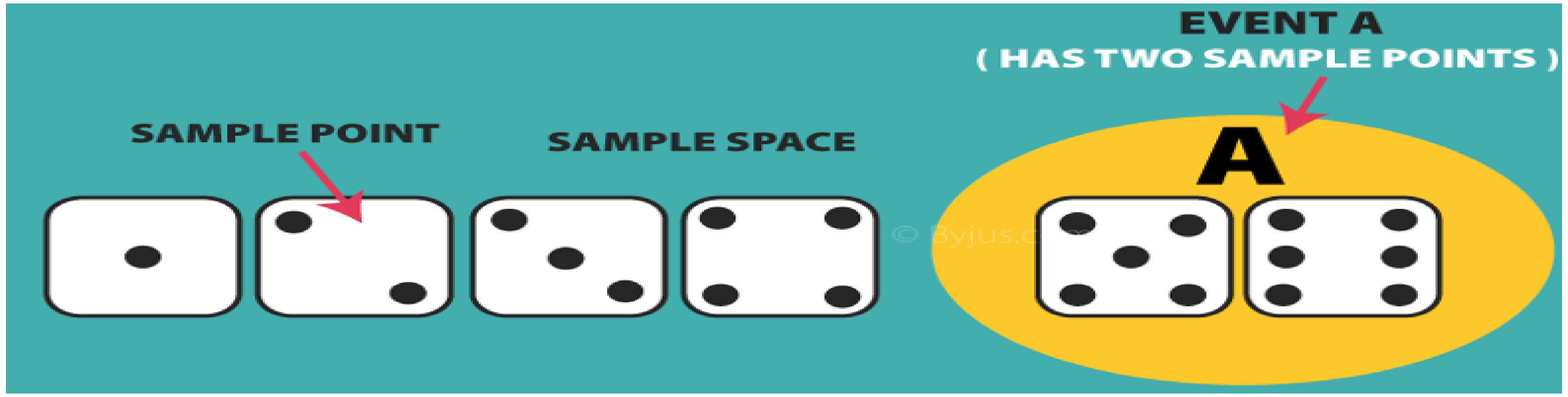
Probability of an event = Number of favorable outcomes / Total number of outcomes.

Probability Tree



Probability Terms and Definition

Term	Definition	Example
Sample Space	The set of all the possible outcomes to occur in any trial	1. Tossing a coin, Sample Space (S) = {H,T} 2. Rolling a die, Sample Space (S) = {1,2,3,4,5,6}
Sample Point	It is one of the possible results	In a deck of Cards: (4 of hearts is a sample point.) (The queen of clubs is a sample point.)
Experiment or Trial	A series of actions where the outcomes are always uncertain.	The tossing of a coin, Selecting a card from a deck of cards, throwing a dice.
Event	It is a single outcome of an experiment.	Getting a Heads while tossing a coin is an event.
Outcome	Possible result of a trial/experiment	T (tail) is a possible outcome when a coin is tossed.
Complimentary event	The non-happening events. The complement of an event A is the event, not A (or A’)	In a standard 52-card deck, A = Draw a heart, then A’ = Don’t draw a heart
Impossible Event	The event cannot happen	In tossing a coin, impossible to get both head and tail at the same time



Finite Sample Spaces

Definition: A sample space with a finite number of outcomes.

Examples:

Tossing a coin: $S=\{H,T\}$

Rolling a die: $S=\{1,2,3,4,5,6\}$

Probability of an Event: If all outcomes are equally likely, the probability of an event E is:

$P(E)=\text{Number of favorable outcomes}/\text{Total number of outcomes}$

Properties:

$0 \le P(E) \le 1$

$P(S)=1$

$P(\emptyset)=0$

Types of Probability

Theoretical

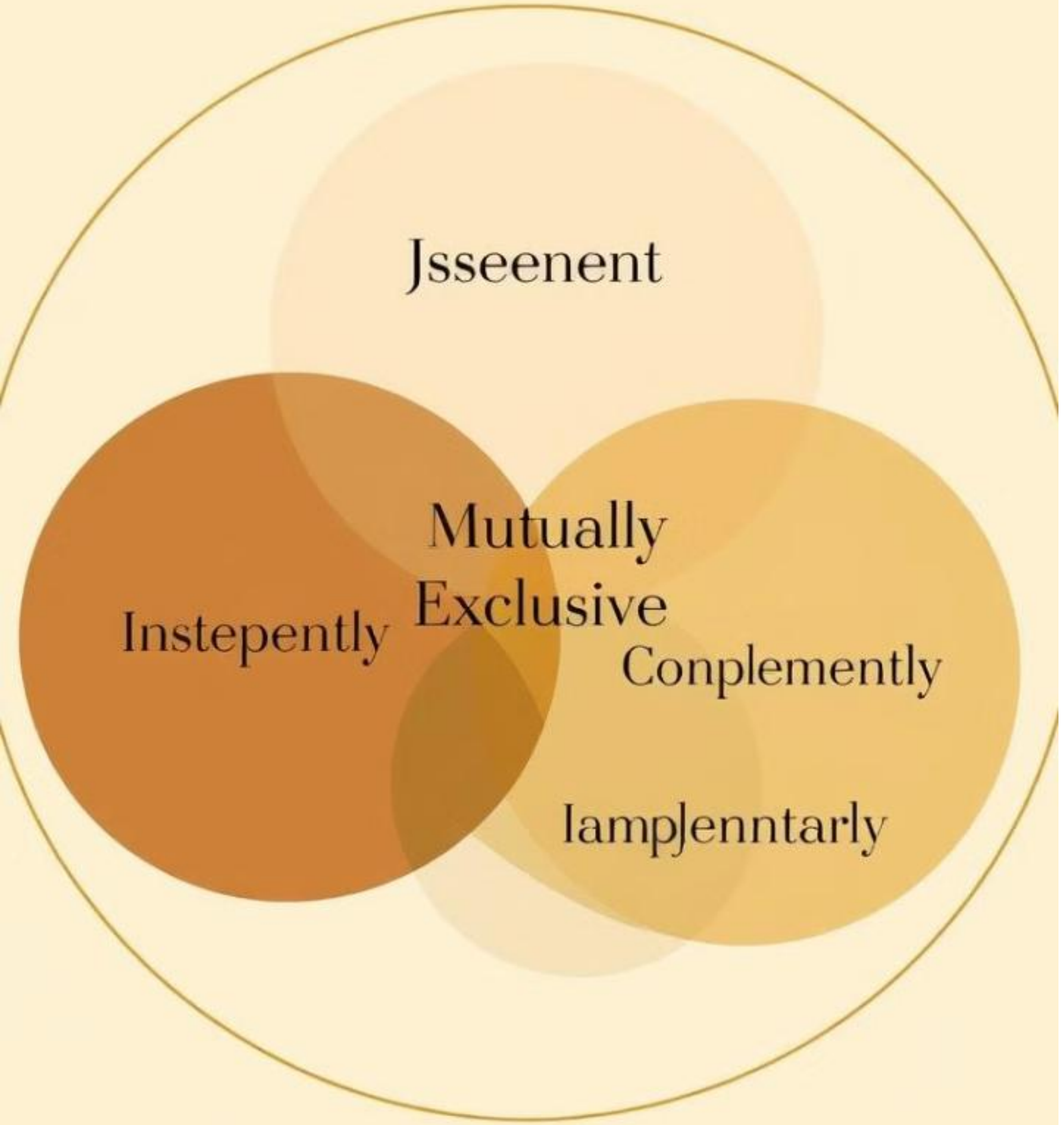
Based on reasoning and possible chances, like the probability of getting heads when flipping a coin is 1/2.

Experimental

Based on observations from experiments, like the probability of getting heads after flipping a coin 10 times and getting heads 6 times is 6/10.

Axiomatic

Based on a set of rules or axioms that apply to all types of probability.



Understanding Events

1

Equally Likely Events

Events with the same probability of occurring, like getting any number on a fair die.

2

Complementary Events

Two outcomes where one event occurring means the other cannot, like winning or losing a lottery.

3

Independent Events

Events where the occurrence of one does not affect the probability of the other, like flipping a coin twice.

4

Mutually Exclusive Events

Events that cannot occur at the same time, like getting heads and tails on a single coin toss.

Probability of Disjoint (or) Mutually Exclusive Event = P (A and B) = 0

How to Find Mutually Exclusive Events?

In probability, the specific addition rule is valid when two events are mutually exclusive. It states that the probability of either event occurring is the sum of probabilities of each event occurring. If A and B are said to be mutually exclusive events then the probability of an event A occurring or the probability of event B occurring that is P (a ∪ b) formula is given by P(A) + P(B), i.e.,

P (A Or B) = P(A) + P(B)

P (A ∪ B) = P(A) + P(B)

If the events A and B are not mutually exclusive, the probability of getting A or B that is P (A ∪ B) formula is given as follows:

P (A ∪ B) = P(A) + P(B) – P (A and B)

Real-life Examples on Mutually Exclusive Events

Some of the examples of the mutually exclusive events are:

- When tossing a coin, the event of getting head and tail are mutually exclusive. Because the probability of getting head and tail simultaneously is 0.
- In a six-sided die, the events "2" and "5" are mutually exclusive. We cannot get both the events 2 and 5 at the same time when we threw one die.
- In a deck of 52 cards, drawing a red card and drawing a club are mutually exclusive events because all the clubs are black.

1. D be the event that outcome is less than 5

   D={1,2,3,4}

   P(D)=4/6=2/3

2. E be the event that outcome is greater than or equal to 3.

   E={3,4,5,6}

   P(E)= 4/6= 2/3

3. F be the event that outcome is less than 8.

   F={1,2,3,4,5,6} = Ω (Sure event)

   P(F)=1

4. G be the event that outcome is greater than 7

   G={ } => Empty set = ∅

   (Impossible event)

   P(G)=0

{P(Ac) A complement= 1- P(A)}

5. A be the event that outcome is divisible by 3

   A={3,6}

   P(A)=2/6=1/3

   Ac(A complement)={1,2,4,5}

   P(Ac)= 2/6=> 1/3 or (6-4)/6 or 1-(2/3)

Set Theory

1. B be the event that outcome is even number of dice

B= {2,4,6}

2. C be the event that outcome is odd number of dice

C={1,3,5}

BUC = {1,2,3,4,5,6}

P (BUC)=?

P(BUC)=1

B Intersection C={ } i.e. Phie
And P(B Intersection C)=0

If intersection of two is phie,It shows that B and C are mutually exclusive events.

Question 1: Find the probability of ‘getting 3 on rolling a die’.

Solution:

Sample Space = S = {1, 2, 3, 4, 5, 6}

Total number of outcomes = n(S) = 6

Let A be the event of getting 3.

Number of favorable outcomes = n(A) = 1 i.e. A = {3}

Probability, P(A) = n(A)/n(S) = 1/6

Hence, P(getting 3 on rolling a die) = 1/6

2) There are 6 pillows in a bed, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow pillow?

Ans: The probability is equal to the number of yellow pillows in the bed divided by the total number of pillows, i.e. 2/6 = 1/3.

•Deck: A pack of playing cards containing 52 cards & 2 jokers.

•Pack: A deck of playing cards, usually 52 cards plus two Jokers.

•Picture cards: Kings, Queens, and Jacks, also called "court cards" or "face cards."

•Pip: The large suit symbols on a card (Spade, Club, Heart, or Diamond).

•Pip value: The numerical value of a card.

•Suit: The 4 suits or tikkas in a deck, i.e., Clubs, Hearts, Spades, and Diamonds.

Question 2: Draw a random card from a pack of cards. What is the probability that the card drawn is a face card?

A standard deck has 52 cards. Total number of outcomes = n(S) = 52

Let E be the event of drawing a face card. Number of favorable events = n(E) = 4 x 3 = 12 (considered Jack, Queen and King only)

Probability, P = Number of Favorable Outcomes/Total Number of Outcomes

P(E) = n(E)/n(S) = 12/52= 3/13

P(the card drawn is a face card) = 3/13

Question 1: What is the probability of a die showing a number 3 or number 5?

Solution: Let,

P(3) is the probability of getting a number 3

P(5) is the probability of getting a number 5

P(3) = 1/6 and P(5) = 1/6

So,

P(3 or 5) = P(3) + P(5)

P(3 or 5) = (1/6) + (1/6) = 2/6

P(3 or 5) = 1/3

Therefore, the probability of a die showing 3 or 5 is 1/3.

Question 2:

Three coins are tossed at the same time. We say A as the event of receiving at least 2 heads. Likewise, B denotes the event of getting no heads and C is the event of getting heads on the second coin. Which of these is mutually exclusive?

Solution:

Firstly, let us create a sample space for each event. For the event 'A' we have to get at least two head. Therefore, we have to include all the events that have two or more heads.

Or we can write:

A = {HHT, HTH, THH, HHH}.

This set A has 4 elements or events in it i.e. n(A) = 4

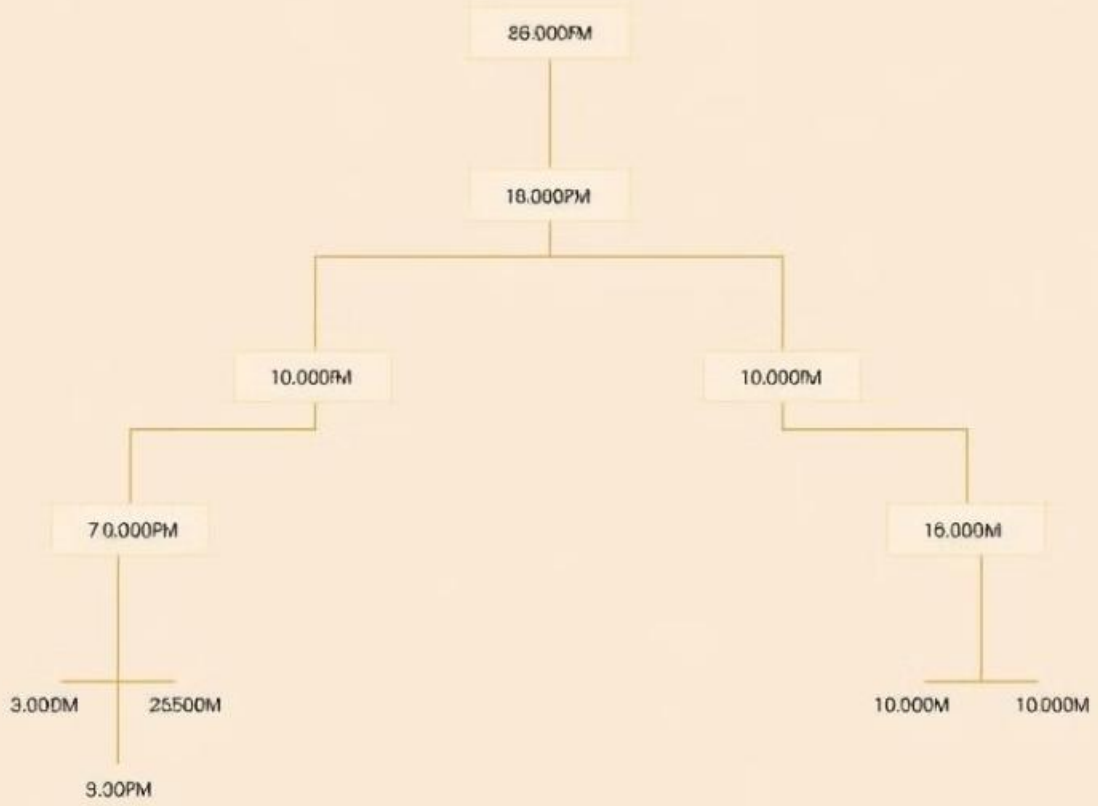
In the same way, for event B, we can write the sample as:

B = {TTT} and n(B) = 1

Again using the same logic, we can write;

C = {THT, HHH, HHT, THH} and n(C) = 4

So B & C and A & B are mutually exclusive since they have nothing in their intersection.



Conditional Probability

Conditional probability is the likelihood of an event occurring given that another event has already happened. It's like saying, "What's the probability of rain today, given that it rained yesterday?"

A company has two types of employees: 60% are full-time employees, and 40% are part-time employees. Among full-time employees, 70% have health insurance. Among part-time employees, only 30% have health insurance.
If a randomly selected employee has health insurance, what is the probability that they are a full-time employee?

Solution:

We are looking for $P(\text{Full-Time}|\text{Health Insurance})$, the probability that an employee is full-time given that they have health insurance. Using the formula for conditional probability:

$$P(\text{Full-Time}|\text{Health Insurance}) = \frac{P(\text{Full-Time and Health Insurance})}{P(\text{Health Insurance})}$$

Step 1: Calculate $P(\text{Full-Time and Health Insurance})$

$$P(\text{Full-Time and Health Insurance}) = P(\text{Full-Time}) \times P(\text{Health Insurance}|\text{Full-Time})$$

$$P(\text{Full-Time and Health Insurance}) = 0.6 \times 0.7 = 0.42$$

Step 2: Calculate $P(\text{Part-Time and Health Insurance})$

$$P(\text{Part-Time and Health Insurance}) = P(\text{Part-Time}) \times P(\text{Health Insurance}|\text{Part-Time})$$

$$P(\text{Part-Time and Health Insurance}) = 0.4 \times 0.3 = 0.12$$

Step 3: Calculate $P(\text{Health Insurance})$

$$P(\text{Health Insurance}) = P(\text{Full-Time and Health Insurance}) + P(\text{Part-Time and Health Insurance})$$

$$P(\text{Health Insurance}) = 0.42 + 0.12 = 0.54$$

Step 4: Calculate $P(\text{Full-Time}|\text{Health Insurance})$

$$P(\text{Full-Time}|\text{Health Insurance}) = \frac{P(\text{Full-Time and Health Insurance})}{P(\text{Health Insurance})}$$

$$P(\text{Full-Time}|\text{Health Insurance}) = \frac{0.42}{0.54} \approx 0.778$$

The probability that an employee is full-time given that they have health insurance is approximately **0.778** or **77.8%**.

Probability in Real Life

Probability is used in many real-life situations, from insurance and finance to weather forecasting and medical research. It helps us make informed decisions and understand the risks involved.

Probability in Everyday Decisions

Weather Forecasts

Probability is used to predict the likelihood of rain, snow, or other weather events.

Games of Chance

Probability helps us understand the odds of winning or losing in games like lottery, poker, or roulette.

Medical Research

Probability is used to analyze clinical trials and determine the effectiveness of treatments.

Probability in Business

1

Market Analysis

Probability helps businesses predict market trends and make informed decisions about investments.

2

Risk Management

Probability is used to assess and manage risks associated with business operations.

3

Product Development

Probability helps businesses evaluate the success rate of new products and services.

Key Takeaways

Probability is a powerful tool for understanding the world around us. It helps us make informed decisions, manage risks, and explore the possibilities of the future.