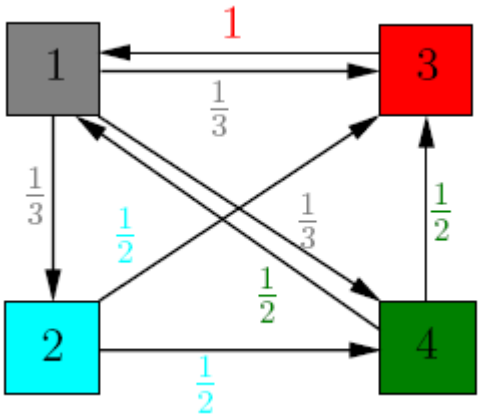
Name: Enrolment No:	

UPES

Assignment I

Programme Name : B.Tech. (SoCS) **Semester : IV**
Course Name : Linear Algebra
Course Code : MATH 2059 **Max. Marks: 10**
Nos. of page(s) : 03

S. No.		Marks	CO
Q 1	The *google page rank* vector is the solution of $Av = v$ where $A = [a_{ij}]_{n\times n}$ is the transition matrix defined as $a_{ij} = p$, where p is the probability of landing on j^{th} website from i^{th} website and $v = (x_1, x_2, x_3, \dots, x_n)^T$ consisting of the $x_i's$ (called as the importance value of the i^{th} website). In any google search, the i^{th} website is listed above the j^{th} website iff $x_i > x_j$.  Consider the above directed graph depicting a situation with 4 nodes, one for each website, where if website *i* references *j*, we add a directed edge between node *i* and node *j* in the graph. The weight on each edge is the value of p. Construct the *transition matrix A* of order 4×4.	1	CO1
Q 2	Ms. Rosy bought 2 pencils, 4 pens, and 2 notebooks that cost Rs. 44 and 1 pencil, 3 pens, and 1 notebook that cost Rs. 27. Describe this problem using a system of linear equations. Hence, a) What price did Rosy pay for each pen?	1	CO1

	b) Is she correct in anticipating the price of each notebook to be Rs. 15? c) In addition to this, if she wishes to spend Rs 50 to buy 3 pencils, 7 pens, and 3notebooks, then will she be able to execute this transaction?		
Q 3	Consider the chemical equation of photosynthesis: $$\alpha\ H_2O + \beta\ CO_2 \longrightarrow \gamma\ C_6H_{12}O_6 + \delta\ O_2$$ where $\alpha, \beta, \gamma, \delta \in \mathbb{R}$. Formulate a homogeneous system of linear equations to balance the above equation. Hence, obtain the values of α, β, γ and δ.	**1**	**CO1**
Q 4	Digitizing the alphabets *A-Z* by numbers 0-25 and digits 0-9 by numbers 26-35. If the number 36 is reserved for blanks between two consecutive words, then the set $S = \{0,1,2,\dots,36\}$ completely describes the character set of English language when only numerals are used as a privilege. Consider a text message "EMPTY SET" which Bob wants to send to Alice. a) Express this linguistic message using a 3×3 matrix M with entries from set S. b) Obtain the *Encrypted matrix* N for the above text matrix M by adding to it the key matrix corresponding to the secret key "LET ME IN" where the addition operation on S is defined as: $a + b = c$ where c is the remainder obtained on dividing $a + b$ by 37.	**1**	**CO1**
Q 5	A real number λ is an eigenvalue of matrix $X = [x_{ij}]_{n\times n}$ if $$\sum_{i=1}^{n} x_{ij} = \lambda, \quad \forall\, j = 1,2,\dots,n$$ Using the above fact, determine whether $\lambda = 1$ is an eigenvalue of *A* obtained in problem 1? Also, compute the *PageRank vector* for the *transition matrix A* and hence, rank the four websites.	**2**	**CO3**
Q 6	Let $P = \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ a & 2 & b \end{bmatrix}$ for some $a, b \in \mathbb{R}$. Suppose that 1 and 2 are eigenvalues of P and $P\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$. Find $P^4\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.	**1**	**CO3**

Q 7	Suppose $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a real matrix with $a_{12} \neq 0, a_{21} \neq 0$. Prove that a) If A has repeated eigenvalues, then $\det(A)$ is non-negative. b) If a_{12} and a_{21} have same sign, then A has real and distinct eigenvalues. Is the converse also true? Give a suitable reason or a counterexample to support your answer.	**2**	**CO3**
Q 8	Suppose S and T are subsets of $\mathbb{R}$ such that $b \in S, c \in T$ and $A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}$ for some $a \in \mathbb{R}$ has eigenvalues not in $\mathbb{R}$. Determine the subsets S and T completely and prove that $S \cap T = \emptyset$.	**1**	**CO3**