Program	B. Tech. (SoCS)	Semester	IV
Course	Linear Algebra	Course Code	MATH 2059
Session	Jan-May 2025	Topic(s)	Rank and System of Linear equations

1. Reduce the following matrices to row echelon form and find their ranks:

a) $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 9 \\ 1 & 5 & 12 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 5 \\ 3 & 6 & 3 & -7 & 7 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 4 & 2 & -2 & 5 & 1 \\ 3 & 6 & 2 & 2 & 0 & 4 \\ 4 & 8 & 2 & 6 & -5 & 7 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 4 & 3 & 5 & 5 & 7 \\ 3 & 6 & 4 & 9 & 10 & 11 \\ 1 & 2 & 4 & 3 & 6 & 9 \end{pmatrix}$

e) $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 3 & 8 & 12 \\ 0 & 0 & 4 & 6 \\ 0 & 2 & 7 & 10 \end{pmatrix}$

f) $\begin{pmatrix} 1 & 3 & 1 & 3 \\ 2 & 8 & 5 & 10 \\ 1 & 7 & 7 & 11 \\ 3 & 11 & 7 & 15 \end{pmatrix}$

2. Solve the following non-homogeneous system of linear equations:

a) $x + 2y - 3z = 4,\ 3y - 4z + 7x = 5,\ 6z + 8x - 9y = 1$

b) $x + 2y - z = 3,\ x + 3y + z = 5,\ 3x + 8y + 4z = 17$

c) $x - 2y + 4z = 2,\ 2x - 3y + 5z = 3,\ 3x - 4y + 6z = 7$

d) $x + y + 3z = 1,\ 2x + 3y - z = 3,\ 5x + 7y + z = 7$

3. Consider the system:

$$x + 2y + z = 3, \qquad ay + 5z = 10, \qquad 2x + 7y + az = b$$

where $a, b \in \mathbb{R}$. Find possible values of a & b for which the system has a unique solution.

4. Determine the possible real values of k for which the system of equations:

$$x + y + z = 1, \qquad 2x + 3y - z = 5, \qquad x + 2y - kz = 4$$

has more than one solution.

5. Let A be a 5×4 matrix with real entries such that $Ax = 0$ if and only if $x = 0$ where x is a 4×1 vector and 0 is a null vector. Then, what can you say about the rank of A?

6. For what values of $a, b, c \in \mathbb{R}$, the following linear system of equations:

$$x + y + z = 1, \qquad ax + by + cz = 1, \qquad a^2x + b^2y + c^2z = 1$$

admits a unique solution.

7. For what real value of k the following system of equations:

$$x + ky + 3z = 0, \qquad 3x + ky - 2z = 0, \qquad 2x + 3y - 4z = 0$$

possess a non-trivial solution. For that value(s) of k, determine all the solutions of the given system.

8. Find the value(s) of k for which the rank of the matrix

$$\begin{pmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{pmatrix}$$ is 3.

9. Find the values of a & b if the matrix $\begin{pmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & 2 & 4 \\ 7 & -1 & a & b \end{pmatrix}$ is of rank 2.

10. Determine the real values of k for which the system of equations

$$x - ky + z = 0, \qquad kx + 3y - kz = 0, \qquad 3x + y - z = 0$$

has *(i)* only trivial solution, *(ii)* non-trivial solution.

11. For what values of $\lambda, \mu \in \mathbb{R}$, the following system of equations:

$$x + 2y + z = 6, \qquad x + 4y + 3z = 10, \qquad x + 4y + \lambda z = \mu$$

admits

a) a unique solution,
b) infinite number of solutions
c) no solution.