Hypothesis testing:

Hypothesis testing is an inferential procedure that uses data from a sample to draw a general conclusion about a population. It is a formal approach and a statistical method that uses sample data to evaluate hypotheses about a population.

When interpreting a research question and statistical results, a natural question arises as to whether the finding could have occurred by chance. Hypothesis testing is a statistical procedure for testing whether chance (random events) is a reasonable explanation of an experimental finding.

Hypothesis testing is an act in statistics whereby an analyst tests an assumption regarding a population parameter. The methodology employed by the analyst depends on the nature of the data used and the reason for the analysis.

Hypothesis testing involves collecting sample data, calculating test statistics, and determining the probability of observing such results if the null hypothesis is true. Based on this probability, we can decide whether to reject the null hypothesis in favor of the alternative or fail to reject it.

Null hypothesis:

In general, the null hypothesis, written H0 ("H-naught"), is the idea that nothing is going on: there is no effect of our treatment, no relation between our variables, and no difference in our sample mean from what we expected about the population mean. The null hypothesis indicates that an apparent effect is due to chance. **This is always our baseline starting assumption, and it is what we (typically) seek to reject.**

Alternative hypothesis:

If the null hypothesis is rejected, then we will need some other explanation, which we call the alternative hypothesis, Ha or H1. The alternative hypothesis is simply the reverse of the null hypothesis, and there are three options, depending on where we expect the difference to lie. We will set the criteria for rejecting the null hypothesis based on the directionality (greater than, less than, or not equal to) of the alternative. **It represents the claim or the new understanding that the researcher wants to prove.**

For example, suppose you want to determine if a new teaching method improves student test scores. You might form the following hypotheses:

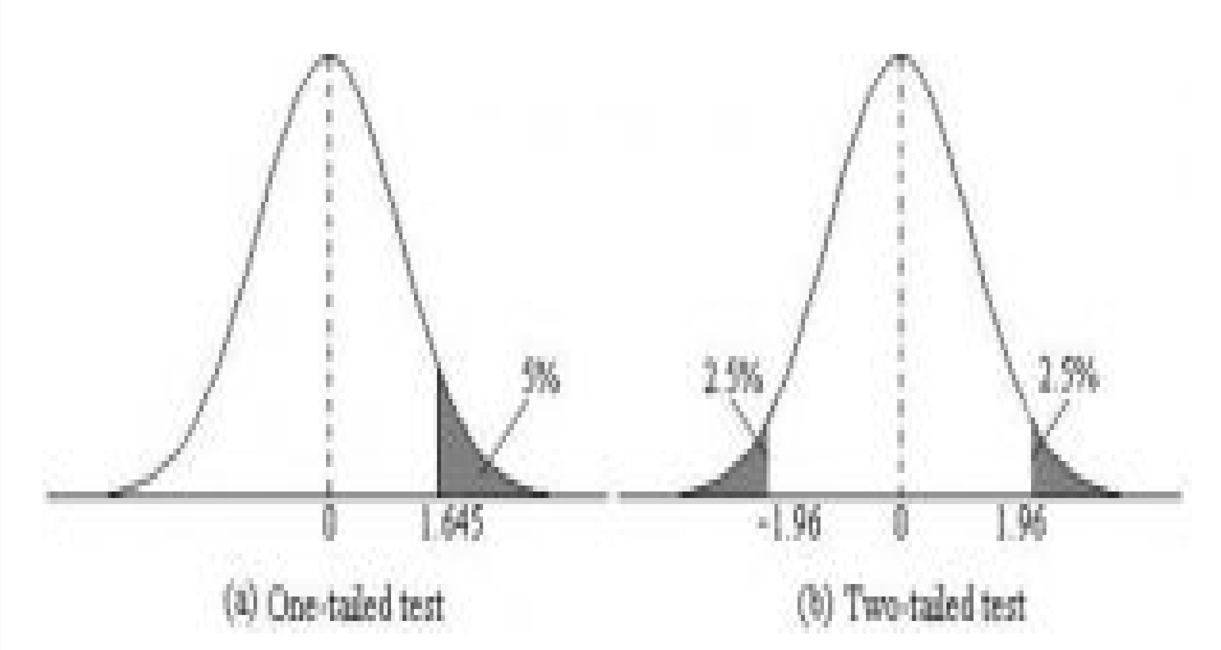
- **Null hypothesis (H_0):** The new teaching method has no effect on student test scores.

- **Alternative hypothesis (H_1):** The new teaching method improves student test scores.

The specific type of hypothesis testing reviewed is specifically known as null hypothesis statistical testing (NHST). We can break the process of null hypothesis testing down into a number of steps:

1. Formulate a hypothesis that embodies our prediction (before seeing the data)
2. Specify null and alternative hypotheses
3. Collect some data relevant to the hypothesis
4. Compute a test statistic
5. Identify the criteria probability (or compute the probability of the observed value of that statistic) assuming that the null hypothesis is true
6. Drawing conclusions. Assess the "statistical significance" of the result

1- tailed and 2-tailed test



The main difference between one-tailed and two-tailed tests is that one-tailed tests will only have one critical region whereas two-tailed tests will have two critical regions. If we require a $100(1-\alpha)$ confidence interval we have to make some adjustments when using a two-tailed test.

The confidence interval must remain a constant size, so if we are performing a two-tailed test, as there are twice as many critical regions then these critical regions must be half the size. This means that when we read the tables, when performing a two-tailed test, we need to consider $\alpha/2$ rather than α.

A one-tailed test may be either left-tailed or right-tailed.

A ***left-tailed*** **test** is used when the alternative hypothesis states that the true value of the parameter specified in the null hypothesis is less than the null hypothesis claims.

A ***right-tailed*** **test** is used when the alternative hypothesis states that the true value of the parameter specified in the null hypothesis is greater than the null hypothesis claims z-score for 5% and 1%, we can identify the critical regions for the critical rejection areas from the unit standard normal table.

A two-tailed test at the 5% level has a critical boundary

Z score of +1.96 and -1.96

A one-tailed test at the 5% level has a critical boundary

Z score of +1.64 or -1.64

A two-tailed test at the 1% level has a critical boundary

Z score of +2.58 and -2.58

A one-tailed test at the 1% level has a critical boundary

Z score of +2.33 or -2.33.

Type I error :

We can reject HO when it is actually true (we call this a false alarm, or Type I error), Type I error means that we have concluded that there is a relationship in the population when in fact there is not. Type I errors occur because even when there is no relationship in the population, sampling error alone will occasionally produce an extreme result.

Type II error:

We can retain HO when it is actually false (we call this a miss, or Type II error). Type II error means that we have concluded that there is no relationship in the population when in fact there is.

Type I and Type II error

The decision is **not to reject *H0*** when ***H0* is true (correct decision).**

The decision is to **reject *H0*** when ***H0* is true** (incorrect decision known as a **Type I error**).

The decision is **not to reject *H0*** when, in fact, ***H0* is false** (incorrect decision known as a **Type II error**).

The decision is to **reject *H0*** when ***H0* is false** (**correct decision**).

Table:
The four possible outcomes in hypothesis testing.

ACTION	*H0* IS ACTUALLY	
	True	False
Do not reject *H0*	Correct Outcome	**Type II error**
Reject *H0*	**Type I Error**	Correct Outcome

One-Sample Z Test

A one-sample z test is used to check if there is a difference between the sample mean and the population mean when the population standard deviation is known. The formula for the z test statistic is given as follows:

$$Z = \frac{(\overline{x}-\mu)}{(\sigma/\sqrt{n})}$$

where

- $\bar{x}$: mean of the sample.
- μ : mean of the population.
- σ : Standard deviation of the population.
- n: sample size.

A low probability value casts doubt on the null hypothesis. How low must the probability value be to conclude that the null hypothesis is false?

Although there is clearly no right or wrong answer to this question, it is conventional to conclude the null hypothesis is false if the probability value is less than 0.05 ($p < .05$).

More conservative researchers conclude the null hypothesis is false only if the probability value is less than 0.01 ($p<.01$).

When a researcher concludes that the null hypothesis is false, the researcher is said to have rejected the null hypothesis.

The probability value below which the null hypothesis is rejected is called the α level or simply α ("alpha"). It is also called the significance level. If α is not explicitly specified, assume that $\alpha = 0.05$.

Example based on Z-Score

Suppose a company claims that their new smartphone has an average battery life of 12 hours. A consumer group tests 100 phones and finds an average battery life of 11.8 hours with a known population standard deviation of 0.5 hours.

Step 1: Hypotheses:

H_0: ? $H_0: \mu=12$

H_1:? $H_1: \mu\neq12$

Assume critical value for $\alpha=0.05$

Rejection Region for $\alpha = 0.05$, $z^* = 1.96$

Step2: Calculate the Z-Score:

we can calculate Z-score using the formula:

$$z = \frac{x-\mu}{\frac{\sigma}{\sqrt{n}}}$$

where $\bar{x}=11.8$, $\mu=12$, $\sigma=0.5$ and $n=100$ after putting the value we get:

$$z = \frac{11.8-12}{\frac{0.5}{\sqrt{100}}} = -4$$

Step3: Decision

Since $|Z|=4>1.96$

we reject H_0 indicate significant evidence against the company's claim.

Example-2

1) The average heights of all residents in a city is 168cm with a population std $\sigma = 3.9$. A doctor believe the mean to be differant. He measured the height of 36 individuals and found the average to be 169.5cm

(a) State Null And Alternate Hypothesis

(b) At a 95% C.I, is there enough evidence to Reject the Null Hypothes

Answer:

μ= 168 cm., σ=3.9 , n=36, $\bar{x}$= 169.5 cm

a) Null hypothesis Ho : μ= 168 cm.

b) Alternate hypothesis H_1: $\mu \neq$ 168 cm. { two tail test}

c) C.I.= 0.95 => 95% ; α= 1-C.I. => 1-0.95=0.05

d) Z test: $Z = \frac{(\bar{x}-\mu)}{(\sigma/\sqrt{n})}$

=>(169.5- 168)/ 3.9/ √36

$\Rightarrow$2.31

$\Rightarrow$2.31> 1.96 i.e. We reject the null hypothesis.

Example-2

A random sample of 50 items gives the mean 6.2 and variance 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance?

Answer:

$n=50$; $\bar{x} = 6.2$; and $\sigma = 10.24$

Null hypothesis (Ho) : ?

Alternate hypothesis (H1): ?

Null hypothesis (Ho) : $\mu=5.4$

Alternate hypothesis (H1): $\mu \neq 5.4$

Test Statistics:

$Z = |(\bar{x} - \mu)/(\sigma/\sqrt{n})|$

$= |(6.2-5.4)/(10.24/\sqrt{50})|$

$=1.77$

Critical value: $\alpha = 0.05 \Rightarrow z=1.96$

$1.77 < 1.96$ i.e. null hypothesis is accepted.

t-test

A t-test is a statistical test used to determine whether there is a significant difference between the means of two groups or between a sample mean and a known value. It is particularly useful when dealing with small sample sizes or when the population standard deviation is unknown.

The t-test statistic for a one sample t-test is calculated using the formula:

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

where:

X¯ is the sample mean

µ is the population mean (or the mean of the comparison group)

s is the sample standard deviation, and

n is the sample size.

Type of t-test:

- One-Sample t-test
- Independent Two-Sample t-test
- Paired t-test

One-Sample t-test: This test compares the mean of a single sample to a known value or population mean. It determines if the sample mean significantly deviates from a specific benchmark.

For example, we can use a one-sample t-test to evaluate whether the average test score of a small class differs from the national average.

- **Independent Two-Sample t-test**: This test compares the means of two independent groups to determine if there is a statistically significant difference between them. It is commonly used in experiments where two groups undergo different treatments or conditions.

For instance, we could use an independent two-sample t-test to compare test scores between students taught using two different teaching methods to see if one method is more effective.

- **Paired t-test**: This test compares means from the same group at different times or under different conditions. It evaluates whether there is a significant change within the same group after an intervention or over time.

An example is measuring student performance before and after implementing a new teaching strategy to assess its impact.

Degrees of Freedom

The *degrees of freedom* refer to the number of independent observations in a set of data. From a single sample of size n, the number of independent values is n−1.

Numbers of observations in the data that are free to vary when we attempt to estimate statistical parameters.

Consider the five person who are free to pick numbers and sum of these number must be 100.

A+B+C+D+E+F=100

20+30+10+15+?=100

Consider the following data set:
75,79,56,81,66,58,77,61,71,76

The mean is
(75+79+56+81+66+58+77+61+71+76)/10=70

Degrees of Freedom…

To calculate the standard deviation, a little care is needed. The deviations of each number are as follows:

$$75-70=5$$
$$79-70=9$$
$$56-70=-14$$
$$81-70=11$$
$$66-70=-4$$
$$58-70=-12$$
$$77-70=7$$
$$61-70=-9$$
$$71-70=1$$
$$76-70=6$$

But these are not independent, they have to add up to 0 because of how the mean is calculated, i.e. the last deviation is dependent on the other 9. So only 9 out of the 10 deviations are independent. In general, $n-1$ out of n deviations will be independent hence we have $n-1$ degrees of freedom.

Key Differences Between t-tests and Z-tests

Sample size considerations

- **t-test**: The t-test is typically used when the sample size is small, generally less than 30. It is designed to be robust when the sample size does not meet the threshold needed for applying the Central Limit Theorem.
- **Z-test**: The Z-test is used when the sample size is large, typically greater than 30. In large samples, the sampling distribution of the mean is approximately normal, which justifies using the Z-test.

Key Differences Between t-tests and Z-tests

Population variance knowledge

- **t-test**: The t-test is used when the population variance is unknown. Instead of the population variance, the sample variance is used to calculate the test statistic. The t-distribution, which has heavier tails than the normal distribution, accounts for the additional uncertainty due to estimating the population variance.
- **Z-test**: The Z-test requires that the population variance is known. This is a key assumption because it allows the use of the standard normal distribution to calculate the test statistic. When the population variance is known, the Z-test provides more precise estimates.

Key Differences Between t-tests and Z-tests

Distribution assumptions

t-test: The t-test assumes that the data within each group are approximately normally distributed. This is particularly important when dealing with small sample sizes. The test statistic in a t-test follows a t-distribution, which has wider tails than the normal distribution. This accounts for the additional variability and uncertainty when estimating the population standard deviation from a small sample.

Z-test: The Z-test assumes that the data are normally distributed or that the sample size is large enough to apply for the Central Limit Theorem. The Central Limit Theorem ensures that, for large samples, the sampling distribution of the mean is approximately normal, even if the underlying data are not perfectly normal.

Key Differences Between t-tests and Z-tests

Practical applications and use cases

- **t-test**: The t-test is commonly used in small-sample studies, such as pilot studies, where the population variance is unknown. Examples include comparing the effectiveness of two treatments in a small group or assessing changes within the same group over time.

- **Z-test**: The Z-test is used in large-sample studies or when dealing with well-established populations where the variance is known. It is often applied in quality control, survey analysis, and large-scale experimental studies.

When to use t-test vs z-test

Do you know the population standard σ

If, no , apply t-test and

if, yes, check sample size, if sample size above 30 apply z-test

If, no apply t-test.

Suppose we have the heights (in centimeters) of five students: 160, 165, 170, 175, and 180. The mean height is 170 cm. Calculate the population variance.

Calculate the squared deviations from the mean:

$$(160-170)^2 = 100$$
$$(165-170)^2 = 25$$
$$(170-170)^2 = 0$$
$$(175-170)^2 = 25$$
$$(180-170)^2 = 100$$

Sum up the squared deviations = 100 + 25 + 0 + 25 + 100 = 250

Divide by the total number of observations (which is 5) = 250 / 5 = 50

Therefore, the population variance for this data set is 50 square centimeters.

Suppose we have the following exam scores (out of 100) for a class of 10 students: 78, 85, 92, 70, 88, 95, 81, 79, 90, and 84. The mean score is 84. Calculate the population variance.

Calculate the squared deviations from the mean:

$$(78-84)^2 = 36$$
$$(85-84)^2 = 1$$
$$(92-84)^2 = 64$$
$$(70-84)^2 = 196$$
$$(88-84)^2 = 16$$
$$(95-84)^2 = 121$$
$$(81-84)^2 = 9$$
$$(79-84)^2 = 25$$
$$(90-84)^2 = 36$$
$$(84-84)^2 = 0$$

Sum up the squared deviations: (36 + 1 + 64 + 196 + 16 + 121 + 9 + 25 + 36 + 0 = 504)

Divide by the total number of observations (which is 10): (504 / 10 = 50.4)

Therefore, the population variance for this data set is 50.4.

The **Chi-Square Test** is a statistical method used to determine whether there is a significant association between categorical variables. In R, the function chisq.test() is used to perform the test.

Chi-square (χ^2)

Table of Observed Values

Qualification / Marital Status	Middle School	High School	Bachelor's	Master's	Ph.D	Total
Never married	18	36	21	9	6	90
Married	12	36	45	36	21	150
Divorced	6	9	9	3	3	30
Widowed	3	9	9	6	3	30
Total	39	90	84	54	33	300

Null hypothesis: There is no relation between the marital status and educational qualification.

Alternate Hypothesis: There is significant relation between the marital status and educational qualification.

Significance level $(\propto) = 0.05$

Table of Observed Values

Qualification / Marital Status	Middle School	High School	Bachelor's	Master's	Ph.D	Total
Never married	18	36	21	9	6	90
Married	12	36	45	36	21	150
Divorced	6	9	9	3	3	30
Widowed	3	9	9	6	3	30
Total	39	90	84	54	33	300

Table of Expected Values

Qualification / Marital Status	Middle School	High School	Bachelor's	Master's	Ph.D
Never Married	$\frac{90 \times 39}{300} = 11.7$	$\frac{90 \times 90}{300} = 27$	25.2	16.2	9.9
Married	19.5	45	42	27	16.5
Divorced	3.9	9	8.4	5.4	3.3
Widowed	3.9	9	8.4	5.4	3.3

Observed Values (O)	Expected Values (E)	$(O-E)$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
18	11.7	6.3	39.69	3.39
36	27	9	81	3
21	25.2	-4.2	17.64	0.7
9	16.2	-7.2	51.84	3.2
6	9.9	-3.9	15.21	1.53
12	19.5	-7.5	56.25	2.88
36	45	-9	81	1.8
⋮	⋮	⋮	⋮	⋮
3	3.3	-0.3	0.09	0.02
				$\sum\frac{(O-E)^2}{E}$ $\chi^2 = 23.57$

Qualification / Marital Status	Middle School	High School	Bachelor's	Master's	Ph.D	Total
Never married	18	36	21	9	6	90
Married	12	36	45	36	21	150
Divorced	6	9	9	3	3	30
Widowed	3	9	9	6	3	30
Total	39	90	84	54	33	300

Degrees of freedom= (columns – 1) (rows-1)

=(5-1) (4-1) =4 × 3 =**12**

$$\chi^2_{calculated} = 23.57$$

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14

Significance level ($\propto$) = 0.05

$$X^2_{tabular} = 21.03$$

$$X^2_{calculated} = 23.57$$

$$X^2_{calculated} > X^2_{tabular}\ (or\ called\ as\ X^2_{critical})$$

$\therefore$ *we reject Null hypothesis, and accept alternate hypothesis*

Alternate Hypothesis: There is significant relation between the marita
status and educational qualification.

F-Test

The F Test Formula is a Statistical Formula used to test the significance of differences between two groups of Data. It is often used in research studies to determine whether the difference in the means of two populations is Statistically significant.

The **F-test** is a statistical test used to compare two variances to determine if they are significantly different from each other. It is commonly used in:

- **Analysis of Variance (ANOVA)** to compare multiple group means.
- **Regression analysis** to test the overall significance of the model.

The name for the test is given in honour of Sir. Ronald A Fisher by George W. Snedecor.

Formula of the F-Test

$$F = \frac{s_1^2}{s_2^2}$$

$s_1^2 =$ larger variance

$s_2^2 =$ smaller variance

Here $S1^2$ and $S2^2$ means variance of samples

Hypotheses for the F-Test

Null Hypothesis (H0): The variances are equal

Alternative Hypothesis (Ha): The variances are not equal

F-test…

The **F-test** uses the **F-distribution** to determine the critical value that will reject or fail to reject the null hypothesis. The critical value depends on:

Degrees of freedom (df) of the numerator (df1) and denominator (df2).

Significance level (α), usually 0.05 or 0.01.

If the calculated F-statistic exceeds the critical value from the F-distribution table for a given significance level, the null hypothesis is rejected.

Example

1

Perform an F Test for the following samples.

2

Sample 1 with variance equal to 109.63 and sample size equal to 41.

3

Sample 2 with variance equal to 65.99 and sample size equal to 21.

Solution:

Step 1:

The hypothesis Statements are written as:

H_0: No difference in variances

H_a: Difference invariances

Step 2:

Calculate the value of F critical. In this case, the highest variance is taken as the numerator and the lowest variance in the denominator.

$$F\_value = \frac{\sigma_1^2}{\sigma_2^2}$$

$$F\_value = \frac{109.63}{65.99}$$

$$F\_value = 1.66$$

Step 3:

The next step is the calculation of degrees of freedom.

- The degrees of freedom is calculated as Sample size - 1
- The degree of freedom for sample 1 is 41 -1 = 40.
- The degree of freedom for sample 2 is 21 - 1 = 20.

Step 4:

There is no alpha level described in the question, and hence a standard alpha level of 0.05 is chosen.

During the test, the alpha level should be reduced to half the initial value, and hence it becomes 0.025.

Step 5:

Using the F table, the critical F value is determined with alpha at 0.025. The critical value for (40, 20) at alpha equal to 0.025 is 2.287.

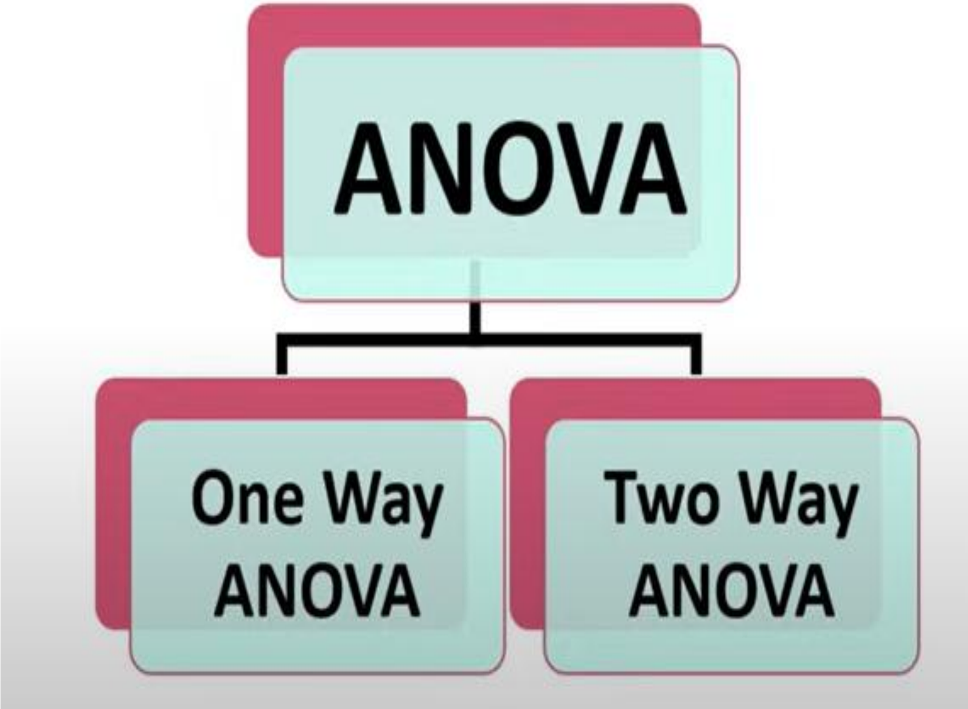
Step 6:

It is now the time for comparing the calculated value with the standard value in the table. Generally, the null hypothesis is rejected if the calculated value is greater than the table value. In this F value definition example, the calculated value is 1.66, and the table value is 2.287.

It is clear from the values that 1.66 < 2.287. Hence, the null hypothesis cannot be rejected.

ANOVA

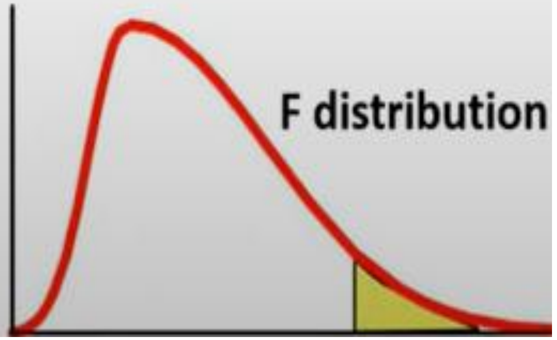
ANOVA stands for Analysis of Variance.
ANOVA enables us to test for significance of difference among more than two sample means.



ANOVA...

- **One way ANOVA**
  - One factor or independent variable.
  - Compares three or more levels of one factor.
- **Two way ANOVA**
  - Extension of One-way Anova
  - More than one factor or independent variable.
  - Compares the effect of multiple levels of two factors.

Test statistics for ANOVA is F-test



Assumption for ANOVA

- Samples follow normal distribution.
- Samples have been selected randomly and independently.
- Each group should have common variance.
- Data are independent.

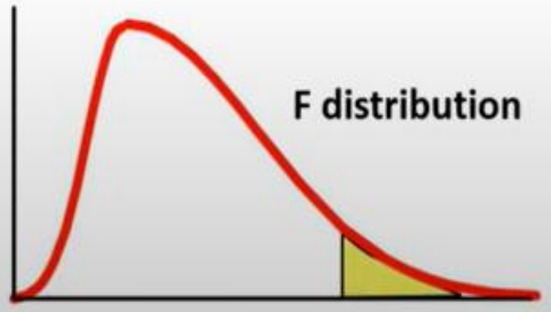
Null Hypothesis – The means for all groups are the same (equal).

$$H_0 : \mu_1 = \mu_2 = \mu_3 \ldots\ldots\ldots\ldots \mu_n$$

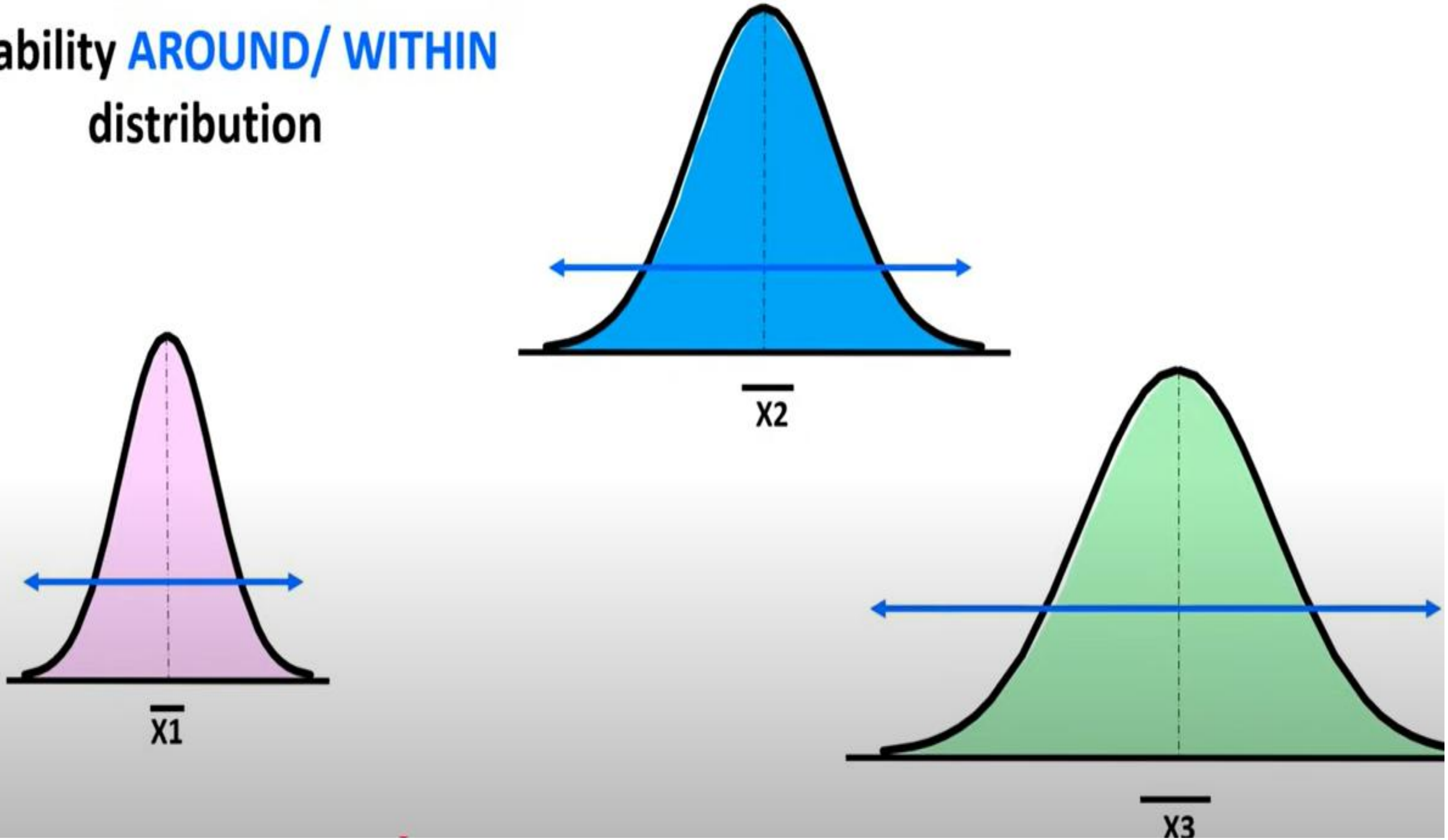
Alternate Hypothesis – The means are different for at least one pair of groups.

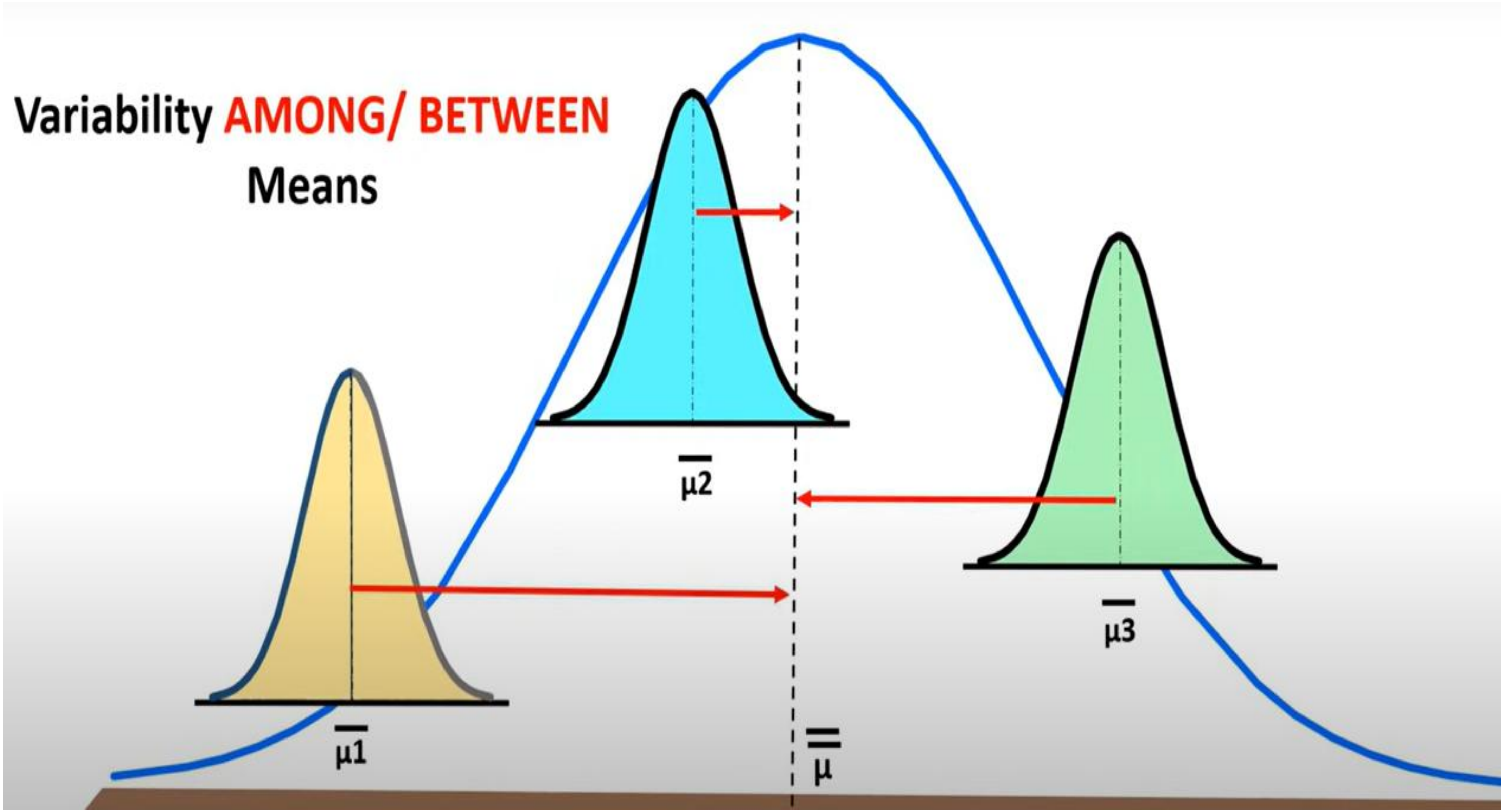
$$H_1 : \mu_1 \neq \mu_2 \neq \mu_3 \neq \ldots\ldots\ldots \mu_n$$

$$\text{ANOVA} = \frac{\text{Variance Between}}{\text{Variance Within}}$$

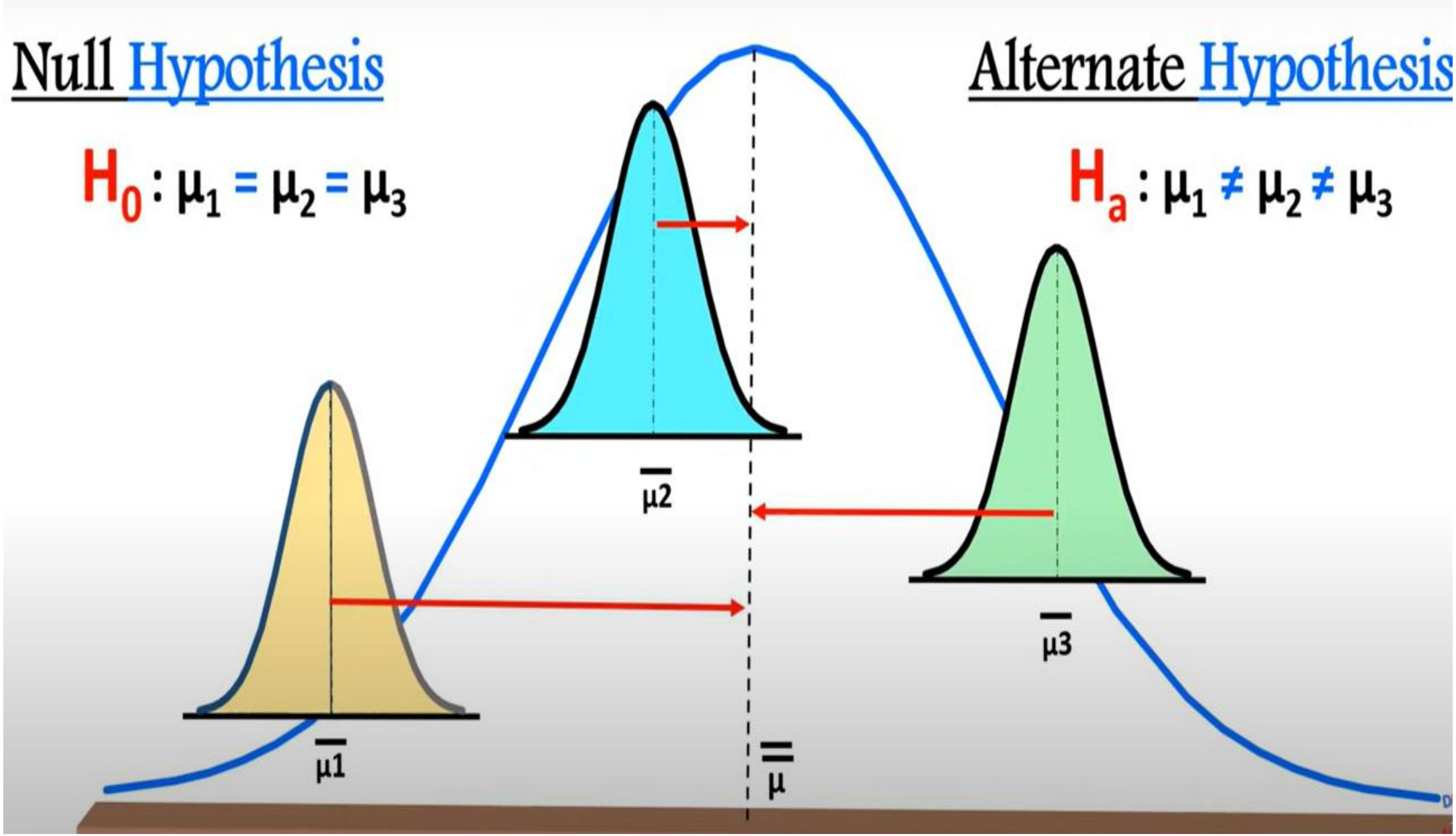


Variability **AROUND/ WITHIN** distribution





Variability AMONG/ BETWEEN
Means
$\overline{\mu 1}$
$\overline{\mu 2}$
$\overline{\mu 3}$
$\overline{\overline{\mu}}$



Null Hypothesis
$H_0 : \mu_1 = \mu_2 = \mu_3$
Alternate Hypothesis
$H_a : \mu_1 \neq \mu_2 \neq \mu_3$
$\overline{\mu 1}$
$\overline{\mu 2}$
$\overline{\mu 3}$
$\overline{\overline{\mu}}$

$$\text{ANOVA} = \frac{\text{Variance Between}}{\text{Variance Within}}$$

$$\text{Total Variance} = \text{Variance Between} + \text{Variance Within}$$

$$\frac{\text{Variance Between}}{\text{Variance Within}} > 1 \quad \text{Reject } H_0$$

$$\frac{\text{Variance Between}}{\text{Variance Within}} < 1 \quad \text{Fail to Reject } H_0$$

$$\frac{\text{Variance Between}}{\text{Variance Within}} = 1 \quad \text{Fail to Reject } H_0$$

We want to see if three different studying methods can lead to different mean exam scores or not. To test this, we select 30 students and randomly assign 10 each to use a different studying method.

Sno	Method A	Method B	Method C
1.	10	8	9
2.	9	9	8
3.	8	10	7
4.	7.5	8	10
5.	8.5	8.5	9
6.	9	7	8
7.	10	9.5	7
8.	8	9	10
9.	8	7	9
10.	9	10	8
Group Mean	8.7	8.6	8.5

Overall Mean 8.6

Between Group Variation =10*(8.7-8.6)^2+10*(8.6- 8.6)^2+10*(8.5-8.6)^2

Between Group Variation = 0.2

Within Group Variation: $\Sigma(X_{ij} - X_j)^2$

Where:

Σ: a symbol that means "sum"
X_{ij}: the i^{th} observation in group j
X_j: the mean of group j

Method A: $(10-8.7)^2 + (9-8.7)^2 + (8-8.7)^2 + (7.5-8.7)^2 + (8.5-8.7)^2 + (9-8.7)^2 + (10-8.7)^2 + (8-8.7)^2 + (8-8.7)^2 + (9-8.7)^2 =$ 6.6

Method B: $(8-8.6)^2 + (9-8.6)^2 + (10-8.6)^2 + (8-8.6)^2 + (8.5-8.6)^2 + (7-8.6)^2 + (9.5-8.6)^2 + (9-8.6)^2 + (7-8.6)^2 + (10-8.6)^2 =$ 10.9

Method C: $(9-8.5)^2 + (8-8.5)^2 + (7-8.5)^2 + (10-8.5)^2 + (9.5-8.5)^2 + (8-8.5)^2 + (7-8.5)^2 + (10-8.5)^2 + (9-8.5)^2 + (8-8.5)^2 =$ 10.5

Within Group Variation: 6.6+10.9+10.5 = 28

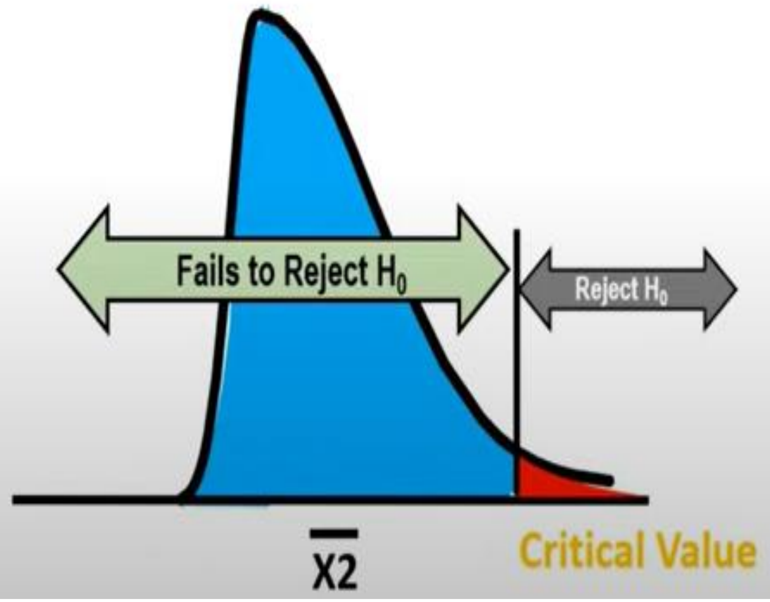
$$\frac{\text{Variance Between}}{\text{Variance Within}} = \frac{0.2}{28} = 0.0071 < 1$$

Fail to Reject H_0

"Means are very close to overall mean and distribution overlap is hard to distinguish".

$F_{Critical} > F_{Stat}$ Fail to Reject H_0

$F_{Critical} < F_{Stat}$ Reject H_0



Assuming $\alpha = 0.05$

$$F_{Stat} = \frac{\text{Variance Between}}{\text{Variance Within}}$$

$$\frac{0.2}{28} = 0.0071$$

$$F_{Critical} = \frac{\text{Numerator Degree of Freedom}}{\text{Denominator Degree of Freedom}}$$

Numerator Degree of Freedom = No. of Samples -1 = 3-1 = 2

Denominator Degree of Freedom = $\sum(nj-1) = n_T - k = 30 - 3 = 27$

	F-table of Critical Values of α = 0.05 for F(df1, df2)																		
	DF1=1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
DF2=1	161.45	199.50	15.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31
2	18.51	19.00	9.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

$F_{Critical} = F_{(2,27)} = 3.35$

$F_{Critical} > F_{Stat}$

Fail to Reject H_0