Program	B. Tech. (SoCS)	Semester	IV
Course	Linear Algebra	Course Code	MATH 2059
Session	Jan-May 2024	Topic	Rank and System of Linear Equations

1. Consider the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{pmatrix}$ where x, y and z are real numbers. Find $Rank(A)$ for the following cases:

(a) $x \neq y \neq z$

(b) $x = y \neq z$

(c) $x = y = z$

2. Suppose A and B are two matrices of same order. Give counterexamples to disprove the following:

(a) $Rank(A + B) = Rank(A) + Rank(B)$

(b) $Rank(A + B) < Rank(A) + Rank(B)$

Suggest an inequality relating the ranks of matrices A, B and $A + B$ that holds correct for all choices of A and B.

3. Consider the matrix $A = [a_{ij}]_{5\times5}$ where $a_{ij} = i^2 - j^2$ for $1 \leq i \leq 5, 1 \leq j \leq 5$. Is it possible that $Rank(A) = 5$? Justify your answer.

4. Let v is a nonzero column vector in 3 −space $\mathbb{R}^3$. Determine the rank of following matrices:

(a) vv^T

(b) $v^T v$

where v^T denotes the transposed vector v.

5. Determine whether or not is it possible to find the value(s) of $\lambda \in \mathbb{R}$ such that the planes P_1, P_2 and P_3 respectively represented by the equations:

$$x - y - \lambda^2 z = 0, \qquad x - y + z = 0, \qquad x + y - z = 0$$

have a common line L of intersection in 3 −space $\mathbb{R}^3$. If the answer is NO, then does there exist at least a common point P of intersection for the three planes? If YES, then find the coordinates of P.

6. For what values of $\lambda \in \mathbb{R}$ and $\mu \in \mathbb{R}$, the planes

$$2x + y + z = 1, 3x - y + \lambda z = 2, x + \mu y + z = 3$$

have a common line L of intersection in 3 −space $\mathbb{R}^3$? Hence find the equation of line L using parameter $t \in \mathbb{R}$.

7. Think of the following geometrically visual concept!!

Two non-parallel planes in three-dimensional space always intersect at a line L.

Design a short mathematical proof for this.

† **Hint:** *Planes* $a_1x + b_1y + c_1z = d_1$ *and* $a_2x + b_2y + c_2z = d_2$ *are parallel if* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

8. Suppose L is the line of intersection of two planes in 3 −space represented as:

$$P_1 \equiv -2x - 3y + z = 0$$

$$P_2 \equiv bz - 5 = 0$$

For what real values of a and b the plane $P_3 \equiv ay + 2z = 5$

(a) intersects L at a unique point.

(b) never intersects L.

(c) contains L.

† **Hint:** *Try to write the equation of planes for your choice in part (b).*

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