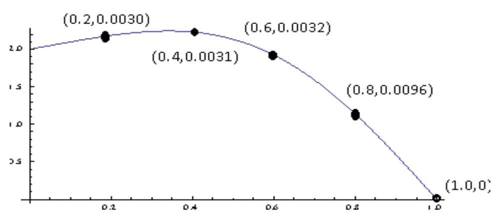
Name:

Enrolment No:

UPES

Assignment-1

Programme Name : B. Tech. CSE | **Semester : II**
Course Name : Advanced Engineering Mathematics-II | **Max. Marks : 100**
Course Code : MATH1065 | **Nos. of page(s): 03**

<table>
<tr><th>S. No.</th><th></th><th>Marks</th><th>CO</th></tr>
<tr><td>1</td><td>In celestial mechanics, mean anomaly is a parameter relating position and time for a body moving in a Kepler orbit and eccentric anomaly is a parameter that defines the position of a body that is moving along an elliptic Kepler orbit. The Kepler equation of motion relating the mean anomaly (M) and the eccentric anomaly (E) of an elliptic orbit with eccentricity e is given by $M = E - e\sin E$. Given $e =$ 0.0167 (Earth's eccentricity) and $M = 1$ (in radians), compute E using (a) Bisection method (b) Newton-Raphson method.</td><td>10</td><td>CO1</td></tr>
<tr><td>2</td><td>The following graph shows the trajectory of an electron in a magnetic field in CRO.


Estimate its position at $x = 0.9$.</td><td>10</td><td>CO1</td></tr>
<tr><td>3</td><td>A robot arm with a rapid laser scanner is doing a quick quality check on holes drilled in a 15" × 10" rectangular plate. The centers of the holes in the plate describe the path the arm needs to take, and the hole centers are located on a Cartesian coordinate system (with the origin at the bottom left corner of the plate) given by the specifications in the following table:
<table>
<tr><td>$x(in.)$</td><td>2.00</td><td>4.25</td><td>5.25</td><td>7.81</td><td>9.20</td><td>10.60</td></tr>
<tr><td>$y(in.)$</td><td>7.2</td><td>7.1</td><td>6.0</td><td>5.0</td><td>3.5</td><td>5.0</td></tr>
</table>
Find the path traversed through the six points using Newton's divided difference method of interpolation.</td><td>10</td><td>CO1</td></tr>
<tr><td>4</td><td>A tank is discharging water through an orifice at a of depth x meter placed below the surface of the water whose area is $A\ m^2$. The following are the values of x for the corresponding values of A:
<table>
<tr><td>A</td><td>1.257</td><td>1.39</td><td>1.52</td><td>1.65</td><td>1.809</td><td>1.962</td><td>2.123</td><td>2.295</td><td>2.462</td><td>2.650</td><td>2.827</td></tr>
<tr><td>x</td><td>1.50</td><td>1.65</td><td>1.80</td><td>1.95</td><td>2.10</td><td>2.25</td><td>2.40</td><td>2.55</td><td>2.70</td><td>2.85</td><td>3.00</td></tr>
</table></td><td>10</td><td>CO1</td></tr>
</table>

<table>
<tr><td></td><td>Using the formula

$$(0.018)T = \int_{1.5}^{3.0} \frac{A}{\sqrt{x}}\, dx \ ,$$

calculate time T, in seconds for the level of the water to drop from 3.0 m to 1.5 m above the orifice.</td><td></td><td></td></tr>
<tr><td>5</td><td>A polluted lake has an initial concentration of a bacteria of 10^7 parts/m^3, while the acceptable level is only 5×10^6 parts/m^3. The concentration of the bacteria will reduce as fresh water enters the lake. The differential equation that governs the concentration C of the pollutant as a function of time (in weeks) is given by

$$\frac{dC}{dt} + 0.06C = 0, \ C(0) = 10^7$$

Using the Euler's method, find the concentration of the pollutant after 7 weeks. Take a step size of 3.5 weeks.</td><td>10</td><td>CO1</td></tr>
<tr><td>6</td><td>To infer the surface shape of an object from images taken of a surface from three different directions, one needs to solve the following set of equations.

$$\begin{bmatrix} 0.2425 & 0 & -0.9701 \\ 0 & 0.2425 & -0.9701 \\ -0.2357 & -0.2357 & -0.9428 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 247 \\ 248 \\ 239 \end{bmatrix}$$

The right hand side values are the light intensities from the middle of the images, while the coefficient matrix is dependent on the light source directions with respect to the camera. The unknowns are the incident intensities that will determine the shape of the object.
Find the values of x_1, x_2 and x_3 using the Gauss-Seidel method. Use

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$$

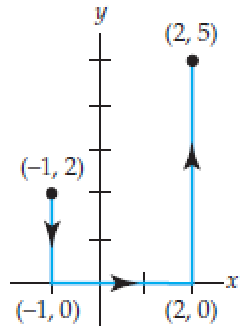
as the initial guess and conduct two iterations.</td><td>10</td><td>CO1</td></tr>
<tr><td>7</td><td>Peter's boats make three different kinds of boats. All can be made profitably in this company, but the company's monthly production is constrained by the limited amount of labour, wood, and screws available each month. The director will choose the combination of boats that maximizes his revenue in view of the information given in the following table:

<table>
<tr><td>Input</td><td>Row Boat</td><td>Canoe</td><td>Keyak</td><td>Monthaly Available</td></tr>
<tr><td>Labour (Hours)</td><td>12</td><td>7</td><td>9</td><td>1.260 hrs.</td></tr>
<tr><td>Wood (Board feet)</td><td>22</td><td>18</td><td>16</td><td>19008 board feet</td></tr>
<tr><td>Screws (kg)</td><td>2</td><td>4</td><td>3</td><td>396 kg.</td></tr>
<tr><td>Selling Price (in Rs.)</td><td>4000</td><td>2000</td><td>5000</td><td></td></tr>
</table>

Solve it by using the simplex method.

Problem formulation:

Let x_1, x_2 and x_3 be the number of Row boats, Canoe and Keyak made every month. Then the above problem can be formulated as:</td><td>10</td><td>CO1</td></tr>
</table>

	Max. Revenue $z = 4000x_1 + 2000x_2 + 5000x_3$ subject to $12x_1 + 7x_2 + 9x_3 \leq 1260$ $22x_1 + 18x_2 + 16x_3 \leq 19008$ $2x_1 + 4x_2 + 3x_3 \leq 396$ $x_1, x_2, x_3 \geq 0.$		
8	If $w(z) = \phi(x, y) + i\psi(x, y)$ represents the complex potential and $\phi(x, y) = x^2 - y^2$. Determine $w(z)$.	**10**	**CO2**
9	Evaluate $\int_C (2x + y)dx + xydy$ on the given curve (see figure) from (−1, 2) to (2, 5) :  	**10**	**CO2**
10	Evaluate: $\int_C \frac{\sinh(z^{2024})}{z^3} dz$, where C is the circle $\|z\| = 1$.	**10**	**CO2**