Program	B. Tech. (SoCS)	Semester	IV
Course	Linear Algebra	Course Code	MATH 2059
Session	Jan-May 2025	Topic(s)	Eigenvectors, Diagonalization and Quadratic forms

1. For each matrix, find all eigenvalues and linearly independent eigenvectors:

a) $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$

b) $\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$

c) $\begin{pmatrix} 5 & -1 \\ 1 & 3 \end{pmatrix}$

2. For each matrix, find a polynomial for which the matrix is a root:

a) $\begin{pmatrix} 3 & -7 \\ 4 & 5 \end{pmatrix}$

b) $\begin{pmatrix} 5 & -1 \\ 8 & 3 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 3 & -2 \\ 0 & 5 & 4 \\ 1 & 0 & -1 \end{pmatrix}$

3. For each of the following matrices, find all the eigenvalues:

a) $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix}$

b) $B = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{pmatrix}$

c) $C = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

4. For each of the matrices in **Q. No. 3**, find invertible matrices P_1, P_2 and P_3 (if possible) such that $P_1^{-1}AP_1,\ P_2^{-1}BP_2$ and $P_3^{-1}CP_3$ are diagonal matrices.

5. Suppose

$$A = \begin{pmatrix} 8 & 12 & 0 \\ 0 & 8 & 12 \\ 0 & 0 & 8 \end{pmatrix}$$

Find a real matrix A such that $B = A^3$.

6. Let $A = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{pmatrix}$. If $aA^{-1} = bA^2 + cA + dI$ where $a,\ b, c, d \in \mathbb{R}$, then determine the value of $ab + cd$.

7. Let A be a 3×3 real matrix such that $\det(A) = 18$ and $\text{trace}(A) = -2$.
If $\det(A + 3I) = 0$ (where I is the identity matrix of order 3), then find the value of $\text{trace}(A^2 - 2A)$.

8. Let the characteristic polynomial of a matrix A be $(x - 2)(x + 1)^2(x + 3)^2$. If A is diagonalizable, then find the rank of $(A + 3I)$ (where I is the identity matrix of order 5).

9. Let $A = \begin{pmatrix} 1 & -3 \\ 0 & 2 \end{pmatrix}$, $B = I + A + \cdots + A^{10}$ and $P^{-1}AP = diag(1, 2)$.
If $\text{trace}(P^{-1}BP) = \alpha + \beta 2^{11}$, then determine the values of α and β.

10. For the following matrices (say A), find the orthogonal matrix P such that $P^T AP = D$ where D is a diagonal matrix having diagonal entries as eigenvalues of A.

a) $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$

b) $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

c) $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

d) $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

11. Reduce the matrix $A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{pmatrix}$ to diagonal form by an orthogonal transformation and hence find A^3.

12. Find the 3×3 symmetric matrix A having eigenvalues $2, 3, 6$ and corresponding eigen vectors $(1\ 0\ -1)^T, (1\ 1\ 1)^T$, and $(1\ -2\ 1)^T$.

13. Verify Cayley-Hamilton theorem for the matrix A and hence, find A^{-1} and A^4.

a) $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$

14. Find the characteristic equation of the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ and hence, find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

15. Find the characteristic roots of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ and verify Cayley-Hamilton theorem for this matrix. Find A^{-1} and, also express $2A^4 - 5A^3 - 7A + 6I$ as a linear polynomial in A and I.