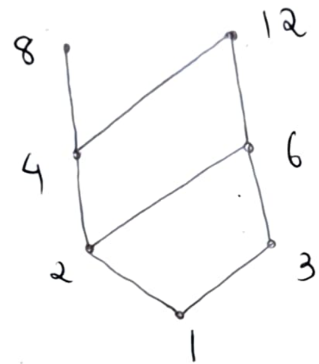
Practice set 4

1) The relation set for $\{(a,b) \mid a \mid b\}$ is

$R = \{(1,2),(1,3),(1,4),(1,6),(1,8),(1,12),$
$(2,4),(2,6),(2,8),(2,12),(3,6),(3,12),$
$(4,8),(4,12),(6,12)\}$



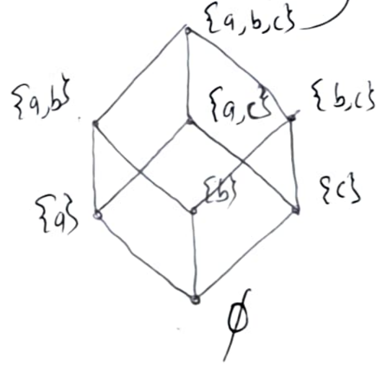
2) $S = \{a, b, c\}$

$P(S) = \{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}, \emptyset\}$

$R = \{(A,B) : A \subseteq B\}$

After removing reflexive and transitive elements, we get the following relation

$\{(\emptyset, \{a\}), (\emptyset, \{b\}), (\emptyset, \{c\}), (\{a\}, \{a,b\}), (\{a\}, \{a,c\}), (\{b\}, \{a,b\}), (\{b\}, \{b,c\}), (\{c\}, \{a,c\}), (\{c\}, \{b,c\}), (\{a,b\}, \{a,b,c\}), (\{b,c\}, \{a,b,c\}), (\{a,c\}, \{a,b,c\})\}$



3) $(\{2,4,5,10,12,20,25\}, |)$

maximal elements (elements which do not divides any other elem

$= 12, 20, 25.$

minimal elements (elements which are not divisible by other elem)

$= 2, 5.$

4) in $(Z^+, |)$,

greatest element = none (no element which is divisible by all elements)

least element = 1 (element which divides all elements).

5)

	lower bounds	upper bounds
$\{a,b,c\}$	a	e, f, h, j
$\{j,h\}$	a, b, c, d, e, f	none
$\{a,c,d,f\}$	a	f, j, h

6) $\{b,d,g\}$: upper bound : g, h

least upper bound : g

lower bound : a, b

greatest lower bound : b

7) (a) Poset is a lattice since every pair of elements have a l.u.b and g.l.b.

(b) Not a lattice, because $\{b,c\}$ ~~and $\{d,e\}$~~ does not have a l.u.b and $\{d,e\}$ does not have a g.l.b.

(c) Is a lattice.

8) $\{3,9,12\}$: glb = gcd (3, 9, 12) = 3 is in the set Z^+

lub = lcm (3, 9, 12) = 36 is in the set Z^+.

$\{1,2,4,5,10\}$: glb = gcd (1, 2, 4, 5, 10) = 1 is in Z^+.

lub = lcm (1, 2, 4, 5, 10) = 20 is in Z^+.

9) $(Z^+, |)$ is a lattice because every pair of elements in Z^+ has a gcd and lcm in Z^+.

10) $(\{1,2,3,4,5\}, |)$ is not a lattice because $\{2,3\}$ does not have an lcm (lub) in the set.

$(\{1,2,4,8,16\}, |)$ is a lattice because every pair has a l.u.b (lcm) and glb (gcd) in the set as 16 is lcm of all numbers and 1 is gcd of all numbers.

~~11) P(S) is a~~

11) $(P(S), \subseteq)$ is a lattice because $P(S)$ contains S to which every ~~subset of~~ element of $P(S)$ is related and also contains $\emptyset$ which is related to every element in $P(S)$

~~12)~~

~~12) {(5,4),(5,3),(5,2),(5,1),(5,0),~~

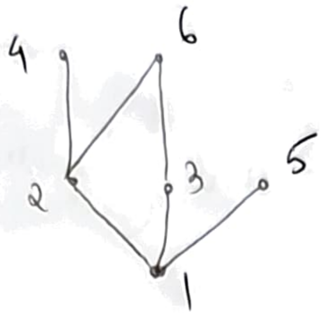
12) After removing all the reflexive and transitive elements,

$\{(5,4), (4,3), (3,2), (2,1), (1,0)\}$

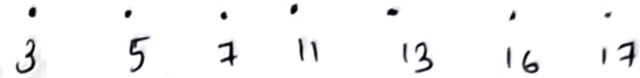
13) After removing transitive and reflexive elements

$\{(0,2), (2,5), (5,10), (10,11), (11,15)\}$

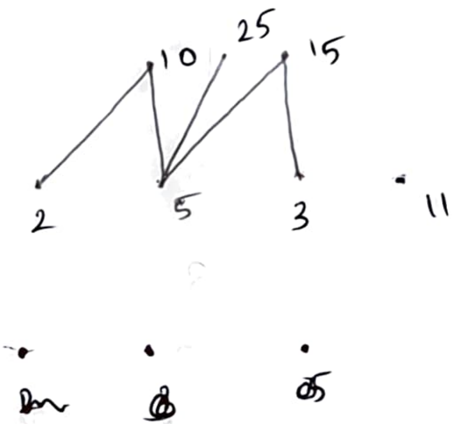
14) a)



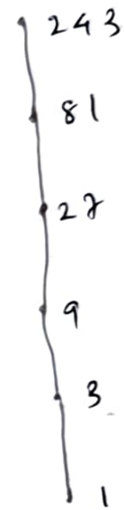
b)



c)



d)



16) a) ✓ b) ✗ ((2,3) and (3,2) is there, so not antisymmetric)

c) ✓ d) ✓

e) ✗ ((0,1), (1,0) is there, so not antisymmetric)

17) similar to 16)

18)

vertex	in-degree	out-degree
a	2	4
b	2	1
c	3	2
d	2	2
e	3	3

19) G - bipartite (done in class)

H - not bipartite (done in class)

20) Regular graph: graph in which every vertex has same degree.

complete graph: Every vertex has degree $n-1$.

$\therefore$ complete graph K_n is regular $\forall n \geq 1$.

cycle: Every vertex has degree 2.

$\therefore$ cycle C_n is regular $\forall n \geq 3$.

21) ~~Regular graph of~~

21) Let 'n' be the number of vertices of the regular graph.

each has degree 4.

By Handshaking Theorem,

sum of degree of vertices = 2 × no of edges.

$$4n = 2 \times 10$$

$$n = \frac{20}{4} = 5.$$

$\therefore$ $\underline{n = 5}$

~~22)~~

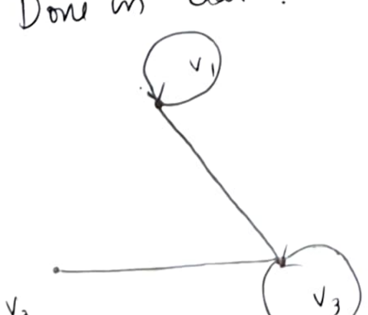
	e_1	e_2	e_3	e_4	e_5	e_6
v_1	1	1	0	0	0	0
v_2	0	0	1	1	0	0
v_3	0	0	0	0	1	1
v_4	1	0	1	0	0	0
v_5	0	1	0	1	0	0

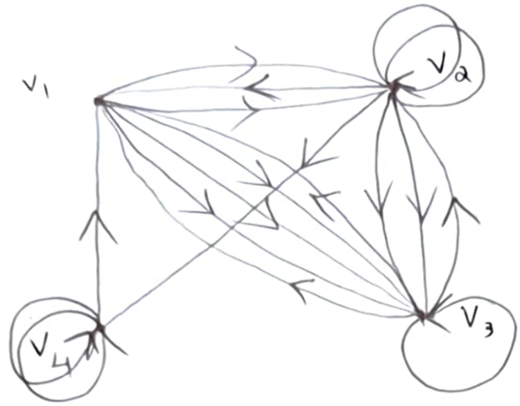
22)

	e_1	e_2	e_3	e_4	e_5	e_6
v_1	1	1	0	0	0	0
v_2	0	0	1	1	0	1
v_3	0	0	0	0	1	1
v_4	1	0	1	0	0	0
v_5	0	1	0	1	1	0

23) Done in class.

24)





25)

	path ~~not~~	simple	circuits	lengths
a) a, e, b, c, b	✓	✗	✗	4
b) a, e, a, d, b, c, a	✗			
c) e, b, a, d, b, e	✗			
d) c, b, d, a, e, c	✓	✓	✓	5

26) Let G be a connected graph with n vertices.

Let us prove this by induction.

Let $P(n)$: ~~gr~~ connected graph with n vertices have atleast $n-1$ edges.

I) it is trivial for $n=1$.

for $n=2$,

The graph is connected only if the 2 vertices are attached to each other by an edge.

$\therefore$ it has atleast 1 edge.

Hence $P(2)$ is true.

II) Let us assume $P(k)$ is true for some $k \geq 2$, ie,

a connected graph with k vertices has atleast $k-1$ edges.

To prove: $P(k+1)$ is true i.e, a connected graph with $k+1$ vertices will have atleast k edges.

Proof: If we have a connected graph with $k+1$ vertices, the subgraph of the same with k vertices will have atleast $k-1$ edges according to our assumption.

Hence, the $k+1^{th}$ vertex will have to be connected to any of the other ~~vert~~ vertices since it is a connected graph.

$\therefore$ there will be atleast one extra edge connecting the $k+1^{th}$ vertex to any of the remaining k vertices.

Hence, there will be a total of atleast k edges. $(k-1+1)$

Hence, $P(k+1)$ is true.

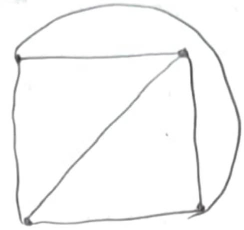
$\therefore$ $P(n)$ is true $\forall n$.

	Hamiltonian circuit	H. path
27) G_1:	a, b, c, d, e, a	a, b, c, d, e
G_2	X	a, b, c, d
G_3	X	X

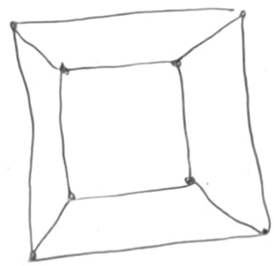
28) a) no Euler circuit because b has odd degree

b) Has Euler circuit since every vertex has even degree.

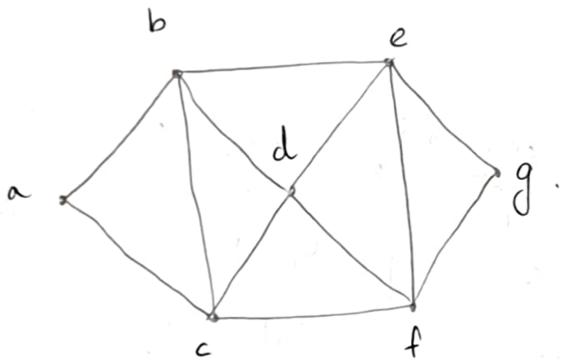
29) a) Planar



b) Planar



30)



I)

vertex	a	b	c	d	e	f	g
degree	2	4	4	4	4	4	2

II) non decreasing order of degrees.

b	c	d	e	f	a	g
C_1	C_2	C_3	C_2	C_1	C_3	C_3.

∴ 3 colours.

Chromatic number = $\underline{\underline{3}}$.