

UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	B.Tech (All SoCSBranches)	Semester	III
Course	Discrete Mathematical Structures	Course Code	CSEG 2006

1. If p be “He is rich” and q be “He is happy”. Write each statement in symbolic form using p and q . Note that “He is poor” and “He is unhappy” are equivalent to $\sim p$ and $\sim q$, respectively.

- (i) If he is rich, then he is unhappy.
 (ii) He is neither rich nor happy.
 (iii) It is necessary to be poor in order to be happy.
 (iv) To be poor is to be unhappy.

2. If p , q and r are three statements, construct the Truth Table of the following propositions

- (i) $p \vee \sim q \wedge r$ (ii) $r \wedge \sim (p \Rightarrow q)$ (iii) $r \Leftrightarrow (p \wedge q)$.

3. If p , q and r are three statements, then check whether the following statements are tautology or not.

- (i) $(p \wedge q) \Rightarrow (p \Rightarrow q)$ (ii) $\sim (p \vee q) \vee [\sim p \wedge q] \vee p$
 (iii) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$.

4. Determine whether the following propositions are a tautology, contingency or contradiction :

- (i) $p \Leftrightarrow (p \wedge q)$ (ii) $p \rightarrow (q \vee p)$ (iii) $(\sim p \wedge q) \wedge p$.

5. Show that the following propositions are equivalent or not?

- (i) $p \Leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ (ii) $[(p \rightarrow q) \rightarrow r] \equiv (p \wedge \sim q) \rightarrow r$.

6. Determine the principal disjunctive normal form (PDNF) and principal conjunctive normal form (PCNF) of the following propositions

- (i) $p \Leftrightarrow q$ (ii) $(q \wedge p) \vee (\sim q \wedge r)$.

7. If p be “He is rich” and q be “He is Generous”. Write in simple sentences the meaning of the following:

- (i) $\sim p$ (ii) $\sim (p \vee q)$ (iii) $\sim (p \wedge q)$
 (iv) $p \rightarrow q$ (v) $\sim (p \rightarrow q)$ (vi) $\sim p \wedge \sim q$.

8. If p , q and r are three statements , construct the Truth Table of the following propositions

- (i) $p \wedge \sim q \vee r$ (ii) $r \wedge \sim (p \vee q)$ (iii) $r \rightarrow (p \wedge q)$.

9. If p , q and r are three statements, then check whether the following statements are tautology or not.

- (i) $p \vee \sim (p \wedge q)$ (ii) $((p \Rightarrow q) \Rightarrow r) \Rightarrow ((p \Rightarrow r) \Rightarrow q)$
 (iii) $(p \Rightarrow q) \Rightarrow (p \Rightarrow (p \wedge q))$ (iv) $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (r \Rightarrow p)$.

10. Determine whether the following propositions are a tautology, contingency or contradiction :

- (i) $p \rightarrow (p \rightarrow q)$ (ii) $p \rightarrow (q \rightarrow p)$ (iii) $(p \rightarrow q) \wedge (p \wedge r) \rightarrow q$.

11. Show that the following propositions are a tautology or not?

- (i) $[(p \vee q) \wedge (\sim p)] \rightarrow q$ (ii) $[(p \rightarrow \sim q) \wedge (r \rightarrow q) \wedge r] \rightarrow \sim p$
 (iii) $[(p \rightarrow q) \wedge (r \rightarrow \sim q)] \rightarrow (p \rightarrow \sim r)$.

11. Determine the principal disjunctive normal form (PDNF) and principal conjunctive normal form (PCNF) of the following propositions

- (i) $p \wedge (p \rightarrow q)$ (ii) $p \vee (q \wedge p)$ (iii) $(p \leftrightarrow q) \vee \sim r$.

12. Determine the validity of the following arguments

- i. $\sim(\sim p \wedge q)$
 $\sim(p \wedge r)$
 $r \vee s$

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- ii. $q \rightarrow s$
 $p \wedge q$
 $p \rightarrow r$
 $s \rightarrow \sim q$

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- iii. $\sim s \wedge r$
 If I study, then I will pass.
 If I do not go to a movie, then I will study.
 I failed.

Therefore, I went to a movie.

- iv. If I like mathematics, then I will study.
 Either I don't study or I pass mathematics.
 If I don't pass mathematics, then I don't graduate.

If I graduate, then I like mathematics.

13. If $D = \{1, 2, 3, \dots, 9\}$. Determine the truth value of each of the following statements.

- i. $(\forall x \in D), x + 4 < 15$,
 ii. $(\exists x \in D), x + 4 = 10$,
 iii. $(\forall x \in D), x + 4 \leq 10$,
 iv. $(\exists x \in D), x + 4 > 15$.

14. Write the negation of the following statements

- (i) All natural numbers are less than 10.
 (ii) For all real numbers x , if $x > 3$ then $x^2 > 9$.

15. Which of the following sentences are propositions? What are the truth values of those that are propositions?

- a. Is this true?
 b. Ram is a name.

- c. Please submit your proposal as soon as possible.
- d. Four is even.
- e. $5 \in \{1, 6, 7\}$.
- f. What a Hit!
- g. Answer this question.
- h. Buy two cinema tickets for Friday.
- i. May God bless you.
- j. $\{1, 3\} \subset \{4, 5, 6\}$

16. Write the negation of the following:

- a. Real number x , if $x > 3$ then $x^2 > 9$.
- b. Integers a, b, c if $a - b$ even and $b - c$ is even, then $a - c$ is even.
- c. No one wants to buy my house.
- d. Some people have no scooter.
- e. Every even integer greater than four is the sum of two primes.
- f. $x \in R$. If $x(x + 1) > 0$ then $x > 0$ or $x < -1$.
- g. If the determinant of a system of linear equations is zero then either the system has no solution or it has an infinite number of solutions.

17. Determine the truth value of the following statements.

- a. $6 + 2 = 7$ and $4 + 4 = 8$.
- b. If $3 \times 5 = 24$, then $3 + 5 = 8$.
- c. If $3 \times 5 = 15$, then $3 + 5 = 12$.
- d. If tigers have wings, then RDX is dangerous.

18. Consider the following

- p: Anil is rich.
- q: Kanchan is poor.

Write each of the following statements in symbolic forms

- a. Neither Anil nor Kanchan is poor.
- b. It is not true that Anil and Kanchan are both rich.
- c. Either Anil is poor or Kanchan is poor.
- d. Anil is not rich and Kanchan is poor.

19. Prove that the following propositions are tautologies.

- a. $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$.
- b. $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$.

20. Establish the equivalence of the following.

$$p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$$

21. Show that each of the following inferences are fallacy

- a. If today is Megha's birthday, then today is September 4. Today is September 4. Hence today is Megha's birthday.
- b. If the client is guilty, then he was at the scene of the crime. The client was at the scene of the crime. Hence the client was not guilty.

22. Consider the following argument:

If 19 is less than 15, then 11 is not a prime number.

19 is not less than 15. So 11 is a prime number.

- (i) Translate the argument into symbolic form, using P and Q to represent statements.
- (ii) Construct a truth table and thereby establish whether the above argument is valid or contains errors.
- (iii) If you decide that the argument contains errors, then reword it in such a way as to make it a valid argument.
- (iv) If you decide that the argument is valid, then reword it in such a way as to make it false.