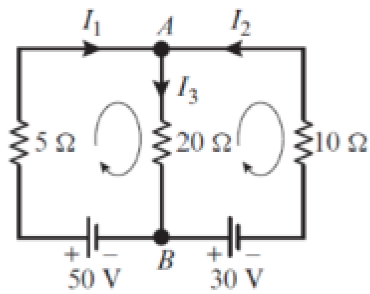
Name: Enrolment No:	

UPES
Assignment II

Programme Name : B.Tech. (SoCS) **Semester : IV**
Course Name : Linear Algebra
Course Code : MATH 2059 **Max. Marks: 10**
Nos. of page(s) : 02

S. No.		Marks	CO
Q 1	Suppose V is a set of all real functions and $\mathcal{F}$ be the field of real scalars. If the sum of the functions $f(x)$ and $g(x)$ in V is defined to be $f(g(x))$, then the zero vector is $g(x) = x$. Keep the usual scalar multiplication $cf(x)$, then find two rules that are broken in order to V be a vector space over the field $\mathcal{F}$.	1	CO2
Q 2	Choose $x = (x_1, x_2, x_3, x_4)$ in R^4. It has 24 rearrangements like (x_2, x_1, x_3, x_4) and (x_4, x_3, x_1, x_2). Those 24 vectors, including x itself, span a subspace S. Find specific vectors x so that the dimension of S is: $(a) 0$ $(b) 1$ $(c) 3$ $(d)\ 4$	1	CO2
Q 3	Find a counterexample to the following statement: If v_1, v_2, v_3, v_4 is a basis for the vector space R^4 and if W is a subspace, then some subset of the v's is a basis for W.	1	CO2
Q 4	Let $P = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.8 & 0.3 \\ 0.2 & 0 & 0.4 \end{bmatrix}$ and $x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Consider a system whose state is described by the Markov chain $x_{k+1} = Px_k$ for $k = 0,1,\cdots$ What happens to the system as time passes? † **Hint: Compute the state vectors $x_1, x_2, \cdots, x_{15}$ to find out.**	1	CO4
Q 5	*For a non-empty subset S of an inner product space V, the orthogonal complement of S is defined as:* $S^{\perp} = \{v \in V \mid < v, s >= 0\ \forall\ s \in S\}$ *which is a subspace of V.* Determine $S^{\perp}$ in each of the following cases: a) $S = \{(1,2,-2),\ (1,-1,3)\}$ in $V = \mathbb{R}^3$ w.r.t. the usual inner product.	1	CO2

	b) $S = \{1 + x,\ x^2\}$ in $V = \mathcal{P}_2(\mathbb{R})$ w.r.t. the inner product $$< p(x), q(x) > = \int_{-1}^{1} p(x)q(x)dx$$		
Q 6	Consider the electric circuit given below:  Determine the values of current I_1, I_2, I_3 by solving a non-homogeneous system of three linear equations in three variables.	**1**	**CO4**
Q 7	Suppose A is a 2×2 symmetric matrix with unit eigenvectors e_1 and e_2. If its eigenvalues are $\lambda_1 = 3$ and $\lambda_2 = -2$, what are U, Σ and V^T?	**2**	**CO4**
Q 8	In Question no. 7, if A changes to $4A$, what is the change in the SVD? What is the SVD for A^T and for A^{-1}?	**2**	**CO4**