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Estimation and Hypothesis Testing

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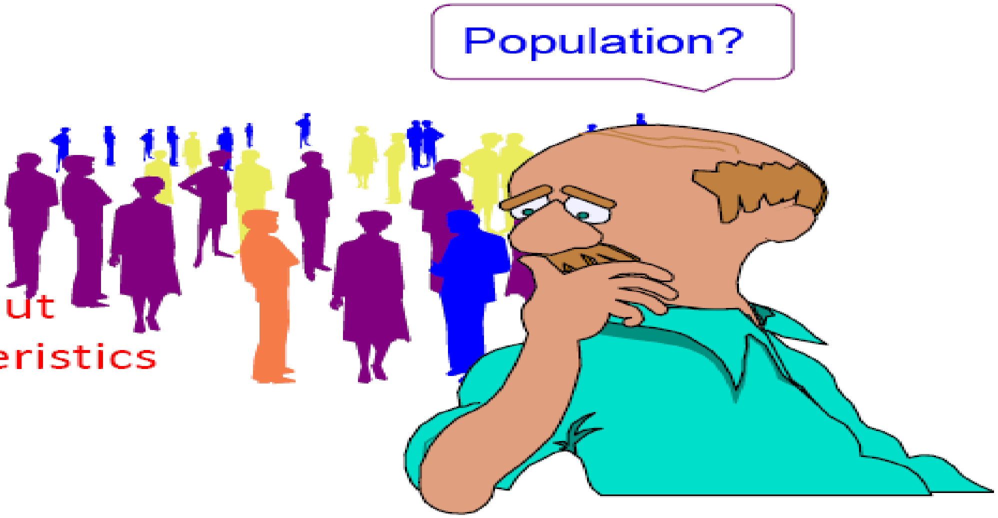
Objectives

- After complete this session you will be able to do
  - Parameter estimations
    - **Point estimate**
    - **Confidence interval**
  - Hypothesis testing
    - Z-test
    - T-test
  - Testing associations
    - Chi-Square test

Introduction # 1

- Inferential is the process of generalizing or drawing conclusions about the target population on the basis of results obtained from a sample.

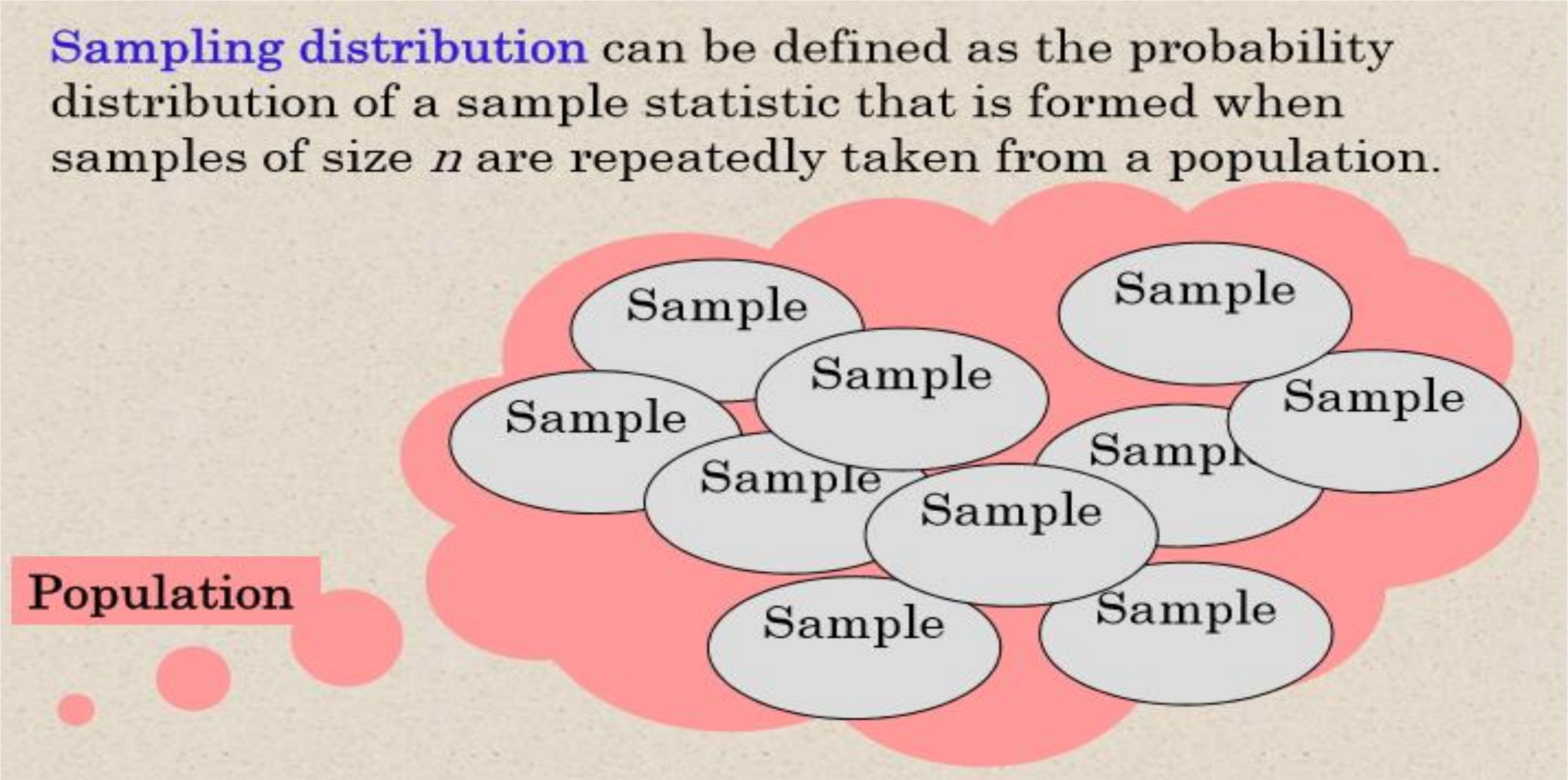
1. Involves
   - Estimation
   - Hypothesis testing
2. Purpose
   - Make decisions about population characteristics



Introduction #2

- Before beginning statistical analyses
  - it is essential to examine the distribution of the variable for skewness (tails),
  - kurtosis (peaked or flat distribution), spread (range of the values) and
  - outliers (data values separated from the rest of the data).
- Information about each of these characteristics determines to choose the statistical analyses and can be accurately explained and interpreted.

Sampling Distribution



Sampling distribution can be defined as the probability distribution of a sample statistic that is formed when samples of size n are repeatedly taken from a population.

- The frequency distribution of all these samples forms the sampling distribution of the sample statistic

Sampling distribution ....

- Three characteristics about sampling distribution of a statistic
    - its mean
    - its variance
    - its shape

- Due to random variation different samples from the same population will have different sample means.
- If we repeatedly take sample of the same size n from a population the means of the samples form a sampling distribution of means of size n is equal to population mean.
- In practice we do not take repeated samples from a population i.e. we do not encounter sampling distribution empirically, but it is necessary to know their properties in order to draw statistical inferences.

The Central Limit Theorem

- Regardless of the shape of the frequency distribution of a characteristic in the parent population,
  - the means of a large number of samples (independent observations) from the population will follow a normal distribution (with the mean of means approaches the population mean μ, and standard deviation of σ/√n).
- Inferential statistical techniques have various assumptions that must be met before valid conclusions can be obtained
  - Samples must be randomly selected.
  - sample size must be greater (n>=30)
  - the population must be normally or approximately normally distributed if the sample size is less than 30.

Sampling Distribution......

Sampling Distribution of the mean: Suppose we choose a random sample of size n, the sampling distribution of the sample mean $\bar{x}$ posses the following properties.

- The sample mean $\bar{x}$ will be an estimate of the population mean μ.
- The standard deviation of $\bar{x}$ is $\sigma/\sqrt{n}$ (called the standard error of the mean).
- Provided n is large enough the shape of the sampling distribution of $\bar{x}$ is normal.

Sampling Distribution ....

- **Proportion**
  - Suppose we choose a random sample of size n, the sampling distribution of the sample proportion p posses the following properties.
  - The sample proportion p will be an estimate of the population mean p.
  - The standard deviation of p is $\sqrt{p(1-p)/n}$ called the standard error of the proportion).
  - Provided n is large enough the shape of the sampling distribution of p is normal.

Standard deviation and Standard error

- **Standard deviation** is a measure of variability between individual observations (descriptive index relevant to mean)
- **Standard error** refers to the variability of summary statistics (e.g. the variability of the sample mean or a sample proportion)
- Standard error is a measure of uncertainty in a sample statistics i.e. precision of the estimate of the estimator

Parameter Estimations

- In parameter estimation, we generally assume that the underlying (unknown) distribution of the variable of interest is adequately described by one or more (unknown) parameters, referred as ***population parameters.***

- As it is usually not possible to make measurements on every individual in a population, parameters cannot usually be determined exactly.

- Instead we estimate parameters by calculating the corresponding characteristics from a random ***sample estimates*** .
- ***the process of estimating the value of a parameter from information obtained from a sample.***

Estimation

- Estimation is a procedure in which we use the information included in a sample to get inferences about the true parameter of interest.

□ An ***estimator*** is a sample statistic that used to estimate the population parameter while an ***estimate*** is the possible values that a given estimator can assume.

Properties of a good estimator

Sample statistic	*Corresponding population parameter*
$\bar{X}$(Sample mean)	μ (population mean)
S^2 (sample variance)	σ^2 (population variance)
S (sample Standard deviation)	σ (population standard deviation)
$\hat{p}$ (Sample proportion)	P (Population proportion)

A desirable property of a good estimator is the following

- It should be ***unbiased*:** The expected value of the estimator must be equal to the parameter to be estimated.
- It should be ***consistent*:** as the sample size increase, the value of the estimator should approaches to the value of the parameter estimated.
- It should be ***efficient*:** the variance of the estimator is the smallest.
- It should be ***sufficient:*** the sample from which the estimator is calculated must contain the maximum possible information about the population.

Types of Estimation

- **There are two types of estimation:**

1. **Point estimation:** It uses the information in the sample to arrive at a single number (that is called an estimate) that is intended to be close to the true value of the parameter.

2. **Interval estimation:** It uses the information of the sample to end up at an interval (i.e. construct 2 endpoints) that is intended to enclose the true value of the parameter.

Point Estimation

- Is a single numerical value used to estimate the corresponding population parameters. A point estimate of some population parameter is a single value of a sample statistic.
- Sample measures (i.e., statistics) are used to estimate population measures (i.e., parameters). These statistics are called estimators.
- Point estimate for population mean μ is

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

- Point estimate for population proportion is given by

$$\hat{\mathrm{p}} = \frac{x}{\mathrm{n}}$$

- Where x is the total number of success (events)

Example

- The population consists of survival times of cancer patients who have been treated with a new drug has SD of 43.3 months. If a random sample of 100 drug-treated patients has a mean survival time of 46.9 months, then

What is the point estimate of the population mean?

Solution: Given: $n = 100$, $\bar{x} = 46.9$, $\sigma = 43.3$, and $= 0.05$.

The point estimates of the population mean is 46.9 months.

Some BLUE estimators

Point Estimates

Estimate Population Parameters ...		with Sample Statistics
Mean	μ	$\bar{X}$
Proportion	p	P_S
Variance	σ^2	S^2
Difference	$\mu_1 - \mu_2$	$\bar{X}_1 - \bar{X}_2$

Interval Estimation

- However the value of the sample statistic will vary from sample to sample therefore, to simply obtain an estimate of the single value of the parameter is not generally acceptable.
  - We need also a measure of how precise our estimate is likely to be.
  - We need to take into account the sample to sample variation of the statistic.
- A confidence interval defines an interval within which the true population parameter is like to fall (interval estimate).

Confidence Intervals…

- Confidence interval therefore takes into account the sample to sample variation of the statistic and gives the measure of precision.
- The general formula used to calculate a Confidence interval is Estimate ± K × Standard Error, k is called reliability coefficient.
- Confidence intervals express the inherent uncertainty in any medical study by expressing upper and lower bounds for anticipated true underlying population parameter.
- The confidence level is the probability that the interval estimate will contain the parameter, assuming that a large number of samples are selected and that the estimation process on the same parameter is repeated.

Confidence intervals…

- Most commonly the 95% confidence intervals are calculated, however 90% and 99% confidence intervals are sometimes used.
- The probability that the interval contains the true population parameter is (1-α)100%.
- If we were to select 100 random samples from the population and calculate confidence intervals for each, approximately 95 of them would include the true population mean B (and 5 would not)

Confidence interval ……

A (1-α) 100% confidence interval for unknown population mean and population proportion is given as follows;

$$\Rightarrow [\bar{x} - z_{\frac{\alpha}{2}} . \frac{\delta}{\sqrt{n}}, \quad \bar{x} + z_{\frac{\alpha}{2}} . \frac{\delta}{\sqrt{n}}]$$

$$\Rightarrow [\hat{p} - z_{\frac{\alpha}{2}} . \sqrt{p(1-p)/n}, \quad \hat{p} + z_{\frac{\alpha}{2}} . \sqrt{p(1-p)/n}]$$

Interval estimation

- Provides range of values
  - Takes into consideration variation in sample statistics from sample to sample
  - Is based on observation from one sample
  - Gives information about closeness to unknown population parameters
  - Is stated in terms of level of confidence
    - Never 100% certain

Elements of Confidence Interval Estimation

- Level of confidence
  - Confidence in which the interval will contain the unknown population parameter
- Precision (range)
  - Closeness to the unknown parameter
- Cost
  - Cost required to obtain a sample of size n

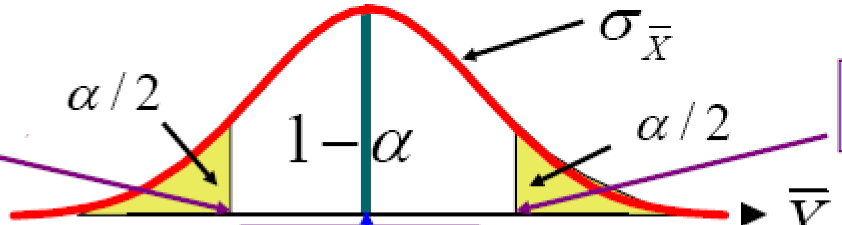
Level of Confidence

- Denoted by $100(1-\alpha)\%$
- A relative frequency interpretation
  - In the long run, $100(1-\alpha)\%$ of all the confidence intervals that can be constructed will contain the unknown parameter
- A specific interval will either contain or not contain the parameter

Interval and Level of Confidence

Sampling Distribution of the Mean

$\mu - Z_{\alpha/2}\sigma_{\bar{X}}$



$\mu + Z_{\alpha/2}\sigma_{\bar{X}}$

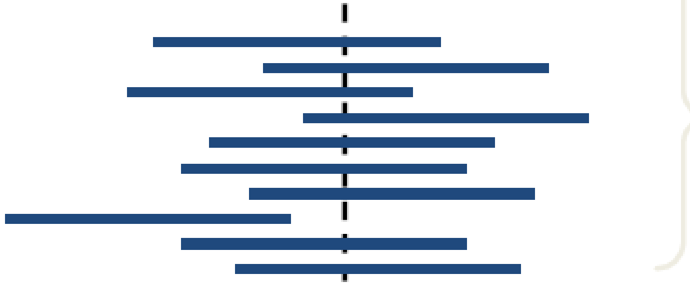
$\mu_{\bar{X}} = \mu$

Intervals extend from

$\bar{X} - Z\sigma_{\bar{X}}$

to

$\bar{X} + Z\sigma_{\bar{X}}$



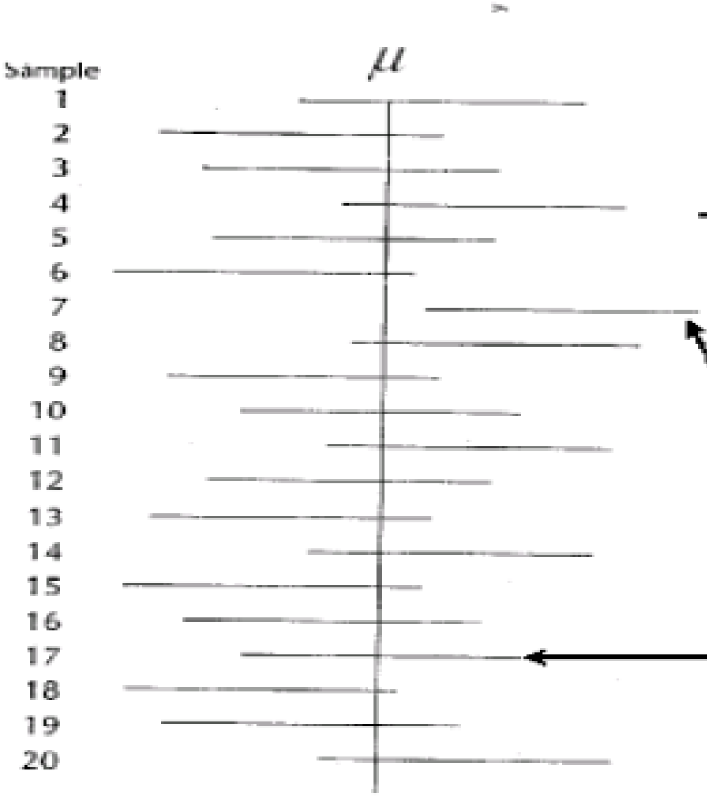
$(1-\alpha)100\%$ of intervals constructed contain μ;

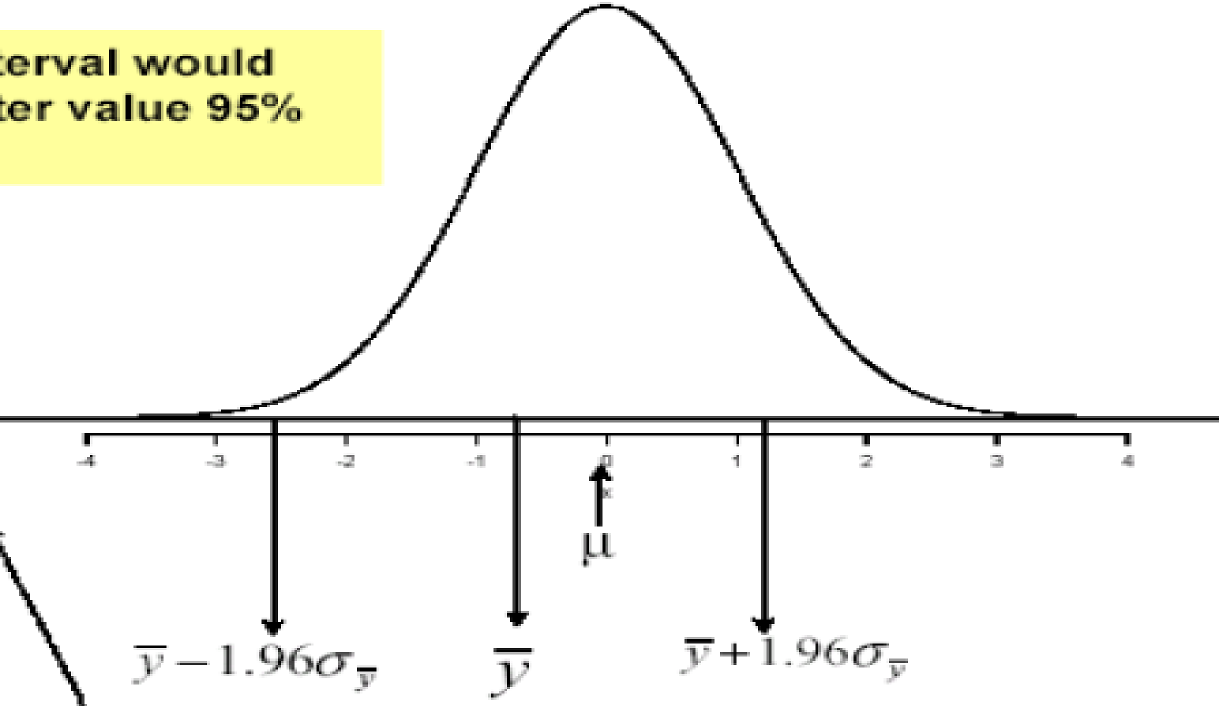
$100\alpha\%$ do not.

Confidence Intervals

Meaning of a 95% Confidence Interval

In repeated sampling, the interval would encompass the true parameter value 95% of the time.





-4
-3
-2
-1
0
1
2
3
4
μ
$\overline{y}-1.96\sigma_{\overline{y}}$
$\overline{y}$
$\overline{y}+1.96\sigma_{\overline{y}}$

5% of the intervals don't include the true mean.

95% of the intervals include the true mean.

Confidence intervals…

- The 95% confidence interval is calculated in such a way that, under the conditions assumed for underlying distribution, the interval will contain true population parameter 95% of the time.
- Loosely speaking, you might interpret a 95% confidence interval as one which you are 95% confident contains the true parameter.
- 90% CI is narrower than 95% CI since we are only 90% certain that the interval includes the population parameter.
- On the other hand 99% CI will be wider than 95% CI; the extra width meaning that we can be more certain that the interval will contain the population parameter. But to obtain a higher confidence from the same sample, we must be willing to accept a larger margin of error (a wider interval).

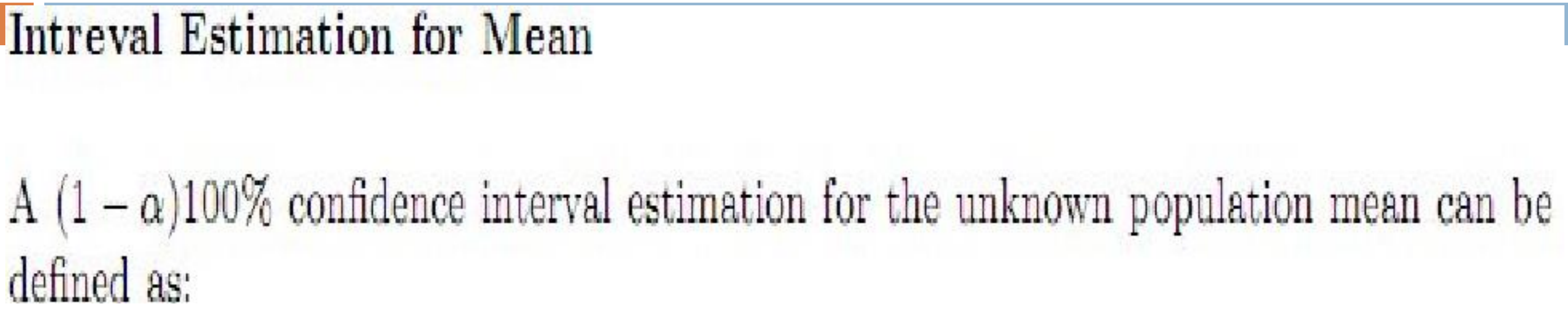
Confidence intervals…

- For a given confidence level (i.e. 90%, 95%, 99%) the width of the confidence interval depends on the standard error of the estimate which in turn depends on the
  - 1. **Sample size**:-The larger the sample size, the narrower the confidence interval (this is to mean the sample statistic will approach the population parameter) and the more precise our estimate. Lack of precision means that in repeated sampling the values of the sample statistic are spread out or scattered. The result of sampling is not repeatable.

Confidence intervals…

- - To increase precision (of an SRS), use a larger sample. You can make the precision as high as you want by taking a large enough sample. The margin of error decreases as $\sqrt{n}$ increases.
- **2. Standard deviation**:-The more the variation among the individual values, the wider the confidence interval and the less precise the estimate. As sample size increases SD decreases.
  - Z is the value from SND
    - 90% CI, z=1.64
    - 95% CI, z=1.96

Confidence interval for a single mean



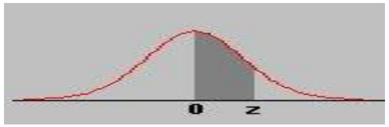
Intreval Estimation for Mean

A $(1-\alpha)100\%$ confidence interval estimation for the unknown population mean can be defined as:

- CI = $$\left(\bar{x} - z_{\alpha/2}\frac{\delta}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\delta}{\sqrt{n}}\right)$$

- Most commonly, we used to compute 95% confidence interval, however, it is possible to compute 90% and 99% confidence interval estimation.

Table 1: Normal distribution



Area between 0 and z

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916

Confidence interval ……

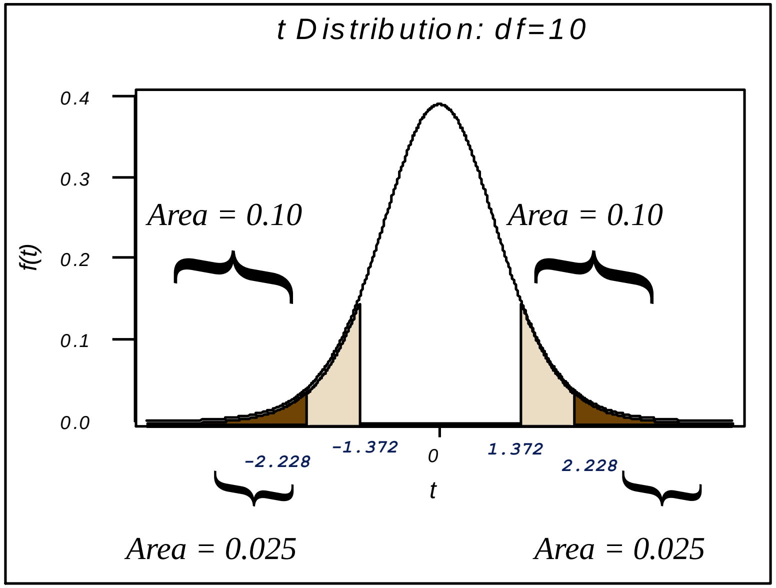
- If the population standard deviation is unknown and the sample size is small (<30), the formula for the confidence interval for sample mean is:

$$\left(\bar{x} - t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}, \bar{x} + t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}\right)$$

  - x is the sample mean
  - s is the sample standard deviation
  - n is the sample size
  - t is the value from the t-distribution with (n-1) degrees of freedom

The t Distribution

df	t0.100	t0.050	t0.025	t0.010	t0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	**1.372**	**1.812**	**2.228**	**2.764**	**3.169**
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	**1.282**	**1.645**	**1.960**	**2.326**	**2.576**



Whenever σ is not known (and the population is assumed normal), the correct distribution to use is the t distribution with n-1 degrees of freedom. Note, however, that for large degrees of freedom, the t distribution is approximated well by the Z distribution.

Point and Interval Estimation of the Population Proportion (p)

We will now consider the method for estimating the binomial proportion p of successes, that is, the proportion of elements in a population that have a certain characteristic. A logical candidate for a point estimate of the population proportion p is the sample proportion $\hat{P} = \frac{x}{m}$ where x is the number of observations in a sample of size n that have the characteristic of interest. As we have seen in sampling distribution of proportions, the sample proportion is the best point estimate of the population proportion.

Proportion…

- *The shape is approximately normal provided n is sufficiently large - in this case, nP > 5 and nQ > 5 are the requirements for sufficiently large n (central limit theorem for proportions) .*
- *The point estimate for population proportion π is given by þ.*
- *A (1-α)100% confidence interval estimate for the unknown population proportion π is given by:*

$$CI = \left(p - Z_{\frac{\alpha}{2}}\sqrt{\pi(1-\pi)/n},\ p + Z_{\frac{\alpha}{2}}\sqrt{\pi(1-\pi)/n} \right)$$

- *If the sample size is small, i.e. np < 5 and nq < 5, and the population standard deviations for proportion are not given, then the confidence interval estimation will take t-distribution instead of z as:*

Example 1:

- A SRS of 16 apparently healthy subjects yielded the following values of urine excreted (milligram per day);

  0.007, 0.03, 0.025, 0.008, 0.03, 0.038, 0.007, 0.005, 0.032, 0.04, 0.009, 0.014, 0.011, 0.022, 0.009, 0.008

Compute point estimate of the population mean

If $x_1, x_2, \ldots, x_n$ are n observed values, then

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{0.295}{16} = 0.01844$$

Construct 90%, 95%, 98% confidence interval for the mean

(0.01844-1.65x0.0123/4, 0.01844+1.65x0.0123/4)=(0.0134, 0.0235)

(0.01844-1.96x0.0123/4, 0.01844+1.96x0.0123/4)=(0.0124, 0.0245)

(0.01844-2.33x0.0123/4, 0.01844+2.33x0.0123/4)=(0.0113, 0.0256)

Example 2

- The mean diastolic blood pressure for 225 randomly selected individuals is 75 mmHg with a standard deviation of 12.0 mmHg. Construct a 95% confidence interval for the mean
- **Solution**

  n=225

  mean =75mmhg

  Standard deviation=12 mmHg

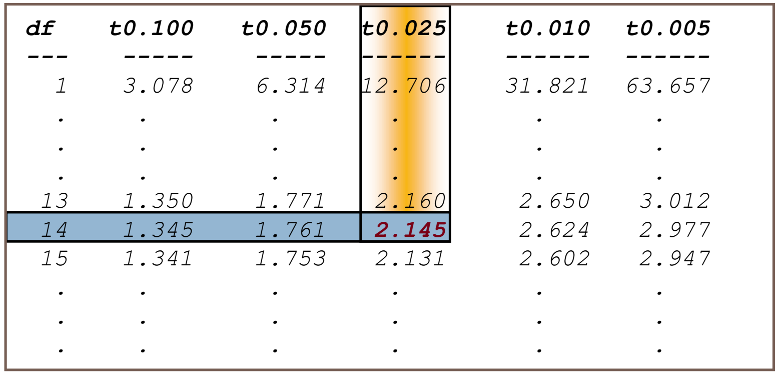
  confidence level 95%

The 95% confidence interval for the unknown population mean is given

- 95%CI = (75 ±1.96x12/15) = (73.432,76.56)

Example 2:

A stock market analyst wants to estimate the average return on a certain stock. A random sample of 15 days yields an average (annualized) return of $\bar{x} = 10.37$ *and a standard deviation of s = 3.5. Assuming a normal population of returns, give a 95% confidence interval for the average return on this stock.*



df	t0.100	t0.050	t0.025	t0.010	t0.005
1	3.078	6.314	12.706	31.821	63.657
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	**2.145**	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

The critical value of t for df = (n -1) = (15 -1) =14 and a right-tail area of 0.025 is:

$$t_{0.025} = 2.145$$

The corresponding confidence interval or interval estimate is: $\bar{x} \pm t_{0.025}\frac{s}{\sqrt{n}}$

$$= 10.37 \pm 2.145\frac{3.5}{\sqrt{15}}$$

$$= 10.37 \pm 1.94$$

$$= [8.43, 12.31]$$

Example 3:

- In a survey of 300 automobile drivers in one city, 123 reported that they wear seat belts regularly. Estimate the seat belt rate of the city and 95% confidence interval for true population proportion.
- Answer : $p = 123/300 = 0.41 = 41\%$

  $n=300$,

  Estimate of the seat belt of the city at 95%

$CI = p \pm z \times(\sqrt{p(1\text{-}p)}\ /n) = (0.35,0.47)$

Example 4:

In a sample of 400 people who were questioned regarding their participation in sports, 160 said that they did participate. Construct a 98 % confidence interval for P, the proportion of P in the population who participate in sports.

Solution:

Let X= be the number of people who are interested to participate in sports.

X=160, n=400, α =0.02, Hence

$$Z_{\alpha/2} = Z_{0.01} = 2.33$$

$$\hat{P} = \frac{X}{n} = \frac{160}{400} = 0.4$$

$$\sigma^2_{\hat{P}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{0.4(0.6)}{400}} = 0.0245$$

As a result, an approximate 98% confidence interval for P is given by:

$$\Rightarrow \hat{P} - Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} < P < \hat{P} + Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}})$$

$$\Rightarrow \left((0.4 - (2.33 * 0.0245)), (0.4 + (2.33 * 0.0245)\right)$$

$$\Rightarrow \left(0.345, 0.457\right)$$

Hence, we can conclude that about 98% confident that the true proportion of people in the population who participate in sports between 34.5% and 45.7%.

HYPOTHESIS TESTING

Introduction

- Researchers are interested in answering many types of questions. For example, A physician might want to know whether a new medication will lower a person's blood pressure.

- These types of questions can be addressed through statistical hypothesis testing, which is a decision-making process for evaluating claims about a population.

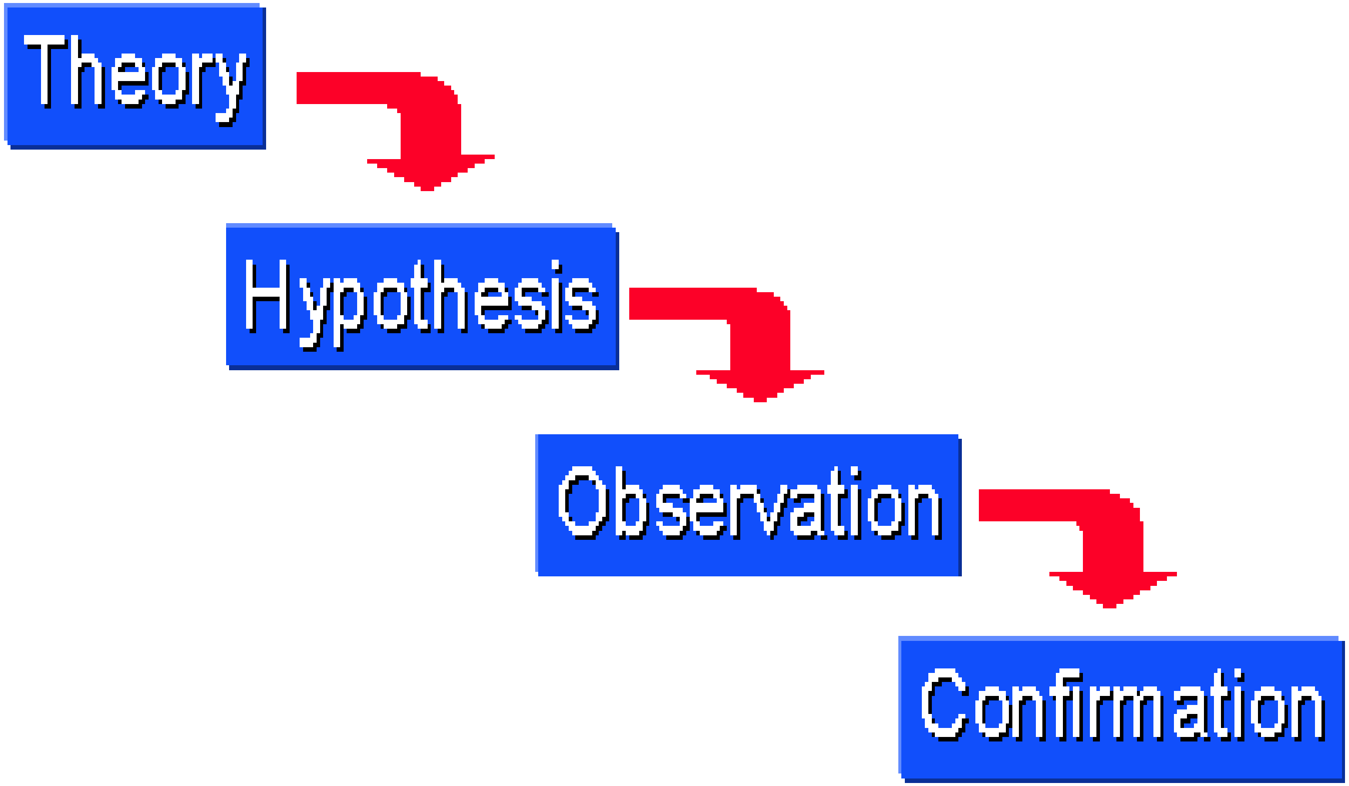
Hypothesis Testing

- The formal process of hypothesis testing provides us with a means of answering research questions.

- Hypothesis is a testable statement that describes the nature of the proposed relationship between two or more variables of interest.

- In hypothesis testing, the researcher must defined the population under study, state the particular hypotheses that will be investigated, give the significance level, select a sample from the population, collect the data, perform the calculations required for the statistical test, and reach a conclusion.

Idea of hypothesis testing



Theory

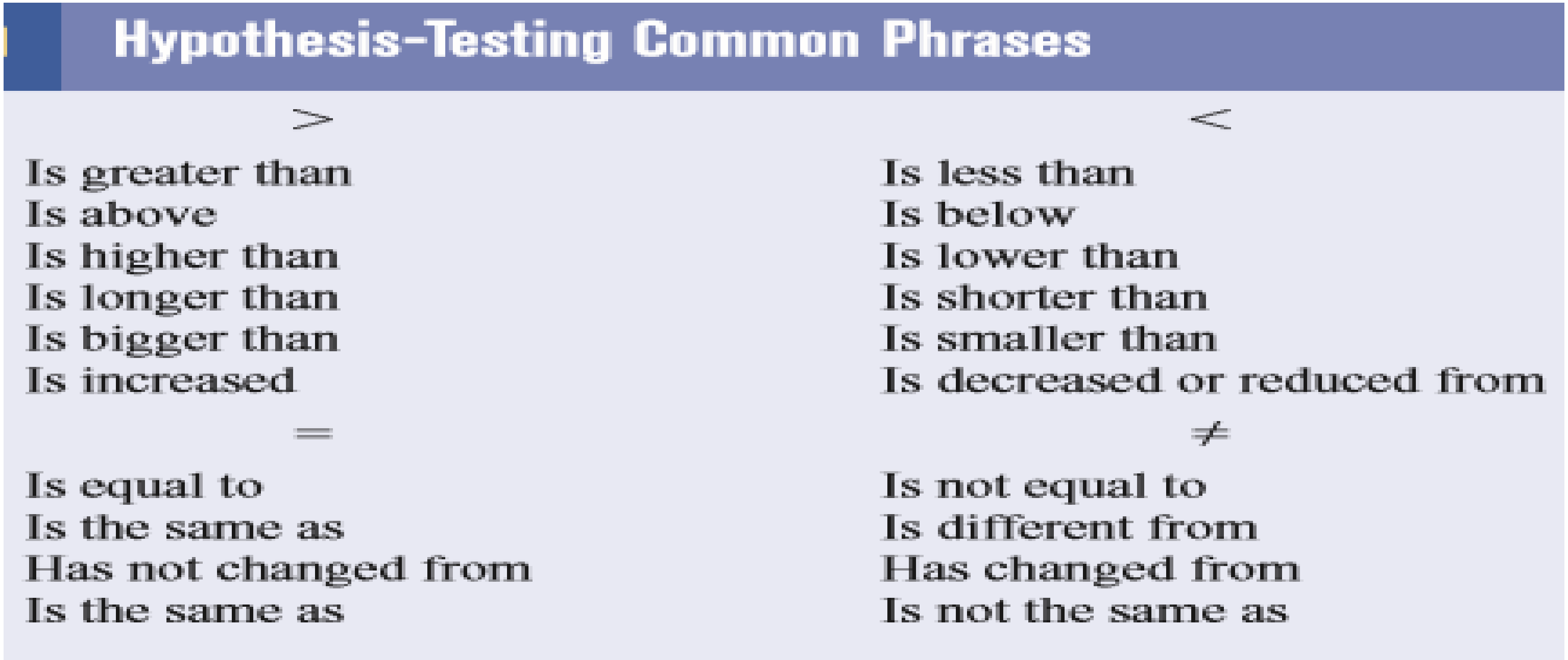
Hypothesis

Observation

Confirmation

type of Hypotheses

- □ ***Null hypothesis*** (represented by H_O) is the statement about the value of the population parameter. That is the null hypothesis postulates that 'there is no difference between factor and outcome' or 'there is no an intervention effect'.
- □ ***Alternative hypothesis*** (represented by H_A) states the 'opposing' view that 'there is a difference between factor and outcome' or 'there is an intervention effect'.



Hypothesis-Testing Common Phrases

>	<
Is greater than	Is less than
Is above	Is below
Is higher than	Is lower than
Is longer than	Is shorter than
Is bigger than	Is smaller than
Is increased	Is decreased or reduced from
=	≠
Is equal to	Is not equal to
Is the same as	Is different from
Has not changed from	Has changed from
Is the same as	Is not the same as

Methods of hypothesis testing

- Hypotheses concerning about parameters which may or may not be true
- Examples
  - The mean GPA of this class is 3.5!
  - The mean height of the Gondar College of Medical Sciences (GCMS) students is 1.63m.
  - There is no difference between the distribution of Pf and Pv malaria in Ethiopia (are distributed in equal proportions.)

Steps in hypothesis testing

1

Identify the null hypothesis H_0 and the alternate hypothesis H_A.

2

Choose α. The value should be small, usually less than 10%. It is important to consider the consequences of both types of errors.

3

Select the test statistic and determine its value from the sample data. This value is called the observed value of the test statistic. Remember that t statistic is usually appropriate for a small number of samples; for larger number of samples, a z statistic can work well if data are normally distributed.

4

Compare the observed value of the statistic to the critical value obtained for the chosen α.

5

Make a decision.

6

Conclusion

Test Statistics

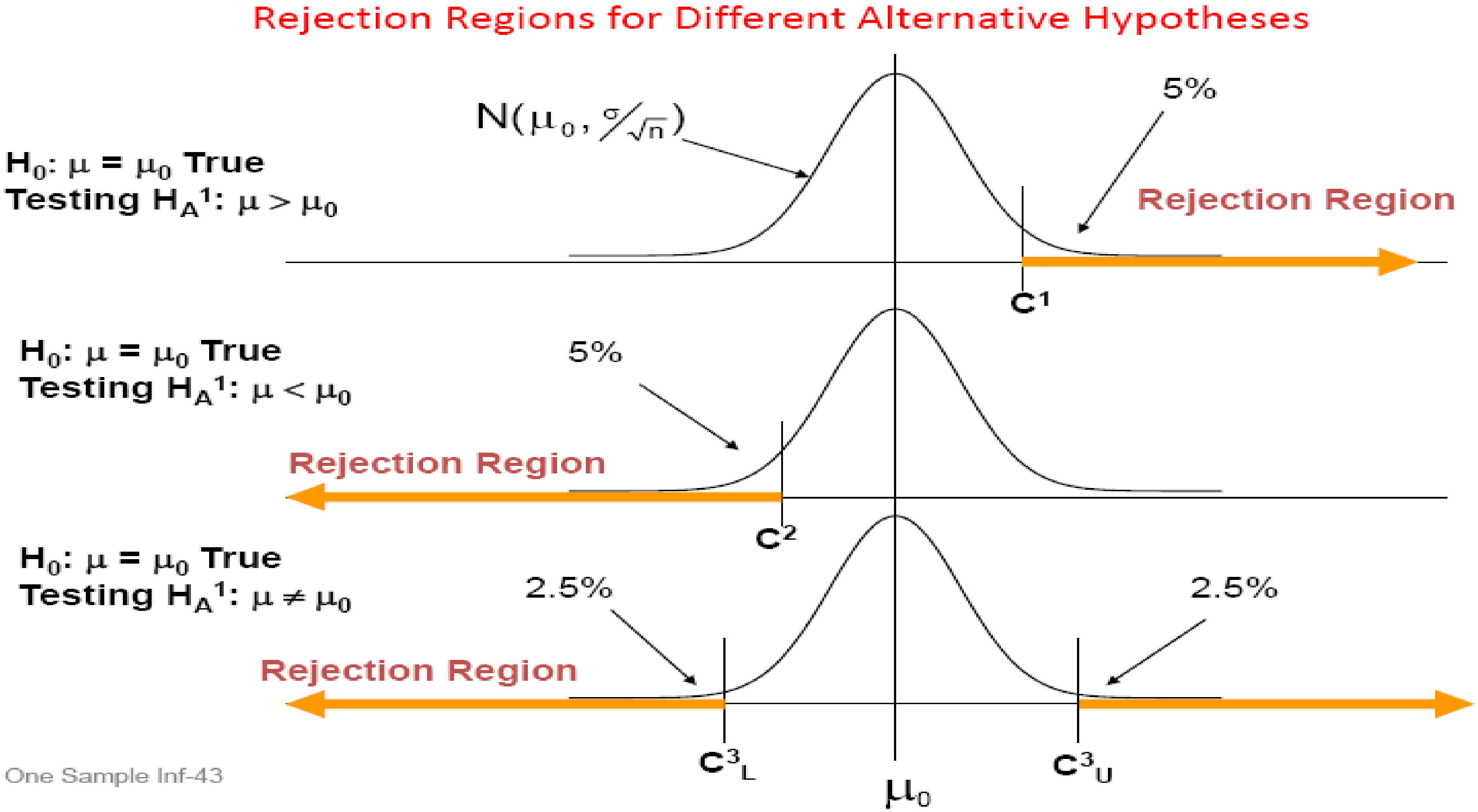
- Because of random variation, even an unbiased sample may not accurately represent the population as a whole.
- As a result, it is possible that any observed differences or associations may have occurred by chance.
- A test statistics is a value we can compare with known distribution of what we expect when the null hypothesis is true.
- The general formula of the test statistics is:

$$\text{Test statistics} = \frac{\left\{\begin{matrix}\text{Observed} \\ \text{value}\end{matrix}\right\} - \left\{\begin{matrix}\text{Hypothesized} \\ \text{value}\end{matrix}\right\}}{\text{Standard error}}$$

- **The known distributions are Normal distribution, student's distribution , Chi-square distribution ….**

Critical value

- The critical value separates the critical region from the noncritical region for a given level of significance


Rejection Regions for Different Alternative Hypotheses
$N(\mu_0, \sigma/\sqrt{n})$
5%
H_0: $\mu = \mu_0$ True
Testing $H_A{}^1$: $\mu > \mu_0$
Rejection Region
C^1
H_0: $\mu = \mu_0$ True
Testing $H_A{}^1$: $\mu < \mu_0$
5%
Rejection Region
C^2
H_0: $\mu = \mu_0$ True
Testing $H_A{}^1$: $\mu \neq \mu_0$
2.5%
2.5%
Rejection Region
$C^3{}_L$
μ_0
$C^3{}_U$
One Sample Inf-43

Decision making

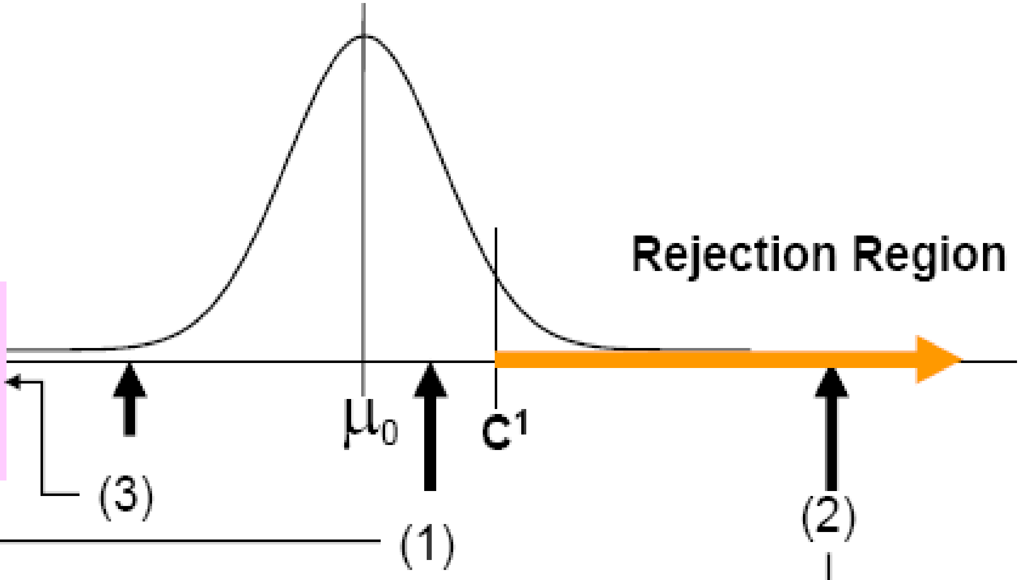
- **Accept or Reject** the null hypothesis
- There are 2 types of errors

Type of decision	H_0 true	H_0 false
Reject H_0	**Type I error (a)**	**Correct decision (1-β)**
Accept H_0	**Correct decision (1-a)**	**Type II error (β)**

- Type I error is more serious error and it is the level of significant
- power is the probability of rejecting false null hypothesis and it is given by 1-β

Type I Error

H_0: True



If the sample mean is at location (1) or (3) we make the correct decision.

If the sample mean is at location (2) **and H_0 is actually true**, we make the wrong decision.

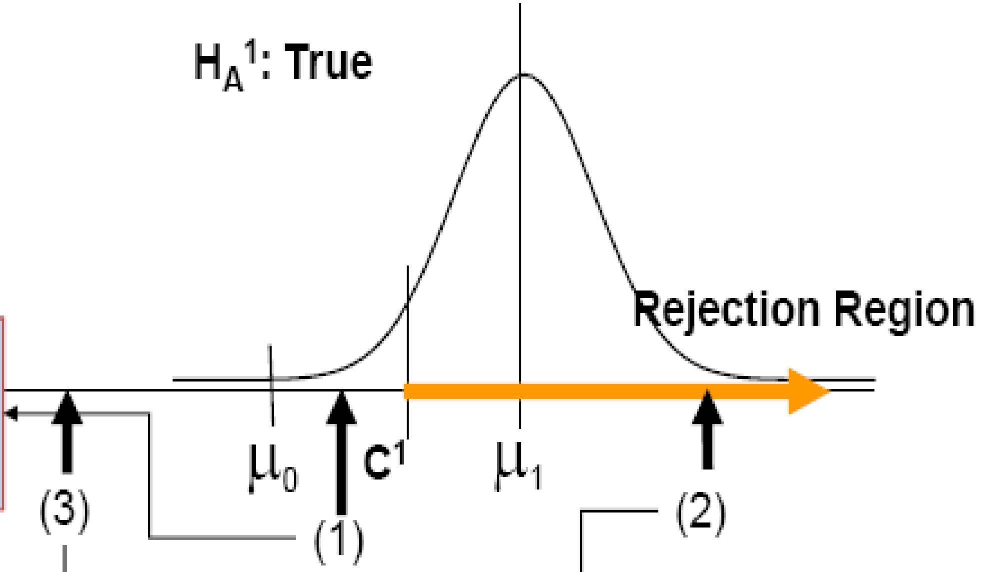
This is called making a **TYPE I error**, and the **probability** of making this error is usually denoted by the Greek letter α.

α = P(Reject H_0 when H_0 is is the true condition)

If $C^1 = \mu_0 + 1.645 \frac{\sigma}{\sqrt{n}}$ then α = 1/20=5/100 or .05

If $C^1 = \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$ then the Type I error is α.

Type II Error



If the sample mean is at location (1) or (3) we make **the wrong decision.**

If the sample mean is at location (2) we make the correct decision.

This is called making a **TYPE II error**, and the probability of making this type error is usually denoted by the Greek letter β.

β = P(Do Not Reject H_0 when H_A is the true condition)

Determining the Critical Value for the Rejection Region

Reject H_0 if the sample mean is larger than “expected”.

If H_0 were true, we would expect 95% of sample means to be less than the upper limit of a 95% CI for μ.

$$C^1 = \mu_0 + 1.645 \frac{\sigma}{\sqrt{n}}$$

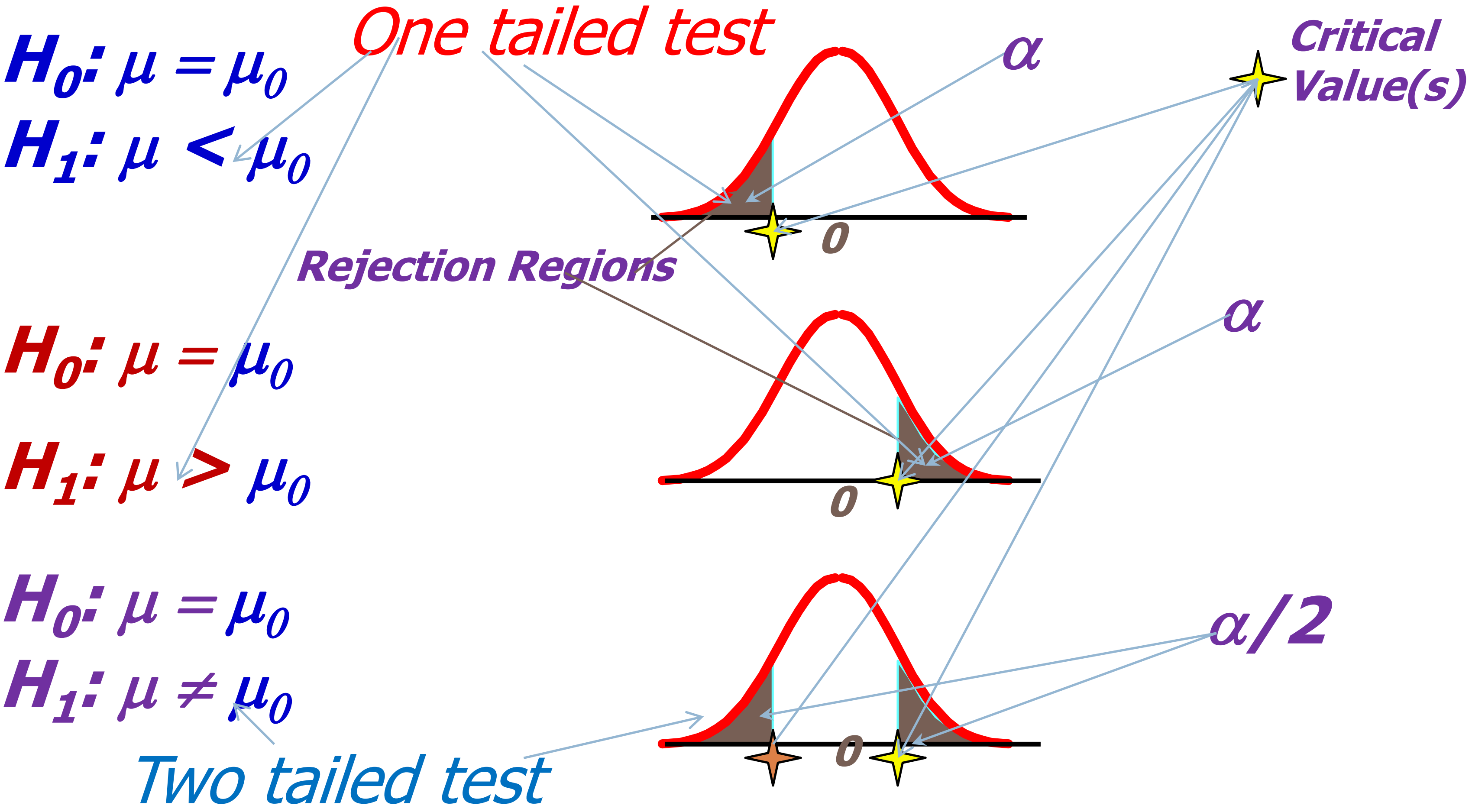
From the standard normal table.

In this case, if we use this critical value, in 5 out of 100 repetitions of the study we would reject H_o incorrectly. That is, we would make an error.

But, suppose $H_A{}^1$ is the true situation, then most sample means will be greater than C^1 and we will be making the correct decision to reject more often.

$$P\left(\frac{\bar{y}-\mu_0}{\sigma/\sqrt{n}} > 1.645\right) = 0.05$$

Types of testes



$H_0: \mu = \mu_0$

$H_1: \mu < \mu_0$

One tailed test

$H_0: \mu = \mu_0$

$H_1: \mu > \mu_0$

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

Two tailed test

Rejection Regions

Critical Value(s)

Hypothesis testing about a Population mean (μ)

Two Tailed Test:

The large sample (n > = 30) test of hypothesis about a population mean μ is as follows

1

$$H_0 : \mu = \mu_0 (\pi = \pi_0)$$

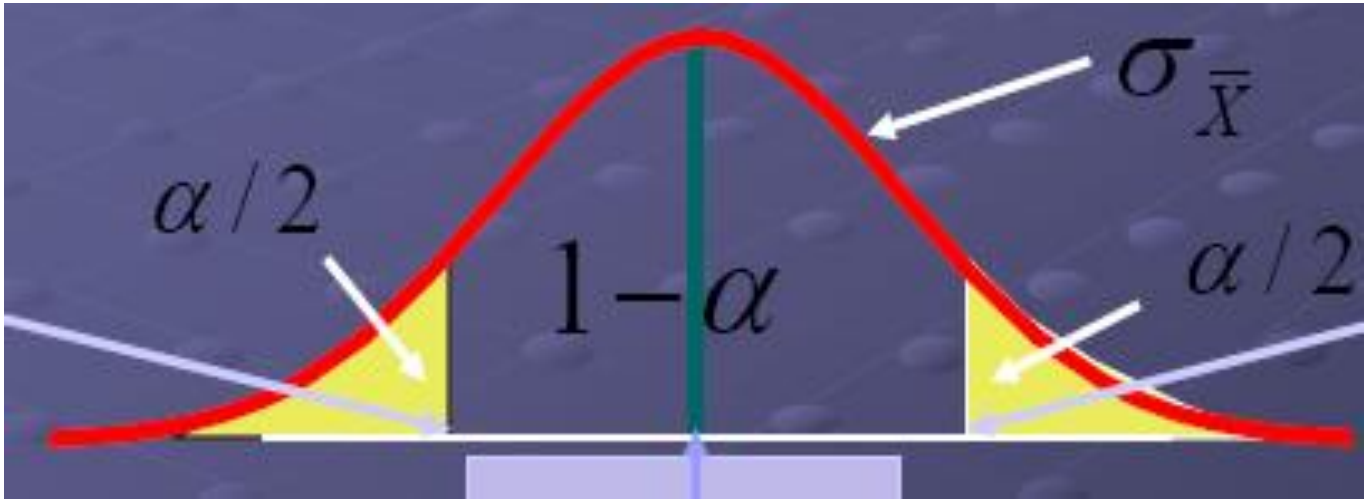
$$H_A : \mu_1 \neq \mu_0 (\pi \neq \pi_0)$$

$$z_{cal} = \frac{\overline{x} - \mu_0}{\frac{\delta}{\sqrt{n}}}$$

$$z_{tabulated} = z_{\frac{\alpha}{2}} \quad for\ two\ tailed\ test$$

$$Decision: \begin{cases} if\ |z_{cal}| > z_{tab}\ reject\ H_o \\ if\ |z_{cal}| < z_{tab}\ do\ not\ reject\ H_o \end{cases}$$

Steps in hypothesis testing…..



If the test statistic falls in the critical region:

Reject H_0 in favour of H_A.

If the test statistic does not fall in the critical region:

Conclude that there is not enough evidence to reject H_0.

One tailed tests

2 $H_0 : \mu = \mu_0 (\pi = \pi_0)$

$H_A : \mu_1 < \mu_0 (\pi < \pi_0)$

$$z_{cal} = \frac{\bar{x} - \mu_0}{\frac{\delta}{\sqrt{n}}}, \quad z_{tabulated} = z_\alpha \quad \text{for one tailed test}$$

$$\text{Decision}: \quad \begin{cases} \text{if } z_{cal} < -z_{tab} \text{ reject } H_o \\ \text{if } z_{cal} > -z_{tab} \text{ do not reject } H_o \end{cases}$$

3 $H_0 : \mu = \mu_0 (\pi = \pi_0)$

$H_A : \mu_1 > \mu_0 (\pi > \pi_0)$

$$\text{Decision}: \quad \begin{cases} \text{if } z_{cal} > z_{tab} \text{ reject } H_o \\ \text{if } z_{cal} < z_{tab} \text{ do not reject } H_o \end{cases}$$

The P- Value

- In most applications, the outcome of performing a hypothesis test is to produce a ***p-value.***
- P-value is the probability that the observed difference is due to chance.

- A large p-value implies that the probability of the value observed, occurring just by chance is low, when the null hypothesis is true.

- That is, a small p-value suggests that there might be sufficient evidence for rejecting the null hypothesis.
- The p value is defined as the probability of observing the computed significance test value or a larger one, if the H0 hypothesis is true. For example, P[Z >=Zcal/H0 true].

P-value……

- A **p-value is the probability of getting the observed difference, or one more** extreme, in the sample purely by chance from a population where the true difference is zero.

- If the p-value is greater than 0.05 then, by convention, we conclude that the observed difference could have occurred by chance and there is no statistically significant evidence (at the 5% level) for a difference between the groups in the population.

How to calculate P-value

- Use statistical software like SPSS, SAS……..
- Hand calculations

—obtained the test statistics (Z Calculated or t-calculated)

—find the probability of test statistics from standard normal table

—subtract the probability from 0.5

—the result is P-value

Note if the test two tailed multiply 2 the result.

P-value and confidence interval

- Confidence intervals and p-values are based upon the same theory and mathematics and will lead to the same conclusion about whether a population difference exists.

- Confidence intervals are referable because they give information about the size of any difference in the population, and they also (very usefully) indicate the amount of uncertainty remaining about the size of the difference.

- When the null hypothesis is rejected in a hypothesis-testing situation, the confidence interval for the mean using the same level of significance will not contain the hypothesized mean.

The P- Value .....

- But for what values of p-value should we reject the null hypothesis?
  - By convention, a p-value of 0.05 or smaller is considered sufficient evidence for rejecting the null hypothesis.
  - By using p-value of 0.05, **we are allowing a 5% chance of wrongly rejecting the null hypothesis when it is in fact true.**

- When the p-value is less than to 0.05, we often say that the result is ***statistically significant*.**

Hypothesis testing for single population mean

EXAMPLE 5: A researcher claims that the mean of the IQ for 16 students is 110 and the expected value for all population is 100 with standard deviation of 10. Test the hypothesis .

- Solution

1. Ho:μ=100 VS HA:$\mu \neq$100
2. Assume α=0.05
3. Test statistics: $z=(110-100)4/10=4$
4. z-critical at 0.025 is equal to 1.96.
5. Decision: reject the null hypothesis since $4 \geq 1.96$
6. Conclusion: the mean of the IQ for all population is different from 100 at 5% level of significance.

Example 6:

Suppose that we have a population mean 3.1 and n=20 people $\bar{x} = 4.5$ and $s = 5.5$ found and , our test statistic is

1. Ho:$\mu = 3.1$

   HA: $\mu \neq 3.1$

2. α = 0.5 at 95% CI

3. $t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{4.5 - 3.1}{5.5 / \sqrt{20}} = 1.14$ $t_{0.05,19} = 2.09$

4. the observed value of the test statistic falls with in the range of the critical values

5. we accept Ho and conclude that there is no enough evidence to reject the null hypothesis.

Cont….

A 95% confidence interval for the mean is

$$\bar{x} \pm t_{0.05,19} s/\sqrt{n} = 4.5 \pm 2.09(5.5/\sqrt{20}) = (1.93, 7.07)$$

Note that this interval includes the hypothesis value of 3.1

Hypothesis testing for single proportions

Example 7: In the study of childhood abuse in psychiatry patients, brown found that 166 in a sample of 947 patients reported histories of physical or sexual abuse.

a) constructs 95% confidence interval

b) test the hypothesis that the true population proportion is 30%?

- Solution (a)
  - The 95% CI for P is given by

$$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$$

$$\Longrightarrow 0.175 \pm 1.96 \times \sqrt{\frac{0.175 \times 0.825}{947}}$$

$$\Longrightarrow 0.175 \pm 1.96 \times 0.0124$$

$$\Longrightarrow [0.151 ; 0.2]$$

Example……

- □ To the hypothesis we need to follow the steps

Step 1: State the hypothesis

Ho: P=Po=0.3

Ha: P≠Po ≠0.3

Step 2: Fix the level of significant (α=0.05)

Step 3: Compute the calculated and tabulated value of the test statistic

$$z_{cal} = \frac{\hat{p} - Po}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.175 - 0.3}{\sqrt{\frac{0.3(0.7)}{947}}} = \frac{-0.125}{0.0149} = -8.39$$

$$z_{tab} = 1.96$$

Example……

- Step 4: Comparison of the calculated and tabulated values of the test statistic
- Since the tabulated value is smaller than the calculated value of the test the we reject the null hypothesis.
- Step 6: Conclusion
- Hence we concluded that the proportion of childhood abuse in psychiatry patients is different from 0.3
- If the sample size is small (if $np<5$ and $n(1-p)<5$) then use student's t- statistic for the tabulated value of the test statistic.

Chi-square test

- In recent years, the use of specialized statistical methods for categorical data has increased dramatically, particularly for applications in the biomedical and social sciences.
- Categorical scales occur frequently in the health sciences, for measuring responses.
- E.g.
  - patient survives an operation (yes, no),
  - severity of an injury (none, mild, moderate, severe), and
  - stage of a disease (initial, advanced).
- Studies often collect data on categorical variables that can be summarized as a series of counts and commonly arranged in a tabular format known as a **contingency table**

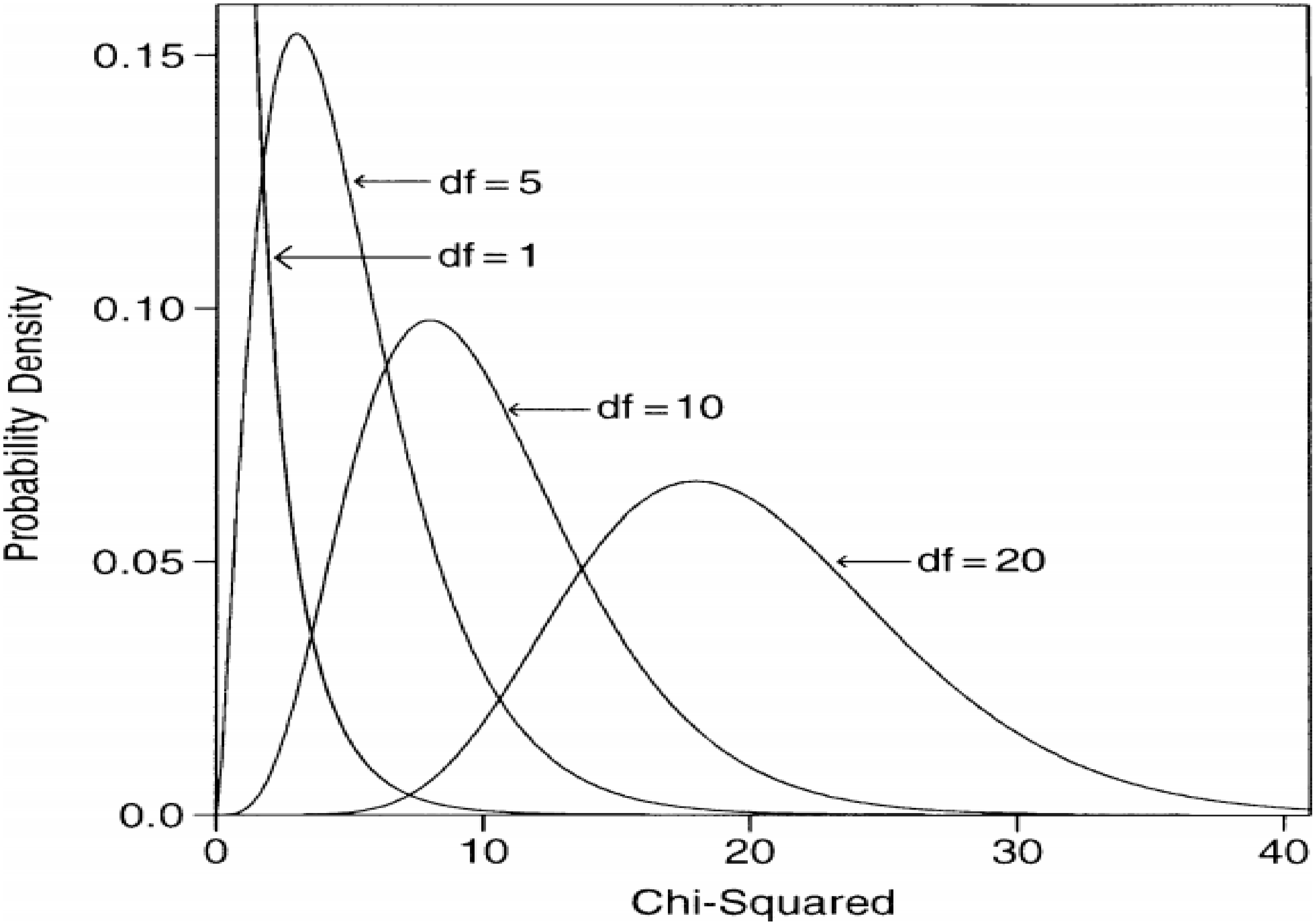
Chi-square Test Statistic cont'd…

- As with the z and t distributions, there is a different chi-square distribution for each possible value of degrees of freedom.

- Chi-square distributions with a small number of degrees of freedom are highly skewed; however, this skewness is attenuated as the number of degrees of freedom increases.

- The chi-squared distribution is concentrated over nonnegative values. It has mean equal to its degrees of freedom (df), and its standard deviation equals √(2df). As df increases, the distribution concentrates around larger values and is more spread out.

- The distribution is skewed to the right, but it becomes more bell-shaped (normal) as df increases.



- The degrees of freedom for tests of hypothesis that involve an rxc contingency table is **equal to (r-1)x(c-1);**

Test of Association

- The chi-squared (χ^2) test statistics is widely used in the analysis of contingency tables.
- It compares the actual observed frequency in each group with the expected frequency (the later is based on theory, experience or comparison groups).
- The chi-squared test (Pearson's χ2) allows us to test for association between categorical (nominal!) variables.
- The null hypothesis for this test is there is no association between the variables. Consequently a significant p-value implies association.

Test of Association

- It is a requirement that a chi-squared test be applied to discrete data. Counting numbers are appropriate, continuous measurements are not. Assuming continuity in the underlying distribution distorts the p value and may make false positives more likely.
- Additionally, chi squared test should not be used when the observed values in a cell are <5. It is, **at times not inappropriate** to pad an empty cell with a small value, though, as one can only assume the result would be more significant with no value there.

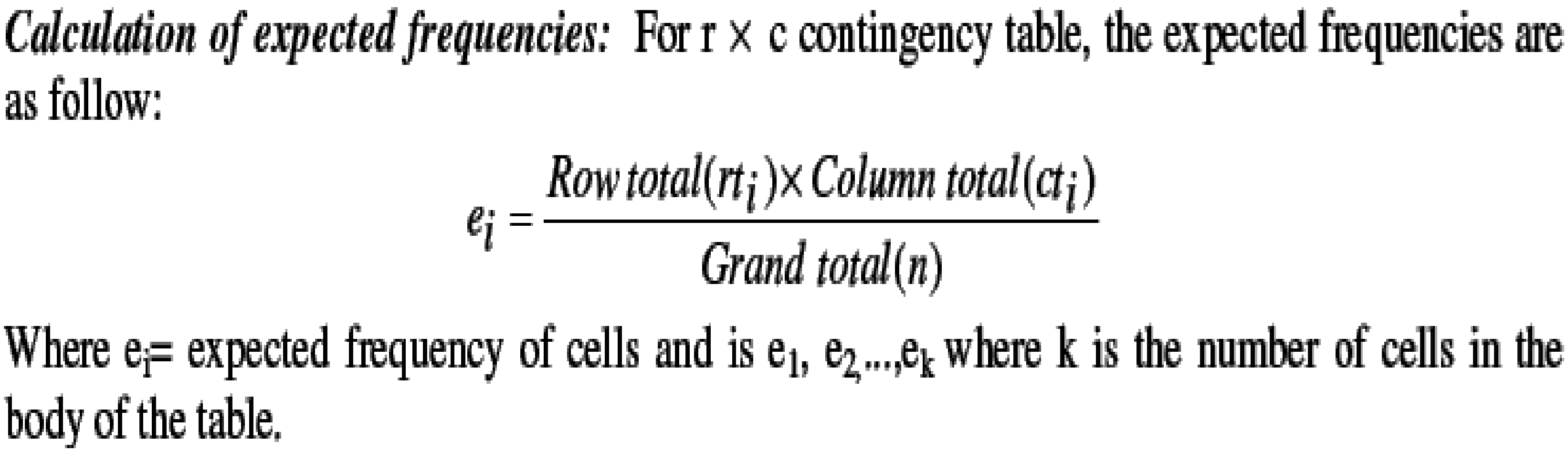
Test Statistic: χ^2-test with d.f. = (r-1)x(c-1)

$$\chi^2 = \sum_{i,j} \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

$$E_{ij} = \frac{i^{th} \text{ raw total} \times \text{ j}^{\text{th}} \text{ column total}}{\text{grand total}} = \frac{R_i \times C_j}{n}$$

O_{ij}=observed frequency, E_{ij}=expected frequency of the cell at the juncture of I th raw & j th column

Chi-square test…

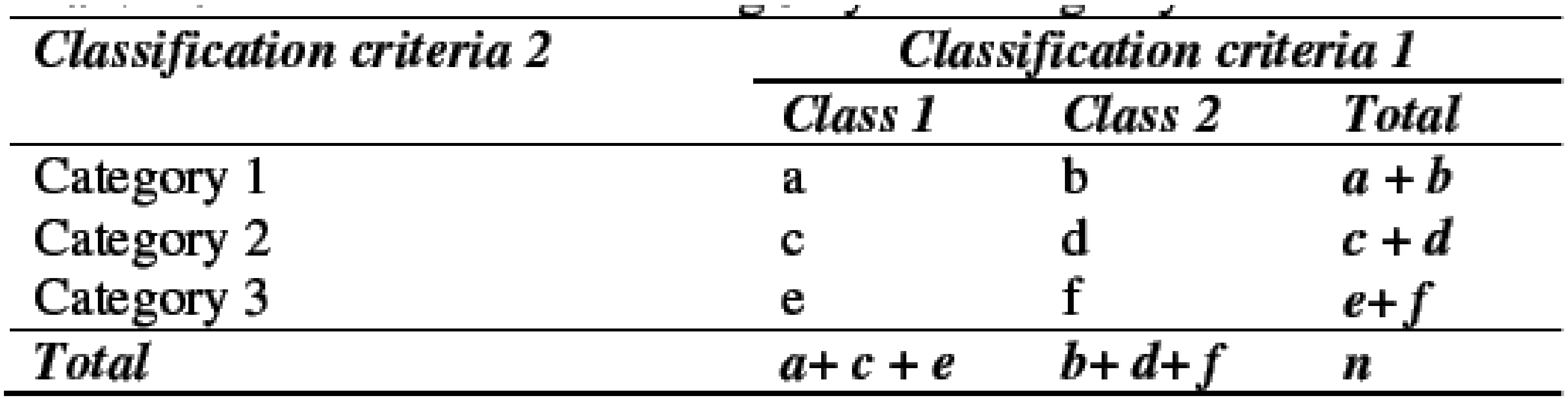


Calculation of expected frequencies: For r × c contingency table, the expected frequencies are as follow:

$$e_i = \frac{Row\ total(rt_i) \times Column\ total(ct_i)}{Grand\ total(n)}$$

Where e_i= expected frequency of cells and is e_1, e_2,…,e_k where k is the number of cells in the body of the table.

Consider the following 3 by 2 contingency table



Classification criteria 2	*Classification criteria 1*		
	Class 1	*Class 2*	*Total*
Category 1	a	b	$a + b$
Category 2	c	d	$c + d$
Category 3	e	f	$e + f$
Total	$a + c + e$	$b + d + f$	n

Chi-square test…

The expected value for the first cell (a), $e_1 = \dfrac{(a+b)(a+c+e)}{n}$

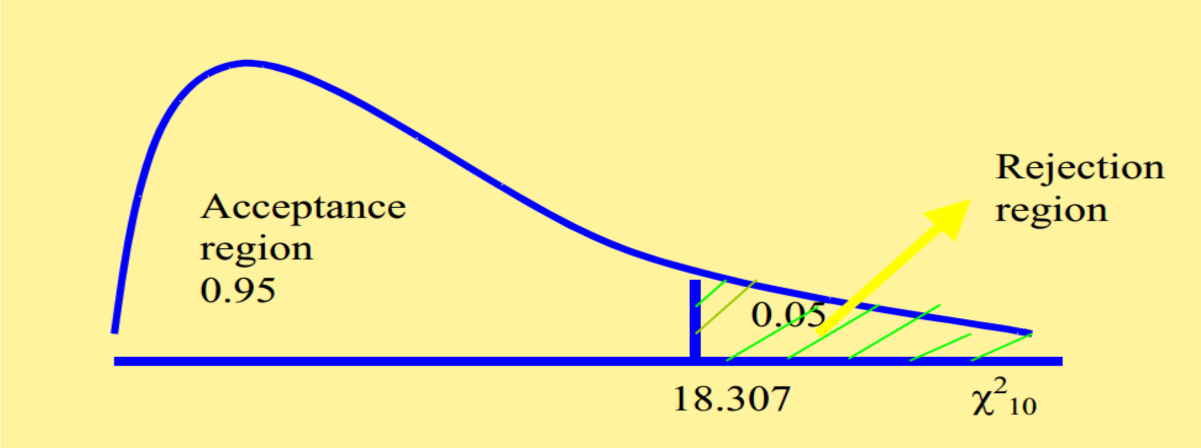
The expected value for the first cell (b), $e_2 = \dfrac{(a+b)(b+d+f)}{n}$

…………………………………………………………………………………:

The expected value for the first cell (f), $e_6 = \dfrac{(e+f)(b+d+f)}{n}$

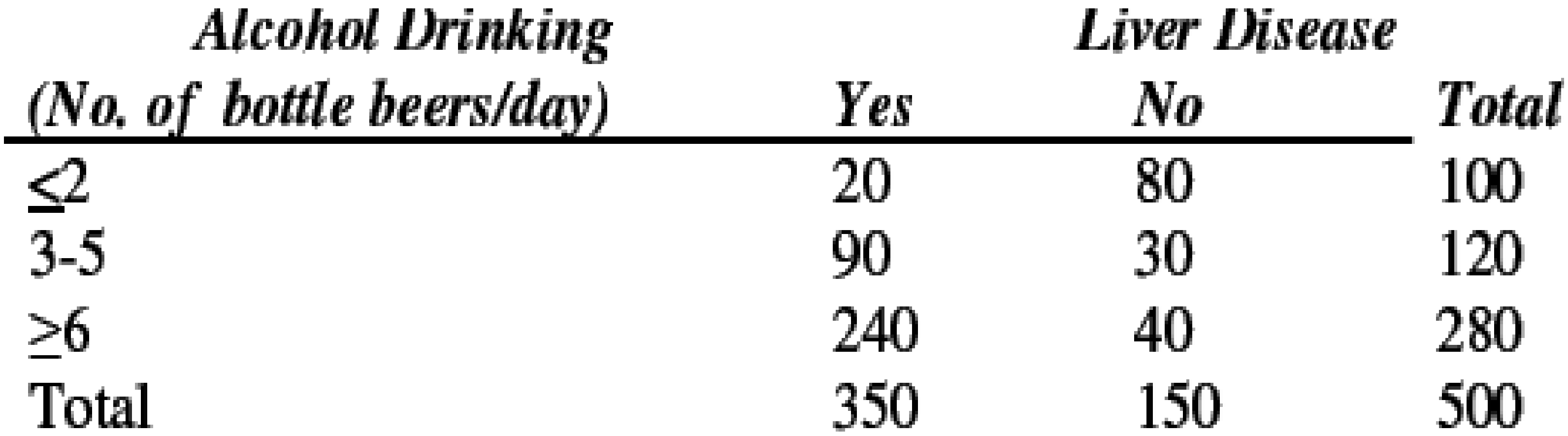
Procedures of Hypothesis Testing

1. State the hypothesis
2. Fix level of significance
3. Find the critical value (x2 (df, α))
4. Compute the test statistics
5. Decision rules; reject null hypothesis if test statistics > table value.



Example 11:

Consider the following 3x2 contingency table



Alcohol Drinking		*Liver Disease*	
(No. of bottle beers/day)	*Yes*	*No*	*Total*
≤2	20	80	100
3-5	90	30	120
≥6	240	40	280
Total	350	150	500

Chi-square test...

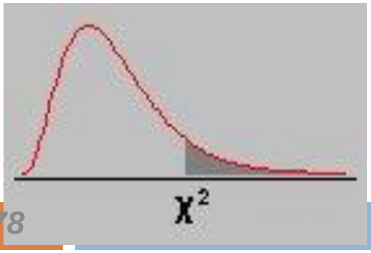
The expected frequencies are

- $e_1 = \dfrac{100x350}{500} = 70;\; e_2 = \dfrac{100x150}{500} = 30;\; e_3 = \dfrac{120x350}{500} = 84;\; e_4 = \dfrac{120x150}{500} = 36$
- $e_5 = \dfrac{280x350}{500} = 196;\; e_6 = \dfrac{280x150}{500} = 84$

For such kind of contingency table, the degree of freedom (*df*) will be (r-1) (c-1) where r is number of rows and c is number of columns. For the above table, the *df*= (3-1) (2-1) =2

$$\Rightarrow X^2 = \sum\left[\frac{(O_i - e_i)^2}{e_i}\right] = \frac{(20-70)^2}{70} + \frac{(80-30)^2}{30} + \frac{(90-84)^2}{84} + \frac{(30-36)^2}{36} + \frac{(240-196)^2}{196} + \frac{(40-84)^2}{84} =$$

$= 153.40$



Chi-square table

Right tail areas for the Chi-square Distribution

df\area	.995	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.5966
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.3448	12.8381
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.1432	13.2767	14.8602
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.0705	12.8325	15.0862	16.7496
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812	7.84080	10.6446	12.5915	14.4493	16.811	18.5475
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.0170	14.0671	16.0127	18.4753	20.2777
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	10.2188	13.3615	15.5073	17.5345	20.0902	21.9549

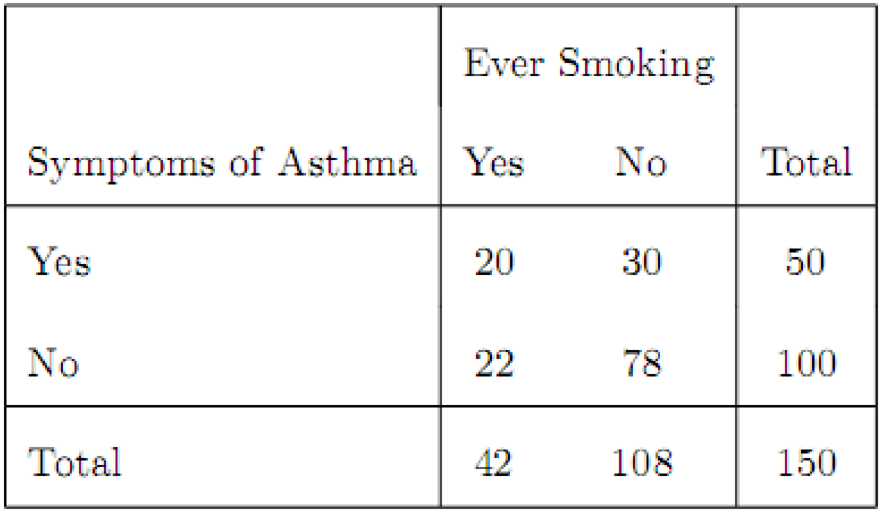
Assumptions of the $\chi2$ - test

The chi-squared test assumes that

- Data must be categorical
- The data be a frequency data
  - the numbers in each cell are 'not too small'. No expected frequency should be less than 1, and
  - no more than 20% of the ***expected*** frequencies should be less than 5.
- If this does not hold row or column variables categories can sometimes be combined (re-categorized) to make the expected frequencies larger or use Yates continuity correction.

Example 12:

- Consider hypothetical example on smoking and symptoms of asthma. The study involved 150 individuals and the result is given in the following table:



	Ever Smoking		
Symptoms of Asthma	Yes	No	Total
Yes	20	30	50
No	22	78	100
Total	42	108	150

The question is, is there associiation between smoking cigarettes and symptoms of asthma at 0.05 level of significance?

Solution

- Hypothesis:
  - H0: there is no association between smoking and symptoms of asthma
  - H0: there is association between smoking and symptoms of asthma

- The critical value is given by X2 (0.05,1) = 3.841
- Test statistics

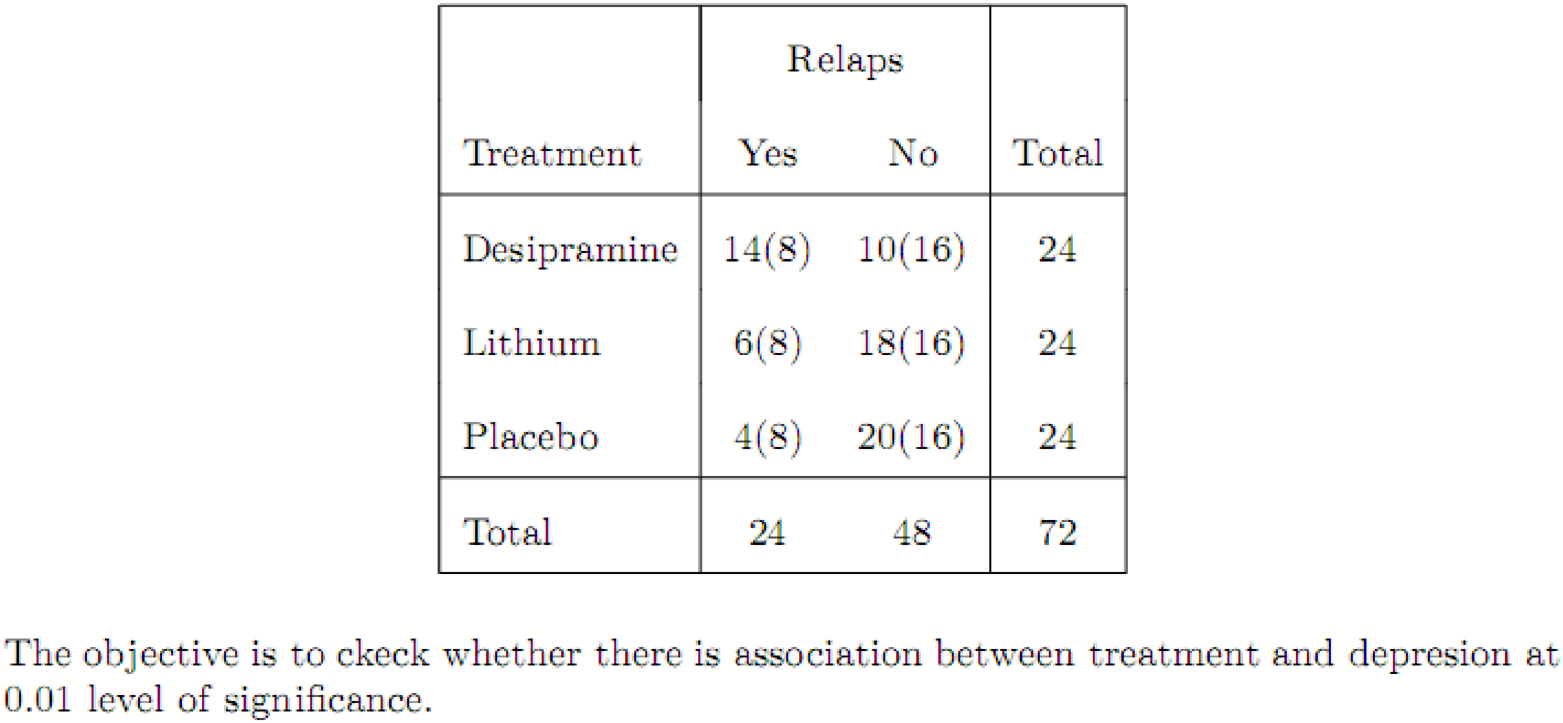
$$\chi^2 = \frac{150(20*78-22*30)^2}{(20+30)(22+78)(20+22)(30+78)} = 5.36$$

And The corresponding p-value to 5.36 at 1 degree of freedom is estimated by 0.02.

- Hence, the decision is reject the null hypothesis and accept the alternative hypothesis
- Conclusion: there is association between smoking and symptoms of asthma).

Example 13:

- Consider the data on the assessment of the effectiveness of antidepressant. The data is given below:

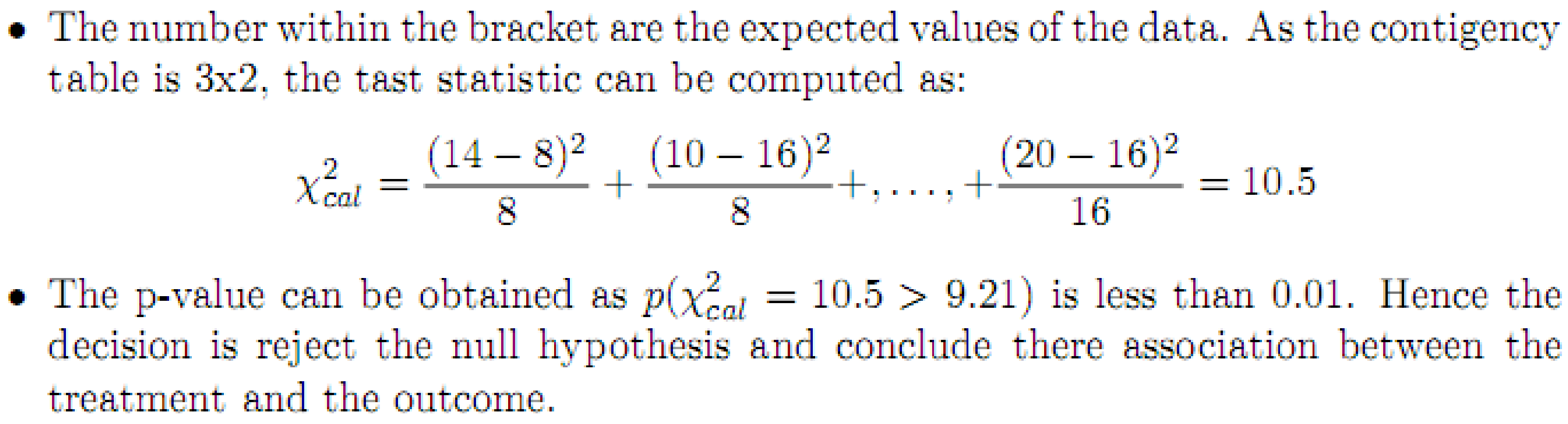


	Relaps		
Treatment	Yes	No	Total
Desipramine	14(8)	10(16)	24
Lithium	6(8)	18(16)	24
Placebo	4(8)	20(16)	24
Total	24	48	72

The objective is to ckeck whether there is association between treatment and depresion at 0.01 level of significance.

Solution

- Hypothesis
  - H0: there is no association between the treatment and relapse
  - H1: there is no association between the treatment and relapse
- The degree of freedom for this table is df = (3-1)(2-1) = 2. thus the critical value from chi-square distribution is given by $\chi^2_{tab} = \chi^2_{0.01,2}$



- The number within the bracket are the expected values of the data. As the contigency table is 3x2, the tast statistic can be computed as:

$$\chi^2_{cal} = \frac{(14-8)^2}{8} + \frac{(10-16)^2}{8} +, \ldots, + \frac{(20-16)^2}{16} = 10.5$$

- The p-value can be obtained as $p(\chi^2_{cal} = 10.5 > 9.21)$ is less than 0.01. Hence the decision is reject the null hypothesis and conclude there association between the treatment and the outcome.

Quiz

- You randomly sampled **286** sexually active individuals and collect information on their HIV status and History of STDs. At the **.05** level, is there evidence of a **relationship between them**?

	HIV		
STDs Hx	No	Yes	Total
No	84	32	116
Yes	48	122	170
Total	132	154	286

Summery

Characteristics χ^2

1. Every χ^2 distribution extends indefinitely to the right from 0.
2. Every χ^2 distribution has only one (right) tail.
3. As df increases, the χ^2 curves get more bell shaped and approach the normal curve in appearance (but remember that a chi square curve starts at 0, not at -∞)
4. If the value of χ^2 is zero, then there is a perfect agreement between the observed and the expected frequencies. The greater the discrepancy between the observed and expected frequencies, the larger will be the value of χ^2.