## UNIT - II

Normal forms - Disjunctive Normal forms - Conjunctive Normal forms - Principal Disjunctive Normal forms - Ordering and uniqueness of Normal forms the theory of inference for the statement calculus - Validity using truth tables - Rules of inference.

(Sections 1.3.1 to 1.3.5, 1.4.1 to 1.4.2)

## DISCRETE MATHEMATICS

UNIT - II

Normal forms - Disjunctive Normal forms - Conjunctive Normal forms - principal Disjunctive Normal forms - ordering and uniqueness of normal forms the theory of inference for the statement calculus - Validity using truth tables - Rules of inference (Sections 1.3.1 to 1.3.5, 1.4.1 to 1.4.2)

UNIT - II

Normal Forms

Satisfable:

A compound proposition A (P1,P2,P3...Pn) is said to be satisfiable it it has a truth value TRUE for atleast one combination of the truth values of P1,P2,....Pn.

Types of Normal forms:

1 Disjunctive Normal forms (DNF)

& Conjunctive Normal forms (CNF)

Elementary product:

A Product of the variables and their negation on a formula is called an elementary product ex:P, PAB, PATP, TPAB, PATPAB, TPATB
Elementary Sum:

A Sum of the variables and their negations in a formula is called an elementary sum. Ex:P, PV70, PV7P, TPV0, PV7PV0, TPV70

Disjunctive Normal Forms: (DNF)

A compound proposition (or) a formula which consists of a sum of elementary products and which is equivalent to the given proposition is called a DNF. Ex: (PAB) V (PATB) V (TPAB)

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conjunctive Normal Forms: (CNF)
             A compound proposition con a formula which.
  consists of a product of elementary sums and which is
  equivalent to the given proposition is called a CNF.
        Ex: (TPVQ) A (PV7Q) A (TQVTP)
 Find the DNF of the following:-
 1. PA (P→Q)
 soln:
     PN(P→Q) ↔ PN(TPVQ)
             ↔ (PATP)V(PAQ)
          which is the required DNF.
2.7(PVQ) ≥ (PAQ)
T(PVQ) \rightleftharpoons (PAQ) \land (T(PVQ)) \land (PAQ)) \lor (T(T(PVQ)) \land T(PAQ))
                   (AFV GE) \land (BEAGE) \lor (BAG) \land (BEAGE) \Leftrightarrow
                   ((BLAde) V((BAd VOLVAL))
                   \Leftrightarrow (TPATBAPAB)V((PVB)ATP)V((PVB)ATB)
                  \Leftrightarrow (TPATRAPAR)V(RATP)V(PATR)V(RATR)
          which is the required DNF.
3.7(7(P≥Q)AR)
               (AV(BLVAL) LV(BVd)L)) L↔
              ↔7(((TPV7Q))(PVQ)))AR)
              \Leftrightarrow \exists ((\exists PA \mid PVQ) \lor (\exists QA \mid PVQ)) \land R)
              \Leftrightarrow T(((3PAP)V(3PAQ)V(3QAP)V(3QAQ)))
             ↔ 7(((FV(7PAQ)V(7QAP)VF)AR)
             \Leftrightarrow \exists ((\exists PAQ) \lor (\exists QAP))) \land R)
             \Leftrightarrow 7(((\neg PV(\neg Q \land P)) \land (Q \lor (\neg Q \land P))) \land R)
             \Leftrightarrow \exists ((\exists PV \exists a) \land (\exists PV P)) \land (\exists V \exists a) \land (\exists V P)) \land R)
             ↔ T(((TPVTQ)) (QVP))) (R)
             ALA(dab)LA(BLAde) L
             ↔ (PNQ)V(JQNJP)VJR
           which is the required DNF.
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A \cdot PV(\exists P \rightarrow (@V(@ \rightarrow \exists R)))

← PV(Jb→ (Øv(JbAJE)))

          E PV(PV(QVTQVTR)))
          PVPVQVTQVTR
          APVAVTAVIR
       which is the required DNF.
E (PVJ(ØVK))∧(b→ø)
         (BVALIBVE)) N(JEVE)
          (avqr)v((grvar) \land q) \leftrightarrow
         ← (PATR)V(PATR)V(TPVQ)
         OVERTAR) V (BLACK)
        which is the regulated DNF.
6 (PATIBUR))V(((PAB)VTR)AP)
         \Leftrightarrow (PA(TBATR))\vee((PAB)AP)\vee(TRAP)
         ♦ (PATRATR)V(PARAP)V(PATR)
         (PATRATR)V(PAR)V(PATR)
        which is the required DNF.
Find the CNF of the following.
I.(PAT(BAR))V(P\rightarrow 0)
         ← (PATR)V(PATR)V(TPVR)
         (BVGL)V((BLVG) \land BL) \lor ((BLVG) \land AL)

⇔ PA(JØVP)A(JØVJR)A (JÞVØ)

⇔ PA(¬RVQV¬RV¬P)

⇔ PA (TV (¬RV¬P)

         FA9 (+)
         \Leftrightarrow p
         which is the required CNF.
@ (RV(PAR)) AT ((PVR) AB)
          ⇔ (av(a∧p))\n((pvR)\a)
         ↔ Q A (¬PA¬R)V¬B

⇔ BA (TRV(TPATR)

⇔ Q∧ (¬RV¬P)∧(¬RV¬R)

        which is the required CNF.
```

Mintorms:

in which each variables (or its negation but not both occurs only one one called minterims.

For two variables P and A the possible minterms

aro

 $m_0 = m_{00} = TPMQ$ 

 $m_1 = m_{01} = \neg P \wedge B$ 

 $m_2 = m_{10} = PATR$  $m_3 = m_{11} = PAR$ 

For 3 varifables P, a and R the possible mintonins are

 $m_0 = m_{000} = TPATRATR$ 

 $m_1 = m_{001} = TP\Lambda TB\Lambda R$ 

 $m_2 = m_{010} = TPARATR$ 

m3 = MOII = TPABAR

 $m_{H} = m_{100} = p_{\Lambda} T R_{\Lambda} T R$ 

m5 = MIDI = PATRAR

my = mm = PNONR

Martons:

in which each variables (or) its negation but not both occurs only once are called marchenes.

For two variables P and a the possible markering are

 $M_0 = M_{00} = PVQ$ 

MI = MoI = PVTA

M2 = M10 = TPVQ

M3= M11= 7PV7Q

For 3 variables P, & and R, the possible maxterine are  $M_0 = M_{000} = PVQVR$ 

 $M_1 = M_{001} = PVQVTR$ 

M2 = MOID = PVTQVR

M3 = MOII = PVTRVTR

M4 = M100 = TPVQVR

M5 = M101 = TPVAVTR

M6 = M110 = TPVTQVR
M7 = M111 = TPVTQVTR
Principal Disjunctive Normal Forms (PDNF)

Sum-of products canonical form

A formula (compound proposition) consisting

Of disjunction of minterms in variables only and equivalent

to the given formula is known as PDNF (or) sum-of
Products canonical form.

Principal conjunctive Normal Forms (PCNF)

Product -of - Sums canonical form

A formula (compound proposition) consisting

et conjunctions of maxterns in variables only and

equivalent to the given formula is known as PCNF (or)

Product of sums canonical form.

Note: 1

If the given formula is a tautology then its PDNF includes all the possible terms of the variables and there is no PCNF.
Note: 2

It the given formula is a contradiction then its and there is no PDNF.

I obtain the PDNF and PCNF of the following formula using truth tables.

 $(i)(TPVTQ) \rightarrow (P \rightleftharpoons TQ)$   $(ii)(PV(TP \rightarrow (QV(TQ \rightarrow R))))$   $(iii)(P \rightarrow (QAR)) \wedge (TP \rightarrow (TPATR)).$ 

1. Lot 5:	(TPV	702)->	(PZ	101)			(DTV70)- BETER
-20	D	9	70	70	(DELALL)	(时分)	(PEGT (BT VQT)
	7	T	F	F	F	F	+
	T	F	F	T	7	+	Ť
	F	F	+	+	Ť	F	F

PONF of  $S \Leftrightarrow (PAR)V(PATR)V(TPAR)$ PONF of  $S \Leftrightarrow (PVR)$ 

PONE Of S (PARAR) V (PARATR) V (PATRAR) V (PATRATR) V (TPARATR) V

PCNF & S (PVRVR)

3. Lots: (P-) (BAR))A (TP-) (TPATR)).

P	0	R	BAR	P->(0/R)	P	TR	TPATR	TP-)(TPATR)	P-> (a/R))/(TP->(TP/TR))
T	T	T	T	T	F	F	7	一丁	T
T	T	F	F	F	F	T	F	T	F
T	F	T	F	LF.	F	F	F	7	E
T	F	F	E	F	F	T	F	7	E .
F	T	T	T	T	T	F	F	F	
F	T	F	F	T	T	T	T	T	
F	F	T	F	T	T	F	F	F	
F	F	F	F	7	T	T	T	T	T

PDNF Of S⇔ (PARAR)V(TPARATR)V(TPATRATR)

PCNF Of S⇔ (TPVTQVR)A(TPVQVTR)A(TPVQVR)A(PVTQVTR)A

(PVQVTR)

4.S: (TP→R) N(QZP)								
PE	R	ПР	7P-JR	Q之 P	(7P-)R)A(Q社P)			
T	TT	F	TT					
T	T F	F	TTT					
T	7	F						
T	F	F		F				
F		1		F				
F	F	7		T				
F	FTF	+	F	T	F			
DONE		[DAR/	DIVIDI	-IVIGEN AI	TONTRAR)			
PONE OF S (PARAR) V (PARATR) V (TPATRAR)  PONE OF S () (TDVAVTR) A (TDVAVR) A (DVTRVR)A								
PCNF of S  Tipvavir) \(Tipvavr) \								

```
→ MOAMIAMHAM5

← Tro,1,4,5 which is the PCNF OB S.

8 (TP+B)A(QZP)
  Let S: (TP→B) N(BZP)
      ((GLUBL) N(GRAD) N(BALL)
      (9TRAFT) V (BAR]) A (BV9)
     ((OLVEL) V(BAG)) ((OLVE)) ((OLVE))
     ( IL BY BY B) A ( BY B) A ( BAB) A ( BAB)
     ↔ (PAR)V(PAR)VF
      O IPARIVE
      # PAB
      < m11
      ↔ m2
    S ( ) Is the PDNF of S.
 PDNF Of TS # E0,1,2
           # mo vm vm2
           ⇔ moo Vmoi Vmio
          (ALVAI) (BVAL) A(BLVAL)
: PCNF Of S # T (PDNF OF TS)
           (PVQ) / (PVTQ) / (TPVQ)

⇔ Moo ∧ Moi ∧ Mio
          MOAMIAMO

⇒ Tro, 1,2 which is the PCNF OBS.

3. (PAT(BAR))V(P-)B)
    Let S: (PAT (BAR)) V(P-)B)
       ↔ (PA (TRVTR))V (TPVA)
      (BV9F) V(RFA9) V(BFA9) ↔

→ ((PATE) NT) V((PATR) NT V((TPVB) NT

      ((BLADY)) N((BLAD)) N(BLAD))
                                  (TPVA) M(RVTR)

← (PATEAR) V(PATEATR) V(PAEATR) V(PATEATR)
                      V((TPA)RVTR)V((QA)RVTR)
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← (PATGAR)V(DATGATR)V(DAGATR)V(DATGATR)V(TPAR)V
                                (TPATR)V(BAR)V(BATR)
   ↔ (PATEMR) V(PATEMTR) V(PATEMTR) V(PATEMTR) V((TPMR)A
        (QVTQ)) V((TPATR)A(BVTQ)) V((QAR)A(PVTP)V
                                          ((BATR)A(PVTP)
   € (PATE AR)V(PATEATR)V(PAEATR)V(TPAEAR)V(TPATEAR)V
         (TPARATR) V(TPATRATR) V(PARAR) V (TPARAR) V
                                  (PABATR) V (TPABATR)
   ↔ [PATGAR) V( PATGATR) V( PAGATR) V(TPAGAR) V(TPATGAR) V
                             (TPABATR)V(TPATBATR)V(PABAR)
   ( Mio) VM100 VM110 VM001 VM010 VM000 VM111

⇔ m5Vm4Vm6Vm3Vm1Vm2VmoVm7

  ⇒ ∑0,1,2,3,4,5,6,7 which is the required PDNF of S.
 4 (QV(PAR)) 17 ((PVR)10)
       € (QVP) A (QVR) A ((TPATR) VTQ)
       \Leftrightarrow (PVB)\Lambda(AVB)\Lambda(AVB)\Lambda(AVB)
       ⇔((PVQ)VF)A((QVRVF)A((¬PV7QVF)A((¬QV¬RVF)
       \Leftrightarrow ((PVR) V(RATR))\Lambda ((Q VR)V(PATP))\Lambda ((TPVTQ)V(RATR)\Lambda
      ↔ (PVQVR)A (PVQVTR)A (PVQVR)A (TPVQVR)A (TPATQVR)A
                              V(SLABLAd)V(SLABLAdL)V
    ↔ (PVQVR)/ (PVQVTR)/ (TPVQVR)/ (TPVQVR)/ (TPVQVR)/
   € MODO AMDOI AMID AMID AMIII AMDII
                                                (PV76VTR)

← Mo MMI AMAA Mb A MT AM3

← Tro,1,3,4,6,7 which is the required pent of S.

  PCNF & 75 AT T2,5
              ⇔ M21M5

⇔ Moio ∧ Mioi

             : PONF Of S ( TIPCNF OF TS)
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A moin / mioi
            # movms
            ⇒ ∑2,5 which is the required PDNF 665.
  5. (TP→R) N(B科P)
     Lot S ↔ (TTPVR) A (a → P) A (P→ a)
            ↔ [PVR)A(TRVP)A(TPVQ)
            ↔ ((PVR)VF) A((TQVP)VF)A((TPVQ)VF)
           \iff ((PVR)V(a\Lambda Ta)\Lambda((TaVP)V(R\Lambda TR))\Lambda((TPVa)V(R\Lambda TR))

⇔ (PVQVR) \( (PVTQVR) \( (PVTQVTR) \\ \)

                                       (TOVAVR) A (TOVAVTR)
           (PVQVR) A (PVTQVR) A (PVTQVTR) A (TPVQVR) A (TPVQVTR)
           HOOD A MOIO A MOII A MIOO A MIDI
           ↔ Mo A Mo AM3 AM4 AM5
         S => TTO, 2, 3, 4, 5 which is the required DCNF of S.
     PCNF Of 7S => TT,,6,7

→ MIYMPYMA

                 MOOIL MIID / MIII
                 (ALVOLVAL) V (ANDLACH) (ALVOND)
  : PDNF of S AT (PCNF of TS)
                (TPATOAR)V(PAGATR)V(PAGAR)
                ↔ moon vmino vmini
                ⇔ m, Vmb Vm
                € ∑1,6,7 which is the required PDNF of S.
6.5: (P \rightarrow (Q \land R)) \land (\neg P \rightarrow (\neg Q \land \neg R))
     S \Leftrightarrow [TPV(QAR)] \land [TTPV(TQATR)]
      \Leftrightarrow (TPV(QAR)) \wedge (PV(TQ\wedgeTR))
      (ALAd) V(BLAd) V(BAdL) V(BAdL) ↔
      ↔ ((¬PVB)VF) \((¬PVR)VF)((PV¬B)VF) \((PV¬RVF)
     \Leftrightarrow ((TPVQ)V(RATR))A ((TPVR)V(QATQ)) A((PVTQ)V(RATR))
     MC(pVTR)V(a)/Ta))
    (TPVQVR) A(TPVQVR) A(TPVQVR) A(TPVTQVR) A(PVTQVR)
                          NLDNJRJN(DVRVJR)N(DVJRVJR)
```

```
← (TPVQVR) \(TPVQVTR) \(TPVTQVR) \(A)

                                          (PVTBVTR) A (PVBVTR)
        MIDD / MIDI / MID / MOID / MOIL / MODIL

← MANM5 NM6 NM2 NM3 NM1

      3 ( TI, 2, 3, 4, 5, 6 which is the required PCNF 96 S.
     PCNF of 73 077
                FMAOM
                HODO A MIII
                ⇔ (PVQVR) ∧ (TPVTQVTR)
    : PDNF of S ( TIPCNF of 73)
                (TPADATR)V(PADAR)
                ⇔ mooo VmIII
               ⇔ movmy

⇒ ∑o,7 which is the required PDNF of S.

               Of Profesione for Statement calculus:
     Pramise (or) Hypothosis:
            A Premise is a statement which is assumed to
    bo true.
    Theorem:
           A theorem consists of a set of premise and
   va conclusion. A theorem is proved by showing that
   The conclusion in true whenever all the premises eve
   assumed to be true.
   Formal Proof:
         The process of determing a conclusion from a set
  of premises by using the accepted rules of reasoning
 is called a formal proof.
 Types of Proofs:
          1 Direct proof
         2. Indirect proof (or) proof by contradiction.
         3. conditional proof (or) proof by deduction.
Implications:
     I_1: PAB \Rightarrow P_2 simplification I_2: PAB \Rightarrow B_2
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I_3: P \rightarrow PVE } oddition I_4: R \rightarrow PVE } oddition
 IE: 16-16-16
 IL: TR=1P18
  T=:7(P+0)=>p
  Is: 7(P18)=>78
 In: P. B => PAR
  In: TP. PVR => (disjunctive syllogism)
  In: P. P-1 => a (Modus Poners)
  In: TR, PAR => TP (Modus tollow)
  II3: P-18, R-1R => P-) R (Hypothetical syllogism)
  TIA: PVR, P->R, R->R (dilemma)
Equivalences:
   E1: 77P ⇔ P (Double Negative)
   Eg: PAR ↔ RAP) (commutative daws)
   E_{5}: (PVB)VR \Leftrightarrow PV(QVR) (associative laws)
   Eb: PA(QVR) ↔ (PAQ)V(PAR)?
(Distributive laws)
   Eq:7(PAR) ↔ 7PVTR } (De Morgan's laws)
   Fip: PVP ↔ P
   En: PAP⇔ P
   E10: RV(PATP) ↔ R, RVIF ↔ R
   E13: RA(PVTP) +R, RAT +R
   Em: RV(PV7P) # 7
   EIB: RA (PATP) + F
   EIL: P-) B + TPVB
```

En: 7(P→Q) ↔ PATR

Ile: P-18 (-) TR-37P

 $E_{19}: P \rightarrow (Q \rightarrow R) \leftrightarrow (P \wedge Q) \rightarrow R$ 

1 20:7(P≥0) ↔ P≥70

 $F_{21}: P \supseteq Q \leftrightarrow (P \rightarrow Q) \land (Q \rightarrow P)$ 

I 22: [PZB) ↔ (PAR) V(TPATR)

Rules of Inference:

A set of premises H1, H2, H3, ..., Hm and a conclusion

Cauce given

we assume that H1, H2, H3..., Hm we all true and we want to conclude the conclusion. That is we want to conclude that the conclusion c follows dogically from the premises H1, H2, H3... Hm.

1. Rule P:

In the derivation.

2. Rule T:

is S is tautology emplood by any one (or) more of the Proceeding formulas in the derivation.

validity using truth table technique:

1 Determine whether the conclusion c follows logically from the premises HI and He.

a)  $H_1: P \rightarrow \mathbb{R}$   $H_2: P$ 

C: Q

b) H<sub>1</sub>: P→Q H<sub>2</sub>: 7P

C: Q

c) H1: P-10 H2: 7(PAQ) C: TP

d) H1: TP H2: P之Q C:T(PAQ)

e) H1: p-1 A H2: Q

C: P

soln:							ļ
	P	0	TP	P→0	P≥a	7(P/0)	
	T	T	F	F	F	FT	
	T T F F T T F T T T T T T T T T T T T T		TFTT	F	T		
							-
a) H1:	P→Q	H2:1		(Q	logically	Jam the	given
Promises	ט שמי. יינג	and H	11 5	40110003	Sugramy	from the	U
b) H1:	P-> Q	H2:	TP C	: ର			
	Dro H	en the	thur	d row,	both the	promises out	e true
and The	e conc	dusion	us a	lso true	and in	the fourth volusion is	row
DOIN II	· The	stumes	lusion	a does	s not tol	low logically	y quise.
from the	io giv	on pr	omisos	p-10 a	nd 7p.	O	J
C) H.:	D-10	Ho:	JIPAR'	) C:7P		han the	สใบอท
Premises	the	conclus	ion 7 p	o follows	Mylcumy	from the g	J. VO.
	P→Q	and -	1(PNQ)				
D) Hi	:7P	H2:	P26	c:7(	PAG)	) (1 )	مك
given po	romises	toricus H <sub>1</sub> 0	ion 7 und Ha	(PAQ) 40	illows Jugi	cally from	Ine
6) H	1: P->6	a H2	:0	C:D			
	Ine	conclu	wion	p does	not follow	os the given	
Peromisos	P>a	and e	ā.				
& Domon	strate	tha	t R	ls a vo	ulid Infor	ence from	the
Premise		) (d ) (d =	→K W	Na P.			
	Promisi	os: P-	30, Q-	+R and	P		
C	onclusio	o					
		(1)	-	RW			
		(0)			11-11		
	21,23	(3)	8	KW	e T, (1),(2), (	and In	

```
Rule P
           643 (4) Q→R
                                 Rule T (37, (4) and In
        $1,2,49 (5) R
            where I_{11}: P, P \rightarrow Q \Rightarrow Q
       i. R is a valid iference from the given premises
  P-1a, Q->R and P.
  3. S.T RVS follows logically from the premises CVD,
  (CVD) → TH, TH → (ANTB) and (ANTB) → RVS
  soln:
    Promises: CVD, (CVD) -> TH, TH-> (ANTB) and (ANTB)-> RVS
  conclusion; RVS
        913 (1) (CVD)→7H Rule P
        128 (2) 7H→(ANTB) Rule P
                                  Rule T (17, (2) and I13
       $1,23 (3) (CVD)→(ANTB)
                                  Rule P
       SA3 (A) (ANTB) -> RVS
                                  Rule Tran, (1) and I13
     $1,2,49 (5) (CVD)-> RVS
                                  Rule P
      264 (6) CVD
                                   Rule T157,16) and I11
61,2,4,6 4 (7) RVS
        where In: P, P -> Q -> Q
              I_{13}: P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R
       .. RVS follows logically from the given premises.
4.3.T SVR is Tautology implied by (PVR) 1/(P-) 1/(R-)5)
 soln:
   Promises: PVQ, P->R, Q->S
  conclusion: SVR
      213 (1) PVR
                                Rule P
                                Rule Tin and E16
      619 (2) TP-10
       434 (3) Q→3
                                Rule P
                                Rule T (2), (3) and I13
      21,34 (4) TP-S
                                Rule TIA) and E18 and E1
      d1,34 (5) 75→P
```

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969 (6) P->R
                           Rule P
 61,3,63 (7) 7S→R
                           Rule T (5), (6) and I13
                           Rule T (7), E16 and E,
  11,3,63 (8) SVR
 where II3: P-10, Q->R => P->R
        E1:77P ↔ P
         E16: P→Q + TOVA
         FI8: P→0 ↔70→7P
    : SVR is tautology implied by
        (PVa) / (P-) R) / (a-)s)
5. S.T I12: 7a, P→a => 7p
 soln:
     Prombos: Ta, P-a
    conclusion: 7p
      {13 (1) P→Q RWQ P
      213 (2) 70->7P Rule T (n and E18
      633 (3) 70 Rule P
      91,34 (4) TP Rule T (27, C37 and I)
    whose
         I_{11}:P,P\rightarrow a\Rightarrow a
         E18: P→A + 70->7P
6. S.T RA(PVQ) is a valid interence from the promises
PVa, a \rightarrow R, P \rightarrow M and \neg M.
 soln:
    Promises: PVQ,Q >R, P >M and TM.
  conclusion: RA(pva)
       fig (1) P->M Rule P
       923 (2) 7M Rule P
       $1,2% (3) TP Rule T(17,127 and I12
       {43 (4) PVQ Rule P
                          Rule T (37, (4) and I 10
     {1,2,4} (5) Q
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```
Rule P
       463 (6) Q→R
                                RuleT (5), (6) and In
 d1,2,4,69 (7) R
                                Rule T (4), (7) and Iq
61,2,4,64 (8) RN(PVQ)
      where, I12: 70, P-10=>TP
              I10: 7P, pva ⇒ a
              I_{11}: P, P \rightarrow Q \Rightarrow Q
              Iq: P, a => PAQ
     Hence RA(PVQ) is a valid interence from the
Promisos PVa, a -> R, P-> M and TM.
7. If there was a ball game then travelling was difficult. If they avoilved on time then travelling was
not difficult. They arrived on time. Therefore there was
no ball game. S.T these statements constitude a valid
orgument.
   soln: Lot us define
     P: There was a ball game
     a: Travelling was difficult
     R: They arrived on time
 Promisos: p-10, R-)70, R
conclusion: 7p
        119 (1) R-> TR
                            Rule P
        623 (2) P→Q
                             Rule P
                              RWETCH and E18
       (1,23 (3) 7a→7P
     {1,2} (A) R→7P
                              Rule T (17, 13) and I13
                               Rule P
       859 (5) R
                               Rule T(47, 15) and III
     $1,2,54 (6) TP
      where, E18: P-10 0 79-7P
              I_{13}: P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R
              I_1:P_2P\rightarrow Q \Rightarrow Q
              given statements constitute a valid
argument.
```

Rule (p (or) conditional Proof:

If we derive s from R and a set of premises, then we can derive  $R \rightarrow s$  from the set of promises alone.

Rule CP is also called the deduction theorem and is generally used if the conclusion is of the form  $R \rightarrow S$ .

In such cases, R is taken as an additional Promise and S is derived from the given promises and R. 1. S.T R  $\rightarrow$  s can be derived from the premises  $P \rightarrow (R \rightarrow S)$ , TRVP and G.

Soln:

Promises: P - (a -> s), 7RVP and a

conclusion: R->S

Since the conclusion is  $R \rightarrow s$ , we use rule CP Therefore we include R as an additional premise and we show s first.

: Promises: P->(a->s), TRVP, 16, and R(additional premise)

conclusion: 5

fig (1) 7RVP Rule P  $\{2\}$  (2) R Rule P (additional premise)  $\{1,2\}$  (3) P Rule  $T_{(1)}$ , (2) and  $I_{10}$ ,  $E_1$   $\{4\}$  (4) P $\rightarrow$ 10 $\rightarrow$ 5) Rule P  $\{1,2,4\}$  (5) Q Rule P  $\{6\}$  (6) Q Rule P Rule P

 $\{1,2,4,6\}$  (7) S Rule (P)

2.5.T the following statements constitute a valid evigument 1) It A works had then either Borc will enjoy themselves.

ii) It B onjoys himself then A will not work hard MI) It D enjoys himself than c will not enjoys himself Therefore if A works hard then D will not enjoy himself. soln: Let us define A: A works hard B: B enjoys himself C: Conjoys himself D: Donfays himselb Paremices: A→(BVC), B→7A, D→7C conclusion: A > 7D Since the conclusion is A-TD, we use rule CP Therfore we include A as an additional premise and we show to first. Promises: A -> (BVC), B -> TA, D -> TC, A condusion: 7D Rule P 113 (1) A -> (BVC) Rule P (additional Premise) (2) A Rule T (1), 127 and III {1,23 (3) BVC Rule P 143 (41 B→7A Rule Turand Eis, Ei 144 (5) A→7B Rule T(2), (Frand I) (2,43 (6) 7B Rule T (3), (6) and I to £1,2,48 (7) C (8) D->7C Rule P 489 Rule T (9), (8) and I 12 DF (1,2,4,8% (9) Rule CP £1,4,88 (10) 7D where, IIo: TP, PVa > a  $I_{11}: P, P \rightarrow 0 \Rightarrow 0$ I12:70, P→0 => TP E18:P-) B ( ) TB-> TP

EL: TIP # P