UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	B.Tech (All SoCSBranches)	Semester	III
Course	Discrete Mathematical Structures	Course Code	CSEG 2006

1. List five integers that are congruent to 4 modulo 12.
2. Find each of these values:
   (a) $(19^2 \; mod \; 41)$
   (b) $(32^3 \; mod \; 13)^2$
3. Prove that there are infinitely many primes.
4. What is the least common multiple of $2^3 3^5 7^2$ and $2^4 3^3$?
5. Find $\gcd(1000{,}625)$ and $\gcd(1000{,}625)$ and verify that $\gcd(1000{,}625) \,.\, lcm(1000{,}625) = 1000 \times 625$.
6. Use the Euclidean algorithm to find
   (a) $\gcd(1{,}5)$
   (b) $\gcd(100{,}101)$
   (c) $\gcd(1529{,}14039)$
7. Show that if a, b, and m are integers such that $m \geq 2$ and $m \; \equiv b(mod\; m)$, then $\gcd(a, m) = \gcd(b, m)$.
8. Show that 15 is an inverse of 7 modulo 26.
9. Find the smallest integer x that leaves a remainder of 3 when divided by 4, a remainder of 4 when divided by 5, and a remainder of 5 when divided by 6.
10. Three traffic lights flash at intervals of 12, 15, and 20 seconds, respectively. If they all flash at the same time now, how long will it take for them to flash together again?
11. Use Fermat's little theorem to find $7^{121} \; (mod \; 13)$ and $23^{1002} (mod \; 41)$.
12. Solve the congruence $4x \equiv 5 \, (mod \; 9)$ using inverse of 4 $(mod \; 9)$.
13. Let a = 8316 and b = 10920. Find integers m and n that satisfies:
    (a). Find d = gcd(a,b), the greatest common divisor of a and b.
    (b). Find integers m and n such that d = ma +nb.
14. Exhibit the addition and multiplication tables for: (a) Z_4; (b) Z_7.
15. Find a^{-1} in Z_m where: (a) a = 37 and m = 249; (b) a = 15 and m = 234.
16. Find all integer solutions to the equation $7x+5y = 1$.
17. Using Fermat's Little Theorem, find the remainder when $3^{100}$is divided by 7.
18. Find the last digit of 7^{83}.
19. Find the general solution of the following recurrence relation(s):
    (a) $a_n = 2a_{n-1} + 3\,a_{n-2}$
    (b) $a_n = a_{n-1} + a_{n-2}$ with $a_0 = 0, \; a_1 = 1$.
    (c) $a_n = 6a_{n-1} - 9\,a_{n-2}$
    (d) $a_n = 11a_{n-1} - 39a_{n-2} + 45a_{n-3}$
    (e) $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ with $a_0 = 3, a_1 = 4, a_2 = 12$.
20. Solve the following recurrence relations:
    (a) $a_k = 5a_{k-1} - 6a_{k-2}$ with initial conditions $a_0 = 6$ and $a_1 = 30$.
    (b) $a_k = 4a_{k-1} - 4a_{k-2} + k^2$ with initial conditions $a_0 = 2$ and $a_1 = 5$.
21. Find an explicit formula for the Fibonacci numbers.

22. A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above.Determine the number of computer programmers that are not proficient in any of these three languages.
23. There are 350 farmers in a large region. 260 farm beetroot, 100 farm yams, 70 farm radish, 40 farm beetroot and radish, 40 farm yams and radish, and 30 farm beetroot and yams. Let B, Y, and R denote the set of farms that farm beetroot, yams and radish respectively. Determine the number of farmers that farm beetroot, yams, and radish.
24. A drawer contains 12 red and 12 blue socks, all unmatched. A person takes socks out at random in the dark. How many socks must he take out to be sure that he has at least two blue socks?
25. The least number of computers required to connect 10 computers to 5 routers to guarantee 5 computers can directly access 5 routers is------.
26. In a group of 267 people how many friends are there who have an identical number of friends in that group?
27. During a month with 30 days, a cricket team plays at least one game a day, but no more than 45 games. There must be a period of some number of consecutive days during which the team must play exactly ______ number of games.
28. Show that the sum of cubes of n natural numbers is equal to $\left(\frac{n(n+1)}{2}\right)^2$ for all $n=1,2,3,\ldots$ natural numbers.
29. Show that the sum of n natural numbers is equal to $\frac{n(n+1)}{2}$ for all $n=1,2,3,\ldots$ natural numbers.
30. Show that the sum of squares of n natural numbers is equal to $\frac{n(n+1)(2n+1)}{6}$ for all $n=1,2,3,\ldots$ natural numbers.