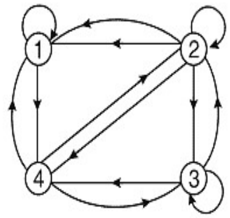
UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B.Tech (All SoCSBranches)**	**Semester**	**III**
Course	**Discrete Mathematical Structures**	**Course Code**	**CSEG 2006**

1. Let $B_n = [n, 2n]$ then find (i) $B_5 \cap B_8$ (ii) $B_1 \cap B_2 \cap B_3$.
2. If $A \subseteq B$ then prove that $(A \times B) \cap (B \times A) = A^2$.
3. Prove that (i) $A \times B = \phi$ if $A = \phi$ or $B = \phi$
   (ii) $A \times B = B \times A$ if and only if $A = B$.
4. In the class of 80 students, 50 students know English, 55 know French and 46 know German Language. 37 students know English and French, 28 know French and German, 25 know English and German and 7 know none of these languages.
   (i). How many students know all the three languages?
   (ii). How many students know exactly two languages?
   (iii). How many know only one language?
5. Consider the following sets A={1,2,3,4,5}, B={4,5,6,7} and C={5,6,7,8,9}.
   Verify distributive laws. Take U={1,2,…,10}. Verify De Morgan's laws.
6. Let A={a,b,c,d,e},B={a,b,d,f,g} and C={b,c,e,y,h}. Find
   (i.). A-(B∪C).
   (ii). (A∪B)-C
   (iii). (A+B)-C

11. If $A = \{x, y, z\}, B = \{X, Y, Z\}, C = \{x, y\}$ and $D = \{Y, Z\}$. R is a relation from A to B defined by $R = \{(x, X), (x, Y), (y, Z)\}$ and S is a relation from C to D defined by $S = \{(x, Y), (y, Z)\}$. Find $R', R \cup S, R \cap S, R - S$ also find their domain and range.
12. Given, set $A = \{1,3,5\}$. Give an example of a relation R defined on the set A, which is:
    (i) unary, (ii) binary, (iii) ternary and (iv) universal.
13. Given, set $A = \{1,2,3\}$. Give an example of a relation R defined on the set A, which is:
    (i) reflexive and transitive but not symmetric
    (ii) symmetric and transitive but not reflexive
    (iii) reflexive and symmetric but not transitive
    (iv) reflexive and transitive but neither symmetric nor antisymmetric
14. Prove that if a relation R on set A is transitive and irreflexive then it is asymmetric.
15. If R is a relation in the set of integers Z defined by
    $R = \{(x, y): x \in Z, y \in Z, (x - y) \text{ is divisible by } 6\}$
    then prove that:
    (i) R is an equivalence relation.
    (ii) R is not a partial order set.
16. Show that the relation '*is congruent modulo 4 to*' on the set of integers $\{0,1,2, \ldots ,10\}$ is an equivalence relation.
17. Let R be a relation on a set A whose directed graph is as shown in below figure. Determine the matrix of R.



18. Let R be a relation on the set $A = \{1,2,3,4,5\}$ whose matrix is given below. Draw the directed graph of R.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

19. Let $A = \{1,2,3\}$ and let R and S be the relations on A such that

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find (i) $M_{R'}$ (ii) $M_{R^{-1}}$ (iii) $M_{R\cup S}$ (iv) $M_{R\cap S}$

20. Let $R = \{(1,2),(3,4),(2,2)\}$ and $S = \{(4,2),(2,5),(3,1),(1,3)\}$. Find $RoS, SoR, Ro(SoR)$, $(RoS)oR, RoR, SoS, RoRoR$ also find their corresponding matrices.

21. Find the domain of the real value function $f(x) = \sqrt{81 - x^2}$.

22. Show that the function $f(x) = x^4$ and $g(x) = x^{1/4}$ for $x \in R$ are inverses of one another.

23. Show that the mapping $f: R \to R$ be defined by $f(x) = ax + b$ is invertible (Take favorable conditions). Define its inverse.

24. Consider the function $f: R \to R$ and $g: R \to R$ defined by $f(a) = 2a + 1,\ g(b) = \frac{b}{3}$. Verify that $(gof)^{-1} = f^{-1}og^{-1}$.

25. Consider the function $f: R \to R$ and $g: R \to R$ defined by $f(x) = x^2,\ g(x) = \sin x$. Show that $(gof) \neq fog$.

26. Let $V = \{1,2,3,4\}$ and let $f = \{(1,3),(2,1),(3,4),(4,3)\}$ and $g = \{(1,2),(2,3),(3,1),(4,1)\}$.

Find: a) fog b) gof