TIC TAC TOE

I. INTRODUCTION :

The purpose of this assignment is to change the rule of tic-tac-toe that would enable the player 1 to win always. A mathematical proof using first Order Logic (FOL) and an exhaustive proof of the same is also given. This has been implemented in C++; which is given at the end.

II. RULES OF THE GAME :

Above the existing rules of tic-tac-toe, the following rules have been added to assure a win for the first player:

→ The players cannot play at the adjacent position (ie. vertically or horizontally adjacent cells) unless:

① He can form a triplet to win
② If all other cells other than the adjacent cells are already occupied.

→ Also the following assumptions are made:

① We assume that both the players play optimally that is the player plays to win or try to force a draw. (More about optimality is discussed in

next section).

(2) From point (1); the first player plays from cell 1 (which is shown to be optimal in section III) without loss of generality.

1	2	3
4	5	6
7	8	9

With respect to cell 1, the cells (2,4), (3,7) and (6,8) are interchangeable (due to symmetry).

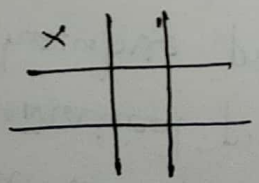
→ The following are the notations used henceforth:

(1) X(i) ⇒ denotes that the first player plays his move at the i^{th} cell.

(2) O(i) ⇒ denotes that the second player plays his move at the i^{th} cell.

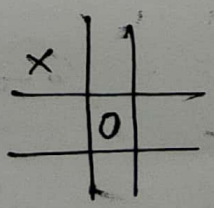
(3) W(A) ⇒ denotes that A wins the game.
'A' can be X (the first player) (or)
O (the second player).

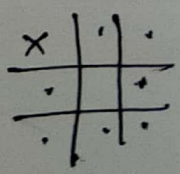
→ The following tree denotes the exhaustive proof.



Leaf 1: If player 2 plays at position 5

Leaf 2: If player 2 player at positions other than 5.





III. OPTIMALITY CONDITIONS :

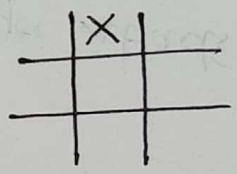
As discussed before, an optimal move refers to either a player winning the game or forcing a draw. The sequence of moves of a player is said to be sub-optimal if it leads to draw or a win.

① Corner is the optimal position :

We shall show that the positions 2, 4, 6, 8 and 5 are not optimal.

(i) The 2, 4, 6, 8 case :

By symmetry, the positions 2, 4, 6 and 8 are same for the player 1 to start. Hence we discuss only about X(2) case.



To show that this is not an optimal strategy for X; we shall show one game exists where O wins or the game leads to a draw.

Consider the following two scenarios :

X(2)	X(2)	{ Also, we get similar
O(5)	O(5)	cases when 7 is
X(7)	X(4)	interchanged with 9 }
O(9)	O(7)	
X(6)	X(9)	
O(1)	O(3)	
W(O)	W(O)	

In both scenarios; both players play optimally and follow the rule imposed.

(ii) The 5 case

Consider the following case:

+X(5)
+O(1)
+X(3) {X(2,4,6,8) are not possible due to the rule}
+O(7)
+X(9)
+O(4)

W(0)

Hence we conclude that the first player starts from corner cells. The next section shows why the cells 1, 3, 7 and 9 are the optimal cases.

(2) Suboptimal cases:

We shall show that; even if first player starts at the corner cell; and if his next consecutive move is not in the corner cells; the game may lead to a draw / losing it.

+X(1)
+O(4)
+X(6)
+O(3)
+X(7)
+O(9).

+X(8)
+O(2)
+X(5)

DRAW

Hence the first player does not choose these strategies to play.

IV. THE STRATEGY :

The following cases show the optimal strategies by which player 1 wins ; depending on the second player's first move.

Case 1 : $\vdash X(1)$ { first move }

The first move of 'O' is 2/4 { interchangeable positions }

STRATEGY :

$\vdash X(1)$
$\vdash O(2)$
$\vdash X(7)$
$\vdash O(4)$
$\vdash X(9)$
$\vdash O(8)$
$\vdash X(5)$

W(X)

(OR)

$\vdash X(1)$
$\vdash O(4)$
$\vdash X(3)$
$\vdash O(2)$
$\vdash X(9)$
$\vdash O(6)$
$\vdash X(5)$

W(X)

Case 2 : The first move of O is 7/3

STRATEGY :

$\vdash X(1)$
$\vdash O(7)$
$\vdash X(3)$
$\vdash O(2)$
$\vdash X(9)$
$\vdash O(6)$
$\vdash X(5)$

W(X)

Case 3 : The first move of O is 6/8

STRATEGY : + X(1)
+ O(6)
+ X(3)
+ O(2)
+ X(7)
+ O(4)
+ X(5)

W(X)

Case 4 : The first move of O is 9

STRATEGY : + X(1)
+ O(9)
+ X(3)
+ O(2)
+ X(7)
+ O(4)
+ X(5)

W(X)

Case 5 : The first move of O is 5

STRATEGY : + X(1)
+ O(5)
+ X(3)
+ O(7)
+ X(2)

W(X)