Homework 1

Problem 1

1. - Approach: To detect the ball we need to segment red color in the video for this we can first convert the image into the HSV format for easier manipulation of the colors. Next we develop a mask to segment the red color for this we set a lower and upper bound in the code. We then set all the pixels in these bounds to 1 and rest to 0. These points are then stored in list.

     To calculate the approximate center of the ball. We can take the mean of all the x values and y values of each frame and plot the $mean\_x$ and $mean\_y$ values to get the trajectory of the center of the ball. We multiply the y values with -1 to get the origin on the top left corner as it is in the case of the video / each frame.

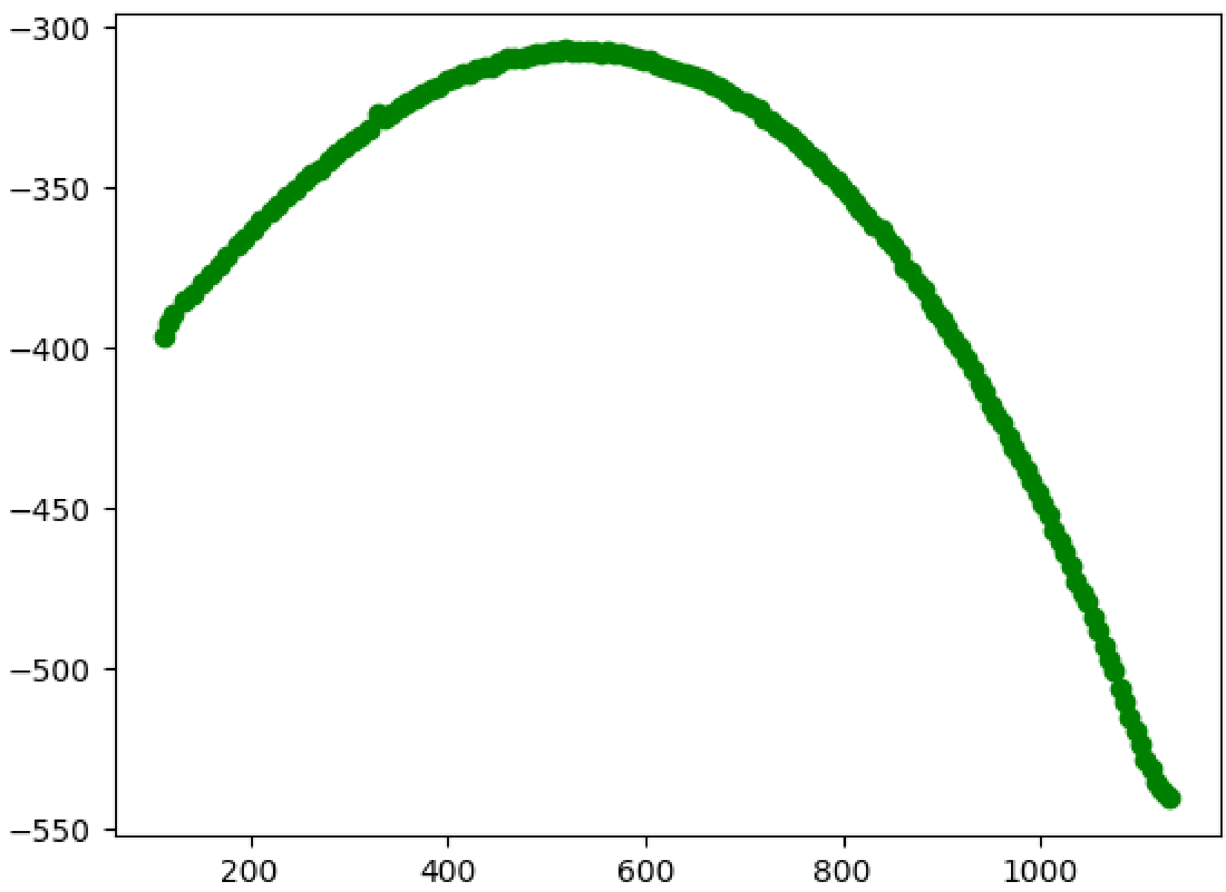

Figure 1: Trajectory of the center of the ball

   - Problems Faced: The hand and the sofa also lies in the same color range as taken for the ball this created error in the mask. To remove this error some rows and columns of the video were replaced with 0.

2. - Consider the general equation of a curved line.

$$a * x^2 + b * x + c = y$$

This equation can also be written as

$$\begin{bmatrix} x^2 & x & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = y$$

which is the form

$$Ax = b$$

We calculate the inverse of A and multiply it with b to get the coefficient matrix using the formula

$$((A^T \cdot A)^{-1} \cdot A^T) \cdot b$$

These coefficients are then put in the equation to get the estimated values of y
The final equation for the estimation is given by

$$Y = -0.0006 * (X^2) + 0.6357 * X - 468.2024$$

- This estimated curve is then plotted against the trajectory of the center of the ball which can be seen below.

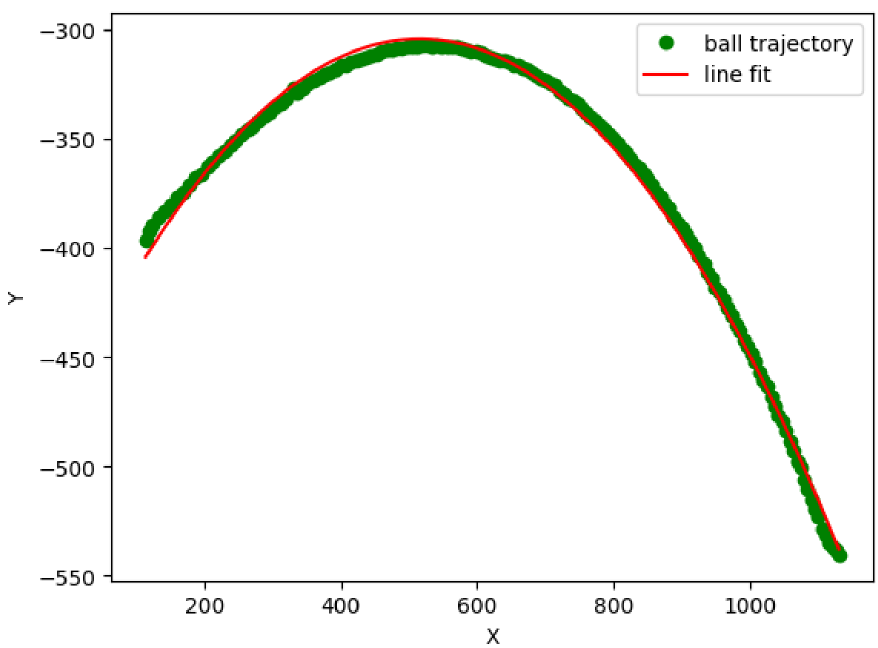

Figure 2: Estimated ball trajectory

3. To calculate the landing spot of the ball we can solve the quadratic equation derived in $Q1.2$ as we already know the y co-ordinate. We calculate it by using the formula

$$x = \pm b - \frac{\sqrt{b^2 - 4 * a * c}}{2 * a}$$

The co-ordinates of the landing spot are: $x = 1312.4759$ and $y = 696.2095$

Problem 2

1. - The covariance matrix is given by

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y & \sum I_x I_z \\ \sum I_x I_y & \sum I_y^2 & \sum I_y I_z \\ \sum I_x I_z & \sum I_y I_z & \sum I_z^2 \end{bmatrix}$$

   Where I is the variance

   The results of the covariance matrix can be found by running the code

   - The surface normal is the eigen vector $[x, y, z]$ that has the smallest eigen value. To calculate the magnitude of the of the normal we take the second norm of the normal vector i.e.:

$$||normal|| = \sqrt{x^2 + y^2 + z^2}$$

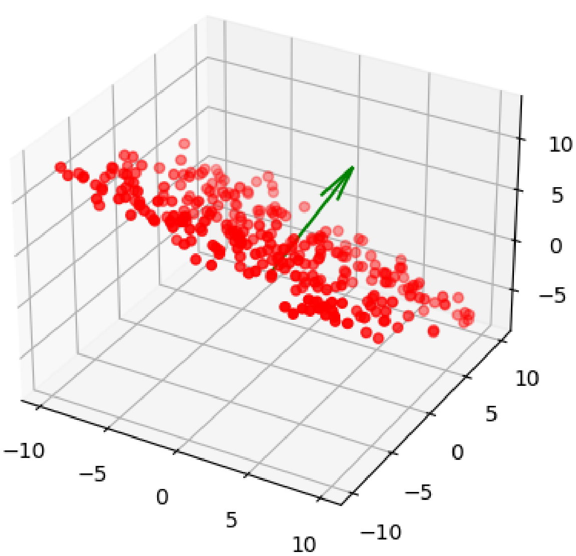

Figure 3: Normal for Point Cloud 1

2. • The general equation of plane is given by

$$a * X + b * Y + c = Z$$

To get get a surface that fits the point cloud using the least square method we first create a matrix A as follows: $\begin{bmatrix} x & y & 1 \end{bmatrix}$ and matrix $b = z$. We the compute the inverse of the matrix A and multiply it with b to get the coefficient matrix. Once we get the coefficients we can put them in the general equation to plot the best fit surface using least square method.

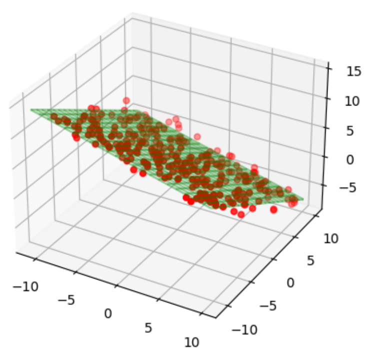

Figure 4: Least Square fitting for Point Cloud 1

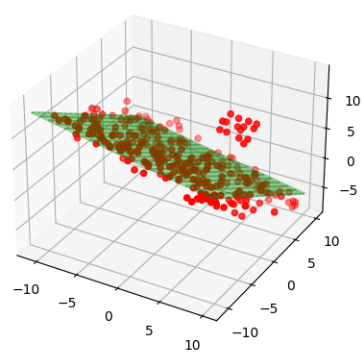

Figure 5: Least Square fitting for Point Cloud 2

- For Total least square we consider the equation of the plane to be

$$a * X + b * Y + c * Z = d$$

We then find the normal to the point cloud to get the coefficients a, b, c. We then multiply these coefficients with the mean values of $x, yandz$ respectively to get d. Once we get these coefficients we substitute them in the general equation to get the estimated plane.

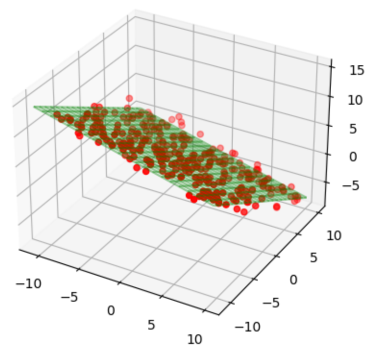

Figure 6: Total Least Square fitting for Point Cloud 1

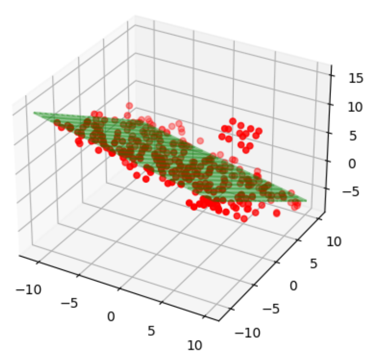

Figure 7: Total Least Square fitting for Point Cloud 2

3. RANSAC: To compute the estimation plane using RANSAC we take 3 random points in the point cloud and calculate the vectors $vector1$ and $vector2$ between the three points. Then we calculate the normal vector between $vector1$ and $vector2$. The values of this normal are the coefficients a, b and c for the estimated plane. we then calculate the d by computing the sum of values of the normal multiplied with components of $point1$.

   Using the 4 coefficients we then compute the distance between each point and the surface by

$$dist = \frac{(a * x + b * y + c * z + d)}{\sqrt{(a^2 + b^2 + c^2)}}$$

   If the distance between the surface and the point is less than the threshold value the point is an inlier else it is an outlier. The plane with the most inliers is considered the best fit for the point cloud.

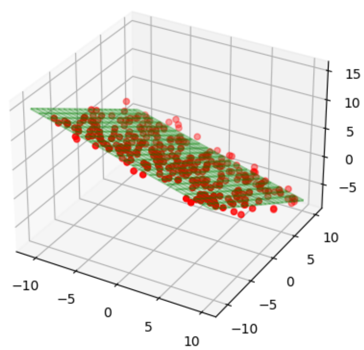

Figure 8: RANSAC fitting for Point Cloud 1

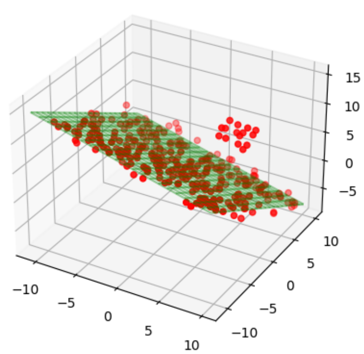

Figure 9: RANSAC fitting for Point Cloud 2

4. Comparison: The surface plot between all three fitting methods for the given data sets were very close especially in the case point cloud 1. We can see from the plots that of point cloud 2 that total least square method considers the outliers in the top right corner and is hence lifted. Where as RANSAC did not account take the outliers into account while finding the best fit for the point cloud.

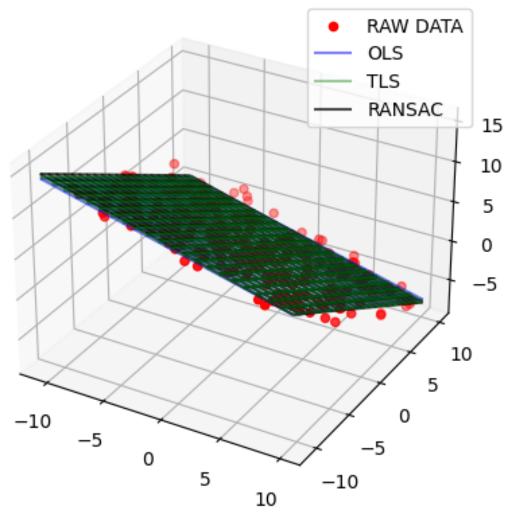

Figure 10: Comparing fits for Point Cloud 1

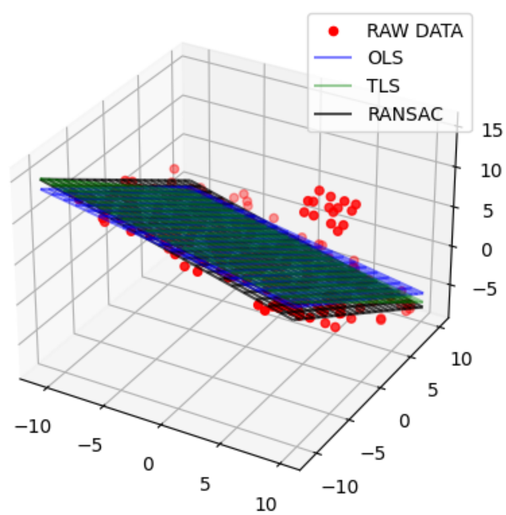

Figure 11: Comparing fits for Point Cloud 2

5. Problems Encountered: While plotting the surface z needed to be 2 dimensional while the z we calculated was 1 dimensional converting z to 2 dimensions was a problem I

faced and solved with some help form stack overflow. The links to the references is given at the end of the code