Optimization I
Project 1 – Linear Programming
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Problem Description:

Assume that your company is deciding how to spend a marketing budget of $10M. You work in the marketing department as a data scientist and the chief marketing officer has asked you write a report recommending how to spread this budget among several marketing mediums. Your department has employed an outside consulting firm to estimate the return on investment (ROI) of each marketing medium under consideration. The results are in the table below, and also in a CSV attached to this assignment:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	3.1%	4.9%	2.4%	3.9%	1.6%	2.4%	4.6%	2.6%	3.3%	4.4%

On top of these ROIs, your boss has decided to constrain your budget as follows:

a. The amount invested in print and TV should be no more than the amount spent on Facebook and Email.
b. The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords.
c. For each platform, the amount invested should be no more than $3M.

Import modules and load the required datasets :

```
# Import required libraries and packages
import pandas as pd
import numpy as np
import gurobipy as gp
from matplotlib import pyplot as plt
%matplotlib inline
```

```
# Import the given CSV files
ROI_data = pd.read_csv('ROI_data.csv')
# Added index_col=0 to make month name as the index of dataframe
roi_mat = pd.read_csv('roi_mat.csv',index_col=0)
ROI_data
```

Question 3:

Formulate the marketing budget allocation problem as a linear program. Use gurobi to find the optimal budget allocation.

Let's formulate the marketing budget allocation problem as a linear program:

Let X_1 be the amount invested in print, X_2 be the amount invested in TV,
X_3 be the amount invested in SEO, X_4 be the amount invested in AdWords,
X_5 be the amount invested in Facebook, X_6 be the amount invested in LinkedIn,
X_7 be the amount invested in Instagram, X_8 be the amount invested in Snapchat,
X_9 be the amount invested in Twitter and X_{10} be the amount invested in Email.

Maximize ROI Using following objective function:

$$0.031X_1 + 0.049X_2 + 0.024X_3 + 0.039X_4 + 0.016X_5 + 0.024X_6 + 0.046X_7 + 0.026X_8 + 0.033X_9 + 0.044X_{10}$$

Constraints:

$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} \leq 10$	(Total Budget Constraints)
$X_5 + X_6 + X_7 + X_8 + X_9 - 2(X_3 + X_4) \leq 0$	(Inter-Platform Budget Constraint)
$X_1 + X_2 - X_5 - X_{10} \leq 0$	(Inter-Platform Budget Constraint)
$X_i \leq 3$, where I ranges from 0 to 9	(Platform Budget Constraint)

Using gurobi to solve the problem,

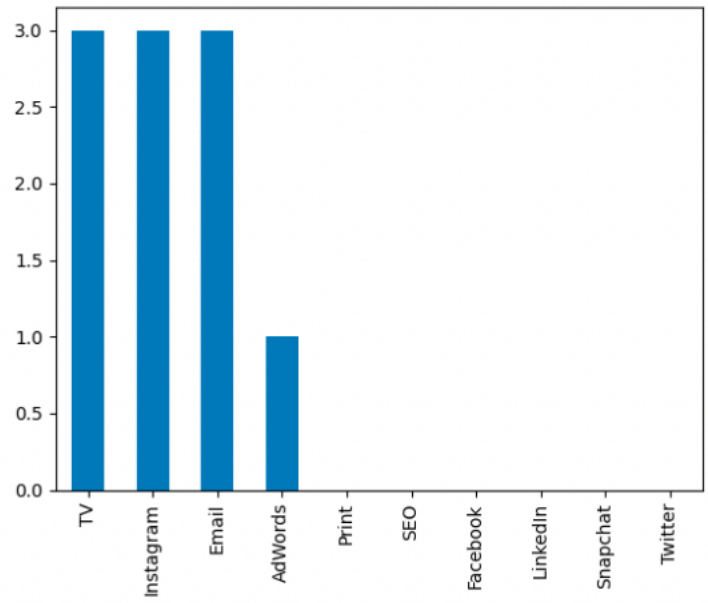
```
# Construct the objective matrix and constraint matrix

# Consider first row of dataframe from column 1(drop platform column) for
 ↪objective matrix
obj = np.array(ROI_data.iloc[0][1:])

# Left hand side of constraits.
A = np.zeros((len(obj)+3, len(obj))) #Initialize matrix with zeroes

# Each medium is budgeted for amount less than or equal to 3M.
A[0:len(obj), 0:len(obj)] = np.identity(len(obj))

#Total budget is of 10M
A[len(obj), :] = 1

#The amount invested in print and TV should be no more than the amount spent on
 ↪Facebook and email
A[len(obj)+1, [medium_dict['Print'], medium_dict['TV']]] = 1
A[len(obj)+1, [medium_dict['Facebook'], medium_dict['Email']]] = -1

#The total amount used in social media (Facebook, LinkedIn, Instagram,
 ↪Snapchat, and
#Twitter) should be at least twice of SEO and AdWords.

A[len(obj)+2, [medium_dict['SEO'], medium_dict['AdWords']]] = 2
A[len(obj)+2, [medium_dict['Facebook'], medium_dict['LinkedIn'],
 ↪medium_dict['Instagram'], medium_dict['Snapchat'], medium_dict['Twitter']]]
 ↪= -1

#Right hand side of contraints

B = np.zeros(len(obj)+3)
B[0:len(obj)] = 3 #Each medium is budgeted for amount less than 3M.
B[len(obj)] = 10 ##Total budget is of 10M

#Sign of contraints we have contructed.
```

```
# Optimize the problem using Gurobi
ojModel = gp.Model() # initialize an empty model

ojModX = ojModel.addMVar(len(obj)) # tell the model how many variables there are
# must define the variables before adding constraints because variables go into␣
 ↪the constraints
ojModCon = ojModel.addMConstrs(A, ojModX, sense, B) # add the constraints to␣
 ↪the model
ojModel.setMObjective(None,obj,0,sense=gp.GRB.MAXIMIZE) # add the objective to␣
 ↪the model...we'll talk about the None and the 0

ojModel.Params.OutputFlag = 0 # tell gurobi to shut up!!
ojModel.Params.TimeLimit = 3600

ojModel.optimize() # solve the LP
```

Solution of the gurobi model is given below:

```
The optimal ROI value is 0.45600000000000007

Marketing budget allocation across given mediums in millions
TV          3.0
Instagram   3.0
Email       3.0
AdWords     1.0
Print       0.0
SEO         0.0
Facebook    0.0
LinkedIn    0.0
Snapchat    0.0
Twitter     0.0
```

We also have plotted the histograms of optimal budget allocation:



From the above graph we can see that, optimal allocation assigns 3M each to TV, Instagram, and Email. Amount of 1 M has been assigned to AdWords medium.

The total expected ROI from the optimal allocation is 0.456. Hence, an investment of 10M $ we will get 10+0.456=10.456 M$.

Question 4:

To be cautious about the decision, your team has decided to get another opinion about the ROI data and rerun the analysis. The second consulting firm returns the estimates of the ROI data in the table below (also in the CSV file mentioned above). You are asked to compare the two optimal allocations from these two ROI estimates.

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	4.9%	2.3%	2.4%	3.9%	4.4%	4.6%	2.6%	1.9%	3.7%	2.6%

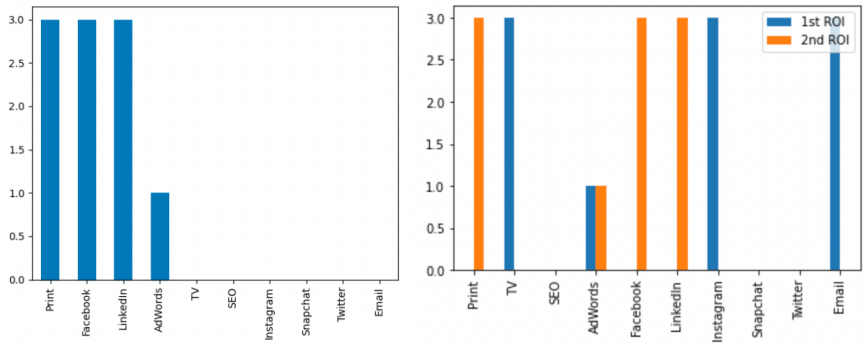
Maximize ROI Using following objective function:

$0.049X_1 + 0.023X_2 + 0.024X_3 + 0.039X_4 + 0.044X_5 + 0.046X_6 + 0.026X_7 + 0.019X_8 + 0.037X_9 + 0.026X_{10}$

Solution of the second gurobi model is given below:

```
The optimal ROI value is 0.45600000000000007

Marketing budget allocation across given mediums in millions
Print        3.0
Facebook     3.0
LinkedIn     3.0
AdWords      1.0
TV           0.0
SEO          0.0
Instagram    0.0
Snapchat     0.0
Twitter      0.0
Email        0.0
```

We also have plotted the histograms of optimal budget allocation:



We can see from the above table, the optimal allocation for AdWords remains the same i.e 1 million. The remaining 9 million is assigned equally to Print, Facebook and LinkedIn (3 million each). The optimal ROI value for the second allocation is the same as the optimal ROI value of the first allocation. Hence, an investment of 10M $ would give us 10+0.456=10.456 M $.

Question 5:

Are the allocations the same? Assuming the first ROI data is correct, if you were to use the second allocation how much lower would the objective be relative to the optimal objective? Assuming the second ROI data is correct, if you used the first allocation how much lower would the objective be relative to the optimal objective? Do you think the third constraint above, based on your boss' experience, is useful?

No, the allocations are not the same. The difference in the objective function for the first and second ROI values is 0. You can assume that any of the two ROI data is correct, this is because the constraints involved are non-binding, so which means that they are not sensitive to changes in the coefficients of the objective value function within a given range.

Assuming the first ROI is correct , with the Allocation of second ROI data:

If the first ROI is correct, we stand to lose 204,000 USD compared to the optimal objective

```
#First ROI is correct and second allocation is used
ojModel.objVal - obj@ojModel2.x
```

```
0.20400000000000007
```

Assuming the second ROI is correct , with the Allocation of second ROI data:

If the second ROI is correct, we stand to lose 192,000 USD compared to the optimal objective

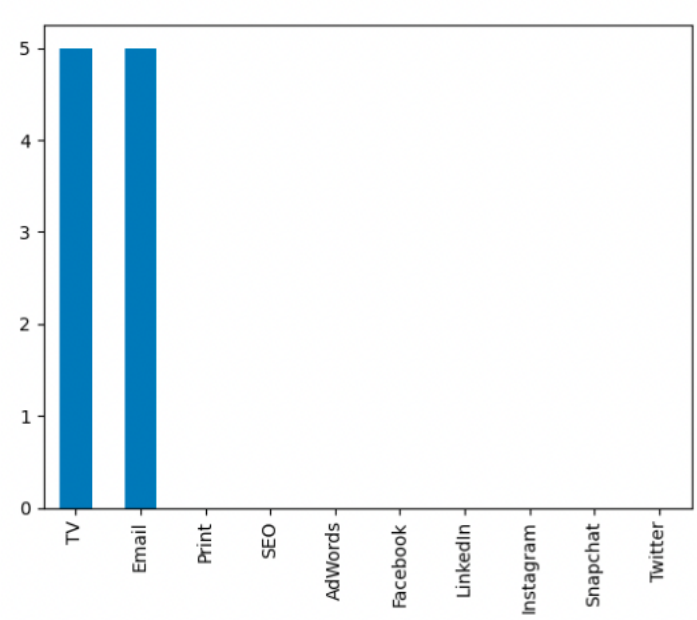
```
#Second ROI is correct and first allocation is used
ojModel2.objVal-obj2@ojModel.x
```

```
0.19200000000000006
```

Removing the third constraint to see if its useful:

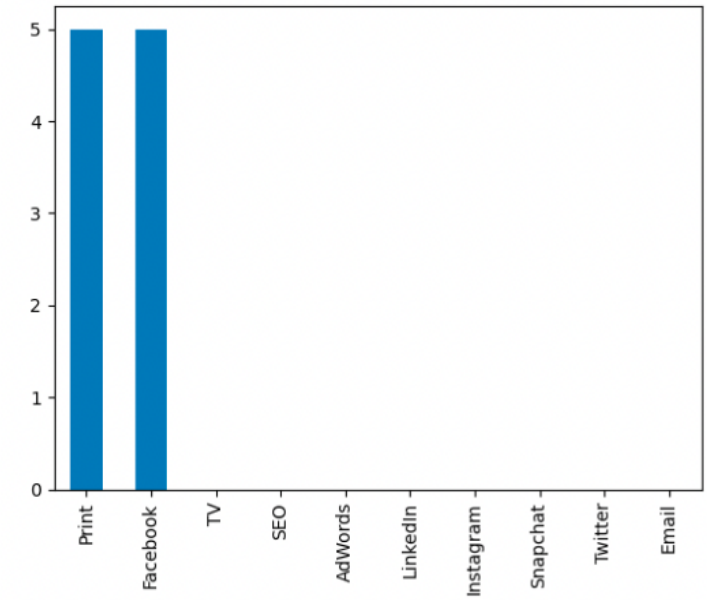
The optimal allocation across platforms for the first ROI estimates after removing the third constraint is:

```
The optimal ROI value is 0.46499999999999997

Marketing budget allocation across given mediums in millions
TV          5.0
Email       5.0
Print       0.0
SEO         0.0
AdWords     0.0
Facebook    0.0
LinkedIn    0.0
Instagram   0.0
Snapchat    0.0
```



The optimal allocation across platforms for the second ROI estimates after removing the third constraint is:

```
Marketing budget allocation across given mediums in millions
Print        5.0
Facebook     5.0
TV           0.0
SEO          0.0
AdWords      0.0
LinkedIn     0.0
Instagram    0.0
Snapchat     0.0
Twitter      0.0
Email        0.0
```



In both ROI estimates, the Optimal ROI without the constraint is $0.465 million. We have seen in the above questions that the optimal ROI with the constraint is $0.456 million. Thus, if we remove the constraint, we gain $0.009 million or $9000 USD extra.

Profit is maximised without the constraint. However, As we can see the budget allocation for media spends is split in only in two media. Adding a constraint maybe a method to avoid distributing budget in only two channels. The boss' third constraint makes sense because we want to distribute our assets among all the outlets. This means that if any one marketing medium fails, we would still gain profits from the others .

Question 6:

To explore this further perform some analysis of how your optimal allocation would change based on changes in the ROI data. Use the first ROI data as your starting point. By how much could each advertising medium's ROI increase or decrease and still result in the same optimal allocation you found in step (3)?

To do this we utilize the Sensitive Analysis properties in the Gurobi optimization model.

```
# We have 13 constraints hence we will get 13 values
print([con.Pi for con in ojModCon])
```

```
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.006999999999999999, 0.0, 0.0, 0.015, 0.039,
0.010000000000000002, 0.0]
```

Here, equations 7,10,11,12 have non zero values. Hence, these equations are binding constraints. The remaining constraints are non-binding constraints.

Hence, keeping all other constraints constant,
Hence, an increase of 1 M in the budget(3M) for Instagram will increase optimal ROI by 0.069.
An increase of 1 M in the budget(3M) for email will increase optimal ROI by 0.015.
An increase of 1 M in the 10M total budget will increase optimal ROI by 0.039.

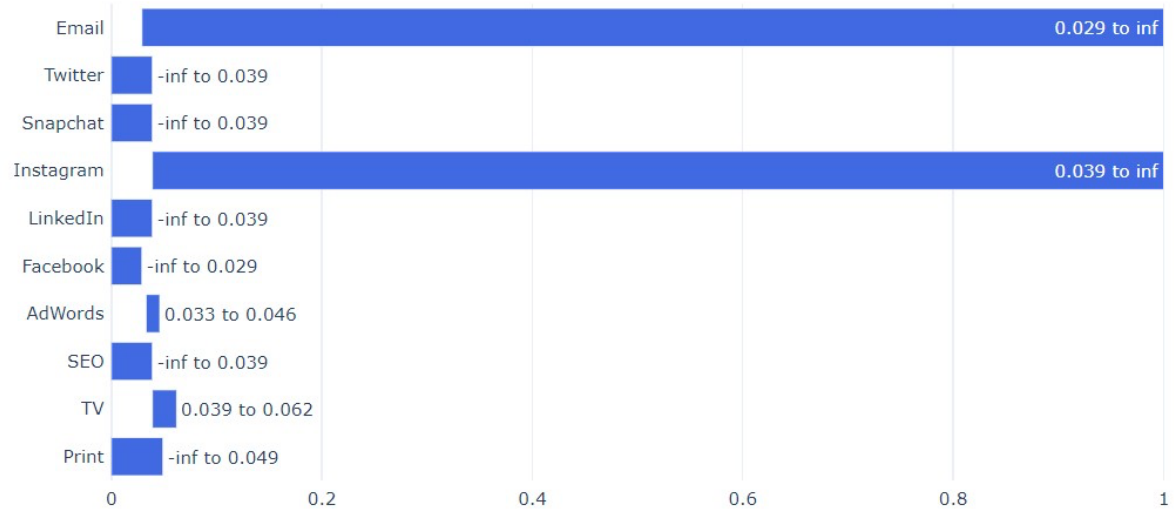
-Inf and Inf values indicate non-binding constraints. This is because of the fact that each medium may have a maximum of 3M in investment.

```
#By how much could each advertising medium's ROI increase or decrease and
#still result in the same optimal allocation you found in step (3)

print("The lowest values of variables are given below")
print(ojModX.SAObjLow)
print("The highest values of variables are given below")
print(ojModX.SAObjUp)
```

```
The lowest values of variables are given below
[ -inf 0.039  -inf 0.033  -inf  -inf 0.039  -inf  -inf 0.029]
The highest values of variables are given below
[0.049 0.062 0.039 0.046 0.029 0.039   inf 0.039 0.039   inf]
```

Below are the tabulated values. If we keep all other values constant, we can change the ROI for the medium of concern in the ranges below, to maintain the same Optimal Allocation.

platform	Minimum	Maximum	current value	Allowed increase	Allowed decrease
Print	0.000	0.049	0.031	0.018	0.100
TV	0.039	0.062	0.049	0.013	0.010
SEO	0.000	0.039	0.024	0.015	0.100
AdWords	0.033	0.046	0.039	0.007	0.006
Facebook	0.000	0.029	0.016	0.013	0.100
LinkedIn	0.000	0.039	0.024	0.015	0.100
Instagram	0.039	0.100	0.046	0.100	0.007
Snapchat	0.000	0.039	0.026	0.013	0.100
Twitter	0.000	0.039	0.033	0.006	0.100
Email	0.029	0.100	0.044	0.100	0.015



Question 7:

Your boss has gained permission to reinvest half of the return. For example, if the marketing obtains a 4% return in January, the budget of February will be $10M + $10M × 4% × 50% = $10.2M. The monthly ROI for next year is given in Project1.Rdata. The three constraints given by your boss are still in place for each month. What is the optimal allocation for each month?

Now, we will calculate the monthly return using the ROI for each respective month. We re-invest half the return of each month and add to the principal amount. This results in increased returns.

Following the above guidelines, we have calculated monthly allocation using ROI data 1.

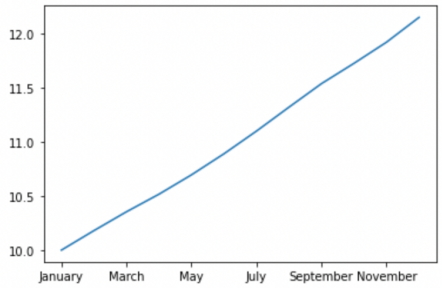
We have used the following formula to calculate the profit based on the allocation received:

```
profit = Adv_x4.x @ np.array(roi_mat.loc[i][0:]) / 100
budget += profit*0.5
```

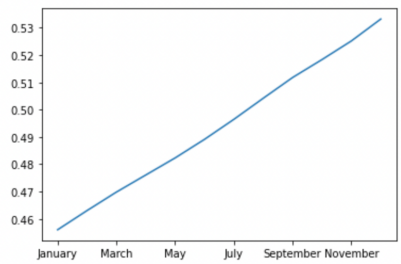
We have presented the monthly budget allocation and the accumulated returns in the tabular form below:

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Total
January	0.0	3.0	0.0	1.000000	0.0	0.0	3.0	0.0	0.000000	3.0	10.000000
February	0.0	3.0	0.0	1.180000	0.0	0.0	3.0	0.0	0.000000	3.0	10.180000
March	0.0	3.0	0.0	1.353920	0.0	0.0	3.0	0.0	0.000000	3.0	10.353920
April	0.0	3.0	0.0	1.505381	0.0	0.0	3.0	0.0	0.010763	3.0	10.516144
May	0.0	3.0	0.0	1.564986	0.0	0.0	3.0	0.0	0.129971	3.0	10.694957
June	0.0	3.0	0.0	1.629829	0.0	0.0	3.0	0.0	0.259657	3.0	10.889486
July	0.0	3.0	0.0	1.699709	0.0	0.0	3.0	0.0	0.399418	3.0	11.099126
August	0.0	3.0	0.0	1.773336	0.0	0.0	3.0	0.0	0.546672	3.0	11.320008
September	0.0	3.0	0.0	1.845758	0.0	0.0	3.0	0.0	0.691517	3.0	11.537275
October	0.0	3.0	0.0	1.908289	0.0	0.0	3.0	0.0	0.816578	3.0	11.724866
November	0.0	3.0	0.0	1.973321	0.0	0.0	3.0	0.0	0.946643	3.0	11.919964
December	0.0	3.0	0.0	2.049697	0.0	0.0	3.0	0.0	1.099393	3.0	12.149090

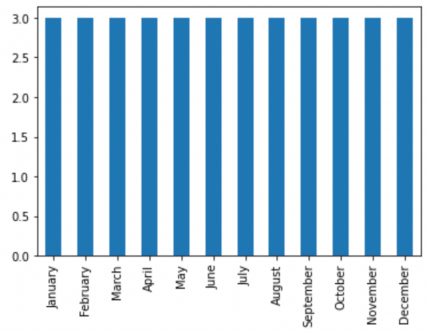
Below, we see the return earned each month.



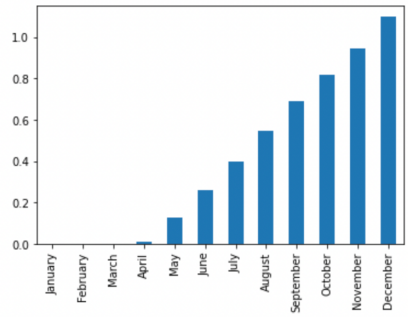
Below, we see the increase in budget across the months.



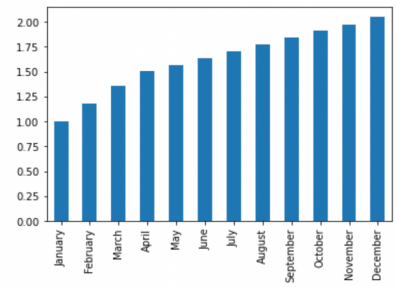
Monthly budget allocation for TV:



Monthly budget allocation for Twitter:



Monthly budget allocation for AdWords:



From the table, we can see that up until March there is only allocation to the 4 Mediums: TV, Instagram, Email and AdWords. From April, however, due to the increased budget, there is a slight investment in Twitter. Twitter becomes the 5th Medium of Investment and is allocated a higher budget as we move across the months.

Question 8:

A stable budget is defined as a monthly allocation such that for each platform the monthly change in spend is no more than $1M. Is the allocation you found stable? If it isn't, you do not need to solve a new optimization model. Describe how my might model this?

From the table, we can see that for each platform the monthly change in spending is no more than $1M. Hence, the allocation we found is stable. If it was not a stable allocation from month to month, we would have created a new matrix with optimal results from the last month. For each iteration of the optimal result from the last month, we would have added a constraint for allocation.

```
#first_ROI_monthly=first_ROI_monthly.drop('Total')
#first_ROI_monthly=first_ROI_monthly.drop('Optimum ROI value')
first_ROI_monthly.diff()
```

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Total	Optimal ROI value
January	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
February	0.0	0.0	0.0	0.180000	0.0	0.0	0.0	0.0	0.000000	0.0	0.180000	0.007020
March	0.0	0.0	0.0	0.173920	0.0	0.0	0.0	0.0	0.000000	0.0	0.173920	0.006783
April	0.0	0.0	0.0	0.151461	0.0	0.0	0.0	0.0	0.010763	0.0	0.162224	0.006262
May	0.0	0.0	0.0	0.059604	0.0	0.0	0.0	0.0	0.119208	0.0	0.178812	0.006258
June	0.0	0.0	0.0	0.064843	0.0	0.0	0.0	0.0	0.129686	0.0	0.194529	0.006809
July	0.0	0.0	0.0	0.069880	0.0	0.0	0.0	0.0	0.139760	0.0	0.209641	0.007337
August	0.0	0.0	0.0	0.073627	0.0	0.0	0.0	0.0	0.147255	0.0	0.220882	0.007731
September	0.0	0.0	0.0	0.072422	0.0	0.0	0.0	0.0	0.144845	0.0	0.217267	0.007604
October	0.0	0.0	0.0	0.062530	0.0	0.0	0.0	0.0	0.125061	0.0	0.187591	0.006566
November	0.0	0.0	0.0	0.065033	0.0	0.0	0.0	0.0	0.130065	0.0	0.195098	0.006828
December	0.0	0.0	0.0	0.076375	0.0	0.0	0.0	0.0	0.152751	0.0	0.229126	0.008019