Practical 1

Aim – Solving problems on strings, sets and binomial coefficients.

Q.1. Write the number of possibilities while picking a team of 3 people from group of 10.

Combination defined as an arrangement of objects where the order in which the objects where are selected does not matter.

$$C(n, r) = \frac{n!}{(n-r)!\,r!}$$

Here, $n = 10$ $r = 3$

$$\therefore \binom{10}{3} = C(10,3) = \frac{10!}{(10-3)!\,3!}$$

$$= \frac{10!}{7!\,3!}$$

$$= 120$$

$\therefore$ The number of possibilities is 120.

Q.2. Picking a President, VP and Waterboy from a group of 10.

Permutation defines an arrangement of objects where the order in which objects are selected matters.

$$P(n, r) = \frac{n!}{(n-r)!}$$ $n = 10$, $r = 3$

$$\therefore P(10, 3) = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 720$$

$\therefore$ There are 720 possibilities.

Q.3. Find expansion of following binomials.

(1) $(x+y)^5$

By Binomial Theorem,

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \cdots$$
$$\cdots + {}^nC_{n-1} x^1 y^{n-1} + {}^nC_n x^0 y^n$$

$$\therefore (x+y)^5 = {}^5C_0 x^5 y^0 + {}^5C_1 x^4 y^1 + {}^5C_2 x^3 y^2$$
$$+ {}^5C_3 x^2 y^3 + {}^5C_4 x^1 y^4 + {}^5C_5 x^0 y^5$$

$$= x^5 + 5x^4 y + 10x^3 y^2 + 10x^2 y^3 + 5xy^4 + y^5$$

$$\therefore \boxed{(x+y)^5 = x^5 + 5x^4 y + 10x^3 y^2 + 10x^2 y^3 + 5xy^4 + y^5}$$

(2) $(2x-y)^4$

By Binomial Theorem,

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \cdots$$
$$\cdots + {}^nC_{n-1} x^1 y^{n-1} + {}^nC_n x^0 y^n$$

$$(2x-y)^4 = {}^4C_0 2x^4 y^0 + {}^4C_1 2x^3 y^1 + {}^4C_2 2x^2 y^2$$
$$+ {}^4C_3 2x^1 y^3 + {}^4C_4 2x^0 y^4$$
$$= (2x)^4 - 4(2x)^3 y - 6(2x)^2 (y)^2 + 4(2x)(y)^3 + y^4$$

$$(2x-y)^4 = 16x^4 - 32x^3 y - 24x^2 y^2 - 8xy^3 - y^4$$

$$\boxed{(2x-y)^4 = 16x^4 - 32x^3 y - 24x^2 y^2 - 8xy^3 - y^4}$$

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Practical 2

Aim - Solving Problems Using Induction.

Q.4. Prove that $3^n > n^2$ for $n=1$, $n=2$ and use mathematical induction to prove that $3^n > n^2$ for any positive integer greater than 2.

By mathematical induction,

$n = 1$

we get, $3^1 > 1^2$

Let $n=2$

we get $3^2 > 2^2$

Assume P holds for $n=k$

$$3^k > k^2$$

Let $n = k+1$

$$3^{k+1} > (k+1)^2$$

$$3 \times 3^k > (k+1)^2$$

$$3^{k+1} = 3 \times 3^k > 3k^2$$

From the assumption. If $k \geq 2$, it follows that $k^2 \geq 2k$, $k^2 > 1$ so,

$$3k^2 = k^2 + k^2 + k^2 > k^2 + 2k + 1 = (k+1)^2$$

So

$$3^{k+1} > 3k^2 > (k+1)^2$$

Hence, proved.

Q.5. Using the principal of mathematical induction prove that $(1 \times 3) + (3 \times 5) + (5 \times 7) + \dots + (2n-1)(2n+1)$

$$= \frac{n(4n^2 + 6n - 1)}{3} \text{ for all } n \in N$$

Let $P(n): 1(3) + (3)(5) + (5)(7) + \dots + (2n-1)(2n+1)$

$$= \frac{n(4n^2+6n-1)}{3}$$

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For $n=1$,

$L.H.S = 1\cdot 3 = 3$

$R.H.S = \frac{1(4\cdot(1)^2+6(1)-1)}{3} = \frac{4+6-1}{3} = \frac{9}{3} = 3$

$\therefore LHS = RHS$

$\therefore$ P(n) is true for $n=1$

Assume P(k) is true

$$1(3)+3(5)+5(7)+\ldots+(2k-1)(2k+1) = \frac{k(4k^2+6k-1)}{3} \quad \text{--(i)}$$

We will prove that P(k+1) is true,

$$1(3)+3(5)+5(7)+\ldots+(2(k+1)-1)(2(k+1)+1)$$
$$= \frac{(k+1)(4(k+1)^2+6(k+1)-1)}{3}$$

$$1(3)+3(5)+5(7)+\ldots+(2k+2-1).(2k+2+1)$$
$$= \frac{(k+1)(4(k^2+1+2k)+6k+6-1)}{3}$$

$$1(3)+3(5)+5(7)+\ldots+(2k+1)(2k+3)$$
$$= \frac{(k+1)(4k^2+4+8k+6k+6-1)}{3}$$

$$1(3)+3(5)+5(7)+\ldots+(2k-1)(2k+1)+(2k+1).(2k+3)$$
$$= \frac{(k+1)(4k^2+4+8k+6k+6-1)}{3}$$

$$= \frac{(k+1)(4k^2+14k+9)}{3}$$

$$= \frac{(k(4k^2+14k+9)+1(4k^2+14k+9))}{3}$$

$$= \frac{(4k^3+18k^2+23k+9)}{3}$$

Thus,

$$P(k+1): 1(3)+3(5)+5(7)+\ldots+(2k-1)(2k+1)+(2k+1).(2k+3)$$
$$= \frac{(4k^3+18k^2+23k+9)}{3} \quad \ldots\ldots \text{(ii)}$$

We have to prove P(k+1) from P(k) i.e (ii) from (i)
From (i)

$$1(3)+3(5)+5(7)+\dots+(2k-1)(2k+1)=\frac{k(4k^2+6k-1)}{3}$$

Adding $(2k+1)(2k+3)$ both sides

$$1(3)+3(5)+5(7)+\dots+(2k-1)(2k+1)+(2k+1)(2k+3)$$

$$=\frac{k(4k^2+6k-1)}{3}+(2k+1)(2k+3)$$

$$=\frac{k(4k^2+6k-1)+3(2k+1)(2k+3)}{3}$$

$$=\frac{k(4k^2+6k-1)+3(2k(2k+3)+1(2k+3))}{3}$$

$$=\frac{k(4k^2+6k-1)+3(4k^2+6k+2k+3)}{3}$$

$$=\frac{k(4k^2+6k-1)+3(4k^2+8k+3)}{3}$$

$$=\frac{k(4k^2+6k-1)+(12k^2+24k+9)}{3}$$

$$=\frac{4k^3+6k^2+12k^2-k+24k+9}{3}$$

$$=\frac{(4k^3+18k^2+23k+9)}{3}$$

Thus,

$$1(3)+3(5)+5(7)+\dots+(2k-1)(2k+1)+(2k+1)(2k+3)$$

$$=\frac{(4k^3+18k^2+23k+9)}{3}$$

which is the same as P(k+1)
P(k+1) is true whenever P(k) is true.
By principle of mathematical induction, P(n) is true ~~for~~
n, where n is a natural number.

Q.6. Using principal of mathematical induction, prove that $3^n - 1$ is divisible by 2, is True for all positive integers.

Let $n = 1$

$P(1) = 3^1 - 1 = 2$

It is divisible by 2

Now assume that $P(k)$ is true or $3^k - 1$ is divisible by 2.

When $P(k+1)$

$3^{k+1} - 1 = 3^k \times 3 - 1 = 3^k \times 3 - 3 + 2 = 3(3^k - 1) + 2$

As $(3^k - 1)$ and 2 both are divisible by 2, it is proved that divisible by 2 is true for all positive integers.

Q.7. Prove using mathematical induction,

$$\left\{ \frac{1}{3(5)} + \frac{1}{5(7)} + \frac{1}{\cancel{5}7(9)} + \cdots + \frac{1}{(2n+1)(2n+3)} \right\} = \frac{n}{3}(2n+3)$$

Let $P(n)$: $\frac{1}{3(5)} + \frac{1}{5(7)} + \frac{1}{7(9)} + \cdots + \frac{1}{(2n+1)(2n+3)}$

$$= \frac{n}{3}(2n+3) = \frac{n}{3(2n+3)}$$

For $n = 1$,

$LHS = \frac{1}{3(5)} = \frac{1}{15}$

$RHS = \frac{1}{3(2(1)+3)} = \frac{1}{3(2+3)} = \frac{1}{3(5)} = \frac{1}{15}$

$\therefore$ $P(n)$ is true for $n = 1$

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Assum P(k) is true

$$\frac{1}{3(5)} + \frac{1}{5(7)} + \frac{1}{7(9)} + \dots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)} \quad \text{---(i)}$$

We will prove that P(k+1) is true

$$RHS = \frac{(k+1)}{3(2(k+1)+3)} = \frac{(k+1)}{3(2k+2+3)} = \frac{(k+1)}{3(2k+5)}$$

$$LHS = \frac{1}{3(5)} + \frac{1}{5(7)} + \frac{1}{7(9)} + \dots + \frac{1}{(2(k+1)+1)(2(k+1)+3)}$$

$$= \frac{1}{3(5)} + \frac{1}{5(7)} + \dots + \frac{1}{(2k+3)(2k+5)}$$

$$= \frac{1}{3(5)} + \frac{1}{5(7)} + \dots + \frac{1}{(2k+1)(2k+3)} + \frac{1}{(2k+3)(2k+5)}$$

From (i),

$$\frac{1}{3(5)} + \frac{1}{5(7)} + \dots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)}$$

$$= \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)}$$

$$= \frac{1}{(2k+3)} + \frac{1}{(2k+3)(2k+5)} = \frac{1}{(2k+3)}\left(\frac{k}{3} + \frac{1}{(2k+5)}\right)$$

$$= \frac{1}{(2k+3)}\left(\frac{k(2k+5)+3}{3(2k+5)}\right)$$

$$= \frac{1}{(2k+3)}\left(\frac{2k^2+2k+3k+3}{3(2k+5)}\right)$$

$$= \frac{1}{(2k+3)}\left(\frac{(2k+3)(k+1)}{3(2k+5)}\right)$$

$$= \frac{(k+1)}{3(2k+5)}$$

$\therefore$ P(k+1) is true whenever P(k) is true.

LHS = R.H.S

$\therefore$ Hence, proved.

Q.4. Prove by mathematical induction –

1) $3+3^2+3^3+\ldots\ldots 3^n = \frac{3}{2}(3^n-1)$

Let $p(n) : 3+3^2+3^3+\ldots\ldots+3^n = \frac{3}{2}(3^n-1)$

For $n=1$

LHS $= 3$

RHS $= \frac{3}{2}(3^n-1) = \frac{3(3-1)}{2} = \frac{3}{2}(2) = 3$

$\therefore$ Statement is true for $n=k, k \in N$

Assume that statement is true for $n=k, k \in N$

$\therefore 3+3^2+3^3+\ldots\ldots+3^k = \frac{3}{2}(3^k-1)$

Now to prove statement for $n=k+1$ i.e to prove

$3+3^2+3^3+\ldots\ldots+3^k+3^{k+1} = \frac{3}{2}(3^{k+1}-1)$

Consider,

LHS $= 3+3^2+3^3+\ldots\ldots+3^k+3^{k+1} = (3+3^2+3^3+\ldots\ldots+3^k)+3^{k+1}$

$= \frac{3}{2}(3^k-1)+3^{k+1}$

$= \frac{3^{k+1}}{2} - \frac{3}{2} + 3^{k+1} = 3^{k+1}\left(\frac{1}{2}+1\right) - \frac{3}{2} = 3^{k+1}\left(\frac{3}{2}\right) - \frac{3}{2}$

$= \frac{3}{2}(3^{k+1}-1)$

$=$ R.H.S.

$\therefore$ Statement is true for $n=k+1$

$\therefore$ By first principle of mathematical induction statement is true for each natural number n.

(2) $1(2) + 2(3) + 3(4) + \dots + n(n+1) = \frac{n(n+1)(n+3)}{3}$

Let $p(n)$: $1(2) + 2(3) + 3(4) + \dots + n(n+1) = \frac{n(n+1)(n+3)}{3}$

For $n=1$

LHS $= n(n+1) = 1\cdot(1+1) = 1\cdot 2 = 2$

RHS $= \frac{n(n+1)(n+2)}{3} = \frac{1(1+1)(1+2)}{3} = \frac{6}{3} = 2$

$\therefore$ LHS = RHS

$\therefore$ Statement is true for $n=1$

Assume that statement is true for $n=k$, $k \in N$

$\therefore$ $1(2) + 2(3) + 3(4) + \dots + k(k+1) = \frac{k(k+1)(k+2)}{3}$

Now to prove statement for $n=k+1$

i.e to prove $1(2) + 2(3) + 3(4) + \dots + k(k+1) + (k+1)(k+2)$

$= \frac{(k+1)(k+2)(k+3)}{3}$

Consider,

$$\text{LHS} = 1(2) + 2(3) + 3(4) + \dots + k(k+1) + (k+1)(k+2)$$

$$= [1(2) + 2(3) + 3(4) + \dots + k(k+1)] + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

$=$ RHS

$\therefore$ Statement is true for $n=k+1$

$\therefore$ By principal of mathematical induction statement is true for any natural number $n \geq 1$

(3) $1(2)(3)+2(3)(4)+3(4)(5)+\dots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

Let $p(n)$: $1(2)(3)+2(3)(4)+\dots+n(n+1)(n+2)$

$$=\frac{n(n+1)(n+2)(n+3)}{4}$$

For $n=1$

$LHS = n(n+1)(n+2) = 1(1+1)(1+2) = 6$

$RHS = \frac{n(n+1)(n+2)(n+3)}{4} = \frac{(1)(2)(3)(4)}{4} = 6$

$\therefore$ LHS = RHS

$\therefore$ Statement is true for $n=1$

Assume that statement is true for $n=k$, $k \in N$

$\therefore$ $1(2)(3)+2(3)(4)+3(4)(5)+\dots+k(k+1)(k+2)$

$$=\frac{k(k+1)(k+2)(k+3)}{4}$$

Now to prove statement for $n=k+1$

i.e to prove $1(2)(3)+2(3)(4)+\dots+k(k+1)(k+2)$

$+(k+1)(k+2)(k+3)$

$$=\frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

Consider,

$LHS = 1(2)(3)+2(3)(4)+\dots+k(k+1)(k+2)+(k+1)(k+2)(k+3)$

$=[1(2)(3)+2(3)(4)+\dots+k(k+1)(k+2)]$

$+(k+1)(k+2)(k+3)$

$$=\frac{k(k+1)(k+2)(k+3)}{4}+(k+1)(k+2)(k+3)$$

$$=\frac{k(k+1)(k+2)(k+3)+4(k+1)(k+2)(k+3)}{4}$$

$$=\frac{(k+1)(k+2)(k+3)(k+4)}{4} = RHS$$

$\therefore$ Statement is true for $n=k+1$

$\therefore$ By principle of mathematical induction statement is true for any natural number $n \geq 1$.

(4) $1+2+2^2+2^3+ \dots +2^n = 2^{n+1}-1$, $n \geq 0$

$P(n)$ i.e $P(n) = 1+2+2^2+ \dots +2^n = 2^{n+1}-1 \; \forall n \in N$

For $n=0$

$LHS = 2^n = 2^0 = 1$

$RHS = 2^{n+1}-1 = 2^{0+1}-1 = 2-1 = 1$

$\therefore$ Statement is true for $n=0$

Assume that statement is true for $n=k$, $k \in N$

$\therefore$ $1+2+2^3+ \dots +2^k = 2^{k+1}-1$

Now to prove statement for $n=k+1$

i.e. to prove $1+2+2^2+ \dots +2^k+2^{k+1} = 2^{k+2}-1$

Consider,

$LHS = 1+2+2^2+2^3+ \dots +2^k+2^{k+1}$

$= (1+2+2^2+2^3+ \dots +2^k) + 3^{k+1}$

$= 2^{k+1}-1+2^{k+1} = 2 \cdot 2^{k+1} = 2^{k+2}-1 = RHS$

$\therefore$ Statement is true for $n=k+1$

$\therefore$ By principle of mathematical induction statement is true $\forall \, n \geq 0$

(5) $1+x+x^2+ \dots +x^n = \frac{1-x^{n+1}}{1-x}$ $\forall \, n \geq 0$

Let $p(n): 1+x+x^2+ \dots +x^n = \frac{1-x^{n+1}}{1-x}$

For $n=0$

$LHS - x^n = x^0 = 1$

$$RHS = \frac{1-x^{n+1}}{1-x} = \frac{1-x^1}{1-x} = \frac{1-x}{1-x} = 1$$

Statement is true for $n=0$

Assume that statement is true for $n=k$, $k \in N$

$$1 + x + x^2 + \dots + x^k = \frac{1-x^{k+1}}{1-x}$$

Now to prove statement for $n=k+1$

i.e to prove $1+x+x^2+\dots+x^k+x^{k+1} = \dfrac{1-x^{k+2}}{1-x}$

Consider,

$$LHS = 1+x+x^2+\dots+x^k+x^{k+1} = (1+x+x^2+\dots+x^k) + x^{k+1}$$

$$= \frac{1-x^{k+1}}{1-x} + x^{k+1} = \frac{1-x^{k+1}+(1-x)x^{k+1}}{1-x}$$

$$= \frac{1-x^{k+1}+x^{k+1}-x^{k+1+1}}{1-x} = \frac{1-x^{k+2}}{1-x} \quad RHS$$

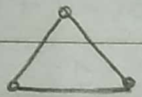
Statement is true for $n=k+1$

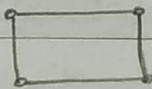
By principle of mathematical induction statement is true for all $n \geq 0$.

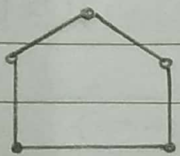
Practical 3

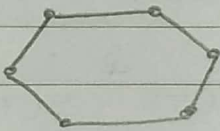
Aim – Solving problems on Chromatic number and coloring.

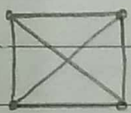
Q.1. Find the chromatic number of following graphs.

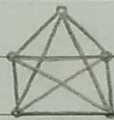
(1)  3

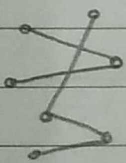
(2)  2

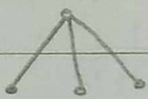
(3)  3

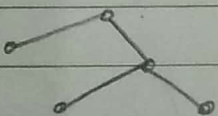
(4)  2

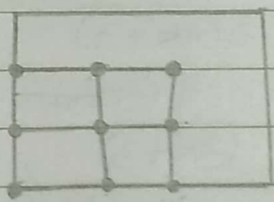
(5)  4

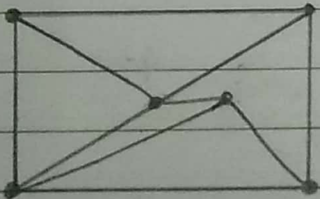
(6)  5

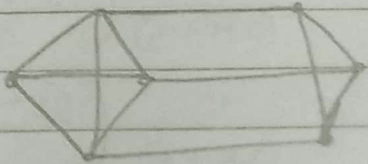
(7)  2

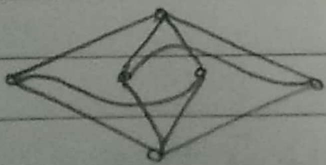
(8)  2

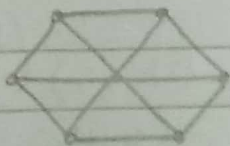
(9)  2

(10)  3

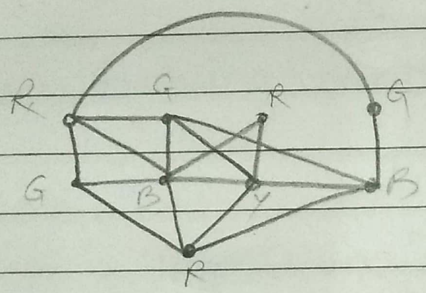
(11)  3

(12)  4

(13)  3

(14)  ~~3~~ 2

(15)



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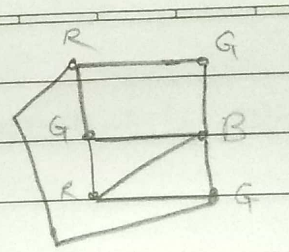

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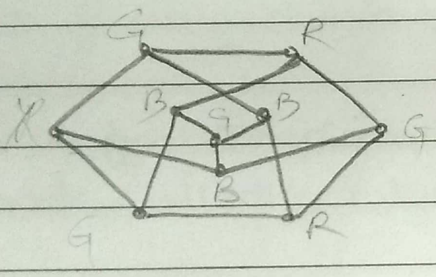

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Practical 5

Aim - Solving problems using Kruskal's Algorithm.

Intro -
Kruskal's Algorithm is one of the best known algorithm for finding minimum weight spanning tree.

Steps for Implementation -

(1) ~~Sotote~~ all the edges from low weight to high weight.

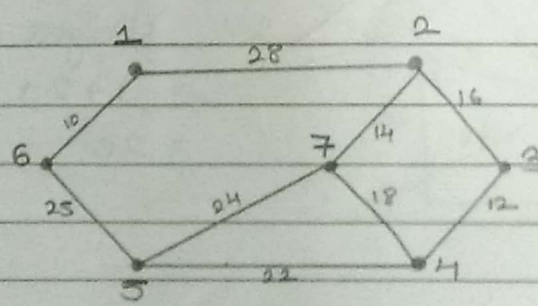
(2) Take the edges with the lowest edge and use it to connect the vertices. (Note - If adding the edges creates a cycle then reject that edge)

(3) Keep adding the edges untill all vertices are connected and you get minimum spanning tree.

(4) Remember, simply drop all the vertices and connect them with edges with minimum weight such that no cycle is form.

Q.1 Solve.

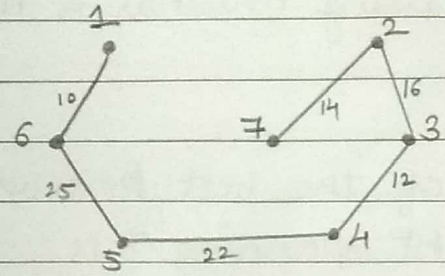
(1)



Step 1 -

× 1-2	28
1-6	10
3-4	12
2-7	14
2-3	16
× 4-7	18
4-5	22
× 5-7	24
5-6	25

Step 2

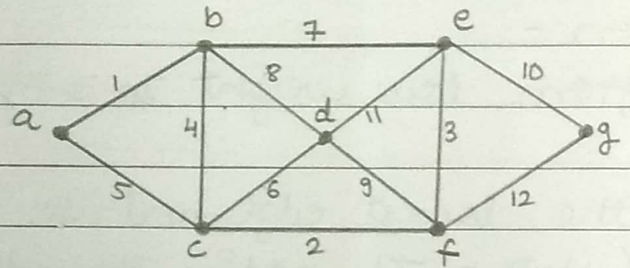


Weight of MST =

= 10 + 12 + 14 + 16 + 22 + 25

= 99

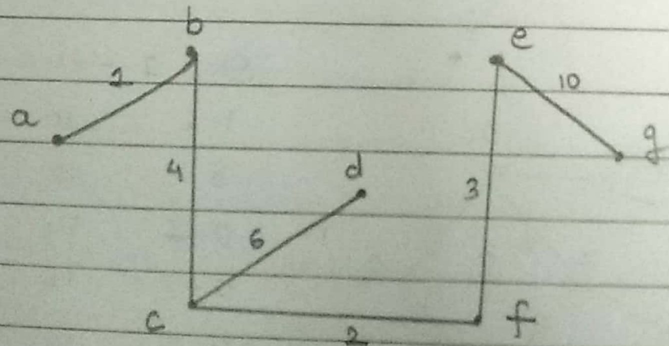
(2)



Step 1 -

a - b	1
c - f	2
e - f	3
b - c	4
× a - c	5
c - d	6
× b - e	7
× b - d	8
× d - f	9
e - g	10
× d - e	11
× g - f	12

Step 2 -

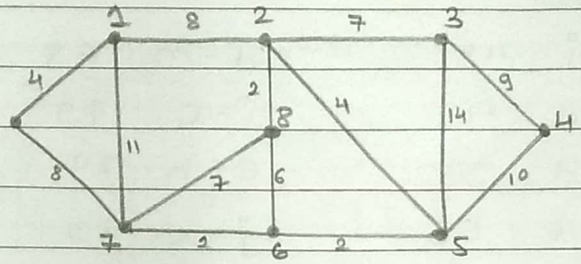


Weight of MST

= 1 + 2 + 3 + 4 + 6 + 10

= 26

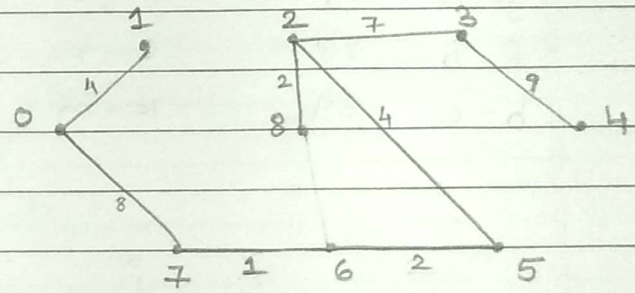
(3)



Step 1

7 - 6	1	0 - 7 -	8
6 - 5	2	× 1 - 2	8
2 - 8	2	3 - 4	9
0 - 1	4	4 - 5	10
2 - 5	4		
8 - 6	6	1 - 7	11
× 7 - 8	7	3 - 5	14
2 - 3	7		

Step 2 -

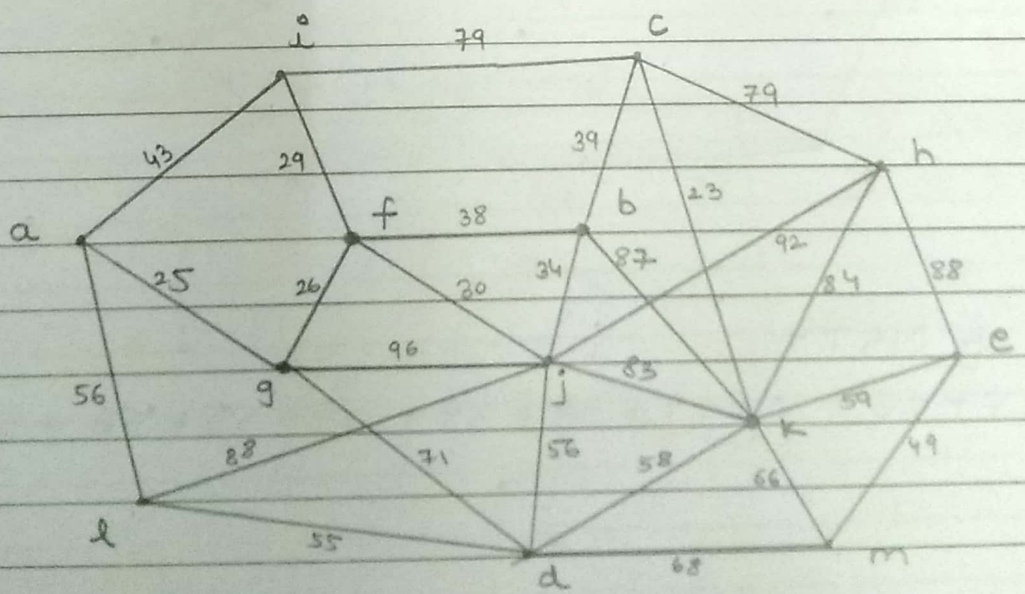


Weight of MST

= 4 + 8 + 1 + 2 + ~~4~~ + 2 + 7 + 9

= 37

(4)



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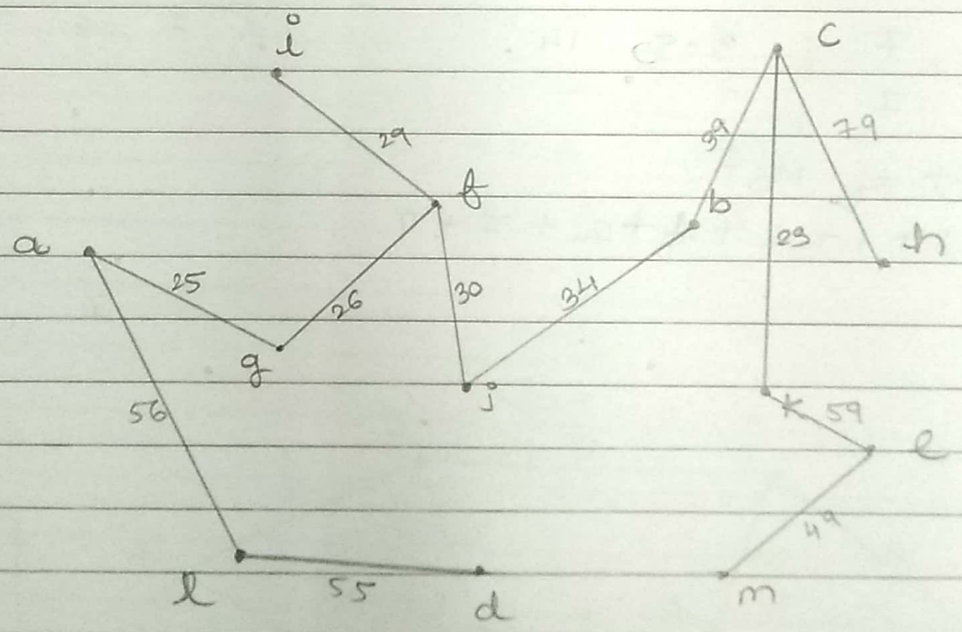
Step 1 -

c-k	23	× a-i	43	× d-m	68	× j-h	92
a-g	25	m-e	49	× i-c	79	× j-g	96
g-f	26	l-d	55	c-h	79	× g-d	71
f-j	30	a-l	56	× j-k	83		
i-f	29	× j-d	56	× j-h	84		
j-b	34	× d-k	58	× b-k	87		
× f-b	38	e-k	59	× h-e	88		
b-c	39	× k-m	66	× j-l	88		

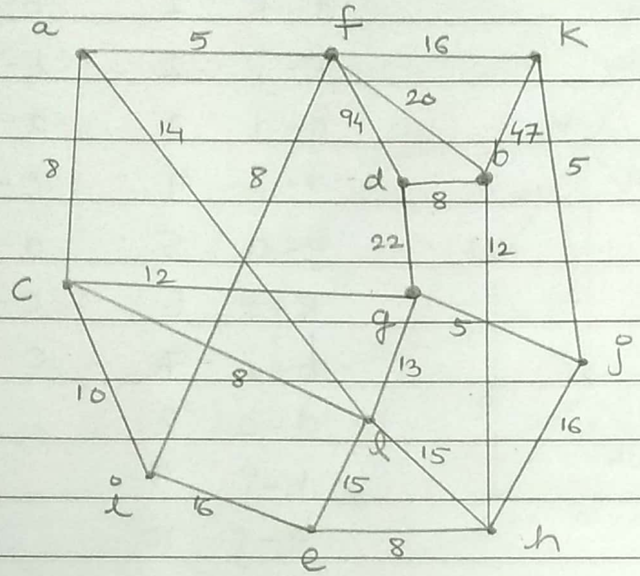


Weight of MST =

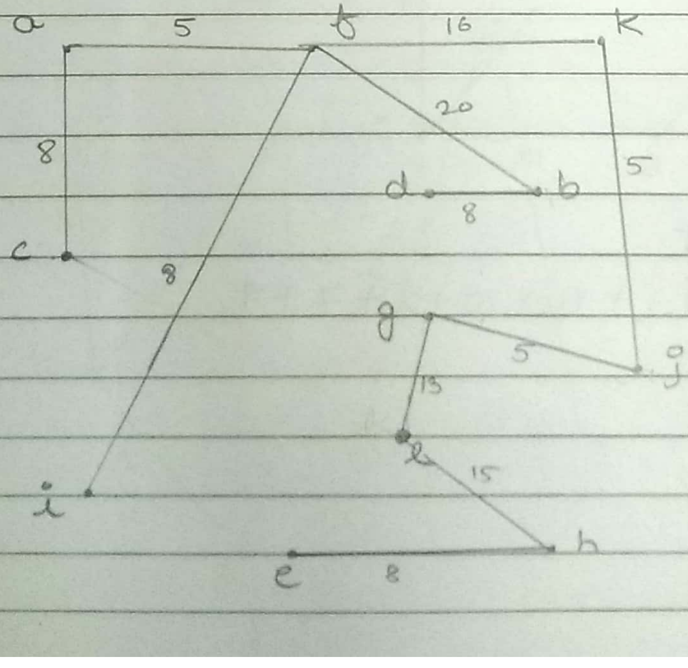
= 23 + 25 + 26 + 30 + 29 + 34 + 39 + 49 + 55 + 56 + 59 + 79

= 504

(5)



k - j	5	×c - l	8	×b - h	12	×i - e	16	×b - k	47
g - j	5	e - h	8	g - l	13	×j - h	16	×f - d	94
a - f	5	d - b	8	×l - a	14	f - k	16		
a - c	8	×c - i	10	l - h	15	f - b	20		
f - i	8	×c - g	12	×l - e	15	×d - g	22		

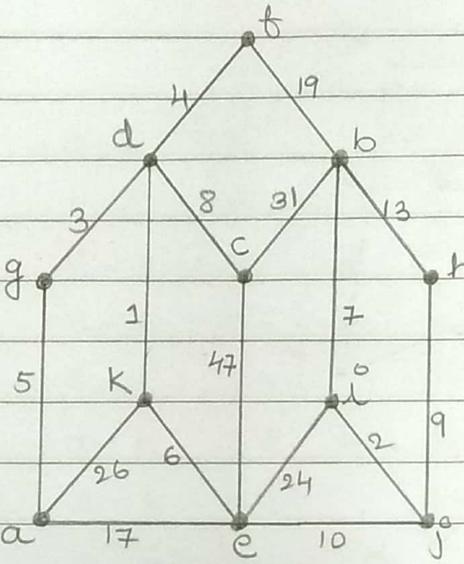


Weight of MST

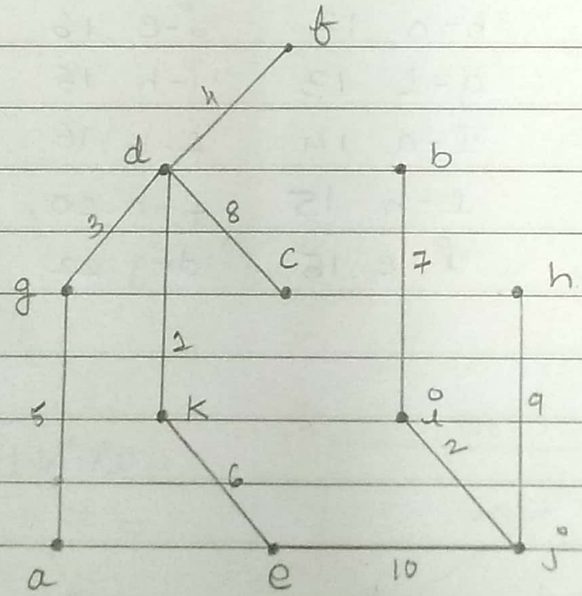
$= 5 + 16 + 8 + 20 + 8 + 5 + 5 + 13 + 15 + 8$

$= 103$

(6)



d - k	1	× b - h	13
i - j	2	× f - b	19
g - d	3	× a - e	17
d - f	4	× e - i	24
g - a	5	× a - k	26
k - e	6	× c - b	31
b - i	7	× c - e	47
d - c	8		
h - j	9		
e - j	10		

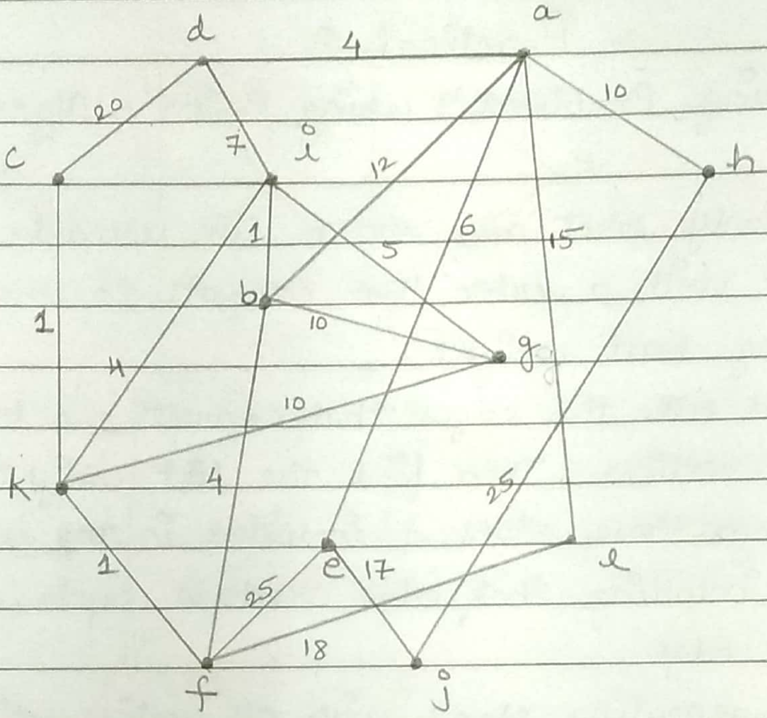


Weight of MST

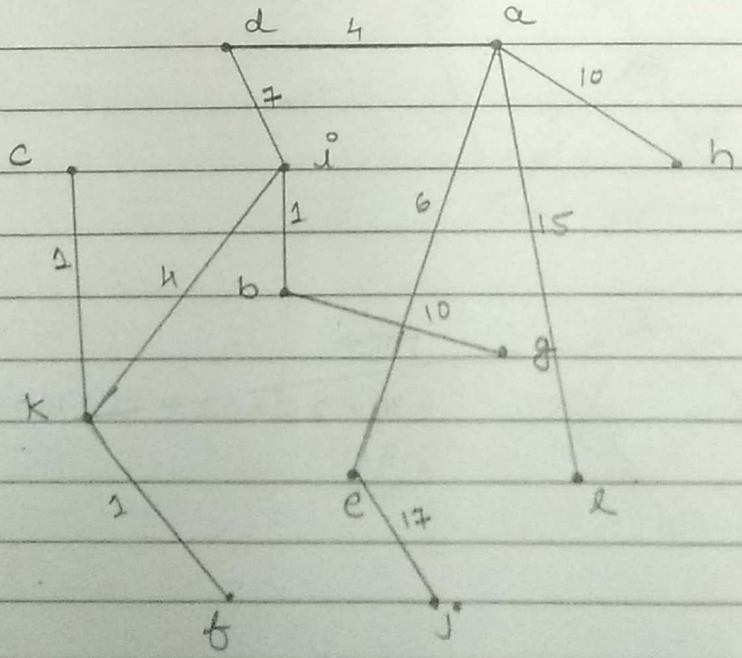
$= 4+3+5+8+1+6+10+2+7+9$

$= 55$

(7)



k - f	1	d-a	4	b-g	10	×f - l	18
c - k	1	×i-g	5	×k-g	10	×c-d	20
b - i	1	a-e	6	×b-a	12	×j-h	25
k - i	4	d-i	7	a-l	15	×e-f	25
×~~b - f~~	4	a-h	10	e-j	17		



Weight of MST

$= 1 + 1 + 1 + 4 + 4 + 6$

$+ 7 + 10 + 10 + 15 + 17$

$= 76$

Practical 6

Aim - Solving Problems's using Prim's Algorithm.

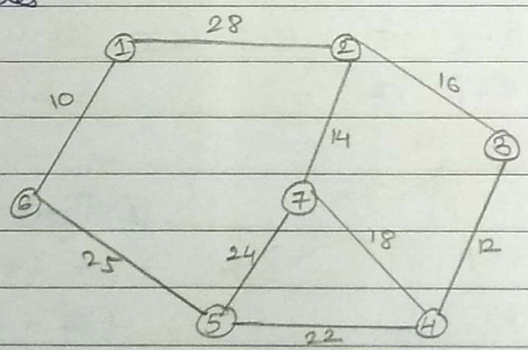
Step 1 - Randomly select any vertex. We usually select & start with a vertex that connects to the edge having least weight.

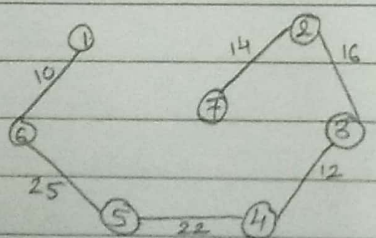
Step 2 - Find all the edges that connect the tree to new vertices. Then find the lest weight edge amongs those edges & included in the existing tree. If including that edge creates a cycle when reject that edge.

Step 3 - Keep repeating step 2 with all vertices until there included & the minimum spanning tree MST obtain.

Examples -

(1)



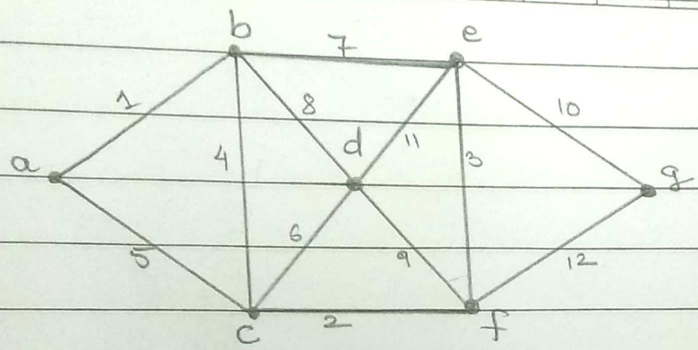


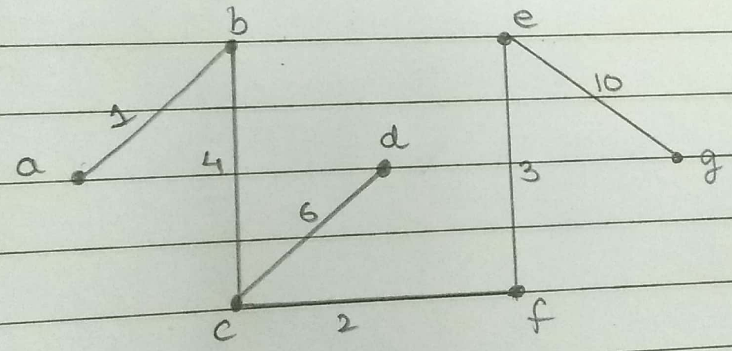
MST

$= 10 + 25 + 22 + 12 + 16 + 14$

$= \underline{99}$

(2)





MST

$= 1+4+2+3+10+6$

$= 26$

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