

INFO 6205

Program Structures & Algorithms

Fall 2020

Assignment No: 4

- **Task:** To perform benchmark test and code the alternative of the already implemented Quick Union method with following conditions:
 - Weighted Quick Union storing Depth instead of Size
 - Weighted Quick Union with path compression with two loops
- **Output:** Below are the average values of depth generated for different values of n as input –

The screenshot shows the Eclipse IDE with the following components:

- Package Explorer:** Displays the project structure, including packages like `edu.neu.coe.info6205` and `edu.neu.coe.info6205.util`.
- Editor:** Shows the file `Alternatives_Benchmark.java` with the following code:


```
1 package edu.neu.coe.info6205.util;
2
3 import edu.neu.coe.info6205.union_find.WQU_Depth;
4 import edu.neu.coe.info6205.union_find.WQU_Size;
5 import edu.neu.coe.info6205.union_find.WQUPC_OneLoop;
6 import edu.neu.coe.info6205.union_find.WQUPC_TwoLoops;
7
8 import java.util.Random;
9 import java.util.function.Consumer;
10 import java.util.function.Supplier;
11
12 public class Alternatives_Benchmark {
13
14     private static int bySizeSum = 0, byDepthSum = 0, oneLoopSum = 0, twoLoopSum = 0;
15
16     //Weighted Quick Union storing size
17     public static void test_WQU_BySize(int n) {
18         // ... (code for WQU_BySize) ...
19     }
20 }
```
- Console:** Displays the output of the benchmark test, showing average times and depths for various configurations and values of n (250, 500, 1000, 2000, 4000, 8000, 16000, 32000, 64000, 128000, 256000, 512000, 1024000, 2048000, 4096000, 8192000, 16384000, 32768000, 65536000, 131072000, 262144000, 524288000, 1048576000, 2097152000, 4194304000, 8388608000, 16777216000, 33554432000, 67108864000, 134217728000, 268435456000, 536870912000, 1073741824000, 2147483648000, 4294967296000, 8589934592000, 17179869184000, 34359738368000, 68719476736000, 137438953472000, 274877906944000, 549755813888000, 1099511627776000, 2199023255552000, 4398046511104000, 8796093022208000, 17592186044416000, 35184372088832000, 70368744177664000, 140737488355328000, 281474976710656000, 562949953421312000, 1125899906842624000, 2251799813685248000, 4503599627370496000, 9007199254740992000, 18014398509481984000, 36028797018963968000, 72057594037927936000, 144115188075855872000, 288230376151711744000, 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Console Output-

```
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing
Size with 30 runs
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing
Depth with 30 runs
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union with One
Loop Path Compression with 30 runs
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union with Two
Loop Path Compression with 30 runs
```

```
-----For N = 250-----
-----
```

```
Average time for QU storing size: 0.5 Average depth for QU storing size: 4.1875
Average time for QU storing Depth: 0.36666666666666664 Average depth for QU storing
Depth: 4.03125
Average time for QUPC with one Loop: 0.43333333333333335 Average depth for QUPC with
one Loop: 3.71875
Average time for QUPC with two Loop: 0.46666666666666667 Average depth for QUPC with
two Loop:1.625
```

```
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing
Size with 30 runs
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing
Depth with 30 runs
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union with One
Loop Path Compression with 30 runs
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union with Two
Loop Path Compression with 30 runs
```

```
-----For N = 500-----
-----
```

```
Average time for QU storing size: 0.5 Average depth for QU storing size: 4.5
Average time for QU storing Depth: 0.43333333333333335 Average depth for QU storing
Depth: 4.34375
Average time for QUPC with one Loop: 0.36666666666666664 Average depth for QUPC with
one Loop: 3.8125
Average time for QUPC with two Loop: 0.43333333333333335 Average depth for QUPC with
two Loop:1.78125
```

```
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing
Size with 30 runs
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing
Depth with 30 runs
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union with One
Loop Path Compression with 30 runs
2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union with Two
Loop Path Compression with 30 runs
```

```
-----For N = 1000-----
-----
```

```
Average time for QU storing size: 0.56666666666666667 Average depth for QU storing
size: 5.0625
Average time for QU storing Depth: 0.43333333333333335 Average depth for QU storing
Depth: 4.78125
Average time for QUPC with one Loop: 0.6 Average depth for QUPC with one Loop:
4.03125
```

Average time for QUPC with two Loop: 0.6666666666666666 Average depth for QUPC with two Loop:1.8125

2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing Size with 30 runs

2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing Depth with 30 runs

2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union with One Loop Path Compression with 30 runs

2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union with Two Loop Path Compression with 30 runs

-----For N = 2000-----

Average time for QU storing size: 0.9333333333333333 Average depth for QU storing size: 5.28125

Average time for QU storing Depth: 1.1 Average depth for QU storing Depth: 4.96875

Average time for QUPC with one Loop: 0.9 Average depth for QUPC with one Loop: 3.78125

Average time for QUPC with two Loop: 1.0 Average depth for QUPC with two Loop:1.84375

2020-10-13 19:20:30 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing Size with 30 runs

2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing Depth with 30 runs

2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union with One Loop Path Compression with 30 runs

2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union with Two Loop Path Compression with 30 runs

-----For N = 4000-----

Average time for QU storing size: 1.4666666666666666 Average depth for QU storing size: 5.6875

Average time for QU storing Depth: 1.6 Average depth for QU storing Depth: 5.5

Average time for QUPC with one Loop: 1.7 Average depth for QUPC with one Loop: 4.1875

Average time for QUPC with two Loop: 1.1666666666666667 Average depth for QUPC with two Loop:1.78125

2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing Size with 30 runs

2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing Depth with 30 runs

2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union with One Loop Path Compression with 30 runs

2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union with Two Loop Path Compression with 30 runs

-----For N = 8000-----

Average time for QU storing size: 2.4333333333333333 Average depth for QU storing size: 6.15625

Average time for QU storing Depth: 2.4 Average depth for QU storing Depth: 5.875

Average time for QUPC with one Loop: 2.0666666666666667 Average depth for QUPC with one Loop: 4.28125

Average time for QUPC with two Loop: 1.7666666666666666 Average depth for QUPC with two Loop:1.90625

```

2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing
Size with 30 runs
2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union Storing
Depth with 30 runs
2020-10-13 19:20:31 INFO Benchmark_Timer - Begin run: Weighted Quick Union with One
Loop Path Compression with 30 runs
2020-10-13 19:20:32 INFO Benchmark_Timer - Begin run: Weighted Quick Union with Two
Loop Path Compression with 30 runs
-----For N = 16000-----
Average time for QU storing size: 6.3 Average depth for QU storing size: 6.4375
Average time for QU storing Depth: 5.466666666666667 Average depth for QU storing
Depth: 6.1875
Average time for QUPC with one Loop: 5.233333333333333 Average depth for QUPC with
one Loop: 3.9375
Average time for QUPC with two Loop: 4.0 Average depth for QUPC with two Loop:1.625

```

- Relationship conclusion:** It can be concluded from the results mentioned above that Weighted Quick Union storing the size and Weighted Quick Union storing depth have almost same benchmark results. This is because tree stores nodes differently and thus tree with n nodes has a height of $\log n$. So, the performance for n nodes will be $O(n \log(n))$.

For Weighted Quick Union with path compression, the performance is significantly better than weighted quick union. This is due to the fact that one and two loops of path compression are reducing the height by setting the parent of node to its grandparent in one loop and setting parent of node to directly its root in two loops. Thus, the performance of two loop path compression is better than one loop compression. The performance can be measured as $O(n + n \log(n))$.

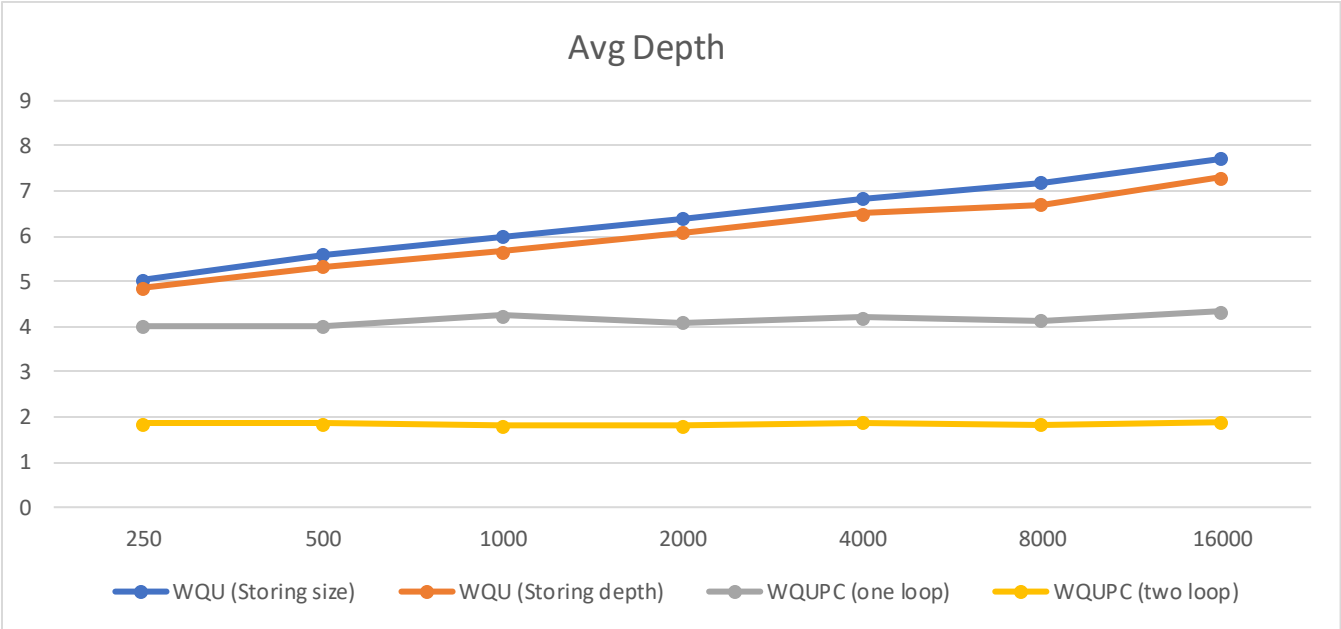
- Evidence to support relationship:** I have attached a chart and a table stating the data of the different output observed for the different set of inputs of n –

Average Run Time:

Value of n	WQU (Storing size)	WQU (Storing depth)	WQUPC (one loop)	WQUPC (two loop)
250	0.3529	0.4004	0.2953	0.3144
500	0.4384	0.4318	0.3729	0.3540
1000	0.6781	0.7091	0.6527	0.5097
2000	1.6321	1.5963	1.2635	1.0878
4000	3.5287	3.4470	2.8497	2.3595
8000	9.1219	7.6977	6.5228	5.5071
16000	17.4690	17.2030	14.1032	11.4350

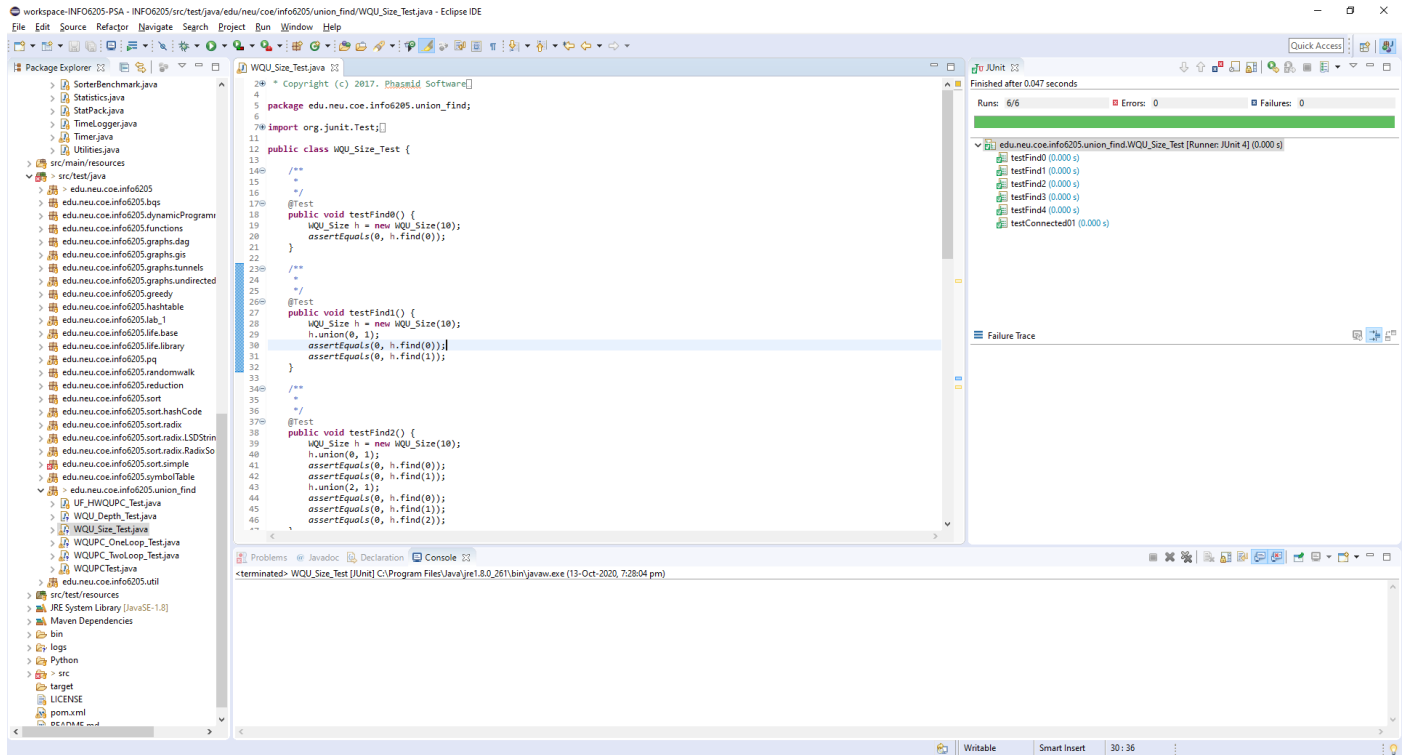
Average Depth:

Value of n	WQU (Storing size)	WQU (Storing depth)	WQUPC (one loop)	WQUPC (two loop)
250	5.039	4.862	4.009	1.852
500	5.588	5.323	4.009	1.862
1000	5.990	5.666	4.235	1.813
2000	6.401	6.088	4.088	1.813
4000	6.833	6.5	4.196	1.872
8000	7.186	6.705	4.127	1.833
16000	7.715	7.313	4.343	1.882

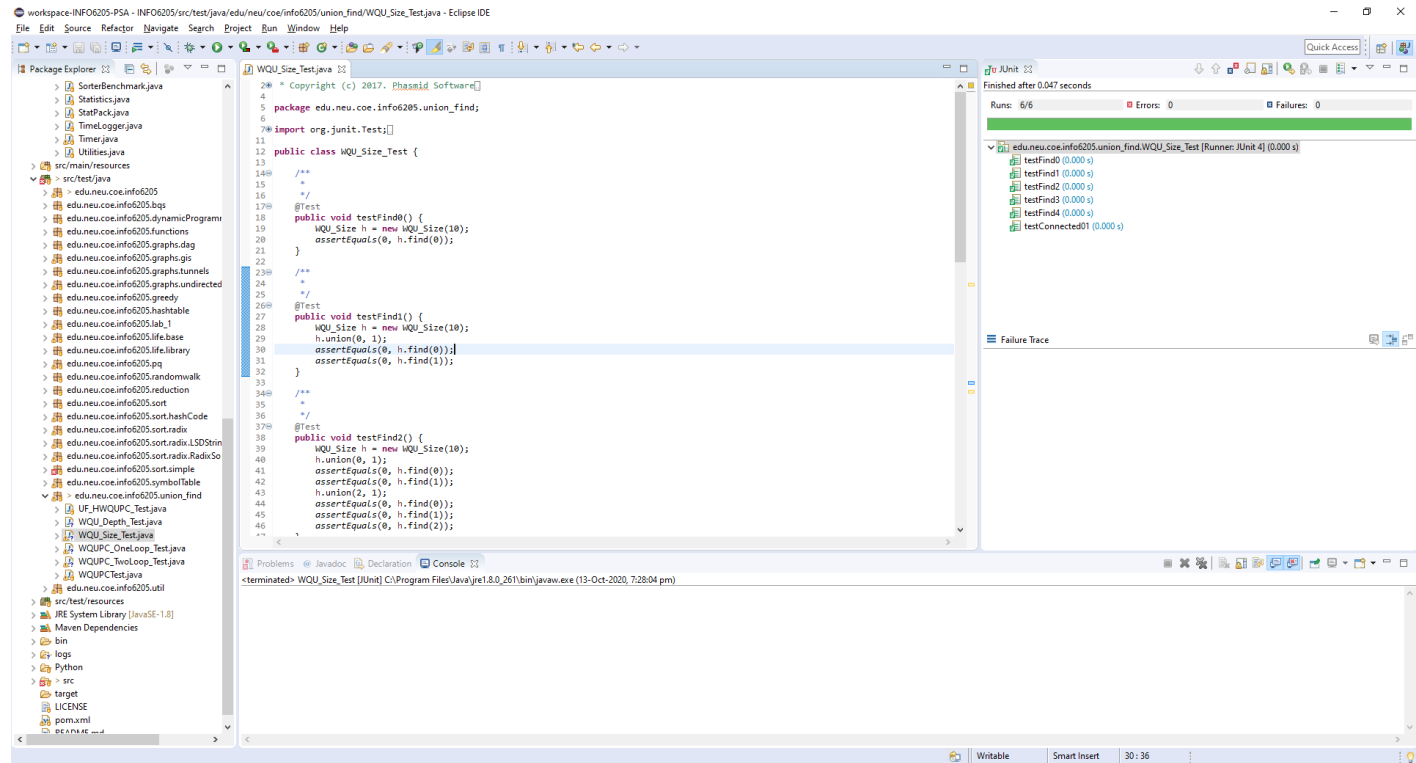


- **Screenshot of Unit test passing:** Below is the screenshot of all the unit tests which ran successfully-

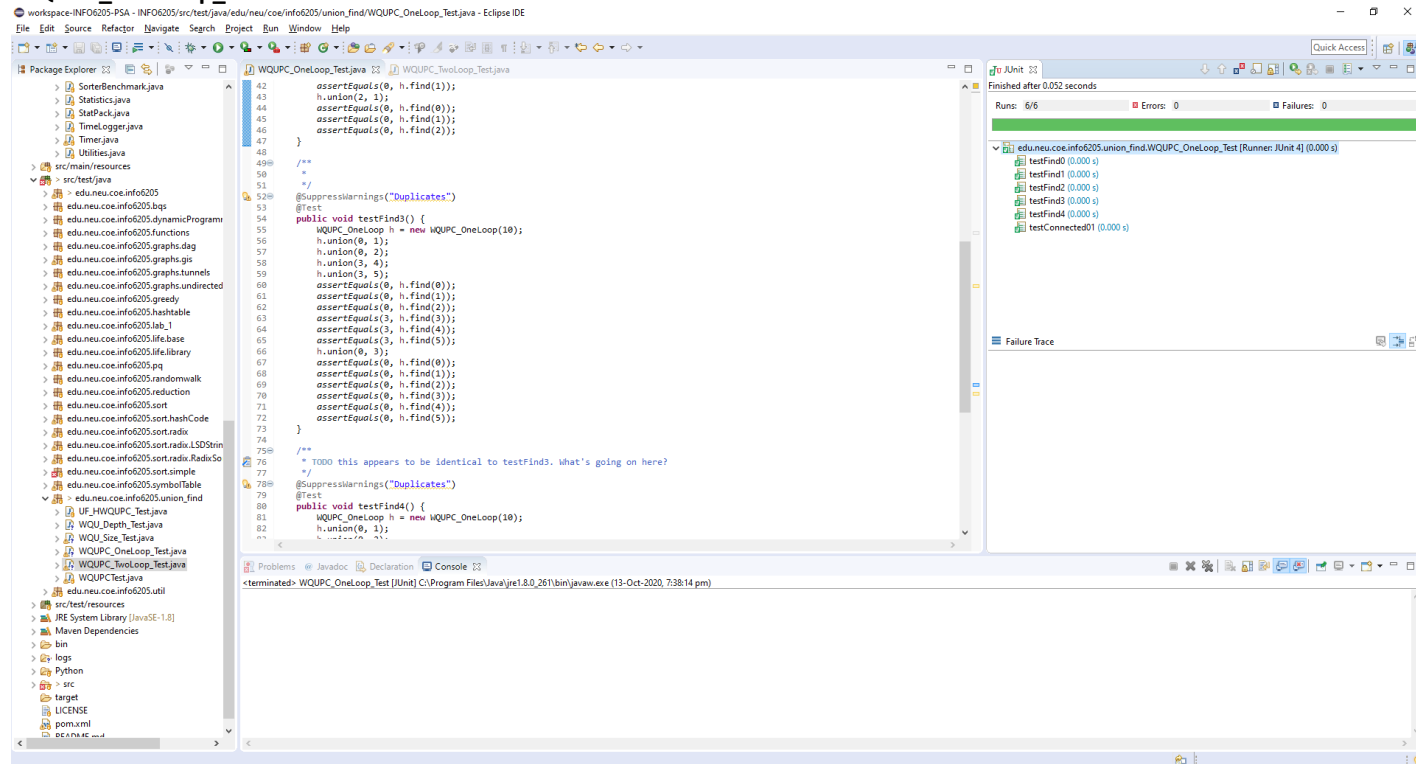
WQU_Size_Test



WQU_Depth_Test



WQUPC_OneLoop_Test



WQUPC_TwoLoop_Test

The screenshot displays the Eclipse IDE interface with the following components:

- Package Explorer:** Shows the project structure with packages like `edu.neu.coe.info6205` and `edu.neu.coe.info6205.union_find`. The `WQUPC_TwoLoop_Test.java` file is selected.
- Editor:** Displays the source code of `WQUPC_TwoLoop_Test.java`. The code includes a package declaration, imports, and several test methods: `testFind0()`, `testFind1()`, and `testFind2()`. Each method uses `assertEquals` to verify the results of the `WQUPC_TwoLoop` class.
- JUnit Runner:** Shows the test results for the `WQUPC_TwoLoop_Test` class. The tests passed successfully, with a total time of 0.007 seconds. The results are listed as: `testFind0 (0.002 s)`, `testFind1 (0.001 s)`, `testFind2 (0.000 s)`, `testFind3 (0.000 s)`, `testFind4 (0.001 s)`, and `testConnected01 (0.001 s)`.
- Console:** Shows the output of the test run, indicating that the tests passed successfully.