Subsymmetries Predict Auditory and Visual Pattern Complexity

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Abstract. A mathematical measure of pattern complexity based on subsymmetries possessed by the pattern, previously shown to correlate highly with empirically derived measures of cognitive complexity in the visual domain, also correlates significantly with empirically derived complexity measures of perception and production of auditory temporal as well as musical rhythmic patterns. Not only does the subsymmetry measure correlate highly with the performance complexity of reproducing the rhythms by tapping after listening to them, but the empirical measures exhibit similar behaviour, for both the visual and auditory patterns, as a function of the relative number of subsymmetries present in the patterns.

Keywords: musical rhythm, auditory patterns, visual patterns, subsymmetries, pattern complexity, perceptual complexity, cognitive complexity, performance complexity

1 Introduction

The correspondence across different modalities of perception, particularly between visual and auditory perception, is a topic that has received considerable attention in the literature for some time (Pitts and McCulloch 1947; Kubovy and Van Valkenburg 2001). Most research on multisensory perception has focused on understanding and modeling the spatial and temporal factors that modulate multisensory integration (Smith et al 2009; Spence 2011). Some studies have suggested that visual temporal structure is automatically transferred from a visual form to an auditory representation (Guttman et al 2005), only to be contradicted in subsequent analyses (McAuley and Henry 2010). In the hopes of obtaining a unified theory across visual and auditory modalities of perception some researchers have analyzed auditory and visual Gestalt groupings that exhibit similar organizational properties (Aksentijević et al 2001). Others have discovered that beat perception can occur in both modalities (Grahn 2012), but with greater affinity in the auditory modality (Patel et al 2005). Research has also shown that auditory rhythms are recognized and reproduced more accurately than visual rhythms (Jokiniemi et al 2008). A stimulus property that has received very little attention is pattern complexity (Toussaint 2012) and its relation to symmetry (Chen et al 2011), in the context of aural perception and production of musical rhythms, the topic of this report.

2 Methods

Alexander and Carey (1968) carried out an experimental study with visual spatial (non-temporal) patterns to determine stimulus properties that could predict empirically obtained differences in pattern complexity. Their paper is concerned with the cognitive complexity of 35 different one-dimensional strip patterns consisting of concatenations of three black and four white squares. The 35 patterns (binary sequences) make up all the permutations of the seven two-color squares, namely (7!/3!4!). These patterns, printed on grey paper, were presented visually to the subjects during the experiments. The authors defined the stimulus property to be a mathematical measure of pattern complexity based on subsymmetries present in the patterns, and explored the question of how the measure predicts the differences in the cognitive complexity of these 35 patterns. The central result of their paper is that the cognitive complexity “is almost perfectly accounted for by the relative numbers of subsymmetries in the different patterns.” A subsymmetry is defined as a contiguous (connected) subsequence of squares of the pattern that possesses (left-right) mirror symmetry. For example in the pattern BBWWW there are three subsymmetries of length two (BB, WW, WW) and one of length three (WWW) for a total of four. This total was used as a mathematical measure of complexity to rank all 35 patterns. Based on a series of five experiments conducted with human subjects the authors obtained an overall empirical ranking of the 35 patterns according to increasing complexity, listed in Figure 1 (left column). The simplest patterns are at the top and most complex at the bottom. The number on the right of each pattern is the number of subsymmetries present in that pattern. The correlation coefficient reported by Alexander and Carey (1968), between the mathematical and empirical rankings was 0.808, significant at the 0.00001 level, which led the authors to conclude with the fundamental result of their paper: “Patterns with many subsymmetries are cognitively simple. Patterns with few subsymmetries are not cognitively simple.”

Fitch and Rosenfeld (2007) conducted experiments to investigate listeners’ abilities to perceive, process, and produce complex syncopated rhythmic patterns. After first listening to the rhythms, the subjects were required to tap the rhythms while they listened to the beat (meter) of the rhythm. The accuracy of reproduction (playback error) was used as an empirical measure of performance complexity. They used 30 rhythms that were generated in such a way as to vary the amount of syncopation contained in the rhythms, ranging along a continuum from unsyncopated to highly syncopated, according to the objective “syncopation index” described by Longuet-Higgins and Lee (1984). The rhythms may be represented by 16-pulse binary sequences in which each pulse is either sounded or silent, and may thus be notated as a sequence of black (sounded) and white (silent) squares. Such a notation is referred to as box-notation (Toussaint 2013). The 30 rhythms in box-notation are shown in Figure 1 (central column). They are ranked in increasing order of performance complexity.

Royer and Garner (1966) performed experiments to test the complexity of auditory patterns by means of the perceptual difficulty encountered by the subjects tested. Two qualitatively dissimilar sounds were used to create different binary sequences of length eight, each yielding either 2, 4, or 8 distinguishably distinct starting points. The subjects were instructed to listen and begin responding by operating a telegraph key in synchrony with the patterns, when they thought they were able. The variability of the point in time at which the subjects began responding (called the *response uncertainty* for a given pattern), the average *delay* before responding, and the average *synchronization errors* produced during responding, were measured, and served as three empirical measures of the stimulus complexity of the temporal patterns. The three measures were found to be highly correlated, leading the authors to the conclusion that “pattern which are easily organized are those which have few alternative modes of organization, and thus can be considered as simple, or good in the Gestalt sense.” The 19 two-tone fundamental sequences used by Royer and Garner (along with the calculations of the three measures) are represented in Figure 1 (right) as binary sequences of black and white squares (box notation), and are ordered according to increasing values of the response uncertainty (pattern complexity). It should be pointed out that these sequences were presented to the subjects in all their eight possible starting points, and the sequences shown in Figure 1 illustrate just one starting point of each fundamental pattern (or necklace).

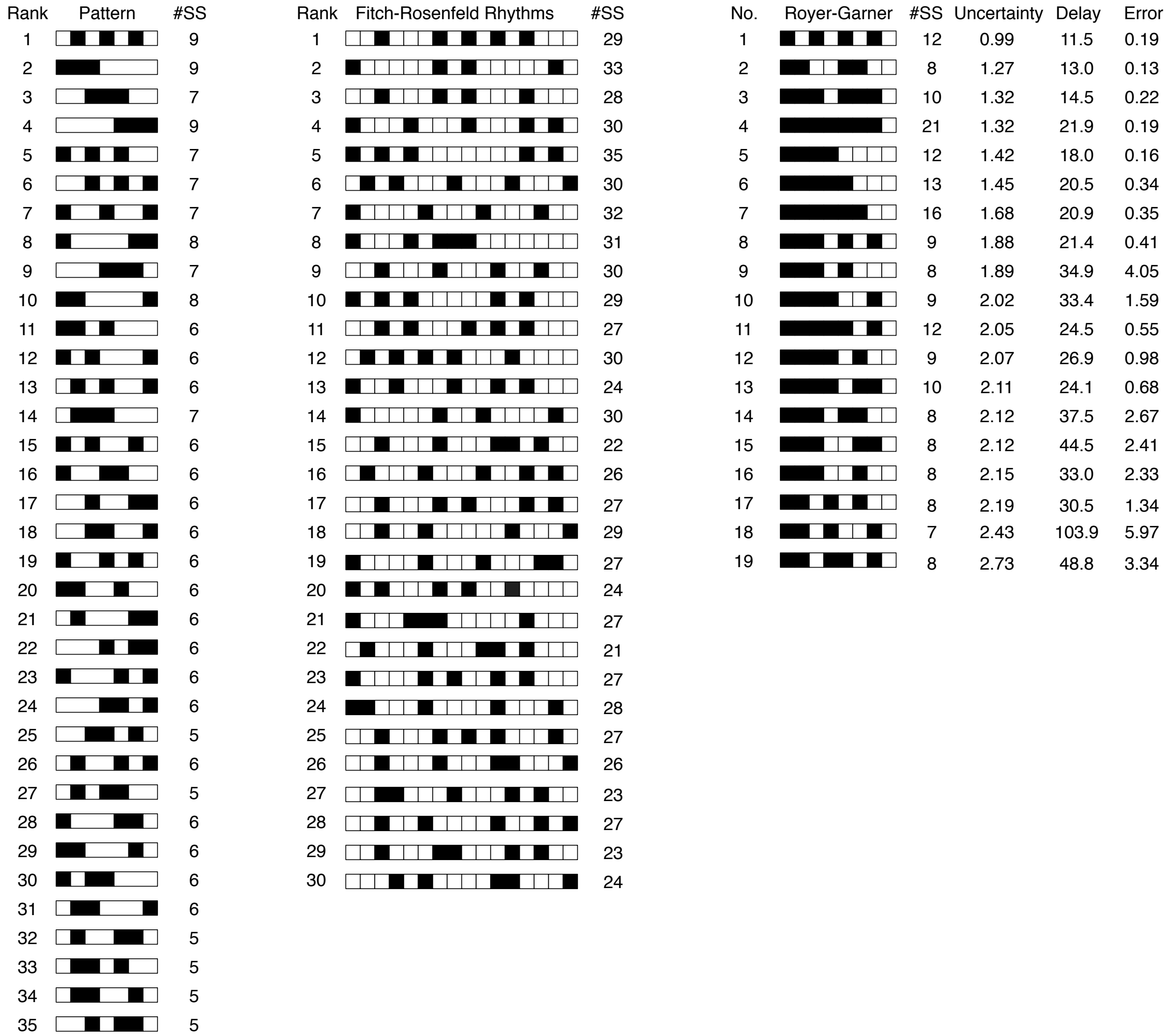

Figure 1. The visual patterns used by Alexander and Carey (left), the auditory rhythms used by Fitch and Rosenfeld in box-notation (center), and the auditory temporal patterns used by Royer and Garner, also expressed in box notation (right).

3 Results

The number of subsymmetries calculated for each rhythm in the Fitch-Rosenfeld data as well as each temporal pattern in the Royer-Garner data, is shown in Figure 1 on the immediate right of each pattern. Spearman rank correlation coefficients were obtained by comparing the empirical rankings based on the various performance complexities with those obtained with the number of subsymmetries. For the Fitch-Rosenfeld data the correlation is 0.719, significant at the 0.000004 level. For the Royer-Garner data the correlations between the number of sub-symmetries and the three empirical complexity measures are: 0.662 with p=0.001 for response uncertainty, 0.679 with p=0.0007 for response delay, and 0.716 with

p=0.0003 for response error. Thus our first result is that rhythmic temporal patterns that have many subsymmetries are simple to reproduce (perform). Our second result is highlighted by the scatter plots of the cognitive and performance complexities of the visual and auditory patterns, respectively, as a function of the number of subsymmetries contained in the patterns (Figures 2 and 3). In all cases for both visual and auditory patterns the plots indicate that a relatively large number of subsymmetries implies that the patterns are simple, but for intermediate and low numbers of subsymmetries, the complexity values vary widely.

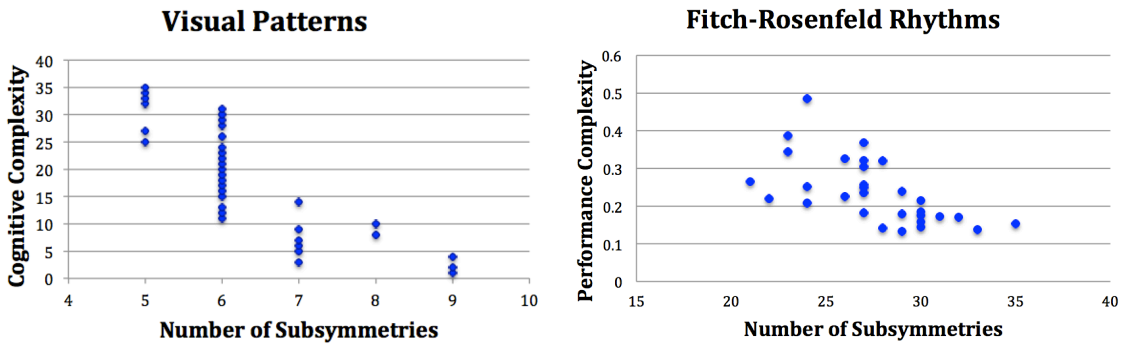

Figure 2. The charts show the cognitive complexity for the Alexander-Carey data (left) and the performance complexity for the Fitch-Rosenfeld data (right) as a function of the subsymmetries contained in the patterns.

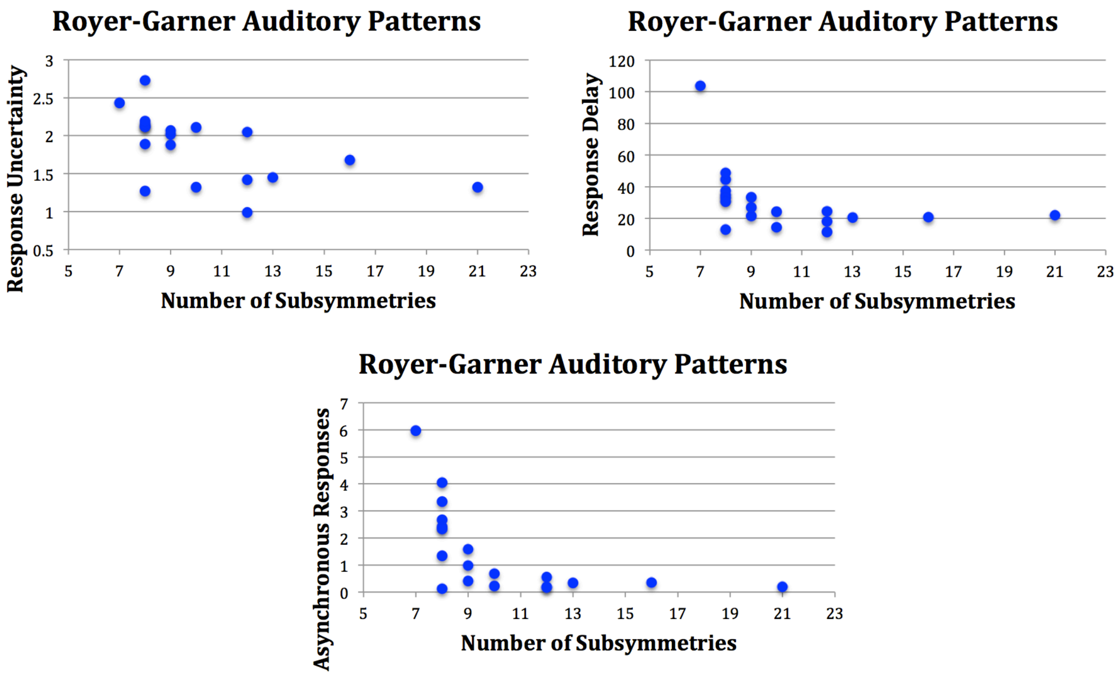

Figure 3. The three charts show the response uncertainty, response delay, and asynchronous responses (errors) for the Royer-Garner data as a function of the number of subsymmetries contained in the patterns.

4 Discussion

It is worth noting that the correlation between the performance complexity and the syncopation index calculated by Fitch and Rosenfeld for the 30 rhythms in Figure 1 is 0.51, significant at the 0.005 level. The results obtained here suggest that in the aural domain the number of subsymmetries present in the stimulus may be better than the syncopation index of Longuet-Higgins and Lee (1984) as a predictor of rhythm performance complexity.

Although the results obtained here provide evidence that for stationary visual patterns as well as auditory temporal patters, pattern complexity is very well accounted for when the number of subsymmetries is relatively high (pattern simplicity), all the charts in Figures 2 and 3 show that the variance in pattern complexity tends to increase monotonically as the number of subsymmetries decreases. Therefore it remains a tantalizing open problem to find stimulus properties that more accurately predict perceptual, cognitive, and performance complexities for those visual and rhythmic patterns that contain relatively few subsymmetries.

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