Vasavi College of Engineering

B.E. II-sem (2021-2022)

Sub: DIFF. EQNS. & VECTOR CAL.

I-Internal Examinations

KEY & SCHEME OF EVALUATION

PART-A

(1) The factor which when multiplied to a non-Exact Diff. Eqn. makes it exact is known as Integ factor. } → (1M)

(2) $IF = \frac{1}{Mx+Ny} = \frac{1}{(x^2y-2xy^2)x-(x^3-3x^2y)y} = \frac{1}{x^2y^2}$

Since M, N are Homog fns. in x & y. → (1M)

(3) A family of curves is said to be self orthogonal if it is its own orthogonal Traj. family. → (1M)

(4) $P.I = \frac{1}{D^2-4}\sin 2x$ put $D^2=-2^2$ $= \cancel{\frac{-x\,1}{2\times 2}\cos 2x}$ → (1M) $= -\frac{1}{8}\sin 2x$ → (1M)

(5) A diff. eqn. is said to be Linear if

(i) the dependent var. & its derivative is of first degree } → (1M)

(ii) No product of dependent var. & its derivative occurs in the Diff. Eqns

(iii) No transcendental fn. of dependent variable exists in the eqn

(6) $AE = f(m) = (m^2+1)^2(m-1) = 0$

$\Rightarrow m = \pm i, \pm i, 1$

$\therefore y_c = ((c_1+c_2x)\cos x + (c_3+c_4x)\sin x) + c_5 e^x$. → (1M)

PART-B

(7) Given $y = xy' - e^{2y'} \rightarrow ①$

$\Rightarrow x = \frac{y + e^{2y'}}{y'}$

Put $y' = p \Rightarrow x = \frac{1}{p}[y + e^{2p}] \rightarrow ②$

diff ② w.r.t 'y' we get,

$\cancel{\frac{1}{p}} = \cancel{1/p} + 1/p \, 2e^{2p} dp/dy - y/p^2 \, dp/dy - 1/p^2 \, e^{2p} \frac{dp}{dy}$

$\Rightarrow \frac{1}{p^2}(2pe^{2p} - y - e^{2p})\frac{dp}{dy} = 0$

$\Rightarrow$ Either $dp/dy = 0$ or $1/p^2(2pe^{2p} - y - e^{2p}) = 0$ → (2M)

when $dp/dy = 0$: on Integ we get, $p = c \Rightarrow y' = c$

Put $y' = c$ in ① we get,

$y = xc - e^{2c}$ → ③ which is the gen. sol of ① → (1M)

when $1/p^2(2pe^{2p} - y - e^{2p}) = 0$:

$\Rightarrow y = e^{2p}(2p-1)$

Sub 'y' in ① we get, $x = 2e^{2p}$

one place and f

X	XI
5	1

$\Rightarrow x = 2e^{2t}$ & $y = e^{2t}(2t-1)$

$\Rightarrow \frac{x}{2} = e^{2t}$

$\Rightarrow 2t = \log x/2$

$\therefore y = \frac{x}{2}\log(x/2) - x/2$

$\Rightarrow y = \frac{x}{2}\left[\log(x/2) - 1\right]$ which is the Singular Sol. of (1). $\to$ 1M

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(8) Given $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1 \to (1)$

Diff (1) w.r.t 'x' we get,

$\frac{2x}{a^2} + \frac{2yy'}{b^2+\lambda}=0$

$\Rightarrow \boxed{b^2+\lambda = -\frac{yy'a^2}{x}}$ $\to$ 1M

Sub $b^2+\lambda$ in (1) we get.

$\frac{x^2}{a^2} - \frac{xy}{a^2y'} = 1 \to (2)$

This is the D.E of (1)

Replacing y' with $-1/y'$ we get,

$\frac{x^2}{a^2}+\frac{xyy'}{a^2}=1$ $\to$ 1M

$\Rightarrow (x^2-a^2)dx + xy \cdot dy = 0 \to (3)$

This is the DE of OT family and is a non-Exact Diff Eqn

Sub. 'C' in ③, we get,

$$i = \frac{L}{R^2+L^2\omega^2}\left[\frac{R}{L}\sin\omega t - \omega\cos\omega t\right] - \frac{\omega L}{R^2+\omega^2L^2} e^{-\frac{R}{L}t}$$

→ 1M

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11) Given $(D^2-1)y = \frac{2}{1+e^x}$ → ①

$AE = m^2-1=0 \Rightarrow m=\pm 1$

$\therefore y_c = c_1 e^x + c_2 e^{-x}$ → ② → 1M

Here $u=e^x$, $v=e^{-x}$

$W=-2$ → 1M

$\therefore y_p = A(x)e^x + B(x)e^{-x}$ → ③

where $A(x) = -\int \frac{v(x)\,r(x)}{W}dx = -\int \frac{e^{-x}\cdot 2\,dx}{-2(1+e^x)}$

$$\boxed{A(x) = \frac{e^{-x}}{-1} + \log(1+e^{-x})}$$ → 2M

Also $B(x) = \int \frac{u(x)\,r(x)}{W}dx$

$B(x) = \int \frac{e^x}{-2}\times\frac{2}{1+e^x}dx$

$\Rightarrow \boxed{B(x) = \log(1+e^x)}$ → 2M

$\therefore y_p = e^x\left[-e^{-x} + \log(1+e^{-x})\right] - e^{-x}\left(\log(1+e^x)\right)$

The Complete sol is $y = y_c + y_p$