Artificial Intelligence

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Adversarial Search

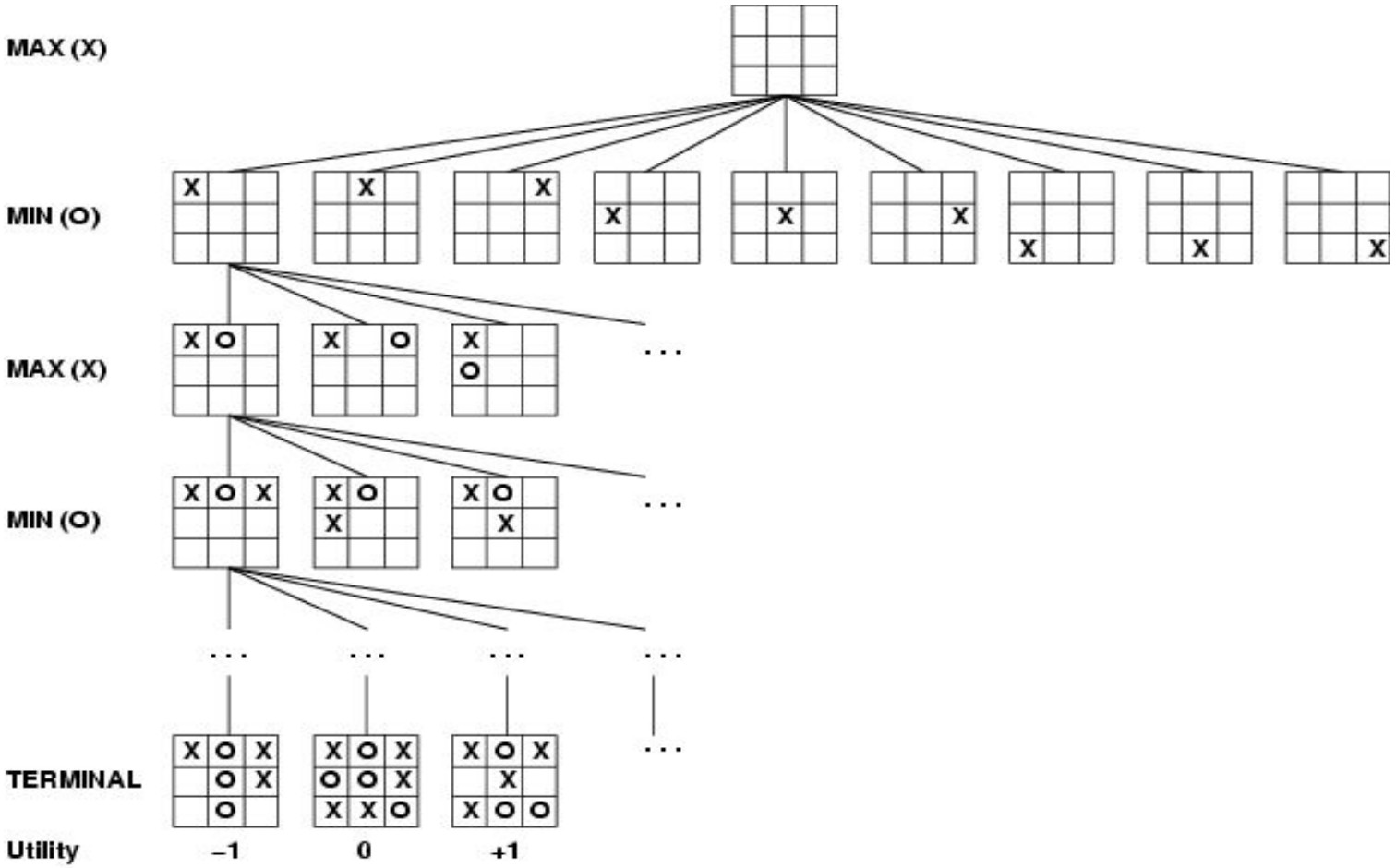
- Games
- Multiagent environments, in which each agent needs to consider the actions of other agents and how they affect its own welfare.
- competitive environments, in which the agents' goals are in conflict, giving rise to adversarial search problems—often known as games.

- In AI, the most common games are of a rather specialized kind—what game theorists call deterministic, turn-taking, two-player, zero-sum games of perfect information (such as chess).
- In our terminology, this means deterministic, fully observable environments in which two agents act alternately and in which the utility values at the end of the game are always equal and opposite.

- We begin with a definition of the optimal move and an algorithm for finding it.
- We then look at techniques for choosing a good move when time is limited.
- Pruning allows us to ignore portions of the search tree that make no difference to the final choice, and heuristic evaluation functions allow us to approximate the true utility of a state without doing a complete search.
- We first consider games with two players, whom we call MAX and MIN for reasons that will soon become obvious. MAX moves first, and then they take turns moving until the game is over. At the end of the game, points are awarded to the winning player and penalties are given to the loser

- A game can be formally defined as a kind of search problem with the following elements:
- S0: The initial state, which specifies how the game is set up at the start.
- PLAYER(s): Defines which player has the move in a state.
- ACTIONS(s): Returns the set of legal moves in a state.
- RESULT(s, a): The transition model, which defines the result of a move.
- TERMINAL-TEST(s): A terminal test, which is true when the game is over and false otherwise. States where the game has ended are called terminal states.
- UTILITY(s, p): A utility function (also called an objective function or payoff function), defines the final numeric value for a game that ends in terminal state s for a player p. In chess, the outcome is a win, loss, or draw, with values +1, 0, or 1/2 .

Fig : A (partial) game tree for the game of tic-tac-toe. The top node is the initial state, and MAX moves first, placing an X in an empty square. We show part of the tree, giving alternating moves by MIN (O) and MAX (X), until we eventually reach terminal states, which can be assigned utilities according to the rules of the game.



Optimal Decision in Games

- In a normal search problem, the optimal solution would be a sequence of actions leading to a goal state—a terminal state that is a win.
- In adversarial search, MIN has something to say about it.
- MAX therefore must find a contingent strategy, which specifies MAX's move in the initial state, then MAX's moves in the states resulting from every possible response by MIN, then MAX's moves in the states resulting from every possible response by MIN to those moves, and so on.

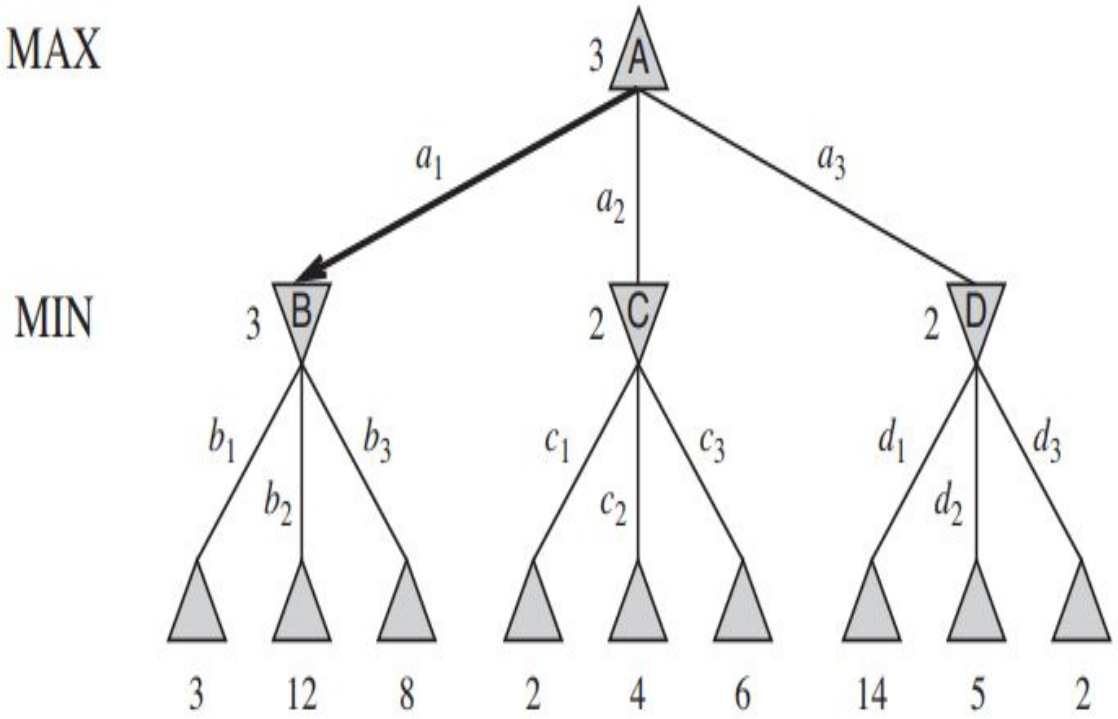


Figure 5.2 A two-ply game tree. The $\triangle$ nodes are "MAX nodes," in which it is MAX's turn to move, and the ∇ nodes are "MIN nodes." The terminal nodes show the utility values for MAX; the other nodes are labeled with their minimax values. MAX's best move at the root is a_1, because it leads to the state with the highest minimax value, and MIN's best reply is b_1, because it leads to the state with the lowest minimax value.

- Given a game tree, the optimal strategy can be determined from the minimax value of each node, which we write as MINIMAX(n).
- The minimax value of a node is the utility (for MAX) of being in the corresponding state, assuming that both players play optimally from there to the end of the game.
- Obviously, the minimax value of a terminal state is just its utility.
- Furthermore, given a choice, MAX prefers to move to a state of maximum value, whereas MIN prefers a state of minimum value. So we have the following

$$\textsc{Minimax}(s) = \begin{cases} \textsc{Utility}(s) & \text{if } \textsc{Terminal-Test}(s) \\ \max_{a \in Actions(s)} \textsc{Minimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{max} \\ \min_{a \in Actions(s)} \textsc{Minimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{min} \end{cases}$$

The minimax algorithm

- The minimax algorithm computes the minimax decision from the current state.
- It uses a simple recursive computation of the minimax values of each successor state, directly implementing the defining equations.
- The recursion proceeds all the way down to the leaves of the tree, and then the minimax values are backed up through the tree as the recursion unwinds.
- For example, in Figure, the algorithm first recurses down to the three bottom left nodes and uses the UTILITY function on them to discover that their values are 3, 12, and 8, respectively.
- Then it takes the minimum of these values, 3, and returns it as the backed-up value of node B.
- A similar process gives the backed-up values of 2 for C and 2 for D. Finally, we take the maximum of 3, 2, and 2 to get the backed-up value of 3 for the root node.

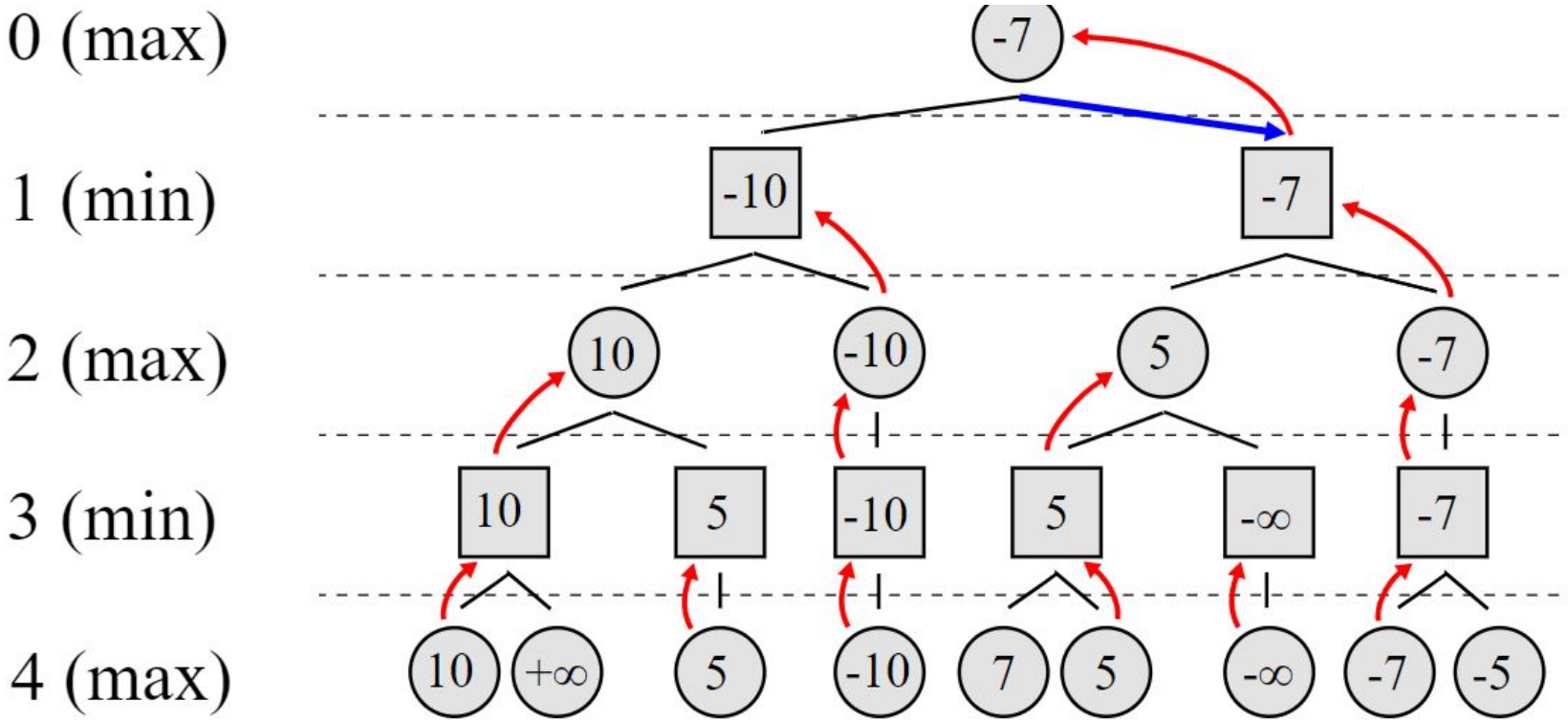
```
function MINIMAX-DECISION(state) returns an action
  return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))
```

```
function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s, a)))
  return v
```

```
function MIN-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← ∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s, a)))
  return v
```

Figure 5.3 An algorithm for calculating minimax decisions. It returns the action corresponding to the best possible move, that is, the move that leads to the outcome with the best utility, under the assumption that the opponent plays to minimize utility. The functions MAX-VALUE and MIN-VALUE go through the whole game tree, all the way to the leaves, to determine the backed-up value of a state. The notation $\text{argmax}_{a \in S} f(a)$ computes the element a of set S that has the maximum value of $f(a)$.

Minmax Example



Optimal decisions in multiplayer games

- Many popular games allow more than two players. Let us examine how to extend the minimax idea to multiplayer games.
- First, we need to replace the single value for each node with a vector of values. For example, in a three-player game with players A, B, and C, a vector {vA, vB, vC} is associated with each node.
- Consider the node marked X in the game tree shown in Figure 5.4. In that state, player C chooses what to do. The two choices lead to terminal states with utility vectors <vA = 1, vB = 2, vC = 6> and <vA = 4, vB = 2, vC = 3>.
- Since 6 is bigger than 3, C should choose the first move.
- This means that if state X is reached, subsequent play will lead to a terminal state with utilities <vA = 1, vB = 2, vC = 6>.
- Hence, the backed-up value of X is this vector. The backed-up value of a node n is always the utility vector of the successor state with the highest value for the player choosing at n.

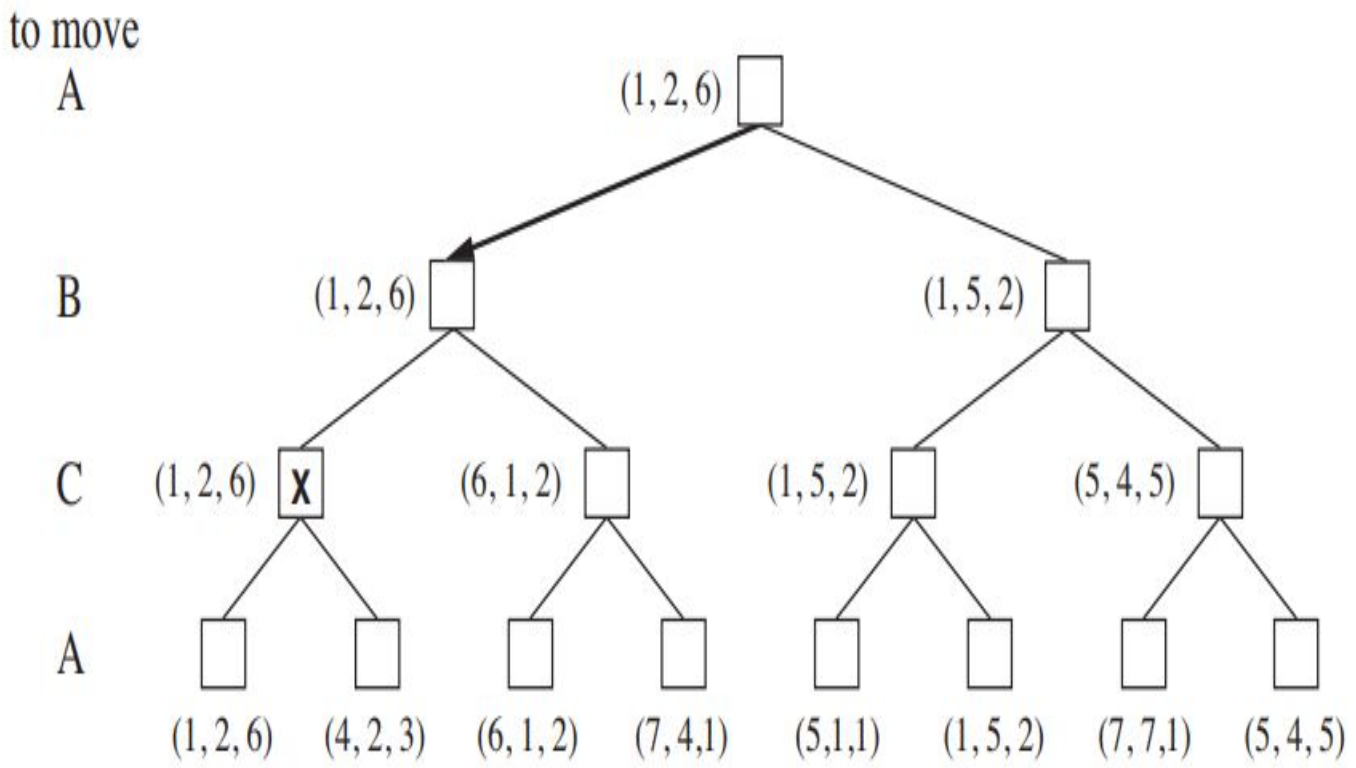


Figure 5.4 The first three plies of a game tree with three players (A, B, C). Each node is labeled with values from the viewpoint of each player. The best move is marked at the root.

ALPHA–BETA PRUNING

- The problem with minimax search is that the number of game states it has to examine is exponential in the depth of the tree.
- Unfortunately, we can't eliminate the exponent, but it turns out we can effectively cut it in half.
- The trick is that it is possible to compute the correct minimax decision without looking at every node in the game tree.
- That is, we can borrow the idea of pruning from Chapter 3 to eliminate large parts of the tree from consideration.
- The particular technique we examine is called alpha–beta pruning.
- When applied to a standard minimax tree, it returns the same move as minimax would, but prunes away branches that cannot possibly influence the final decision.

- Let the two unevaluated successors of node C in Figure 5.5 have values x and y. Then the value of the root node is given by

$$\begin{aligned}
\textsc{Minimax}(root) &= \max(\min(3,12,8), \min(2,x,y), \min(14,5,2)) \\
&= \max(3, \min(2,x,y), 2) \\
&= \max(3,z,2) \qquad \text{where } z = \min(2,x,y) \leq 2 \\
&= 3.
\end{aligned}$$

- In other words, the value of the root and hence the minimax decision are independent of the values of the pruned leaves x and y.
- Alpha–beta pruning can be applied to trees of any depth, and it is often possible to prune entire subtrees rather than just leaves.

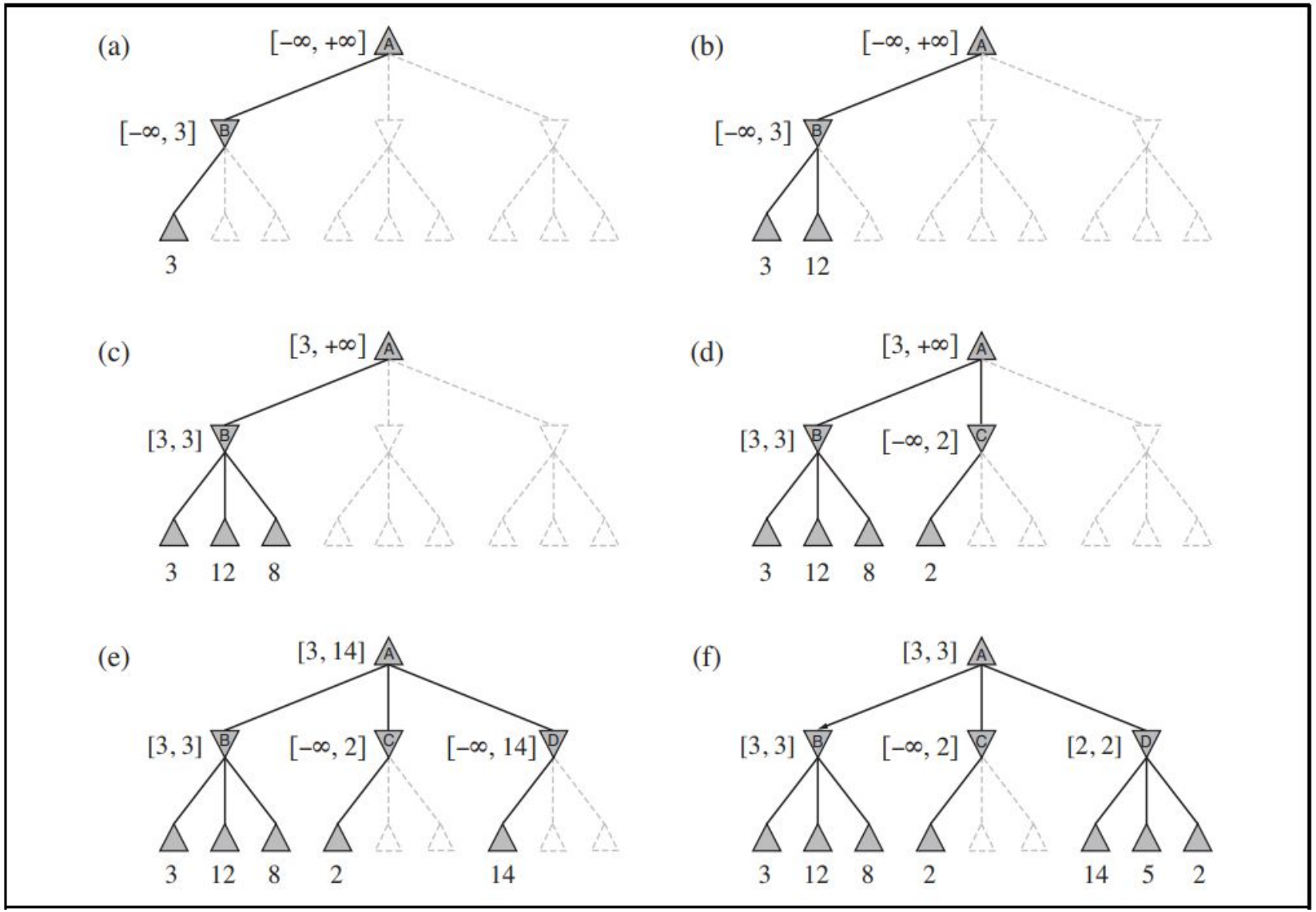

(a)
[−∞, +∞]
A
[−∞, 3]
B
3
(b)
[−∞, +∞]
A
[−∞, 3]
B
3
12
(c)
[3, +∞]
A
[3, 3]
B
3
12
8
(d)
[3, +∞]
A
[3, 3]
B
[−∞, 2]
C
3
12
8
2
(e)
[3, 14]
A
[3, 3]
B
[−∞, 2]
C
[−∞, 14]
D
3
12
8
2
14
(f)
[3, 3]
A
[3, 3]
B
[−∞, 2]
C
[2, 2]
D
3
12
8
2
14
5
2

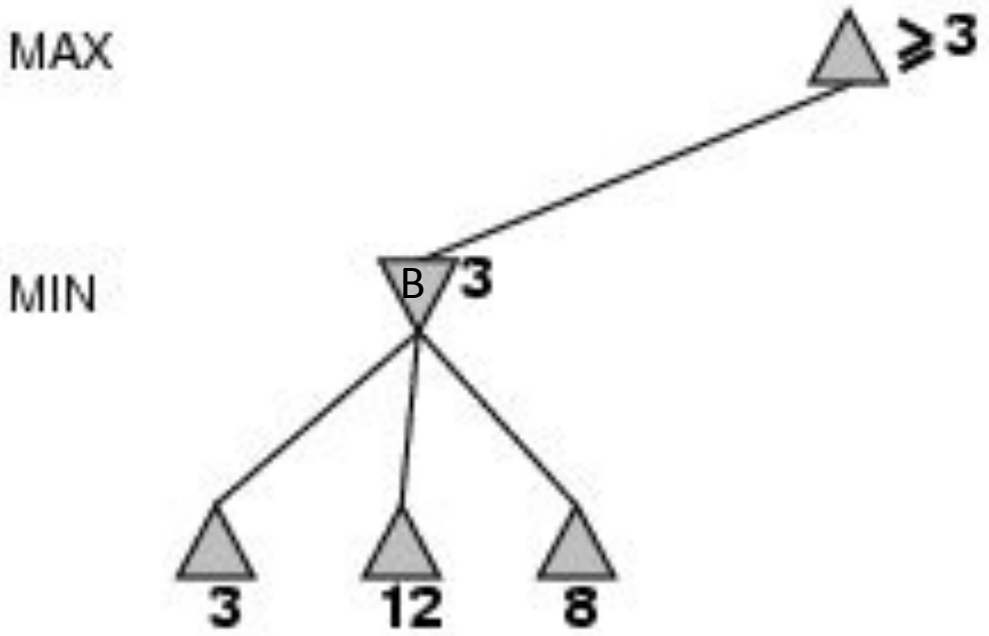
Figure 5.5 Stages in the calculation of the optimal decision for the game tree in Figure 5.2. At each point, we show the range of possible values for each node. (a) The first leaf below B has the value 3. Hence, B, which is a MIN node, has a value of *at most* 3. (b) The second leaf below B has a value of 12; MIN would avoid this move, so the value of B is still at most 3. (c) The third leaf below B has a value of 8; we have seen all B's successor states, so the value of B is exactly 3. Now, we can infer that the value of the root is *at least* 3, because MAX has a choice worth 3 at the root. (d) The first leaf below C has the value 2. Hence, C, which is a MIN node, has a value of *at most* 2. But we know that B is worth 3, so MAX would never choose C. Therefore, there is no point in looking at the other successor states of C. This is an example of alpha–beta pruning. (e) The first leaf below D has the value 14, so D is worth *at most* 14. This is still higher than MAX's best alternative (i.e., 3), so we need to keep exploring D's successor states. Notice also that we now have bounds on all of the successors of the root, so the root's value is also at most 14. (f) The second successor of D is worth 5, so again we need to keep exploring. The third successor is worth 2, so now D is worth exactly 2. MAX's decision at the root is to move to B, giving a value of 3.

- Remember that minimax search is depth-first, so at any one time we just have to consider the nodes along a single path in the tree. Alpha–beta pruning gets its name from the following two parameters that describe bounds on the backed-up values that appear anywhere along the path:
- α = the value of the best (i.e., highest-value) choice we have found so far at any choice point along the path for MAX.
- β = the value of the best (i.e., lowest-value) choice we have found so far at any choice point along the path for MIN.
- Alpha–beta search updates the values of α and β as it goes along and prunes the remaining branches at a node (i.e., terminates the recursive call) as soon as the value of the current node is known to be worse than the current α or β value for MAX or MIN, respectively.
- The complete algorithm is given in Figure 5.7.
- We encourage you to trace its behavior when applied to the tree in Figure 5.5

```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, −∞, +∞)
  return the action in ACTIONS(state) with value v

---

function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v

---

function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← +∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if v ≤ α then return v
    β ← MIN(β, v)
  return v
```

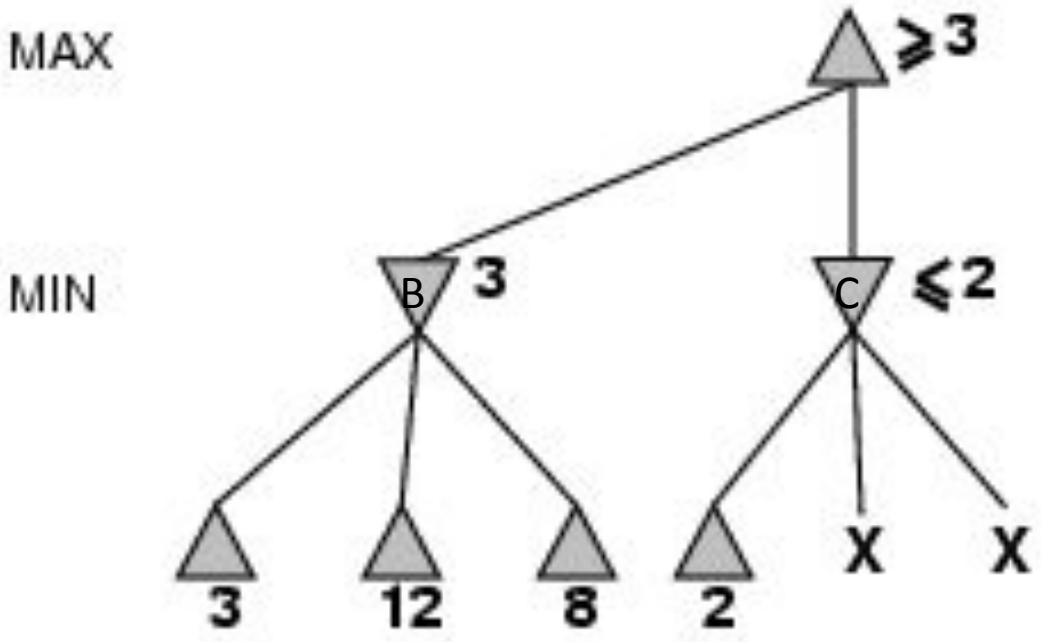
Figure 5.7 The alpha–beta search algorithm. Notice that these routines are the same as the MINIMAX functions in Figure 5.3, except for the two lines in each of MIN-VALUE and MAX-VALUE that maintain α and β (and the bookkeeping to pass these parameters along).

Alpha-beta pruning



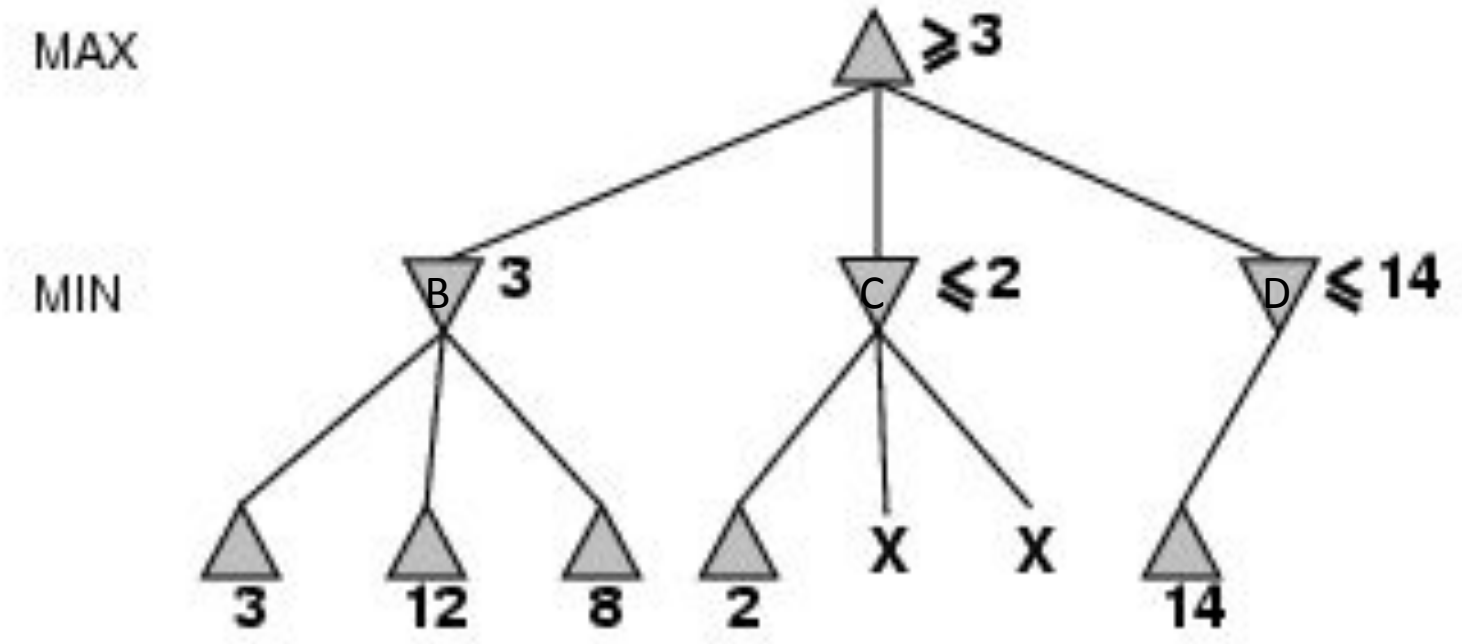
- It can be inferred that the value at the root is *at least* 3, because MAX has a choice worth 3.

Alpha-beta pruning



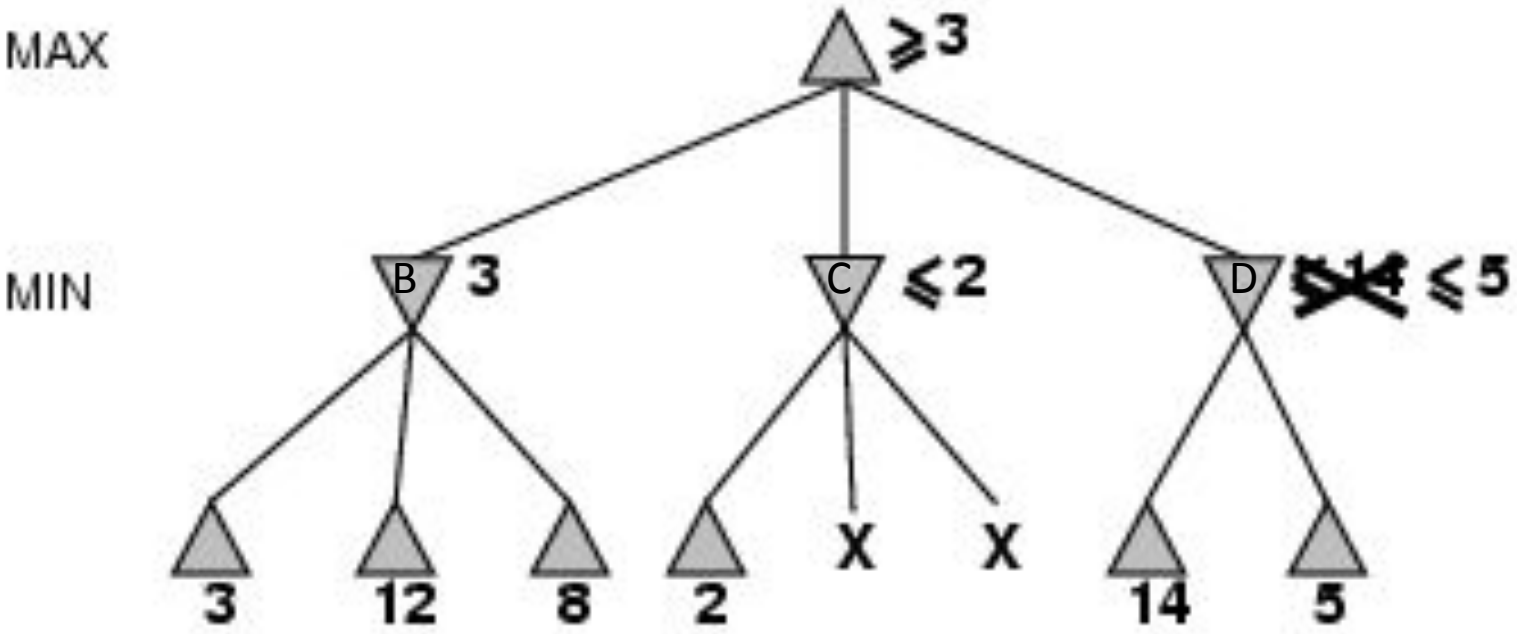
- Therefore, there is no point in looking at the other successors of *C*.

Alpha-beta pruning



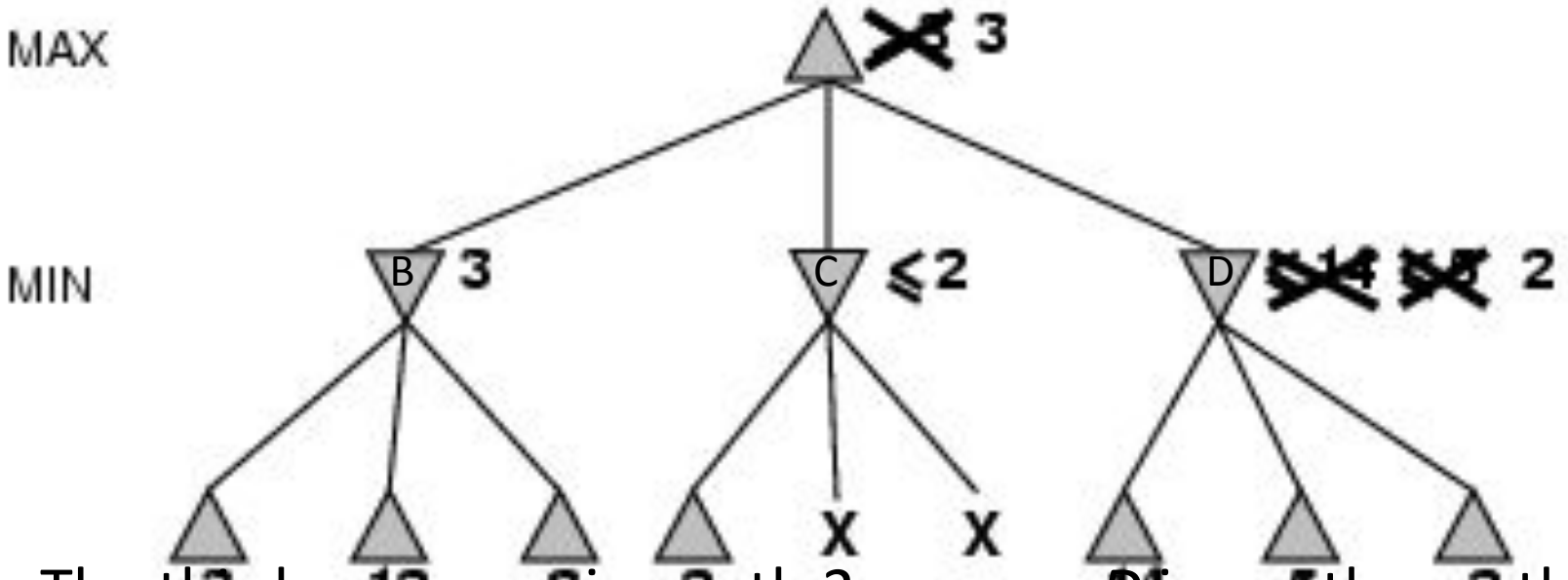
- This is still higher than MAX's best alternative (i.e., 3), so *D*'s other successors are explored.

Alpha-beta pruning

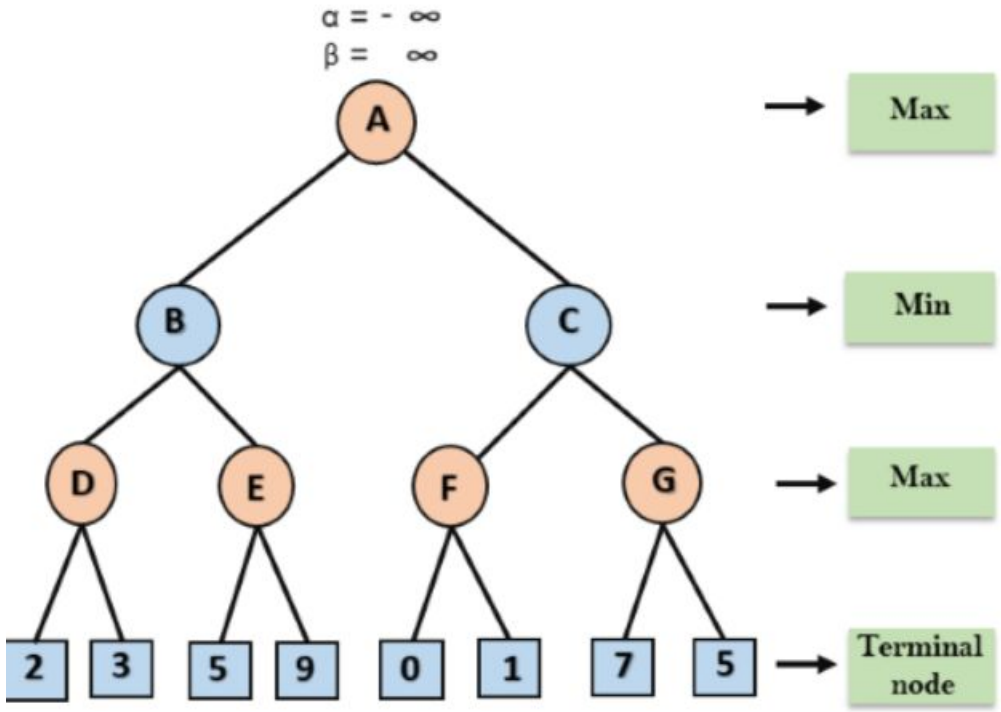


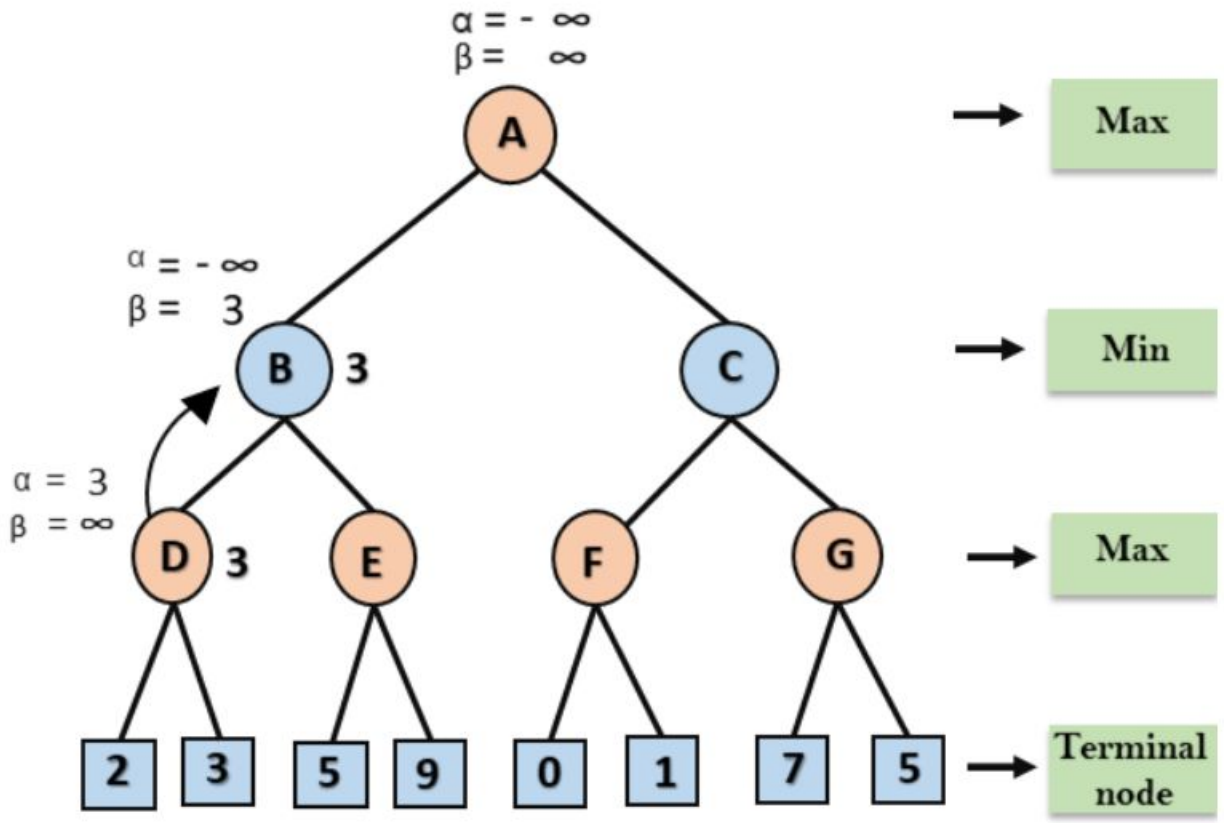
- The second successor of *D* is worth 5, so the exploration continues.

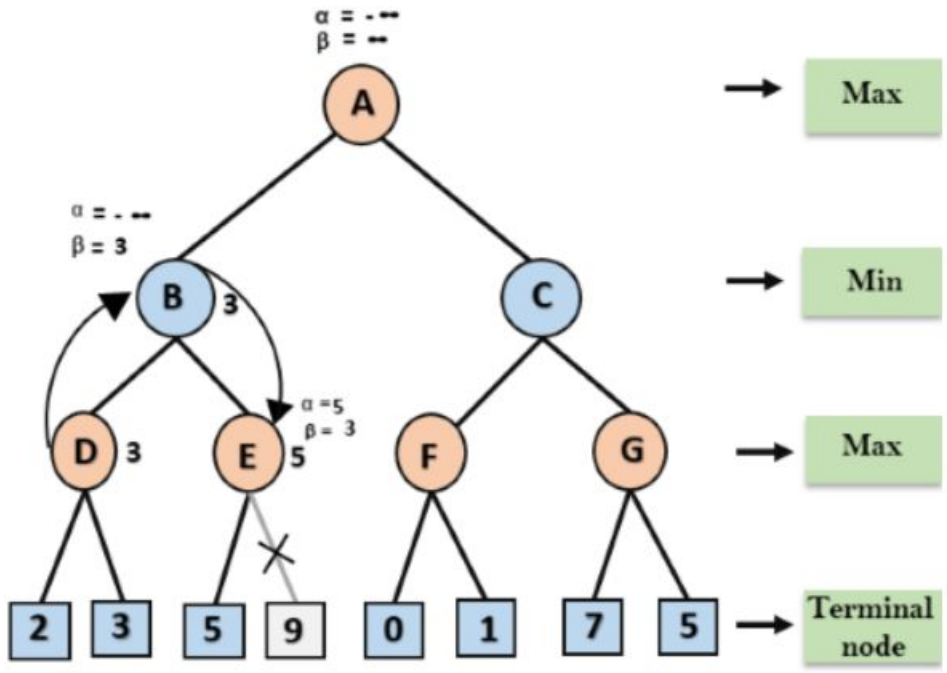
Alpha-beta pruning



- The third successor is worth 2, so now *D* is worth exactly 2.
- MAX's decision at the root is to move to *B*, giving a value of 3


α = - ∞
β = ∞
A
B
C
D
E
F
G
2
3
5
9
0
1
7
5
Max
Min
Max
Terminal node


α = - ∞
β = ∞
A
Max
α = - ∞
β = 3
B
3
C
Min
α = 3
β = ∞
D
3
E
F
G
Max
2
3
5
9
0
1
7
5
Terminal node


α = -∞
β = ∞
A
Max
α = -∞
β = 3
B
3
C
Min
α =5
β = 3
D
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E
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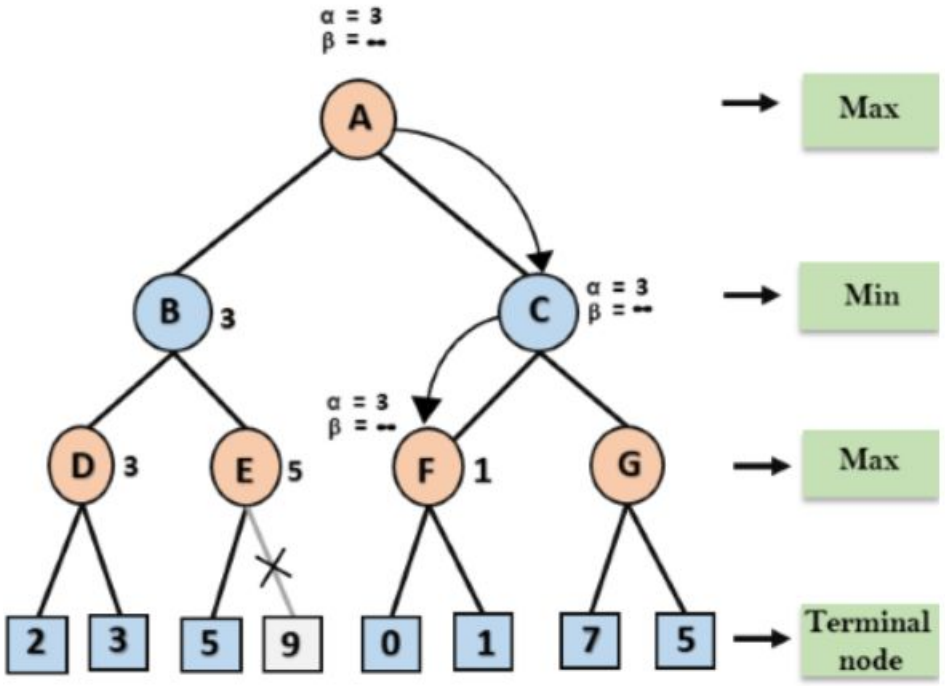
F
G
Max
2
3
5
9
0
1
7
5
Terminal node


α = 3
β = ∞
A
Max
B
3
C
α = 3
β = ∞
Min
α = 3
β = ∞
D
3
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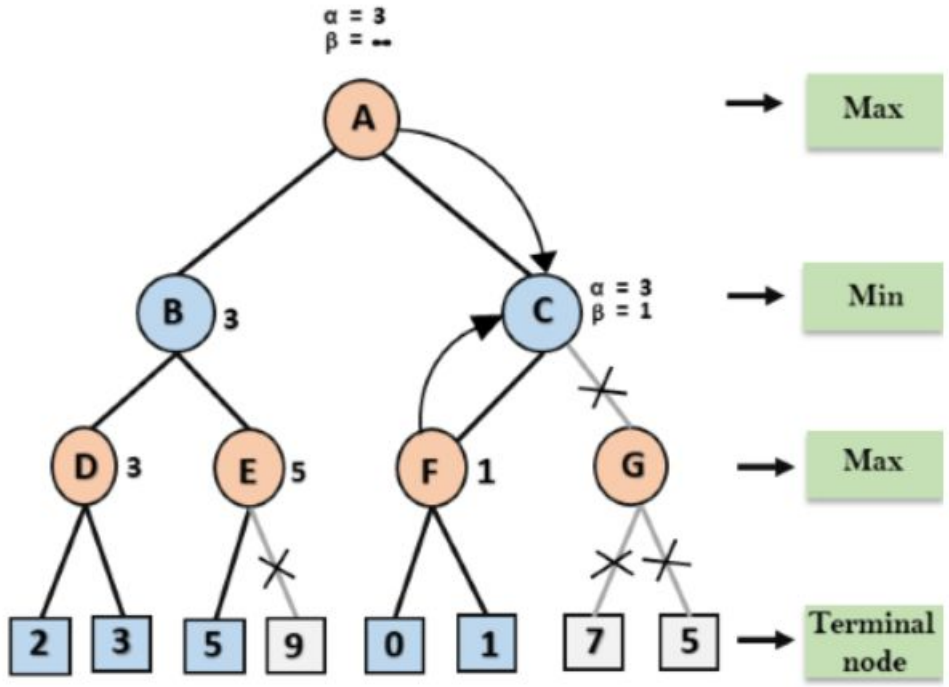
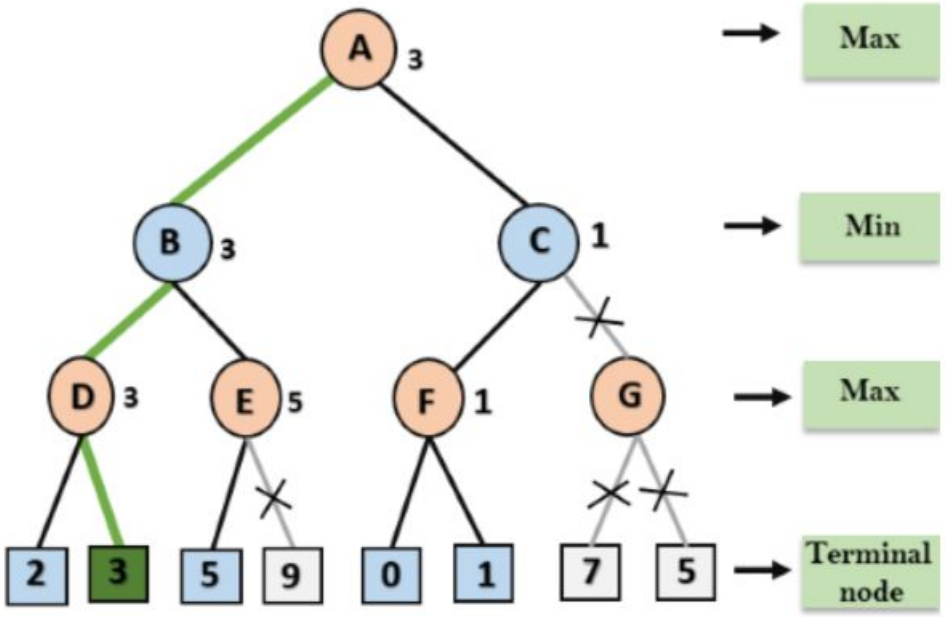
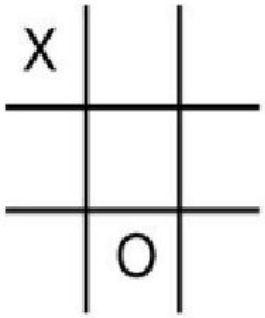
5
F
1
G
Max
2
3
5
9
0
1
7
5
Terminal node

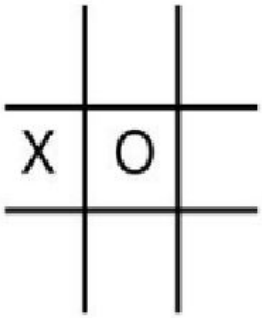

α = 3
β = ∞
A
Max
B
3
C
α = 3
β = 1
Min
D
3
E
5
F
1
G
Max
2
3
5
9
0
1
7
5
Terminal node


A
3
B
3
C
1
D
3
E
5
F
1
G
2
3
5
9
0
1
7
5
Max
Min
Max
Terminal node

Applying minimax to tic-tac-toe

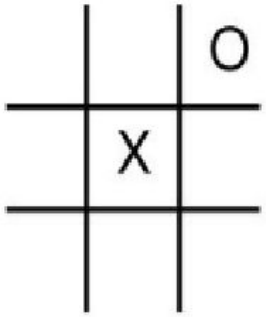
- The static heuristic evaluation function:
  - Count the number of possible win lines





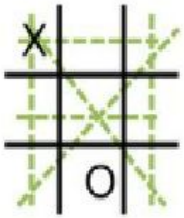
X has 4 possible wins
O has 6 possible wins

E(n) = 4 – 6 = -2

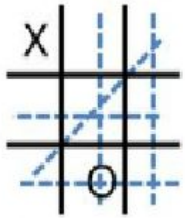


X has 5 possible wins
O has 4 possible wins

E(n) = 5 – 4 = 1



X has 6 possible win paths



O has 5 possible win paths

E(s) = 6 – 5 = 1