

* BOOLEAN ALGEBRA:

⇒ Conversions:

1) Decimal to Binary:

$$(63)_{10} = (111111)_2$$

$$\begin{array}{r} 2 \overline{) 63 - 1} \\ 2 \overline{) 31 - 1} \\ 2 \overline{) 15 - 1} \\ 2 \overline{) 7 - 1} \\ 2 \overline{) 3 - 1} \\ 1 \end{array}$$

2) Decimal to Octal:

$$(63)_{10} = (77)_8$$

$$\begin{array}{r} 8 \overline{) 63 - 7} \\ 7 \end{array}$$

3) Decimal to Hexadecimal:

$$(63)_{10} = (3F)_{16}$$

$$\begin{array}{r} 16 \overline{) 63 - 15} \\ 3 \end{array}$$

4) Binary to Decimal:

$$\begin{array}{ccccccc} (11010)_2 & = & (16 + 8 + 0 + 2)_{10} & = & (26)_{10} \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

~~5) Binary to Octal:~~

~~$$(11010)_8 = 4096 + 8 + 0 + 8 + 0 = 4112$$~~

6) Octal to Decimal:

$$\begin{array}{ccc} (765)_8 & = & (64 \times 7 + 8 \times 6 + 5)_{10} = (501)_{10} \\ 8^2 & 8^1 & 8^0 \end{array}$$

7) Hexadecimal to Decimal:

$$\begin{array}{ccc} (1B2)_{16} & = & 256 + 16 \times 11 + 2 \times 1 = (210)_{10} \\ 16^2 & 16^1 & 16^0 \end{array}$$
$$= (434)_{10}$$

* Properties of Boolean Algebra:

1) Closure:

$$B = \{0, 1\}$$

$$x, y, z \in B$$

x, y is closed under $+$ (or) \cdot (and)

2) Commutative Law:

$$x + y = y + x$$

$$x \cdot y = y \cdot x$$

3) Distributive Law:

$$x(y + z) = xy + xz$$

$$x + yz = (x + y)(x + z)$$

4) Unary Operator:

$$x \in B$$

$$\exists x' \in B$$

$$x + x = x$$

$$x \cdot x = x$$

$$x + x' = 1$$

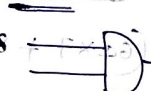
$$x \cdot 1 = x$$

$$xy + yx = xy$$

* OR



AND



NOT



* De-Morgan's

$$\rightarrow (x + y)' = x' y'$$

$$(xy)' = x' + y'$$

* Consensus Theorem:

$$xy + x'z + yz = xy + x'z$$

* Duality Principle:

The dual of the algebraic expression is given by interchanging OR and AND operators and replace 1's by 0's and 0's by 1's.

Precedence: NOT \rightarrow AND \rightarrow OR



MINTERM: Output of a single cell = 1

MAX TERM: Output of a single cell = 0

\rightarrow K-MAP \Rightarrow 1 is mapped

\rightarrow K-MAP \Rightarrow 0 is mapped

SOP \rightarrow sum of products: 2 or more product terms are summed up.

POS \rightarrow product of sums: 2 or more sum terms are multiplied.

* Canonical SOP: Sum of minterms.

Canonical POS: Product of maxterms.

* Conversion of SOP form to standard form: SOP:

1) By multiplying each product term with sum of missing variable and its complement, due to which the no. of terms gets double.

Ex: $F = xy + yz + xz$
 $F = xy(z+z') + yz(x+x') + xz(y+y')$

$$F = xyz + xy'z + x'yz + xy'z$$

* Conversion of POS form to standard POS:

By adding each sum term with the product of missing variable & its complement and using $A+BC = (A+B) * (A+C)$

Ex: $F = (A'+B+C) * (B+C+D) * (A+B'+C+D)$

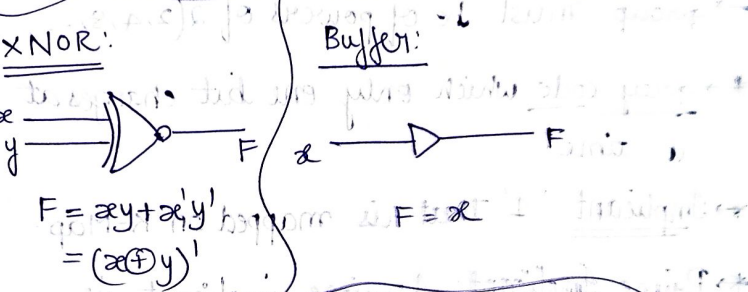
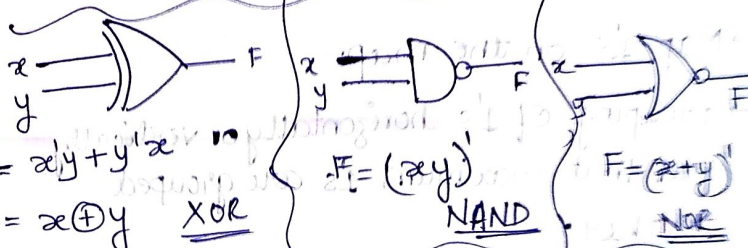
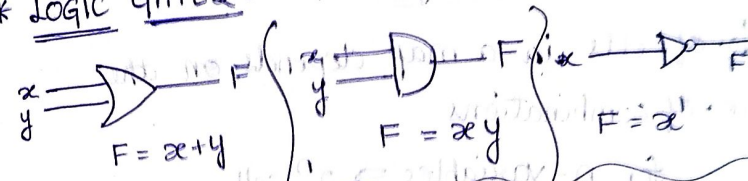
$$F = (A'+B+C+D) * (A'+A'+B+C+D) * (A+B'+C+D)$$

$$F = (A'+B+C+D)(A'+B+C+D')(A+B'+C+D)(A+B'+C+D')(A+B'+C+D)$$

$$F = (A'+B+C+D)(A'+B+C+D')(A+B'+C+D)(A+B'+C+D')$$

~~* Karnaugh Maps~~

* LOGIC GATES:



SOP \rightarrow NAND gates } Universal Gates
 POS \rightarrow NOR gates }
 XOR \rightarrow Odd function (odd no. of 1's)
 XNOR \rightarrow Even function (even no. of 1's)

- 1 \rightarrow High level \rightarrow +ve logic system
- 0 \rightarrow Low level \rightarrow -ve logic system

* Gate-Level Minimisation

K-Map (or) Karnaugh Map

No. of cells in a map depends on the no. of combinations for n -variables $\Rightarrow 2^n$ cells.

- \rightarrow Map 1's on the map
- \rightarrow Grouping of 1's horizontally or vertically such that maximum 1's are grouped together.
- \rightarrow Group must be of powers of 2 (2, 4, 8, ...)

* Gray code: which only one bit changes at a time.

* Implicant: '1' that is mapped on K-Map.

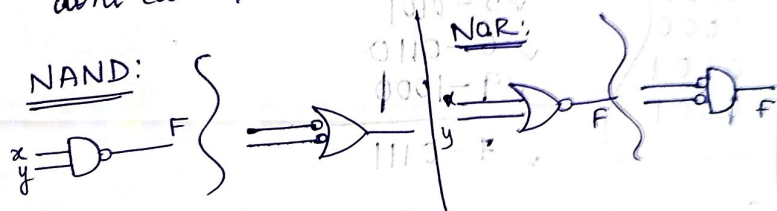
* Prime Implicant: A prime implicant is a product term obtained by combining maximum possible no. of adjacent squares on map.

* Essential Prime Implicants

If a minterm in a square is covered by only one prime implicant, then that prime implicant is said to be essential.

for SOP terms, we map 1 on K-Map and for POS terms, we map 0 on K-Map.

* Combination of inputs which give unspecified output are said to be don't care conditions.



① Expressing SOP using NAND gates:

- 1) Simplify function & express using SOP form.
- 2) Draw simple OR & AND gates and convert them to NAND by adding appropriate nots.

② Expressing POS using NOR gates:

- 1) Simplify function & express using POS form.
- 2) Draw simple OR & AND gates and convert to NOR gates.

* TABULATION METHOD:

* Ex.

$$\Sigma m(0, 1, 3, 6, 7) + d(4, 5, 8, 9)$$

Step-1: Group according to no. of 1's

0000	✓ 0 - 0000
0001	✓ 1 - 0001
0011	✓ 4 - 0101
0110	✓ 8 - 1000
0111	✓ 3 - 0011
0100	✓ 5 - 0101
1000	✓ 6 - 0110
0101	✓ 9 - 1001
0001	✓ 7 - 0111

Step-2: Group adjacent elements (only one bit change group).

✓ 0, 1 - 000 -	✓ 3, 7 - 0 - 11
✓ 0, 4 - 0 - 00	✓ 5, 7 - 01 - 10
✓ 0, 8 - - 000	6, 7 - 011 -
✓ 1, 3 - 00 - 1	
✓ 1, 5 - 0 - 01	
✓ 1, 9 - - 001	
✓ 4, 5 - 010 -	
✓ 4, 6 - 01 - 0	
✓ 8, 9 - 100 -	

Step-3: Group further if possible having same ~~empty~~ blank and one bit change.

$$0, 1, 4, 5 = 0 - 0 - \checkmark$$

$$0, 1, 8, 9 = - 00 -$$

$$0, 4, 1, 5 = 0 - 0 - \checkmark$$

$$0, 8, 1, 9 = - 00 -$$

$$1, 5, 3, 7 = 0 - - 1 \checkmark$$

$$1, 3, 5, 7 = 0 - - 1 \checkmark$$

$$4, 6, 5, 7 = 01 - -$$

Step-4: Prime Implicant Chart:
If possible group further else.

	0	1	3	6	7	
a'b'c				x	x	
a'c'	x	x				
b'c'	x	x			x	
a'd		x	x			
a'b				x	x	

P.I =
column with only 1
'x'

$$F = a'd + a'b + a'c' = a'd + a'b + b'c'$$

$$= a'd + a'bc + b'c' = a'd + a'bc' + a'c'$$

* The amount of time taken to give the output for given inputs is called propagation delay.

* If gates are running parallelly the propagation delay will remain same.

* Parity Checker & Generator:

XOR gates are used for error detection and correction codes.

→ parity bit \Rightarrow used for error detection

- Included with binary message to make the no. of 1's either even or odd.

- Circuit generating parity bit: parity generator.

- Circuit checking parity bit: parity checker.

* Even parity Generator Truth Table:

3-bit

x	y	z	P _g
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$P_g = \sum m(1, 2, 4, 7)$$

1	1	1	1
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* Even parity checker for above generator:

x	y	z	P _g	P _c
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$$P_c =$$

$$\sum m(1, 2, 4, 7, 8, 11, 13, 14)$$

If even parity checker & generator no. of 1's is even and odd parity checker & generator no. of 1's is odd.

Ex

$$F = ab + a'b'$$

$$F^D = (a+b) \cdot (a'+b') \Rightarrow F' = (a'+b')(a+b)$$

$$F^D = ((a+b)(a'+b'))' = (a+b)' + (a'+b')'$$

$$= a'b' + ab$$