Numerical Problems

Unit-II

By: Dr. Vanita Thakur

1. **Calculate the de Broglie wavelength of a proton moving with velocity equal to 1/20 of the velocity of light.**

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.62 \times 10^{-34}}{1.67 \times 10^{-27} \times \left(\frac{1}{20} \times c\right)}$$

$$= \frac{6.62 \times 10^{-34} \times 20}{1.67 \times 10^{-27} \times 3 \times 10^{8}}$$

2. Find the wavelength of neutron of energy 12.8 MeV.

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 12.8 \times 10^{6} \times 1.6 \times 10^{-19}}}$$

$$\lambda = 8.04 \times 10^{-15}\ m.$$

3. An electron is accelerated through a potential of 54Volt. Calculate its wavelength.

$$\lambda = \frac{12.27}{\sqrt{V}} = 1.67\ \mathring{A}$$

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4. Calculate velocity and K.E of neutron having de Broglie wavelength 1Å.

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{1.67 \times 10^{-27} \times v}$$

$$v = 3.96 \times 10^{3} \text{ m/s.}$$

$$E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times (3.96 \times 10^{3})^2$$

$$E = 0.082 \text{ eV}$$

5. Calculate de Broglie wavelength of alpha particle accelerated through a potential difference 200V.

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$q = 2e$

$m = 4m_p$

$$= \frac{h}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$$

$$= 0.00715\,Å$$

6. An electron is accelerated from rest through a potential difference of 200V.

i. **Calculate the associated wavelength.**
ii. **This beam is passed through diffraction grating of spacing 3A. At what angle of deviation from the incident direction will be the first maximum observed?**

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.602 \times 10^{-19} \times 200}}$$

$$\lambda = 0.86\ \mathring{A}$$

$$2d \sin\theta = n\lambda$$

$$n = 1, \quad d = 3\mathring{A}$$

$$\therefore \quad 2d \sin\theta = \lambda$$

$$\sin\theta = \frac{\lambda}{2d}$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right)$$

$$\theta = \sin^{-1}\left(\frac{0.86\mathring{A}}{2 \times 3\mathring{A}}\right) = 8.31^{\circ}$$

7. An enclosure filled with He is heated to 400K. A beam of He atoms emerges out of the enclosure. Calculate the de Broglie wavelength corresponding to He atoms. Mass of He atom is 6.67×10^{-27}kg.

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

8. Find the de Broglie wavelength of

i. **An electron accelerated through a potential difference of 182 Volts.**
ii. **A 1kg object moving with a speed 1m/s.**
iii. **Comparing the results explain why the wave nature of matter is not more apparent in daily observations.**

(3) $$\lambda_e = \frac{h}{\sqrt{2meV}}$$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times (1.602 \times 10^{-19}) \times (9.1 \times 10^{-31}) \times 182}}$$

$$\lambda = 9.1 \times 10^{-11} m$$

$$\lambda = 0.91 \, Å$$

ii) $$\lambda_m = \frac{h}{mv}$$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{1 \times 1}}$$

$$\lambda_m = 6.6 \times 10^{-34} m$$

So, we can clearly see that the λ of accelerated e^- is about $10^{?}$ times larger than its own size and is $\therefore$ significant.

But the λ associated with macroscopic object is negligibly small & is thus not apparent in its interactions with other objects.

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9. For an electron and photon each having a wavelength of 1Å. Compare their

i. **Momentum**
ii. **Total energy**
iii. **Ratio of K.E**

Sol.

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$

for e^-, $$p_e = \frac{6.62 \times 10^{-34}}{1 \times 10^{-10}} = 6.63 \times 10^{-24} \text{ kg m/s}$$

for photon, $$p_p = \frac{h}{\lambda} = \frac{h}{c/\nu} = \frac{h\nu}{c}$$

$$= \frac{6.62 \times 10^{-34}}{1 \times 10^{-10}} = 6.63 \times 10^{-24} \text{ kg m/s.}$$

So, for same λ, e^- & photon have same momentu

ii> Total energy$_{(e)}$ = K.E + Rest mass energy

$$= \frac{p_e^2}{2m_0} + m_0 c^2$$

$$= \frac{(6.63 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}} + \frac{9.1 \times 10^{-31} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}}$$

$$T.E = 0.512 \times 10^6 \text{ eV} = 0.512 \text{ MeV}$$

for Photon Total Energy $\Rightarrow$ $E = h\nu = \frac{hc}{\lambda} = pc$

$$E = pc = 6.63 \times 10^{-24} \times 3 \times 10^8$$

$$E = 1.24 \times 10^4 \text{ eV}$$

$$E = 12.4 \text{ keV}$$

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iii) $K.E$ of $e^- = \frac{p_e^2}{2m_e} = \frac{(6.63\times10^{-24})^2}{2\times9.1\times10^{-31}}$

$K_e = 1.51\times10^2 eV$

$K_e = 0.151 keV$

$K.E$ of photon $= K_p = h\nu = \frac{hc}{\lambda} = pc$

$K_p = 6.63\times10^{-24}\times3\times10^8$

$K_p = 12.4 keV$

$$\frac{K_e}{K_p} = \frac{0.151}{12.4} = 1.23\times10^{-2}$$

10. Calculate de Broglie wavelength of

i. **A ball of mass 20kg moving with speed of 5m/s**
ii. **Ii. An electron travelling with a speed of 10^6m/s**

Sol. i> $\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{20 \times 5} = 6.62 \times 10^{-32}$ m.

or $\lambda = 6.62 \times 10^{-22}$ Å

ii> $\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{6}} = 7.27 \times 10^{-10}$ m

$\lambda = 7.27$ Å

11. Compare the wavelengths of a photon and an electron having

i. Same momentum

ii. Same energy

Sol. i) $\lambda_e = \frac{h}{p_e}$ (for e^-)

$p_e = \frac{h}{\lambda_e}$

$\lambda_p = \frac{h}{p_p}$ (for photon)

$p_p = \frac{h}{\lambda_p}$

Since, momentum of e^- & photon are same.

i.e. $p_e = p_p$

$\frac{h}{\lambda_e} = \frac{h}{\lambda_p}$

$\Rightarrow \boxed{\lambda_e = \lambda_p}$ So, there wavelengths are also same.

ii) <u>for same Energy</u>

$\lambda_e = \frac{h}{\sqrt{2mE}}$ (for e^-) ——— ①

$\lambda_p = \frac{h}{p}$ (for photon)

$\lambda_p = \frac{hc}{E}$ ——— ②

as $E = h\nu = \frac{hc}{\lambda}$

$E = pc$

or $p = \frac{E}{c}$

Comparing ① & ②

$$\frac{\lambda_p}{\lambda_e} = \frac{hc}{E} \times \frac{\sqrt{2mE}}{h}$$

$$= c\sqrt{\frac{2m}{E}}$$

$$\frac{\lambda_p}{\lambda_e} = \sqrt{\frac{2mc^2}{E}}$$

12. A particle confined to move along the x-axis has wave function $\psi = ax$ between $x = 0$ and $x = 1.0$ and $\psi = 0$ everywhere. Find the probability of finding the particle between $x = 0.35$ to $x = 0.45$.

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$$P = \int_{x_1}^{x_2} |\psi(x)|^2 dx = a^2 \int_{0.35}^{0.45} x^2 dx$$

$$= a^2 \frac{x^3}{3}\Big|_{0.35}^{0.45} = \frac{a^2}{3}\left[(0.45)^3 - (0.35)^3\right]$$

$$\boxed{P = 0.0161\, a^2} \quad \text{— (1)}$$

Using Normalization condition,

$$\int_0^1 a^2 x^2 dx = 1$$

$$a^2 \frac{x^3}{3}\Big|_0^1 = 1$$

$$a^2 \frac{1}{3} = 1$$

$$\boxed{a^2 = 3}$$

using in (1)

$$P = 0.0161 a^2$$

$$= 0.0161 \times 3$$

$$\boxed{P = 0.048}$$

13. An electron is confined to move between two rigid walls separated by 1nm. Find the de Broglie wavelength representing the first two allowed energy states of the electron and the corresponding energies.

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2}$$

$$E_1 = \frac{h^2}{8mL^2} = \frac{(6.62\times10^{-34})^2}{8\times9.1\times10^{-31}\times(1\times10^{-9})^2} = 6.02\times10^{-20}\,J$$

$$E_2 = \frac{4h^2}{8mL^2} = 4\times6.02\times10^{-20} = 2.4\times10^{-19}\,J$$

$$\lambda_1 = \frac{h}{\sqrt{2mE_1}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times6.02\times10^{-20}}}$$

$$\lambda_1 = 2\,nm.$$

$$\lambda_2 = \frac{h}{\sqrt{2mE_2}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times2.4\times10^{-19}}}$$

$$\lambda_2 = 1\,nm.$$

14. Find the probability that a particle trapped in a box L wide can be found between 0.45L and 0.55L for the ground state and first excited state.

Sol. $P = \int_{x_1}^{x_2} |\psi(x)|^2 dx$

$$= \frac{2}{L}\int_{0.45L}^{0.55L} \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{L}\times\frac{1}{2}\int_{0.45L}^{0.55L}\left(1 - \cos\frac{2n\pi x}{L}\right) dx$$

$$= \frac{1}{L}\left[x - \frac{1}{2n\pi}\left(\sin\frac{2n\pi x}{L}\right)\right]_{0.45L}^{0.55L}$$

for state $n=1$

$$P = \frac{1}{L}\left[x - \frac{1}{2\pi}\left(\sin\frac{2\pi x}{L}\right)\right]_{0.45L}^{0.55L}$$

$$= \frac{1}{L}\left[(0.55L - 0.45L) - \frac{1}{2\pi}\left(\sin\frac{2\times180\times0.55L}{L} - \sin\frac{2\times180\times0.45L}{L}\right)\right]$$

$$= 0.1 - \frac{1}{2\times3.14}(-0.309 - 0.309)$$

$$= 0.198$$

or $P = 19.8\%$

IIly for $n=2$

$P = 0.0065$ or 0.65%

15. Find the lowest energy state of an electron confined in a cubical box each of side 1 Å.

Sol.

$$E_n = \frac{n^2 \pi^2 \hbar^2}{8 m L^2}$$

for 3-D

$$E_n = \frac{\pi^2 \hbar^2}{8 m L^2} \left(n_x^2 + n_y^2 + n_z^2 \right)$$

for lowest energy state, $n_x = 1 = n_y = n_z$

$$\therefore E_1 = \frac{\pi^2 \hbar^2}{8 m L^2} (1 + 1 + 1)$$

$$= \frac{3 \times (3.14)^2 \times (6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$= \frac{18.03 \times 10^{-18}}{1.6 \times 10^{-19}} \text{ J}$$

$$E_1 = 112.6 \text{ eV}$$

A particle is in 1-dim. Box of width 30A^o. Calculate the probability of finding the particle within the interval of 2A^o at the centre of box when it is in the state of least energy.

$$\psi_n(x) = \sqrt{\frac{2}{L}}\ \sin\frac{n\pi x}{L}$$

for least energy, $n = 1$

$$\psi_1(x) = \sqrt{\frac{2}{L}}\ \sin\frac{\pi x}{L}$$

at the centre of box, $x = \frac{L}{2}$

Prob. in unit interval

$$|\psi(x)|^2 = \left|\sqrt{\frac{2}{L}}\ \sin\frac{\pi L/2}{L}\right|^2 = \frac{2}{L}\sin^2\frac{\pi}{2} = \frac{2}{L}$$

Prob. in interval Δx,

$$P = |\psi(x)|^2\Delta x = \frac{2}{L}\Delta x$$

given, $L = 30\ \text{Å}$, $\Delta x = 2\ \text{Å}$

$$\therefore P = \frac{2}{30}\times 2 = \cancel{0.16}\ 0.13$$

$$\text{or}\quad P = \cancel{16\%}\ 13\%$$

16. An electron microscope is used to locate an electron in an atom within a distance 0.2 Å. What is the uncertainty in momentum of electron location?

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{h}{2\pi\, 2\,\Delta x} = \frac{6.62 \times 10^{-34}}{2 \times 3.14 \times 2 \times 0.2 \times 10^{-10}}$$

$$\Delta p = 2.64 \times 10^{-24}\ \text{kg m/s}.$$

17. Find the smallest possible uncertainty in momentum of electron for which uncertainty in its position is $4 x 10^{-10}$m.

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$$\Delta x \cdot \Delta p_r = \frac{h}{2}$$

$$\Delta p_r = 1.32 \times 10^{-25}\ \text{kgm/s}$$

18. An electron has speed 1.05×10^4 m/s with an accuracy of 0.02%. Calculate the uncertainty in position of electron.

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Sol

$$\Delta x \cdot \Delta p = \hbar$$

$$p = mv = 9.45 \times 10^{-27} \text{ kg m/s.}$$

$$\Delta p = \frac{0.02}{100} \times p = 1.89 \times 10^{-30}$$

$$\Delta x = \frac{\hbar}{\Delta p} = 5.58 \times 10^{-5} \text{ m.}$$

That's all!!!!!!!

Thank you……….