10/06/2023

UNIT-IV

TEST OF HYPOTHESIS

→ Introduction

→ Test of Hypothesis

→ Null Hypothesis and Alternative Hypothesis

→ Errors – Type1 error and Type2 error

→ Test of Significance Of small samples.

→ t-test for single mean.
(or)
students t-test

→ F-test (or) Fisherman-test for comparision of variance

→ chisquare-Test for goodness of fit.
(χ^2)

→ Critical Region and level of Significance.

→ Confidence Intervals.

$$N \begin{cases} \mu \text{(mean)} \\ \sigma \text{(SD)} \end{cases} \text{Population size.}$$

$$n \begin{cases} \bar{x} \text{(mean)} \\ s \text{(S.D)} \end{cases} \text{samplesize.}$$

→ INTRODUCTION:

→ In real life when the population is large and information is to be obtained about population. Then we cannot test each and every product to get its result.

→ But we take a sample out of the whole population. The sample taken is then tested to know about the result of entire population.

Ex: A manufacturing company which produces thousands of beauty products per year. But we cannot test each and every product. If it is the case then no product would be sent to the market.

→ So a sample of 40 beauty products are taken and they are tested and its result/performance is considered as result of all thousands of beauty products.

* **Sample:** A finite subset of the population is known as a sample.

Ex: Selecting 2 girls in a class of 20

students is a sample.

Sample size: No. of items in a sample.

Sampling: Process of selecting samples.

* Population: Population is a collection of objects, population may be finite (or) infinite depending upon size of the population. Here size refers to total no. of objects in the population and is denoted by N.

Ex: 1) No. of students in a class is 50. Here N = 50 which is finite i.e. the population is finite.

2) The no. of stars in the sky; Here N is infinite. Therefore, the population is infinite population.

3) Suppose if we want to buy a bag of rice, we first check small portion of it by cooking, then we buy it. Complete rice in a bag is infinite population.

Note: In the example of rice; small portion is the sample.

1. A sample with less than 30 items is called small sample.
2. A sample with more than 30 items is called large sample.

Objectives of sampling.

→ sampling aims at gathering the max information about the population with the minimum effort.

The object of sampling studies is to obtain the best possible values of the parameters under specific conditions.

Statistic: The statistical constants of the population such as mean (μ), Standard deviation (σ) etc.. are called population parameters. Similarly constants for the sample drawn from the given population i.e mean ($\bar{x}$), Standard deviation (S) etc.. are called sample statistics.

→ The logic of the sampling theory is the logic of induction in which we pass from a particular sample to general population. Such a generalization from sample to population is called statistical inference.

Standard Error: The standard deviation of sampling distribution of a statistic is known as its standard error and is denoted by S.E.

∴ The Standard error of means = $\frac{\sigma}{\sqrt{n}}$.

→ Sampling Distribution: Sampling Distribution of a statistic helps us to get information about the corresponding population parameter.

Def: The probability distribution of a statistic is called as sampling distribution.

→ If we draw a sample of size 'n' from a given finite population of size 'N' then total no. of possible samples is

$$^{N}C_{n} = \frac{N!}{n!(N-n)!}$$

Another Def of testing a Hypothesis:- Testing a hypothesis is meant a process for deciding whether to accept or reject the hypothesis.

Null Hypothesis (H_0): A hypothesis which is definite statement about the population parameter is called Null hypothesis denoted by

H_0: Usually hypothesis of no diff.

Alternative hypothesis (H_1): Any hypothesis which is complementary to Null hypothesis is called Alternative Hypothesis (H_1).

Ex:

Null Hypothesis (H_0): $\mu = 10$

Alternative Hypothes(H_1): $\mu \neq 10$ (or) $\mu > 10$ (or) $\mu < 10$

For Example,

If we test the Null Hypothesis that the population has a specified mean μ_0, then we have

$H_0: \mu = \mu_0$

Alternative Hypothesis will be

(i) $H_1: \mu \neq \mu_0$ ($\mu > \mu_0$ (or) $\mu < \mu_0$) (two tailed alternative hypothesis)

(ii) $H_1: \mu > \mu_0$ (right tailed alternative hypothesis (or) single tailed)

(iii) $H_1: \mu < \mu_0$ (left tailed alternative hypothesis (or) single valued).

→ Errors in sampling:

The main aim of the sampling theory is to draw a valid conclusion about the population parameters, on the basis of the sample results the following 2 types of errors occurs.

→ TYPE-I Error: (Reject H_0 when it is true).

If the NULL hypothesis H_0 is true but it is rejected by test procedure, then the error is called TYPE-I Error (or) α error (or) the error made in rejection of null hypothesis even it is true.

→ TYPE-II Error (Accept H_0 when it is wrong)

If the NULL hypothesis H_0 is false but it is accepted by the test, then error is called TYPE II Error (or) β Error

(or) the error made in acceptance of NULL hypothesis even it is false.

	Accept H_0	Reject H_0
H_0 is true	Correct decision	TYPE-I error
H_0 is false	TYPE-II error	Correct decision

* **Degrees of freedom**: It is the number equals to the total no. of observations less than the no. of indepedent constraints imposed on the observations.

i.e. $\boxed{\nu = n-k}$ $n \to$ no. of observations, $k \to$ no. of constraints.

NOTE:

1) The probability of committing type-I error is known as "level of significance" and it is denoted by Greek letter 'α'.

2) The probability of committing type-II error is denoted by the Greek letter 'β'.

$$\beta = P(\text{type-II error})$$

Here $1-\beta$ is known as "power of the test".

* **Level of significance**:

The area of critical region is equal to the level of significance.

(OR)

The probability level below which we reject the hypothesis is known as level of significance.

(OR)

probability of the value of the variate falling in the critical region is the level of significance.

* **CRITICAL REGION**:

The region in which a sample value falling is rejected is known as the critical region (OR) It is the region of rejection of null hypothesis.

A region corresponding to a statistic 't' in the sample space 's' which leads to the rejection of H_0 is called critical region. Those region which lead to the acceptance of the H_0 given us a region called acceptance region.

Note:

In general we take 2 critical region which cover 5% and 1% areas of the normal curve.

* **Critical values (or) significant values**:

The value of the test statistic; which separates the critical region & the acceptance region is called critical value (or) significant value.

This value is dependent on.
The level of significance used.
(i) The level of significance
(ii) The alternative hypothesis; whether it it is one tailed (or) ~~depending on the~~ 2-tailed.
~~nature of the~~ ~~alternative~~

Note: Depending on the nature of the alternative hypothesis, critical region may lie on the oneside / both sides of the probability distribution curve.

ONE TAILED TEST:

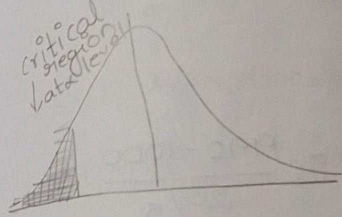
If the alternative hypothesis H_1 in a test of a statistical hypothesis be one-tailed (either right tailed (or) left tailed but not both)
then the test is called a onetailed test
(OR) If critical region lies on the one side then the corresponding test is said to be one-tailed test.

For Ex: to test whether the population mean $\mu = \mu_0$. Here it is 2 types:
Right tailed test

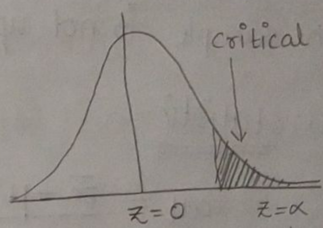
consider NULL hypothesis $H_0 : \mu = \mu_0$
$H_1 : \mu > \mu_0$ (right tailed)
$H_1 : \mu < \mu_0$ (left tailed)
corresponding test is a single tailed or onetailed (or) one sided.

*Right Tailed Test:

In the right tailed test $H_1 : \mu > \mu_0$ → the critical region $z > z_\alpha$ lies entirely in the right tail of the sampling distribution of sample mean $\bar{x}$ with area equal to the level of significance α.



Left tailed test



Right tailed test.

In the left tailed test $H_1 : \mu < \mu_0$ the critical region $z < -z_\alpha$ lies entirely in the left tail of the sampling distribution of sample mean $\bar{x}$ with area equal to the level of significance α.

* A sample of 26 bulbs gives a mean life of 990 hrs with S.D. of 20 hours. The manufacturer claims that the mean life of 100hrs.

Sol^n $\mu = 1000$ $\bar{x} = 990$

sample size $n = 26 < 30$ (small sample)

SD = $s = 20$; degrees of freedom (ν) $= n-1$
$= 25$

level of significance $= 5$.

Null hypothesis (H_0): sample is upto the standard

Alternative hypothesis (H_1) $\mu < 1000$

(the sample is not upto the standard)

Test statistic:

$$t_{cal} = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{990-1000}{20/5}$$
$$= -2.5$$

from the tabulated value of tcal at 5% level with 25 of freedom

$$t_{tab} = 1.708 = 25_{0.05}.$$

$\therefore$ $t_{cal} > t_{tab}$; we reject the NULL hypothesis (H_0) and conclude that the sample is not upto the standard.

Q) A machine is designed to produce insulating washers for electrical devices of average thickness 0.025cm, a random sample of 10 washers was found that have thickness of 0.024cm with SD of 0.002cm test the significance of deviation value of t for 50% level is 2.262.

Null hypothesis (H_0); There is no significance diff b/w means $\mu = 0.025$; $\bar{x} = 0.024$

$s = 0.002$ $n = 10$; $\nu = 9$

$t_{cal} = (0.024 - 0.025)/(0.002/3)$

$= -1.5$

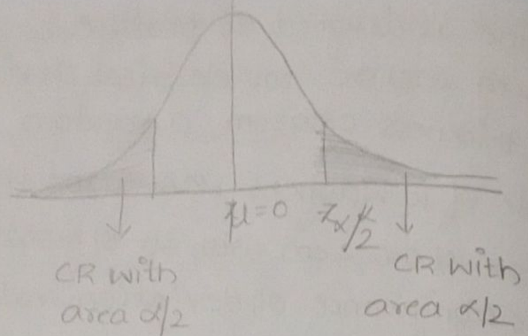
$|t| = 1.5$ $t_{cal} < t_{tab}$

Null hypothesis (H_0): $\bar{x} = \mu$

Alternative hypothesis (H_1):
$\bar{x} \neq \mu$

*Two tailed test:

When the alternative hypothesis H_1 is of the not equal type i.e. $H_1: \mu \neq \mu_0$ then the entire critical region of area 'α' lies on both sides tails of probability density curve shown in the following figure.



* Procedure for testing of hypothesis:

From the given problem context; identify the appropriate parameter of your interest. Then follow the steps given below.

1) State the NULL hypothesis H_0
2) Speify an appropriate alternative hypothesis H_1.
3) Set up the level of significance (α)
(Note: If 'α' is not given in the problem always we consider it as $\alpha = 5\%$)
4) Set up the test statistic as
$|t_{cal}| < |t_{tab}|$ then accept H_0
$|t_{cal}| > |t_{tab}|$ then reject H_0.

* Testing of Hypothesis: (Small sample tests)

Intro: In the earlier chapter; We considered certain tests of hypothesis on the theory of the normal distribution. The assumptions made in deriving the tests of hypothesis will be valid only for large samples. Here in this chapter we will discuss the tests of hypothesis when sample sizes are small (i.e.) $n<30$);

→ Small Sample tests.
1) t-test for single mean ($n<30$)
2) χ^2-test for goodness of fit
3) F-test for equality of population variance.

Note:
Before starting the t-tests, chi-square test & F-test; we shall first discuss about t, chi-square and F-distributions.

*t-distribution: Consider a small sample of size 'n', drawn from a normal population with mean (μ) and S.D (σ). If $\bar{x}$ and s be the sample mean and S.D. of sample then the statistic t is defined as

$$\boxed{t = \frac{\bar{x}-\mu}{s/\sqrt{n}} \quad \text{(or)} \quad t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}}}$$

Here 's' is the estimated standard deviation calculate through given sample items $x_1, x_2, \ldots\ldots, x_n$ using the formula

$$\boxed{s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

Here the 't' statistic is a random variable follows t-distribution with $\nu = n-1$ degrees of freedom with the probability density function.

$$f(t) = y_0\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}} \quad \text{where}$$

$y_0 \rightarrow$ const

It is again called as students t-distribution graph.

* **t-test for a single mean:** $(n<30)$

This is used to test the following.

a) If a random sample of size 'n' has been drawn from a 'normal population with specified mean μ.

b) If the sample mean differs significantly from the hypothesis value μ of the population mean

In this case the statistic is given by

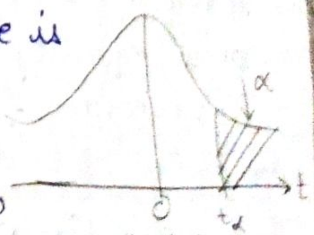
$$\boxed{t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} \quad (\text{or}) \quad t = \frac{\bar{x}-\mu}{s/\sqrt{n}}}$$

- In t-test for single mean the table values are observed at $n-1=\nu$ degrees of freedom
- Here the null hypothesis is $H_0: \mu = \mu_0$ and μ_0 is the specified value of population mean μ.

- **Properties of t-distribution:**

1) The shape of t-distribution is bell-shaped, which is similar to that of a normal distribution and is symmetrical about the mean.

2) The t-distribution curve is also asymptotic to the t-axis. i.e. 2 tails of the curve on both sides of $t=0$ extends to infinite.



3) It is symmetrical about the line $t=0$

4) The form of the probability curve varies with degrees of freedom i.e. with sample size.

5) It is unimodal with mean = median = mode.

6) The mean of standard normal distribution and as well as t-distribution is zero but the variance of t-distribution depends upon the parameter ν which is called the degree of freedom.

7) The variance of t-distribution exceeds 1; but approaches 1 as $n \to \infty$. Infact the t-distribution with ν degrees of freedom approaches standard normal distribution as $\nu = (n-1) \to \infty$

8) The t-distribution is extensively used in hypothesis about one mean (or) about equality of 2 means when σ is unknown.

*Applications of t-distribution:

The t-distribution has a wide no. of application in statistics. some of them are given below:

1) To test the significance of the sample mean, where population variance is not given.

2) To test the significance of the mean of the sample i.e. to test if the sample mean differs significantly from the population mean.

3) To test the significance of the diff. b/w 2 sample means (or) to compare 2 samples.

4) To test the significance of an observed sample correlation coeff and sample regression coeff.

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→ A random sample of 8 envelopes is taken from the letter box of a post office & their weights in grams are found to be 12·1, 11·9, 12·4, 12·3, 11·5, 11·6, 12·1 and 12·4. Does this sample indicate at 1% level that the average weight of envelopes received at a post office 12·35g (μ)?

* Estimate calculated value of t:

$\mu = 12{\cdot}35g$ (population mean).

$n = 8$ (Sample size)

Sample Mean: $\bar{x} = \frac{1}{n}\Sigma x = \frac{1}{8}(96{\cdot}6)$

$\bar{x} = 12{\cdot}075$

$$SD = \sqrt{\frac{1}{n-1}\left(\Sigma(x-\bar{x})^2\right)}$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
12.1	12.1 − 12.03 = 0.07	0.0049
11.9	−0.13	0.0169
12.4	0.37	0.1369
12.3	0.27	0.0729
11.5	−0.53	0.2809
11.6	−0.43	0.1849
12.1	0.07	0.0049
12.4	0.37	0.1369

$$\Sigma(x-\bar{x})^2 = 0.8392$$

$$S.D = \sqrt{\frac{1}{7}(0.8392)} = \sqrt{0.119885}$$

$$SD = 0.346$$

Sample mean = 12.03

Sample SD = 0.346

$$|t|_{cal} = \frac{\bar{x}-\mu}{S/\sqrt{n-1}}$$

$$= \frac{12.03-12.35}{0.346/\sqrt{7}} = \left|\frac{-0.32}{0.1307}\right|$$

$$\boxed{|t|_{cal} = 2.44}$$

Estimate tabulated value:

Null hypo (H_0) : $\mu = \mu_0$

i.e. $\mu = 12.35$

Alternative Hypothesis (H_1) : $\mu \neq \mu_0$

i.e. $\mu \neq 12.35$ (2 tail test).

d.f (ν) = $n-1 = 7$ = degree of freedom.

level of significance = 1% = 0.01

But we consider LS = 0.005

(∵ test is 2 tailed).

$$t_{tab} = (7)_{0.005}$$

$$\boxed{t_{tab} = 3.499}$$

$t_{cal} < t_{tab}$ → ∴ H_0 is accepted.

Yes, the sample indicate at 1% level that the average weight of envelopes received at the post office is 12.35g.

* Find 95% confidence limits for the mean weight of the envelopes received at the post office.

* confidence limit : $\bar{x} \pm (t_{\alpha/2})\frac{s}{\sqrt{n}}$

Confidence limits

$$= \left[\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} , \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \right]$$

$$t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = \cancel{t_{1/2} \cdot \frac{0.346}{\sqrt{8}}}$$

$$\cancel{= 3.499 \times \frac{0.346}{\sqrt{8}}}$$

$$= t_{5/2} \cdot \frac{0.346}{\sqrt{8}}$$

$$= t_{0.025} \times \frac{0.346}{\sqrt{8}}$$

$$= \frac{2.365 \times 0.346}{\sqrt{8}} = \frac{0.81829}{\sqrt{8}}$$

$$= 0.2893$$

$$= [12.03 - 0.2893 ; 12.03 + 0.2893]$$

$$= \boxed{[11.7407, 12.319]}$$

* A random sample of 10 boys IQ^s 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Does this

(i) data supports the assumption of the population mean IQ of 100.

(ii) Find a reasonable range in which most of the mean IQ values of samples of 10 boys.

* To estimate tabulated value of t:

① Null hypothesis (H_0): $\mu = 100$.

② Alternative hypothesis (H_1): $\mu \neq 100$.

(Two tailed test)

③ d.f (ν) = $n - 1 = 9$

④ Level of significance $= 5\% = 0.05$

Consider L.S = $\frac{0.05}{2}$ = 0.025. (2.5%)

($\because$ 2 tailed test).

t_{tab} = 9 d.f at 0.025

$$\boxed{t_{tab} = 2.262}$$

* To estimate calculated value of t:

x	$x - \bar{x}$	$(x - \bar{x})^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84

$\frac{972}{10}$

$\boxed{\bar{x} = 97.2}$

$\Sigma(x - \bar{x})^2 = 1833.6$.

$$S \cdot D(S) = \sqrt{\frac{1}{n-1}\left(\Sigma(x-\bar{x})^2\right)}$$

$$= \sqrt{\frac{1}{9}(1833 \cdot 6)}$$

$$\boxed{S = 14 \cdot 27}$$

* Sample Mean $(\bar{x}) = \frac{1}{n}\Sigma x = 97 \cdot 2$

Sample size $(n) = 10$.

$S = 14 \cdot 27$ (Sample SD)

$$t_{cal} = \frac{\bar{x} - \mu}{S/\sqrt{n-1}} = \frac{97 \cdot 2 - 100}{14 \cdot 27} \times 3.$$

$$= -0 \cdot 5886$$

$$\boxed{|t_{cal}| = 0 \cdot 5886.}$$

$t_{tab} > t_{cal} \Rightarrow H_0$ is accepted.

$\Rightarrow$ Yes, this data supports the assumption of the population mean IQ of 100

(ii) Confidence limits:

$$t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}, \ \bar{x} + t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}]$$

$$t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} = t_{5/2} \cdot \frac{12 \cdot 7}{\sqrt{10}}$$

$$= t_{0 \cdot 025} \cdot \frac{12 \cdot 7}{\sqrt{10}}$$

$$= 2 \cdot 262 \cdot \frac{12 \cdot 7}{\sqrt{10}}$$

$$= 9 \cdot 08.$$

$$= [97 \cdot 2 - 9 \cdot 08 ; \ 97 \cdot 2 + 9 \cdot 08]$$

$$= [88 \cdot 12, \ 106 \cdot 28]$$

* 2) Random sample from companies very extensi files shows that the orders for a certain kind of machinary were filled respectively 10, 12, 19, 14, 15, 18, 11, 13. Use the level of significance $\alpha = 0 \cdot 01$. To test the client that on the average of such orders are filled in 10·5 days. Choose the alternative hypothesis so that rejection of null hypothesis $\mu = 10 \cdot 5$ da implies that it takes longer than indicated.

* **To estimate calculated value of t**

x	$(x-\bar{x})$	$(x-\bar{x})^2$
10	-4	16
12	-2	4
19	5	25
14	0	0
15	1	1
18	4	16
11	-3	9
13	-1	1

$\bar{x} = 14.$ $\quad \Sigma(x-\bar{x})^2 = 72.$

$$(s) = \sqrt{\frac{1}{7}(72)} = 3.207.$$

$$t_{cal} = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = \frac{(14-10.5)\sqrt{7}}{3.207}$$

$$= 2.88$$

* **To estimate tabulated value of t:**

Null Hypothesis$(H_0) \Rightarrow \mu = 10.5$

Alternative Hyp $H_1 \Rightarrow \mu_0 \neq 10.5$

$\mu > 10.5$ (One tail test).

$d.f(\nu) = n-1 = 7$

Level of significance $= 1\%$

$\alpha = 0.01$

$t_{tab} = 7$ d.f at 0.01

$= 2.998$

$t_{tab} > t_{cal} \Rightarrow H_0$ is accepted.

* **F-distribution test:**

Let $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots y_n$ be the values of 2 independent random samples drawn from the normal population σ^2 having equal variances.

→ Let $\bar{x}, \bar{y}$ be the sample means

and $\boxed{s_1^2 = \frac{1}{n_1-1}\sum_{i=1}^{n}(x_i-\bar{x})^2}$

$\boxed{s_2^2 = \frac{1}{n_2-1}\sum_{i=1}^{n}(y_i-\bar{y})^2}$ be the sample variance

Then we define F by the relation

$$F = \frac{s_1^2}{s_2^2} \quad (s_1 > s_2)$$

(or) $$F = \frac{s_2^2}{s_1^2} \quad (s_2 > s_1).$$

with d.f $\nu_1 = n_1 - 1$; $\nu_2 = n_2 - 1$

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* $n = 60$
  $\bar{x} = 33.8$ min
  $s = 6.1$ min

Can we reject the null hypothesis $\mu = 32.6$ mm in favour of alternative hypothesis $\mu > 32.6$ at $\alpha = 0.025$ level of significance?

Sol^n: Null hypothesis $\mu = 32.6$ min.

alternative hypothesis : $\mu > 32.6$
(1 tail test)

$d.f(\nu) = \sqrt{n-1} = \sqrt{59}$

$L.S = 0.025$ (given)

$t_{tab} = t_{59\,0.025} = 2.00$ (app)

$t_{cal} = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}} = \dfrac{33.8 - 32.6}{6.1/\sqrt{59}}$

to test means − T test
to test variance − F-test
population mean − F-test

Note: To test the population mean (or) normal population (or) variance i.e. comparison between the variance, then we and F-test

F-Tabular value:

→ Null Hypothesis (H_0): There is no diff b/w variances (OR) there is no diff b/w normal population i.e. $\mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$

* alternate hypothesis (H_1) ⇒ $\sigma_1^2 \neq \sigma_2^2$
* $d.f(\nu_1, \nu_2) = (n_1 - 1, n_2 - 1)$
* Level of significance = 5% (or) 1%.

* Properties of F-test:

→ The F-distribution curve lies entirely in 1st quadrant and unimodal.
→ The F-distribution is independent of the population variance σ^2 and depend on ν_1 and ν_2 only.
→ $F_\alpha(\nu_1, \nu_2)$ is the value of F for ν_1 and ν_2 such that area of the right F_α is α
→ It can be shown that the mode distribution is less than unity.

* The measurements of the O/P of 2 units have given the following results. Assuming that both samples have been obtained from the normal populations at 5% significant level. Test whether 2 populations have the same variance.

Sample	Size	Sum of squares of deviation from the mean
1	10	90
2	12	108

Estimate value for F

$\Sigma(x-\bar{x})^2 = 90$; $\Sigma(y-\bar{y})^2 = 108$

$S_1^2 = \frac{1}{n_1-1}(90)$ $\quad S_2^2 = \frac{1}{n_2-1}(108)$

$= \frac{1}{9}(90) = 10$ $\quad S_2^2 = \frac{1}{11}(108)$

$= 9.8$

$S_1^2 > S_2^2$

$F_{cal} = \frac{10}{9.8} = 1.01$

$H_0 \Rightarrow \mu_1 = \mu_2$; $\sigma_1^2 = \sigma_2^2$

$H_1 \Rightarrow \sigma_1^2 \neq \sigma_2^2$

$d.f = (n_1-1, n_2-1) = (9, 11)$

Level of significance $= 5\% = 0.05$

$F_{tab} = F_{0.05}(9, 11)$

$\boxed{F_{tab} = 2.90}$

$F_{tab} > F_{cal} \rightarrow$ Null hypothesis is accepted.

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→ F-distributions:

* $\Sigma(x-\bar{x})^2$ and $\Sigma(y-\bar{y})^2$ are not given but SDs are given

$S_1^2 = \frac{n_1 s_1^2}{n_1-1}$ $\quad S_2^2 = \frac{n_2 s_2^2}{n_2-1}$

* It is known that mean diameters of rivets produced by 2 firms A & B are practically same but SD may differ for 22 rivets produced by firm A; SD is 2.9mm while for 16 rivets manufactured by

firm B ; the SD is 3·8mm Compute the statistics to tell whether the product of A have some variability of B & test its significance:

$H_0 \Rightarrow \sigma_1^2 = \sigma_2^2$

variance of firm A = variance of firm B.

$H_1 \Rightarrow \sigma_1^2 \neq \sigma_2^2$

LDS = 5%

Fcal:

$n_1 = 22$; $n_2 = 16$

$S_1 = 2.9$ $\quad S_2 = 3.8mm$

$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = 8.81$

$S_2^2 = \frac{16(3.8)^2}{15} = 15.4$

$S_2^2 > S_1^2 \Rightarrow Fcal = \frac{S_2^2}{S_1^2} = 1.148$

Ftab > Fcal ⟶ Null hypothesis is accepted.

Pumpkins are grown under 2 experimental conditions. 2 random samples of 7 & 11 pumpkins. Show the sample S.D. of their weights are 0.8 & 0.5 respectively. Assuming that the ~~variance~~ weight distributions are normal. Test hypothesis that the variance are equal.

$H_0 \rightarrow \sigma_1^2 = \sigma_2^2$

$H_1 \rightarrow \sigma_1^2 \neq \sigma_2^2$

$S_1 = 0.8$ $\quad S_2 = 0.5$

$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$ $\quad S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$

$= \frac{7 \times 0.8^2}{6}$ $\quad = \frac{10 \times 0.5^2}{10}$

$= 0.74$ $\quad = 0.25$

$\frac{S_1^2}{S_2^2} = 2.96$ $\quad \nu_1 = 6$, $\nu_2 = 10$

$F_{cal} = 1.86$ $\quad F_{tab} = 3.22$

→ The magnitude of discrepancy between observed (experimental) and expected (hypo… frequency is given by the quantity χ^2 and with n-1 degrees of freedom

$$\chi^2_{cal} = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

χ^2 is a measure of correspondance b/w theory & observation

$\nu = n-1 \rightarrow$ degrees of freedom.

*Conditions:

Following are sufficient condition that should be satisfied before χ^2 test to be applied:

→ Sample observation should be independent.

→ total frequency is large i.e. $N > 50$

→ The constraints on the cell frequencies are linear

→ If $\chi^2_{cal} > \chi^2_{tab}$ at α level the null hypothesis is rejected otherwise Ho is accepted.

* Applications:

→ χ^2 distribution is continuous probabity distribution & it is used in both large & small test.

→ it is mainly used in

* to test goodness of fit
* to test independence of attributes
* to test if the pop has specified value of variance σ^2.

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*Problems

→ χ^2 test for ~~the fit of the curve~~:

1) The following figures show distributions of disease in number choosen at random from a telephone directory. Test whether the digits may be taken to occur equally, frequently in the directory.

Digits	frequency (O_i)	E_i	$O_i - E_i$	$(O_i-E_i)^2/E_i$
0	1026	1000	26	0.676.
1	1107	1000	107	11.449
2	997	1000	-3	0.009
3	966	1000	-34.	1.156
4	1075	1000	75	5.625
5	933	1000	-67	4.489
6	1107	1000	107	11.449
7	972	1000	-28	0.784
8	964	1000	-36	1.296
9	853	1000	-147.	21.609

$\Sigma O_i = 10,000$

$E(x_i) = \frac{1}{n} \sum O_i = \frac{1}{10}(10,000)$

$= 1000$

$$\sum \frac{(O_i - E_i)^2}{E_i} = 58.542$$

$$\boxed{\chi^2_{cal} = \sum \frac{(O_i - E_i)^2}{E_i} = 58.542}$$

* χ^2_{tab}

Null Hyp (H_0) = The digits occur equally, frequently in the directory.

Alternative Hyp (H_1) = The digits ~~occur~~ do not occur equally, frequently in the directory

d.f $(\nu) = n-1 = 9$

Level of significance = ~~[illegible]~~ $5\% = 0.05$

$\chi^2_{0.05}(9) = 16.919$

$\chi^2_{tab} < \chi^2_{cal}$

Null hypothesis is not accepted.

* A pair of dice are thrown 360 times & frequency of each sum is indicated below. Would you say that the dice are fair on the basis of the χ^2 test on 0.05 level of significance.

Sum	O_i	E_i	$(O_i - E_i)$	$(O_i-E_i)^2/E_i$
2	8	10	−2	0.4
3	24	20	4	0.8
4	35	30	5	0.83
5	37	40	−3	0.225
6	44	50	−6	0.72
7	65	60	5	0.416
8	51	50	1	0.02
9	42	40	2	0.1
10	26	30	−4	0.53
11	14	20	−6	1.8
12	14	10	4	1.6

$\sum O_i = 360$

Pdf:

$x = x_i$	2	3	4	5	6	7	8	9	10	11	12
$P(x=x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\boxed{E_i = 360 \times P(x = x_i)}$$