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UNIT-2

LINEAR DIFFERENTIAL EQUATIONS OF SECOND AND HIGHER ORDER:

→ Homogeneous linear differential equations with constant coefficients.
- → Complementary function
- → Problems.

→ Non-Homogeneous linear DE with constant coefficients
- → Complementary function
- → Particular Integral
- → Problems

→ Method of variation of parameters - problems

→ Applications of second order equations - LCR circuits.

$$\boxed{D=\frac{d}{dx}}$$

$$* \quad \frac{d^ny}{dx^n}+K_1\frac{d^{n-1}y}{dx^{n-1}}+\cdots+K_n y=0 \quad \text{—} (1)$$

$$\left[D^n+K_1D^{n-1}+\cdots+K_n\right]y=0 \quad \text{—} (2)$$

(2) is the operator | standard form.

Let $y=e^{mx}$

$Dy=me^{mx}$

$D^2y=m^2e^{mx}$

$\vdots$

$D^ny=m^ne^{mx}$

Auxiliary equation:

$$e^{mx}\left[m^n+K_1m^{n-1}+K_2m^{n-2}+\cdots+K_n\right]=0 \quad \text{—} (3)$$

$e^{mx}\neq 0$

$\cancel{f(D)=0}$ $f(m)=0$

$$[(D-m_1)(D-m_2)\cdots(D-m_n)]y=0$$

$$\left[(D-m_1)(D-m_2)\right]y=0.$$

O.E:

$f(D)\cdot y=0$

A.E:

$f(m)=0$

(i) $(D-m_1)\,y=0$

$$\frac{dy}{dx}-m_1 y=0$$

$$y=c_1e^{m_1x}$$

$$\frac{dy}{dx}=m_1 y$$

$$\frac{dy}{y}=m_1 dx$$

$$\log y=m_1x+\log c$$

$$y=c_1e^{m_1x}$$

(ii) $(D-m_2)\,y=0$

$$y=c_2e^{m_2x}$$

$$y=c_3e^{m_3x}\ \ldots\ldots$$

$$\boxed{y_c=c_1e^{m_1x}+c_2e^{m_2x}+\ldots}$$

General solution of linear Homogeneous Differential Equation is a complementary function only.

Roots:
- → Real and distinct
- → Real and equal
- → Imaginary roots

* If $m_1=m_2$

$$\left[(D-m_1)(D-m_1)\right]y=0$$

$$(D-m_1)\,z=0$$

*Assume

$$(D-m_1)\,y=z$$

$$\frac{dz}{dx}-m_1z=0$$

$$z=c_1e^{m_1x}$$

$$(D-m_1)y=z$$

$$\frac{dy}{dx}-m_1y=c_1e^{m_1x}$$

$$I.F=e^{-m_1x}$$

$$ye^{-m_1x}=\int c_1e^{m_1x}\cdot e^{-m_1x}\,dx+c_2$$

$$ye^{-m_1x}=c_1x+c_2$$

$$\boxed{y=(c_1x+c_2)e^{m_1x}}$$

If roots are 3, 3, 3

$$y=(c_1x^2+c_2x+c_3)e^{3x}$$

If roots are $-1,-1,2,4$

$$y=(c_1x+c_2)e^{-x}+c_3e^{2x}+c_4e^{4x}$$

*Imaginary Roots: $a\pm ib$

$a+ib,\ a-ib$

$$y=c_1e^{(a+ib)x}+c_2e^{(a-ib)x}$$

$$=c_1e^{ax}\cdot e^{ibx}+c_2e^{ax}\cdot e^{-ibx}$$

$$y=c_1e^{ax}(\cos bx+i\sin bx)+c_2e^{ax}(\cos bx-i\sin bx)$$

$$y=e^{ax}\left[\cos bx(c_1+c_2)+\sin bx(ic_1-ic_2)\right]$$

$$\boxed{y=e^{ax}\left[c_1\cos bx+c_2\sin bx\right]}$$

Nature of Roots:	Complementary Function
1) m_1, m_1, m_2, m_2, m_5.	$(c_1x+c_2)e^{m_1x} + (c_3x+c_4)e^{m_2x} + c_5e^{m_5x}$
2) m_1, m_1, m_1, m_2	$(c_1x^2+c_2x+c_3)e^{m_1x} + c_4e^{m_2x}$
3) $m_1, m_2, m_3, m_3 \ldots$	$c_1e^{m_1x}+c_2e^{m_2x} + (c_3x+c_4)$
4) $(a\pm ib)^2$	$\cancel{e^{ax}(c_1\cos bx + c_2\sin bx) + e^{ax}(c}$ $e^{ax}\left[(c_1x+c_2)\cos bx + (c_3x+c_4)\sin bx\right]$

* **2nd order DE: (or) higher order DE:**

A differential equation is said to be linear differential equation if its dependent variable and its derivatives occurs in the first degree and are not multiplied together.

$\Rightarrow$ **Operator (D):**

$$\frac{d}{dx} \to D \; ; \; \frac{dy}{dx} = Dy$$

$$\frac{d^2y}{dx^2} \to D^2y$$

$$\frac{d^ny}{dx^n} \to D^ny$$

where

$D \to$ differential operator.

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* Solve the following DE:

1) $y'' + 36y = 0$

$$\frac{d^2y}{dx^2} + 36y = 0 \quad -(1)$$

Operator form of given DE is

$$(D^2+36)y = 0 \quad -(2)$$

Auxiliary equation of (2) is

$$(m^2+36)e^{mx} = 0 \qquad e^{mx} \neq 0$$

$$m^2+36 = 0$$

$$m^2 = -36$$

$$m = \pm 6i$$

$$(D-6i)(D+6i) = 0$$

$\therefore$ Here the roots are imaginary roots.

Complementary Function:

Roots are imaginary.

$$C.F(y_c) = e^{ax}(c_1\cos bx + c_2\sin bx)$$

$$= e^{0(x)}(c_1\cos 6x + c_2\sin 6x)$$

$$\boxed{C.F = c_1\cos 6x + c_2\sin 6x}$$

2) $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 0 \quad -(1)$$

Operator form of given DE is

$$(D^2+2D+4)y = 0 \quad -(2)$$

Auxiliary equation of (2) is

$(D^2+2D+4)y=0$

$m^2+2m+4=0$ $\quad e^{mx}\neq 0$

$m=\frac{-2\pm\sqrt{4-16}}{2}$

$=\frac{-2\pm\sqrt{-12}}{2}=\frac{-2\pm 2\sqrt{3}i}{2}$

$m=-1\pm\sqrt{3}i$ Roots are imaginary

C.F $=\boxed{y_c=e^{-x}(c_1\cos\sqrt{3}x+c_2\sin\sqrt{3}x)}$

3) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$

O.F: $D^2+2D+1=0$

A.F: $m^2+2m+1=0$

$(m+1)^2=0$

$m=-1,-1$

Roots are real and equal

$\boxed{y_c=(c_1x+c_2)e^{-x}}$

4) $(D^2+3D+2)y=0$

O.F $=(D^2+3D+2)y$

A.F $=\boxed{m^2+3m+2=0}$

$m=-2,-1$

Required General Solution:

$\boxed{y_c=c_1e^{-x}+c_2e^{-2x}}$

1) $4y''-8y'+3y=0$ $\quad y(0)=1,\ y'(0)=3$

$4\frac{d^2y}{dx^2}-8\frac{dy}{dx}+3y=0$

O.F. of given DE

$(4D^2-8D+3)y=0$

A.F of given DE:

$4m^2-8m+3=0$

$4m^2-6m-2m+3=0$

$2m(2m-3)-(2m-3)=0$

$(2m-3)(2m-1)=0$

$m_1=\frac{3}{2}$ $\quad m_2=\frac{1}{2}$

$\boxed{C.F=c_1e^{3/2x}+c_2e^{1/2x}}$ ⇒ General Solⁿ.

2) ~~4y''+4y'+y=0~~

$y(0)=1$

$x=0;\ y=1$

$\boxed{c_1+c_2=1}$

$\frac{dy}{dx}=c_1\cdot\frac{3}{2}e^{3/2x}+\frac{c_2}{2}e^{1/2x}$

$3=\frac{3}{2}c_1+\frac{c_2}{2}$

$3c_1+c_2=6$

$c_1+c_2=1$

$2c_1=5$

$c_1=5/2$

$c_2=-\frac{3}{2}$

$\boxed{y(x)=\frac{5}{2}e^{3/2x}-\frac{3}{2}e^{1/2x}}$

2) $4y'' + 4y' + y = 0$

$$4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$$

$(4D^2 + 4D + 1)y = 0 \Rightarrow$ O.F of given DE

A.F $\Rightarrow 4m^2 + 4m + 1 = 0$

$(2m+1)^2 = 0$ Roots are real & equal

$2m + 1 = 0$

$m = -1/2$

$m_1 = m_2 = -1/2$

$$\boxed{y_c = (c_1 x + c_2)e^{-x/2}}$$

3) $y'' + 4y' + 13y = 0 \quad y(0) = 0; y'(0) = 1$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$$

O.F $= (D^2 + 4D + 13)y = 0$

A.F $= m^2 + 4m + 13 = 0$

$$m = \frac{-4 \pm \sqrt{16 - 52}}{2}$$ Roots are imaginary

$$= \frac{-4 \pm \sqrt{36}}{2}$$

$$= \frac{-4 \pm 6i}{2} = -2 \pm 3i$$

$$y_c = e^{ax}[c_1 \cos bx + c_2 \sin bx]$$

$$y_c = e^{-2x}[c_1 \cos 3x + c_2 \sin 3x]$$

$x = 0; y = 0$

$\boxed{c_1 = 0}$

$$y'(x) = e^{-2x}[c_1(-\sin 3x)3 + c_2(3)(\cos 3x)] + e^{-2x}(-2)[c_1 \cos 3x + c_2 \sin 3x]$$

$y'(0) = 1$

$x = 0; y = 1$

$1 = [0 + 3c_2] - 2(0 + 0)$

$$\boxed{c_2 = +\frac{1}{3}}$$

$$y_c = e^{-2x}\left[\frac{+1}{3}\sin 3x\right]$$

4) $(D^2 + 9D)y = 0$

O.F $\Rightarrow (D^2 + 9D)y = 0$

A.F $\Rightarrow m^2 + 9m = 0$

$m(m + 9) = 0$

$m = 0, -9$

$y_c = c_1 e^{(0)x} + c_2 e^{-9x}$

$\boxed{y_c = c_1 + c_2 e^{-9x}} \Rightarrow$ CF

5) $y'' + 2y' + 2y = 0 \quad y(0) = 1 \; ; \; y(\pi/2) = e^{-\pi/2}$

$O \cdot F = (D^2 + 2D + 2)y = 0$

$A \cdot F = m^2 + 2m + 2 = 0$

$$m = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$= \frac{-2 \pm 2i}{2} = -1 \pm i$$

$C \cdot F = e^{-x}[c_1 \cos x + c_2 \sin x]$

$y(0) = 1 \qquad y(\pi/2) = e^{-\pi/2}$

$x = 0 ; \; y = 1 \qquad C_2 = 1$

$1 = c_1 + 0 \quad \boxed{c_1 = 1}$

$y'(c) = e^{-x}[-c_1 \sin x + c_2 \cos x]$

$\cancel{- e^{-x}[c_1 \cos x + c_2 \sin x]}$

$\cancel{x = \pi/2 \quad y = e^{-\pi/2}}$

$\cancel{e^{-\pi/2} = e^{-\pi/2}[-1 + c_2(0)]}$

$\cancel{- e^{-\pi/2}[0 + c_2]}$

$\cancel{1 = -1 - c_2}$

$\cancel{\boxed{c_2 = -2}}$

$\boxed{y_c = e^{-x}[\cos x + \sin x]}$

6) $(D^2 + 4)y = 0$

$\cancel{O \cdot F \Rightarrow (D^2 + 4)y = 0}$

$\cancel{A \cdot F \Rightarrow m^2 + 4 = 0} \quad \boxed{m^2 = \pm 2i}$

$\cancel{m = \pm 2i}$

$\cancel{y_c = e^{0(x)}[c_1 \cos 2x + c_2 \sin 2x]}$

$\cancel{y_c = c_1 \cos 2x + c_2 \sin 2x}$

$m^2 = -2i \; x$

$m^2 = 2i$

$m = \pm\sqrt{2i}$

$\cancel{y_c = e^{0x}[c_1 \cos \sqrt{2i}\, x + c_2 \sin \sqrt{2i}\, x]}$

6) $(D^4 + 4)y = 0$

$y(D^4 + 4 + 4D^2 - 4D^2) = 0$

$y[(D^2 + 2)^2 - (2D)^2] = 0$

$A \cdot F \Rightarrow y[(m^2 + 2)^2 - (2m)^2] = 0$

$(m^2 + 2 - 2m)(m^2 + 2 + 2m) = 0$

$m^2 - 2m + 2 = 0 \qquad m^2 + 2m + 2 = 0$

$$m = \frac{2 \pm \sqrt{4-8}}{2} \qquad \boxed{m = -1 \pm i}$$

$$= \frac{2 \pm \sqrt{4}}{2}$$

$$= \frac{2 \pm 2i}{2} = 1 \pm i$$

$y_c = e^{x}[c_1 \cos x + c_2 \sin x] + e^{-x}[c_1 \cos x + c_2 \sin x]$

$\boxed{y_c = (e^{x}+e^{-x})c_1 \cos x + (e^{x}+e^{-x}) c_2 \sin x}$

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1) $(4D^2+12D+9)y=0$

$O.F = (4D^2+12D+9)y=0$

$A.E = 4m^2+12m+9=0$

$4m^2+6m+6m+9=0$

$2m(2m+3)+3(2m+3)=0$

$(2m+3)(2m+3)=0$

$m_1=m_2=-3/2$ Nature of roots are real & equal

C.F: General sol^n

$\boxed{y_c = (c_1 x + c_2)e^{-3/2 x}}$

2) $y''' - 6y'' + 11y' - 6y = 0$

$y(0)=0 \quad y'(0)=-4$

$y''(0)=-18$

$O.F (D^3-6D^2+11D-6)y=0$

$A.E = m^3-6m^2+11m-6=0$

$m_1=1, m_2=2, m_3=3$

Nature of roots are real and unequal.

$\Rightarrow$ C.F: General Sol^n

$\boxed{y_c = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x}}$

$x=0; y=0$

$\boxed{0 = c_1+c_2+c_3}$ $\quad c_1=-c_2-c_3$

$x=0; y=-4$

$\boxed{-4 = c_1+2c_2+3c_3}$

$\boxed{-18 = c_1+4c_2+9c_3}$

$-4 = c_2+2c_3 \times 3 \Rightarrow -12 = 3c_2+6c_3$

$-18 = 3c_2+8c_3 \qquad \underline{-18 = 3c_2+8c_3}$

$6 = -2c_3$

$c_2 = -4+6 \qquad c_3=-3$

$c_2 = 2$

$c_1 = 1$

3) $y^{iv} - 5y'' + 4y = 0$

$O.F = (D^4 - 5D^2+4)y=0$

$A.F = m^4-5m^2+4=0$

$m^4-4m^2-m^2+4=0$

$m^2(m^2-4)-1(m^2-4)=0$

$m^2=4; \quad m^2=1$

$m=\pm 2 \quad m=\pm 1$

Roots are real & distinct

C.F: General Sol^n

$$\boxed{y_c = c_1 e^{x} + c_2 e^{-x} + c_3 e^{2x} + c_4 e^{-2x}}$$

4) Solve the boundary value problem.

$y'' + w^2 y = 0 \qquad y(0) = 0 \; ; \; y(l) = 0$

$(D^2 + w^2) y = 0 \Rightarrow$ standard Form

A.E: $(m^2 + w^2) = 0$

$m^2 = -w^2$

$m = \pm wi$

Roots are imaginary

C.F: General Sol^n

C.F = $y_c(x) = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

$y_c(x) = e^{0(x)}(c_1 \cos wx + c_2 \sin wx)$

$y_c(x) = c_1 \cos wx + c_2 \sin wx$

$\rightarrow x = 0 ; y = 0$

$0 = c_1 + c_2(0)$

$c_1 = 0$

$\rightarrow x = l \; ; y = 0$

$0 = c_1 \cos wl + c_2 \sin wl$

$c_2 (\sin wl) = 0$

if $c_2 = 0 \Rightarrow y_c(x) = 0$ × (Trivial Sol^n)

∵ Every system has Non-trivial

Sol^n:

ii) $\Rightarrow \sin wl = 0$

$wl = \pm n\pi \qquad n = 1, 2, 3, \ldots$

$\boxed{l = \pm \frac{n\pi}{w}}$

$\boxed{w = \pm \frac{n\pi}{l}}$

$\boxed{y_c(x) = c_2 \sin \frac{n\pi}{l} x}$

By super-position principle:

$\boxed{y(x) = \sum_{n=1}^{\infty} c_K \sin \frac{n\pi x}{l}}$

5) Find the non-trivial solution of boundary value problems (BVP) $y^{IV} - w^4 y = 0$.

$y(0) = 0 \; ; \; y''(0) = 0 \quad y(l) = 0 \quad y''(l) = 0$

Operator Form:

$(D^4 - w^4) y = 0$

A.E: $m^4 - w^4 = 0$

$(m^2 + w^2)(m^2 - w^2) = 0$

$m^2 = -w^2 \qquad m^2 = w^2$

$m = \pm wi \qquad m = \pm w$

~~$y_c(x) \Rightarrow$ C.F~~ 2 Roots are real

~~$y_c(x) = c_1 e^{wix}$~~ and 2 roots imaginary

$y_c x =$

C.F $\Rightarrow$ General Solution

$$y_c = c_1 e^{\omega x} + c_2 e^{-\omega x} + e^{(0x)}[c_3 \cos\omega x + c_4 \sin\omega x]$$

$$\boxed{y_c = c_1 e^{\omega x} + c_2 e^{-\omega x} + c_3 \cos\omega x + c_4 \sin\omega x}$$

$x = 0 ; y = 0$

$0 = c_1 + c_2 + c_3$

$\boxed{c_1 + c_2 + c_3 = 0}$

$\cancel{c_2 = 0}$

$\boxed{c_1 + c_2 + c_3 = 0}$ — (1)

$x = 0 \quad y'(0) = 0$

$\cancel{c_1\omega + c_2(-\omega) - \omega c_1(0) + \omega c_2 = 0}$

$c_1\omega = 0$

$\cancel{c_1 = 0}$

$y'(x) = c_1 \omega e^{\omega x} - c_2 \omega e^{-\omega x} - c_3 \omega \sin\omega x + c_4 \omega \cos\omega x$

$y''(x) = c_1\omega^2 e^{\omega x} + c_2 \omega^2 e^{-\omega x} - c_3 \omega^2 \cos\omega x - c_4 \omega^2 \sin\omega x$

$x = 0 \quad y''(0)$

$\boxed{0 = c_1\omega^2 + c_2\omega^2 - c_3\omega^2}$ — (3)

$x = l ; y = 0$

$\boxed{c_1 e^{\omega l} + c_2 e^{-\omega l} + c_3 \cos\omega l + c_4 \sin\omega l}$ — (4)

$x = l ; y''(l)$

$\boxed{c_1\omega^2 e^{\omega l} + c_2 \omega^2 e^{-\omega l} - c_3\omega^2 \cos\omega l - c_4 \omega^2 \sin\omega l}$ — (3)

$\omega^2(c_1 + c_2 - c_3) = 0$ — (1)

$c_1 + c_2 + c_3 = 0$ — (3)

$\Rightarrow \boxed{c_1 + c_2 = 0}$ — (5) $\times e^{\omega l}$

(5) & (4)

$\boxed{c_1 e^{\omega l} + c_2 e^{-\omega l} = 0}$ — (6)

$\cancel{c_1 e^{\omega l}} + c_2 e^{\omega l} = 0$

$\cancel{c_1 e^{\omega l}} + c_2 e^{-\omega l} = 0$

$\boxed{c_2(e^{\omega l} - e^{-\omega l}) = 0}$

if $c_2 = 0 ; c_1 = 0 \quad c_3 = 0$

$y_c(x) = c_4 \sin\omega x$ — (7)

$y'(x) = c_4 \cos\omega x \cdot \omega$

$y''(x) = -c_4\omega^2 \sin\omega x$

$x = 0 ; y = 0$ in (7)

$x = l ; y = 0$

$c_4 \sin\omega l = 0$

$c_4 \neq 0 \quad \omega l = n\pi$

$\boxed{\omega = \frac{n\pi}{l}}$

$\boxed{y_c(x) = c_4 \sin\frac{n\pi x}{l}}$

**

NON-HOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS OF HIGHER ORDER WITH CONSTANT COEFFICIENTS:

N.H.L.D.E are always having complete (general) solution i.e. [complementary function and particular integral].

$$\boxed{\text{Complete Solution} = \text{Complementary Function} + \text{Particular Integral}}$$

General form of NHLDE:

$$\frac{d^n y}{dx^n} + K_1 \frac{d^{n-1} y}{dx^{n-1}} + K_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + K_n y = Q(x) \quad -(1)$$

$$Q(x) \neq 0$$

Operator Form:

$$(D^n + K_1 D^{n-1} + K_2 D^{n-2} + \cdots + K_n) y = Q(x) \quad -(2)$$

To find C.F:

H.L.DE:

$$(D^n + K_1 D^{n-1} + K_2 D^{n-2} + \cdots + K_n) y = 0$$

$$f(m) = 0$$

Find the roots and follow the previous method.

To find Particular Integral:

$$\boxed{f(D) y = Q(x)}$$

- $Q(x) = e^{ax}$
- $Q(x) = \sin ax / \cos ax$
- $Q(x) = x^K$
- $Q(x) = e^{ax} \cdot V(x)$
  - $\sin ax$
  - $\cos ax$
  - x^K
- $Q(x) = x^K V(x)$ ($K = 1$)
  - $\sin ax$
  - $\cos ax$

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Working procedure to find particular integral:

* P.I. is only possible to solve for LNHDE

Consider

$$\frac{d^n y}{dx^n} + K_1 \frac{d^{n-1} y}{dx^{n-1}} + K_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + K_n y = Q(x) \quad -(1)$$

⇒ **Step 1:** Operator form of (1)

$$[D^n + K_1 D^{n-1} + \cdots + K_n] = Q(x)$$

$$\boxed{f(D) y = Q(x)} \quad -(2)$$

$$\boxed{\text{P.I.} (y_P) = \frac{1}{f(D)} \cdot Q(x)}$$

(i) where $\boxed{Q(x) = e^{ax}}$

$$y_P = \frac{1}{f(D)} e^{ax} \qquad \text{put } \boxed{D = a}$$

$$\boxed{y_P = \frac{e^{ax}}{f(a)}} \quad \text{if } f(a) \neq 0$$

if $f(a)=0$

Special Case: $D=a$

$$\boxed{y_p = \frac{x e^{ax}}{f'(a)}}$$ if $f'(a) \neq 0$

if $f'(D) = 0$

$$\boxed{y_p = \frac{x^2 e^{ax}}{f''(D)}}$$ if $f''(a) \neq 0$

(ii) where $\boxed{Q(x) = \sin ax / \cos ax}$

$$y_p = \frac{1}{f(D)} \sin ax$$

put only $\boxed{D^2 = -a^2}$ only in Dr

$$\boxed{y_p = \frac{1}{f(-a^2)} \sin ax}$$ if $f(-a^2) \neq 0$

if only 'D' term is in Dr, then rationalise.

Ex: $\frac{1}{D^3+2} \sin 2x$

$a=2$

$$\frac{1}{D^2 \cdot D+2} \sin 2x \qquad D^2 = -4$$

$$= \frac{1}{-4D+2} \sin 2x$$

$$= \frac{\sin 2x}{2(-2D+1)} = \frac{\sin 2x\,(1+2D)}{2(1-2D)(1+2D)}$$

$$= \frac{\sin 2x\,(1+2D)}{2(1+16)} = \frac{\sin 2x(1+2D)}{34}$$

$$= \frac{\sin 2x + 4\cos 2x}{34}$$

if $f(-a^2)=0$

$$\boxed{y_p = \frac{x \sin ax}{f'(D)}}$$ put $D^2 = -a^2$

$$\boxed{y_p = \frac{x \sin ax}{f'(-a^2)}}$$ $f'(-a^2) \neq 0$

if $f'(-a^2) = 0$

$$\boxed{y_p = \frac{x^2 \sin ax}{f''(-a^2)}}$$ $f''(-a^2) \neq 0$

$\Rightarrow$ until dr is non zero

(iii) where $\boxed{Q(x) = x^k}$ $\boxed{k>0}$

$$\cancel{y_p = \frac{1}{f(D)} \sin ax}$$

$$y_p = \frac{1}{f(D)} x^k$$ make dr first element as 1

$$y_p = [f(D)]^{-1} x^k$$

using binomial expansion

we get y_p.

$Dx^2 = 2x$
$D^2x^2 = 2$

$$\frac{1}{D+5} x^2$$

$$\frac{1}{5\left(1+\frac{D}{5}\right)} \cancel{x^2}$$

$$\frac{1}{5}\left(1+\frac{D}{5}\right)^{-1} x^2$$

$$\frac{1}{5}\left(1-\frac{D}{5}+\frac{D^2}{25}\right)x^2$$

$$\frac{1}{5}\left(x^2 - \frac{2x}{5} + \frac{2}{25}\right)$$

25/06/2022

* Solve the following DE:

1) $(4D^2+4D-3)y = e^{2x}$

~~Ans~~ Linear Non-Homogeneous DE with 2^{nd} order.

To find C.F:

Operator Form:

$(4D^2+4D-3)y = 0$

A.E:

$(4m^2+4m-3) = 0$

$4m^2+6m-2m-3=0$

$2m(2m+3)-1(2m+3)=0$

$m = \frac{1}{2}, \frac{-3}{2}$

$y_c = c_1 e^{x/2} + c_2 e^{-3/2 x}$

To find P.I:

$y_p = \frac{e^{2x}}{4D^2+4D-3}$

put $D=2$

$y_p = \frac{e^{2x}}{4(2)^2+4(2)-3}$

$= \frac{e^{2x}}{21}$

$\boxed{y_p = \frac{e^{2x}}{21}}$

General Solution:

$y_g = c_1 e^{x/2} + c_2 e^{-3x/2} + \frac{e^{2x}}{21}$

2) Solve. $(D^2+3D+2)y = 5$.

To find C.F:

O.F $\Rightarrow (D^2+3D+2)y = 0$

AE $\Rightarrow (m^2+3m+2) = 0$

$(m+1)(m+2) = 0$

$m = -1, -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$

To find PI:

$y_p = \frac{5}{D^2+3D+2}$

put $D=0$.

$\boxed{y_p = \frac{5}{2}}$

General Solution:

$y = c_1 e^{-x} + c_2 e^{-2x} + \frac{5}{2}$

3) $(D^3-3D^2+4)y = e^{2x}$

To find C.F:

Operator Form:

$(D^3-3D^2+4)y = 0$

A.E:

$m^3 - 3m^2+4 = 0$

$m=-1$

$$\require{cancel}\begin{array}{r} m^2-2m+2 \\ m+1 \overline{) m^3-3m^2+4} \\ m^3+m^2 \\ \hline -2m^2+0m+4 \\ -2m^2-2m+0 \\ \hline 2m+4 \\ 2m+ \end{array}$$

$m=-1$

$$\begin{array}{r} m^2-4m+4 \\ m+1 \overline{) m^3-3m^2+4} \\ m^3+m^2 \\ \hline -4m^2+0m+4 \\ -4m^2-4m+0 \\ \hline 4m+4 \\ 4m+4 \\ \hline 0 \end{array}$$

$m = -1, 2, 2$.

To find P.I:

$y_p = \frac{e^{2x}}{D^3-3D^2+4}$

$\boxed{D=2}$

$y_p = \frac{e^{2x}}{8-12+4}$

$\neq 0$

$\Rightarrow y_p = \frac{e^{2x}}{D^3-3D^2+4}$

$y_p = \frac{xe^{2x}}{3D^2-6D+0}$

$D=2$

$= \frac{xe^{2x}}{12-12}$

$=0$

$y_p = \frac{x^2e^{2x}}{6D-6}$

$y_c = (c_1x+c_2)e^{2x} + c_3e^{-x}$ $\quad y_p = \frac{x^2e^{2x}}{6}$

General sol^n

$\Rightarrow y = (c_1x+c_2)e^{2x} + c_3e^{-x} + \frac{x^2e^{2x}}{6}$

4) $(D^3-6D^2+11D-6)y = e^{-2x} + e^{-3x}$

To find CF:

$y(D^3-6D^2+11D-6) = 0 \Rightarrow$ Operator Form.

A.E:

$m^3-6m^2+11m-6=0$

$m = 1, 2, 3$

$y_c = c_1e^x + c_2e^{2x} + c_3e^{3x}$

To find P.I 1:

$(D^3-6D^2+11D-6)y = e^{-2x}$

$y = \frac{e^{-2x}}{D^3-6D^2+11D-6}$ $\quad -8-6(4) +22-6$

$D = -2$

$y_p = \frac{e^{-2x}}{-8-6(4)-22-6}$ $\quad -32 \; -28$

$= \frac{e^{-2x}}{-60}$

$\boxed{y_p = \frac{-e^{-2x}}{+60}}$

To find PI: 2:

$(D^3-6D^2+11D-6)y = e^{-3x}$

$y_p = \frac{-e^{-3x}}{-27-6(9)-33-6}$

$= \frac{-e^{-3x}}{+120}$ $\quad -81 \; -39 \; -120$

General Solution:

$c_1e^x + c_2e^{2x} + c_3e^{3x} - \frac{e^{-2x}}{60} - \frac{e^{-3x}}{120}$.

27/06/2022

* Find the PI of $(D^2+4)y = \cos 2x$

$y_p = \frac{1}{D^2+4}\cos 2x$ $\quad$ put $D^2 = -(a^2) = -(4)$

$y_p = \frac{\cos 2x}{0}$ $\times$ fails

$\Rightarrow$ diff $\Rightarrow y_p = \frac{x\cos 2x}{2D} \Rightarrow$ put $D^2 = -(a^2)$

$= \frac{x}{2}\int \frac{\cos 2x}{D}$

$y_p = \frac{x}{2} \cdot \frac{\sin 2x}{2}$

$\boxed{y_p = \frac{x\sin 2x}{4}}$

* Find P.I. of $(D^2+1)y = \sin x \sin 2x$

$= \frac{1}{2}(2\sin 2x \sin x$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

$(D^2+1)y = \frac{1}{2}[\cos(x) - \cos 3x]$

$(D^2+1)y = \frac{\cos x}{2}$

$y_P = \frac{\cos x}{2(D^2+1)}$

put $\Rightarrow D^2 = -1$

$y_P = \frac{\cos x}{2(0)}$

fails.

$y_P = \frac{x\cos x}{2(2D)}$

$\boxed{y_{P_1} = \frac{x \sin x}{4}}$

$(D^2+1)y = -\frac{\cos 3x}{2}$

$y_P = \frac{-\cos 3x}{2(D^2+1)}$

$D^2 = -9$.

$y_P = -\frac{\cos 3x}{2(-8)}$.

$\boxed{y_{P_2} = \frac{\cos 3x}{16}}$

$\boxed{y_P = \frac{x\sin x}{4} + \frac{\cos 3x}{16}}$

28/06/2022

* Solve $(D^2+2D+2)y = e^{-x} + \sin 2x$.

C.F:

$(D^2+2D+2)y = 0$

$m^2+2m+2 = 0$

$m = \frac{-2 \pm \sqrt{4-8}}{2}$

$= \frac{-2 \pm 2i}{2}$

$= -1 \pm i$

$y_c = e^{-x}(C_1 \cos x + C_2 \sin x)$

y_{P_1}:

$(D^2+2D+2)y = e^{-x}$

$y_P = \frac{e^{-x}}{D^2+2D+2}$

put $D = -1$

$y_{P_1} = \frac{e^{-x}}{1+2(-1)+2} = e^{-x}$.

$\boxed{y_{P_1} = e^{-x}}$

$$y_{P_2} = \frac{1}{D^2+2D+2} \sin 2x$$

put $D^2 = -4$

$$y_{P_2} = \frac{\sin 2x}{-4+2D+2} = \frac{\sin 2x}{2D-2} = \frac{(\sin 2x)(D+1)}{2(D^2-1)}$$

put $D^2 = -4$

$$y_{P_2} = \frac{\sin 2x(D+1)}{2(-4-1)} = \frac{-1}{10} \sin 2x (D+1)$$

$$\boxed{y_{P_2} = \frac{-1}{10}(2\cos 2x + \sin 2x)}$$

$$\boxed{y = e^{-x}(c_1 \cos x + c_2 \sin x) + e^{-x} \frac{-1}{10}(2\cos 2x + \sin 2x)}$$

* $y'' + 4y' + 20y = 23\sin t - 15\cos t$, $y(0) = 0$, $y'(0) = -1$

Operator Form:

$(D^2 + 4D + 20)y = 23\sin t - 15\cos t$

A.E: $(m^2 + 4m + 20) = 0$

C.F: $m = \frac{-4 \pm \sqrt{16-80}}{2} = \frac{-4 \pm 8i}{2} = -2 \pm 4i$

$y_c = e^{-2x}(c_1 \cos 4x + c_2 \sin 4x)$

~~$x = 0;\ y = 0$~~

~~$y_c = e^{(0)}[c_1(1) + c_2(0)] = 0$~~

~~$c_1 = 0$~~

~~$y'_c = e^{-2x}(-4c_1 \sin 4x + 4c_2 \cos 4x)$~~

~~$-2e^{-2x}[c_1 \cos 4x + c_2 \sin 4x]$~~

~~$x = 0;\ y'_c = -1$~~

~~$-1 = e^{-2(0)}[0 + 4c_2] - 2e^{(0)}[c_1]$~~

~~$-1 = 4c_2 - 0$~~

~~$4c_2 = -1$~~

~~$c_2 = \frac{-1}{4}$~~

$$\boxed{y_c = e^{-2x}\left(\frac{-1}{4} \sin 4x\right)}$$

y_{P_1}:

$(D^2 + 4D + 20)y_{P_1} = 23\sin t$

$$y_{P_1} = \frac{23\sin t}{D^2+4D+20}$$

put $D^2 = -1$

$$y_{P_1} = \frac{23\sin t}{-1+4D+20} = \frac{23\sin t}{4D+19}$$

$$y_{p_1} = \frac{23\sin t\,(4D-19)}{16D^2-361} \qquad D^2=-1$$

$$= \frac{23\sin t\,(4D-19)}{-16-361} \qquad 361$$

$$= \frac{92\cos t - 437}{-377} = \frac{437 - 92\cos t}{377}$$

sint

y_{p_2}:

$$(D^2+4D+20)\,y_{p_2} = -15\cos t$$

$$y_{p_2} = \frac{-15\cos t}{D^2+4D+20} \qquad D^2=-1$$

$$= \frac{-15\cos t}{-1+4D+20}$$

$$= \frac{-15\cos t}{4D+19} \times \frac{4D-19}{4D-19}$$

19 × 15 = 95, 19, 285

$$y_{p_2} = \frac{+\cancel{6}0\sin t + 285\cos t}{16D^2-361} \qquad D^2=-1$$

$$= \frac{60\sin t + 285\cos t}{-377}$$

$$y = e^{-2x}\left[c_1\cos 4\cancel{x} + c_2\sin 4\cancel{x}\right] + \frac{437 - 92\cos t}{377} - \frac{60\sin t + 285\cos t}{377}$$

sint

$$y = e^{-2\cancel{t}}\left[c_1\cos 4\cancel{t} + c_2\sin 4\cancel{t}\right] + \cancel{\frac{722 - 92\cos t - 60\sin t}{377}}$$

$y' =$

$$y = e^{-2t}\left[c_1\cos 4t + c_2\sin 4t\right] + \frac{\cancel{3}77\sin t - 3\cancel{9}7\cos t}{377}$$

118, 285, 92, 193, 285, 92, 77

$$\boxed{y = e^{-2t}\left[c_1\cos 4t + c_2\sin 4t\right] + \sin t - \cos t}$$

$y=0 \quad t=0$

$$y = 1\left[c_1 + 0\right] + 0 - 1$$

$$0 = c_1 - 1$$

$$c_1 = 1$$

$$y' = e^{-2t}\left[-4\sin 4t + 4c_2\cos 4t\right] - 2e^{-2t}\left[\cos 4t + c_2\sin 4t\right] + \cos t + \sin t$$

$\cancel{t}=0 \quad y=-1$

$$-1 = 1\left[0 + 4c_2\right] - 2\left[1+0\right] + 1$$

$-1 = 4C_2 - 2 + 1$

$\frac{h}{4} = C_2 \quad (C_2 = 0)$

$y = e^{-2t}\left[C_1\cos 4t + \frac{1}{4}\cancel{\sin 4t}\right] + \sin t - \cos t$

$*(D^3+1)\, y = \cos(2x-1)$

$m^3 + 1 = 0.$

$(m+1)(m^2 - m + 1) = 0$

$m = -1\ ;\ \frac{-1 \pm \sqrt{3}i}{2}.$

C.F:

$$\boxed{y_c = C_1 e^{-x} + e^{-x/2}\left[C_1\cos\frac{\sqrt{3}}{2}x + C_2\sin\frac{\sqrt{3}x}{2}\right]}$$

$y_p = \frac{1}{D^3+1}\cos(2x-1).$

$D^2 = -4$

$= \frac{\cos(2x-1)}{-4D+1}$

$= \frac{[\cos(2x-1)](1+4D)}{1-16D^2} \quad D^2 = -4$

$= \frac{\cos(2x-1) + 4\times 2(-\sin(2x-1))}{1+64}$

$$\boxed{y_p = \frac{\cos(2x-1) - 8\sin(2x-1)}{65}}$$

$y = C_1 e^{-x} + e^{-x/2}\left[C_1\cos\frac{\sqrt{3}}{2}x + C_2\sin\frac{\sqrt{3}}{2}x\right]$

$+ \frac{1}{65}\left[\cos(2x-1) - 8\sin(2x-1)\right]$

$*\ f(D)\,y = Q(x)$

$Q(x) = x^k$; k is a +ve integer.

P.I. $y_p = \frac{1}{f(D)} Q(x).$

$= \frac{1}{1 \pm \phi(D)} Q(x).$

$= [1 + \phi(D)]^{-1} Q(x)$

Using binomial expansion expand the function with $Q(x)$.

$\Rightarrow (1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$

$(1+x)^{-2} = 1 - 2x + 3x^2 + \cdots$

$(1-x)^{-2} = 1 + 2x + 3x^2 + \cdots$

$(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \cdots$

$(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \cdots$

* Solve $(D^2+D+1)y = x^3$

C.F

$m^2+m+1=0$

$m = \frac{-1 \pm \sqrt{3}i}{2}$

$y_c = e^{-x/2}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}x}{2}\right]$

$y_p = \frac{x^3}{D^2+D+1}$

$y_p = \frac{x^3}{1+(D+D^2)}$

$D(x^3) = 3x^2$
$D^2(x^3) = 6x$
$D^3(x^3) = 6$

$y_p = (1+(D+D^2))^{-1} x^3$

$y_p = (1 - D - D^2 + (D+D^2)^2 - (D+D^2)^3 + \cdots) x^3$

$= (1 - D - D^2 + D^2 + 2D^3 - D^3) x^3$

$y_p = (1 + D^3 - D) x^3$

$\boxed{y_p = x^3 + 6 - 3x^2}$

$\boxed{y = y_c + y_p}$

* $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2+2x+4$

Operator form: $(D^2+D)y = x^2+2x+4$

A.E. $m^2+m=0$

$\boxed{m=0,-1}$

$\boxed{y_c = c_1 + c_2 e^{-x}}$

$y_p = \frac{1}{D^2+D}(x^2+2x+4)$

$y_p = \frac{1}{D(1+D)}(x^2+2x+4)$

$= \frac{1}{D}(1+D)^{-1}(x^2+2x+4)$

$D(x^2+2x+4) = 2x+2$

$D^2(x^2+2x+4) = 2.$

$= \frac{1}{D}(1+D)^{-1}(x^2+2x+4)$

$= \frac{1}{D}(1-D+D^2)(x^2+2x+4)$

$= \left(\frac{1}{D} - 1 + D\right)(x^2+2x+4)$

$= \frac{x^3}{3} + \cancel{x^2} + 4x$
$- \cancel{x^2} - \cancel{2x} - 4$
$+ \cancel{2x} + 2)$

$\boxed{y_p = \frac{x^3}{3} + 4x - 2}$

$\boxed{y = c_1 + c_2 e^{-x} + \frac{x^3}{3} + 4x - 2}$

* Solve $(D^3+2D^2+D)y = e^{2x} + x^2 + x + \sin x.$

C.F:

$m^3+2m^2+m=0$

$m(m^2+2m+1)=0$

$m = 0, -1, -1$

$\boxed{y_c = c_1 + (c_2 x + c_3) e^{-x}}$

y_{P_1}:

$(D^3+2D^2+D)y = e^{2x}$

$y = \dfrac{e^{2x}}{D^3+2D^2+D}$ put $D=2$

$= \dfrac{e^{2x}}{8+8+2} = \dfrac{e^{2x}}{18}$

$y_{P_1} = \dfrac{e^{2x}}{18}$

y_{P_2}

$(D^3+2D^2+D)y = \sin 2x$

$y_{P_2} = \dfrac{\sin 2x}{D^3+2D^2+D}$ $D^2=-4$

$= \dfrac{\sin 2x}{-4D-8+D} = \dfrac{\sin 2x}{-3D-8}$

$D^2=-4$ $= \dfrac{-\sin 2x\,(3D-8)}{9D^2-64}$

$= \dfrac{-6\cos 2x + 8\sin 2x}{-100}$

$= \dfrac{-8\sin 2x + 6\cos 2x}{100}$

$\boxed{y_{P_2} = \dfrac{-4\sin 2x + 3\cos 2x}{50}}$

y_{P_3}:

$(D^3+2D^2+D)y = x^2+x$

$y_{P_3} = \dfrac{x^2+x}{D(D^2+2D+1)}$

$D(x^2+x) = 2x+1$

$D^2(x^2+x) = 2$

$= \dfrac{1}{D}\left(1+(D^2+2D)\right)^{-1}(x^2+x)$

$= \dfrac{1}{D}\left[1-(D^2+2D)+(D^2+2D)^2 + \cdots\right]^{-1}$

$y_{P_3} = \dfrac{1}{D}\left[1-D^2-2D+4D^2+\cdots\right](x^2+x)$

$= \dfrac{1}{D}\left[1-2D+3D^2\right](x^2+x)$

$= \left(\dfrac{1}{D} - 2 + 3D\right)(x^2+x)$

$= \dfrac{x^3}{3} + \dfrac{x^2}{2} - 2x^2 - 2x + 6x + 3$

$\boxed{y_{P_3} = \dfrac{x^3}{3} - \dfrac{3x^2}{2} + 4x + 3}$

$y = c_1 + (c_2x + c_3)e^{-x} + \dfrac{e^{2x}}{18}$

$+ \dfrac{x^3}{3} - \dfrac{3x^2}{2} + 4x + 3.$

$+ \left(\dfrac{-4\sin 2x + 3\cos 2x}{50}\right)$

* $\boxed{f(D)\, y = Q(x)}$ TYPE: 4

$Q(x) = e^{ax} V(x)$

$\sin bx \quad \cos bx \quad x^n$

* $\boxed{y = \frac{1}{f(D)} e^{ax} \sin bx}$

put $D = D+a$

$y = \frac{e^{ax} \sin bx}{\phi(D)}$

put $D^2 = -b^2$

$= e^{ax}\left[\frac{\sin bx}{\phi(D)}\right]$

$\Rightarrow$ follow type: 2

* $y = \frac{1}{f(D)} e^{ax} \cdot x^k$

put $D = D+a$

$y = e^{ax}\left[\frac{x^k}{\phi(D)}\right]$

follow type: 3

* $(D^4-1)y = e^x \cos x$

$y = \frac{e^x \cos x}{(D^4-1)}$ put $\Rightarrow D = D+1$

$y = e^x\left[\frac{\cos x}{(D+1)^4 - 1}\right]$

$= e^x\left[\frac{\cos x}{4D^3+6D^2+4D+D^4}\right]$ put $D^2 = -1$

$= \cancel{e^x} \frac{\cos x}{\cancel{-4+6-4+1}}$

$= e^x \frac{\cos x}{-4D-6+4D+1}$

$\boxed{y_p = \frac{-e^x \cos x}{5}}$

* $(D^3-3D^2+3D-1)y = x^2 e^x$ $\quad y_c = (C_1x^2 + C_2x + C_3)e^x$

$y = \frac{(1)\, e^x x^2}{(D^3-3D^2+3D-1)}$

put $D = D+1$

$= \frac{x^2 e^x}{D^3+\cancel{1}+\cancel{3D^2}+\cancel{3D}-3(\cancel{D^2}+\cancel{2D}+1)+\cancel{3D}+3-\cancel{1}}$

$= \frac{x^2 e^x}{D^3}$

$= e^x\left[\frac{1}{D(D^2)} x^2\right] = \cancel{\frac{e^x x^3}{D\,3}}$

$\int x^2 = \frac{x^3}{3}$

$\int \frac{x^3}{3} = \frac{x^4}{12}$

$\int \frac{x^4}{12} = \frac{x^5}{60}$

$$\boxed{y_p = \frac{e^{x} x^5}{60}}$$

* $(D^3 - 4D^2 - D + 4)y = e^{3x}\cos 2x$

$$y = \frac{e^{3x}\cos 2x}{D^3 - 4D^2 - D + 4}$$

put $D = D+3$

$$y = \frac{e^{3x}\cos 2x}{(D+3)^3 - 4(D+3)^2 - D - 3 + 4}$$

$$= \frac{e^{3x}\cos 2x}{D^3 + 27 + 9D^2 + 27D - 4D^2 - 36 - 24D - D + 1}$$

$$= \frac{e^{3x}\cos 2x}{D^3 + 5D^2 + 2D - 8}$$

put $D^2 = -4$

$$y = e^{3x}\left[\frac{\cos 2x}{-4D - 20 + 2D - 8}\right]$$

$$= e^{3x}\frac{\cos 2x}{-2D - 28}$$

$$= \frac{-e^{3x}}{2}\frac{\cos 2x}{D+14} \times \frac{D-14}{D-14}$$

$$= \frac{-e^{3x}}{2}\frac{\cos 2x}{D^2 - 196}(D-14)$$

$$= \frac{e^{3x}}{200}(-2\sin 2x + 14\cos 2x)$$

$$\boxed{y_p = \frac{+e^{3x}}{100}(\sin 2x + 7\cos 2x)}$$

$m^3 - 4m^2 - m + 4 = 0$

$m = 1$

$$m-1 \overline{)\, m^3 - 4m^2 - m + 4 \,}\ (m^2 - 3m - 4$$

$m^3 - m^2$

$-3m^2 - m + 4$

$-3m^2 + 3m$

$-4m + 4$

$-4m + 4$

0

$m^2 - 4m + m - 4 = 0$

$(m-4)(m+1) = 0$

$$\boxed{m = 1, -1, 4}$$

$$\boxed{y_c = c_1 e^{x} + c_2 e^{-x} + c_3 e^{4x}}$$

* **TYPE : 5**

$Q(x) = x\,V(x)$

$V(x)$: $\sin ax$, $\cos ax$

$$y_p = \left[x - \frac{f'(D)}{f(D)}\right] V(x)$$

Suppose $V(x) = \sin ax$

$$y_p = \left[x - \frac{f'(D)}{f(D)}\right]\sin ax$$

first simplify the 'D' operations and simplify the particular integral.

* $(D^2+2D+1)\,y = x\cos x$

$$y_p = \left(x - \frac{2D+2}{D^2+2D+1}\right)\cos x$$

$$= \left[x - \frac{2(D+1)}{(D+1)^2}\right]\cos x$$

$$= \left[x - \frac{2}{D+1}\right]\cos x$$

$$= x\cos x - \frac{2}{(D+1)}\cos x$$

$$= x\cos x - \frac{2(D-1)\cos x}{(D+1)(D-1)}$$

$$= x\cos x - \frac{2(-\sin x - \cos x)}{D^2-1} \qquad D^2 = -1$$

$$\boxed{y_p = x\cos x - (\sin x + \cos x)}$$

30/06/2022

* Solve $(D^2+1)x = t\cos 2t$; given that $x = 0$; $\frac{dx}{dt} = 0$ at $t = 0$.

$x(0) = 0 \qquad x'(0) = 0$

C.F:

$(D^2+1)x = 0$

$m^2+1 = 0$

$m^2 = -1$

$m = \pm i$

$x_c = C_1\cos x + C_2\sin x$

P.I:

$$x_p = \left(t - \frac{2D}{D^2+1}\right)\frac{\cos 2t}{D^2+1} \qquad D^2 = -4$$

$$= \left(t - \frac{2D}{D^2+1}\right)\frac{\cos 2t}{-3}$$

$$= \frac{-t\cos 2t}{3} + \frac{2D\cos 2t}{3(D^2+1)}$$

$$= -\frac{t\cos 2t}{3} + \frac{2}{3}\,\frac{D\cos 2t}{-3}$$

$$x_p = -\frac{t\cos 2t}{3} - \frac{2}{9}\,2(-\sin 2t)$$

$$= -\frac{t\cos 2t}{3} + \frac{4}{9}\sin 2t$$

$x = x_c + x_p$

$x = C_1\cos t + C_2\sin t - \frac{t\cos 2t}{3} + \frac{4}{9}\sin 2t$

$x = 0;\ t = 0$

$0 = C_1 + 0 - 0 + 0$

$C_1 = 0$

$x' = -C_1\sin t + C_2\cos t - \frac{1}{3}(t(-2\sin 2t) + \cos 2t) + \frac{4}{9}\cdot 2\cos 2t$

$= -C_1\sin t + C_2\cos t + \frac{2t\sin 2t}{3} - \frac{1}{3}\cos 2t + \frac{8}{9}\cos 2t$

$t = 0;\ x' = 0$

$0 = -C_1(0) + C_2 + 0 - \frac{1}{3} + \frac{8}{9}$

$C_2 = \frac{1}{3} - \frac{8}{9}$

$= \frac{-5}{9}$

$x = \frac{-5}{9}\sin t - \frac{t\cos 2t}{3} + \frac{4}{9}\sin 2t$

* $(D^2+2D+1)\,y = x\cos x$

$y_c = (C_1 x + C_2)e^{-x}$

$y_p = \left[x - \frac{2D+2}{(D+1)^2}\right]\frac{\cos x}{(D+1)^2}$

$= \left[x - \frac{2}{D+1}\right]\frac{\cos x}{\cancel{D^2}-1+2D+1}$

$= \frac{x\cos x}{2D} - \frac{2\cos x}{(D+1)D}$

$= \left[x - \frac{2}{D+1}\right]\frac{\sin x}{2}$

$= \frac{x\sin x}{2} - \frac{\sin x\,(D-1)}{D^2-1}$ $\quad D^2 = -1$

$= \frac{x\sin x}{2} + \frac{\sin x\,(D-1)}{2}$

$= \frac{x\sin x}{2} + \frac{\cos x - \sin x}{2}$

$y = y_c + y_p$

$$\boxed{y = (C_1 x + C_2)e^{-x} + \frac{x\sin x}{2} + \frac{\cos x - \sin x}{2}}$$

* $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x \sin x$

Operator form: $(D^2+3D+2)y = xe^x \sin x$.

$\underline{\text{A.E}}$: $m^2+3m+2=0$

$m=-1,-2$.

$C.F = C_1e^{-x} + C_2e^{-2x}$

$\underline{\text{P.I}}$:

$$y = \frac{xe^x \sin x}{D^2+3D+2}$$

$$= \frac{e^x(x\sin x)}{D^2+3D+2} \quad \text{put } D = D+1$$

$$= \frac{xe^x \sin x}{D^2+1+2D+3D+3+2}$$

$$= e^x\left(\frac{x\sin x}{D^2+5D+6}\right)$$

$$= e^x\left[x - \frac{2D+5}{D^2+5D+6}\right]\frac{\sin x}{D^2+5D+6}$$

$$= e^x\left[x - \frac{2D+5}{D^2+5D+6}\right]\frac{\sin x}{5(D+1)} \quad D^2=-1$$

$$= e^x\left[x - \frac{2D+5}{(D+2)(D+3)}\right]\frac{\sin x\,(D-1)}{5(D+1)(D-1)}$$

$$= e^x\left[x - \frac{2D+5}{(D+2)(D+3)}\right]\frac{(D-1)\sin x}{5(D^2-1)} \quad D^2=-1$$

$$= e^x\left[x - \frac{2D+5}{(D+2)(D+3)}\right]\frac{(D-1)\sin x}{-10}$$

$D(\cos x) = -\sin x$
$D^2(\cos x) = -\cos x$
$D^3(-\cos x) = \sin x$

$$= \frac{-e^x}{10}\left[x - \frac{2D+5}{(D+2)(D+3)}\right]\cos x - \sin x$$

$D(\sin x) = \cos x$
$D^2(\sin x) = -\sin x$
$D^3(\sin x) = -\cos x$

$$= \frac{-e^x}{10}\left[x\cos x - x\sin x - \frac{2D+5(D-2)(D-3)}{(D^2-2)(D^2-3)}\cos x + \frac{(2D+5)(D^2-5D+6)}{(D^2-2)(D^2-3)}\sin x\right.$$

$$= \frac{-e^x}{10}\left[x\cos x - x\sin x - \frac{(2D+5)(D^2-5D+6)\cos x}{-3x-4} + \frac{(2D+5)(D^2-5D+6)\sin x}{-12}\right.$$

$$= \frac{-e^x}{10}\left[x\cos x - x\sin x + \frac{1}{12}\left[2D^3-5D^2-13D+30\right](\cos x - \sin x)\right.$$

$$= \frac{-e^x}{10}\left[x\cos x - x\sin x + \frac{1}{12}\left[2\sin x + 5\cos x + 13\sin x + 30 - (-2\cos x + 5\sin x - 13\cos x + 30)\right]\right.$$

$$=\frac{-e^x}{10}\left[x\sin x - x\cos x + \frac{1}{12}\left[2(\sin x+\cos x) + 5(\cos x - \sin x) + 13(\cos x+\sin x)\right]\right.$$

$$=\frac{-e^x}{10}\left[x\sin x - x\cos x\right] + \frac{-e^x}{120}\left[10\sin x + 20\cos x\right]$$

$$=-\frac{e^x}{10}\left[x\sin x - x\cos x\right) - \frac{e^x}{12}\left[\sin x + 2\cos x\right]$$

$$=\frac{e^x}{10}\left[x - \frac{2D+5}{D^2+5D+6}\right](\sin x - \cos x)$$

$$=\frac{e^x}{10}\left\{\left[x(\sin x - \cos x)\right] - \frac{2D+5}{5(D+1)}(\sin x - \cos x)\right\}$$

$$\frac{-e^x}{10}\left[x(\sin x - \cos x)\right] - \frac{(2D+5)(D-1)}{5(-2)}(\sin x - \cos x)$$

$$=\frac{e^x}{10}(x\sin x - x\cos x) + \frac{e^x}{100}\left[2(-\sin x) + 2\cos x + 3\cos x + 3\sin x - 5\sin x + 5\cos x\right]$$

$$y = c_1e^{-x} + c_2e^{-2x} + \frac{xe^x}{10}(\sin x - \cos x) + \frac{e^x}{100}(-4\sin x + 10\cos x)$$

$$*(D^2-4D+4)y = x^2\sin x + e^{2x} + 3$$

$$y_c \Rightarrow m^2 - 4m + 4 = 0$$

$$m = 2, 2$$

$$y_c = (c_1x + c_2)e^{2x}$$

① $$y_p = \frac{x^2\sin x}{D^2-4D+4} = \frac{x^2\sin x}{(D-2)^2} \qquad \sin x = \text{I.P. of } e^{ix}$$

$$= \text{I.P. of } \frac{1}{(D-2)^2} x^2e^{ix} \qquad \boxed{D = D+i}$$

$$= \text{I.P. of } \frac{x^2e^{ix}}{(D+i-2)^2}$$

$$= \text{I.F. } e^{ix}\frac{1}{4}\left[\frac{x^2e^{ix}}{\left[1+\left(\frac{D+i}{2}\right)\right]^2}\right]x^2$$

$$= \text{I.P. of } \frac{e^{ix}}{4}\left[1 + 2\left(\frac{D+i}{2}\right) + 3\left(\frac{D+i}{2}\right)^2 + 4\left(\frac{D+i}{2}\right)^3 \cdots\right]x^2$$

* Direct Method to find PI:

If Q is a function of x and α is a constant; then particular value of $\frac{1}{D-\alpha}Q$ is equal to: $\boxed{e^{\alpha x}\int Q(x)e^{-\alpha x}dx}$

i.e. PI of $\frac{1}{D-\alpha}Q = e^{\alpha x}\int Qe^{-\alpha x}dx$

PI of $\frac{1}{D+\alpha}Q = e^{-\alpha x}\int Qe^{\alpha x}dx$

If $\frac{1}{D-\beta}, \frac{1}{D-\alpha}$ are 2 inverse operators, then we define:

$$\boxed{\frac{1}{(D-\beta)(D-\alpha)}Q = \frac{1}{(D-\beta)}\left[\frac{1}{D-\alpha}Q\right]}$$

$\alpha, \beta \to$ constants ; Q is a function of x

i.e. $\frac{1}{(D-\beta)(D-\alpha)}Q = \frac{1}{D-\beta}\left[e^{\alpha x}\int Qe^{-\alpha x}dx\right]$

$$= e^{\beta x}\int e^{-\beta x}\left\{e^{\alpha x}\int Qe^{-\alpha x}dx\right\}dx$$

Ex:

1) $\frac{1}{D}x^2 = \frac{x^3}{3}$ 2) $\frac{1}{D^3}(\cos x) = -\sin x$

3) $\frac{1}{D+1}(x) = e^{-x}\int xe^{x}dx$

$= e^{-x}(xe^{x} - e^{x}) = \underline{x-1}$

4) $\frac{1}{(D-2)(D-3)}e^{2x} = \frac{1}{D-2}\left(\frac{1}{D-3}e^{2x}\right)$

$= \frac{1}{D-3}e^{2x} = e^{3x}\int e^{2x}\cdot e^{-3x}dx = e^{3x}\cdot -e^{-x} = -e^{2x}$

$= \frac{1}{D-2}(-e^{2x}) = -e^{+2x}\int e^{2x}\cdot e^{-2x}dx = -xe^{2x}$

* Rules for finding PI in special cases:

1) PI of $F(D)y = \phi(x)$ when $\phi(x) = e^{ax}$ ($a \to$ const)

Case(i): $f(D)y = e^{ax}$ then

$$y_P = \frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax} \quad (f(a) \neq 0)$$

∵ Multiplying by $f(D)$ on both sides

$$e^{ax} = \frac{f(D)}{f(a)}e^{ax}$$

$\therefore \frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$ if $f(a) \neq 0$

Case(ii): Let $f(a) = 0$ then $(D-a)$ is a factor of $f(D)$ if a is a root repeated k times for $f(a)=0$ then $f(D) = (D-a)^k\phi(D)$ where $\phi(a) \neq 0$ then we have $\frac{1}{f(D)}e^{ax} = \frac{1}{(D-a)^k}\cdot\frac{1}{\phi(D)}e^{ax} = \frac{1}{\phi(a)}\cdot\frac{e^{ax}}{(D-a)^k}$

$$= \frac{1}{\phi(a)}e^{ax}\cdot\frac{x^k}{k!}$$

The reason is as follows.

$$* \quad \frac{1}{(D-a)} e^{ax} = xe^{ax} \; ; \left[(D-a)(xe^{ax})\right] = e^{ax}$$

$$* \quad \frac{1}{(D-a)^2} e^{ax} = \frac{x^2}{2!} e^{ax} \; ; \left[(D-a)^2 \left(\frac{x^2 e^{ax}}{2!}\right)\right] = e^{ax}$$

$$* \quad \frac{1}{(D-a)^k} e^{ax} = \frac{x^k}{k!} e^{ax} \; ; \left[(D-a)^k \; \frac{x^k e^{ax}}{k!}\right] = e^{ax}$$

$$* \quad (D^2+4)\, y = \tan 2x$$

$$y = \frac{1}{(D^2+4)} \tan 2x$$

$$= \frac{1}{(D+2i)(D-2i)} \tan 2x$$

Resolving into partial fraction

$$\frac{1}{(D-2i)(D+2i)} = \frac{A}{D-2i} + \frac{B}{D+2i}$$

$$\frac{\tan 2x}{(D-2i)(D+2i)} = \frac{1}{4i}\left[\frac{1}{D-2i} - \frac{1}{D+2i}\right] \tan 2x \quad \text{—(1)}$$

Consider; $\frac{1}{D-2i} \tan 2x = e^{i(2x)} \int \tan 2x \cdot e^{-2ix}\, dx$

$$= e^{i(2x)} \int \left[\cos 2x - i \sin 2x\right] \tan 2x \, dx$$

$$= e^{i(2x)} \left[\int \sin 2x \, dx - \int i \sin 2x \tan 2x \, dx\right]$$

$$= e^{i(2x)} \left[\int \sin 2x \, dx - i \int (\sec 2x - \cos 2x)\, dx\right]$$

$$= e^{i(2x)} \left[-\frac{\cos 2x}{2} - i \log\left(\frac{\sec x + \tan x}{2}\right) + \frac{i \sin 2x}{2}\right]$$

* METHOD OF VARIATION OF PARAMETERS (MVP's)

$$(D^2+4)\,y = \tan 2x.$$

C.F:

$m^2+4=0$

$$\boxed{m = \pm 2i}$$

$y = C_1\cos 2x + C_2\sin 2x.$

$y = C_1(u(x)) + C_2(v(x))$

$u(x) = \cos 2x$

$u'(x) = -2\sin 2x$

$v(x) = \sin 2x$

$v'(x) = 2\cos 2x.$

$$W(x) = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$W(x) = 2 \neq 0.$

u, v are L.I.

$$A(x) = -\int \frac{vR}{W(x)}\,dx$$

$$= -\int \frac{\sin 2x \tan 2x}{2}$$

$$B(x) = \int \frac{uR}{W(x)}\,dx.$$

$$= \int \frac{\cos 2x \tan 2x}{2}\,dx$$

$$\boxed{y_p = A(x)\,u(x) + B(x)\cdot v(x)}$$

$$\boxed{y = C_1\cos 2x + C_2\sin 2x + y_p}$$

5/07/2022

Working procedure to find solution of D.E. by using M.V.P:

*Step 1:

Find C.F (y_c)
- → Real & equal : $(c_1 x + c_2) e^{mx}$
- → Real & distinct : $c_1 e^{m_1 x} + c_2 e^{m_2 x}$
- → Imaginary : $e^{ax}[c_1 \cos bx + c_2 \sin bx]$

Consider coeff of $c_1 = u(x)$.
$c_2 = v(x)$.

General solution: $y = c_1 u(x) + c_2 v(x)$
(C.F)

known functions.

where $u(x) = \_\_\_$
$v(x) = \_\_\_$
$u'(x) = \_\_\_$
$v'(x) = \_\_\_$

Step:2: Consider Wronskain function.

$$W(x) = \begin{vmatrix} u(x) & v(x) \\ u'(x) & v'(x) \end{vmatrix}$$

$$W(x) = u(x) \cdot v'(x) - v(x) \cdot u'(x) \neq 0.$$

$u(x)$ & $v(x)$ are linearly independent functions

Step3: Find P.I

known functions

$y_p = A(x) \cdot u(x) + B(x) \cdot v(x)$.

unknown functions

90% imaginary roots
5% equal
5% distinct

Step 4: $$\boxed{A(x) = -\int \frac{V(x)\,R(x)}{W(x)}\,dx}$$

$$R(x) = \frac{d^2y}{dx^2} + \frac{dy}{dx}(P(x)) + Q(x)\cdot y$$

$$\boxed{B(x) = \int \frac{u(x)\,R(x)}{W(x)}\,dx}$$

* Complete (or) General Solution :

$$y = y_c + y_p$$

$$\boxed{y = C_1 u(x) + C_2 V(x) + A(x)u(x) + B(x)V(x)}$$

* Using method of variation of parameters; find the solution of the following.

1) $\frac{d^2y}{dx^2} + y = x$

$\cancel{(D^2+1)}$ Comparing with

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)\,y = R(x)$$

$P(x) = 0$, $Q(x) = 1$, $R(x) = x$

$$\boxed{R(x) = x}$$

C.F $(D^2+1)y = 0$

$m^2+1=0$

$m = \pm i$

$$\boxed{C.F = y_c = C_1\cos x + C_2\sin x}$$

Coeff of C_1 $u(x) = \cos x$

C_2 $v(x) = \sin x$

$u'(x) = -\sin x$ $\quad v'(x) = \cos x$

$W(x) = \cos^2 x + \sin^2 x = 1 \neq 0$

P.I :

$y_P = A(x)\cdot\cos x + B(x)\sin x$ $\quad A(x) = -\int \frac{V(x)\cdot R(x)}{W(x)}dx$

$A(x) = -\int \frac{\sin x\cdot x}{1}dx$ $\quad$ $\boxed{ILATE}$

$= -[x(-\cos x) + \sin x]$ $\quad \int u\cdot v = u\int v - \int u\cdot\frac{d}{d}v$

$= x\cos x - \sin x$ $\quad \int u\cdot v = u\int v - \int u'\int v\,dx$

$B(x) = \int \frac{u(x)\cdot R(x)}{W(x)}dx$

$= \int \frac{\cos x\cdot x}{1}dx$

$= x\sin x + \cos x$

$y_P = (x\cos x - \sin x)\cos x + (x\sin x + \cos x)\sin x$

$= x - \sin x\cos x + \sin x\cos x = x$

$$\boxed{y = y_c + y_P = C_1\cos x + C_2\sin x + x}$$

2) $(D^2-1)y = \frac{2}{1+e^x}$

C.F: $(D^2-1)y = 0$

$m^2-1=0$

$m=\pm 1$

$y_c = c_1 e^x + c_2 e^{-x}$

$u(x) = e^x$

$v(x) = e^{-x}$

$u'(x) = e^x$

$v'(x) = -e^{-x}$

$W(x) = -1-1 = -2 \neq 0$

P.I: $A(x)\cdot u(x) + B(x)\cdot v(x) = y_p$

$A(x) = -\int \frac{v(x)\cdot R(x)}{W(x)}$

$= -\int \frac{e^{-x}}{-\cancel{2}} \cdot \frac{\cancel{2}}{1+e^x}$

$= \int \frac{e^{-x}}{1+e^x} dx$

$e^{-x} = t$

$-e^{-x} dx = dt$

$= \cancel{\int \frac{1}{e^x+e^{2x}} dx}$

$= \int \frac{-dt}{1+1/t} = \int -\frac{t\,dt}{1+t} = -\int \frac{t+1}{t+1} dt - \int \frac{1}{t+1}$

$A(x) = -t + \log(t+1)$

$= -e^{-x} + \log(e^{-x}+1)$

$B(x) = \int \frac{u(x)\cdot R(x)}{W(x)} dx$

$= \int \frac{e^x}{1+e^x} dx = \log(1+e^x)$

$y_p = -e^{-x} + \log(1+e^{-x}) + \log(1+e^x)$

$y = y_c + y_p$

$\boxed{y = c_1 e^x + c_2 e^{-x} - e^{-x} + \log[(1+e^{-x})(1+e^x)]}$

* $y'' - 2y' + y = e^x \log x$
* $y'' + y = \operatorname{cosec} x$
* $y'' + 16y = 32 \sec 2x$

i) $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = e^x \log x$

$y(D^2 - 2D + 1) = e^x \log x$

$m^2 - 2m + 1 = 0$

$\boxed{m = 1, 1}$

$\boxed{y_c = (c_1 x + c_2)e^x}$

coeff of $c_1 = xe^x = u(x)$ $\quad u'(x) = xe^x + e^x$

$c_2 = e^x = v(x)$ $\quad v'(x) = e^x$

$W(x) = xe^{2x} - e^{2x}(x+1)$

$= -e^{2x}$

$y_p = A(x)\cdot u(x) + B(x)\cdot v(x)$

$$B(x) = \int \frac{u(x)\cdot R(x)}{W(x)}\, dx$$

$$= \int \frac{\cos 4x \cdot \cancel{32}^{\,8} \sec 2x}{\cancel{4}}\, dx$$

$$= 8\int \frac{1+\cos^2 2x}{2} \cdot \frac{1}{\cos 2x}\, dx$$

$$= 4\left[\int \{\sec 2x + \cos 2x\}\, dx\right]$$

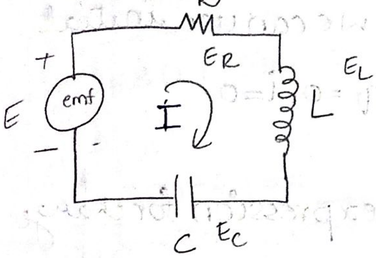
$$= 4\left[\frac{\log(\tan 2x + \sec 2x)}{2} + \frac{\sin 2x}{2}\right]$$

$$= 2\log(\tan 2x + \sec 2x) + 2\sin 2x$$

$$y = c_1 \cos 4x + c_2 \sin 4x + 8\cos 2x + 2\log(\tan 2x + \sec 2x) + 2\sin 2x$$

06/07/2022

* Applications of higher order differential equation



$E_R = IR$

$E_L = L\frac{di}{dt}$

$E_C = \frac{q}{C}$

$i = \frac{dq}{dt}$

$E = E_R + E_L + E_C$

$E = IR + L\frac{di}{dt} + \frac{q}{C}$ — (1) $i = \frac{dq}{dt}$

* LCR CIRCUIT

$$E = \frac{dq}{dt}R + L\frac{d^2q}{dt} + \frac{1}{C}q$$

$$L\frac{d^2q}{dt} + R\frac{dq}{dt} + \frac{1}{C}q = E \quad \text{— (2)}$$

(2) is the linear differential equation with constant coefficients for LCR circuits.

$D = \frac{d}{dt}$

$$LD^2q + RDq + \frac{1}{C}q = E$$

$$\left(LD^2 + RD + \frac{1}{C}\right)q = E \quad \text{— (3)}$$

2nd order LDE with const. coefficients

⇒ Operator form of DE:

∵ L, C, R, are constant for any given circuit

→ The general solution of DE contains 2 arbitrary constants c_1 & c_2. To find the arbitrary constants; we can use initial conditions i.e. $t=0;\ q=0;\ i=0$.

→ Finally we get an expression for charge in terms of 't'.

* $L=2H$ and $R=16\Omega$ and $C=0.02\mu F$ are connected in series with a battery of emf $E=100\sin 3t$. Initially the charge on the capacitor & current in circuit are zero. then find the charge & current for $t>0$.

$E=100\sin 3t \quad L=2H;\ R=16\Omega$

$C=0.02\mu F$

$i=0,\ q=0 \quad at\ t=0$

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=E$$

$$\frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{1}{CL}q=\frac{E}{L}$$

$$\frac{R}{L}=8 \qquad \frac{1}{CL}=\frac{100}{4}=25$$

$$\frac{E}{L}=\frac{100\sin 3t}{2}=50\sin 3t$$

$$\boxed{\frac{d^2q}{dt^2}+\frac{8dq}{dt}+25q=50\sin 3t}$$

Operator form:

$$q(D^2+8D+25)=50\sin 3t$$

C.F:

$$m^2+8m+25=0$$

$$m=\frac{-8\pm\sqrt{64-100}}{2}$$

$$=\frac{-8\pm 6i}{2}$$

$$=-4\pm 3i$$

$$q_c=e^{-4t}(c_1\cos 3t+c_2\sin 3t)$$

P.I.

$$q_p=\frac{50\sin 3t}{D^2+8D+25} \qquad \text{put } D^2=-9$$

$$=\frac{50\sin 3t}{-9+8D+25}$$

$$=\frac{50\sin 3t}{8(D+2)}=\frac{25}{4}\frac{\sin 3t\,(D-2)}{D^2-4}$$

$$=\frac{25}{4}\left(\frac{-1}{13}\right)(3\cos 3t-2\sin 3t)$$

$$q_p=-\frac{25}{52}(3\cos 3t-2\sin 3t)$$

$$q=e^{-4t}(c_1\cos 3t+c_2\sin 3t)-\frac{25}{52}(3\cos 3t-2\sin 3t)$$

$q_c = e^{-50t}[c_1 \cos 218t + c_2 \sin 218t]$

P.I.

$$q = \frac{1}{D^2+100D+50000} \times 2200.$$

put $D=0$

$$q_p = \frac{22\cancel{00}}{500\cancel{00}} = \frac{11}{250}.$$

$q = q_c + q_p$

$$= e^{-50t}[c_1 \cos 218t + c_2 \sin 218t] + \frac{11}{250}.$$

at $t=0$; $q=0$

$$0 = [c_1 + 0] + \frac{11}{250}.$$

$$\boxed{c_1 = -\frac{11}{250}}$$

$$q = e^{-50t}\left[\frac{-11}{250}\cos 218t - \frac{11}{1090}\sin 218t\right] + \frac{11}{250}$$

$$i = \frac{dq}{dt}$$

$$= e^{-50t}(-218c_1 \sin 218t + 218c_2 \cos 218t) - 50e^{-50t}(c_1 \cos 218t + c_2 \sin 218t)$$

$t=0$; $i=0$

$$0 = [0 + 218c_2] - 50[c_1 + 0]$$

$$50c_1 = 218c_2$$

$$c_2 = \frac{\cancel{50}}{218}\left(\frac{-11}{\cancel{250}}\right)$$

$$\boxed{c_2 = \frac{-11}{1090}}$$