# Method of Sections

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## 6.4 The Method of Sections

When we need to find the force in only a few members of a truss, we can analyze the truss using the *method of sections*. It is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium. For example, consider the two truss members shown on the left in Fig. 6–14. If the forces within the members are to be determined, then an imaginary section, indicated by the blue line, can be used to cut each member into two parts and thereby "expose" each internal force as "external" to the free-body diagrams shown on the right. Clearly, it can be seen that equilibrium requires that the member in tension (T) be subjected to a "pull," whereas the member in compression (C) is subjected to a "push."

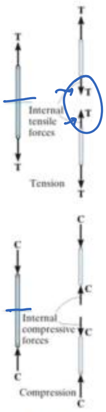
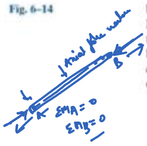


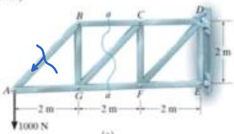

Fig. 6–14



The method of sections can also be used to "cut" or section the members of an entire truss. If the section passes through the truss and the free-body diagram of either of its two parts is drawn, we can then apply the equations of equilibrium to that part to determine the member forces at the "cut section." Since only *three* independent equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$) can be applied to the free-body diagram of any segment, then we should try to select a section that, in general, passes through not more than *three* members in which the forces are unknown. For example, consider the truss in Fig. 6–15*a*. If the forces in members *BC*, *GC*, and *GF* are to be determined, then section *aa* would be appropriate. The free-body diagrams of the two segments are shown in Figs. 6–15*b* and 6–15*c*. Note that the line of action of each member force is specified from the *geometry* of the truss, since the force in a member is along its axis. Also, the member forces acting on one part of the truss are equal but opposite to those acting on the other part — Newton's third law. Members *BC* and *GC* are assumed to be in *tension* since they are subjected to a "pull," whereas *GF* in *compression* since it is subjected to a "push."

The three unknown member forces $\mathbf{F}_{BC}$, $\mathbf{F}_{GC}$, and $\mathbf{F}_{GF}$ can be obtained by applying the three equilibrium equations to the free-body diagram in Fig. 6–15*b*. If, however, the free-body diagram in Fig. 6–15*c* is considered, the three support reactions $\mathbf{D}_x$, $\mathbf{D}_y$ and $\mathbf{E}_x$ will have to be known, because only three equations of equilibrium are available. (This, of course, is done in the usual manner by considering a free-body diagram of the *entire truss*.)




(a)

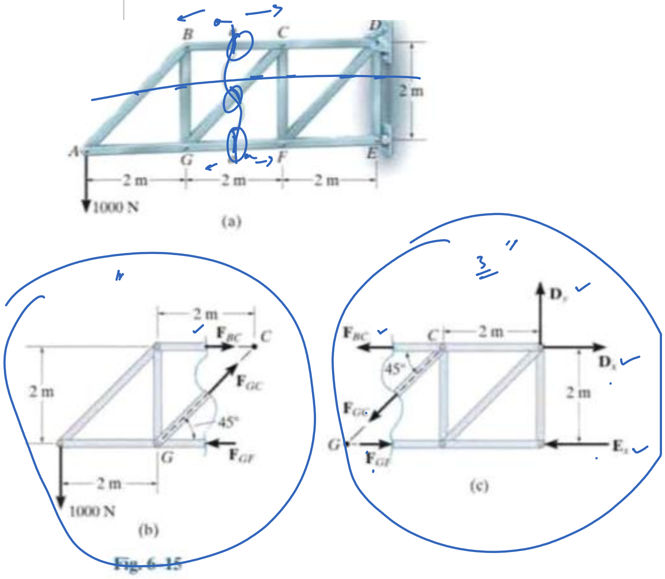



Fig. 6–15

$\Sigma M_z = 0$
$\Sigma F_x = 0$
$\Sigma F_y = 0$

Alternate
$\Sigma M_A = 0$
$\Sigma M = 0$
$\Sigma M = 0$

When applying the equilibrium equations, we should carefully consider ways of writing the equations so as to yield a *direct solution* for each of the unknowns, rather than having to solve simultaneous equations. For example, using the truss segment in Fig. 6–15*b* and summing moments about $C$ would yield a direct solution for $\mathbf{F}_{GF}$ since $\mathbf{F}_{BC}$ and $\mathbf{F}_{GC}$ create zero moment about $C$. Likewise, $\mathbf{F}_{BC}$ can be directly obtained by summing moments about $G$. Finally, $\mathbf{F}_{GC}$ can be found directly from a force summation in the vertical direction since $\mathbf{F}_{GF}$ and $\mathbf{F}_{BC}$ have no vertical components. This ability to *determine directly* the force in a particular truss member is one of the main advantages of using the method of sections.*

As in the method of joints, there are two ways in which we can determine the correct sense of an unknown member force:

- The correct sense of an unknown member force can in many cases be determined "by inspection." For example, $\mathbf{F}_{BC}$ is a tensile force as represented in Fig. 6–15*b* since moment equilibrium about $G$ requires that $\mathbf{F}_{BC}$ create a moment opposite to that of the 1000-N force. Also, $\mathbf{F}_{GC}$ is tensile since its vertical component must balance the 1000-N force which acts downward. In more complicated cases, the sense of an unknown member force may be *assumed*. If the solution yields a *negative* scalar, it indicates that the force's sense is *opposite* to that shown on the free-body diagram.
- *Always assume* that the unknown member forces at the cut section are *tensile* forces, i.e., "pulling" on the member. By doing this, the numerical solution of the equilibrium equations will yield *positive scalars for members in tension and negative scalars for members in compression.*

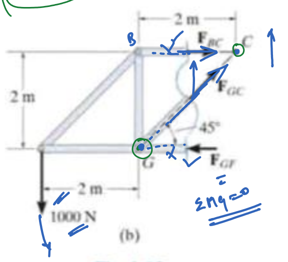
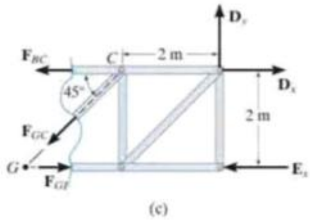



Fig. 6–15




*Notice that if the method of joints were used to determine, say, the force in member $GC$, it would be necessary to analyze joints $A$, $B$, and $G$ in sequence.

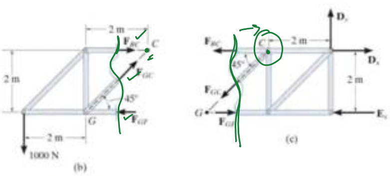



Fig. 6–15

Simple trusses are often used in the construction of large cranes in order to reduce the weight of the boom and tower.

6

## Procedure for Analysis

The forces in the members of a truss may be determined by the method of sections using the following procedure.

**Free-Body Diagram.**

2

- Make a decision on how to "cut" or section the truss through the members where forces are to be determined.

1

- Before isolating the appropriate section, it may first be necessary to determine the truss's support reactions. If this is done then the three equilibrium equations will be available to solve for member forces at the section.
- Draw the free-body diagram of that segment of the sectioned truss which has the least number of forces acting on it.
- Use one of the two methods described above for establishing the sense of the unknown member forces.

**Equations of Equilibrium.**

Strategy

- Moments should be summed about a point that lies at the intersection of the lines of action of two unknown forces, so that the third unknown force can be determined directly from the moment equation.
- If two of the unknown forces are *parallel*, forces may be summed *perpendicular* to the direction of these unknowns to determine *directly* the third unknown force.

## EXAMPLE 6.5

Determine the force in members $GE$, $GC$, and $BC$ of the truss shown in Fig. 6–16$a$. Indicate whether the members are in tension or compression.

### SOLUTION

Section $aa$ in Fig. 6–16$a$ has been chosen since it cuts through the *three* members whose forces are to be determined. In order to use the method of sections, however, it is *first* necessary to determine the external reactions at $A$ or $D$. Why? A free-body diagram of the entire truss is shown in Fig. 6–16$b$. Applying the equations of equilibrium, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 400\text{ N} - A_x = 0 \qquad A_x = 400\text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -1200\text{ N}(8\text{ m}) - 400\text{ N}(3\text{ m}) + D_y(12\text{ m}) = 0$$

$$D_y = 900\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 1200\text{ N} + 900\text{ N} = 0 \qquad A_y = 300\text{ N}$$

**Free-Body Diagram.** For the analysis the free-body diagram of the left portion of the sectioned truss will be used, since it involves the least number of forces, Fig. 6–16$c$.

**Equations of Equilibrium.** Summing moments about point $G$ eliminates $\mathbf{F}_{GE}$ and $\mathbf{F}_{GC}$ and yields a direct solution for $F_{BC}$.

$$\circlearrowleft + \Sigma M_G = 0; \qquad -300\text{ N}(4\text{ m}) - 400\text{ N}(3\text{ m}) + F_{BC}(3\text{ m}) = 0$$

$$F_{BC} = 800\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$

In the same manner, by summing moments about point $C$ we obtain a direct solution for $F_{GE}$.

$$\circlearrowleft + \Sigma M_C = 0; \qquad -300\text{ N}(8\text{ m}) + F_{GE}(3\text{ m}) = 0$$

$$F_{GE} = 800\text{ N} \quad (\text{C}) \qquad \textit{Ans.}$$

Since $\mathbf{F}_{BC}$ and $\mathbf{F}_{GE}$ have no vertical components, summing forces in the $y$ direction directly yields $F_{GC}$, i.e.,

$$+\uparrow \Sigma F_y = 0; \qquad 300\text{ N} - \tfrac{3}{5}F_{GC} = 0$$

$$F_{GC} = 500\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$

**NOTE:** Here it is possible to tell, by inspection, the proper direction for each unknown member force. For example, $\Sigma M_C = 0$ requires $\mathbf{F}_{GE}$ to be *compressive* because it must balance the moment of the 300-N force about $C$.

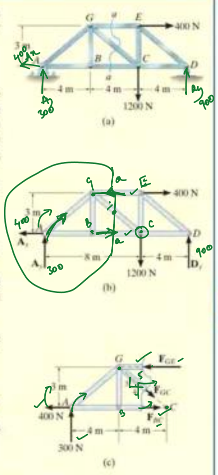



Fig. 6–16

Support reactions

Consider whole truss as single rigid body and apply equilibrium equations

$\Sigma MA = 0$ ↺+ $(400 \times 3) + 1200 \times 8 - Dy \times 12 = 0$

$Dy = 900\text{ N}$

$\Sigma Fx = 0 \rightarrow +$ $+400 - Ax \Rightarrow Ax = 400\text{ N}$

$\Sigma Fy = 0 \uparrow +$ $Ay - 1200 + \cancel{Dy}^{900} = 0 \Rightarrow Ay = 300\text{ N}$

$\Sigma M G = 0$

$\Sigma Mc = 0$ ↺+

$(300 \times 8) - FGE \times 3 = 0 \Rightarrow FGE = 800\text{N (C)}$

$\Sigma M G = 0$ ↺+

$(400 \times 3) + (300 \times 4) - FBC \times 3 = 0$

$FBC = 800\text{ N (T)}$

$\Sigma Fy = 0\ (\uparrow +)$

$300 - FGC\left(\frac{3}{5}\right) = 0$

$FGC = 500\text{N (T)}$

## EXAMPLE 6.6

Determine the force in member *CF* of the truss shown in Fig. 6–17*a*. Indicate whether the member is in tension or compression. Assume each member is pin connected.

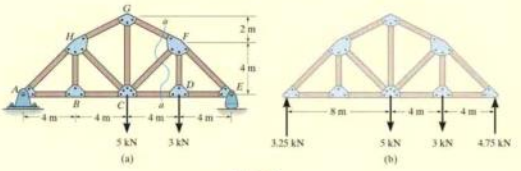
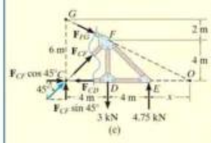



Fig. 6–17

### SOLUTION

**Free-Body Diagram.** Section *aa* in Fig. 6–17*a* will be used since this section will "expose" the internal force in member *CF* as "external" on the free-body diagram of either the right or left portion of the truss. It is first necessary, however, to determine the support reactions on either the left or right side. Verify the results shown on the free-body diagram in Fig. 6–17*b*.




The free-body diagram of the right portion of the truss, which is the easiest to analyze, is shown in Fig. 6–17*c*. There are three unknowns, $F_{FG}$, $F_{CF}$, and $F_{CD}$.

**Equations of Equilibrium.** We will apply the moment equation about point $O$ in order to eliminate the two unknowns $F_{FG}$ and $F_{CD}$. The location of point $O$ measured from $E$ can be determined from proportional triangles, i.e., $4/(4 + x) = 6/(8 + x)$, $x = 4$ m. Or, stated in another manner, the slope of member $GF$ has a drop of 2 m to a horizontal distance of 4 m. Since $FD$ is 4 m, Fig. 6–17*c*, then from $D$ to $O$ the distance must be 8 m.

An easy way to determine the moment of $\mathbf{F}_{CF}$ about point $O$ is to use the principle of transmissibility and slide $\mathbf{F}_{CF}$ to point $C$, and then resolve $\mathbf{F}_{CF}$ into its two rectangular components. We have

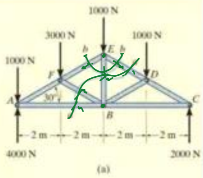
$\zeta + \Sigma M_O = 0;$

$$-F_{CF} \sin 45^\circ (12 \text{ m}) + (3 \text{ kN})(8 \text{ m}) - (4.75 \text{ kN})(4 \text{ m}) = 0$$

$$F_{CF} = 0.589 \text{ kN} \quad (\text{C}) \qquad \textit{Ans.}$$

## EXAMPLE 6.7

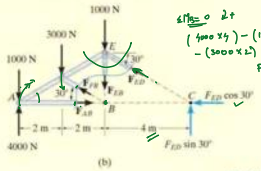
Determine the force in member $EB$ of the roof truss shown in Fig. 6–18$a$. Indicate whether the member is in tension or compression.




(a)

### SOLUTION

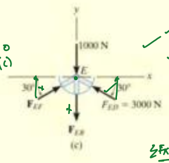
**Free-Body Diagrams.** By the method of sections, any imaginary section that cuts through $EB$, Fig. 6–18$a$, will also have to cut through three other members for which the forces are unknown. For example, section $aa$ cuts through $ED$, $EB$, $FB$, and $AB$. If a free-body diagram of the left side of this section is considered, Fig. 6–18$b$, it is possible to obtain $\mathbf{F}_{ED}$ by summing moments about $B$ to eliminate the other three unknowns; however, $\mathbf{F}_{EB}$ cannot be determined from the remaining two equilibrium equations. One possible way of obtaining $\mathbf{F}_{EB}$ is first to determine $\mathbf{F}_{ED}$ from section $aa$, then use this result on section $bb$, Fig. 6–18$a$, which is shown in Fig. 6–18$c$. Here the force system is concurrent and our sectioned free-body diagram is the same as the free-body diagram for the joint at $E$.




(b)

$\Sigma M_B = 0$ ↺+

$(4000 \times 4) - (1000 \times 4)$
$- (3000 \times 2) - (F_{ED} \sin 30 \times 4) = 0$
$F_{ED} = \cancel{10/000}$ N (C)
3000




(c)

✓ $\Sigma F_x = 0$
✓ $\Sigma F_y = 0$

$\Sigma F_y = 0$
$-1000 + F_{ED} \sin 30$ (3000)
$+ F_{EF} \sin 30 - F_{EB} = 0$
$F_{EB} = 2000$ N (T)

$\Sigma F_x = 0$
$F_{EF} \cos 30 - F_{ED} \cos 30 = 0$ (3000)
$F_{EF} = 3000$ N (C)

**Fig. 6–18**

**Equations of Equilibrium.** In order to determine the moment of $\mathbf{F}_{ED}$ about point $B$, Fig. 6–18$b$, we will use the principle of transmissibility and slide the force to point $C$ and then resolve it into its rectangular components as shown. Therefore,

$$\circlearrowleft + \Sigma M_B = 0; \qquad 1000\text{ N}(4\text{ m}) + 3000\text{ N}(2\text{ m}) - 4000\text{ N}(4\text{ m}) + F_{ED} \sin 30°(4\text{ m}) = 0$$

$$F_{ED} = 3000\text{ N} \quad \text{(C)}$$

Considering now the free-body diagram of section $bb$, Fig. 6–18$c$, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{EF} \cos 30° - 3000 \cos 30°\text{ N} = 0$$

$$F_{EF} = 3000\text{ N} \quad \text{(C)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 2(3000 \sin 30°\text{ N}) - 1000\text{ N} - F_{EB} = 0$$

$$F_{EB} = 2000\text{ N} \quad \text{(T)} \qquad \textit{Ans.}$$