$$= \sqrt{\left(\frac{60.32 \times 10^{-3}}{54.31 \times 10^{-3}}\right)^2 - 1} = 0.4833$$

To find transformer secondary voltage rating

We know that, $$V_{\text{d.c.}} = \frac{2V_m}{\pi} - I_{\text{d.c.}}\,(r_s + r_f)$$

where r_f is the diode forward resistance and r_s is the transformer secondary winding resistance.

$$55.4 = \frac{2V_m}{\pi} - 54.31 \times 10^{-3} \times 20 = \frac{2V_m}{\pi} - 1.086$$

$$56.49 = \frac{2V_m}{\pi}$$

Therefore, $$V_m = 56.49 \times \frac{\pi}{2} = 88.73 \text{ V}$$

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} = \frac{88.73}{\sqrt{2}} = 62.74 \text{ V}$$

Hence, transformer secondary voltage rating is 65 V – 0 = 65 V

Bridge rectifier

The need for a center tapped transformer in a full-wave rectifier is eliminated in the bridge rectifier. As shown in Fig. 18.5, the bridge rectifier has four diodes connected to form a bridge. The a.c. input voltage is applied to the diagonally opposite ends of the bridge. The load resistance is connected between the other two ends of the bridge.

For the positive half-cycle of the input a.c. voltage, diodes D_1 and D_3 conduct, whereas diodes D_2 and D_4 do not conduct. The conducting diodes will be in series through the load resistance R_L. So the load current flows through R_L.

During the negative half-cycle of the input a.c. voltage, diodes D_2 and D_4 conduct, whereas diodes D_1 and D_3 do not conduct. The conducting diode D_2 and D_4 will be in series through the load R_L and the current flows through R_L in the same direction as in the previous half-cycle. Thus a bidirectional wave is converted into an unidirectional one.

The average values of output voltage and load current for bridge rectifier are the same as for a center-tapped full-wave rectifier. Hence,

$$V_{\text{d.c.}} = \frac{2V_m}{\pi} \quad \text{and} \quad I_{\text{d.c.}} = \frac{V_{\text{d.c.}}}{R_L} = \frac{2V_m}{\pi R_L} = \frac{2I_m}{\pi}$$

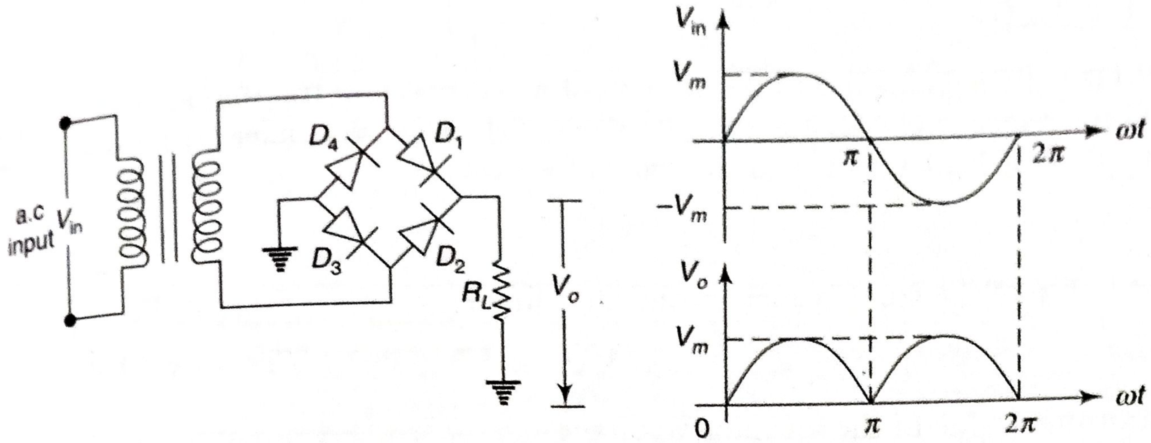
If the values of the transformer secondary winding resistance (r_s) and diode forward resistance (r_f) are considered in the analysis, then

$$V_{\text{d.c.}} = \frac{2V_m}{\pi} - I_{\text{d.c.}}\,(r_s + r_f)$$

$$I_{\text{d.c.}} = \frac{2I_m}{\pi} = \frac{2V_m}{\pi(r_s + r_f + R_L)}$$

The maximum efficiency of a bridge rectifier is 81.2% and the ripple factor is 0.48. The PIV is V_m.

Advantages of the bridge rectifier In the bridge rectifier, the ripple factor and efficiency of the rectification are the same as for the full-wave rectifier. The PIV across either of the non-conducting diodes



(a) (b)

Fig. 18.5 *Bridge rectifier*

s equal to the peak value of the transformer secondary voltage, V_m. The bulky center tapped transformer is not required. Transformer utilisation factor is considerably high. Since the current flowing in the transformer secondary is purely alternating, the TUF increases to 0.812, which is the main reason for the popularity of a bridge rectifier. The bridge rectifiers are used in applications allowing floating output terminals, i.e. no output terminal is grounded.

The bridge rectifier has only one disadvantage that it requires four diodes as compared to two diodes for center-tapped full-wave rectifier. But the diodes are readily available at cheaper rate in the market. Apart from this, the PIV rating required for the diodes in a bridge rectifier is only half of that for a center tapped full-wave rectifier. This is a great advantage, which offsets the disadvantage of using extra two diodes in a bridge rectifier.

Comparison of rectifiers The comparison of rectifiers is given in Table 18.1.

Table 18.1 *A comparison of rectifiers*

$$(k+1)(k-1)\Big]$$

The angular frequency of the power supply is the lowest angular frequency present in the above expression. All the other terms are the even harmonics of the power frequency.

The full-wave rectifier consists of two half-wave rectifier circuits, arranged in such a way that one circuit conducts during one half cycle and the second circuit operates during the second half. Therefore, the currents are functionally related by the expression $i_1(\alpha) = i_2(\alpha + \pi)$. Thus, the total current of the full-wave rectifier is $i = i_1 + i_2$ as expressed by

$$i = I_m \left[\frac{2}{\pi} - \frac{4}{\pi} \sum_{\substack{k=\text{even} \\ k \neq 0}} \frac{\cos k\omega t}{(k+1)(k-1)} \right]$$

From the above equation, it can be seen that the fundamental angular frequency is eliminated and the lowest frequency is the second harmonic term 2ω. This is the advantage that the full-wave rectifier presents in filtering of the output. Additionally, the current pulses in the two halves of the transformer winding are in such directions that the magnetic cycles formed through the iron core is essentially of the alternating current. This avoids any d.c. saturation of the transformer core that could give rise to additional harmonics at the output.

18.2.4 Filters

The output of a rectifier contains d.c. component as well as a.c. component. Filters are used to minimise the undesirable a.c., i.e. ripple leaving only the d.c. component to appear at the output.

The ripple in the rectified wave being very high, the factor being 48% in the full-wave rectifier, most of the applications which cannot tolerate this, will need an output which has been further processed

Figure 18.6 shows the concept of a filter, where the full-wave rectified output voltage is applied at its input. The output of a filter is not exactly a constant d.c. level. But it also contains a small amount of a.c. component. Some important filters are:

(i) Inductor filter
(ii) Capacitor filter
(iii) LC or L-section filter
(iv) CLC or π-type filter

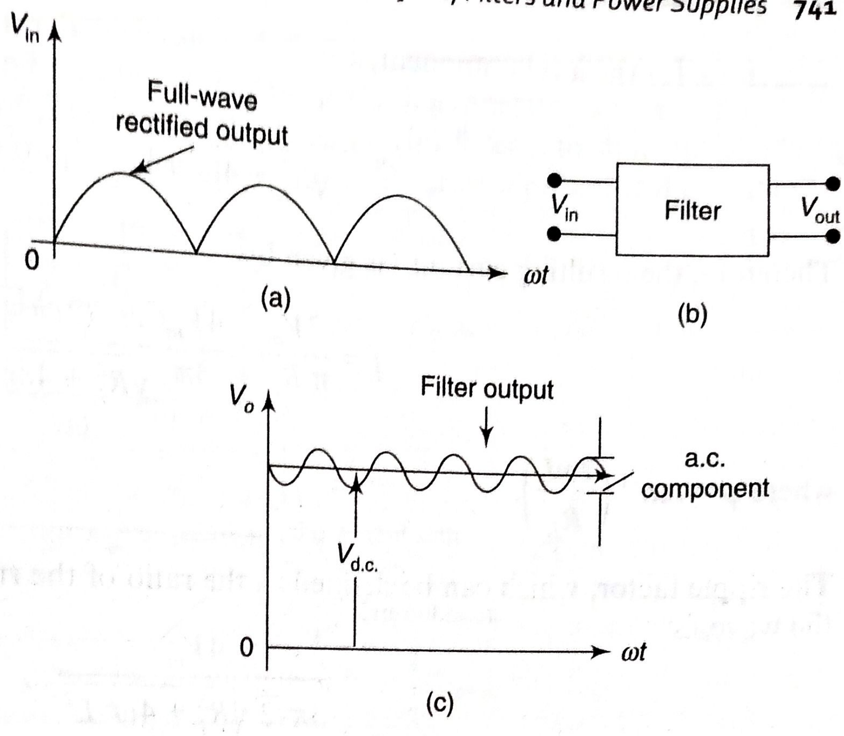


Fig. 18.6 *Concept of a filter*

Inductor filter Figure 18.7 shows the inductor filter. When the output of the rectifier passes through an inductor, it blocks the a.c. component and allows only the d.c. component to reach the load.

The ripple factor of the Inductor filter is given by

$$\Gamma = \frac{R_L}{3\sqrt{2}\,\omega L}$$

It shows that the ripple factor will decrease when L is increased and R_L is decreased. Clearly, the inductor filter is more effective only when the load current is high (small R_L). The larger value of the inductor can reduce the ripple and at the same time the output d.c. voltage will be lowered as the inductor has a higher d.c. resistance.

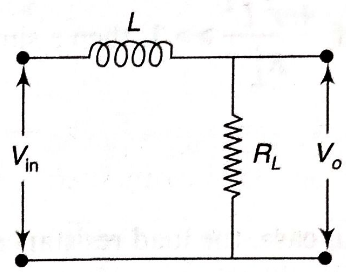


Fig. 18.7 *Inductor filter*

The operation of the inductor filter depends on its well known fundamental property to oppose any change of current passing through it.

To analyse this filter for a full-wave, the Fourier series can be written as

$$V_o = \frac{2V_m}{\pi} - \frac{4V_m}{\pi}\left[\frac{1}{3}\cos 2\,\omega t + \frac{1}{15}\cos 4\omega t + \frac{1}{35}\cos 6\,\omega t + \cdots\right]$$

The d.c. component is $\frac{2V_m}{\pi}$.

Assuming the third and higher terms contribute little output, the output voltage is

$$V_o = \frac{2V_m}{2\pi} - \frac{4V_m}{3\pi}\cos 2\,\omega t$$

The diode, choke and transformer resistances can be neglected since they are very small as compared with R_L. Therefore, the d.c. component of current $I_m = \frac{V_m}{R_L}$. The impedance of series combination of L and R_L at 2ω is

$$Z = \sqrt{R_L^2 + (2\,\omega L)^2} = \sqrt{R_L^2 + 4\omega^2 L^2}$$

Therefore, for the a.c. component,

$$I_m = \frac{V_m}{\sqrt{R_L^2 + 4\omega^2 L^2}}$$

Therefore, the resulting current i is given by,

$$i = \frac{2V_m}{\pi R_L} - \frac{4V_m}{3\pi}\frac{\cos(2\omega t - \varphi)}{\sqrt{R_L^2 + 4\omega^2 L^2}}$$

where $\varphi = \tan^{-1}\left(\frac{2\omega L}{R_L}\right)$.

The ripple factor, which can be defined as the ratio of the rms value of the ripple to the d.c. value of the wave, is

$$\Gamma = \frac{\dfrac{4V_m}{3\pi\sqrt{2}\sqrt{R_L^2 + 4\omega^2 L^2}}}{\dfrac{2V_m}{\pi R_L}} = \frac{2}{3\sqrt{2}} \cdot \frac{1}{\sqrt{1 + \dfrac{4\omega^2 L^2}{R_L^2}}}$$

If $\frac{4\omega^2 L^2}{R_L^2} >> 1$, then a simplified expression for Γ is

$$\Gamma = \frac{R_L}{3\sqrt{2}\omega L}$$

In case, the load resistance is infinity, i.e. the output is an open circuit, then the ripple factor is

$$\Gamma = \frac{2}{3\sqrt{2}} = 0.471$$

This is slightly less than the value of 0.482. The difference being attributable to the omission of higher harmonics as mentioned. It is clear that the inductor filter should only be used where R_L is consistently small.

EXAMPLE 18.20

Calculate the value of inductance to use in the inductor filter connected to a full-wave rectifier operating at 60 Hz to provide a d.c. output with 4% ripple for a 100 Ω load.

Solution We know that the ripple factor for inductor filter is $\Gamma = \frac{R_L}{3\sqrt{2}\,\omega L}$

Therefore,

$$0.04 = \frac{100}{3\sqrt{2}\,(2\pi \times 60 \times L)} = \frac{0.0625}{L}$$

$$L = \frac{0.0625}{0.04} = 1.5625 \text{ H}$$

Capacitor filter An inexpensive filter for light loads is found in the capacitor filter which is connected directly across the load, as shown in Fig. 18.8(a). The property of a capacitor is that it allows a.c. component and blocks the d.c. component. The operation of a capacitor filter is to short the ripple to ground but leave the d.c. to appear at the output when it is connected across a pulsating d.c. voltage.

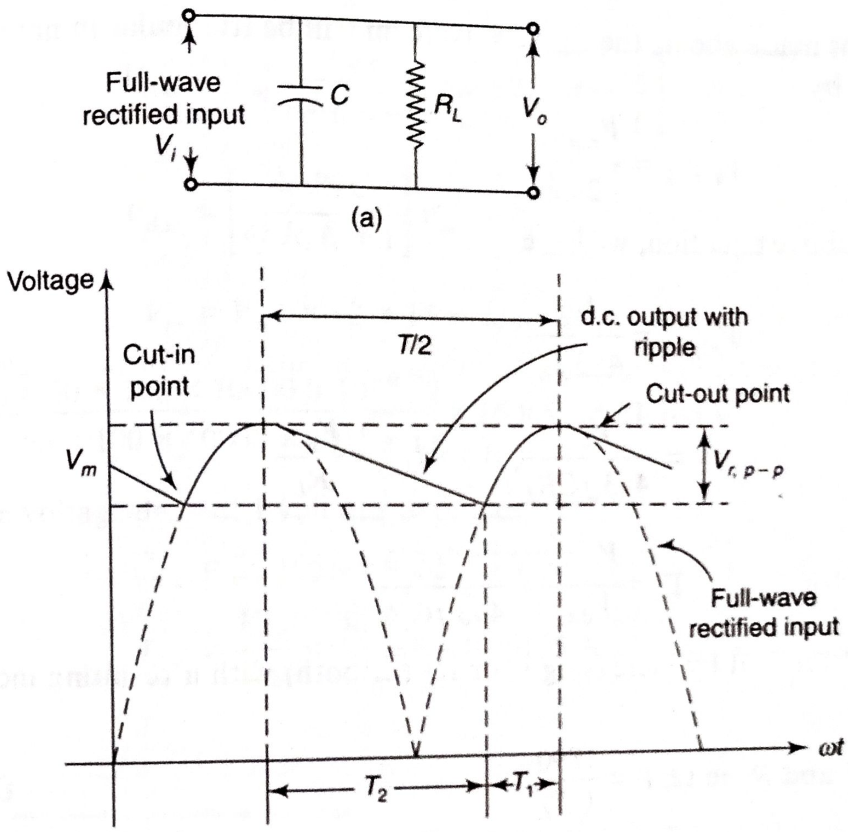


Fig. 18.8 *(a) Capacitor filter, (b) Ripple voltage triangular waveform*

During the positive half-cycle, the capacitor charges up to the peak value of the transformer secondary voltage, V_m, and will try to maintain this value as the full-wave input drops to zero. The capacitor will discharge through R_L slowly until the transformer secondary voltage again increases to a value greater than the capacitor voltage (equal to the load voltage). The diode conducts for a period which depends on the capacitor voltage. The diode will conduct when the transformer secondary voltage becomes more than the 'cut-in' voltage of the diode. The diode stops conducting when the transformer voltage becomes less than the diode voltage. This is called cut-out voltage.

Referring to Fig. 18.8(b) with slight approximation, the ripple voltage waveform can be assumed as triangular. From the cut-in point to the cut-out point, whatever charge the capacitor acquires is equal to the charge the capacitor has lost during the period of non-conduction, i.e. from cut-out point to the next cut-in point.

The charge it has acquired $= V_{r,p\text{-}p} \times C$

The charge it has lost $= I_{\text{d.c.}} \times T_2$

Therefore,

$= I_{\text{d.c.}} \times T_2$

If the value of the capacitor is fairly large, or the value of the load resistance is very large, then it can be assumed that the time T_2 is equal to half the periodic time of the waveform.

i.e.
$$T_2 = \frac{T}{2} = \frac{1}{2f}, \quad \text{then} \quad V_{r,p-p} = \frac{I_{d.c.}}{2fC}$$

With the assumptions made above, the ripple waveform will be triangular in nature and the rms value of the ripple is given by

$$V_{r,\,rms} = \frac{V_{r,p-p}}{2\sqrt{3}}$$

Therefore from the above equation, we have

$$V_{r,\,rms} = \frac{I_{d.c.}}{4\sqrt{3}\,fC}$$

$$= \frac{V_{d.c.}}{4\sqrt{3}\,fCR_L}, \text{ since } I_{d.c.} = \frac{V_{d.c.}}{R_L}$$

Therefore, ripple factor
$$\Gamma = \frac{V_{r,\,rms}}{V_{d.c.}} = \frac{1}{4\sqrt{3}\,fCR_L}$$

The ripple may be decreased by increasing C or R_L (or both) with a resulting increase in d.c. output voltage.

If $f = 50$ Hz, C in μF and R_L in Ω, $\Gamma = \dfrac{2890}{CR_L}$.

EXAMPLE 18.21

Calculate the value of capacitance to use in a capacitor filter connected to a full-wave rectifier operating at a standard aircraft power frequency of 400 Hz, if the ripple factor is 10% for a load of 500 Ω.

Solution We know that the ripple factor for capacitor filter is

$$\Gamma = \frac{1}{4\sqrt{3}\,fCR_L}$$

Therefore,
$$0.01 = \frac{1}{4\sqrt{3} \times 400 \times C \times 500} = \frac{0.722 \times 10^{-6}}{C}$$

$$C = \frac{0.722 \times 10^{-6}}{0.01} = 72.2\ \mu\text{F}$$

EXAMPLE 18.22

A15-0-15 Volts (rms) ideal transformer is used with a full-wave rectifier circuit with diodes having forward drop of 1 Volt. The load is a resistance of 100 Ohm and a capacitor of 10,000 μF is used as a filter across the load resistance. Calculate the d.c. load current and voltage.

Solution Given transformer secondary voltage = 15-0-15 V (rms);

Diode forward drop = 1 V; $R_L = 100\ \Omega$; $C = 10{,}000\ \mu F$

We know that,

$$V_{d.c.} = V_m - \frac{V_{repp}}{2} = V_m - \frac{I_{d.c.}}{4fC}$$

Therefore,

$$V_{d.c.} = V_m - \frac{V_{d.c.}}{R_L\,4fC}, \left[\text{since } I_{d.c.} = \frac{V_{d.c.}}{R_L}\right]$$

Simplifying, we get

$$V_{d.c.} = \left[\frac{4f R_L C}{4f R_L C + 1}\right] V_m$$

We know that

$$V_m = V_{rms} \times \sqrt{2} = 15 \times \sqrt{2}$$

Therefore, $V_{d.c.} = \left[\dfrac{4 \times 50 \times 100 \times 10000 \times 10^{-6}}{4 \times 50 \times 100 \times 10000 \times 10^{-6} + 1}\right] \times 15 \times \sqrt{2} = 21.105$ V

Considering the given voltage drop of 1 volt due to diodes,

$$V_{d.c.} = 21.105 - 1 = 20.105 \text{ V}$$

$$I_{d.c.} = \frac{V_{d.c.}}{R_L} = \frac{20.105}{100} = 0.20105 \text{ A}$$

EXAMPLE 18.23

A full-wave rectified voltage of 18 V peak is applied across a 500 μF filter capacitor. Calculate the ripple and d.c. voltages if the load takes a current of 100 mA.

Solution Given

$$V_m = 18 \text{ V}, C = 500\ \mu\text{F and } I_{d.c.} = 100 \text{ mA}$$

$$V_{d.c.} = V_m - \frac{I_{d.c.}}{4fC} = 18 - \frac{100 \times 10^{-3}}{4 \times 50 \times 500 \times 10^{-6}} = 17 \text{ V}$$

$$V_{r,rms} = \frac{I_{d.c.}}{4\sqrt{3}fC} = \frac{100 \times 10^{-3}}{4\sqrt{3} \times 50 \times 500 \times 10^{-6}} = 0.577 \text{ V}$$

Therefore, ripple

$$\Gamma = \frac{V_{rms}}{V_{d.c.}} = \frac{0.577}{17} \times 100 = 3.39\%$$

EXAMPLE 18.24

A bridge rectifier with capacitor filter is fed from 220 V to 40 V step down transformer. If average d.c. current in load is 1 A and capacitor filter of 800 μF, calculate the load regulation and ripple factor. Assume power line frequency of 50 Hz. Neglect diode forward resistance and d.c. resistance of secondary of transformer.

Solution

$$V_{rms} = 40 \text{ V}, I_{d.c.} = 1\text{A}, C = 800\ \mu\text{F and } f = 50 \text{ Hz}$$

$$V_m = \sqrt{2}\, V_{\text{rms}} = \sqrt{2} \times 40 = 56.5685 \text{ V}$$

$$V_{\text{d.c.}(FL)} = V_m - \frac{I_{\text{d.c.}}}{4fC} = 56.5685 - \frac{1}{4 \times 50 \times 800 \times 10^{-6}} = 50.3185 \text{ V}$$

On no load,
$$I_{\text{d.c.}} = 0$$

Hence,
$$V_{\text{d.c.}(NL)} = V_m = 56.5685 \text{ V}$$

Therefore, percentage of regulation $= \dfrac{V_{\text{d.c.}(NL)} - V_{\text{d.c.}(NL)}}{I_{\text{d.c.}(NL)}} \times 100$

$$= \frac{56.5685 - 50.3185}{50.1385} \times 100 = 12.42\%$$

$$R_L = \frac{V_{\text{d.c.}}}{I_{\text{d.c.}}} = \frac{50.1385}{1} = 50.1385\ \Omega$$

$$\Gamma = \frac{1}{4\sqrt{3} fCR_L} = \frac{1}{4\sqrt{3} \times 50 \times 800 \times 10^{-6} \times 50.3185} = 0.0717, \text{ i.e. } 7.1$$

LC filter We know that the ripple factor is directly proportional to the load resistance R_L in the ind tor filter and inversely proportional to R_L in the capacitor filter. Therefore, if these two filters are combined as LC filter or L-section filter as shown in Fig. 18.9, the ripple factor will be independent of R_L.

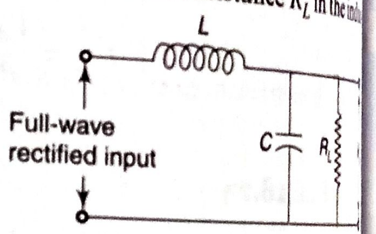


Fig. 18.9 *LC filter*

If the value of the inductance is increased, it will increase the time of conduction. At some critical value of inductance, one diode, either D_1 or D_2 in full-wave rectifier, will always be conducting.

From Fourier series, the output voltage can be expressed as

$$V_o = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi} \cos 2\,\omega t$$

The d.c. output voltage,
$$V_{\text{d.c.}} = \frac{2V_m}{\pi}$$

Therefore,
$$I_{\text{rms}} = \frac{4V_m}{3\pi\sqrt{2}} \cdot \frac{1}{X_L} = \frac{\sqrt{2}}{3} \cdot \frac{V_{\text{d.c.}}}{X_L}$$

This current flowing through X_c creates the ripple voltage in the output.

Therefore,
$$V_{r,\text{rms}} = I_{\text{rms}} \cdot X_C = \frac{\sqrt{2}}{3} \cdot V_{\text{d.c.}} \cdot \frac{X_C}{X_L}$$

The ripple factor,
$$\Gamma = \frac{V_{r,\text{rms}}}{V_{\text{d.c.}}} = \frac{\sqrt{2}}{3} \cdot \frac{X_C}{X_L}$$

$$= \frac{\sqrt{2}}{3} \cdot \frac{1}{4\omega^2 C_L}, \text{ since } X_C = \frac{1}{2\omega C} \text{ and } X_L = 2\omega L$$

If $f = 50$ Hz, C is in μF and L is in Henry, ripple factor $\Gamma = \dfrac{1.194}{LC}$.

Bleeder resistor It was assumed in the analysis given above that for a critical value of inductor, either of the diodes is always conducting, i.e. current does not fall to zero. The incoming current consists of two components:

(i) $I_{d.c.} = \dfrac{V_{d.c.}}{R_L}$ and (ii) a sinusoidal varying components with peak value of $\dfrac{4V_m}{3\pi X_L}$. The negative peak of the a.c. current must always be less than d.c., i.e., $\sqrt{2}\, I_{rms} \le \dfrac{V_{d.c.}}{R_L}$.

We know that for LC filter, $\quad I_{rms} = \dfrac{\sqrt{2}}{3} \times \dfrac{V_{d.c.}}{X_L}$

Hence $\dfrac{2V_{d.c.}}{X_L} \le \dfrac{V_{d.c.}}{R_L}$, $\quad$ i.e. $\quad X_L \ge \dfrac{2}{3} R_L$

i.e., $L_C = \dfrac{R_L}{3\omega}$, where L_C is the critical inductance.

It should be noted that the condition $X_L \ge \dfrac{2}{3} R_L$ cannot be satisfied for all load requirements. At no load, i.e. when the load resistance is infinity, the value of the inductance will also tend to be infinity. To overcome this problem, a bleeder resistor R_B, is connected in parallel with the load resistance as shown in Fig. 18.10. Therefore, a minimum current will always be present for optimum operation of the inductor. It improves voltage regulation of the supply by acting as the pre-load on the supply. Also, it provides safety by acting as a discharging path for capacitor.

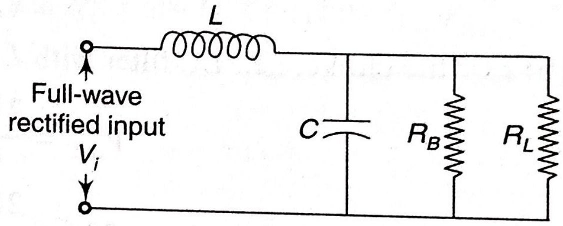


Fig. 18.10 *Bleeder resistor connected at the filter output*

EXAMPLE 18.25

Design a filter for full-wave circuit with LC filter to provide an output voltage of 10 V with a load current of 200 mA and the ripple is limited to 2%.

Solution The effective load resistance $R_L = \dfrac{10}{200 \times 10^{-3}} = 50\ \Omega$

We know that the ripple factor, $\Gamma = \dfrac{1.194}{LC}$

i.e.
$$0.02 = \frac{1.194}{LC}$$

i.e.
$$LC = \frac{1.194}{0.02} = 59.7$$

Critical value of $L = \dfrac{R_L}{3\omega} = \dfrac{50}{3 \times 2\pi f} = 53$ mH

Taking $L = 60$ mH (about 20% higher), C will be about 1000 μF.

The a.c. voltage across C_2 and hence across the load is given by

Fig. 18.11 A multiple (two-section) L-section filter

$$V_{33'} = I_2 X_{C2} = I_1 \frac{X_{C2} X_{C1}}{X_{L2}} = \frac{\sqrt{2}\, V_{\text{d.c.}}}{3} \frac{X_{C2}}{X_{L2}} \frac{X_{C1}}{X_{L1}}$$

The ripple factor is obtained by dividing the above equation by $V_{\text{d.c.}}$. Hence

$$\Gamma = \frac{\sqrt{2}}{3} \frac{X_{C1}}{X_{L1}} \frac{X_{C2}}{X_{L2}}$$

The generalized expression for any number of sections can be obtained by comparing the above equation with that of a single L-section. For example, the ripple factor of a multiple L-section filter (Γ_n) is given by

$$\Gamma_n = \frac{\sqrt{2}}{3} \left(\frac{X_C}{X_L}\right)^n = \frac{\sqrt{2}}{3} \frac{1}{(16\pi^2 f^2 LC)^n}$$

where n is the number of similar L-sections.

CLC or π-section filter Figure 18.12 shows the CLC or π-type filter which basically consists of a capacitor filter followed by an LC section. This filter provided a fairly smooth output, and characterized by a highly peaked diode currents and poor regulation.

The action of a π-section filter can best be understood by considering the inductor and the second capacitor as an L-section filter that acts upon the triangular output-voltage wave from the first capacitor. The output voltage is then approximately that from the input capacitor, decreased by the d.c. voltage drop in the inductor. The ripple contained in this output is reduced by the L-section filter.

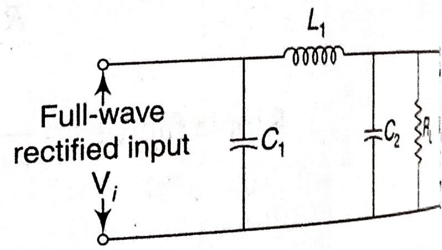


Fig. 18.12 CLC or π-type filter

The ripple voltage can be calculated by analyzing the triangular wave into a Fourier series and then multiplying each component by X_{C2}/X_{L1} for this harmonic. The Fourier analysis of this waveform is given by

$$v = V_{\text{d.c.}} - \frac{V_r}{\pi}\left(\sin 2\omega t - \frac{\sin 2\omega t}{2} + \frac{\sin 6\omega t}{3} - \cdots\right)$$

We know that

$$V_r = \frac{I_{\text{d.c.}}}{2fC_1}$$

The rms second-harmonic voltage is

$$V_{\text{rms}} = V_2' = \frac{V_r}{\pi\sqrt{2}} = \frac{I_{\text{d.c.}}}{2\pi f C_1 \sqrt{2}} = \sqrt{2} I_{\text{d.c.}} X_{C1}$$