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UNIT-1

FUNDAMENTALS OF LOGIC

→ Basic Connectives
→ Truth Table
→ Logical Equivalence
→ Logical Implication
→ Use of quantifiers.
→ Definitions and proofs of theorems.

*→ Functions :

→ Cartesian product
→ One-One & Onto functions
→ Special Functions
→ Pigeonhole Theorem
→ Composite and Inverse Functions

* **Logic:**

It is derived from greek word "Logos" which means reason (or) principle. It provides rules by which one can determine whether an argument is valid or not.

* To design computer circuits, construction of computer programs and for the verification of correctness of the programs.

* **Proposition:** Proposition (or) statement is a declarative sentence which is either true (or) false but not both.

- If a proposition is true : T (Truth)
- If a proposition is false : F (Falsitivity)
- Propositional variables are denoted by p, q, r, s, t, etc.
- T, F are called Truth values.
- Sometimes it is also denoted as T(1) and F(0).

* Which of the following statements are propositions?

1) 5 is a prime number : P(T)
2) Delhi is capital of India : P(T)
3) 2 + 2 = 5 : P(F)
4) Wish u a Happy Birthday : Not P.
5) How beautiful you are? : Not P.

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* **CONNECTIVES:**

1) Negation : 'NOT' [$\sim$ (OR) $\neg$]
2) Conjunction : 'AND' / 'BUT' [$\wedge$]
3) Disjunction : 'OR' [$\vee$]
4) Conditional (Implication) : [$\rightarrow$ (OR) $\Rightarrow$] : 'if then'
5) Bi-conditional : [$\leftrightarrow$ (OR) $\Leftrightarrow$] : 'iff'

* **Propositional Calculus:**

It is a statement calculus in which logical equivalence are used.

** A logical equivalence consists of
→ Universe of proportions.
→ Truth tables for logical operations
→ Definition that explains equivalence and implication of proportions

* **Connectives:**

The words (or) phrases (or) symbols which are used to make proposition by 2 (or) more propositions are called logical connectives.

* There are 5 basic connectives called

conditional and bi-conditional

* Truth Tables:

→ Negation:

The negation of a statement is generally formed by writing the word 'NOT' at a proper place in the statement (proposition) (or) by prefixing the statement with the phrase:

'It is not the case that' ⇒

Ex:
p: Hyderabad is a city
$\sim p$: Hyderabad is not a city.

- p: statement

$\neg p$ / $\sim p$: Negation of statement

- Also called 'denial'

Truth Table:

p	$\sim p$
T	F
F	T

Note:
unless, not we use.
$\sim$ / $\neg$

p	q	$\sim p$	$\sim q$
T	T	F	F
T	F	F	T
F	T	T	F
F	F	T	T

p	q	r	$\sim p$	$\sim q$	$\sim r$
T	T	T	F	F	F
T	T	F	F	F	T
T	F	T	F	T	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	T

* Conjunction:

The conjunction is a compound proposition formed by 2 (or) more propositions: p, q using connective 'AND' denoted $p \wedge q$ and read as p and q.

Conjunction is true only when p & q are true and otherwise all other cases are false.

Truth Table:

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

$\wedge$ ⟶ and/but/conjunction.
$\vee$ ⟶ or/disjunction.

p: It is raining today
q: There are 50 students in the classroom.

$p \wedge q$: It is raining today and there are 50 students in the class.

Disjunction:

The disjunction is a compound statement formed by using 2 (or) more propositions p, q using connective 'OR' denoted by $p \vee q$ and read p (or) q.

Disjunction is False when both ~~the~~ p & q are false else it is true.

Truth Table:

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

p: There is something wrong with the system.

q: There is something wrong with the internet.

p∨q: There is something wrong with the system (or) there is something wrong with the internet.

possible cases are

Inclusive OR: →

Impossible ~~OR~~: Cases: Exclusive OR.

p and q are two propositions p⊕q is given by exclusive OR.

p: Mr. Rahul will go to Banglore

q: Mr. Rahul will go to Hyderabad.

p⊕q: Mr. Rahul will go to Banglore or Hyderabad.

p	q	p⊕q
T	T	F
T	F	T
F	T	T
F	F	F

* Conditional proposition (Implication):

If p and q are any 2 statements then the statement p → q which read as 'if p, then q' is called conditional statement or implication and the connective is conditional connective.

The conditional proposition is false when p is true and q is false and true otherwise.

∴ p → hypothesis/antecedent/premise.

q → conclusion/consequence.

Note: whenever 'nevertheless' we use p → q.

Truth Table:

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Ex:

p: You drive over 70km per hour.

q: You get a speeding ticket.

p→q: If you drive over 70km per hour, then you get a speeding ticket.

* Different expressions [illegible]

1) If p then q : p implies q
2) If p, q" : p only if q.
3) p is sufficient for q : a sufficient condition for q is p
4) q if p
→ q whenever p.
5) q when p → q is necessary for p.
6) a necessary condition for p is q
q follows ~~from~~ p.
7) q is consequence of p.

p: You drive over 70km/hr.
q: You get a speeding ticket

* Write the following statements into symbolic forms:

1) You will get a speed indicate, if you drive over 70km/hr.
$\Rightarrow p \rightarrow q$

2) Driving over 70km/hr is sufficient for getting a speeding ~~indicator~~ ticket.
$\Rightarrow p \rightarrow q$

3) If you do not drive over 70km/hr then you will not get a speed indicator.
$\Rightarrow \sim p \rightarrow \sim q$.

4) whenever we get speed indicator; you drive over 70km/hr.
$\Rightarrow q \rightarrow p$

5) You drive over 70km/hr but you do not get speeding ticket
$\Rightarrow p \wedge \sim q$

* Transform the following statements into symbolic statements using conditional connectives:

1) The crop will be destroyed if there is a flood.
- $p \rightarrow q$
- If there is a flood then the crop will be destroyed.

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* Bi-conditional Propositions:

If p and q are two propositions 'p iff q' (or) p if and only if q is called bi-implication of p & q denoted by
$p \Leftrightarrow q$, (or) $p \leftrightarrow q$

Read as p implies by q / p is necessary and sufficient for q

(or) if p then q and conversely.

* A biconditional is true when both p and q are true (or) when both p & q are false

Ex:1 If you finish your meal, then you can have desert.

(OR)

You can have desert iff you finish your meal.

Ex:2 p: Triangle ABC is right angled.

q: $(AC)^2 = (AB)^2 + (BC)^2$.

$p \leftrightarrow q$: Triangle ABC is right angled iff $(AC)^2 = (AB)^2 + (BC)^2$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

* Types of Conditional propositions: $\boxed{p \rightarrow q}$

Converse $[q \rightarrow p]$

Inverse $[\sim p \rightarrow \sim q]$

Contrapositive $[\sim q \rightarrow \sim p]$

Truth Table:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$q \rightarrow p$ (Converse)	$\sim p \rightarrow \sim q$ (Inverse)	$\sim q \rightarrow \sim p$ (Contrapositive)
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

* Write converse, inverse & contrapositive of following.

1) If it rains then the crop will grow.

C: If the crop grows then it rains.

I: If it do not rains then the crop will not grow.

CP: If the crop will not grow then it do not rain.

2) If the triangle is equilateral then it is isosceles.

C: If the triangle is isosceles then it is equilateral.

I: If the triangle is not equilateral then it is not isosceles.

CP: If the triangle is not isosceles then it is not equilateral.

3) A positive integer is prime only if it has no divisors other than one & itself.

~~C: If a number~~

C: If a positive integer has no divisors other than one & itself then it is prime.

I: If a positive integer is not prime then it has divisors other than one & itself.

CP: If a positive integer has divisors other than one & itself then it is not prime.

4) I go to beach whenever it is a sunny summer day.

p = It is a sunny summer day.

q = I go to beach

C: If I go to beach then it is a sunny summer day.

I: If it is not a sunny summer day then I will not go to beach.

CP: If I do not go to beach then it is not a sunny summer day.

5) When I stay up late it is necessary that ~~I wake up late~~ I sleep until noon.

C: If I sleep until noon then I stay up late.

I: If I do not stay up late then I will not sleep until noon.

CP: If I will not sleep until noon then I do not stay up late.

6) You can access the internet from campus only if you are a computer science major or you are not a freshman.

: $p \rightarrow q \vee \sim r$

7) You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.

: $\sim p \wedge r \rightarrow \sim q$

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* How many rows appear in this truth table for each of this compound proposition?

a) $q \rightarrow \neg q$: $2^1 = 2$

no. of rows: 2^n where n is no. of variables.

b) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t) = 2^4 = 16$

c) $(p \vee \neg t) \wedge (p \vee \neg s) = 2^3 = 8$

* All are true: Tautology
* All are false: Contradiction
* Combination of true & false: Contingency

* Construct a truth table for each of these compound propositions:

1) $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

2) $p \vee \sim p$

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

3) $(p \vee \neg q) \rightarrow q$

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

4) $(p \vee q) \rightarrow (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

5) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

~~p∼q∼q~~

p	$\sim p$	q	$\sim q$	$p \rightarrow q$	$\sim q \rightarrow \sim p$	$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
T	F	T	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	T	F	T	~~F~~	T	T

5) $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

6) $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p\to q)\to(q\to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

7) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$\neg q$	$p \leftrightarrow \neg q$	$(p\leftrightarrow q) \oplus (p \leftrightarrow \sim q)$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	F	F	T	T
F	F	T	T	F	T

8) $(q \to \neg p) \longleftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$(q \to \neg p)$	$p \leftrightarrow q$
T	T	F	F	T
T	F	F	T	F
F	T	T	T	F
F	F	T	T	T

$(q \to \neg p) \longleftrightarrow (p \leftrightarrow q)$

F
F
F
T

* Tautology : A statement formula which is true always true regardless of the truth values of the statements which replace the variables in it is called a universally valid formula (or) a logical truth or a tautology.

* Contradiction: A statement formula which is always false regardless of truth values of statements which replace the variables in it is said to be a contraction.

*Contingency: A statement formula which is neither a tautology nor a contradiction is known as a contingency.

* $(p \to q) \leftrightarrow (\neg q \to \neg p)$

p	q	$\neg p$	$\neg q$	$\neg q \to \neg p$	$p \to q$	*
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

* $(p \oplus q) \wedge (p \oplus \neg q)$

p	q	$p \oplus q$	$\neg q$	$p \oplus \neg q$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	T	F	F	F
F	F	F	T	T	F

* $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

p	q	r	$\neg p$	$\neg q$	$(\neg p \leftrightarrow \neg q)$	$(q \leftrightarrow r)$	*
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F

F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

* **SUBSTITUTION INSTANCE**: A formula A is called a substitution instance of another formula B if A can be obtained from B by substituting formulas some variable of B; with the condition that same formula is substituted for the same variable each time it occurs.

* Substitution instance is used in programming and mathematical reasoning.

Ex: Let $B: p \to (J \wedge p)$

Substitute $R \leftrightarrow S$ for p in B we get

$A: (R \leftrightarrow S) \to (J \wedge (R \leftrightarrow S))$

A is a substitution instance of B.

Note that $(R \leftrightarrow S) \to (J \wedge p)$ is not a substitution instance of B because the variables p in $J \wedge p$ was not replaced by $R \leftrightarrow S$.

* **Logical Equivalence:**

Two properties p and q are said to be logically equivalent if they have identical truth tables.

The propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology.

If p and q are logically equivalent then it is denoted by $p \Leftrightarrow q$ (or) $p \equiv q$

There are two ways to determine whether two propositions are equivalent.

*** Note:**

We can construct truth table using symbol.

1) $\Leftrightarrow$ is only symbol, but not connective
2) $p \Leftrightarrow q$ is a tautology iff truth tables of p and q are the same.
3) Equivalence relation is symmetric and transitive.
4) p & q are said to be equivalent to each other iff $p \Leftrightarrow q$ is a tautology.

*** Method-I: Truth Table Method:**

One method to determine whether any two statement formulas are equivalent is to construct their truth tables.

Ex: Prove that $(p \vee q) \Leftrightarrow \neg(\neg p \wedge \neg q)$

p	q	$p \vee q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$	$p \vee q \Leftrightarrow \neg(\neg p \wedge \neg q)$
T	T	T	F	F	F	T	T
T	F	T	F	T	F	T	T
F	T	T	T	F	F	T	T
F	F	F	T	T	T	F	T

$(p \vee q)$, $\neg(\neg p \wedge \neg q)$ are having same truth values;

$(p \vee q) \Leftrightarrow \neg(\neg p \wedge \neg q)$ is a tautology.

* Prove That $(p \to q) \Leftrightarrow (\neg p \vee q)$

p	q	$p \to q$	$\neg p$	$\neg p \vee q$	$(p \to q) \Leftrightarrow (\neg p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

$(p \to q) \Leftrightarrow (\neg p \vee q)$ is a tautology.

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* If connectives are same; it is tautology. i.e. if logical equivalence; it is tautology.

→ A tautology may or may not be logical equivalent.

***Standard Equivalence:**

1) Idempotent laws: (a) $(p \vee p) \Leftrightarrow p$
   (b) $(p \wedge p) \Leftrightarrow p$
2) Associative laws: (a) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$
   (b) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$
3) Commutative laws: (a) $(p \vee q) \Leftrightarrow (q \vee p)$
   (b) $(p \wedge q) \Leftrightarrow (q \wedge p)$

4) Identity Laws: (a) $p \vee F \Leftrightarrow p$

(b) $p \wedge T \Leftrightarrow p$

5) Domination laws: (a) $p \vee T \Leftrightarrow T$

(b) $p \wedge F \Leftrightarrow F$

6) Double Negative laws: $\neg(\neg p) \Leftrightarrow p$

7) Law of disjunction: (a) $p \vee \neg p \Leftrightarrow T$

(b) $p \wedge \neg p \Leftrightarrow F$

8) Absorption Laws: (a) $p \vee (p \wedge q) \Leftrightarrow p$

(b) $p \wedge (p \vee q) \Leftrightarrow p$

9) Demorgan's Laws: (a) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

(b) $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

10) Distributive laws:

$$\left.\begin{array}{l}(a)\ p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r) \\ (b)\ p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)\end{array}\right\} \text{Left distributive}$$

$$\left.\begin{array}{l}(c)\ (p \vee q) \wedge r = (p \wedge r) \vee (q \wedge r) \\ (d)\ (p \wedge q) \vee r = (p \vee r) \wedge (q \vee r)\end{array}\right\} \text{Right distributive}$$

11) Law of Implication:

(a) $p \to q \Leftrightarrow \neg p \vee q$

* $(p \vee q) \Leftrightarrow \neg(\neg p \wedge \neg q)$

Using logical equivalence.

RHS: $\neg(\neg p \wedge \neg q)$

$\Leftrightarrow \neg(\neg(p \vee q))$

$\Leftrightarrow (p \vee q)$

By Demorgan's

- $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
- $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

* Show that $(p \wedge q) \to (p \vee q)$ is tautology.

p	q	$p \wedge q$	$p \vee q$	*
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

* Prove that $p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$

LHS: $p \to (q \to r)$

$\Leftrightarrow \neg p \vee (q \to r)$ } By law of implication

$\Leftrightarrow \neg p \vee (\neg q \vee r)$

$\Leftrightarrow (\neg p \vee \neg q) \vee r \to$ Associative law.

$\Leftrightarrow \neg (p \wedge q) \vee r$ } By Law of implication

$\Leftrightarrow (p \wedge q) \to r$

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T.

* $(p \to q) \wedge (r \to q) \Leftrightarrow (p \vee r) \to q$.

* $[\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

$[r \wedge \neg(q \vee p)] \vee [r \wedge (q \vee p)]$

$\Leftrightarrow r \wedge (\neg(q \vee p) \vee (q \vee p))$

$\Leftrightarrow r \wedge T \Leftrightarrow \underline{r}$

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** $\neg(p \wedge q) \longrightarrow (\neg p \vee (\neg p \vee q)] \Leftrightarrow (\neg p \vee q)$

LHS: $\neg(p \wedge q) \longrightarrow (p \longrightarrow (\neg p \vee q))$ } Law of Implication

$(p \wedge q) \vee (p \rightarrow (\neg p \vee q))$.

$(p \wedge q) \vee (\neg p \vee (\neg p \vee q))$

$(p \wedge q) \vee ((\neg p \vee \neg p) \vee q)$ By associative law.

$\Rightarrow (p \wedge q) \vee \underbrace{(\neg p \vee q)}_{r}$

By distributive law.

$(p \wedge q) \vee r = (p \vee r) \wedge (q \vee r)$

$\Rightarrow \cancel{[(p \wedge q) \vee (\neg p \vee q)] \wedge [}$

$\Leftrightarrow (p \vee (\neg p \vee q)) \wedge (q \vee (\neg p \vee q))$

$\Leftrightarrow$ By Associative law.

$\Rightarrow ((p \vee \neg p) \vee q) \wedge ((q \vee \neg p) \vee q)$

$\Rightarrow (T \vee q) \wedge ((q \vee \neg p) \vee q)$.

$\Leftrightarrow T \wedge ((\neg p \vee q) \vee q)$

$T \wedge (\neg p \vee (q \vee q)) \Leftrightarrow T \wedge (\neg p \vee q)$

$\Leftrightarrow \underline{(\neg p \vee q)}$ (RHS)

** $(p \wedge q) \rightarrow (p \vee q)$ is tautology.

By implication law.

$\sim(p \wedge q) \vee (p \vee q)$

$\Leftrightarrow (\sim p \vee \sim q) \vee (p \vee q)$. (By Demorgan's law

$\Leftrightarrow \sim p \vee (\sim q \vee p \vee q)$ Associative law.

$\Leftrightarrow \sim p \vee (T \vee p)$ Law of disjunction

$\Leftrightarrow (\sim p \vee T) \Rightarrow T$

** Show that

$[(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))] \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r$

(By implication)

$[(p \vee q) \wedge \neg(\neg p \wedge (q \rightarrow \neg r))]$

$\vee (\neg p \wedge \neg q) (\neg p \wedge \neg$

$[(p \vee q) \wedge (p \vee \sim(q \rightarrow \neg r))]$ Distributive law

$\vee [(\neg p \wedge (\neg q \vee \neg r))]]$

$(p \vee q) \wedge (p \vee (q \wedge r)) \vee$ (Double Negation)

$[\neg p \wedge (\neg q \vee \neg r)]$

$(p \vee q) \wedge (p \vee (q \wedge r)) \vee \neg (p \vee (q \wedge r))$

$(p \vee q) \wedge T = (p \vee q)$

$* [(p \vee q) \wedge p \vee (q \wedge r)] \vee \neg p \wedge (\neg q \vee \neg r)$ Distributive law.

By Negation

Demorgan's law.

$(p \vee q) \wedge (p \vee (q \wedge r)) \vee \neg (p \vee (q \wedge r))$

$(p \vee q) \wedge (p \vee q) \wedge (p \vee r) \vee \neg [(p \vee q) \wedge (p \vee r)]$

$(p \vee q) \wedge (p \vee r) \vee \neg [(p \vee q) \wedge (p \vee r)]$

$p \vee (q \wedge r) \vee \neg (p \vee (q \wedge r))$

$= \underline{\underline{T}}$

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* Write the negation of the following statements:

a) Jam will take a job in industry or go to graduate school

(p: Jam will take a job in industry; q: go to graduate school)

* $p \vee q$

* $\sim p \wedge \sim q$

* Jam will not take a job in industry and do not go to graduate school

b) James will bicycle or run tomorrow.

* $p \vee q$ * $\sim p \wedge \sim q$

* James will not bicycle and he will not run tomorrow.

c) If the processor is fast, then the printer is slow.

* $p \rightarrow q$ * $\sim(p \rightarrow q)$ * $\sim(\sim p \vee q)$

* $p \wedge \sim q$

* The processor is fast and the printer is not slow.

* Use Demorgan's laws to write the negation of each statement:

1) I want a car and worth a cycle

$\sim p \vee \sim q$

→ I don't want a car or not worth cycle

2) My cat stays outside or it make a mess

$\sim p \wedge \sim q$

→ My cat do not stay outside and it does not make a mess

3) I've fallen and I can't get up $p \wedge \sim q$

→ I have not fallen $\sim p \vee q$

or I can get up.

4) You study or you don't get a good grade

$p \vee \sim q$

You do not study & you get a good grade

$\sim p \wedge q$

* Write the negation of the following

a) If it is raining, then the game is cancelled.

$p \rightarrow q \Leftrightarrow \sim(\sim p \vee q)$

$\sim(\sim p \vee q) \Leftrightarrow p \wedge \sim q$

It is raining and the game is not cancelled.

b) ~~If he~~ ~~studies~~ ~~then~~ or ~~he will pass the examinations~~ doesnt

b) If he studies then he will pass the examinations : $p \to q$

$\sim(\sim p \vee q) \Leftrightarrow p \wedge \sim q$

He doesn't study & he will not pass the examinations.

* $(p \to q) \to r$ and $p \to (q \to r)$ logically equivalent ?

Justify your answer using expressions of logic as well as truth tables:

$(p \to q) \to r : (\sim p \vee q) \to r$

$\Leftrightarrow \sim(\sim p \vee q) \vee r$

$\Leftrightarrow (p \wedge \sim q) \vee r$

$(p \vee r) \wedge (\sim q \vee r)$

$p \to (q \to r) : p \to (\sim q \vee r)$

$\Leftrightarrow \sim p \vee (\sim q \vee r)$

$\Leftrightarrow \cancel{(\sim p \vee \sim q) \vee (\sim p \vee r)}$

$\because$ LHS $\neq$ RHS ; they are not logically equivalent

Truth Table:

p	q	r	$(p \to q)$	$(p \to q) \to r$	$q \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

$p \to (q \to r)$ and $(p \to q) \to r$ are not logically equivalent ; since they are not having same truth values.

* If you study hard, then you will excel.

Converse:

If you will excel, then you study hard.

Contrapositive

If you will not excel, then you will not study hard.

logic negation : $\sim(p \to q)$

$\sim(\sim p \vee q) \Leftrightarrow p \wedge \sim q$

*Duality law:

Two formulas A and A* are said to be duals of each other if either one can be obtained from the other by replacing $\boxed{\wedge \text{ by } \vee}$ and $\boxed{\vee \text{ by } \wedge}$. The connectives $\vee$ and $\wedge$ are called duals of each other. If the formula A contains the special variable T/F then A* its dual is obtained by replacing T by F and F by T in addition to the above mentioned interchanges.

→ Write the dual of the following formulae.

(i) $(p \vee q) \wedge r \Rightarrow (p \wedge q) \vee r$

(ii) $(p \wedge q) \vee T \Rightarrow (p \vee q) \wedge F$

(iii) $(p \wedge q) \vee (p \vee \neg(q \wedge \neg s)) \Rightarrow (p \vee q) \wedge (p \wedge \neg(q \vee \neg s))$

*Result:

The negation of the formula is equivalent to its dual in which every variable is replaced by its negation.

We can prove:

$\neg A(P_1, P_2, \cdots P_n) \Leftrightarrow A^*(\neg P_1, \neg P_2, \cdots \neg P_n)$

* Prove that.

a) $\neg(p \wedge q) \rightarrow (\neg p \vee (\neg p \vee q)) \Leftrightarrow (\neg p \vee q)$.

LHS:

$\neg(p \wedge q) \longrightarrow (\neg p \vee (\neg p \vee q))$

By Implication condition

$\Leftrightarrow \neg(\neg(p \wedge q)) \vee (\neg p \vee (\neg p \vee q))$.

By associative law.

$\Leftrightarrow (p \wedge q) \vee ((\neg p \vee \neg p) \vee q)$

By Idempotent Law

$\Leftrightarrow (p \wedge q) \vee (\neg p \vee q)$

~~$\Leftrightarrow (q \wedge p) \vee (q \vee \neg p)$~~

$\Leftrightarrow ((p \wedge q) \vee \neg p) \vee q$

$(p \vee \neg p) \wedge (q \vee \neg p) \vee q$

$\Leftrightarrow T \wedge (q \vee \neg p) \vee q$

$\Leftrightarrow (q \vee \neg p) \Leftrightarrow (\neg p \vee q)$

(b) $(p \vee q) \wedge (\neg p \wedge (\neg p \wedge q)) \Leftrightarrow (\neg p \wedge q)$

from (a)

$(p \wedge q) \vee (\neg p \vee (\neg p \vee q)) \Leftrightarrow (\neg p \vee q)$

Dual:

$(p \vee q) \wedge (\neg p \wedge (\neg p \wedge q)) \Leftrightarrow (\neg p \wedge q)$

*Predicate:

A predicate is a declarative sentence which:

1. contain one (or) more variables.
2. It is not a proposition
3. produces a proposition when each of its variables is replaced by specific value from universe of discourse.

Ex: $P(x)$: x is a rational number.

*UNIVERSE OF COURSE:

It specifies the possible values of variables is a propositional function.

* I^{st} part x is a subject
  II^{nd} part is called predicate. [rational number]
  ($\because$ it refers the property of a subject)

$P(x)$ is a propositional statement.

*PROPOSITIONAL FUNCTION:

A prepositional function defined on a given set A is an expression $p(x)$ which has the property that $P(a)$ is true (or) false for each $a \in A$ i.e. $P(x)$ becomes a proposition whenever any element is assigned for x.

*Quatifiers: If all the variables in a propositional function are assigned values then the resulting will have a truth ~~table~~ value i.e. It becomes a proposal creating a propositional function while assigning values to a subject is called Quantifiers.

Quantifiers:
- Universal Quantifiers
- Existential Quantifiers.

UNIVERSAL QUANTIFIER:

Universal Quantification of a propositional function $p(x)$ is the proposition "$p(x)$ is true for all values of x". in the universe of discourse.

The notation '$\forall x, P(x)$' denotes universal quantification of $P(x)$ | $\forall$ → Universal Quantifier |

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The proposition is also expressed as "for all x", $P(x)$ is true" (or) "for every x $P(x)$ is true" (or) "for each x, $P(x)$ is true".

* **Express the statements using Quantifiers:**

Ex: Every student in CSE have studied JAVA language

- Here universe of discourse is set of students of CSE class.
- x has studied JAVA language.

$p(x)$: 'x' is a student of CSE class

$q(x)$: x has studied JAVA language $\forall x$,

$p(x) \rightarrow q(x)$.

Ex: If $p(x)$ is a statement $p(x): x+1 > x$ then what is truth value of quantification $\forall x, P(x)$ when the universe of discourse is a set of real numbers.

Ex: What is the truth value of $\forall x, P(x)$ where $p(x)$ denotes $x^2 < 10$ and universe of discourse is +ve integers not exceeding 4

$P(x)$ is true for 0,1,2,3 and false for 4

$4^2 < 10$ is false

$\therefore$ Quantification $\forall x$ $P(x)$ is false.

* **Existential Quantifier:**

The existential quantification of $p(x)$ is a proposition there exists an element x in the universe of discourse such that $P(x)$ is true.

* The notations "$\exists x, P(x)$" denotes the existential quantification of $P(x)$.

Here $\exists$ is known as existential quantifier it can also be expressed as "There is an x such that $P(x)$ is true"

(OR)

"There is atleast one x such that $P(x)$ is true".

(OR)

"for some x $p(x)$ is true".

Ex: $P(x) : x > 3$

Universe of discourse is set of real numbers.

∴ The above proposition fn can be written as "There exists x, $P(x)$" i.e. '$\exists x, P(x)$', $P(x)$ is true if $x > 3$.

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*

* "Every student in this class attended either DIS or DS." Using quantifiers write the argument / symbolic form of given statement.

→ UOD: set of students in the class, say x

$\forall x [p(x) \vee q(x)]$ where

$p(x) \rightarrow x$ has attended DIS

$q(x) \rightarrow$ " DS

* "No one is perfect."

"Not everyone is perfect."

"All of your friends are perfect."

"One of your friend is perfect."

"Everyone of your friend and is perfect."

"Not everybody is friends or someone is not perfect."

$x \rightarrow$

UOD: set of friends, say x.

→ $\boxed{p(x) \rightarrow x \text{ is perfect.}}$ where x is a person.

$f(x,y) \rightarrow$ friends ; x & y are friends.

→ $\boxed{f(x,y) : y \text{ is a friend of } x}$

1) $\forall x [\sim p(x)]$

2) $\exists x [\sim p(x)]$

3) $\forall x, \forall y [f(x,y) \rightarrow p(x)]$

4) $\forall x, \exists y [f(x,y) \rightarrow p(x)]$

5) $\forall x, \forall y [f(x,y) \wedge p(x)]$

6) $\exists x \exists y [\sim f(x,y) \vee \sim p(x)]$

$\boxed{\text{'are'} : \rightarrow}$

*

Translate the statement:

$\exists x \forall y \forall z [F(x,y) \wedge F(x,z) \wedge y \neq z \rightarrow \sim F(y,z)]$

$F(a,b) \rightarrow a, b$ are friends

UOD: set of students in the college.

* For some students, ~~for every students~~ y, z ~~such that x, y are friends and~~

There are some students ; for all students such that x, y are friends and x, z are friends but y, z are strangers then y, z are not friends.

*1) All humming birds are richly coloured.

~~No... is richly coloured~~

2) No large birds live on honey.

3) Birds that do not live on honey ~~and~~ are dull in colour.

4) Humming birds are small.

UOD: set of birds, say x

$z(x) \rightarrow$ Humming bird

$q(x) \rightarrow$ Richly coloured

$p(x) \rightarrow$ Large

$r(x) \rightarrow$ Live on honey.

~~1) $\forall x [q(x)]$~~

1) $\forall x [z(x) \rightarrow q(x)]$

2) $\exists x [\sim p(x) \wedge r(x)]$

3) $\forall x [\sim r(x) \rightarrow \sim q(x)]$

4) $\forall x [z(x) \rightarrow \sim p(x)]$

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* PROOF STRATEGY:

Argument: An argument in mathematics are logic is a finite sequence of statements $P_1, P_2 \ldots P_n$ such that $(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \rightarrow P$

Each statement in the sequence $P_1, P_2 \ldots P_n$ is called as premise or assumption (or) hypothesis and P is called as conclusion logical implication. The prop P logically follows from the props $P_1, P_2 \ldots P_n$ if P is true whenever $P_1, P_2, \ldots P_n$ are true i.e. $(P_1 \wedge P_2 \wedge \ldots P_n) \rightarrow P$

Theorem: A property that is proved to be true is known as a theorem.

Proof (or) disproof: A proof of the property 'P' is a mathematical argument consisting of a sequence of statements $P_1, P_2 \ldots P_n$ from which p logically follows so p is called as conclusion of the argument.

It is also said as a mathematically

acceptable evidence that shows that a property is universally true.

If instead of showing a property is true if we prove that as false (i.e. $\sim p$ is true) then such a proof is called direct proof.

* Conjectors:

A mathematical proof which can neither be proved nor disproved is called conjector. If a conjector is proved then it would be a theorem and if not proved it is negation.

* Methods of proving theorems:

There are 2 different methods of proving theorem

(i) Direct proof

(ii) Indirect proof.

A direct proof of implication $p \to q$ is a logically valid argument that begins with the assumptions that p is true and in one or more applications of the law of detachment concludes that q must be true.

In a direct proof of $p \to q$ we start by assuming that p is true then in one or more steps we conclude that q is true.

Indirect proof:

There are 2 ways to prove theorem by indirect proof method:

(i) proof by contrapositive

(ii) proof by contradiction.

(i) Proof by contrapositive:

Since the implication $p \to q$ is equivalent to its contrapositive $\sim q \to \sim p$ then the $p \to q$ can be proved by showing that its contrapositive $\sim q \to \sim p$ is true. An argument of this type is called indirect proof by contrapositive.

(ii) Proof by contradiction:

In this method to prove is true in an implication $p \to q$ we start assuming that q is false i.e. $\sim q$ is true, then by a logical argument we arrive at a situation where a propostion is true as well as false i.e. we reach a contradiction. This can happen only when $\sim q$ is false. Therefore, q must be true.

This method is called proof by contradiction or reductive absurdum. It is so called as it relies on reducing a given assumption to an absurdity.

Eg: Give a direct proof of the theorem "if n is odd n^2 is odd".

Sol: Let 'n' be odd.

i.e. $n = 2k+1$ — ① $k \in \mathbb{Z}$

We have prove n^2 is odd.

consider $n^2 = (2k+1)^2$

$= 2(2k^2) + 4k + 1$

$= 2(2k^2 + 2k) + 1$

$= 2q + 1$

where $q = 2k(k+1) \in \mathbb{Z}$

$\Rightarrow n^2$ is odd.

* Eg: Let n be an odd integer such that "sum of two-odd integers is even".

Let n_1 be a odd integer

$n_1 = 2k+1$

let n_2 be an other odd integer

$n_2 = 2m+1$

$n_1 + n_2 = 2k+1+2m+1$

$= 2(k+m+1) = 2q$ $\quad q = k+m+1$

$\therefore n_1 + n_2$ is even

* Use the direct proof to show that "the product two odd nos is odd".

* Let n_1 be an odd no

$n_1 = 2k+1$;

n_2 be an odd no.

$n_2 = 2m+1$

$n_1 \times n_2 = (2k+1)(2m+1)$

$= 4km + 2k + 2m + 1$

$= 2(2km + k + m) + 1$

$= 2q + 1$ $\quad q = 2km + k + m$.

$n_1 \times n_2$ is odd.

* S.T. the square of an even no. is an even no.

Let n be an even no $n = 2k$

Square of $n = n^2 = (2k)^2 = 4k^2$

$= 2(2k^2)$

$= 2q$.

n^2 is even.

* ~~P.T. if n is a +ve integer then n is odd if 5n+6 is odd (proof by contradiction.~~

~~n is odd iff 5n+6 is odd~~

~~Firstly, suppose 'n' is odd $\Rightarrow n = 2k+1, k \in \mathbb{Z}$~~

~~& 5n+6 is odd.~~

~~$5n+6 = 5(2k+1) + 6 = 10k+11$~~

~~$= 2(5k+5) + 1 = 2l+1$.~~

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* FUNCTIONS:

Let A & B be any 2 non-empty sets, a function f from A to B is an assignment of exactly one element of B to each element of A.

* We write $f(a) = b$ if b is a unique element of B assigned by the function f to the element a of A.

* If f is a function from A to B; we write $f: A \to B$ function is also known as mapping or tansformation.

* If f is func from A to B, we say that A is a domain of f and B is the codomain of f.

* If $f(a) = b$, we say that b is image of a and a is the pre-image of b.

* Let f_1, f_2 be the functions from A to R then $f_1 + f_2$ & $f_1 f_2$ are also functions from A to R defined by

(i) $f(x_1 + x_2) = f(x_1) + f(x_2)$

(ii) $f(x_1 \cdot x_2) = f(x_1) \cdot f(x_2)$

Proof:

1) Let f_1 and f_2 be functions of R to R such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What are the functions $f_1 + f_2$ & $f_1 f_2$

$f_1 + f_2 = x^2 + x - x^2 = x$

$f_1 f_2 = x^2(x - x^2) = x^3 - x^4$

2) Let $A = \{a, b, c, d, e\}$ & $B = \{1, 2, 3, 4\}$ with $f(a) = 2$; $f(b) = 1$; $f(c) = 4$; $f(d) = 1$; $f(e) = 1$. The image of subset of $S = \{b, c, d\}$ is the set $f(S)$ is 1 & 4.

$t = \{b, d, e\} \Rightarrow f(t) = 1$

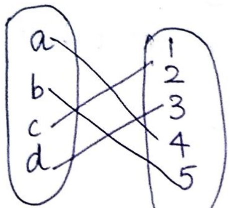
* One to one function (1-1):

A function f is said to be 1-1 (or) injective if & only if $f(a) = f(b) \Rightarrow a = b \ \forall a$ in the domain of f and function is said to be an injection (or) 1-1.

* Note: A func f is 1-1 iff $f(a) \neq f(b)$ whenever $a \neq b$ f is 1-1 obtained by taking contrapositive of the implicat

*→ Determine whether the func 'f' from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) =$

$f(b)=5$; $f(c)=1$ & $f(d)=3$ is 1-1 (or) not.



The func f is 1-1 because f takes diff. values of the 4 elements from the domain.

2) Determine whether the func $f(x)=x+1$ from the set of R to itself is one to one.

Yes f is one to one function because

$f(x) \neq f(y)$

$x+1 \neq y+1$

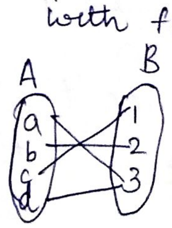
$x \neq y$

* Onto function (or) Surjection:

A function f from A to B is called onto iff $\forall\ b \in B\ \exists\ a \in A$ with $f(a)=b$ then the function f is called surjection.

Consider $A=\{a,b,c,d\}$ & $B=\{1,2,3\}$

with $f(a)=3$, $f(b)=2$; $f(c)=1$ & $f(d)=3$



Every element in the codomain B have pre-images in A.

* Is the func $f(x)=x^2$ from the set of integers to $\mathbb{Z}$?

* No, the func f is not onto because there are no integers in domain whose square is negative number in codomain.

* Is the func $f(x)=x+1$ from $\mathbb{Z} \to \mathbb{Z}$ is onto?

Yes, f is onto.

* Bijection (or) one-one and onto:

If func $f: A \to B$ is called bijection iff it satisfies one-one & onto, such mapping is also called a 1-1 correspondance b/w A & B.

1) If $f(x)=2x+1$ for $x \in R$ & $f: R \to R$ is f a bijection;

By the def of 1-1;

$f(x)=f(y)$

$2x+1=2y+1$

$2x=2y$

$x=y$

$\therefore f$ is one-one.

$f(x)=y$

$2x+1=y$

$x=\frac{y-1}{2} \in R$

f is onto $\because$ every element in codomain has pre-image in domain.

$\therefore f$ is bijection.

* Identity Function:

Let A be any set & f be a function such that $f: A \to A$ is defined by $f(a) = a$ $\forall a \in A$; then f is called as identity function (or) identity transformation.

* Note: Identity fn is always a bijection

$I_x(x) = I_y(y) \Rightarrow x = y$.

I_x is also onto since $I_x = x \ \forall x$

Ex:

1) $f: Z \to Z$ with $f(z) = x^2$ is not one-one & onto.

* Composite Functions:

Let $f: X \to Y$; $g: Y \to Z$ be 2 functions then the composition of f & g is fog (or) gof is the function from x to z and given as

$gof = g[f(x)]$

~~1) Let $x = \{1, 2, 3\}$ & f, g, h,~~

* $f(x) = x+2$; $g(x) = x-2$ & $h(x) = 3x$ for $x \in R$.

Find gof, fog, fof, gog, foh, hog, hof & fohog.

$gof = g(x+2) = x+2-2 = x$

$fog = f(x-2) = x-2+2 = x$

$fof = f(x+2) = x+2+2 = x+4$

$gog = g(x-2) = x-2-2 = x-4$.

$foh = f(3x) = 3x+2$

$hog = h(x-2) = 3x-6$

$hof = h(x+2) = 3x+6$

$fohog = f(h(x-2)) = f(3x-6) = 3x-6+2$
$= 3x-4$

* Inverse:

A function $f: X \to Y$ is said to be invertible of its inverse func f^{-1} is also func from the range of x x f.

A function $f: X \to Y$ is invertible $\Leftrightarrow$ f is bijecti

1) Let $X = \{a, b, c, d\}$; $Y = \{1, 2, 3, 4\}$ &

$f: X \to Y$ be given by

$f = \{(a,1), (b,2), (c,2), (d,3)\}$ Is f^{-1} a func (invertible) }

f is not one-one & onto $\Rightarrow f^{-1}$ is not a fu

f is not invertible.

* In addition to the above functions; another 2 important functions are useful to analyze; the ~~sto~~ data to solve the problems of a particular size in no. theory

→ These functions are

- Floor function
- ceiling function.

FLOOR FUNCTION

Let x be a real number; the floor function rounds x to the closest integer less than or equal to x.

Denoted by : $\lfloor x \rfloor$

CEILING FUNCTION

Let x be a real number; the ceiling function rounds x ~~to~~ upto the closest integer greater than or equal to x.

Denoted by : $\lceil x \rceil$

*>

Floor value of half : $\lfloor 1/2 \rfloor = 0$

$-1/2$ $\lfloor -1/2 \rfloor = -1$

3.1 : $\lfloor 3.1 \rfloor = 3$

7 : $\lfloor 7 \rfloor = 7$

$$\left.\begin{array}{l} \lceil 1/2 \rceil = 1 \\ \lceil -1/2 \rceil = 0 \\ \lceil 3.1 \rceil = 4 \\ \lceil 7 \rceil = 7 \end{array}\right\} \text{Ceiling}$$

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* PIGEON - HOLE PRINCIPLE:

It states that if there are more pigeons than pigeon holes then there must be 1 hole with atleast more than 1 pigeon in it.

* In general; if k is real; if $k+1$ (or) more objects are placed into k boxes then there is atleast 1 box with 2 or more of the objects.

→ If possible suppose that none of the k boxes can contain more than 1 object. Then the total no. of objects would be atmost k. This is a contradiction since there are $k+1$ objects.

This principle is also known as n Dirichlet-drawer principle.

* Generalised Pigeon-Hole principle:

If n objects are placed into k boxes then there is atleast 1 box containing $\lceil n/k \rceil$ no. of objects in it.

Here 2 cases arise:

Case1: When n isn't multiple of k;

in this case an assignment is made into the box to minimise the no. of obj per box by spreading the objects around evenly.

Suppose m obj's are placed per box where $m*k<n$ then the remaining are placed in $(n-m*k)$ boxes i.e. $m = \lfloor \frac{N}{K} \rfloor$ objects are placed in the boxes.

Also some of the boxes will have one more than that i.e. some of the boxes will have more $\lceil \frac{N}{K} \rceil$ no. of objects.

∴ There exists atleast one box containing $\lceil \frac{N}{K} \rceil$ no. of

Eg: If $N=10$; $K=4$

$2 \times 4 < 10$

$8 < 10$

So there are atleast 2 objects in each box

left over obj's $= N - m \times n = 10-8 = 2$

* The boxes with even distribution of no. of obj $= \lfloor \frac{N}{K} \rfloor = \lfloor \frac{10}{4} \rfloor = 2$

→ Move the 2 objects $= \lceil \frac{N}{K} \rceil = \lceil \frac{10}{4} \rceil = 3$

Case: 2:

When N is a multiple of K.

In this case when assignment of objects is made we have $m = \lfloor \frac{N}{K} \rfloor$ objects $= \lceil \frac{N}{K} \rceil$

i.e. there exists atleast 1 box containing $\lceil \frac{N}{K} \rceil$ no. of objs. Hence proved.

* Show that there are atleast 9 people who were born in the same month if there are 100 people.

→ It can be reframed as distributing 100 obj's (people) into 12 boxes (months).

By pigeon-hole principle; there will be atleast $\left\lceil \frac{100}{12} \right\rceil$ people born in the same month i.e. 9 people born in same month.

* What is the min no. of students required in DS class to show that atleast 6 will recieve the same grade if there 5 possible grades A, B, C, D, E

→ By pigeon hole principle we have

$\left\lceil \frac{N}{5} \right\rceil = 6$ where N is the smallest integer i.e. min no. of students required in DS class.

$\boxed{N = 26}$

* P.T. if 'n' is a +ve integer then n is odd iff 5n+6 is odd"

n is odd ⇒ $n = 2k+1$; $k \in \mathbb{Z}$ and we prove 5n+6 is odd.

$5n+6 = 5(2k+1) + 6$
$= 10k + 11$
$= 2(5k+5)+1$
$= 2l+1$ $\quad 5k+5 \in \mathbb{Z}^+$

5n+6 is odd

Conversely suppose that 5n+6 is odd.

$5n+6 = 5(2l) + 6$
$= 10l + 6 = 2(5l+3) = 2k$. $\quad k = 5l+3 \in \mathbb{Z}$

∴ 5n+6 is even.

→ Contradiction to the hypothesis that 5n+6 is odd. This contradiction is of our false supposition that n is not odd.

* Give an indirect proof of if 3n+2 is odd then n is odd. (By contrapositive position)

Sol: $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Let 3n+2 is odd.

We have to proof is n is odd.

We know that by indirect proof i.e. by contrapositive

i.e. If n is even then 3n+2 is even;

so let 'n' is even $n = 2k$; $k \in \mathbb{Z}$.

consider, $3n+2 = 3(2k)+2 = 6k+2$
$= 2(3k+1) = 2l$.
$l = 2k+1 \in \mathbb{Z}$.

* Prove that the integer n is odd iff n is odd. S: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

We have to prove that if n is odd then n^2 is odd.

firstly let n is odd; $n = 2k+1$

Consider $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$

$=2(2k^2+2k)+1 = 2l+1 \quad l = 2k^2+2k \in \mathbb{Z}$

n^2 is odd.

Now, conversely suppose that n^2 is odd;

we have to prove n is odd.

If possible, suppose that n is not odd i.e.

n is even $\Rightarrow n = 2k$.

$$n^2 = (2k)^2 = 4k^2 = 2(2k^2) = 2l.$$

$l = 2k^2 \rightarrow n^2$ is even.

This is the contradiction to the hypothesis n^2 is odd. This is because of our false suppostion that n is even which is false

$\therefore$ n is odd is true.

* S.T. "if n is an integer and n^3+5 is odd then n is even" using (a) proof by contrapositive (b) proof by contradiction.

Let n^3+5 is odd and n is integer

We have to p.t. n is even.

By contrapositive; suppose n is odd.

& P.T. n^3+5 is even. $\quad$ n is odd $\Rightarrow n = 2k+1$

consider $n^3+5 = (2k+1)^3+5$.

$= 2(4k^3+2k^2+k+3) = 2l$

$\Rightarrow n^3+5$ is even

By contradiction; suppose n is odd

$n^3+5 = (2k+1)^3+5 = 2(4k^3+2k^2+k+5)$

$= 2l. \quad l \in \mathbb{Z}$

$\therefore$ n^3+5 is even.

This is the contradiction to the hypothesis that n^3+5 is odd. This is because of our false supposition that n is odd, which is false.

$\therefore$ n is even is true.

* P.T. $\sqrt{2}$ is irrational by giving a proof by contradiction (reductive absurdum)

S: Let p: $\sqrt{2}$ is irrational

we have to prove 'p' is true.

Now we prove this by contradiction proof.

if possible, let p is not irrational i.e. p is rational.

$\sqrt{2} = \frac{a}{b}$ — (1) where $a, b \in \mathbb{Z}$ and $b \neq 0$

a & b are in lowest terms.

$\sqrt{2}b = a$ $\quad$ i.e. they don't have common terms.

$a^2 = 2b^2$ — (2)

$2/a^2$ [$\because$ by divisibility]

$\frac{2}{a^2} = \frac{2}{a}$ — (3)

$a = 2c \quad c \in \mathbb{Z}$.

SOBS $\Rightarrow a^2 = 4c^2 \quad 2b^2 = 4c^2$

$\boxed{b^2 = 2c^2}$