Unit-1- BEM

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1-1 SCOPE OF MECHANICS

The subject of mechanics occupies a unique position in the physical sciences because it is fundamental to so many fields of study. In its broadest sense, mechanics may be defined as the science which describes and predicts the conditions of rest or motion of bodies under the action of forces. In a narrower sense, mechanics may be divided into areas which, with some overlap, are of separate interest to physicists and to engineers.

- Engineering Mechanics
  - Mechanics of solids
    - Rigid bodies
      - Statics
      - Dynamics
        - Kinematics
        - Kinetics
    - Deformable bodies
      - Strength of materials
      - Theory of elasticity
      - Theory of plasticity
  - Mechanics of fluids
    - Ideal fluids
    - Viscous fluids
    - Compressible fluids

Figure 1-1.1 Abbreviated outline of engineering mechanics.

1.3 FUNDAMENTAL CONCEPTS AND AXIOMS

RIGID BODY

A rigid body is defined as a definite amount of matter the parts of which are fixed in position relative to one another. Actually, solid bodies are never rigid; they deform under the action of applied forces. In many cases, this deformation is negligible compared to the size of the body and the body may be *assumed* rigid. Bodies made of steel or cast iron, for example, are of this type. The study of *strength of materials*, however, is based on the deformation (however small) of such bodies.

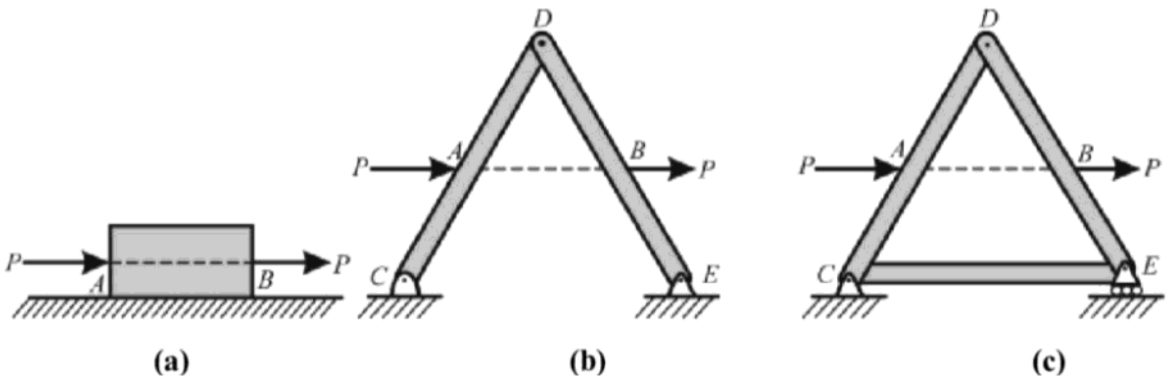
FORCE

Force is the action exerted by one body upon another. Its *external* effect upon a body is manifested by a change in, or a tendency to change, the state of motion of the body upon which it acts. The *internal* effect of a force is to produce stress and deformation in the body. The *characteristics* of a force are:

1. its magnitude,
2. the position of its line of action, and
3. the direction (or sense) in which the force acts along its line of action.

The *unit* of force commonly used in India is Newton (N) or multiples of the Newton such as the kN (i.e., 1 kN = 1000 N).

The *principle of transmissibility* of a force states that the *external* effect of a force on a rigid body is the same for all points of application along its line of action; i.e., it is independent of the point of application. Its *internal* effect, however, definitely is associated with the point of application of the force. Thus, in Fig. l-3,l(a), the motion of the block will be the same whether it is pushed at *A* or pulled at *B*. The local internal effects at *A* and *B*, however, will be quite different. Note that the principle of transmissibility applies only to the external effect of a force on the *same* rigid body. Observe that the system of two bars in Fig. 1-3.l(b), hinged together at *D*, is not inherently rigid; it is the hinge supports at *C* and *E* that restrict relative movement of the bars. In this case,



therefore, since we do not have a single rigid body, both the external reactions and the internal effects will be different if *P* is applied first at *A* and then at *B*. However, if the hinge support at *E* is replaced by a roller support and another bar *CE* added as in Fig. 1-3.l(c), we shall have a triangular truss equivalent to a single rigid body. Here the external reactions will be the same whether *P* is applied at *A* or at *B*, although the internal effects will differ since *P* applied at *A* tends to bend bar *CD* while *P* acting at *B* tends to bend bar *DE*.

AXIOMS OF MECHANICS

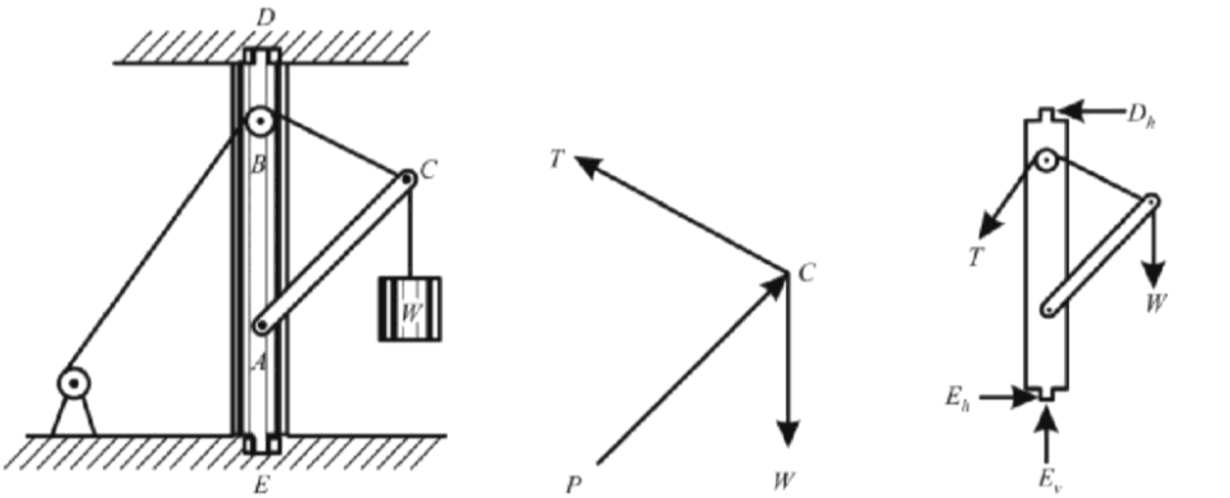
The principles of mechanics are postulated upon several more or less self-evident facts which cannot be proved mathematically but can only be demonstrated to be true. We shall call these facts the axioms of mechanics. They will be discussed at length in subsequent sections as they are used. At this time, we merely collate them for reference and state them in the following form:

1. The parallelogram law: The resultant of two forces is the diagonal formed on the vectors of these forces.
2. Two forces are in equilibrium only when equal in magnitude, opposite in direction, and collinear in action.
3. A set of forces in equilibrium may be added to any system of forces without changing the effect of the original system.
4. Action and reaction forces are equal but oppositely directed.

1-5 INTRODUCTION TO FREE-BODY DIAGRAMS

One of the most important concepts in mechanics is that of the free-body diagram. This concept is discussed in detail in Chapter 3 where we first really use it. It is introduced here to help you distinguish between action and reaction forces. To do so, it is necessary to *isolate* the body being considered by removing all elements that act upon the isolated body. A sketch of the isolated body which shows only the forces acting *upon it* by the removed elements is defined as a *free-body diagram*. The forces acting on the isolated or free body are the action forces, also called the applied forces. The reaction forces are those exerted by the free body upon other bodies. One exception to this concept concerns ground supports whose actions on the free body are commonly called reactions.

The free body may consist of an entire assembled structure or an isolated part of it, depending on which quantities are to be determined. For example, consider the derrick shown in Fig. 1-5.1(a). To determine the force in the boom AC, we would isolate pin C whose free-body diagram (Fig. l-5.l(b)) exposes this force as one of those acting upon it. These forces consist of the gravitational attraction of the weight W, the pull T exerted on the pin by the cable, and the force P exerted by the boom. If the effect of the supports at D and E were desired, a free-body diagram of the entire derrick, showing it isolated from these supports as in Fig l-5.l(c), would expose their action upon the derrick. Observe that the force P in the boom is not shown here since the boom is part of the entire derrick. Remember that a free-body diagram shows only forces exerted by the removed elements that are necessary to isolate the body. Therefore, since P acts in an element that was not removed, its action is internal and is not exposed.



(a) Derrick

(b) Free-body diagram of pin C

(c) Free-body diagram of derrick

Figure 1-5.1 Free-body diagrams.

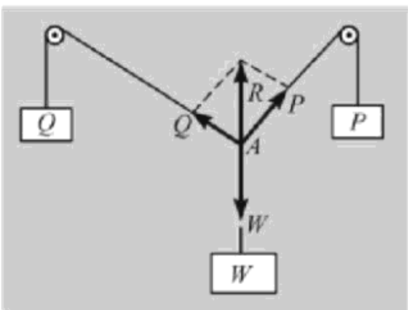
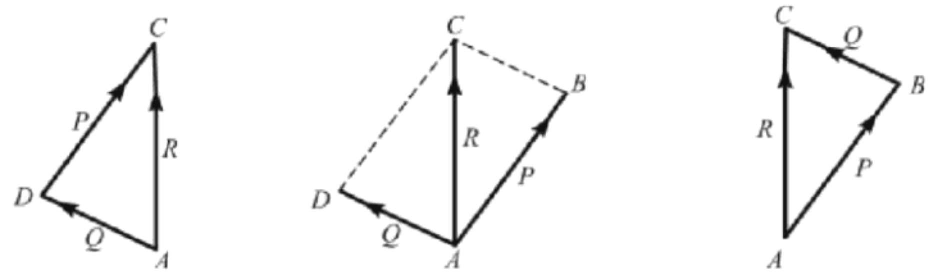


Figure 2-1.1 Parallelogram law.

The tensions in these cords will then be equal to the weights *P* and *Q*. Draw vectors *P* and *Q* to scale from point A where the cords are tied together and construct a parallelogram with these vectors as the initial sides. It will be found that the diagonal *R* of the parallelogram scales exactly to the value of *W* and is in line with the vector representing *W*.

Since it is axiomatic that two equal, opposite, collinear forces are in equilibrium, we conclude that weight *W* will be perfectly supported by the force *R*. In other words, the net effect of the forces *P* and *Q* may be replaced by the single force *R*. Such a force is called a resultant. The parallelogram law may now be stated as follows: *The resultant of two forces is the diagonal of the parallelogram whose initial sides are the vectors of these forces*. The diagonal to be used is that which emanates from the intersection of the initial sides.

The parallelogram formed by two vectors P and Q is divided by their resultant R into two congruent triangles as is evident in Fig. 2-2.2(b). If the triangle ABC were drawn alone as in part (c), the vector joining the tail of P to the tip of Q would have the same magnitude and direction as the resultant R defined by the parallelogram law. In this instance, however, force Q has been represented by the free vector BC. A *free vector* is defined as one which may be freely moved in space, as distinguished from a *localized vector* which is fixed or bound to a specific point of application. It is also evident that the triangle ADC in part (a) may also be used to determine R. In this case, P is taken as the free, vector whereas Q is the localized vector.

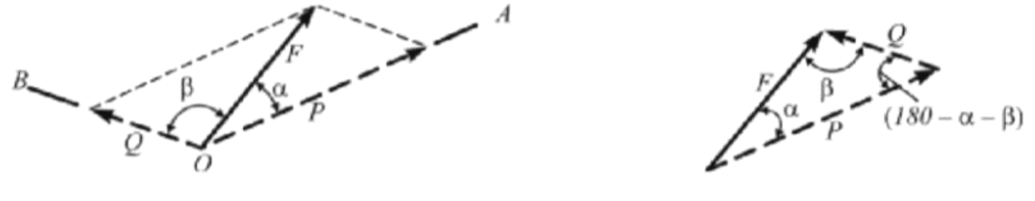


(a) (b) (c)

We may now state the *triangle law* as a convenient corollary of the parallelogram law: *If two forces are represented by their free vectors placed tip-to-tail, their resultant is the vector directed from the tail of the first vector to the tip of the second vector.*

2-3 FORCES AND COMPONENTS

The parallelogram law shows how to combine two forces into a resultant force. Of equal importance is the inverse operation, called resolution, in which a given force is replaced by two components which are equivalent to the given force. The method is demonstrated in Fig. 2-3.l(a) in which we are to replace force F by components directed along the lines OA and OB radiating from the tail O of F. We need to merely draw lines from the tip of F parallel to the specified directions to form the parallelogram shown. The initial sides P and Q of this parallelogram are the desired components. Obviously for the parallelogram law to be applied the components P and Q must intersect on F.



(a) (b)

This graphical construction can also be made by the triangle law as shown in part (b) which is convenient for slide rule solution using the sine law relation:[1]

$$\frac{F}{\sin(180-\alpha-\beta)}=\frac{P}{\sin\beta}=\frac{Q}{\sin\alpha}$$

Although the triangle law is more convenient for analytical solution, it is the localized components shown in part (a) which completely replace F. Recall that the basic demonstration of the parallelogram law required that the components of the resultant (F in this instance) must intersect on the line of action of the resultant force.

Depending on the directions specified, there are an infinite number of pairs of oblique components of F that may be found. Such nonorthogonal components, however, are of limited use. Analytically, it is much more convenient to resolve a force into a pair of perpendicular components. Such rectangular components are then readily combined with similarly oriented rectangular components of other forces by adding these components algebraically as we shall show in the next section.

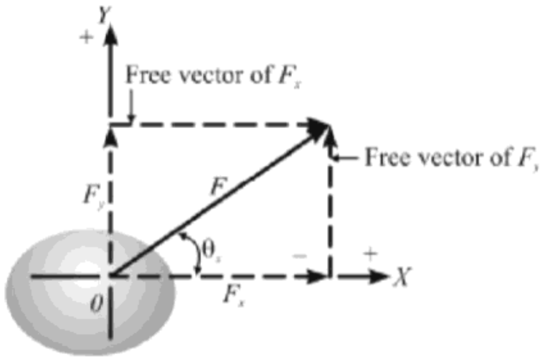


Figure 2-3.2 Rectangular components.

Consider now Fig. 2-3.2 in which force F acts upon the given body. The effect of the force is to move the body rightward and upward. Choosing these directions as the positive directions of perpendicular X and Y reference axes, we project the force F upon them to obtain the perpendicular components F_x and F_y. More precisely, we should draw parallels to the X and Y axes to obtain the basic parallelogram, but when the reference axes are perpendicular, the projected length of the force yields the same components. The relations between these components and F is determined by the basic definitions of sine and cosine of the angle θ_x between F and the X axis; i.e., $\sin\theta_x = F_y/F$ and $\cos\theta_x = F_x/F$ which are usually rewritten in the following form.

$$\begin{aligned} F_x &= F\cos\theta_x \\ F_y &= F\sin\theta_x \end{aligned} \qquad (2\text{-}3.1)$$

The components F_x and F_y are considered positive if they act in the positive directions of the X and Y axes and negative if directed oppositely. Usually the X and Y axes are horizontal and vertical, and their positive directions are those of the common Cartesian coordinate axes. However, the orientation of the X and Y axes is arbitrary; their directions are adapted to the situation. The relations given above are independent of the orientation of the X axis. If desired, the angle θ_y between F and the Y axis may also be used; the components are then given by $F_x = F \sin \theta_y$, and $F_y = F \cos \theta_y$.

It is, obvious that when the rectangular components of a force are known, they completely specify the magnitude, inclination, and direction of the force. For example, assuming values of F_x and F_y to be known, we obtain from Fig. 2-3.2 the following equations:

$$\left.\begin{aligned} F &= \sqrt{(F_x)^2 + (F_y)^2} \\ \tan \theta_x &= \frac{F_y}{F_x} \end{aligned}\right\} \qquad (2\text{-}3.2)$$

The direction of F is determined by the signs of its components and its inclination by the acute angle it makes with the X axis; this is further explained in the illustrative problems.

2-3.1 Determine the X and Y components of the forces P and F shown in Fig. 2-3.3.

Solution

Since P is directed upward to the left, its components act in these directions as shown. With respect to the positive directions of the reference axes, the sign of P_y is plus and of P_x is minus. Applying Eq. (2-3.1). we obtain

$[F_x = F \cos \theta_x\}$ $\quad P_x = -200 \cos 60^\circ = -200(0.5)$

$= -100 \text{ N} = 100 \text{ N}\leftarrow$ ***Ans.***

$[F_y = F \sin \theta_x\}$ $\quad P_y = 200 \sin 60^\circ = 200(0.866)$

$= 173.2 \text{ N} = 173.2 \text{ N}\uparrow$ ***Ans.***

$P_y = 173.2$

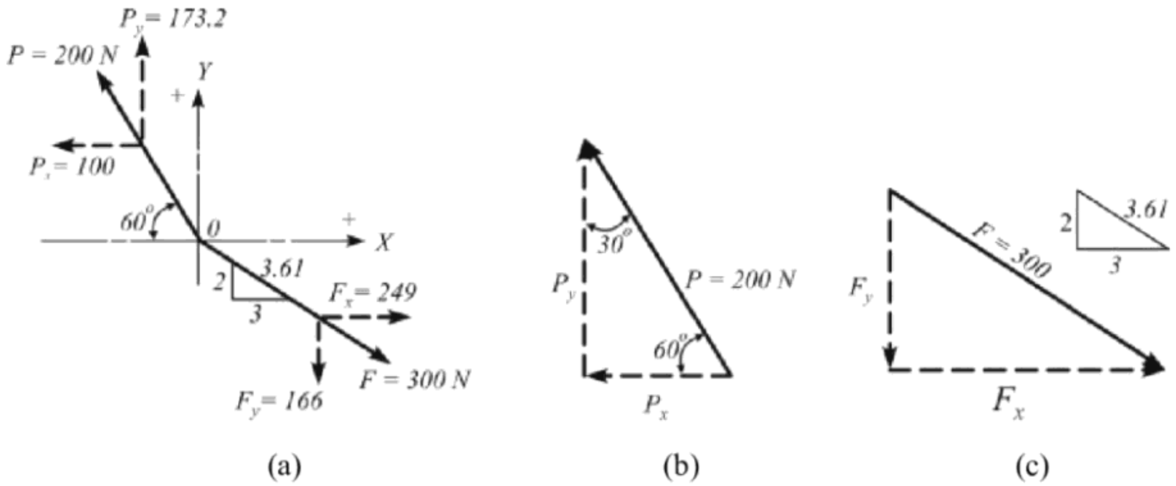


Figure 2-3.3

2-3.3 Determine the X and Y components of each of the forces shown in Fig. P-2-3.3.

For T: $T_x = -307$ N; $T_y = -257$ N ***Ans.***

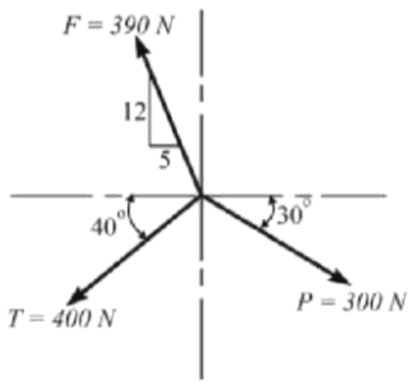


Figure P-2-3.3

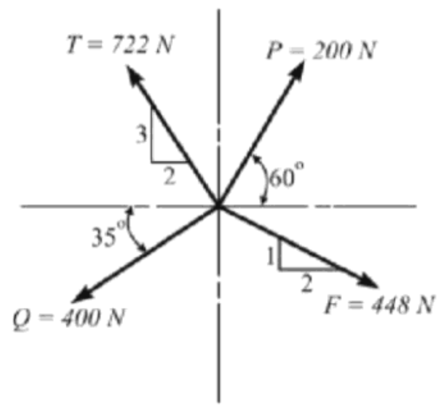


Figure P-2-3.4

2-3.4 Compute the X and Y components of each of the four forces shown in Fig. P-2-3.4.

2-3.5 The body on the incline in Fig. P-2-3.5 is subjected to the vertical and horizontal forces shown. Find the components of each force along X-Y axes oriented parallel and perpendicular to the incline.

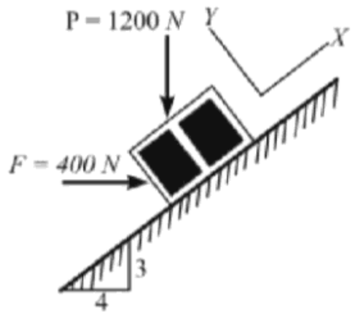
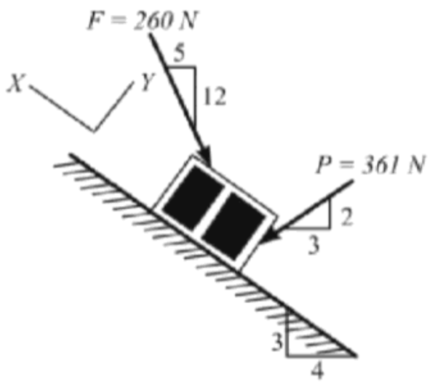


Figure P-2-3.5

2-3.7 Referring to Fig. P-2-3.7, determine the components of force P along the X-Y axes which are parallel and perpendicular to the incline.

Hint: First determine the horizontal and vertical components of the force and then combine the parallel and perpendicular effects of these components.



2-4.1 Determine completely the resultant of the coplanar concurrent force system shown in Fig. 2-4.4.

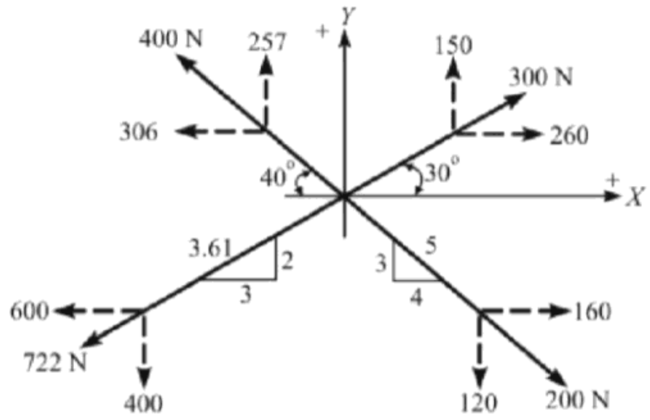


Figure 2-4.4 Components denoted by dashed vectors.

2-4.4 The force system shown in Fig. P-2-4.4 has a resultant of 200 N pointing up along the Y axis. Compute the values of F and θ required to give this resultant.

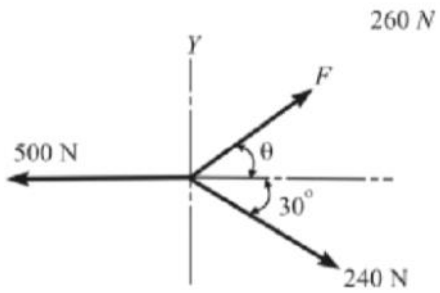


Figure P-2-4.4

F = 369 N at $\theta_x = 47.6°$ ***Ans.***

2-4.6 The block shown in Fig, P-2-4.6 is acted on by its weight $W = 400$ N. a horizontal force $F = 600$ N, and the pressure P exerted by the inclined plane. The resultant R of these forces is parallel to the incline. Determine P and R. Does the block move up or down the incline?

Hint: Take one reference axis parallel to the incline.

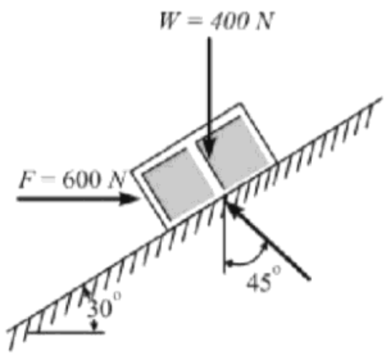


Figure P-2-4.6

$R = 146.5$ N **Ans.**

2-4.7 Two locomotives on opposite banks of a canal pull a vessel moving parallel to the banks by means of two horizontal ropes. The tensions in these ropes are 2000 N and 2400 N while the angle between them is 60°. Find the resultant pull on the vessel and the angle between each of the ropes and the sides of the canal.

$R = 3820$ N; 33° and 27° **Ans.**

2-4.8 Rework Prob. 2-4.7 if the cable tensions are 2000 N and 1500 N.

2-5 COMPONENTS OF FORCES IN SPACE

The spatial counterpart of the parallelogram law is that the resultant R of three concurrent spatial forces P, Q, and F is the body diagonal of the rhomboid shown in Fig. 2-5.1 formed by using these three forces as its initial sides. This conclusion is the result of two applications of the basic parallelogram law; one applied to find an intermediate resultant R_1 of P and Q, and the other combining this intermediate resultant R_1 with F to determine the final resultant R. Except for the fact that the forces are not coplanar, this procedure is the exact parallel of that described in Fig. 2-4.1 and it should be clear that the resultant R could also be formed by the tip-to-tail sum of P, Q, and F taken as free vectors in any order.

Although this construction is straightforward, actually doing it on paper is difficult to perform. Practically, we resort to the concept of finding the components of each force along mutually perpendicular directions, whence combining these components algebraically, we obtain three mutually perpendicular components of the resultant force which is then readily found. This procedure, of course, is the analytical spatial counterpart of Section 2-4.

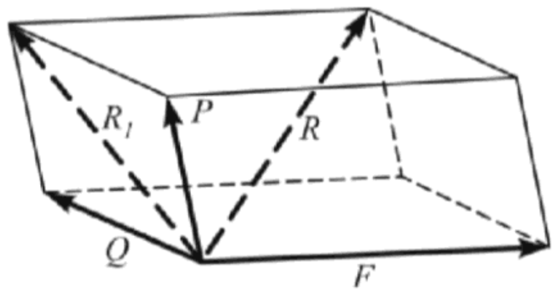


Figure 2-5.1 R is body diagonal of rhomboid having *P*, *Q*, and *F* as initial sides.

To determine the rectangular components of a space force, consider Fig. 2-5.2 showing a force F whose direction is specified by two points along its line of action. The rectangular components of the force are directly proportional to the rectangular components of the distance d separating the two points. This proportionality is an extension of the relations between the sides of a force triangle and its corresponding slope triangle that was discussed in Illus. Prob. 2-3.1. It is expressed by

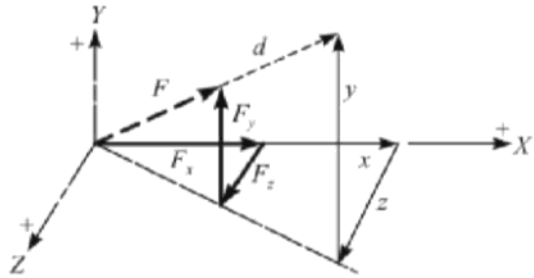


Figure 2-5.2 Proportionality of force components to distance components.

$$\frac{F_x}{x}=\frac{F_y}{y}=\frac{F_z}{z}=\frac{F}{d}=F_m \qquad (2\text{-}5.1)$$

where F_m is the value of the equal ratios and is known as the multiplier of the force F (or more briefly, as the force multiplier) expressed in units of force per unit of length.

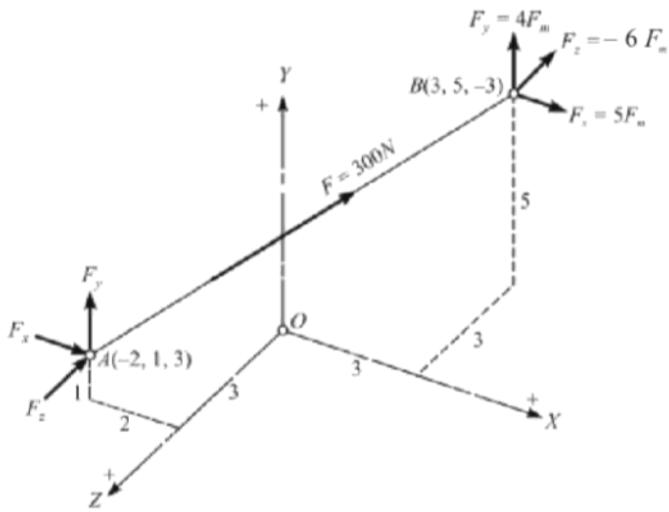


Figure 2-5.3 The components of a force as determined by the coordinates of two points on its line of action.

It is easy to show by repeated applications of the Pythagorean theorem that the magnitudes of F and d are given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \text{ and } d = \sqrt{x^2 + y^2 + z^2} \qquad (2\text{-}5.2)$$

As an example, let us determine the components of a force F = 300 N whose line of action is directed from point A (–2, 1, 3) toward point B (3, 5, –3) as shown in Fig. 2-5.3. From this figure, or by taking the difference in the coordinates of B and A, the components of the distance d between A and B are $x = 5$, $y = 4$, and $z = –6$; applying Eq. (2-5.2), the length of d is found to be

$$\left[d = \sqrt{x^2 + y^2 + z^2}\right] \quad d = \sqrt{(5)^2 + (4)^2 + (6)^2} \quad d = 8.78$$

and from Eq. (2-5.1), the components of F are

$$\left[\frac{F_x}{x} = \frac{F_y}{y} = \frac{F_z}{z} = \frac{F}{d}\right] \qquad \frac{F_x}{5} = \frac{F_y}{4} = \frac{F_z}{-6} = \frac{300}{8.78}$$

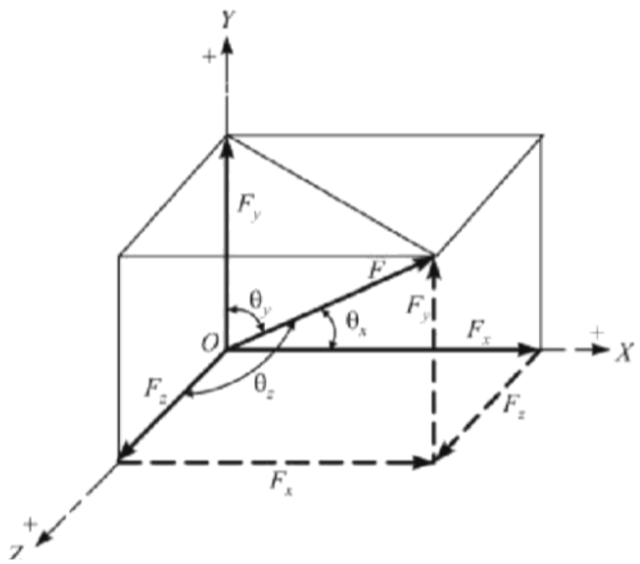
which gives

$$F_x = 171 \text{ N};\ F_y = 137 \text{ N};\ F_z = -205 \text{ N}$$

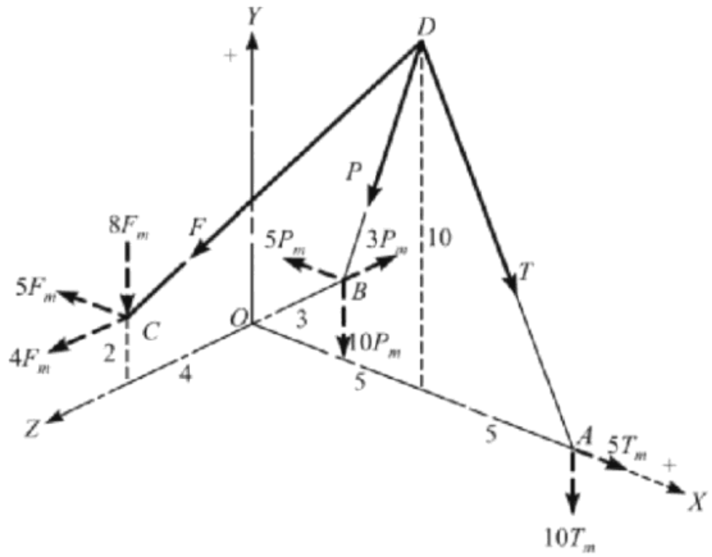
$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1 \qquad (2\text{-}5.4)$$

Figure 2-5.4 also verifies the extension to spatial systems regarding the conclusion obtained in Section 2-4 that the resultant force can be found from any sequence of tip-to-tail addition of its components. The resultant of a system of concurrent space forces may now be found in a fashion similar to that used for coplanar concurrent forces. The X component of the resultant is equal to the algebraic summation of the X components of the forces comprising the system, and the same holds true for the Y and Z components as well, i.e.,

$$R_x = \Sigma X \qquad R_y = \Sigma Y \qquad R_z = \Sigma Z$$



2-5.1 Find the resultant of the concurrent force system shown in Fig. 2-5.5 which consists of the forces $T = 300$ N, $P = 200$ N, and F = 500 N directed from D toward A, B, and C, respectively.



Solution

It is convenient to tabulate the computations as shown below. The table is almost self-explanatory The components of distance are the signed lengths traveled from D along coordinate paths to A, B, and C whence the terms in the column headed Distance are found from d = $\sqrt{x^2 + y^2 + z^2}$. The force multiplier is the ratio of force to distance which is then multiplied into the respective components of distance to find the corresponding force components.

Force (N)	**Comps. of distance (m)**			**Distance *d***	**Force multiplier (N/m)**	**Force components (N)**		
	x	**y**	**z**			***X* comp.**	***Y* comp.**	***Z* comp.**
T = 300	5	– 10	0	11.18	26.8	+ 134	– 268	0
P = 200	– 5	– 10	– 3	11.57	17.3	– 86.4	– 173	– 51.9
F = 500	– 5	– 8	4	10.25	48.8	– 244	– 390	+ 195
Totals						– 196.4	– 831	+ 143.1

An alternate plan is to draw the force components on the space diagram as shown by the dashed vectors, the direction of each component corresponding with the direction taken in moving along paths parallel to the reference axes from D to A, B, and C. Each component is shown as the product of the force multiplier by the absolute length of the coordinate paths, and the sign of each component is evident by comparison with the positive senses of the reference axes. The coefficient of each component then corresponds to the components of distance in the table above, whence the distance and force multiplier are evaluated as before and used to find the force components.

From the algebraic summation of the force components, we obtain

$$R_x = \sum X = -\ 196.4 \text{ N}$$

$$R_y = \sum Y = -\ 831 \text{ N}$$

$$R_z = \sum Z = +\ 143.1 \text{ N}$$

By visualizing a vector addition of these terms, we see that the resultant points to the left, down, and forward. The magnitude of the resultant and its direction cosines and direction angles are determined as follows;

$$\left[R = \sqrt{R_x^2 + R_y^2 + R_z^2}\right] \qquad \text{R} = \sqrt{(196.4)^2 + (831)^2 + (143.1)^2}$$

$$= 865 \text{ N} \qquad \textbf{\textit{Ans.}}$$

$$\left[\frac{R}{1} = \frac{R_x}{\cos\theta_x} = \frac{R_y}{\cos\theta_y} = \frac{R_z}{\cos\theta_z}\right]$$

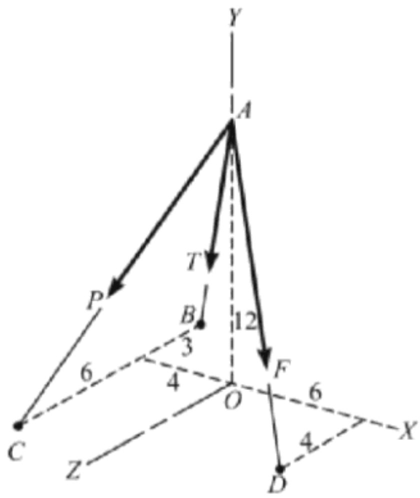
$$\frac{865}{1} = \frac{196.4}{\cos\theta_x} = \frac{831}{\cos\theta_y} = \frac{143.1}{\cos\theta_z}$$

whence

$$\theta_x = 76.88^\circ; \qquad \theta_y = 16.1^\circ; \qquad \theta_z = 80.48^\circ \qquad \textbf{\textit{Ans.}}$$

2-5.4 Find the resultant of the force system shown in Fig. P-2-5.4 in which $P = 280$ N, $T = 260$ N, and $F = 210$ N.

$R = 674$ N pointing forward, down, and to the left ***Ans.***



2-5.5 In Fig. P-2-5.5, a vertical boom AE is supported by guy wires from A to B, C, and D. If the tensile load in $AD = 252$ N, find the forces in AC and AB so that the resultant force on A will be vertical.

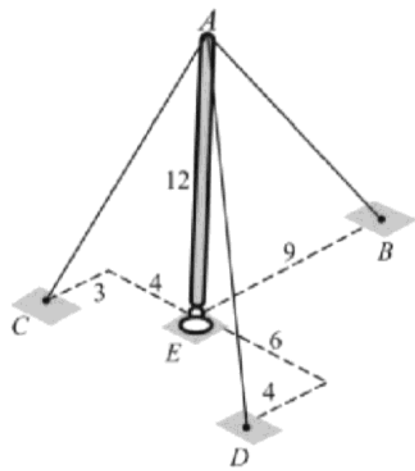
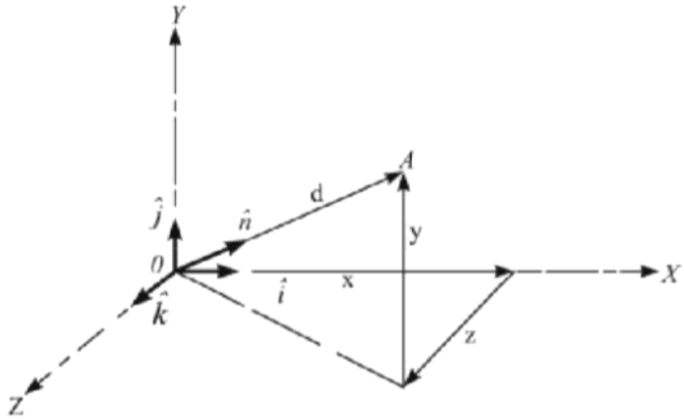


Figure P-2-5.5

$AB = 255$ N; $AC = 351$ N ***Ans.***

A unit vector is defined as a vector of unit magnitude in a specified direction. We shall denote them by placing a circumflex (^) over them as well as printing them in boldface. For longhand writing, the circumflex is sufficient to denote a unit vector; it need not also be underlined. Multiplying a unit vector by a scalar denotes a vector having the direction of the unit vector and a magnitude equal to that of the scalar. In this connection, a negative scalar denotes a vector direction opposite to that of the unit vector.



It is conventional to let $\hat{\mathbf{i}}, \hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ represent vectors of unit length directed along the positive senses of the *X*, *Y*, *Z* coordinate axes respectively as shown in Fig. 2-6.1. Applying this convention, the vector component $\mathbf{F}_x$ may be written as $\mathbf{F}_x\hat{\mathbf{i}}$ (meaning that the vector $\mathbf{F}_x$ has the scalar magnitude F_x in the + *X* direction). Thus an alternate form of writing Eq. (*a*) in this new notation is

$$\mathbf{F} = \mathbf{F}_x\hat{\mathbf{i}} + \mathbf{F}_y\hat{\mathbf{j}} + \mathbf{F}_z\hat{\mathbf{k}} \qquad \text{(c)}$$

This is known as the standard Cartesian form of representing a vector. It does not appear to be much of an improvement over Eq. (*a*) but its value will be apparent after we learn how to correlate a unit vector $\hat{\mathbf{n}}$ in any direction with the $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ unit vectors.

Consider now a unit vector $\hat{\mathbf{n}}$ in an arbitrary direction such as $\overrightarrow{OA}$ in Fig. 2-6.1. By definition, the unit vector $\hat{\mathbf{n}}$ in the direction from O to A is the vector **d** divided by the magnitude d of the distance from O to A. Thus,

$$\hat{\mathbf{n}} = \frac{\mathbf{d}}{d} = \frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}}{d} = \frac{1}{d}\left(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}\right) \tag{2-6.1}$$

where $\mathrm{d} = \sqrt{x^2 + y^2 + z^2}$

In particular, the proportionality ratios between a force **F** parallel or coincident with **d** discussed in Section 2-5 which are,

$$\frac{F_x}{x} = \frac{F_y}{y} = \frac{F_z}{z} = \frac{F}{d} = F_m \tag{2-5.1}$$

permit the force **F** to be expressed in any of the following variations:

$$\mathbf{F} = \mathbf{F}\hat{n} = \frac{\mathbf{F}}{\mathbf{d}}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \mathbf{F_m}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) \tag{2-6.2}$$

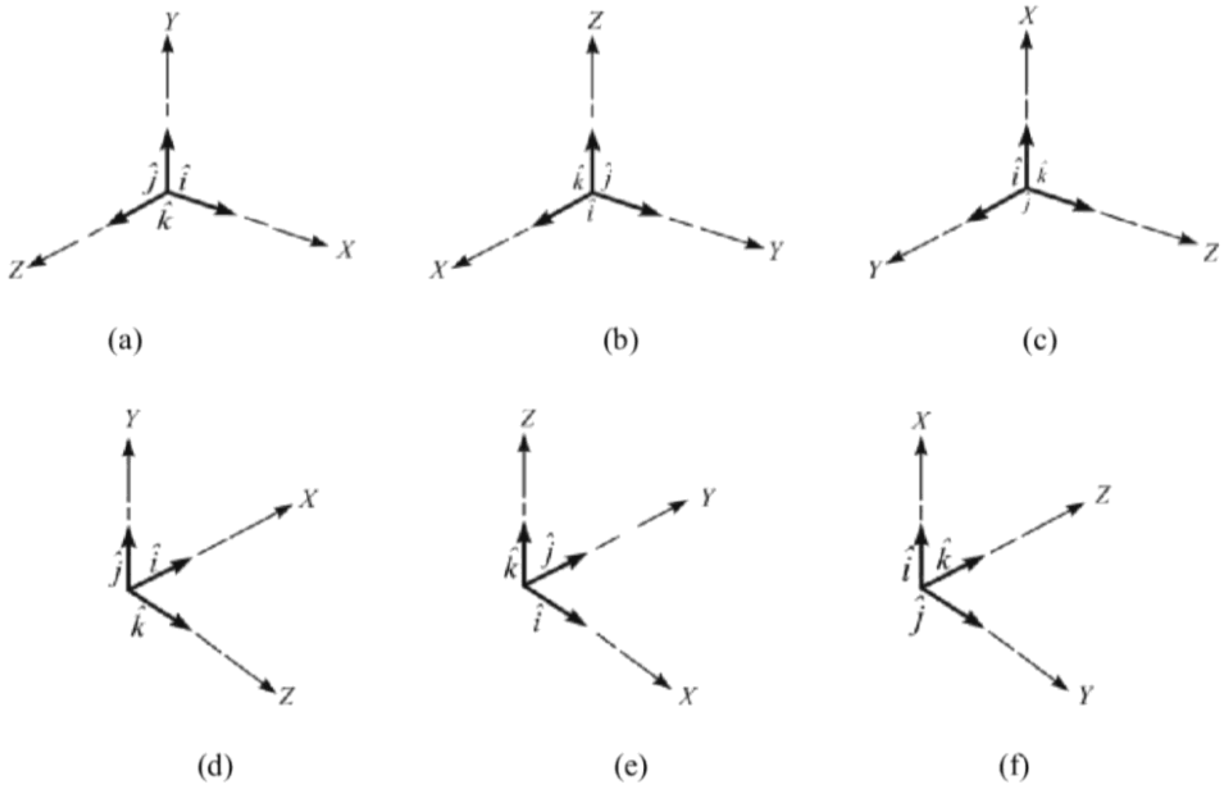


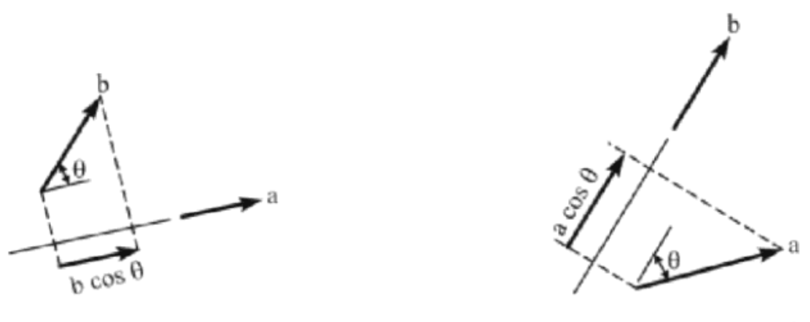
Figure 2-6.2 Various arrangements of right-handed coordinate system obtained by cyclic interchange of the coordinate directions.

There are two types of vector multiplication which seem almost to have been invented to find the component of a vector along any spatial direction, or the moment of the vector about any center[4]. These types are known as the scalar (or dot) product and the vector (or cross) product. In each case, the result will be the product of the magnitudes of the vectors times respectively the cosine or the sine of their included angle.

Let us consider first the *dot product.* It is defined to be a scalar quantity determined by multiplying the magnitudes of two vectors by the cosine of their included angle. The name comes from using a dot between two vectors to denote their multiplication. Thus, the dot product of multiplying **a** by **b** is

$$\mathbf{a} \cdot \mathbf{b} = ab \cos \theta \qquad (2\text{-}7.1)$$

This product is interpreted geometrically in Fig. 2-7.1. It shows how to obtain the component of one vector along the direction of another vector. The angle θ is the angle smaller than 180° included between the vectors (or their directions if they do not have a common origin). As shown in parts (a) and (b), the dot product may be used to find either the orthogonal component of **b** upon **a** or the orthogonal of **a** upon **b**. Either component is obtained by dividing the dot product by the magnitude of the vector in whose direction the component is desired. Observe that **a**/a and **b**/b are actually unit vectors $\hat{\boldsymbol{n}}_a$ and $\hat{\boldsymbol{n}}_b$ in the respective directions of **a** and **b**. This observation is formalized by this rule: *The component of a vector in any direction is the dot product of the vector with a unit vector in the desired direction*. Its application is discussed in Illus. Prob. 2-7.1.



(a) $b \cos\theta = \dfrac{a.b}{a} = \hat{n}_a.b$ (b) $a \cos\,\theta = \dfrac{a.b}{b} = a.\hat{n}_b$

Figure 2-7.1 Interpretation of dot product **a. b** = ab cos θ.

As a preliminary step to applying the dot product, consider the dot products of various combinations of the orthogonal unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. From its definition, the dot product of a unit vector with itself is unity since it is the product of unit magnitudes by the cosine of the zero angle between their directions; however, all the other combinations of dot products of the unit vectors will be zero because their included angle is 90° whose cosine is zero. Thus, we obtain

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = \mathbf{1}$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = \mathbf{0} \qquad (2\text{-}7.2)$$

Some other properties of the dot product, as a consequence of its definition, are the following:

Commutative: $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$

Associate: $m\mathbf{A} \cdot n\mathbf{B} = mn\mathbf{A} \cdot \mathbf{B}$

Distributive $\mathbf{C} \cdot (\mathbf{A} + \mathbf{B}) = \mathbf{C} \cdot \mathbf{A} + \mathbf{C} \cdot \mathbf{B}$

These properties of the dot product enable us to evaluate the expansion of **a . b** in terms of their scalar components as follows:

$$\mathbf{a} \cdot \mathbf{b} = \left(a_x\hat{\boldsymbol{i}} + a_y\hat{\mathbf{j}} + \mathrm{a}_z\hat{\boldsymbol{k}}\right) \cdot \left(b_x\hat{\boldsymbol{i}} + b_y\hat{\boldsymbol{j}} + b_z\hat{\boldsymbol{k}}\right)$$

$$= a_x b_x \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} + a_x b_y \underbrace{\cancel{\hat{\mathbf{i}} \cdot \hat{\mathbf{j}}}}_{=0} + \mathrm{a}_x b_z \underbrace{\cancel{\hat{\mathbf{i}} \cdot \hat{\mathbf{k}}}}_{=0}$$

$$+ a_y b_x \underbrace{\cancel{\hat{\mathbf{j}} \cdot \hat{\mathbf{i}}}}_{=0} + a_y b_y \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} + a_y b_z \underbrace{\cancel{\hat{\mathbf{j}} \cdot \hat{\mathbf{k}}}}_{=0}$$

$$+ a_z b_x \underbrace{\cancel{\hat{\mathbf{k}} \cdot \hat{\mathbf{i}}}}_{=0} + a_z b_y \underbrace{\cancel{\hat{\mathbf{k}} \cdot \hat{\mathbf{j}}}}_{=0} + a_z b_z \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}$$

Since the dot product of a unit vector with itself is unity, all other dot products of unit vectors being zero as indicated, this expansion reduces to

$$\mathbf{a} \cdot \mathbf{b} = \boldsymbol{a_x b_x} + \boldsymbol{a_y b_y} + \boldsymbol{a_z b_z} \qquad (2\text{-}7.3)$$

which is summarized by the rule: *The dot product of two vectors is equal to the sum of the products of their respective scalar components*. In particular,

$$\mathbf{a} \cdot \mathbf{a} = a_x^2 + a_y^2 + a_z^2 = a^2$$

Further, if **a** . **b** = 0 and it is known that neither **a** nor **b** are zero, then the cosine of their included angle must be zero; i.e., θ must be 90°, so that two vectors are perpendicular when their dot product is zero. Conversely, if two nonzero vectors are known to be perpendicular, their dot product must be zero.

The following illustrative problem discusses the application of the dot product.

2-7.1 In the tripod shown in Fig. 2-7.2, forces **F** and **P** act as shown along the legs *DC* and *DB*. In terms of its force multiplier, determine the component of **F** along the direction of **P** and also the angle between **F** and **P**. Note that points *B* and *C* are in the same vertical plane and that *C* is located 2 m above the horizontal plane containing *B* and *A*.

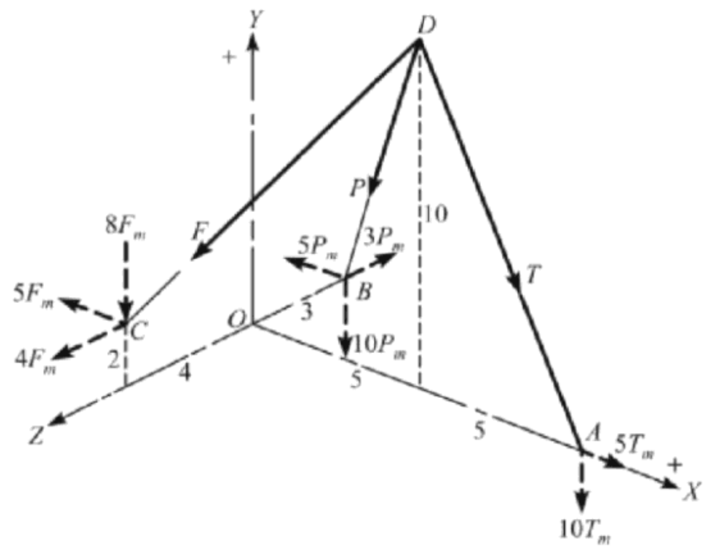


Figure 2-7.2

Solution

Following the method discussed on p. 39, the components of **F** and **P**, acting as shown, have been expressed as the product of their respective force multipliers and the components of the distances traveled in moving along coordinate directions from D to C and from D to B. On comparing the directions of these components with the positive directions of the coordinate axes, we obtain

$$\mathbf{F} = F_m\,(-5\,\hat{\mathbf{i}} - 8\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}); \qquad F = F_m\,\sqrt{105}$$

and

$$\mathbf{P} = P_m\,(-5\,\hat{\mathbf{i}} - 10\,\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}); \qquad P = P_m\,\sqrt{134}$$

The component F_{DB} of **F** along the direction of DB (i.e., along **P**) may now be found by applying the rule deduced on p. 40 that the component of a vector in any direction is the dot product of the vector with a unit vector in the desired direction. The unit vector $\hat{\mathbf{n}}_p$ along **P** being

$$\hat{\mathbf{n}}_p = \frac{\mathbf{P}}{P} = \frac{1}{\sqrt{134}}\left(-5\hat{\mathbf{i}} - 10\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)$$

the component F_{DB} of **F** along **P** is

$[F_{DB} = \mathbf{F} \cdot \hat{\mathbf{n}}_p]$

$$F_{DB} = F_m\left(-5\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 4\hat{\mathbf{k}}\right) \cdot \frac{1}{\sqrt{134}}\left(-5\hat{\mathbf{i}} - 10\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)$$

whence, summing the algebraic products of the scalar coefficients of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ as in Eq. (2-7.3), we obtain

$$F_{DB} = \frac{F_m}{\sqrt{134}}(25 + 80 - 12) = \frac{93}{\sqrt{134}}F_m \qquad \textbf{\textit{Ans.}}$$

The positive value of this result indicates that the component of **F** upon **P** acts in the direction of $\hat{\mathbf{n}}_p$; i.e., from D to B.

By reapplying the definition of $\mathbf{F} \cdot \hat{\mathbf{n}}_p$, we may also use the preceding result to find the angle between **F** and **P**. Thus;

$$[F \cos\theta = \mathbf{F} \cdot \hat{\mathbf{n}}_p] \qquad F_m \sqrt{105}\ \cos\theta = \frac{93}{\sqrt{134}} F_m$$

whence
$$\cos\theta = \frac{93}{\sqrt{(105)(134)}} = 0.785; \ \theta = 38.2^\circ \qquad \textbf{\textit{Ans.}}$$

A general interpretation of this procedure is that the cosine of the angle between two directions equals the dot product of the unit vectors along the specified directions.

2.7.2 In Fig. P-2-7.2, a boom *AC* is supported by a ball-and-socket joint at C and by the cables *BE* and *AD*. If the force multiplier of a force **F** acting from *B* to *E* is F_m = 10 N/m and that of a force **P** acting from *A* to *D* is P_m = 20 N/m, find the component of each force along *AC*.

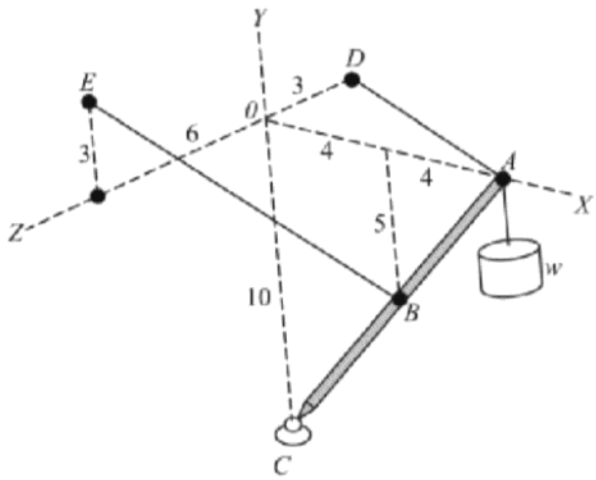


Figure P-2-7.2

F_{AC} = –37.5 N; P_{AC} = 100 N ***Ans.***

2.7.4 In the system shown in Fig. P.2-74, it is found that the force multiplier of force F acting from B to D is F_{m}, = 150 N/m and that of force **P** acting from *A* to *E* is P_m 100 N/m. Find the component of force along AC. What angle does each force make with *AC*?

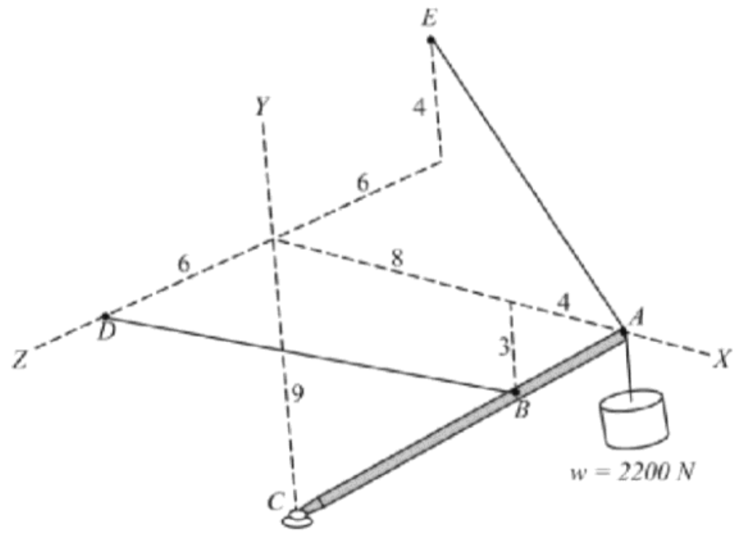


Figure P-2-7.4

F_{AC} = 690 N; P_{AC} = 720 N; θ_F = 63.85°; θ_P = 59.1° ***Ans.***

2-8 VECTOR ALGEBRA, CROSS PRODUCT

The second type of vector multiplication is known as the cross product because of the symbol × used to denote this type of multiplication. The cross product of two vectors **a** and **b** (written as **a** × **b**) is defined as the product of their magnitudes by the sine of their included angle. In addition, this result is a new vector (hence the alternate name of vector product) acting perpendicular to the plane of **a** and **b** in the direction a right-hand screw would advance as **a** is turned toward **b**. An alternate to the right-hand screw rule is the three-finger right-hand rule: Place the thumb in the direction of **a**, the first finger in the direction of **b**; then the second finger points in the direction of **a** × **b**. Perhaps best of all is the right-hand rule in which the fingers of the right-hand curl in the direction of **a** rotating toward **b**; then the extended thumb points in the positive direction of the cross product. Denoting $\hat{\mathbf{n}}$ as the unit vector of this positive direction, the explicit form of the cross product is

$$\mathbf{a} \times \mathbf{b} = ab \sin \boldsymbol{\theta}\, \hat{\mathbf{n}} \qquad (2\text{-}8.1)$$

which is interpreted geometrically in Fig. 2-8.1. We see that the magnitude of the cross product is equal to the area of a parallelogram constructed with initial sides **a** and **b**. It is also equal to twice the area of the triangle formed by taking either diagonal of this parallelogram with two of its adjacent sides. These rather obvious comments are frequently used to compare geometrically the cross products of different sets of vectors.

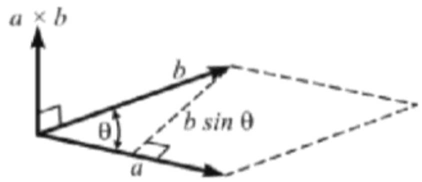


Figure 2-8.1 **a** × **b** is perpendicular to plane of **a** and **b**.

Due to the right-hand rule, note that **b** × **a** acts in the opposite sense to **a** × **b**, i.e.,

$$\mathbf{b} \times \mathbf{a} = -\,\mathbf{a} \times \mathbf{b} \tag{2-8.2}$$

Therefore, the vector product is *not* commutative; the order of the terms may not be interchanged without also changing the sign of the cross product.

Consider now the cross products of various combinations of the orthogonal unit vectors. A cross product of a unit vector with itself (or of any vector with itself) is zero because the sine of the included angle is zero. A convenient scheme to remember other combinations is that if any two follow each other alphabetically (i.e., in the clockwise cyclic order shown below), their cross product is the positive value of the third. Thus, we obtain

$$\begin{array}{lll} \hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0 & & \\ \hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\ ; & \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}}\ ; & \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}} \\ \hat{\mathbf{j}}\times\hat{\mathbf{i}} = -\,\hat{\mathbf{k}}\ ; & \hat{\mathbf{k}}\times\hat{\mathbf{j}} = \hat{\mathbf{i}}\ ; & \hat{\mathbf{i}}\times\hat{\mathbf{k}} = -\,\hat{\mathbf{j}} \end{array} \tag{2-8.3}$$

These properties of the cross product of the orthogonal unit vectors enable us to evaluate the expansion of **a** × **b** in terms of their scalar components as follows:

$$\mathbf{a}\times\mathbf{b} = \left(a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}\right)\times\left(b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}} + b_z\hat{\mathbf{k}}\right)$$

$$= a_x b_x \cancelto{=0}{(\hat{\mathbf{i}}\times\hat{\mathbf{i}})} + a_x b_y \cancelto{=\hat{\mathbf{k}}}{(\hat{\mathbf{i}}\times\hat{\mathbf{j}})} + a_x b_z \cancelto{=-\hat{\mathbf{j}}}{(\hat{\mathbf{i}}\times\hat{\mathbf{k}})}$$

$$+ a_y b_x \cancelto{=-\hat{\mathbf{k}}}{(\hat{\mathbf{j}}\times\hat{\mathbf{i}})} + a_y b_y \cancelto{=0}{(\hat{\mathbf{j}}\times\hat{\mathbf{j}})} + a_y b_z \cancelto{=\hat{\mathbf{i}}}{(\hat{\mathbf{j}}\times\hat{\mathbf{k}})}$$

$$+ a_z b_x \cancelto{=\hat{\mathbf{j}}}{(\hat{\mathbf{k}}\times\hat{\mathbf{i}})} + a_z b_y \cancelto{=-\hat{\mathbf{i}}}{(\hat{\mathbf{k}}\times\hat{\mathbf{j}})} + a_z b_z \cancelto{=0}{(\hat{\mathbf{k}}\times\hat{\mathbf{k}})}$$

which reduces to

$$\mathbf{a}\times\mathbf{b} = \hat{\mathbf{i}}\left(a_y b_z - a_z b_y\right) + \hat{\mathbf{j}}\left(a_z b_x - a_x b_z\right) + \hat{\mathbf{k}}\left(a_x b_y - a_y b_x\right) \qquad (2\text{-}8.4)$$

Instead of this direct expansion of the cross product, the same result is usually easier to obtain by evaluating the third-order determinant formed by taking the scalar components of the firsts and second terms of the cross product as respectively the elements the first and second rows and using $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ as the elements the third row. Thus, we get for a × b,

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \end{vmatrix} = \hat{\mathbf{i}}\begin{vmatrix} a_y & a_z \\ by & b_z \end{vmatrix} + \hat{\mathbf{j}}\begin{vmatrix} a_z & a_x \\ b_z & bx \end{vmatrix} + \hat{\mathbf{k}}\begin{vmatrix} ax & a_y \\ bx & b_y \end{vmatrix}$$

$$= \hat{\mathbf{i}}\,(a_y b_z - a_z b_y) + \hat{\mathbf{j}}\,(a_z b_x - a_x b_z) + \hat{\mathbf{k}}\,(a_x b_y - a_y b_x)$$

Some further properties of the cross product are that it is associative when multiplied by a scalar, namely

$$m\,(\mathrm{a}\times\mathrm{b}) = m\mathrm{a}\times\mathrm{b} = \mathrm{a}\times m\mathrm{b} \qquad (2\text{-}8.5)$$

but it is *not* associative when cross-multiplied by a vector; i.e.,

$$\mathrm{a}\times(\mathrm{b}\times\mathrm{c}) \neq (\mathrm{a}\times\mathrm{b})\times\mathrm{c}$$

The inequality of these vector triple products is evident by comparing the following reduction formulas for a vector triple product:

$$\begin{aligned} a\times(b\times c) &= (a\;.\;c)b - (a\;.\;b)c \\ (a\times b)\times c &= (c\;.\;a)b - (c\;.\;b)a \end{aligned} \qquad (2\text{-}8.6)$$

In these reduction formulas, the vector triple product is equal to the difference of the vectors enclosed in the parentheses; each of these vectors being multiplied by the scalar dot product of the remaining two others. In writing the difference of these terms, give the positive sign to that vector enclosed in the parentheses which next to the vector outside these parentheses.

We have stated these reduction formulas to demonstrate the nonassociative property mentioned above. Although we have no immediate need for these reduction formulas at this introductory stage, they can be so useful in later work that we present them here for future reference.

Another property of the cross product is that it is distributive over vector addition; thus

$$c \times (a + b) = c \times a + c \times b \qquad (2\text{-}8.7)$$

but we must be careful to maintain the original order of the vectors.

For some applications of the cross product, consider the following illustrative problems.

ILLUSTRATIVE PROBLEM

2-8.1 Find the shortest distance from the origin to the line passing through points A (– 2, 1, 3) and B (4, 5, 0).

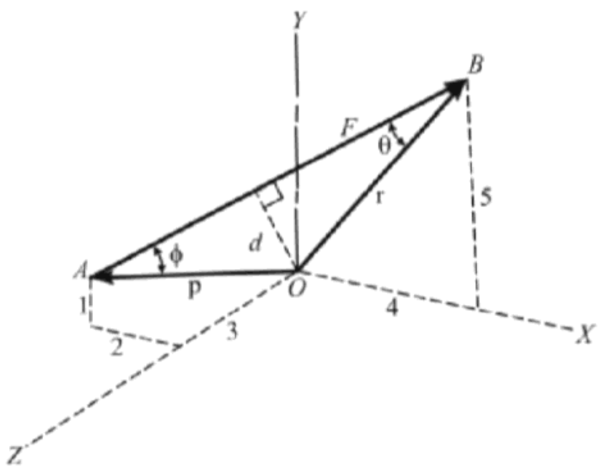


Figure 2-8.2

Solution

Figure 2-8.2 represents the given data and will aid the analysis. Let the directed line segments *AB*, *OB*, and *OA* be denoted by **F**, **r**, and **p**, respectively. Then.

$$\mathbf{F} = 6\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}} \qquad \mathrm{F} = \sqrt{36+16+9} = \sqrt{61}$$

$$\mathbf{r} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} \qquad \mathrm{r} = \sqrt{16+25} = \sqrt{41}$$

$$\mathbf{p} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}} \qquad \mathrm{p} = \sqrt{4+1+9} = \sqrt{14}$$

As shown in the figure, the required distance is $d = r \sin \theta$. If we recall that $r \sin \theta$ is part of the magnitude of $r \times \mathrm{F}$, i.e., $|\mathbf{r} \times \mathbf{F}| = rF \sin \theta$, our procedure is clear. First, we evaluate $\mathrm{r} \times \mathrm{F}$ as explained on p. 48 to obtain

$$\mathbf{r} \times \mathbf{F} = \begin{vmatrix} 4 & 5 & 0 \\ 6 & 4 & -3 \\ \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \end{vmatrix} = \hat{\mathbf{i}} \begin{vmatrix} 5 & 0 \\ 4 & -3 \end{vmatrix} + \hat{\mathbf{j}} \begin{vmatrix} 0 & 4 \\ -3 & 6 \end{vmatrix} + \hat{\mathbf{k}} \begin{vmatrix} 4 & 5 \\ 6 & 4 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(-15) + \hat{\mathbf{j}}(12) + \hat{\mathbf{k}}(16 - 30) = -15\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - 14\hat{\mathbf{k}}$$

Computing its magnitude as the square root of the sum of the squares of its orthogonal components, we have

$$|\mathbf{r} \times \mathbf{F}| = rF \sin \theta = \sqrt{225 + 144 + 196} = \sqrt{565}$$

whence

$$d = r \sin \theta = \frac{|\mathbf{r} \times \mathbf{F}|}{F} = \frac{\sqrt{565}}{\sqrt{61}} = 3.04 \qquad \textbf{\textit{Ans.}}$$

The, distance d could also be found from $d = \mathrm{p} \sin \phi$ which suggests the use of $\mathbf{p} \times \mathbf{F}$. Thus, we obtain

$$\mathbf{p} \times \mathbf{F} = \begin{vmatrix} -2 & 1 & 3 \\ 6 & 4 & -3 \\ \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \end{vmatrix} = \hat{\mathbf{i}} \begin{bmatrix} \hat{\mathbf{i}}(-3-12) \\ +\hat{\mathbf{j}}(18-6) \\ +\mathbf{k}(-8-6) \end{bmatrix} = -15\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - 14\hat{\mathbf{k}}$$

Observe the somewhat surprising result here that $\mathbf{p} \times \mathbf{F}$ is identical with $\mathbf{r} \times \mathbf{F}$. The physical significance of this equality will become clear when we discuss the concept of the moment of a force in the next section. There we shall see that the cross product of *any* vector from *O* to the vector *F* defines the moment of **F** about *O* and consequently produces the same result. Continuing the solution, it is evident that the magnitude of $\mathbf{p} \times \mathbf{F}$ is

$$| \mathbf{p} \times \mathbf{F} | = pF \sin \phi = \sqrt{565}$$

Whence as before

$$d = p \sin \phi = \frac{|\mathbf{p} \times \mathbf{F}|}{F} = \frac{\sqrt{565}}{\sqrt{61}} = 3.04 \qquad \textbf{\textit{Check}}$$

2-9 MOMENT OF FORCE. PRINCIPLE OF MOMENTS

The moment of a force (or more generally, of any vector) about point is defined as the product of the magnitude of the force by the perpendicular distance from the point to the action line of the force. In Fig. 2-9.1, this perpendicular distance d is called the moment arm whence the magnitude of the moment of a force F about a center O is expressed by

$$M_O^F = Fd \tag{a}$$

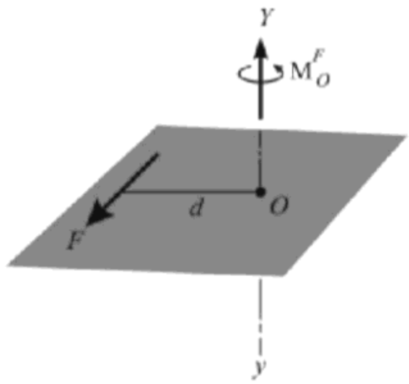


Figure 2-9.1 Moment of a force.

The moment of a force about a point represents the tendency to rotate the moment arm (or the body on which the force acts) about an axis which is perpendicular to the plane defined by the force and its moment arm. The moment axis passes through the moment center. By curling the fingers of the right hand about the moment axis in the sense of this rotation, the extended thumb represents the direction of the moment vector. The moment vector may be considered to be a sliding vector whose line of action coincides with the moment axis. Because a moment is the product of a force and a distance, the unit of moment is their dimension product e.g., Newton-meter (N-m), gram-centimeters (gm-cm), etc[5]. The moment vector is usually denoted by a double-line arrow to distinguish it from a force vector, or by a curved arrow about the moment axis.

The vector nature of moment can be deduced from Fig. 2-9.2 where we have drawn position vectors **r** and **p** from the moment center O to any two points on the action line of **F**. Since **r** and **p** lie in the plane defined by **F** and O; it is evident that the moment arm d is given by either $d = r \sin \theta$ or $d = p \sin \phi$. Applying the definition of moment, we then have

$$M_O^F = Fd = Fr \sin \theta = Fp \sin \phi \qquad (b)$$

where, from the definition of a cross product, the last two terms represent the magnitudes of **r x F** and **p x F**. (This explains the observation made in Prob. 2-8.1). Either cross product represents a vector perpendicular to the plane defined by **F** and d, the direction of which is consistent with the moment vector of ***F*** about O described by the right-hand rule of the preceding paragraph. We conclude that the moment of **F** about O is given by

$$\boldsymbol{M_O^F} = \mathbf{r} \times \mathbf{F} \qquad (2\text{-}9.1)$$

where **r** is *any* position vector extending from the moment center to any point on the action line of the force.

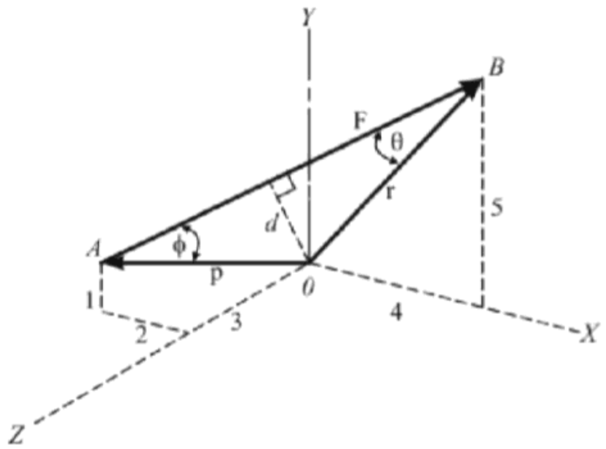


Figure 2-9.2

Note that the moment cross product must be written as **r** × **F** and *not* as **F** × **r** so that consistency with the right-hand rule for a moment vector can be maintained. This statement is so important that we restate it explicitly thus: The moment of a force about a moment center is the product of the position vector from the moment center to *any* point on the line of action of the force crossed with the force vector.

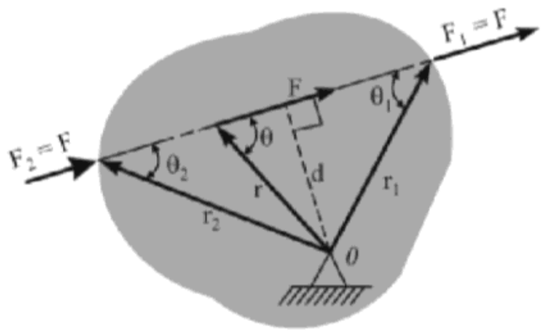


Figure 2-9.3 The moments of **F**, **F₁**, and **F₂** about *O* are all equal since $d = r \sin\theta = r_1 \sin\theta_1 = r_2 \sin\theta_2$.

This general definition of moment as a cross product can be used to demonstrate the *principle of transmissibility*, namely that the external effect of a force on a rigid body is independent of where it is applied along its line of action. Thus, in Fig. 2-9.3, the rotational effect of force **F** upon a body free to rotate about an axle at *O* is equivalent to that of a numerically equal force $\mathbf{F}_1$ applied along the action line of **F** at the right edge of the body or to that of $\mathbf{F}_2$ applied at the left edge inasmuch as

$$M_O^F = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}_1 \times \mathbf{F}_1| = |\mathbf{r}_2 \times \mathbf{F}_2| = Fd \qquad (c)$$

PRINCIPLE OF MOMENTS

We now consider a very important concept known as the principle of moments which states that the moment of a force is equal to the moment sum of its components. This almost self-evident statement is known as Varignon's theorem[6] and is demonstrated as follows:

In Fig. 2-9.4, let **R** be the resultant of the concurrent, forces **P**, **F**, and **T**. The force system may be either coplanar or spatial, *but it must be concurrent*.

We have seen that $\mathbf{R} = \mathbf{P} + \mathbf{F} + \mathbf{T}$. About any point O as moment center, the moments of these forces is

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{F} + \mathbf{T}) = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{F} + \mathbf{r} \times \mathbf{T}$$

which proves the theorem. Note carefully that the resultant and its components must be concurrent or it will be impossible to draw a common position vector to them from the common moment center.

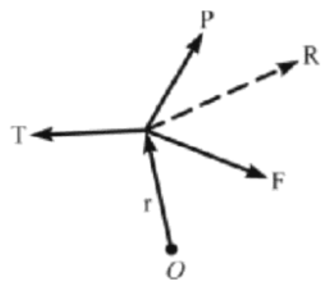


Figure 2-9.4

A general symbolic statement of the theorem is

$$\boldsymbol{M_o^F} = \Sigma \boldsymbol{M_o} = \Sigma(\boldsymbol{r} \times \boldsymbol{F}) \qquad (2\text{-}9.2)$$

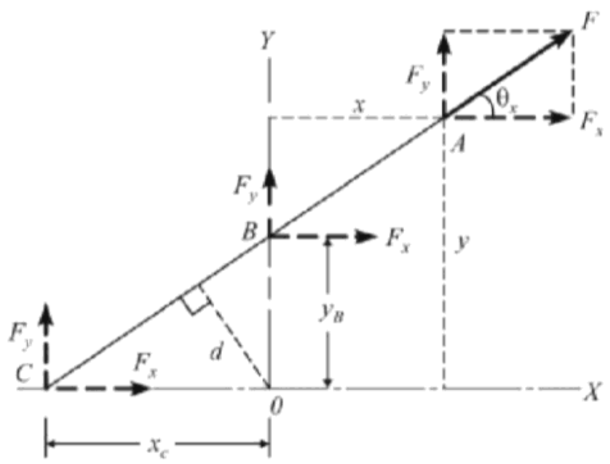


Figure 2-10.1 Coplanar application of the principle of moments.

ILLUSTRATIVE PROBLEMS

2-10.1 In Fig. 2-10.3, a force **F** passing through *C* causes a clockwise moment of 120 N-m about A and a clockwise moment of 70 N-m about *B*. Determine the force and its *x* intercept.

Solution

This problem demonstrates how to interpret and apply the physical meaning of moment. By resolving the force into its components at *C*, we observe that since F_y passes through *A*, the moment of **F** about *A* is due only to F_x which must act leftward as shown in order to cause the specified clockwise moment about *A*. Next, in order to create the specified clockwise moment about *B*, the action line of **F** must intersect the *X axis* to the left of *B* at *D* with F_y acting upward as shown.

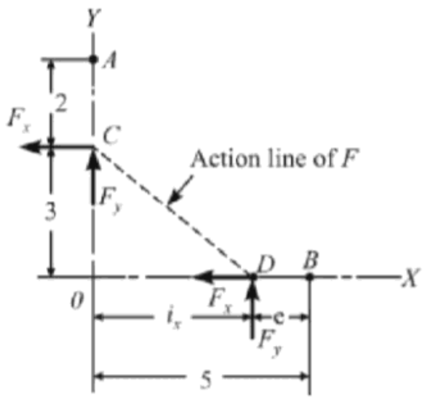


Figure 2-10.3

We can now apply the principle of moments to obtain

$[\circlearrowleft + \; M_A^P = yFx]$ $\quad$ $120 = 2F_x$ $\quad$ $F_x = 60\text{ N} \leftarrow$

$[\circlearrowleft + \; M_B^F = xF_y - yF_x]$ $\quad$ $70 = 5F_y - 3(60)$ $\quad$ $F_y = 50\text{ N} \uparrow$

With F_y now known, we use the components acting at *D* to get

$[\circlearrowleft + \; M_B^F = eF_y]$ $\quad$ $70 = e(50)$ $\quad$ $e = 1.4\text{ m}$

whence the *x* intercept from *O* is

$$i_x = 5 - e = 5 - 1.4 = 3.6\text{ m}$$ ***Ans.***

This procedure illustrates the application of Varignon's theorem, but it may be simpler in this instance to determine i_x directly, using the slope of the action line of **F** as specified by its components. Doing this yields

$\left[\dfrac{3}{i_x} = \dfrac{F_y}{F_x}\right]$ $\quad$ $\dfrac{3}{i_x} = \dfrac{50}{60}$ $\quad$ $\boldsymbol{i_x} = 3.6\text{ m}$ $\quad$ ***check***

2-10.2 In Fig. P-2-10.2 a force **F** passing through *C* causes a clockwise moments as positive, compute the moment of force $F = 450$ N and of force $P = 361$ N about points *A, B, C* and *D*.

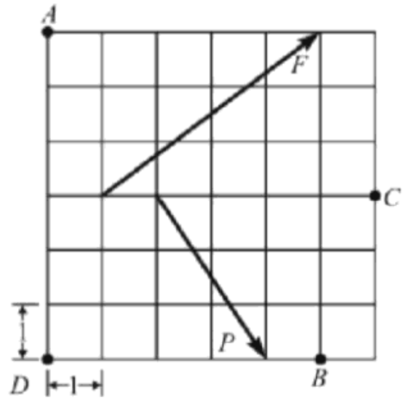


Figure P-2-10.2

For F: $M_D^F = 810$ N-m; for P: $M_C^P = -1200$ N-m ***Ans.***

2.10.4 In Fig. P-2-10.4, find the *y* coordinate of point *A* so that the 361N force will have a clockwise moment of 400 N-m about *O*. Also determine the *Y* intercept of the action line of the force.

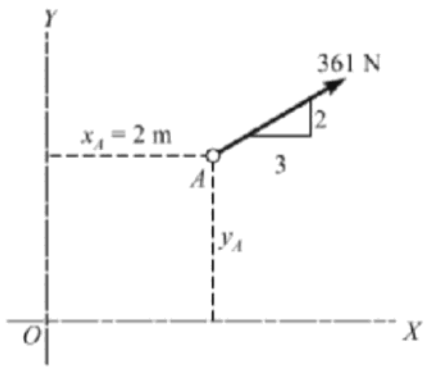


Figure P-2-10.4

$Y_A = 2.67$ m; $i_y = 1.33$ m above O ***Ans.***

2-10.6 In the rocker arm shown in Fig. P-2-10.6, the moment of F about O balances that of P about O. Find F.

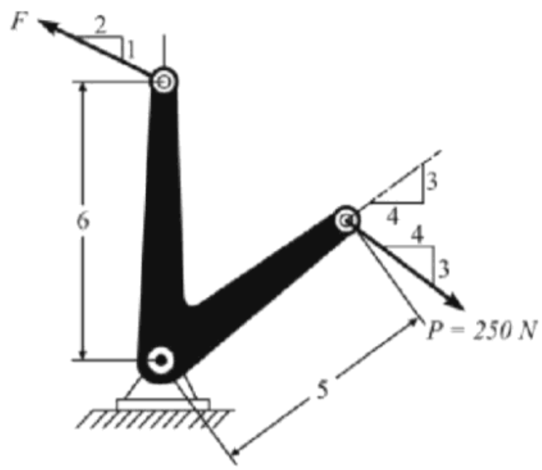


Figure P-2-10.6

$F = 224$ N **Ans.**

2-10.7 In Fig. P-2-10.7, the moment of a certain force F is 180 N-m clockwise about O and 90 N-m counterclockwise about B. If its moment about A is zero, determine the force.

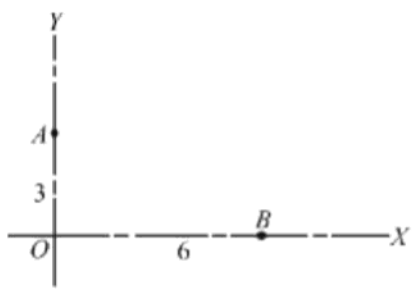


Figure P-2-10.7

F = 75 N down to the right at $\theta_x = 36.9^{\circ}$ **Ans.**

moment about B is 40 N-m clockwise, determine its y intercept.

2-10.9 In Fig. P-2-10.9, a force F passing through C produces a clockwise moment of 600 N-m about A and a counterclockwise moment of 300 N-m about B. Determine the moment of F about O.

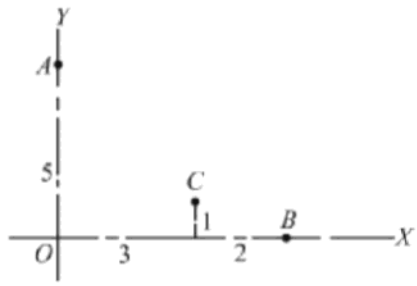
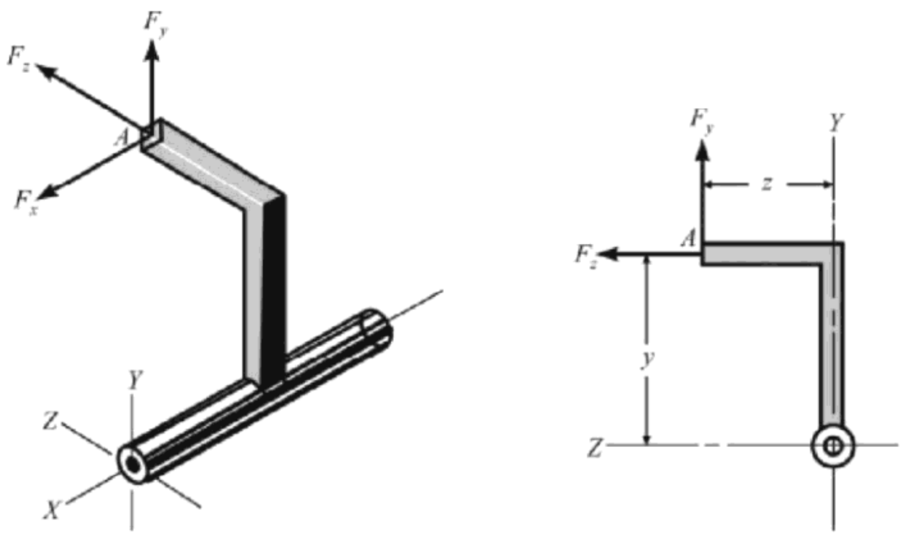


Figure P-2-10.9

$$M_C^F = 300 \text{ N-m clockwise} \qquad \textbf{\textit{Ans.}}$$



(a) (b)

Figure 2-11.1 Moment of a force about the X axis.

relying on the rules for evaluating a determinant. There is an alternate geometric approach for finding the moment of a force which is also useful. However, this alternate method requires a three-dimensional sketch and the ability to visualize its spatial aspects. We shall now discuss it. It is recommended that the student gain familiarity with both methods by applying them alternately in assignments.

To understand the physical significance of the moment of any force about any axis, let us imagine a device like that shown in Fig. 2-11.1. Assume the axis (the X axis in this instance) to be a taut wire on which is mounted a structure free to slide and rotate about the axis. A force **F**, represented by its rectangular components, is applied at point A. The component F_x which is parallel to the axis (i.e., the wire) can only slide the structure along the wire; it is evidently impossible for F_x to rotate the structure about the axis. From another point of view, we could say that F_x intersects the *X axis* at infinity and therefore has no moment about it. In vector terms, we would note that $\boldsymbol{x} \times \boldsymbol{F_x} = 0$. In any event, we conclude that a force has no moment about a parallel axis.

A different situation exists, however, with respect to both F_y and F_z. Looking at the end view of the structure, we recognize a condition similar to that for moment of coplanar forces. In fact, F_y and F_z *are* coplanar, and the X axis appears as a point which is the center of moments. Assuming the counterclockwise sense of rotation to be positive, the moment of **F** about the X axis is given by

$$+\uparrow M_x = yF_z - zF_y$$

A similar analysis may be made to determine the moment M_y of **F** about the Y axis or the moment M_z of **F** about the ***Z*** axis. *In general, the moment of a force about any axis is due to the components of the force lying in the plane perpendicular to the axis of moments.* Thus, as developed above, the moment of a force about the X axis is due solely to the Y and Z components of the force. It is helpful to observe that there is always one X term, one Y term, and one Z term involved in the moment of a force component about a coordinate axis; for example, the moment arm of the Y component of a force about the X axis is its Z coordinate.

To conclude this discussion, consider Fig. 2-11.2 which shows the components of a force **F** acting at a point A whose position vector from a moment center at O is **r**. The vector form of the moment of **F** about O is

$$\boldsymbol{M_O^F} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} x & y & z \\ F_x & F_y & F_z \\ \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \end{vmatrix}$$

which, on expanding the determinant, becomes

$$\boldsymbol{M}_O^F = \hat{\imath}\,(yF_z - zF_y) + \hat{\jmath}(zF_x - xF_z) + \hat{k}\,(xF_y - yF_x)$$

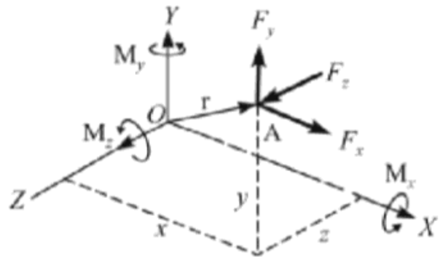


Figure 2-11.2 M_O^F = r × F

The coefficients of the unit vectors are respectively the moment of **F** about the coordinate axes, or

$$M_x = yF_z - zF_y$$

$$M_y = zF_x - xF_z$$

$$M_z = xF_y - yF_x$$

thereby giving results which correspond exactly with the geometric approach discussed above.

Since moment is a vector, its component (i.e., rotational effect) about any arbitrary axis through the moment center can be found as the dot product of the moment **M** with a unit vector $\hat{\mathbf{n}}$ which specifies the direction of this arbitrary axis. This concept will be applied in Illus. Prob. 2-11.1.

2-11.1 A tripod consists of three bars joined at *D* as shown schematically in Fig. 2-11.3. If 10 N/m is the value of the force multiplier of **F** acting along bar *DA* from *D* to *A*, find the moment of **F** about point *C* and about the line *CB*. Also find the value of **P** acting in bar *DB* from *D* to *B* that will cause the moment of **P** about the axis *AC* to be 2000 N-m.

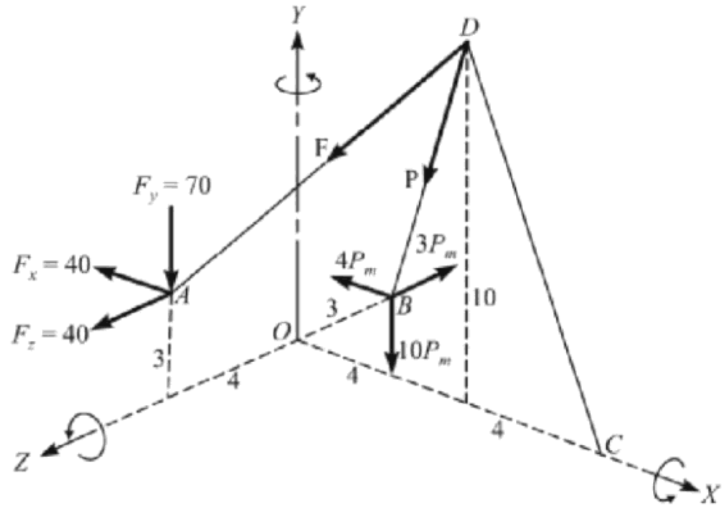


Figure 2-11.3

Solution

The moment of **F** about C is $\mathbf{M}_C^F = \mathbf{r}_{CD} \times \mathbf{F}$ where $\mathbf{r}_{CD} = -4\,\hat{\mathrm{i}} + 10\,\hat{\mathrm{j}}$ is the most convenient position vector from C to a point D on **F**. Express this cross product in determinant form as shown below, noting that the first row is the *xyz* order of the components of $\mathbf{r}_{CD}$; the second row is the *xyz* order of the components of **F**; and the third row consists of the unit vectors in the same *xyz* order. By following this scheme, the vector form of $\mathbf{r}_{CD}$ and **F** need not be written out; the respective components with their signs being obtained directly from the space diagram. Thus, we obtain

$$\mathbf{M}_C^F = \mathbf{r}_{CD} \times \mathbf{F} = \begin{vmatrix} -4 & 10 & 0 \\ -40 & -70 & 40 \\ \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \end{vmatrix} = \begin{vmatrix} & \hat{\mathbf{i}}(400) \\ + & \hat{\mathbf{j}}(160) \\ + & \hat{\mathbf{k}}(280+400) \end{vmatrix}$$

$$\mathbf{M}_C^F = 400\,\hat{\mathbf{i}} + 160\,\hat{\mathbf{j}} + 680\,\hat{\mathbf{k}} \text{ N-m} \qquad \textit{Ans.}$$

The coefficients of these unit vectors represent respectively the magnitude of the moment of **F** about the *X, Y*, and *Z* coordinate directions through *C*. Combining these components, the magnitude of the moment is $\mathbf{M}_C^F$ = 805 N-m.

The moment component of $\mathbf{M}_C^F$ about the direction *CB* is expressed by $\mathbf{M}_{CB}^F = \mathbf{M}_C^F \cdot \hat{\mathbf{n}}_{CB}$ where the unit vector along *CB* is

$$\hat{\mathbf{n}}_{CB} = \frac{\vec{CB}}{CB} = \frac{-8\hat{\mathbf{i}} - 3\hat{\mathbf{k}}}{\sqrt{73}}$$

Recalling that the dot product is the summation of the products of the respective coefficients of the unit vectors, we obtain

$$M_{CB}^F = \mathbf{M}_C^F \cdot \hat{\mathbf{n}}_{CB} = (400\,\hat{\mathbf{i}} + 160\,\hat{\mathbf{j}} + 680\,\hat{\mathbf{k}}) \cdot \left(\frac{-8\hat{\mathbf{i}} - 3\hat{\mathbf{k}}}{\sqrt{73}}\right)$$

$$= \frac{1}{\sqrt{73}}(-3200 - 2040) = -614 \text{ N-m} \qquad \textit{Ans.}$$

The negative sign of this result denotes that the vector corresponding to this moment component is directed opposite to the direction *CB*, i.e., $\mathbf{M}_{CB}^F = -614\,\hat{\mathbf{n}}_{CB}$. Of course, a positive component would be obtained if the unit vector had been $\hat{\mathbf{n}}_{BC}$ directed from *B* to *C*.

Now to find the value of **P** to cause its moment about the axis *AC* to be 2000 N-m, refer again to Fig. 2-11.3 and, by using the scheme explained previously, first find the moment of **P** about a center on *AC* as follows:

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$$\mathbf{M}_C^P = \mathbf{r}_{CD} \times \mathbf{P} = P_m \begin{vmatrix} -4 & +10 & 0 \\ -4 & -10 & -3 \\ \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \end{vmatrix}$$

$$= P_m\,[\,\hat{\mathbf{i}}(-30) + \hat{\mathbf{j}}(-12) + \hat{\mathbf{k}}(40 + 40)] \text{ N-m}$$

The data imply that the moment component of **P** about *AC* is 2000 N-m in the direction *AC*, so we use a unit vector $\hat{\mathbf{n}}_{AC}$ in forming the following dot product:

$$[\,M_{AC}^P = \mathbf{M}_C^P \cdot \hat{\mathbf{n}}_{AC}\,]$$

$$2000 = P_m(-30\hat{\mathbf{i}} - 12\hat{\mathbf{j}} + 80\hat{\mathbf{k}}).\left(\frac{8\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}}{\sqrt{89}}\right)$$

$$= \frac{P_m}{\sqrt{89}}(-240 + 36 - 320)$$

$$= -\frac{524}{9.43}P_m, \quad P_m = -36$$

Finally, multiplying this value of P_m into the second row of the determinant written above (since this row represents the scalar components of **P**), we obtain the vector form of **P** to be

$$\mathbf{P} = 144\hat{\mathbf{i}} + 360\hat{\mathbf{j}} + 108\hat{\mathbf{k}} \text{ and } P = 403 \text{ N} \qquad \textbf{\textit{Ans.}}$$

An alternate procedure, preferred by some, is to write $M_{AC}^{P} = \mathbf{M}_C^P \cdot \hat{\mathbf{n}}_{AC}$ as $(\mathbf{r}_{CD} \times \mathbf{P}) \cdot \hat{\mathbf{n}}_{AC}$ which is a scalar triple product. The value of this scalar triple product is the expansion of the third-order determinant as written below, in which the first, second, and third rows are the *xyz* order of the scalar components of the respective order of the vectors in the triple product. Doing this gives

$$M_{AC}^{P} = \frac{P_m}{\sqrt{89}}\begin{vmatrix} -4 & -10 & 0 \\ -4 & -10 & -3 \\ 8 & -3 & -4 \end{vmatrix}$$

$$= \frac{P_m}{\sqrt{89}}[8(-30) - 3(-12) - 4(40 + 40)]$$

$$= \frac{P_m}{\sqrt{89}}[-240 + 36 - 320]$$

as before.

2-11.5 In Fig. P-2-11.5, a boom *AC* is supported by a ball-and-socket joint at *C* and by the cables *BE* and *AD*. If the force multiplier of force **F** acting from *B* to *E* is F_m = 10 N/m, find (a) the moment of **F** about point *C*, (b) the moment of **F** about point *D*, (c) the moment of **F** about a line directed from *C* to *D*.

(a) $\mathbf{M}_C^F = 300\hat{\mathbf{i}} - 240\hat{\mathbf{j}} + 520\hat{\mathbf{k}}$ N-m;

(b) $\mathbf{M}_D^F = -540\hat{\mathbf{i}} - 360\hat{\mathbf{j}} + 120\hat{\mathbf{k}}$ N-m;

(c) $M_{CD}^F = -379$ N-m. (What does the minus sign mean?) ***Ans.***

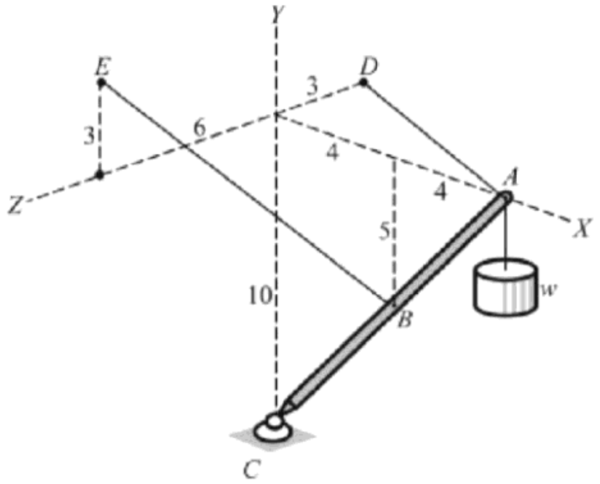


Figure P-2-11.5

2-11.7 For the system shown in Fig. P-2-11.7, the force multiplier of **P** acting from A to E is $P_m = 100$ N/m. Find the moment of **P** about C and about D. What is the moment of **P** about an axis directed from C to D?

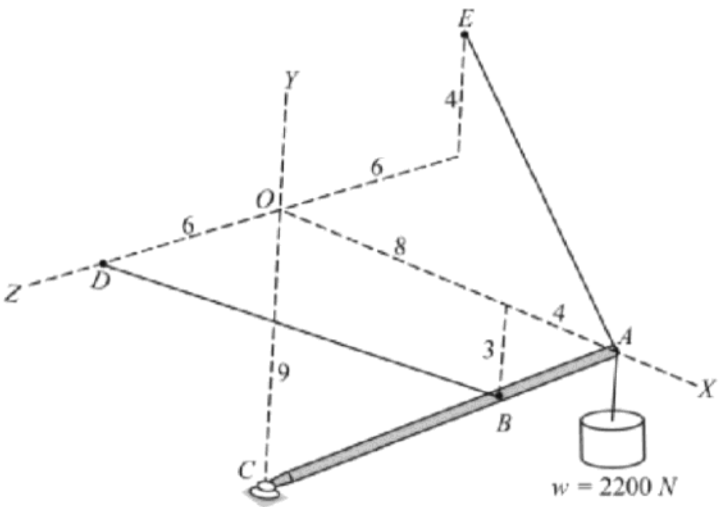


Figure P-2-11.7

2-12 COUPLES

A couple is defined as a pair of equal, parallel, oppositely directed forces. As shown in Fig. 2-12.1, the perpendicular distanced d between the action lines of the forces is called the moment arm of the couple. The vector sum of these two forces is zero, but their moment sum is not zero. The only effect of a couple on a body is a tendency to rotate the body about an axis perpendicular to the plane of the couple. *A unique property of a couple is that the moment sum of its forces is constant and independent of any moment center*. This is proved by selecting moment centers at A and B to give respectively

$$\circlearrowleft + \Sigma M_A = Fd$$

$$\circlearrowleft + \Sigma M_B = F(d + a) - Fa = Fd$$

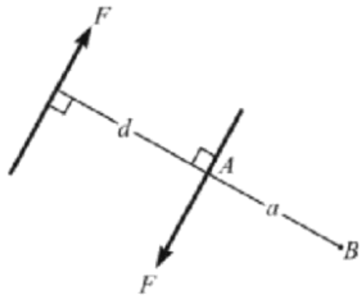


Figure 2-12.1 Moment of a couple is independent of moment center.

We conclude that the moment of a couple **C** is equal to the product of one of the forces composing the couple multiplied by the perpendicular distance between their action lines. This relation is expressed by the equation

$$|\mathrm{C}| = Fd$$

For the general case of two parallel, equal, oppositely directed spatial forces **F** and –**F** shown in Fig. 2-12.2, their moment sum also is constant and independent of any moment center. Thus, with respect to any origin O, draw position vectors $\mathbf{r}_A$ to any point A on **F** and draw $\mathbf{r}_B$ to any point B on –**F**. Adding the moment of each force about O, we have

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F} = \rho \times \mathbf{F}$$

from which we conclude that the moment vector of a couple is perpendicular to the plane of the couple, and independent of any moment center since the magnitude of $\boldsymbol{\rho} \times \mathbf{F}$ is always equal to Fd where d is the perpendicular distance between the action lines of the forces. Furthermore, since the origin and points A and B may be chosen arbitrarily, it follows that the moment vector of a couple is a free vector; i.e., it may be placed any where parallel to itself. The position vector $\boldsymbol{\rho}$ may be drawn from either of the forces to the other and the resultant cross product used to determine the couple vector. Usually the sense of rotation of the couple is evident by inspection.

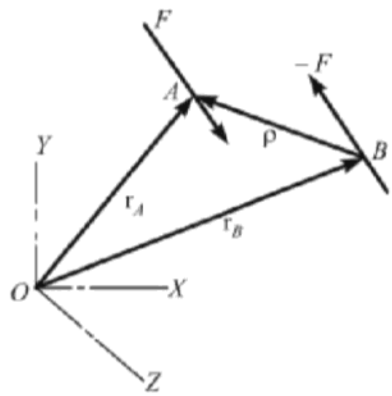


Figure 2-12.2 Moment of a spatial couple is independent of any movement center.

Since the only effect of a couple is to produce a moment that is independent of the moment center, the effect of a couple is unchanged if

(a) the couple is rotated through any angle in its plane

(b) the couple is shifted to any other position in its plane

(c) the couple is shifted to a parallel plane

(d) the couple is replaced by another pair of forces in its plane whose product Fd and sense of rotation is unchanged.

For the somewhat special case in which a given force system is composed entirely of couples in the same or parallel planes, the resultant couple will consist of another couple equal to the algebraic summation of the moment sum of the original couples. For all other cases, the resultant couple will be the geometric summation of the moment vectors of the original couples.

2-13 RESULTANT OF ANY FORCE SYSTEM

In this section, we shall show how to replace any force system by single resultant force acting through any arbitrary point plus a single resultant couple which is the moment sum of the original force system about that arbitrary point. This reduction of a given force system to an equivalent force-couple system determines either the criteria for equilibrium or else the type of motion it imparts to a rigid body.

The transformation of a given force system into an equivalent force-couple system is accomplished as shown in Fig. 2-13.1 where the original spatial force system in (a) is modified to that in (b) by adding balanced pairs of forces through the chosen reference point O. Each force of these balanced pairs has a magnitude equal to one of the original forces. Obviously, the addition of these balanced pairs of forces does not alter the effect of the original force system on the state of motion of the body. System (b) is now separated into that shown in (c) where we see that each of the original forces is equivalent to the same force now acting through O plus a couple equal to the moment of that force about O. The resultant of the concurrent system of the left portion of (c) is the resultant force **R** acting through O in part (d) whereas the system of couples acting on the right portion of (c) produces the resultant couple shown as $C = \Sigma M_O$, in (d).

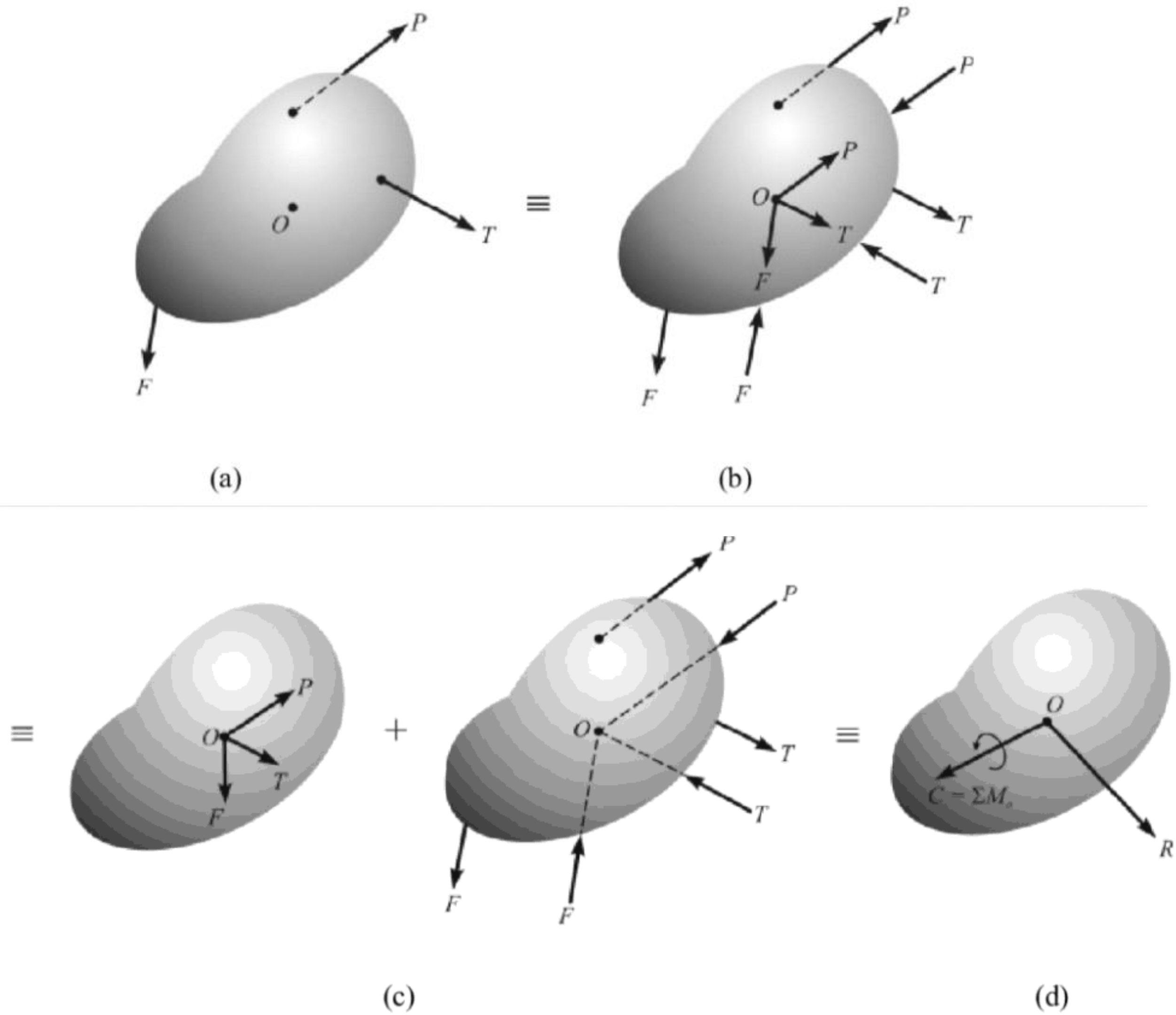


Figure 2-13.1 Reduction of a nonconcurrent system to a force-couple system.

This discussion is valid for either coplanar or spatial systems. For coplanar systems, if the resultant is not merely a couple, the reduction may be extended one more step to consist of a single force so located that it causes the same rotational effect about O as the couple $\mathbf{C} = \Sigma \mathbf{M}_O$, but for spatial systems the reduction usually ends with the force-couple system. It is possible to reduce spatial systems one step further to a *wrench* consisting of the resultant force and a parallel couple vector, but this has limited interest. Refer to the hint in Prob. 2-13.24 for the procedure of determining a wrench.

Summarizing this discussion, we compute the resultant force as though the original forces were concurrent using the methods of Sections 2-4 and 2-5. The value of the resultant couple is equal to the moment sum of the original forces about the selected origin. Note that the resultant couple depends on the position of the selected origin, but that the resultant force is independent of the position of the origin. In vector form we have

and
$$\begin{aligned} \mathbf{R} &= \mathbf{P} + \mathbf{F} + \mathbf{T} + \ldots = \Sigma \mathbf{F} \\ \mathbf{M}_O^R &= \mathbf{r}_1 \times \mathbf{P} + \mathbf{r}_2 \times \mathbf{F} + \mathbf{r}_3 \times \mathbf{T} + \ldots = \Sigma \mathbf{M}_O \end{aligned} \qquad (2\text{-}13.1)$$

which are equivalent to these six scalar equations and magnitudes:

$$\begin{aligned} &\mathbf{R}_x = \Sigma \mathbf{X}; \quad \mathbf{R}_y = \Sigma \mathbf{Y}; \quad \mathbf{R}_z = \Sigma \mathbf{Z} \\ &\mathbf{M}_x^R = \Sigma \mathbf{M}_x; \quad \mathbf{M}_y^R = \Sigma \mathbf{M}_y; \quad \mathbf{M}_Z^R = \Sigma \mathbf{M}_z \\ &\mathbf{R} = \sqrt{(\Sigma \mathbf{X})^2 + (\Sigma \mathbf{Y})^2 + (\Sigma \mathbf{Z})^2}; \\ &\mathbf{M}_O^R = \sqrt{(\Sigma \mathbf{M}_x)^2 + (\Sigma \mathbf{M}_y)^2 + (\Sigma \mathbf{M}_z)^2} \end{aligned} \qquad (2\text{-}13.2)$$

2-13.1 A flat plate is subjected to the coplanar system of forces shown in Fig. 2-13.2(a). The inscribed grid with each square having a length of 1 m locates each force and its slope. Determine the resultant and its x and y intercepts.

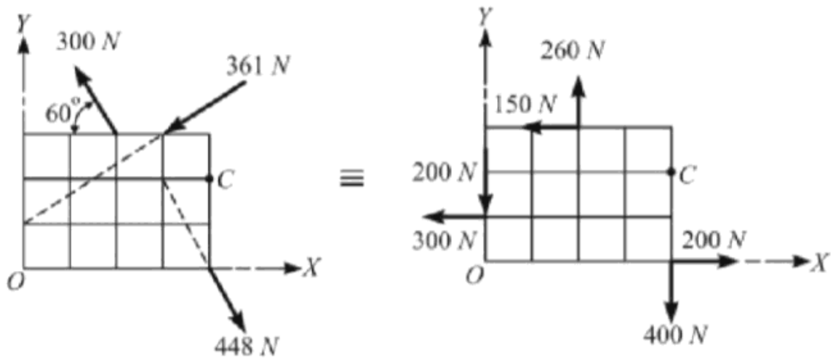


Figure 2-13.2

Solution

We start by resolving each force into its X and Y components. These components may be drawn as dashed vectors on the original diagram, but they are shown here on a duplicate diagram in (b) to help follow the computations. Each set of components must intersect on the action line of the force they replace (see Sections 2-9 and 2-10). By judiciously locating these components as shown, we can simplify the moment summation about O.

The effects of the resultant are now found to be

$$[\overset{+}{\rightarrow} R_x = \Sigma X] \qquad R_x = 200 - 150 - 300 = -250 \text{ N} \leftarrow$$

$$[+\uparrow R_y = \Sigma Y] \qquad R_y = 260 - 200 - 400 = -340 \text{ N} \downarrow$$

$$[\circlearrowleft + M_O^R = \Sigma M_O] \qquad M_O^R = 400(4) - 260(2) - 150(3) - 300(1)$$

$$= 330 \text{ N-m}$$

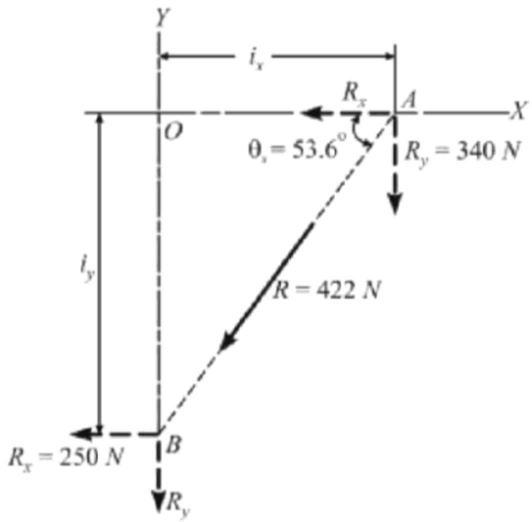


Figure 2-13.3 Location of resultant.

Combining these effects, the resultant has a magnitude of 422 N directed down to the left at $\theta_x = 53.6°$ with the X axis as shown in Fig. 2-13.3. The position of the reference origin must be above R as shown in order for R to have the clockwise moment effect about O found above. Using this figure, we apply the principle that the moment of a force equals the moment sum of its parts by resolving R into its components at A and B. From the components located at A, we find the x intercept to be

$$[M_O^R = (R_y)\, i_x] \qquad 330 = 340\, i_x \qquad i_x = 0.97 \text{ m right of } O$$

and similarly from the components located at B, the y intercept is

$[M_O^R = (R_x)\, i_y]$ $\qquad 330 = 250\, i_y \qquad i_y = 1.32$ m below O

Note that at point A, R_x passes through O and hence has no moment about it; likewise at B, R_y has no moment about O.

In the case of coplanar forces, it is possible to reduce a force-couple system to a single resultant force as we have done above because the vector of the resultant couple is perpendicular to the resultant force. This usually does not occur in a spatial force system except for parallel spatial forces.

2-13.5 Replace the system of forces acting on the frame in Fig. P-2-13.5 by a resultant force R through A and a couple acting horizontally through B and C.

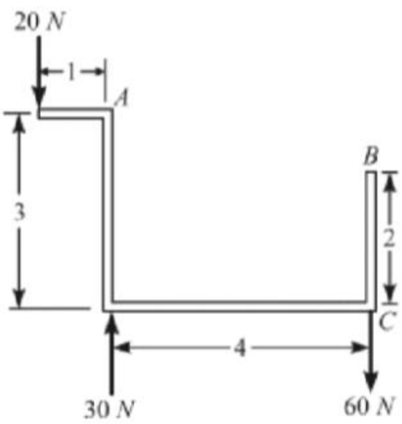


Figure P-2-13.5

$R = 50$ N down; $B = 110$ N right; $C = 110$ N left ***Ans.***

2-13.6 Fig. P-2.13.6 shows a plate subjected to loads. Replace the loads by an equivalent force through C and a couple acting through A and B. Solve if the forces of the couple are (a) horizontal and (b) vertical.

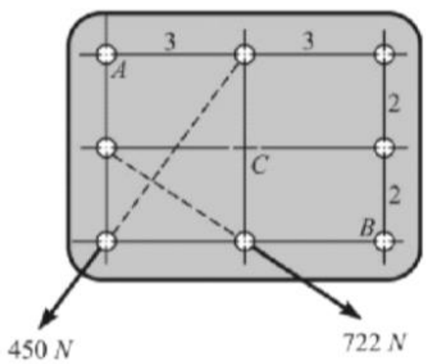


Figure P-2-13.6

2-13.7 A vertical force P at A and another vertical force F at B which act on the bar shown in Fig. P-2-13.7 produce a resultant force of 150 N down at D and a counterclockwise couple C = 300 N-m. Find the magnitude and direction of forces P and F.

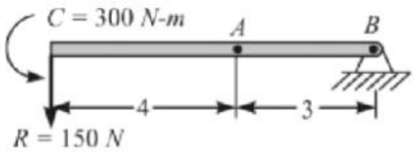


Figure P-2-13.7

2-13.8 Determine the amount and position of the resultant of the loads acting on the Fink truss shown in Fig. P-2-13.8.

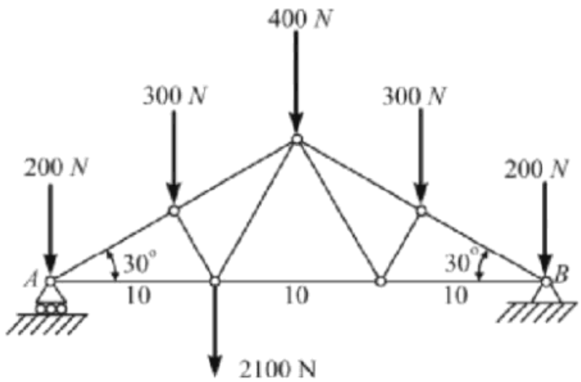


Figure P-2-13.8

R = 3500 N down at 12 m to right of A ***Ans.***

2-13.9 Find the values of P and F so that the four forces shown in Fig. P-2-13.9 produce an upward resultant of 300 N acting at 4 m from the left end of the bar.

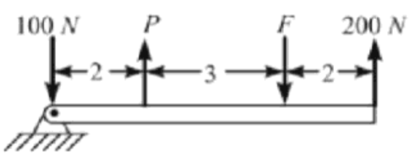
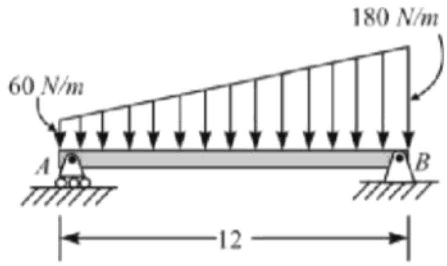


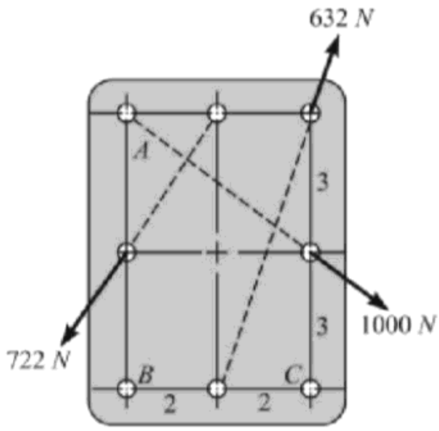
Figure P-2-13.9

2-13.11 The beam *AB* in Fig. P-2-13.11 supports a load which varies uniformly from an intensity of 60 N/m to 180 N/m. Calculate the magnitude and position of the resultant load.

Hint: Replace the given loading by two equivalent triangular loadings each similar to the loading of Prob. 2-13.10.



2-13.13 Compute the resultant of the three forces acting on the plate shown in Fig. P-2-13.13. Locate its intersection with *AB* and *BC*.



$R = 776$ N down to the right at $\theta_h = 40.6°$
acting 3.43 m. above *B* and 4 m. right of *B* ***Ans.***

2-13.15 The Howe roof truss shown in Fig. P-2-13.15 carries the given loads. The wind loads are perpendicular to the inclined members. Determine the resultant and its intersection with *AB*.

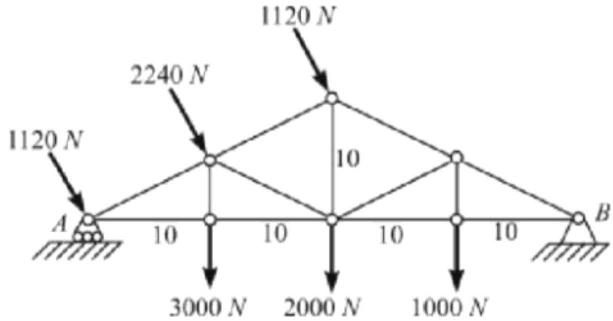


Figure P-2-13.15

2-13.16 The three forces shown on the grid in Fig. P-2-13.16 produce a horizontal resultant force through point A. Find the magnitude and sense of *P* and *F*.

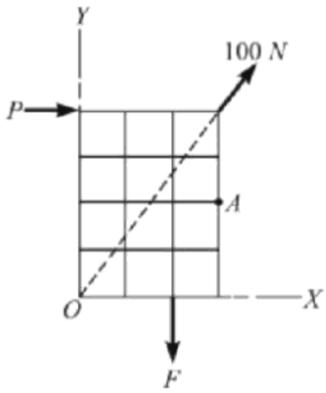


Figure P-2-13.16

2.13.18 The three forces shown in Fig. P-2-13.18 are required to cause a horizontal resultant acting through point *A*. If $T = 316$ N, determine the values of *P* and *F*.

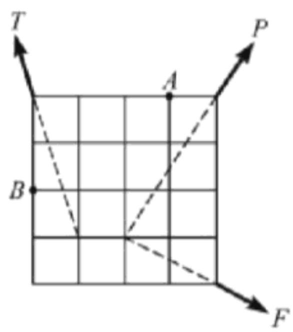


Figure P-2-13.18

P = – 180.5 N; F = 336 N ***Ans.***

2-13.22 In Fig. P-2-13.22, a force **T** acts along *BE* and a force **P** acts along *AD*. Assuming their force multipliers to be $T_m = 20$ N/m and $P_m = 10$ N/m, find the force **F** to be applied at *C* to reduce their resultant to a couple. What is this resultant couple?

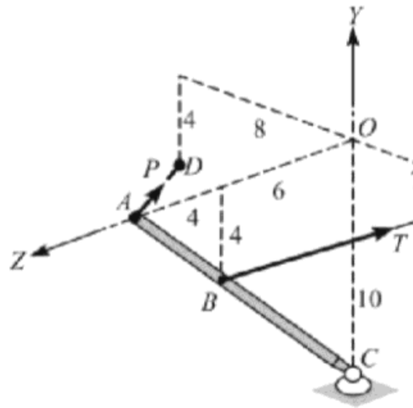


Figure P-2-13.22

$$F = -60\,\hat{\mathbf{i}} - 40\,\hat{\mathbf{j}} + 220\,\hat{\mathbf{k}} \text{ N;}$$

$$C = -1800\,\hat{\mathbf{i}} + 40\,\hat{\mathbf{j}} - 40\,\hat{\mathbf{k}} \text{ N-m}$$ ***Ans.***

2.13.23 In Fig. P-2-13.23, the boom *AC* is acted upon by a vertical load W = 600 N, a force $T = 440$ N directed from *B* to *D*, and a force $P = 280$ N directed from *A* to *E*. Find the force **F** to be applied at *C* to reduce the resultant on the boom to a couple. What is this couple?

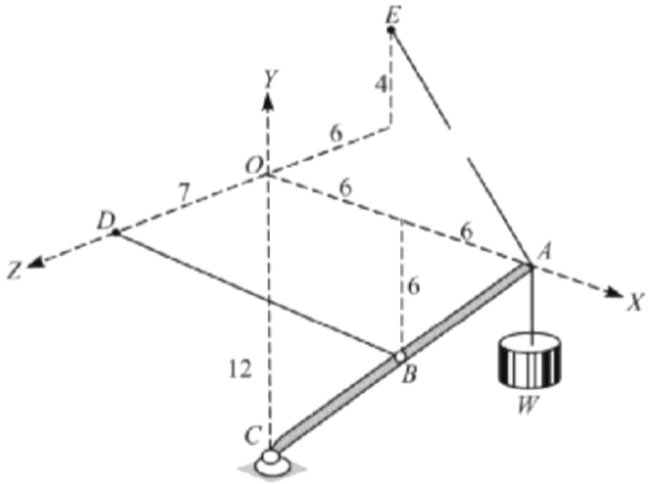


Figure P-2-13.23