Chapter 6

CURVES USED IN ENGINEERING PRACTICE

6-0. INTRODUCTION

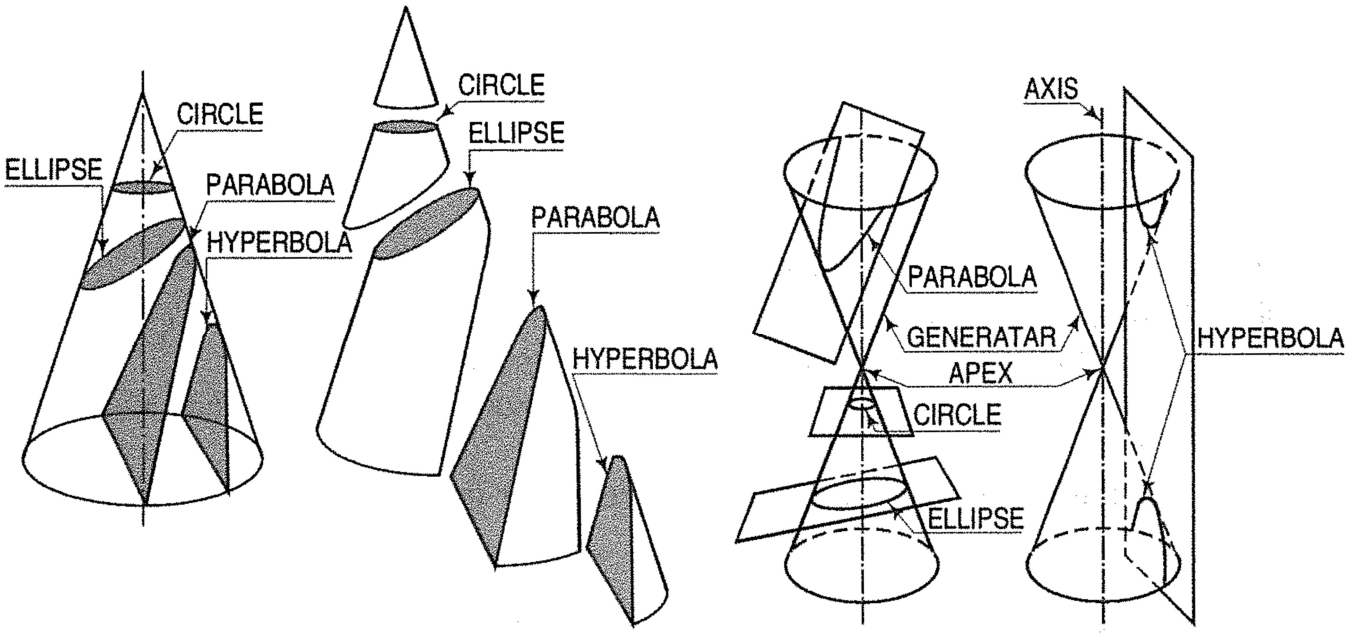
The profile of number of objects consists of various types of curves. This chapter deals with various types of curves which are commonly used in engineering practice as shown below:

1. Conic sections
2. Cycloidal curves
3. Involute
4. Evolutes
5. Spirals
6. Helix.

We shall now discuss the above in details with reference to their construction and applications.

6-1. CONIC SECTIONS

The sections obtained by the intersection of a right circular cone by a plane in different positions relative to the axis of the cone are called *conics*. Refer to fig. 6-1.



Conic sections

FIG. 6-1

(i) When the section plane is inclined to the axis and cuts all the generators on one side of the apex, the section is an *ellipse* [fig. 6-1].

(ii) When the section plane is inclined to the axis and is parallel to one of the generators, the section is a *parabola* [fig. 6-1].

(iii) A *hyperbola* is a plane curve having two separate parts or branches, formed when two cones that point towards one another are intersected by a plane that is parallel to the axes of the cones.

The conic may be defined as the locus of a point moving in a plane in such a way that the ratio of its distances from a fixed point and a fixed straight line is always constant. The fixed point is called the *focus* and the fixed line, the *directrix*.

The ratio $\dfrac{\text{distance of the point from the focus}}{\text{distance of the point from the directrix}}$ is called *eccentricity* and is denoted by e. It is always less than 1 for ellipse, equal to 1 for parabola and greater than 1 for hyperbola i.e.

(i) ellipse : $e < 1$

(ii) parabola : $e = 1$

(iii) hyperbola : $e > 1$.

The line passing through the focus and perpendicular to the directrix is called the *axis*. The point at which the conic cuts its axis is called the *vertex*.

6-1-1. ELLIPSE

Use of elliptical curves is made in arches, bridges, dams, monuments, manholes, glands and stuffing-boxes etc. Mathematically an ellipse can be described by equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. Here '$a$' and '$b$' are half the length of major and minor axes of the ellipse and x and y co-ordinates.

(1) General method of construction of an ellipse:

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This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 12 for the following problem.

Problem 6-1. (fig. 6-2): *To construct an ellipse when the distance of the focus from the directrix is equal to 50 mm and eccentricity is* $\frac{2}{3}$.

(i) Draw any vertical line AB as directrix.

(ii) At any point C on it, draw the axis perpendicular to the AB (directrix).

(iii) Mark a focus F on the axis such that $CF = 50$ mm.

(iv) Divide CF into 5 equal divisions (sum of numerator and denometer of the eccentricity.).

(v) Mark the vertex V on the third division-point from C.

Thus, eccentricity, $e = \dfrac{VF}{VC} = \dfrac{2}{3}$.

(vi) A scale may now be constructed on the axis (as explained below), which will directly give the distances in the required ratio.

(vii) At V, draw a perpendicular VE equal to VF. Draw a line joining C and E.

Thus, in triangle CVE, $\dfrac{VE}{VC} = \dfrac{VF}{VC} = \dfrac{2}{3}$.

(viii) Mark any point 1 on the axis and through it, draw a perpendicular to meet C*E*-produced at 1'.

(ix) With centre *F* and radius equal to 1-1', draw arcs to intersect the perpendicular through 1 at points P_1 and P'_1.

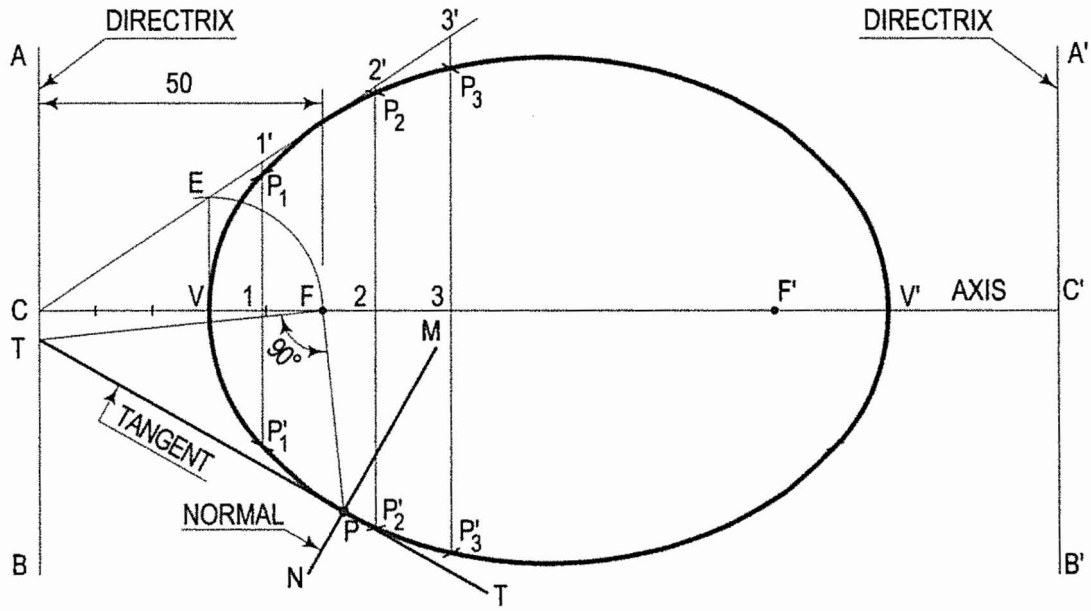
These are the points on the ellipse, because the distance of P_1 from *AB* is equal to *C*1,

$$P_1 F = 1\text{-}1'$$

and
$$\frac{1-1'}{C1} = \frac{VF}{VC} = \frac{2}{3}.$$

Similarly, mark points 2, 3 etc. on the axis and obtain points P_2 and P'_2, P_3 and P'_3 etc.

(x) Draw the ellipse through these points. It is a closed curve having two foci and two directrices.



Directrix and focus

FIG. 6-2

(2) Construction of ellipse by other methods:

Ellipse is also defined as a curve traced out by a point, moving in the same plane as and in such a way that the sum of its distances from two fixed points is always the same.

(i) Each of the two fixed points is called the *focus*.

(ii) The line passing through the two foci and terminated by the curve, is called the *major axis*.

(iii) The line bisecting the major axis at right angles and terminated by the curve, is called the *minor axis*.

Conjugate axes: Those axes are called conjugate axes when they are parallel to the tangents drawn at their extremities.

In fig. 6-3, *AB* is the major axis, *CD* the minor axis and F_1 and F_2 are the foci. The foci are equidistant from the centre *O*.