Algebraic structures

Fields: The standard properties of real numbers with respect to addition and Multiplication are so fundamental in Mathematics that whenever any set X with two binary operations '+' and '.' (called addition and multiplication) satisfies these properties ~~is~~ is called a Field.

The following properties holds for a field

Let R be the set of Real numbers and the operation $+$ and $\cdot$ in R

1) Addition is associative.
2) There Exists an Element $0 \in R$ with the property
$$0 + a = a = a + 0 \quad \text{for all} \quad a \in R$$
3) For each $a \in R$, There exists $-a \in R$ such that
$$a + (-a) = 0 = (-a) + a$$
4) Addition is Commutative
5) Multiplication is associative
6) There exists an element $1 \in R$ such that $1 \cdot a = a = a \cdot 1$ for all $a \in R$.
7) For each $a (\neq 0)$ in R, There exists an element $\frac{1}{a} \in R$ Such that $a \cdot \frac{1}{a} = 1 = a \cdot \frac{1}{a}$
8) Multiplication is Commutative.
9) Multiplication is distributive over addition that is for all $a, b, c \in R$ Such that
$$a \cdot (b + c) = a \cdot b + a \cdot c$$
and $$(a + b) \cdot c = a \cdot c + b \cdot c$$

Ex: 1) The set Q of Rational Numbers under $+$ and $\cdot$ is a field R (Real numbers), C (Complex numbers) are fie

2) The set of Z (Integer) is not a field under addition and Multiplicatio (∵ prop 7 is not satisfied