$$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)}$$

$$y_2 = \frac{x'(\theta)\, y''(\theta) - y'(\theta)\, x''(\theta)}{[x'(\theta)]^2}.$$

$$p = \frac{\left\{1 + \left[\frac{y'(\theta)}{x'(\theta)}\right]^2\right\}^{3/2} [x'(\theta)]^2}{x'(\theta)\cdot y''(\theta) - y'(\theta)\, x''(\theta)}$$

$$\boxed{p = \frac{\left\{[x'(\theta)]^2 + [y'(\theta)]^2\right\}^{3/2}}{x'(\theta)\cdot\left[x'(\theta)\cdot y''(\theta) - y'(\theta)\, x''(\theta)\right]}}$$

* Asteroid Equation: $\boxed{x^{2/3} + y^{2/3} = a^{2/3}}$

14|12|2021

* TAYLOR'S SERIES:

If $f : [a\ b] \longrightarrow R$ satisfies

(i) $f^{(n-1)th}$ derivatives exists & are continuous in [a

ii) $f^{(n-1)th}$ ~~derivable~~ derivative exists and derivable in (a x)

Then the expansion of f(x) in the [a b] is given by.

$$f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!} f''(a)$$

$$+ \frac{(b-a)^3}{3!} f'''(a) + \dots + \frac{(b-a)^{n-1}}{} f^{(n-1)}(a) +$$

ontinuous in [a x]
erivable in (a x)
[a b] is given

$$-a)^2 \over 2! \; f''(a)$$

$$+ \frac{(b-a)^{n-1}}{(n-1)!} f^{(n-1)}(a) +$$

$$\cdots + \frac{(b-a)^n}{n!} f^n(a) + \cdots \quad \text{— (1)}$$

* Another form of Taylor's Series:

put $b = a+h$ in eq: 1.

If $f : [a, a+h] \rightarrow R$ satisfies

(i) $f^{(n-1)th}$ derivative exists & are continuous in $[a\ x]$

(ii) $f^{(n-1)th}$ derivative exists & derivable in $(a\ x)$.

Then the expansion of $f(x)$ in the $[a\ a+h]$ is given by:

$$f(a+h) = f(a) + (h) f'(a) + \frac{h^2}{2!} f''(a) + \cdots + \frac{h^{n-1}}{(n-1)!} f^{n-1}(a) + \cdots + \frac{h^n}{n!} f^n(a) \quad \text{— (2)}$$

* put $a+h = x$ in eq 2

$a = x - h$. $\quad h = x - a$

$$f(x) = f(a) + (x-a) \cdot f'(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots + \frac{(x-a)^{n-1}}{(n-1)!} f^{n-1}(a) + \cdots + \frac{(x-a)^n f^n(a)}{n!} \quad \text{— (3)}$$

* put $a=0$ in eq (3)

$$f(x) = f(0) + x \cdot f'(0) + \frac{x^2}{2!} f''(0) + \cdots + \frac{x^{n-1}}{(n-1)!} f^{(n-1)}(0) + \cdots \frac{x^n f^n(0)}{n!}$$

* Statement of Maclaurin's Series:

If $f:[0\ x] \longrightarrow R$ satisfies

i) $f^{(n-1)th}$ derivative exists & are continuous in $[0\ x]$

) $f^{(n-1)th}$ derivative exists & derivable in $(0\ x)$.

1/12/2021

Find the Taylor Series of $f(x) = \sin x$ about $x =$

$$f(x) = \sin x \qquad f(\pi/3) = \frac{\sqrt{3}}{2}$$

$$f'(x) = \cos x \qquad f'(\pi/3) = \frac{1}{2}$$

$$f''(x) = -\sin x \qquad f''\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$f'''(x) = -\cos x \qquad f'''\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

Taylor's Series:

ansion of $f(x)$ about $x = a$

$$) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots$$

$$= \frac{\sqrt{3}}{2} + \cancel{\frac{\pi}{3}} \left(x - \frac{\pi}{3}\right) \cancel{f'} \left(\frac{1}{2}\right) + \frac{\left(x - \frac{\pi}{3}\right)^2}{2!} \left(-\frac{\sqrt{3}}{2}\right)$$

$$+ \frac{\left(x - \frac{\pi}{3}\right)^3}{3!} \left(-\frac{1}{2}\right).$$

2) Expand ~~the~~ sin x in powers of x

$$\Rightarrow \quad f(x) = \sin x \qquad f(0) = 0$$
$$f'(x) = \cos x \qquad f'(0) = 1$$
$$f''(x) = -\sin x \qquad f''(0) = 0$$
$$f'''(x) = -\cos x \qquad f'''(0) = -1$$

By Maclaurin's Series:

Expansion of f(x) about $x = 0$

- $$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots$$

$$\Rightarrow \sin x = 0 + x(1) + \frac{x^2}{2!}(0) + \frac{x^3}{3!}(-1)$$

$$= \cancel{x}\ \cancel{0}\ x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

3) Expand log(1+x) in powers of x

$$f(x) = \log(1+x) \qquad f(0) = 0$$
$$f'(x) = \frac{1}{1+x} \qquad f'(0) = 1$$
$$f''(x) = \frac{(1+x)(0) - 1}{(1+x)^2} \qquad f''(0) = -1$$
$$= \frac{-1}{(1+x)^2}$$
$$f'''(x) = -\left[\frac{(1+x)^2(0) - 2(1+x)}{(1+x)^4}\right] \qquad f'''(0) = 2$$
$$= \cancel{-}\ \frac{2}{(1+x)^3}$$

By Maclaurin's Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \cdots$$

$\Rightarrow$

$$\log(1+x) = 0 + x(1) + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(2)$$

$$= x - \frac{x^2}{2!} + \frac{2x^3}{3!}$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

Expand $\log(1-x)$ in powers of x.

$f(x) = \log(1-x)$ $\qquad f(0) = 0$

$f'(x) = \frac{-1}{1-x}$ $\qquad f'(0) = -1$

$$f''(x) = -\frac{[(1-x)(0) - (-1)]}{(1-x)^2} \qquad f''(0) = -1$$

$$= \frac{-1}{(1-x)^2}$$

$$f'''(x) = -\frac{[(1-x)^2(0) - (-1)]}{(1-x)^4} \qquad f'''(0) = -1.$$

$$= \frac{-1}{(1-x)^4}$$

$\frac{x^2}{2!} f''(0) + \cdots$

$(-1) + \frac{x^3}{3!}(2)$

$\frac{x^3}{3!}$

$- \frac{x^4}{4} + \cdots$

ers of x.

$f(0) = 0$

$f'(0) = -1$

$1)]$

$f''(0) = -1$

$(-1)]$ $f'''(0) = -1$.

$$\log(1-x) = -\left[x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right]$$

5) Write the expansion $2x^3+7x^2+x-6$ in powers of $x-2$.

6) Expand $\log_e x$ in powers of $x-1$ & hence evaluate $\log_e 1.1$ correct to 4 decimal places.

7) Obtain the expansion of $\cosh x$ about $x=0$

8) Expand $\sec x$ by Taylor's Series in powers of x.

5) $f(x) = 2x^3 + 7x^2 + x - 6$

$f'(x) = 6x^2 + 14x + 1$

$f''(x) = 12x + 14$

$f'''(x) = 12$

$f(2) = 2(8) + 7(4) + 2 - 6$

$= 16 + 28 - 4$

$= 40$

$f'(2) = 6(4) + 14(2) + 1$

$= 25 + 28$

$= 53$

$f''(2) = 38$

$f'''(2) = 12$

By Taylor's Series:

~~$2x^3+7x^2+x-6 = 24$~~

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots$$

$$2x^3+7x^2+x-6 = 40 + \frac{(x-2)}{1!}(53) + \frac{(x-2)^2}{2!}(38) + \frac{(x-2)^3}{3!}(12) + \cdots$$

$$42 + (x-2)(53) + \frac{(x-2)^2}{\cancel{4}\,2}(\cancel{38}\;19) + \frac{(x-2)^3}{6}$$
$$+ \cdots$$

6) $f(x) = \log_e x$ $\qquad f(1) = 0$

$f'(x) = \frac{1}{x}$ $\qquad f'(1) = 1$

$f''(x) = \frac{x(0)-1}{x^2}$ $\qquad f''(1) = -1$

$= \frac{-1}{x^2}$

$f'''(x) = \frac{-[x^2(0) - 2x]}{x^4}$ $\qquad f'''(1) = 2$

$= \frac{2}{x^3}$

aylor's Series:

$$= f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a)$$

$$= 0 + (x-1)(1) + \frac{(x-1)^2}{2!}(-1) + \frac{(x-1)^3}{6}(2)$$

$$= (x-1) - \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3} + \cdots$$

$$= (1{\cdot}1-1) - \frac{(1{\cdot}1-1)^2}{2} + \frac{(1{\cdot}1-1)^3}{3} + \cdots$$

$(0{\cdot}1) - (0{\cdot}1)^2/2 + (0{\cdot}1)^3/3 \cdots$

$+\frac{(x-2)^3}{6}\left(\frac{2}{12}\right)$

$= 0.1 - \frac{0.01}{2} + \frac{0.001}{3} + \cdots$

$= 0.1 - 0.005 + 0.00033$

$= 0.10033 - 0.005 = 0.09533$

7) $f(x) = \cosh x$ $\quad f(0) = 1$

$f'(x) = \sinh x$ $\quad f'(0) = 0$

$f''(x) = \cosh x$ $\quad f''(0) = 1$

$f'''(x) = \sinh x$ $\quad f'''(0) = 0.$

$) = 2$

By Maclaurin's Series;

$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) \cdots$

$\cosh x = 1 + (x-0)f(0) + \frac{x^2}{2!}(1) + \frac{x^3}{3!}(0)$

$) + \frac{(x-a)^3}{36}f'''(a)$

$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

$) + \frac{(x-1)^3}{6}(2)$

8) $f(x) = \sec x$ $\quad f(0) = 1$

$f'(x) = \tan x \sec x$ $\quad f'(0) = (0)$

$f''(x) = \sec^3 x + \tan^2 x \sec x$ $\quad f''(0) = 1 + 0$

$f'''(x) = 3\sec^2 x \cdot \sec x \tan x + \tan^3 x \sec x + \sec x \cdot 2\tan x \sec^2 x$ $= 3\sec^3 x \tan x + \sec x \tan^3 x + 2\tan x \sec^3 x$

$)^3$

$+ \cdots$

$f'''(0) = 3(0) + 0 + 0 = 0.$

$f''''(x) = 3(3\sec^2 x)\sec x \tan^2 x + 3\sec^5 x + \sec x \cdot 3\tan^2 x \cdots$

By Maclaurin's Series:

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots$$

$$\sec x = 1 + x(0) + \frac{x^2}{2!}(1) + \frac{x^3}{3!}(0) + \cdots$$

$$= 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \cdots$$

16/12/2021

* Without applying Taylor's or Maclaurin's Series Expand the following in powers of x.

1) $f(x) = e^x \cdot \sin x$

$$f(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)$$

$$= x - \frac{x^3}{3!} + x^2$$

2) $f(x) = \frac{\sin x}{x}$

$$= (\sin x)\frac{1}{x}$$

$$= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)\left(\frac{1}{x}\right)$$

$$= 1 - \frac{x^2}{6} + \frac{x^4}{5!}$$

3) $f(x) = \sin^2 x$

$$= (\sin x)^2 = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)^2$$

* Expand $f(x) = \log(\sin x)$ about $x = a$

$$f(x) = \log(\sin x)$$
$$f'(x) = \cot x$$
$$f''(x) = -\operatorname{cosec}^2 x$$
$$f'''(x) = -2\operatorname{cosec}^2 x \cot x$$

$$f(a) = \log(\sin a)$$
$$f'(a) = \cot a$$
$$f''(a) = -\operatorname{cosec}^2 a$$
$$f'''(a) = +2\operatorname{cosec}^2 a \cot a$$

By Taylor's Series:

$$f(x) = f(a) + (x-a) f'(a) + (x-a)^2 f''(a) + \cdots$$

$$\log(\sin x) = \log(\sin a) + \frac{(x-a)}{1!}\cot a + \frac{(x-a)^2}{2!}(-\operatorname{cosec}^2 a) + \frac{(x-a)^3}{3!}(+2\operatorname{cosec}^2 a \cot a)$$

$$= \log(\sin a) + (x-a)\cot a - \frac{(x-a)^2}{2!}\operatorname{cosec}^2 a + \frac{(x-a)^3}{3!}(2)\operatorname{cosec}^2 a \cot a + \cdots$$

* Expand $\sin^{-1} x$ in powers of x:

$$f(x) = \sin^{-1} x$$
$$f'(x) = \frac{+1}{\sqrt{1-x^2}}$$
$$f''(x) = \frac{-1}{2}(1-x^2)^{-3/2} \cdot -2x$$
$$= x(1-x^2)^{-3/2}$$
$$f'''(x) = -\frac{3}{\cancel{2}} x (1-x^2)^{-5/2} \cdot -\cancel{2}x + (1-x^2)^{-3/2}$$
$$= 3x^2(1-x^2)^{-5/2} + (1-x^2)^{-3/2}$$

$$f(0) = 0$$
$$f'(0) = 1$$
$$f''(0) = 0$$
$$f'''(0) = 1$$

By Maclaurin's Series:

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots$$

$$\sin^{-1}x = 0 + x(1) + \frac{x^2}{2!}(0) + \frac{x^3}{3!}(1) + \cdots$$

$$\sin^{-1}x = x + \frac{x^3}{3!} + \cdots$$

* Expand 2^x in powers of x.

$$f(x) = 2^x \qquad f(0) = 2^0 = 1$$

$$f'(x) = 2^x \log 2 \qquad f'(0) = \log 2$$

$$f''(x) = (2^x)^2(\log 2)^2 \qquad f''(0) = (\log 2)^2$$

$$f'''(x) = 2^x(\log 2)^3 \qquad f'''(0) = (\log 2)^3$$

By Maclaurin's Series:

$$f(x) = f(0) + (x \cancel{-a}) f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f''(0) + \cdots$$

$$2^x = 1 + x(\log 2) + \frac{x^2}{2!}(\log 2)^2 + \frac{x^3}{3!}(\log 2)^3 + \cdots$$

* Expand $f(x) = \cos x$ in powers of $x = -\pi/4$

$$f(x) = \cos x \qquad f(-\pi/4) = \frac{1}{\sqrt{2}}$$

$$f'(x) = -\sin x \qquad f'(-\pi/4) = 1/\sqrt{2}$$

$$f''(x) = -\cos x \qquad f''(-\pi/4) = -1/\sqrt{2}$$

$$f'''(x) = \sin x \qquad f'''(-\pi/4) = -1/\sqrt{2}$$

By Taylor's Series:

$$f(x) = \cos x = \frac{1}{\sqrt{2}} + (x + \pi/4)\left(\frac{1}{\sqrt{2}}\right) + \frac{(x+\pi/4)^2}{2!}\left(\frac{-1}{\sqrt{2}}\right) +$$

$) + \frac{x^3}{3!} f'''(0) + \cdots$

$\frac{x^3}{3!}(1) + \cdots$

$0) = 2^0 = 1$

$(0) = \log 2$

$'(0) = (\log 2)^2$

$(0) = (\log 2)^3$

$'(0) + \frac{x^3}{3!} f''(0) + \cdots$

$2)^2 + \frac{x^3}{3!}(\log 2)^3 + \cdots$

owers of $x = -\pi/4$

$\sqrt{2}$

$/\sqrt{2}$

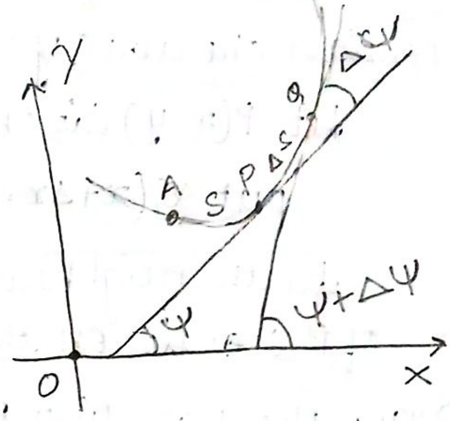
$-1/\sqrt{2}$

$-1/\sqrt{2}$

$+ \frac{(x+\pi/4)^2}{2!}\left(\frac{-1}{\sqrt{2}}\right) +$

$\frac{(x+\pi/4)^3}{3!}\left(\frac{-1}{\sqrt{2}}\right) + \cdots\cdots$

* CURVATURE:



Curvature of a curve is a measure of rate of change of bending moment.

Let AP = arc length $= S$

$PQ = \Delta S$ $\qquad AQ = S + \Delta S$

The total curvature of arc PQ is defined to be the angle $\Delta\psi$.

Angle b/w PQ is $\boxed{\lim_{\Delta s \to 0} \frac{\Delta\psi}{\Delta S}}$

$$\boxed{k = \frac{d\psi}{ds}}$$

* Find the curvature of $\psi = \sec s$

$k = \sec s \cdot \tan s$.

→ Radius of Curvature is the reciprocal of curvature at any point of a curve and it is denoted by ρ / ρ (rho)

$$\boxed{\rho = \frac{1}{k}} \qquad \boxed{\rho = \frac{ds}{d\psi}}$$

* Reason: The curvature at any point of the circle is equal to reciprocal of its radius.

3/12/2021

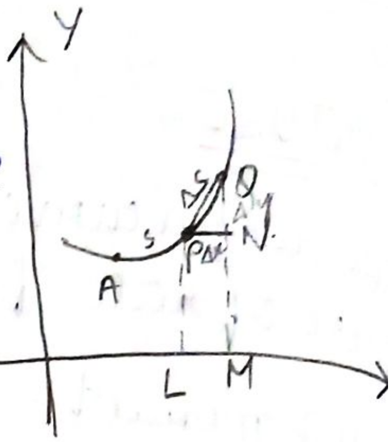
Derivation of length of an arc:

Consider the curve $y = f(x)$

Let $P(x, y)$ lie on the curve

put $Q(x+\Delta x, y+\Delta y)$

be the neighbouring point of P i.e. lie on the curve



Draw the $\perp$ar lines PL and QM which meets the x-axis & also draw PN is a $\parallel$el of x-axis.

In ΔPQN $(PQ)^2 = (PN)^2 + (QN)^2$

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2$$

Dividing by $(\Delta x)^2$ on both sides

$$\left(\frac{\Delta s}{\Delta x}\right)^2 = 1 + \left(\frac{\Delta y}{\Delta x}\right)^2$$

$$\frac{\Delta s}{\Delta x} = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}$$

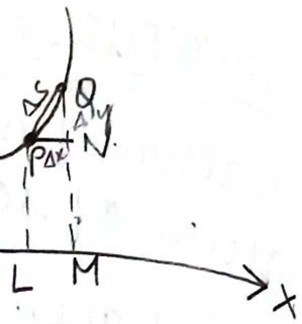
$$\lim_{\Delta x \to 0} \frac{\Delta s}{\Delta x} = \lim_{\Delta x \to 0} \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\boxed{\frac{ds}{dx} = \sqrt{1 + y_1^2}}$$

* $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

* $$\lim_{\Delta y \to 0} \frac{V[x, y+\Delta y] - V[x, y]}{\Delta y}$$

$$\Rightarrow \frac{\partial^2 v}{\partial x \partial y} = \frac{\partial}{\partial x}(V_y) = \lim_{\Delta x \to 0} \frac{V_y[x+\Delta x, y] - V_y[x, y]}{\Delta x}.$$



meets the x-axis

$(\ldots)^2 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

* Derivation of radius of curvature in cartesian form:

ρ [Radius]

- Cartesian
- polar (Not in syllabus)
- parametric

⇒DERIVATION :

Given curve $y = f(x)$ (explicit) — ①

implicit
$f(x, y) = C$ (or)
$f(x, y) = 0$

We have slope of the tangent of given curve

$$\tan\psi = y_1 \quad — ②$$

Diff. 2 w.r.to 'x' on both sides

length of arc:
$$\frac{ds}{dx} = \sqrt{1 + y_1^2}$$

$$y_2 = \sec^2\psi \frac{d\psi}{dx}$$

$$y_2 = (1 + \tan^2\psi)\frac{d\psi}{ds} \cdot \frac{ds}{dx}$$

$$y_2 = (1 + y_1^2)\, k \cdot \sqrt{1 + y_1^2}$$

$$y_2 = k(1 + y_1^2)^{3/2}$$

$$\boxed{\frac{1}{k} = \frac{(1 + y_1^2)^{3/2}}{|y_2|} = \text{radius of curvature } \rho}$$

ivation of radius of curvature in parametric

n:

$= x(t) \quad , \quad y = y(t)$

$\frac{dx}{dt} = x'(t) \qquad \frac{dy}{dt} = y'(t)$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

$$\frac{d^2y}{dx^2} = \frac{\dfrac{x'(t)\cdot y''(t) - y'(t)\cdot x''(t)}{[x'(t)]^2}}{x'(t)}$$

$\cancel{y_2 = x'(t)}$

$$y_2 = \frac{x'(t)\cdot y''(t) - y'(t)\cdot x''(t)}{[x'(t)]^3}$$

We have ρ in cartesian form.

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|}$$

$$= \left[1 + \left(\frac{y'(t)}{x'(t)}\right)^2\right]^{3/2} \times \frac{(x'(t))^3}{x'(t)\cdot y''(t) - y'(t)\cdot}$$

$$= \frac{\left[(x'(t))^2 + (y'(t))^2\right]^{3/2}}{\cancel{(x'(t))^3}} \cdot \frac{\cancel{(x'(t))^3}}{x'(t)\cdot y''(t) - y'(t)\cdot}$$

ture in parametric

$y'(t)\cdot x''(t)$

$]^2$

$)\cdot x''(t)$

$$\frac{(x'(t))^3}{x'(t)\cdot y''(t) - y'(t)\cdot x''(t)}$$

$]^{3/2}$

$$\frac{(x'(\cancel{t}))^3}{x'(t)\cdot y''(t) - y'(t)\cdot x''(t)}$$

$$\boxed{\rho = \frac{\left[[x'(t)]^2 + [y'(t)]^2\right]^{3/2}}{|x'(t)\cdot y''(t) - y'(t)\cdot x''(t)|}}$$

$x^2+y^2=a^2$

* Find the radius of curvature of a circle.

$x^2+y^2=a^2$.

$2x+2yy_1=0$

$$y_1 = -\frac{x}{y}$$

$$y_2 = -\frac{[y(1) - xy_1]}{y^2} = \frac{xy_1 - y}{y^2}$$

$$y_2 = \frac{-\frac{x^2}{y} - y}{y^2} = \frac{-x^2-y^2}{y^3} = -\frac{(x^2+y^2)}{y^3} = \frac{-a^2}{y^3}$$

$$\rho = \frac{\left(1+\frac{x^2}{y^2}\right)^{3/2}}{a^2/y^3} = \frac{a^3}{a^2} = \underline{\underline{a}}$$

$x = a\cos\theta$; $y = a\sin\theta$

$x'(\theta) = -a\sin\theta$; $y'(\theta) = a\cos\theta$

$x''(\theta) = -a\cos\theta$

$y''(\theta) = -a\sin\theta$

$y_1 = -\cot\theta$

$$\rho = \frac{\left[a^2\sin^2\theta + a^2\cos^2\theta\right]^{3/2}}{|a^2\sin^2\theta + a^2\cos^2\theta|} = \frac{a^3}{a^2} = \underline{\underline{a}}$$

$$\boxed{\rho = a}$$

20/12/2021

If $y_1 = \infty$; $y_2 = 0$ [Special case].
then differentiate the given equation w.r.to. 'x' ; find x_1 & x_2

$x_1 = \frac{dx}{dy}$

$$\Rightarrow \rho = \frac{(1+x_1^2)^{3/2}}{|x_2|}.$$

$x_2 = \frac{d^2x}{dy^2}$

*) Find the radius of curvature of the following:

a) $x^2+y^2 = 16$ at any point on this curve.

$x = 4\cos\theta$; $y = 4\sin\theta$

$x^2+y^2 = 16$. — (1)   Let point $P(x, y)$.

diff. w.r.to 'x' on both sides.

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{xy_1 - y}{y^2}.$$

$$= \frac{-(x^2+y^2)}{y^3}$$

$$\frac{d^2y}{dx^2} = -\frac{16}{y^3}$$

wrto.

wing:

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|}$$

$$= \frac{\left(1+\frac{x^2}{y^2}\right)^{3/2}}{16/y^3} = \cancel{(y^2+x^2)^{3/2}}$$

$$= \frac{(y^2+x^2)^{3/2}}{16} = \frac{4^3}{16} = \underline{\underline{4}}$$

b) ρ for $y^2 = \frac{a^2(a-x)}{x}$ at $(a,0)$

$xy^2 = a^3 - a^2x$ — ①

Let $P(a,0)$

diff ① wrto 'x' on both sides

$x2yy_1 + y^2 = 0 - a^2$

$$y_1 = \frac{-a^2 - y^2}{2xy}.$$

$$y_1 = \frac{-a^2 - 0}{2(0)} = \infty$$

diff① wrto 'y' on both sides

$x2y + y^2x_1 = 0 - a^2x_1$

$$x_1 = \frac{-2xy}{a^2+y^2} = -2(0) = \underline{\underline{0}}$$

$$x_1 = \frac{-2xy}{a^2+y^2}$$

$$\cancel{x_2 = -[(a^2+y^2)(2)][x+yx_1]}$$

$$x_1(a^2+y^2) = -2xy \quad \text{--- (2)}$$

diff (2) w.r.to 'x'.

$$x_1(2y) + x_2(a^2+y^2) = -2[x + yx_1]$$

$$2yx_1 + a^2x_2 + y^2x_2 = -2(x+0).$$

$$0 + a^2x_2 + y^2x_2 = -2x$$

$$x_2 = \frac{-2x}{a^2+y^2}$$

$$x_2 = \frac{-2a}{a^2} = \frac{-2}{a}$$

Radius of curvature

$$\rho = \frac{(1+x_1^2)^{3/2}}{|x_2|}$$

$$= \frac{(1+0)^{3/2}a}{2} = \frac{a}{2}.$$

$$\boxed{\rho = \frac{a}{2}}$$

c) $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}y_1 = 0.$$

$$y_1 = -\frac{\sqrt{y}}{\sqrt{x}} = -1.$$

$$y_1 \sqrt{x} = -\sqrt{y}.$$

$$y_1 \frac{1}{2\sqrt{x}} + \sqrt{x}\, y_2 = -\frac{1}{2\sqrt{y}} \cdot y_1.$$

$$\frac{-1}{2\sqrt{x}} + \sqrt{x}\, y_2 = \frac{1}{2\sqrt{y}}$$

$$\sqrt{x}\, y_2 = \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}.$$

$$\cancel{y_2 = \frac{1}{2x} + \frac{1}{2x\sqrt{y}}.}$$

$$\cancel{= \frac{2}{a} + \frac{2 \times 2}{a\sqrt{a}}}$$

$$\cancel{y_2 = \frac{2}{a}\left[1 + \frac{2}{\sqrt{a}}\right]}$$

$$\cancel{\rho = \sqrt{1+}}$$

$$y_2 = \frac{1}{2\sqrt{x}}\left[\frac{\sqrt{a}}{\sqrt{xy}}\right]$$

$$= \frac{2}{2\sqrt{a}}\left[\frac{\sqrt{a} \cdot 42}{a}\right]$$

$$y_2 = \frac{4}{a}$$

$$\rho = \frac{(1+1)^{3/2}}{4} \cdot a.$$

$$= \frac{2\sqrt{2}}{4} a = \frac{\sqrt{2}\, a}{2} = \boxed{\frac{a}{\sqrt{2}} = \rho}$$

$$\boxed{\rho = \frac{(x^2+y^2)^{3/2}}{2c^2}}$$

21/12/2021.

* Find the radius of curvature of $y = c\cosh x/c$

$$y = c\cosh x/c$$

$$y_1 = \frac{c\sinh x/c}{c}$$

$$y_2 = \frac{\cosh x/c}{c}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|}$$

$$\rho = \frac{(1+\sinh^2 x/c)^{3/2} \cdot c}{\cosh x/c}$$

$$= \frac{(\cosh^2 x/c)^{3/2} \cdot c}{\cosh x/c}$$

$$\boxed{\rho = c\cosh^2 x/c}$$

$$\rho = c\frac{y^2}{c^2} = \frac{y^2}{c}$$

* ~~Find~~ Show the radius of curvature of the curve $s^2 = 8ay$ is $\rho = 4a\sqrt{1-\frac{y}{2a}}$

$$s^2 = 8ay$$

$$2s\frac{ds}{dy} = 8a$$

$$\frac{ds}{dy} = \frac{4a}{s} =$$

$$\cancel{\frac{d^2s}{dy^2} = \frac{4a(-1)}{s^2} = \frac{-4a}{s^2}}$$

$$\cancel{\rho = \frac{(1+y_1^2)^{3/2}}{y_2}}$$

$$\cancel{= \frac{\left(1+\frac{16a^2}{s^2}\right)^{3/2}}{\frac{-4a}{s^2}}}$$

$$\cancel{= \frac{-(s^2+16a^2)s^2}{s^3(-4a)}}$$

$$\cancel{= \frac{s^2+16a^2}{-s\,4a}}$$

$$s^2 = 8ay.$$

$$2s\frac{ds}{dx} = 8a\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{s}{4a}\cdot\frac{ds}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{4a}\cdot\frac{d^2s}{dx^2}.$$

$$\frac{ds}{dy} = \frac{4a}{s}.$$

$$\frac{d^2s}{dy^2} = -\frac{4a}{s^2}$$

$$\frac{dy}{ds} = \frac{s}{4a}.$$

$$\frac{dy}{ds}\cdot\frac{ds}{dx} = \tan\psi$$

$$\frac{s}{4a}\cdot\frac{ds}{dx} = \tan\psi$$

$$\frac{s}{4a}\cdot\sec\psi = \tan\psi$$

$$\frac{dy}{ds} = \sin\psi$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|}$$

$$\frac{s}{4a} = \sin\psi.$$

$$s = 4a\sin\psi.$$

$$\frac{ds}{d\psi} = 4a\cos\psi$$

$$\rho = 4a\sqrt{1-\frac{s^2}{(4a)^2}}$$

$$= 4a\sqrt{1-\frac{8ay}{16a^2}} = 4a\sqrt{1-\frac{y}{2a}}$$

* Radius of curvature of $x = a(\cos t + t\sin t)$

$y = a(\sin t - t\cos t).$

$$\frac{dx}{dt} = a(-\sin t + t\cos t + \sin t)$$

$$= \underline{at\cos t}.$$

$$\frac{dy}{dt} = a(\cos t - \cos t + t\sin t)$$
$$= at\sin t$$

$x'(t) = at\cos t$. $\quad y'(t) = at\sin t$.

$x''(t) = a[-t\sin t + \cos t] \quad y''(t) = a[t\cos t + \sin t]$

$$\rho = \frac{\left[(x'(t))^2 + (y'(t))^2\right]^{3/2}}{|x'(t)\,y''(t) - x''(t)\,y'(t)|}$$

$$= \frac{\left[a^2t^2(1)\right]^{3/2}}{|a^2[t^2\cos^2 t + t\sin t\,\cancel{\cos t} + t^2\sin^2 t - t\,\cancel{\sin t}\cos t}$$

$$= \frac{a^3t^3}{a^2t^2} = \underline{\underline{at}}$$

<u>22/12/2021</u>

1) $x = e^t + e^{-t}$ $\quad$ at $t = 0$. Find ρ.

$y = e^t - e^{-t}$

$x'(t) = e^t - e^{-t}$. $\quad x''(t) = e^t + e^{-t}$

$y'(t) = e^t + e^{-t}$. $\quad y''(t) = e^t - e^{-t}$

$\cancel{\rho = [(x'(t)]^2 +}$

$$\frac{dy}{dx} = \frac{e^t + e^{-t}}{e^t - e^{-t}} = \frac{1+1}{1-1} = \infty$$

$$x_1 = \frac{dx}{dy} = \underline{\underline{0}}.$$

$$x_1 = \frac{e^t - e^{-t}}{e^t + e^{-t}}$$

$$\cancel{x_2 = e^t + e^{-t}}$$

$$x_2 = \frac{(e^t + e^{-t})^2 - (e^t - e^{-t})^2}{(e^t + e^{-t})^3}$$

$$x_2 = \frac{dx_1/dt}{dy/dt}$$

$$= \frac{(e^t + \cancel{e^{-t}} + e^t - \cancel{e^{-t}})(\cancel{e^t} + e^{-t} - \cancel{e^t} + e^{-t})}{(e^t + e^{-t})^3}$$

$$x_2 = \frac{(2)(2)}{2^3} = \frac{4}{8} = \frac{1}{2}$$

$$\rho = \frac{(1+x_1^2)^{3/2}}{|x_2|} = \frac{(1+0)^{3/2}}{1/2} = \underline{\underline{2}}$$

Parametric form.

$$\rho = \frac{\left[[x'(t)]^2 + [y'(t)]^2\right]^{3/2}}{|x'(t) \cdot y''(t) - y'(t)\, x''(t)|}$$

$$* \quad = \frac{\left[2\,[e^{2t} + e^{-2t}]\right]^{3/2}}{|(\cancel{e^{2t}} + \cancel{e^{-2t}} - 2) - (\cancel{e^{2t}} + \cancel{e^{-2t}} + 2)|}$$

$$= \frac{8}{4} = \underline{\underline{2}}$$

$x_1 = e^t$

$\frac{dx}{dt} = e$

$\frac{dy}{dx} =$

$\frac{d^2y}{dx^2} =$

$=$

* Find ρ of

$x(\theta) =$

$x'(\theta)$

$x''(\theta)$

$y(\theta) = a$

$y'(\theta)$

$y''(\theta)$

$$x_1 = e^t - e^{-t}$$

$$\frac{dx}{dt} = e^t - e^{-t} \qquad \frac{dy}{dt} = e^t + e^{-t}$$

$$\frac{dy}{dx} = \frac{e^t + e^{-t}}{e^t - e^{-t}} = \frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y - xy'}{y^2}$$

$$= \frac{y - \frac{x^2}{y}}{y^2} = \frac{y^2 - x^2}{y^3}$$

* Find ρ of curve $x = a\log(\sec\theta + \tan\theta)$.
$y = a\sec\theta$

$$x(\theta) = a\log(\sec\theta + \tan\theta)$$

$$x'(\theta) = \frac{a(\sec\theta\tan\theta + \sec^2\theta)}{(\sec\theta + \tan\theta)}$$

$$= a\sec\theta$$

$$\rho = (x'(t))^2 +$$

$$x''(\theta) = a\sec\theta\tan\theta$$

$$y(\theta) = a\sec\theta$$

$$y'(\theta) = a\sec\theta\tan\theta$$

$$y''(\theta) = a[\sec^3\theta + \sec\theta\tan^2\theta]$$

$$\rho = \frac{\left[a^2\sec^2\theta + a^2\sec^2\theta\tan^2\theta\right]^{3/2}}{a^2\left[\sec^4\theta + \sec^2\theta\tan^2\theta\right] - a^2\sec^2\theta\tan^2\theta}$$

$$= \frac{\left[a^2\sec^4\theta\right]^{3/2}}{a^2\sec^4\theta} = \frac{a^3\sec^6\theta}{a^2\sec^4\theta}$$

$$\boxed{\rho = a\sec^2\theta}$$

* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a,0)$ and $(0,b)$

(i) $(a,0)$

$$\frac{\cancel{2}x}{a^2} + \frac{\cancel{2}y}{b^2}y_1 = 0$$

$$y_1 = -\frac{b^2x}{a^2y}$$

$$y_2 = \frac{-b^2}{a^2}\left[\frac{y - xy_1}{y^2}\right]$$

$$= \frac{-b^2}{a^2}\left[\frac{y + \frac{b^2x^2}{a^2y}}{y^2}\right]$$

$$= \frac{-b^2}{a^2}\left[\frac{a^2y^2 + b^2x^2}{a^2y^3}\right]$$

*

$\rho =$

$=$

$=($

$\rho\,(a,0)$

2) $x^3 + y$

3) $y =$

4) $x =$

5) $x = a$

$\rho(0,b) =$

$$= \frac{-b^2}{a^2} \cdot \frac{\cancel{a^2} b^2}{\cancel{a^2} y^3} = \frac{-b^4}{a^2 y^3}.$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|}$$

$$= \frac{\left(1 + \frac{b^4 x^2}{a^4 y^2}\right)^{3/2} a^2 y^3}{b^4}$$

$$= \frac{(a^4 y^2 + b^4 x^2)^{3/2} \cancel{a^2 y^3}}{a^6 y^3 \cdot b^4}$$

$$\rho_{(a,0)} = \frac{(a^4(0) + b^4 a^2)^{3/2}}{a^4 b^4} = \frac{\cancel{a^6} b^6 a^3}{a^4 b^4} = \frac{b^2}{a}$$

2) $x^3 + y^3 = 2a^3$ at (a,a)

3) $y = \frac{\log x}{x}$ at $x = 1$.

4) $x = \log t \; ; \; y = \frac{1}{2}\left(t + \frac{1}{t}\right)$

5) $x = \frac{a \cos\theta}{\theta}$, $y = \frac{a \sin\theta}{\theta}$.

$$\rho(0,b) = \frac{(a^4 b^2)^{3/2}}{a^4 b^4} = \frac{a^6 \cdot b^3}{a^4 b^4} = \frac{a^2}{b}$$

$y_1 = \infty$

$\frac{2x}{a^2} \cdot x_1 + \frac{2y}{b^2} = 0$

$x_1 = -\frac{a^2 y}{b^2 x}$

2) $x^3 + y^3 = 2a^3$ at (a,a)

$3x^2 + 3y^2 y_1 = 0$

$y_1 = -\frac{x^2}{y^2} = -1$

$y_2 = -\frac{[y^2 \cdot 2x - x^2 \cdot 2y y_1]}{y^4}$

$= \cancel{\frac{2(x^2 y - x y^2)}{y^4}}$

$= \cancel{2(}$

$= -\frac{[2xy^2 + \frac{2x^4}{y}]}{y^4}$

$= -\frac{[2a^3 + 2a^3]}{a^4}$

$= -\frac{4}{a}$

$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|}$

* $= \frac{(1+1)^{3/2} a}{4} = \frac{2\sqrt{2}\, a}{4}$

$= \frac{a}{\sqrt{2}}$

3) $y = \frac{\log x}{x}$

$y_1 = \frac{1 - \ldots}{x^2}$

$y_2 = x \ldots$

$= - \ldots$

$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|} = \ldots$

$\rho = (1 + \ldots$

4) $x = \log t$

$x_1 = \frac{1}{t}$

$x_2 = -\frac{1}{t \ldots}$

$\rho = \frac{[x_1^2 + \ldots}{|x_1 y_2 \ldots}$

3) $y = \frac{\log x}{x}$ $\qquad x = 1$

$$y_1 = \frac{1 - \log x}{x^2} = 1$$

$$y_2 = \frac{x^2\left(0 - \frac{1}{x}\right) - (1 - \log x)\,2x}{x^2}$$

$$= \frac{-x - 2x + 2x\log x}{x^2}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|} \qquad = \frac{2(0) - 3}{} = \underline{-3}$$

$$\rho = \frac{(1+1)^{3/2}}{3} = \frac{2\sqrt{2}}{3}$$

$\left[\frac{c^4}{y}\right]$

3

4) $x = \log t \qquad y = \frac{1}{2}\left(t + \frac{1}{t}\right)$

$$x_1 = \frac{1}{t} \qquad y_1 = \frac{1}{2}\left(1 - \frac{1}{t^2}\right)$$

$$x_2 = \frac{-1}{t^2} \qquad y_2 = \frac{1}{\cancel{2}}\left(0 + \frac{\cancel{2}}{t^3}\right) \quad \frac{2}{t^3}$$

$$\rho = \frac{\left[x_1^2 + y_1^2\right]^{3/2}}{|x_1 y_2 - y_1 x_2|} = \frac{\left[\frac{1}{t^2} + \frac{1}{4}\left(1 + \frac{1}{t^4} - \frac{2}{t^2}\right)\right]^{3/2}}{\frac{1}{t^4} + \frac{1}{2t^2}\left(1 - \frac{1}{t^2}\right)}$$

$$= \frac{\left(\frac{1}{t^2} + \frac{1}{4} + \frac{1}{4t^4} - \frac{1}{2t^2}\right)^{3/2}}{\frac{1}{t^4}\left(\frac{1}{2}\right) + \frac{1}{2t^2}}$$

$$\frac{\left(\frac{1}{4}+\frac{1}{4t^4}+\frac{1}{2t^2}\right)^{3/2}}{\left|\frac{1}{2}\left(\frac{1}{t^4}+\frac{1}{t^2}\right)\right|}$$

$2 \cdot \frac{1}{2} \cdot \frac{1}{2t^2}$

$$= \frac{\left(\frac{1}{2}+\frac{1}{2t^2}\right)^{3/2}}{\frac{1}{2}\cdot\frac{1}{t^2}\left(\frac{1}{1}+\frac{1}{t^2}\right)}$$

$$= \frac{\left(\frac{1}{2}\right)^3\left(1+\frac{1}{t^2}\right)^{\cancel{3}\,2}}{\frac{1}{2t^2}\cancel{\left(1+\frac{1}{t^2}\right)}}$$

$$= \frac{t^2}{4}\left(1+\frac{1}{t^2}\right)^2 = \frac{t^2(t^2+1)^2}{4.t^{\cancel{4}}} = \frac{(t^2+1)^2}{4t^2}$$

5) $x = \frac{a\cos\theta}{\theta}$ $\quad y = \frac{a\sin\theta}{\theta}$

$\cancel{x_1 = a\left[\theta \sin\theta\right.}$

$$x_1 = a\left[\frac{-\theta\sin\theta-\cos\theta}{\theta^2}\right]$$

$$x_1 = -a\left[\frac{\theta\sin\theta+\cos\theta}{\theta^2}\right]$$

$$x_2 = -a\left[\frac{\theta^2(\theta\cos\theta+\sin\theta)+2\theta(\theta\sin\theta+\cos\theta)}{\theta^4}\right]$$

$y_1 = a\left[\frac{\theta\cos\right.$

$\frac{dy}{dx} = y' = \frac{\sin}{\theta}$

$y_2 = \frac{\frac{dy'}{d\theta}}{\frac{dx}{d\theta}} = (\theta s$

$\frac{\cancel{\theta}[\theta^2\sin^2\theta}{-a}$

$= \frac{\theta^2[\theta^2](}{a(\theta\sin\theta+\cos}$

$= \frac{-\theta^4}{\theta\sin\theta+co}$

$\rho = \frac{(1+y_1^2)^{3/}}{|y_2|}$

$$y_1 = a\left[\frac{\theta\cos\theta - \sin\theta}{\theta^2}\right]$$

$$\frac{dy}{dx} = y' = \frac{\sin\theta - \theta\cos\theta}{\theta\sin\theta + \cos\theta}.$$

$$y_2 = \frac{\frac{dy'}{d\theta}}{\frac{dx}{d\theta}} = \frac{\dfrac{(\theta\sin\theta+\cos\theta)(\cancel{\cos\theta}+\theta\sin\theta-\cancel{\cos\theta}) - \theta(\sin\theta-\theta\cos\theta)(\theta\cos\theta+\cancel{\sin\theta}-\cancel{\sin\theta})}{(\theta\sin\theta+\cos\theta)^2}}{-a\dfrac{(\theta\sin\theta+\cos\theta)}{\theta^2}.}$$

$$\frac{\theta^2\left[\theta^2\sin^2\theta + \theta\cos\theta\cancel{\sin\theta} - \theta\cos\theta\cancel{\sin\theta} + \theta^2\cos^2\theta\right]}{-a\ (\theta\sin\theta + \cos\theta)^3}$$

$$= \frac{\theta^2\left[\theta^2\right](-1)}{a(\theta\sin\theta+\cos\theta)^3}$$

$$= \frac{-\theta^4}{\theta\sin\theta+\cos\theta}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|} = \frac{\left(\dfrac{\theta^2\sin^2\theta+\cos^2\theta+\cancel{\sin 2\theta\cdot\theta} + \sin^2\theta+\theta^2\cos^2\theta - \cancel{\sin 2\theta\cdot\theta}}{\cancel{(\theta\sin\theta+\cos\theta)^2}}\right)^{3/2}}{\dfrac{\theta^4}{a(\theta\sin\theta+\cos\theta)^3}}$$

$$\frac{125}{\cancel{16}} = \frac{(y^2+16)^{3/2}}{\cancel{16}}$$

$$5^{\cancel{3}\times\frac{2}{\cancel{3}}} = y^2+16.$$

$$\boxed{\begin{aligned} y &= \pm 3 \\ x &= \frac{9}{8} \end{aligned}}$$

* Find the ρ of the curve $y^2 = 4ax$. ~~The~~ Prove that square of ρ at any point of the curve varies as the ~~se~~ cube of the focal distance of the point.

$P(at^2, 2at)$

$$2yy_1 = 4a$$

$$yy_1 = 2a.$$

Consider the parametric equations of the parabola.

$x = at^2$; $y = 2at$

$\frac{dx}{dt} = 2at$ $\qquad$ $\frac{dy}{dt} = 2a$

$x_2 = 2a$ $\qquad$ $y_2 = 0$

$$\rho = \frac{(4a^2t^2+4a^2)^{3/2}}{|0-4a^2|} = \frac{(2a)^3(t^2+1)^{3/2}}{(2a)^2}$$

$$\boxed{\rho = 2a(1+t^2)^{3/2}}$$

Let $P(at^2, 2at)$ be a point on the parabola

Let $S(a, 0)$ be the focus of parabola.

$$SP = \sqrt{a^2(t^2-1)^2 + 4a^2t^2}$$

$$\rho^2 = 4a^2(1+t^2)^3$$

$$(SP)^3 = (a^2)^{3/2}\left(t^4+1-2t^2+4t^2\right)^{3/2}$$

$$(SP)^3 = a^3(t^2+1)^3.$$

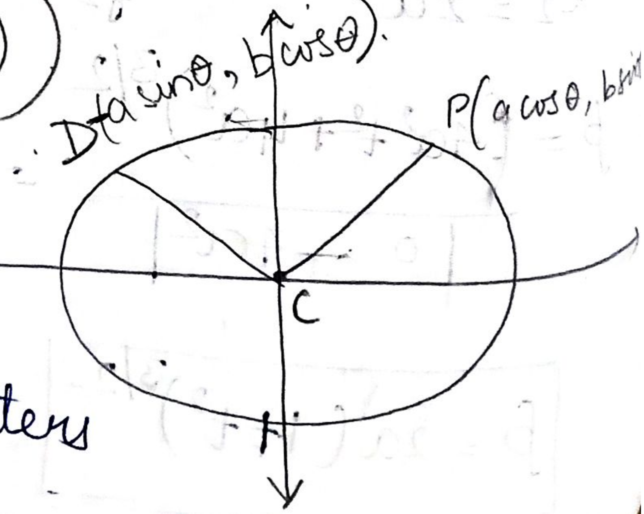
$$\frac{\rho^2}{(SP)^3} = \frac{4}{a}.$$

$$\rho^2 \propto (SP)^3$$

Hence square of radius of curvature at any point varies with cube of the focal distance of the point.

* CP & CD be a pair of conjugate semi diameters of an ellipse. Prove that ρ at P is $\frac{(CD)^3}{ab}$, a & b being the lengths of semi-axis of ellipse.

$D\left(a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right)$



We know that the co-ordinates of end points of semi diameters are $D(-a\sin\theta, b\cos\theta)$ & $P(a\cos\theta, b\sin\theta)$.

Let $C(0,0)$ be the centre of an ellipse.

$$CD = \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$$

$$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$(CD)^3 = (a^2\sin^2\theta + b^2\cos^2\theta)^{3/2} \quad \text{--- (1)}$$

$x = a\cos\theta$	$y = b\sin\theta$
$x_1 = -a\sin\theta$	$y_1 = b\cos\theta$
$x_2 = -a\cos\theta$	$y_2 = -b\sin\theta$

$$\rho = \frac{(x_1^2 + y_1^2)^{3/2}}{|x_1y_2 - x_2y_1|}.$$

$$\rho = \frac{\left[a^2\sin^2\theta + b^2\cos^2\theta\right]^{3/2}}{|ab\sin^2\theta + ab\cos^2\theta|}$$

$$= \frac{(a^2\sin^2\theta + b^2\cos^2\theta)^{3/2}}{ab} \quad \text{--- (2)}$$

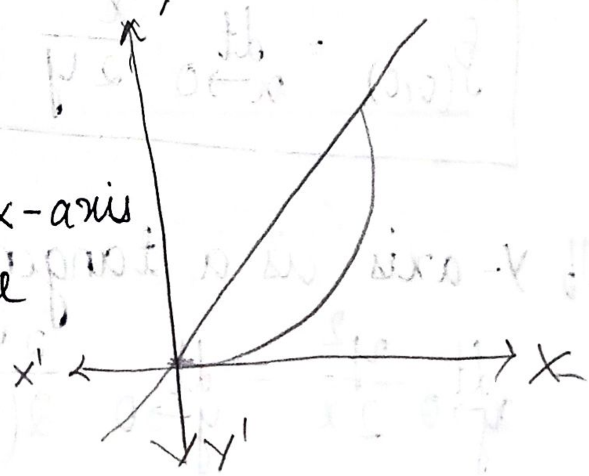
from (1) & (2)

$$\Rightarrow \boxed{\rho = \frac{(CD)^3}{ab}}$$

28/12/2021.

*NEWTONIAN METHOD:

Suppose a curve is passing through the origin and x-axis (or) Y-axis is tangent to the curve at the origin



(i) $\rho_{at(0,0)} = \underset{\substack{x\to 0\\ y\to 0}}{\mathrm{lt}} \dfrac{x^2}{2y} = \underset{x\to 0}{\mathrm{lt}} \dfrac{x^2}{2y}$.

(ii) If y-axis is tangent at (0,0)

$$\rho_{at(0,0)} = \underset{\substack{x\to 0\\ y\to 0}}{\mathrm{lt}} \frac{y^2}{2x} = \underset{y\to 0}{\mathrm{lt}} \frac{y^2}{2x}.$$

<u>Proof:</u>

If x-axis is a tangent at (0,0) $\dfrac{dy}{dx} = 0$

$$\underset{x\to 0}{\mathrm{lt}} \frac{x^2}{2y} = \underset{x\to 0}{\mathrm{lt}} \frac{2x}{2\left(\frac{dy}{dx}\right)}$$

$$= \underset{x\to 0}{\mathrm{lt}} \left(\frac{1}{\frac{d^2y}{dx^2}}\right) \quad \text{—} (1)$$

$$\rho_{(0,0)} = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}_{(0,0)}}{\left(\frac{d^2y}{dx^2}\right)_{(0,0)}} = \frac{1}{\left(\frac{d^2y}{dx^2}\right)_{(0,0)}} \quad \text{—} (2) \qquad \frac{dy}{dx}=0$$

from (1) & (2)

$$\boxed{\rho_{(0,0)} = \underset{x\to 0}{\mathrm{lt}} \frac{x^2}{2y}}$$

If y-axis is a tangent at (0,0) $\dfrac{dx}{dy} = 0$.

$$\underset{y\to 0}{\mathrm{lt}} \frac{y^2}{2x} = \underset{y\to 0}{\mathrm{lt}} \frac{2y}{2\left(\frac{dx}{dy}\right)}$$

$$= \lim_{y \to 0} \left(\frac{1}{\frac{d^2x}{dy^2}} \right) \quad \text{— (1)}$$

$$\rho_{(0,0)} = \frac{\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{3/2}_{(0,0)}}{\left(\frac{d^2x}{dy^2}\right)_{(0,0)}} = \frac{1}{\left(\frac{d^2x}{dy^2}\right)_{(0,0)}} \quad \text{— (2)} \qquad \frac{dx}{dy} = 0$$

from (1) & (2)

$$\boxed{\rho_{(0,0)} = \lim_{y \to 0} \frac{y^2}{2x}}$$

* Find the $\rho_{(0,0)}$ for the curve $2x^4 + 3y^4 + 4x^2y + xy - y^2 + 2x = 0$

Curve: $2x^4 + 3y^4 + 4x^2y + xy - y^2 + 2x = 0$

Equating the lowest degree term $(x) = 0$:

$\Rightarrow$ y-axis is a tangent at $(0,0)$.

$$\lim_{\substack{x \to 0 \\ y \to 0}} \frac{y^2}{2x}.$$

$$\Rightarrow \lim_{\substack{x \to 0 \\ y \to 0}} \frac{2x^4}{2x} + \frac{3y^4}{2x} + \frac{4x^2y}{2x} + \frac{xy}{2x} - \frac{y^2}{2x} + \frac{2x}{2x} = 0$$

$$\Rightarrow x^3 + \frac{3y^4}{2x} + 2xy + \frac{y}{2} - \frac{y^2}{2x} + 1 = 0$$

$$\lim_{\substack{x \to 0 \\ y \to 0}} \left(x^3 + \frac{3y^4}{2x} + 2xy + \frac{y}{2} - \frac{y^2}{2x} + 1 \right) = 0$$

$$0 + 0 + 0 + 0 - \frac{y^2}{2x} + 1 \Rightarrow \rho + 1 = 0$$

$$\boxed{\rho = 1}$$

$2x^4+3y^4+4x^2y+xy-y^2+2x=0$

$8x^3+12y^3y_1+4x^2y_1+8xy+xy_1+y-2yy_1$

$8x^3+8xy+y+2$

$+y_1(12y^3+4x^2+\cancel{8xy}-2y)=0$

$$y_1=-\frac{(8x^3+8xy+y+2)}{(12y^3+4x^2+x-2y)}$$

$y_1=\infty$

* Find $\rho_{(0,0)}$ for the curve $x^3+y^3-2x^2+6xy=0$

$y=0 \Rightarrow x$-axis is tangent

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^3}{2y}+\frac{y^3}{2y}-\frac{2x^2}{2y}+\frac{6x\cancel{y}}{2\cancel{y}}=0$$

$$0+0-2\left(\frac{x^2}{2y}\right)+3=0$$

$2\rho=3$

$\rho=\frac{3}{2}$

* Find $\rho_{(0,0)}$ for $x^4-y^4+x^3-y^3+x^2-y^2+y=0$

* Find $\rho_{(0,0)}$ for the curve $y-x=x^2+2xy+y^2$

$\boxed{y=xy_1+\ldots}$

1) $x^4-y^4+x\ldots$

$y=0$

$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^4}{2y}-\frac{y^4}{2y}\ldots$

$0-0+\ldots$

2) $y-x=x^2+2\ldots$

$xy_1+\frac{x^2}{2!}y_2\ldots$

$\boxed{y=xy_1\ldots}$

$xy_1+\frac{x^2y_2}{2}-x\ldots$

Coeff of y_1

(y_1-1)

$y_1=1$

$\rho=\frac{(1+1)^{3/2}}{2\ldots}$

$=2^{3/2}\ldots$

$$\boxed{y = xy_1 + \frac{x^2}{2!}y_2}$$

$-2yy_1+2$

$f)=0$

$y=0$

$=0$

1) $x^4 - y^4 + x^3 - y^3 + x^2 - y^2 + y = 0$

$y = 0 \Rightarrow$ X-axis is tangent.

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^4}{2y} - \frac{y^4}{2y} + \frac{x^3}{2y} - \frac{y^3}{2y} + \frac{x^2}{2y} - \frac{y^2}{2y} + \frac{y}{2y} = 0$$

$$0 - 0 + 0 - 0 + \rho - 0 + \frac{1}{2} = 0$$

$$\rho = \frac{1}{2}$$

2) $y - x = x^2 + 2xy + y^2 \Rightarrow$ Neither X-axis nor Y-axis are tangent.

$$xy_1 + \frac{x^2}{2!}y_2 - x = x^2 + 2x\left(xy_1 + \frac{x^2}{2}y_2\right) + \left(xy_1 + \frac{x^2 y_2}{2}\right)^2$$

$$\boxed{y = xy_1 + \frac{x^2}{2}y_2}$$

$$xy_1 + \frac{x^2 y_2}{2} - x = x^2 + 2x^2 y_1 + x^3 y_2 + x^2(y_1)^2 + \frac{x^4}{4}(y_2)^2 + x^3 y_1 y_2$$

Coeff of y_1 compare x & x^2 terms

$(y_1 - 1) = 0$

$y_1 = 1$

$$\frac{y_2}{2} = 1 + 2y_1 + y_1^2$$

$$y_2 = (1+2+1)2$$

$$y_2 = 8$$

$$\rho = \frac{(1+1)^{3/2}}{2^3} = 2^{3/2-3} = 2^{-3/2} = \frac{1}{2\sqrt{2}}$$

29/12/2021

* Find the radius of curvature at the origin

$$y^2 = \frac{x^2(a+x)}{(a-x)}$$

∵ Neither ~~x~~ nor y-axes are tangent.

$$y^2(a-x) = x^2(a+x)$$

$$\boxed{y = xy_1 + \frac{x^2}{2}y_2}$$ ⇒ By Maclaurin's Series

$$\left(x^2(y_1)^2 + \frac{x^4}{4}(y_2)^2 + x^3 y_1 y_2\right)(a-x) = x^2(a+x)$$

Compare x^2, x, x^3

$$y_1^2 a = a$$

$$y_1 = 1$$

$$y_1 y_2 a - y_1^2 = 1$$

$$ay_2 - 1 = 1$$

$$y_2 = \frac{2}{a}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|}$$

$$= \frac{2\sqrt{2}\,a}{2} = a\sqrt{2}$$

* CENTRE OF

The centre o
is a point
normal at

- Consider t
- C is the
(X, Y)

* Draw a ⊥
to x-axis
Join PN.
$CP = \rho$

⇒ From the

$X = OM = O$
$= x$
$= x$
$= x$

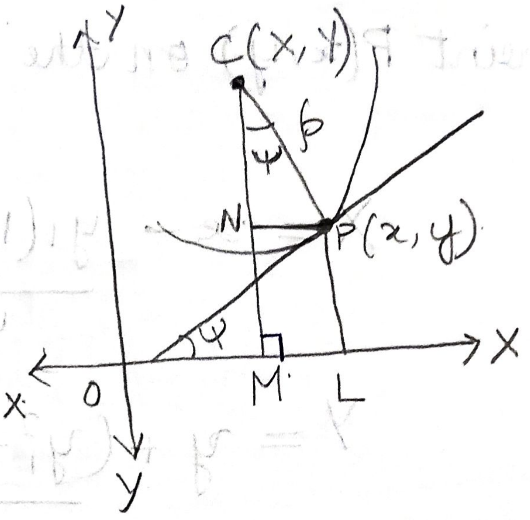
$X = x$

$Y = CM =$
$= y$
$= y$

$Y = y +$

*CENTRE OF CURVATURE:

The centre of curvature at any $P(x,y)$ of a curve is a point which lies on the +ve direction of normal at P and is at a distance ρ from it.



- Consider the curve $f(x)$
- C is the centre (X,Y)

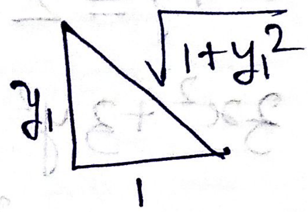
* Draw a $\perp$ar from C to X-axis

Join PN.

$CP = \rho$

$\Rightarrow$ From the figure:

$\tan\psi = y_1$



$$X = O\cancel{M} = OL - LM$$
$$= x - PN$$
$$= x - \rho\sin\psi$$
$$= x - \frac{(1+y_1^2)^{3/2}}{y_2} \cdot \frac{y_1}{\sqrt{1+y_1^2}}$$

$$\boxed{X = x - \frac{y_1(1+y_1^2)}{y_2}}$$

$$Y = \cancel{O}CM = MN + NC$$
$$= y + \rho\cos\psi$$
$$= y + \frac{(1+y_1^2)^{3/2}}{y_2} \cdot \frac{1}{\sqrt{1+y_1^2}}$$

$$\boxed{Y = y + \frac{(1+y_1^2)}{y_2}}$$

* Find the centre of curvature at the point (1, 1) on the curve $x^3+y^3=2$

If (X, Y) is the coordinates of centre of curvature at any point $P(x, y)$ on the curve $y=f(x)$ where

$$X = x - \frac{y_1(1+y_1^2)}{y_2}$$

$$Y = y + \frac{(y_1^2+1)}{y_2}$$

$$x^3+y^3=2$$

$$3x^2+3y^2y_1=0$$

$$y_1 = \frac{-x^2}{y^2} = -1 = y_1$$

$$y^2\cdot y_2 + 2y(y_1)^2 = -2x$$

$$y^2(y_2)+2=-2$$

$$\boxed{y_2=-4}$$

$$X = 1 - \frac{(-1)(1+1)}{-4} = 1-\frac{1}{2} = \frac{1}{2}$$

$$X = 1/2$$

$$Y = 1 + \frac{2}{-4} = 1-\frac{1}{2}$$

$$Y = \frac{1}{2}$$

* Find co-ordin... rectangular ...

* $a^2y=x^3$

* $xy=x^2+4$...

1) $xy=c^2$

$xy_1+y=0$

$y_1=-\frac{y}{...}$

$X = x + ...$

$= x + ...$

$Y = y + ...$

$= y + ...$

$c(\frac{3}{...}$

* Find co-ordinates of COC at any point on rectangular ~~parabola~~ hyperbola; $xy = c^2$

* $a^2 y = x^3$

* $xy = x^2 + 4$; $(2,4)$

1) $xy = c^2$

$xy_1 + y = 0$

$y_1 = -\frac{y}{x}$

$xy_2 + y_1 + y_1 = 0$

$y_2 = \frac{-2y_1}{x}$

$y_2 = +\frac{2y}{x^2}$

$$\boxed{\begin{aligned} X &= x - \frac{y_1(1+y_1^2)}{y_2} \\ Y &= y + \frac{y_1^2+1}{y_2} \end{aligned}}$$

$$X = x + \frac{\frac{y}{x}\left(1+\frac{y^2}{x^2}\right)}{\frac{2y}{x^2}}$$

$$= x + \frac{\frac{\cancel{y}}{x}\left(\frac{x^2+y^2}{\cancel{x^2}}\right)}{\frac{2\cancel{y}}{\cancel{x^2}}}$$

$$= x + \frac{x^2+y^2}{2x}$$

$$= \frac{2x^2+x^2+y^2}{2x}$$

$$= \frac{3x^2+y^2}{2x}$$

$$Y = y + \frac{\left(\frac{y^2}{x^2}+1\right)}{\frac{2y}{x^2}}$$

$$= y + \frac{x^2+y^2}{2y} = \frac{3y^2+x^2}{2y}$$

$$C\left(\frac{3x^2+y^2}{2x}, \frac{3y^2+x^2}{2y}\right)$$

2) $a^2 y = x^3$

$a^2 y_1 = 3x^2$

$$\boxed{y_1 = \frac{3x^2}{a^2}}$$

$a^2 y_2 = 6x$

$$\boxed{y_2 = \frac{6x}{a^2}}$$

$$X = x - \frac{y_1(1+y_1^2)}{y_2} \qquad Y = y + \frac{(y_1^2+1)}{y_2}$$

$$X = x - \frac{\frac{\cancel{3}x^{\cancel{2}}}{\cancel{a^2}}\left(1+\frac{9x^4}{a^4}\right)}{\frac{\cancel{6x}}{\cancel{a^2}}}$$

$$= x - \frac{x}{2}\left(\frac{a^4+9x^4}{a^4}\right)$$

$$= \frac{2a^4x - xa^4 - 9x^5}{2a^4}$$

$$X = \frac{a^4x - 9x^5}{2a^4}$$

$$Y = y + \frac{\frac{9x^4}{a^4}+1}{\frac{6x}{a^2}}$$

$$= y + \frac{9x^4+a^4}{6xa^2}$$

$$= \frac{6a^2xy + 9x^4 + a^4}{6xa^2} = \frac{6a^2x \cdot \frac{x^3}{a^2} + 9x^4 + a^4}{6xa^2}$$

$$= \frac{15x^4+a^4}{6xa^2}$$

$$C\left(\frac{a^4x - 9x^5}{2a^4}, \frac{15x^4+a^4}{6xa^2}\right)$$

3) $xy = x^2 + 4$ P

$xy_1 + y = 2x$

$$y_1 = +\frac{2x-y}{x}$$

$$y_1 = 0$$

$$X = x - \frac{y_1(1+y_1^2)}{y_2}$$

$$= 2 - \frac{0(1+\ldots)}{1\ldots}$$

$$X = 2$$

$C(2,5)$

04/01/2022

* Find the co-ordinat... a given curve $\sqrt{x}+\sqrt{\ldots}$ find circle of curvatu...

$$\sqrt{x} + \sqrt{y} = \sqrt{\ldots}$$

$$\frac{1}{\cancel{2}\sqrt{x}} + \frac{1}{\cancel{2}\sqrt{y}} \ldots$$

y

$$= \frac{15x^4 + a^4}{6xa^2}$$

$$C\left(\frac{ax^4 - 9x^5}{2a^4}, \frac{15x^4 + a^4}{6xa^2}\right)$$

3) $xy = x^2 + 4$ — $P(2, 4)$

$xy_1 + y = 2x$

$y_1 = +\frac{2x - y}{x}$

$y_1 = 0$

$xy_2 + y_1 + y_1 = 2$

$y_2 = \frac{2 - 2y_1}{x}$

$= \frac{2}{2} = 1$

$X = x - \frac{y_1(1 + y_1^2)}{y_2}$ $\quad Y = y + \frac{y_1^2 + 1}{y_2}$

$= 2 - \frac{0(1+0)}{1}$ $\quad Y = 4 + \frac{0+1}{1}$

$= 4 + 1$

$X = 2$ $\quad Y = 5$

$C(2, 5)$

04/01/2022

* Find the co-ordinates of centre of curvature of a given curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$. hence find circle of curvature

$\sqrt{x} + \sqrt{y} = \sqrt{a}$.

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} y_1 = 0$

$y_1 = -\frac{\sqrt{y}}{\sqrt{x}} = -1 = y_1$

$$y_1 = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\sqrt{x}\, y_1 = -\sqrt{y}$$

$$\sqrt{x}\, y_2 + \frac{1}{2\sqrt{x}} y_1 = -\frac{1}{2\sqrt{y}} y_1$$

$$\frac{\sqrt{a}}{2} y_2 = \frac{1\times \cancel{2}}{\cancel{2}\sqrt{a}} - \frac{(-1)\times \cancel{2}}{\cancel{2}\sqrt{a}}$$

$$\boxed{y_2 = \frac{4}{\sqrt{a}}}$$

Circle of curvature at a point $P(x, y)$ is an equation with centre $C(X, Y)$ at a distance ρ from the curve; therefore circle of curvature is $\boxed{(x-X)^2 + (y-Y)^2 = \rho^2}$

$$X = x - \frac{y_1(1+y_1^2)}{y_2}$$

$$= \frac{a}{4} + \frac{1(\cancel{2})\sqrt{a}}{\cancel{4}}\,a$$

$$X = \frac{a}{4} + \frac{a}{2} = \frac{3a}{4}$$

$$Y = y + \frac{(1+y_1^2)}{y_2}$$

$$= \frac{a}{4} + \frac{\cancel{2}\sqrt{a}}{\cancel{2}}\frac{a}{2}$$

$$Y = \frac{a}{4} + \frac{a}{2}$$

$$= \frac{3a}{4}$$

$$C\left(\frac{3a}{4}, \frac{3a}{4}\right)$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|}$$

$$= \frac{(2)^{3/2}\cancel{4}a}{\cancel{2}} = \cancel{\sqrt{a}}\,\frac{a}{\sqrt{2}}$$

$$\cancel{\left(x - \frac{a}{4} - \sqrt{a}\right)^2 + \left(y - \frac{a}{4} - \sqrt{a}\right)^2 = \frac{a^2}{4}}$$

$$\left(x - \frac{3a}{4}\right) \ldots$$

$$x^2 + y^2 - \ldots$$

$$x^2 + y^2 - 3\ldots$$

$$8x^2 + 8y^2 - \ldots$$

* Find the circl... of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \ldots$$

$$\frac{2x}{a^2} + \frac{2y}{b^2} \ldots$$

$$a^2 y \ldots$$

$$a^2[y y_2 \ldots$$

$$a^2[b y \ldots$$

$$\rho = \frac{1(a^2)}{b} = \ldots$$

$$x^2 + \left(y - b + \frac{a^2}{b}\right) \ldots$$

$$x^2 + y^2 + b^2 + \frac{a^4}{b^2} - \ldots$$

$$x^2 + y^2 - 2by + \ldots$$

$\frac{1}{2\sqrt{y}}\, y_1$

$\frac{(-1)\times \cancel{2}}{\cancel{2}\sqrt{a}}$

...nt $P(x,y)$ is an ...at a distance ρ ...cle of curvature

$\boxed{\ldots = \rho^2}$

$Y = y + \frac{(1+y_1^2)}{y_2}$

$= \frac{a}{4} + \frac{\cancel{2}\;\frac{a}{2}}{\cancel{2}}$

$Y = \frac{a}{4} + \frac{a}{2}$

$= \frac{3a}{4}$

$C\left(\frac{3a}{4}, \frac{3a}{4}\right)$

$\cancel{\left(\ldots - \sqrt{a}\right)^2 = \frac{a^2}{4}}$

$$\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \frac{a^2}{2}$$

$$x^2 + y^2 - \frac{3ax}{2} - \frac{3ay}{2} + \frac{\cancel{18}^{\,9} a^2}{\cancel{16}_{\,8}} = \frac{a^2}{2}$$

$$x^2 + y^2 - \frac{3ax}{2} - \frac{3ay}{2} + \frac{5a^2}{8} = 0$$

$$8x^2 + 8y^2 - 12ax - 12ay + 5a^2 = 0$$

* Find the circle of curvature at a point $(0, b)$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{2x}{a^2} + \frac{2y}{b^2} y_1 = 0$$

$$y_1 = -\frac{b^2 x}{a^2 y} = \underline{0} \; ; \; y_2 =$$

$$a^2 y y_1 = -b^2 x$$

$$a^2\left[y y_2 + y_1^2\right] = -b^2$$

$$a^2\left[b y_2 + 0\right] = -b^2 \qquad \boxed{y_2 = \frac{-b}{a^2}}$$

$$\rho = \frac{1(a^2)}{b} = \frac{a^2}{b} \qquad X = 0 - 0 = 0$$

$$Y = b + \frac{(1)\, a^2}{-b}$$

$$= b - \frac{a^2}{b}$$

$$x^2 + \left(y - b + \frac{a^2}{b}\right)^2 = \frac{a^4}{b^2}$$

$$x^2 + y^2 + b^2 + \cancel{\frac{a^4}{b^2}} - 2by - 2a^2 + \frac{2a^2 y}{b} = \cancel{\frac{a^4}{b^2}}$$

$$x^2 + y^2 - 2by + \frac{2a^2 y}{b} - 2a^2 + b^2 = 0$$

* Find the eq. of circle of curvature of curve $x = a(\cos\theta + \theta\sin\theta)$ & $y = a(\sin\theta - \theta\cos\theta)$

$$x_1 = a(-\sin\theta + \theta\cos\theta + \sin\theta)$$
$$= a\theta\cos\theta$$

$$y_1 = a(\cancel{\cos\theta} - \theta(-\sin\theta) - \cancel{\cos\theta})$$
$$y_1 = +a\theta\sin\theta.$$

$$x_2 = a[-\theta\sin\theta + \cos\theta]$$
$$x_2 = a(\cos\theta - \theta\sin\theta)$$

$$y_2 = +a[\theta\cos\theta - \sin\theta]$$

$$\rho = \frac{(x_1^2 + y_1^2)^{3/2}}{|x_1 y_2 - y_1 x_2|}$$

$$\cancel{= \frac{a^2\theta^2}{a^2|\theta^2\cos^2\theta - \theta\sin\theta\cos\theta - \theta\sin\theta\cos\theta - \theta^2\sin\theta|}}$$

$\cancel{\rho =}$

y_1: $y' = \tan\theta$

$$y'' = \sec^2\theta \cdot \frac{d\theta}{dx} = \sec^2\theta \cdot \frac{\sec\theta}{a\theta} = \frac{\sec^3\theta}{a\theta}$$

$$X = x - \frac{y_1(1+y_1^2)}{y_2}$$
$$= x - \frac{\tan\theta(\sec^2\theta)}{\sec^3\theta}\, a\theta$$
$$= x - \sin\theta(a\theta)$$
$$\boxed{X = x - a\theta\sin\theta}$$

$$X = a\cos\theta + a\theta\sin\theta - a\theta\sin\theta$$
$$\boxed{X = a\cos\theta}$$

$Y = y + \ldots$

$= a\sin\ldots$

$Y = a\ldots$

$\rho = (1+\ldots$

$(x - a\cos\ldots$

$x^2 + y^2 \ldots$

* EVOLUTES:

Locus of center… point $P(x,y)$ on … curve.

The curve itself…

Note: Working P…

Step-1: Find …

2: Elimina…

x, y …

3: We get…

Consider para…

* Find an evol…

y^2 …

$x = at^2$

$$Y = y + \frac{\cancel{y_1}(1+y_1^2)}{y_2}$$

$$= a\sin\theta - \theta a\cos\theta + \frac{\sec^2\theta}{\sec^3\theta}\, a\theta$$

$$\boxed{Y = a\sin\theta} \qquad C(a\cos\theta, a\sin\theta).$$

$$\rho = \frac{(1+\tan^2\theta)^{3/2}}{\sec^3\theta} \cdot a\theta = a\theta$$

$$(x - a\cos\theta)^2 + (y - a\sin\theta)^2 = a^2\theta^2$$

$$x^2 + y^2 - 2ax\cos\theta - 2ay\sin\theta + a^2 - a^2\theta^2 = 0$$

* EVOLUTES: **

Locus of centre of curvature $C(X, Y)$ of a variable point $P(x, y)$ on a curve is called the evolute of a curve.

The curve itself is called involute of the evolute.

Note: Working procedure:

Step - 1: Find the centre of curvature (X, Y)

2: Eliminate (x, y); we get relation b/w X, Y

3: We get evolute

Consider parametric forms.

* Find an evolute of the parabola $y^2 = 4ax$

$y^2 = 4ax$

$x = at^2 \qquad y = 2at$

$x_1 = 2at \qquad y_1 = 2a$

$x_2 = 2a \qquad y_2 = 0$

$$y' = \frac{1}{t}$$

$$y'' = \frac{-1}{t^2}\frac{dt}{dx} = \frac{-1}{t^2}\cdot\frac{1}{2at} = \frac{-1}{2at^3}$$

$$X = x - \frac{(1+y_1^2)y_1}{y_2}$$

$$Y = y + \frac{(1+y_1^2)}{y_2}$$

$$X = at^2 + \frac{\frac{1}{t}\left(1+\frac{1}{t^2}\right)}{\frac{1}{2at^3}}$$

$$= at^2 + \frac{\frac{t^2+1}{\cancel{t^3}}}{\frac{1}{2a\cancel{t^3}}}$$

$$X = at^2 + 2a(t^2+1)$$

$$\boxed{X = 3at^2 + 2a}$$

$$\left(\frac{X-2a}{3a}\right)^{1/2} = t$$

$$Y = 2at + \frac{\left(1+\frac{1}{t^2}\right)}{\frac{-1}{2at^3}}$$

$$= 2at - \frac{t^2+1}{\cancel{t^2}} \times \frac{2at^3}{1}$$

$$= 2at - (t^2+1)(2at)$$

$$= \cancel{2at} - 2at^3 - \cancel{2at}$$

$$\boxed{Y = -2at^3}$$

$$\cancel{t^3 = \frac{Y}{-2a}}$$

$$Y = -2a\left(\frac{X-2a}{3a}\right)^{3/2}$$

$$= \frac{-2\cancel{a}}{3\sqrt{3}\,\cancel{a}\sqrt{a}}(X-2a)^{3/2}$$

$$Y = \frac{-2}{3\sqrt{3a}}(X-2a)^{3/2}$$

$$Y^2 = \frac{4}{9(3a)}(X-2a)^3$$

~~$9y^2 = 4$~~

$$\boxed{27ay^2 = 4(x-2a)^3}$$

$y^2 = 4ax$

$2yy_1 = 4a$

$\boxed{y_1 = \frac{2a}{y}}$

$y_2 y + y_1^2 = 0$

$y_2 = \frac{-y_1^2}{y}$

$= -\frac{4a^2}{y^2} \times \frac{1}{y}$

$\boxed{y_2 = -\frac{4a^2}{y^3}}$

$X = x - \frac{y_1(1+y_1^2)}{y_2}$

$X = x + \frac{2a}{y}\frac{\left(1+\frac{4a^2}{y^2}\right)}{\frac{4a^2}{y^3}}$

$= x + \frac{y^2+4a^2}{2a}$

$$\boxed{X = x + \frac{y^2+4a^2}{2a}}$$

$= x + \frac{4ax+4a^2}{2a}$

$= \frac{6ax+4a^2}{2a}$

$= \frac{6x+4a}{2}$

$\boxed{X = 3x+2a}$

$x = \frac{X-2a}{3}$

$Y = y + \frac{1+y_1^2}{y_2}$

$= y + \frac{1+\frac{4a^2}{y^2}}{-\frac{4a^2}{y^3}}$

$= y - \frac{(y^2+4a^2)y}{4a^2}$

$= \frac{4a^2y - y^3 - 4a^2y}{4a^2}$

$Y = \frac{-y^3}{4a^2} = -\frac{y\,4ax}{4a^2} = -\frac{xy}{a}$ $\quad y^2 = 4ax$

$y = (-4aY)^{1/3}$

~~$y = (-4a^2X)^{1/3}$~~

$$Y = -\frac{xy}{a}$$

$$= -\left(\frac{x-2a}{3}\right)\frac{y}{a}$$

$$3aY = -(x-2a)\,2\sqrt{ax}$$

$$9a^2Y^2 = (x-2a)^2(4ax)$$

$$\frac{9a^2Y^2}{4a} = (x-2a)^2\left(\frac{x-2a}{3}\right)$$

$$\boxed{27a^2y^2 = 4a(x-2a)^3}$$

05/01/22

Find the evolute of given curve ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$x = a\cos\theta \qquad y = b\sin\theta$

$x_1 = -a\sin\theta \qquad y_1 = b\cos\theta$

$y' = \frac{-b}{a}\cot\theta \qquad \cancel{\rho = \frac{(1+y'^2)^{3/2}}{y''} = \frac{\left(1+\frac{b^2}{a^2}\cot^2\theta\right)^{3/2}}{\frac{b}{a}\operatorname{cosec}^2\theta}}$

$y'' = \frac{+b}{a}\operatorname{cosec}^2\theta\,\frac{d\theta}{dx}$

$$X = x - \frac{y'(1+y'^2)}{y''}$$

$$= a\cos\theta + \frac{\frac{b}{a}\cot\theta\left(1+\frac{b^2}{a^2}\cot^2\theta\right)}{\frac{b}{a}\operatorname{cosec}^2\theta}$$

$$\cancel{= a\cos\theta + \frac{\sin\theta\cos\theta\left(a^2+b^2\cot^2\theta\right)}{a^2}}$$

$$\frac{\cos\theta/\sin\theta}{1/\sin^2\theta}$$

$$\boxed{y' = \frac{-b}{a}\cot\theta}$$

$$y'' = \frac{b}{a}\cdot\frac{-1}{a\sin\theta}\times \operatorname{cosec}^2\theta$$

$$\boxed{y'' = \frac{-b}{a^2}\operatorname{cosec}^3\theta}$$

$$X = x - \frac{y'(1+(y')^2)}{y''}$$

$$= a\cos\theta + \frac{\frac{b}{a}\cot\theta\left(1+\frac{b^2}{a^2}\cot^2\theta\right)}{\frac{-b}{a^2}\operatorname{cosec}^3\theta}$$

$$= a\cos\theta - \frac{a\cot\theta}{a^2\operatorname{cosec}^3\theta}\left(a^2+b^2\cot^2\theta\right)$$

$$\cancel{X = a\cos\theta\left[a - \sin^2\theta\left(a^2+b^2\cot^2\theta\right)\right]}$$

$$\cancel{X = a\cos\theta\left[1 - \left(a^2\sin^2\theta + b^2\cos^2\theta\right)\right]}$$

$$\boxed{X = \frac{a^2\cos\theta - a^2\cos\theta\sin^2\theta - b^2\cos^3\theta}{a}}$$

$$Y = y + \frac{(1+(y')^2)}{y''}$$

$$= b\sin\theta + \frac{\left(1+\frac{b^2}{a^2}\frac{\cos^2\theta}{\sin^2\theta}\right)}{\frac{-b}{a^2}\operatorname{cosec}^3\theta}$$

$$= b\sin\theta + \frac{a^2\sin^2\theta + b^2\cos^2\theta}{-b\,a^2\sin^2\theta}\times a^2\sin^3\theta$$

$$\left(\ldots{}^2\theta\right)^{3/2}$$

$$\ldots\operatorname{cosec}^2\theta$$

$$Y = b\sin\theta - \frac{\sin\theta}{b}\left(a^2\sin^2\theta + b^2\cos^2\theta\right)$$

$$Y = \frac{b^2\sin\theta - a^2\sin^3\theta - b^2\sin\theta\cos^2\theta}{b} \quad (1-\sin^2\theta)$$

$$bY = b^2\cancel{\sin\theta} - b^2\cancel{\sin\theta} + b^2(1-\sin^2\theta)$$

$$-a^2\sin^3\theta$$

$$bY = +b^2\sin^3\theta - a^2\sin^3\theta$$

$$\sin\theta = \left(\frac{-bY}{\cancel{\cos}\, a^2-b^2}\right)^{1/3}$$

$$ax = \cancel{a^2\cos\theta} - b^2\cos^3\theta - \cancel{a^2\cos\theta}(1 - \cos^2\theta)$$

$$= \cancel{a^2\cos\theta} - b^2\cos^3\theta + a^2\cos^3\theta$$

$$ax = (a^2-b^2)\cos^3\theta$$

$$\cos\theta = \left(\frac{ax}{a^2-b^2}\right)^{1/3}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{(ax)^{2/3} + (bY)^{2/3}}{(a^2-b^2)^{2/3}} = 1$$

$$(ax)^{2/3} + (bY)^{2/3} = (a^2-b^2)^{2/3}$$

Find an evolu

$$\frac{x^2}{a^2} -$$

$$\frac{dx}{d\theta} = a\,se$$

$$\frac{dy}{d\theta}$$

y_2

$X = as$

$= ase$

$= as$

$= a$

$Y = b\tan$

Q: Show that evolute of the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ is a circle.

$C(a\cos\theta, a\sin\theta)$

$X = a\cos\theta$ | $Y = a\sin\theta$

$\cos\theta = \frac{X}{a}$ | $\sin\theta = \frac{Y}{a}$

$\cos^2\theta + \sin^2\theta = 1$

$X^2 + Y^2 = a^2$

Q: ~~Find~~ that evolute of the curve: cycloid $x = a(\theta - \sin\theta)$; $y = a(1-\cos\theta)$ is another cycloid

$\cancel{\frac{dx}{d\theta}} = \quad x = a(\theta - \sin\theta) \qquad y = a(1-\cos\theta)$

$\frac{dx}{d\theta} = a(1-\cos\theta) \qquad \frac{dy}{d\theta} = a(\sin\theta)$

$\frac{dy}{dx} = \frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta}{2\sin^2\theta/2} = \cot\theta/2$

$\frac{d^2y}{dx^2} = -\frac{\csc^2\theta/2}{2} \cdot \frac{1}{a\, 2\sin^2\theta/2}$

$= -\frac{\csc^4\theta/2}{4a}$

$y_1 = \cot\theta/2 \qquad y_2 = \frac{-\csc^4\theta/2}{4a}$

$X = x - \frac{y_1(1+y_1^2)}{y_2}$

$= a\theta -$

$= a\theta - a$

$= a[\theta -$

$= a[\theta -$

$\boxed{X = a(\theta +}$

$Y = y +$

$= a -$

$= a -$

$= a - a$

$= a$

$= a[$

$= a($

$\boxed{Y = -a(}$

Hence the l
is anothe

$$= a\theta - a\sin\theta + \frac{\cot\theta/2 \cdot (1+\cot^2\theta/2)}{\csc^4\theta/2}\, 4a$$

$$= a\theta - a\sin\theta + 4a\,\frac{\cot\theta/2}{\csc^2\theta/2}$$

$$= a\left[\theta - \sin\theta + 4\,\sin\theta/2\cos\theta/2\right]$$

$$= a\left[\theta - \sin\theta + 2\sin\theta\right]$$

$$\boxed{X = a(\theta + \sin\theta)}$$

$$Y = y + \frac{y_1(1+y_1^2)}{y_2}$$

$$= a - a\cos\theta + \frac{1+\cot^2\theta/2}{-\csc^4\theta/2}(4a)$$

$$= a - a\cos\theta - \frac{4a}{\csc^2\theta/2} \qquad \sin^2\theta/2 = \frac{1-\cos\theta}{2}$$

$$= a - a\cos\theta - 4a\sin^2\theta/2$$

$$= a\left[1 - \cos\theta - \cancel{4}2\frac{(1-\cos\theta)}{\cancel{2}}\right]$$

$$= a\left[1 - \cos\theta - 2 + 2\cos\theta\right] \qquad -a(1-\cos\theta)$$

$$= a(-1 + \cos\theta).$$

$$\boxed{Y = -a(1-\cos\theta)}$$

Hence the locus of $x = a(\theta - \sin\theta)$; $y = a(1-\cos\theta)$ is another cycloid with $X = a(\theta+\sin\theta)$

$Y = a(\cos\theta - 1)$

Find $\frac{dx}{dt}$ of $x = a(\cos t + \log(\tan t/2))$

$$\frac{dx}{dt} = a\left[-\sin t + \frac{\sec^2 t/2}{\tan t/2} \cdot \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{\sin t}\right]$$

$$\frac{\frac{1}{\cos^2}}{\frac{\sin}{\cos}}$$

$$= -a\left[\frac{\sin^2 t - 1}{\sin t}\right]$$

$$\cancel{= \frac{a}{\sin t}\left[1 - \frac{1}{\sin^2 t}\right] \sin^2 t}$$

$$\cancel{= a\left[1 - \right.}$$

$$= a\,\frac{\cos^2 t}{\sin t} = a \cos t \cot t$$