Method of Joints

Friday, June 11, 2021 9:28 AM

Structural Analysis

6

CHAPTER OBJECTIVES

- To show how to determine the forces in the members of a truss using the method of joints and the method of sections.
- To analyze the forces acting on the members of frames and machines composed of pin-connected members.

6.1 Simple Trusses

A *truss* is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts or metal bars. In particular, *planar* trusses lie in a single plane and are often used to support roofs and bridges. The truss shown in Fig. 6–1*a* is an example of a typical roof-supporting truss. In this figure, the roof load is transmitted to the truss *at the joints* by means of a series of *purlins*. Since this loading acts in the same plane as the truss, Fig. 6–1*b*, the analysis of the forces developed in the truss members will be two-dimensional.

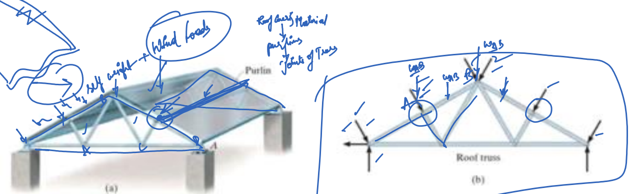
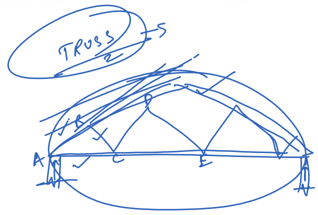
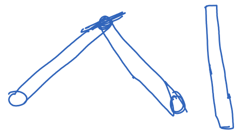
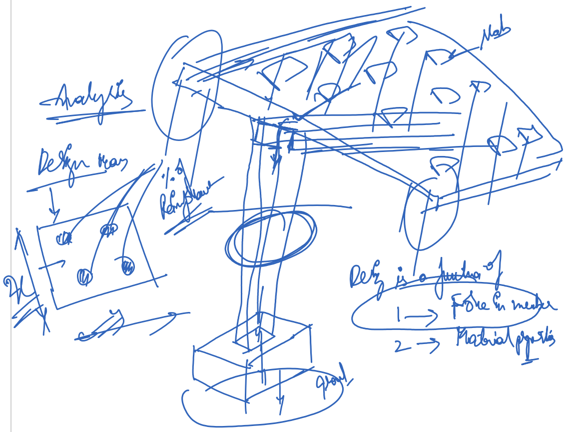


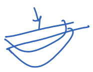
Fig. 6–1







Defn is a function of
1 → Force in member
2 → Material properties



planar Truss → loading & Members in same plane

Spatial truss

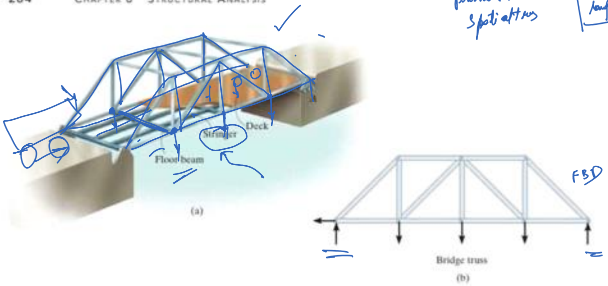


Fig. 6–2

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In the case of a bridge, such as shown in Fig. 6–2*a*, the load on the deck is first transmitted to *stringers*, then to *floor beams*, and finally to the *joints* of the two supporting side trusses. Like the roof truss, the bridge truss loading is also coplanar, Fig. 6–2*b*.

When bridge or roof trusses extend over large distances, a rocker or roller is commonly used for supporting one end, for example, joint *A* in Figs. 6–1*a* and 6–2*a*. This type of support allows freedom for expansion or contraction of the members due to a change in temperature or application of loads.

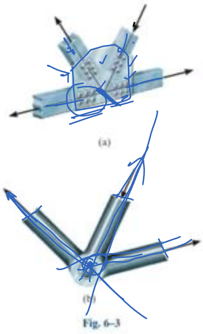
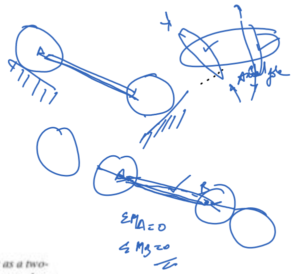


Fig. 6–3

Assumptions for Design. To design both the members and the connections of a truss, it is necessary first to determine the *force* developed in each member when the truss is subjected to a given loading. To do this we will make two important assumptions:

- ***All loadings are applied at the joints.*** In most situations, such as for bridge and roof trusses, this assumption is true. Frequently the weight of the members is neglected because the force supported by each member is usually much larger than its weight. However, if the weight is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, with half of its magnitude applied at each end of the member.
- ***The members are joined together by smooth pins.*** The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a *gusset plate*, as shown in Fig. 6–3*a*, or by simply passing a large bolt or pin through each of the members, Fig. 6–3*b*. We can assume these connections act as pins provided the center lines of the joining members are *concurrent*, as in Fig. 6–3.

Fig. 6–4



$\Sigma M_A = 0$

$\Sigma M_B = 0$

Because of these two assumptions, *each truss member will act as a two-force member*, and therefore the force acting at each end of the member will be directed along the axis of the member. If the force tends to *elongate* the member, it is a *tensile force* (T), Fig. 6–4*a*; whereas if it tends to *shorten* the member, it is a *compressive force* (C), Fig. 6–4*b*. In the actual design of a truss it is important to state whether the nature of the force is tensile or compressive. Often, compression members must be made *thicker* than tension members because of the buckling or column effect that occurs when a member is in compression.

The use of metal gusset plates in the construction of these Warren trusses is clearly evident.

Simple Truss. If three members are pin connected at their ends they form a *triangular truss* that will be *rigid*, Fig. 6–5. Attaching two more members and connecting these members to a new joint D forms a larger truss, Fig. 6–6. This procedure can be repeated as many times as desired to form an even larger truss. If a truss can be constructed by expanding the basic triangular truss in this way, it is called a *simple truss*.

2 Members are added for each joint

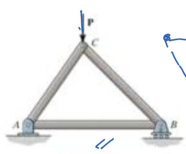


Fig. 6–5

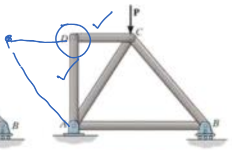
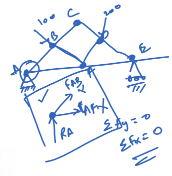


Fig. 6–6

Analysis of Truss

MOJ MOS



At least 1 known
Max -2 Unknown

6.2 The Method of Joints (MOJ)

In order to analyze or design a truss, it is necessary to determine the force in each of its members. One way to do this is to use the method of joints. This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium. Therefore, if the free-body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint. Since the members of a *plane truss* are straight two-force members lying in a single plane, each joint is subjected to a force system that is *coplanar and concurrent*. As a result, only $\Sigma F_x = 0$ and $\Sigma F_y = 0$ need to be satisfied for equilibrium.

For example, consider the pin at joint B of the truss in Fig. 6–7*a*. Three forces act on the pin, namely, the 500-N force and the forces exerted by members BA and BC. The free-body diagram of the pin is shown in Fig. 6–7*b*. Here, $\mathbf{F}_{BA}$ is "pulling" on the pin, which means that member BA is in *tension;* whereas $\mathbf{F}_{BC}$ is "pushing" on the pin, and consequently member BC is in *compression*. These effects are clearly demonstrated by isolating the joint with small segments of the member connected to the pin, Fig. 6–7*c*. The pushing or pulling on these small segments indicates the effect of the member being either in compression or tension.

When using the method of joints, always start at a joint having at least one known force and at most two unknown forces, as in Fig. 6–7*b*. In this way, application of $\Sigma F_x = 0$ and $\Sigma F_y = 0$ yields two algebraic equations which can be solved for the two unknowns. When applying these equations, the correct sense of an unknown member force can be determined using one of two possible methods.

On FBD of joint → if force is away from the joint → Tension in member

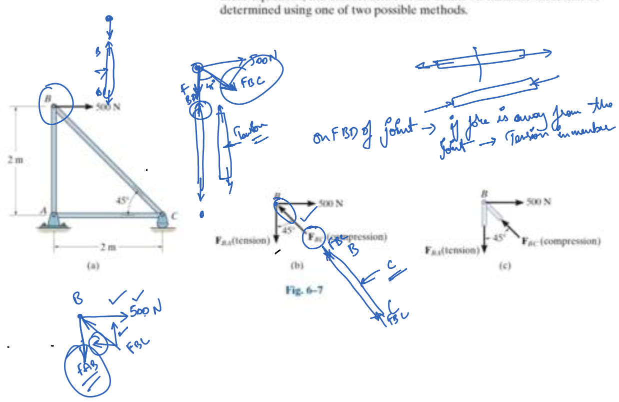


Fig. 6–7

- The *correct* sense of direction of an unknown member force can, in many cases, be determined "by inspection." For example, $\mathbf{F}_{BC}$ in Fig. 6–7*b* must push on the pin (compression) since its horizontal component, $F_{BC} \sin 45°$, must balance the 500-N force ($\Sigma F_x = 0$). Likewise, $\mathbf{F}_{BA}$ is a tensile force since it balances the vertical component, $F_{BC} \cos 45°$ ($\Sigma F_y = 0$). In more complicated cases, the sense of an unknown member force can be *assumed;* then, after applying the equilibrium equations, the assumed sense can be verified from the numerical results. A *positive* answer indicates that the sense is *correct*, whereas a *negative* answer indicates that the sense shown on the free-body diagram must be *reversed.*
- *Always assume* the *unknown member forces* acting on the joint's free-body diagram to be in *tension;* i.e., the forces "pull" on the pin. If this is done, then numerical solution of the equilibrium equations will yield *positive scalars for members in tension and negative scalars for members in compression.* Once an unknown member force is found, use its *correct* magnitude and sense (T or C) on subsequent joint free-body diagrams.

The forces in the members of this simple roof truss can be determined using the method of joints.

6

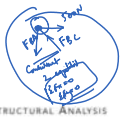
Procedure for Analysis

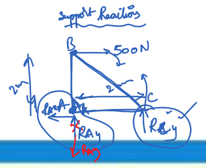
support

The following procedure provides a means for analyzing a truss using the method of joints.

1) Determine Reactions assuming truss as a Whole Rigid Body

- Draw the free-body diagram of a joint having at least one known force and at most two unknown forces. (If this joint is at one of the supports, then it may be necessary first to calculate the external reactions at the support.)
- Use one of the two methods described above for establishing the sense of an unknown force.
- Orient the x and y axes such that the forces on the free-body diagram can be easily resolved into their x and y components and then apply the two force equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Solve for the two unknown member forces and verify their correct sense.
- Using the calculated results, continue to analyze each of the other joints. Remember that a member in *compression* "pushes" on the joint and a member in *tension* "pulls" on the joint. Also, be sure to choose a joint having at most two unknowns and at least one known force.





AB
BC
AC

① $\Sigma F_x = 0$ →+

$+500 - R_{Ax} = 0$

$R_{Ax} = \underline{500\,N}$

$\Sigma M_A = 0$; ↺+

$500 \times 2 - R_{Cy} \times 2 = 0$

$R_{Cy} = \underline{500\,N}$

$\underline{\Sigma F_y = 0}$ ↑+

$R_{Ay} + R_{Cy} = 0$

$R_{Ay} = -R_{Cy}$

$R_{Ay} = -\underline{500\,N}$

EXAMPLE 6.1

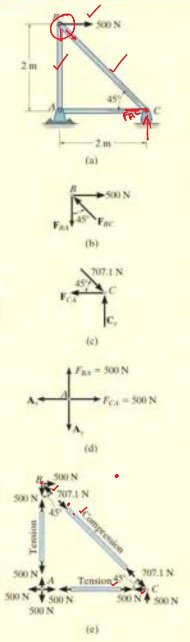


Fig. 6–8

Determine the force in each member of the truss shown in Fig. 6–8*a* and indicate whether the members are in tension or compression.

SOLUTION

Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint *B*.

Joint B. The free-body diagram of the joint at *B* is shown in Fig. 6–8*b*. Applying the equations of equilibrium, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 500\text{ N} - F_{BC} \sin 45^\circ = 0 \qquad F_{BC} = 707.1\text{ N (C)} \quad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} \cos 45^\circ - F_{BA} = 0 \qquad F_{BA} = 500\text{ N (T)} \quad \textit{Ans.}$$

Since the force in member *BC* has been calculated, we can proceed to analyze joint *C* to determine the force in member *CA* and the support reaction at the rocker.

Joint C. From the free-body diagram of joint *C*, Fig. 6–8*c*, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{CA} + 707.1 \cos 45^\circ\text{ N} = 0 \qquad F_{CA} = 500\text{ N (T)} \quad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad C_y - 707.1 \sin 45^\circ\text{ N} = 0 \qquad C_y = 500\text{ N} \quad \textit{Ans.}$$

Joint A. Although it is not necessary, we can determine the components of the support reactions at joint *A* using the results of F_{CA} and F_{BA}. From the free-body diagram, Fig. 6–8*d*, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 500\text{ N} - A_x = 0 \qquad A_x = 500\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 500\text{ N} - A_y = 0 \qquad A_y = 500\text{ N}$$

NOTE: The results of the analysis are summarized in Fig. 6–8*e*. Note that the free-body diagram of each joint (or pin) shows the effects of all the connected members and external forces applied to the joint, whereas the free-body diagram of each member shows only the effects of the end joints on the member.

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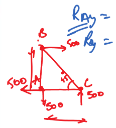


Joint B

$\Sigma F_y = 0$

$-F_{BA} + 707.1 \sin 45 = 0$

$F_{BA} = \underline{500\,N}$ (T)

Joint C

$\Sigma F_y = 0$

$-F_{BC} \sin 45 + 500 = 0$

$F_{BC} = \underline{707.1\,N}$ (C)

$\Sigma F_x = 0$ (→+)

$-F_{AC} + F_{BC} \cos 45 = 0$

$F_{AC} = 707.1 \times \cos 45$

$= \underline{500.0\,N}$ (T)

Force member	Magnitude	Nature
AB	500	T
BC	707.1	C
CA	500	T

EXAMPLE 6.2

Determine the force in each member of the truss in Fig. 6–9*a* and indicate if the members are in tension or compression.

SOLUTION

Since joint C has one known and only two unknown forces acting on it, it is possible to start at this joint, then analyze joint D, and finally joint A. This way the support reactions will not have to be determined prior to starting the analysis.

Joint C. By inspection of the force equilibrium, Fig. 6–9*b*, it can be seen that both members BC and CD must be in compression.

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} \sin 45^\circ - 400\text{ N} = 0$$

$$F_{BC} = 565.69\text{ N} = 566\text{ N (C)} \qquad \textit{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CD} - (565.69\text{ N}) \cos 45^\circ = 0$$

$$F_{CD} = 400\text{ N (C)} \qquad \textit{Ans.}$$

Joint D. Using the result $F_{CD} = 400\text{ N (C)}$, the force in members BD and AD can be found by analyzing the equilibrium of joint D. We will assume $\mathbf{F}_{AD}$ and $\mathbf{F}_{BD}$ are both tensile forces, Fig. 6–9*c*. The x', y' coordinate system will be established so that the x' axis is directed along $\mathbf{F}_{BD}$. This way, we will eliminate the need to solve two equations simultaneously. Now $\mathbf{F}_{AD}$ can be obtained *directly* by applying $\Sigma F_{y'} = 0$.

$$+\nearrow \Sigma F_{y'} = 0; \qquad -F_{AD} \sin 15^\circ - 400 \sin 30^\circ = 0$$

$$F_{AD} = -772.74\text{ N} = 773\text{ N (C)} \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{F}_{AD}$ is a compressive force. Using this result,

$$+\searrow \Sigma F_{x'} = 0; \qquad F_{BD} + (-772.74 \cos 15^\circ) - 400 \cos 30^\circ = 0$$

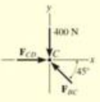
$$F_{BD} = 1092.82\text{ N} = 1.09\text{ kN (T)} \qquad \textit{Ans.}$$

Joint A. The force in member AB can be found by analyzing the equilibrium of joint A, Fig. 6–9*d*. We have

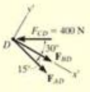
$$\xrightarrow{+} \Sigma F_x = 0; \qquad (772.74\text{ N}) \cos 45^\circ - F_{AB} = 0$$

$$F_{AB} = 546.41\text{ N (C)} = 546\text{ N (C)} \qquad \textit{Ans.}$$

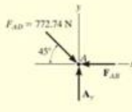
(a)



(b)



(c)



(d)

Fig. 6–9

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EXAMPLE 6.3

Determine the force in each member of the truss shown in Fig. 6–10*a*. Indicate whether the members are in tension or compression.

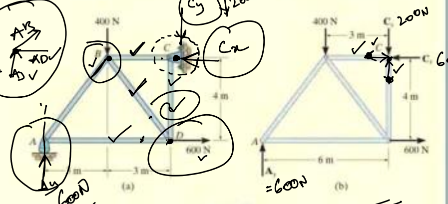


Fig. 6–10

SOLUTION

Support Reactions. No joint can be analyzed until the support reactions are determined, because each joint has more than three unknown forces acting on it. A free-body diagram of the entire truss is given in Fig. 6–10*b*. Applying the equations of equilibrium, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 600\text{ N} - C_x = 0 \qquad C_x = 600\text{ N}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -A_y(6\text{ m}) + 400\text{ N}(3\text{ m}) + 600\text{ N}(4\text{ m}) = 0$$

$$A_y = 600\text{ N}$$

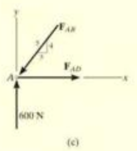
$$+\uparrow \Sigma F_y = 0; \qquad 600\text{ N} - 400\text{ N} - C_y = 0 \qquad C_y = 200\text{ N}$$

The analysis can now start at either joint *A* or *C*. The choice is arbitrary since there are one known and two unknown member forces acting on the pin at each of these joints.

Joint A. (Fig. 6–10*c*). As shown on the free-body diagram, $\mathbf{F}_{AB}$ is assumed to be compressive and $\mathbf{F}_{AD}$ is tensile. Applying the equations of equilibrium, we have

$$+\uparrow \Sigma F_y = 0; \qquad 600\text{ N} - \tfrac{4}{5}F_{AB} = 0 \qquad F_{AB} = 750\text{ N (C)} \qquad \textit{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AD} - \tfrac{3}{5}(750\text{ N}) = 0 \qquad F_{AD} = 450\text{ N (T)} \qquad \textit{Ans.}$$



① $\Sigma M_C = 0$ ✓ ↺+

$+(A_y \times 6) - 400 \times 3 - 600 \times 4 = 0$

$A_y = 600\text{N}$

② $\Sigma F_y = 0$ ↑ +ve

$+600 - 400 - C_y = 0$

$C_y = 200\text{N}$ ✓

③ $\Sigma F_x = 0$ → +

$600 - C_x = 0$

$C_x = 600\text{N}$

Which joint to select

→ where at least 1 known force and not more than 2 unknowns are

Joint C

$\Sigma F_x = 0$

$-F_{CB} - 600 = 0$

$F_{CB} = 600\text{N (C)}$

$\Sigma F_y = 0$

$-200 + F_{CD} = 0$

$F_{CD} = 200\text{N (C)}$

Joint D

$\Sigma F_y = 0$

$F_{DB}\left(\frac{4}{5}\right) - 200 = 0$

$F_{DB} = \frac{200 \times 5}{4}$

$\boxed{F_{DB} = 250\text{N (T)}}$

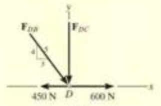
$\Sigma F_x = 0$ Assume. F_{AD} (T)

$-F_{AD} - \cancel{F_{AB}}\,F_{DB}\left(\frac{3}{5}\right) + 600 = 0$

$F_{AD} = 450\text{ (T)}$

Joint D. (Fig. 6–10*d*). Using the result for F_{AD} and summing forces in the horizontal direction, Fig. 6–10*d*, we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad -450\text{ N} + \tfrac{3}{5}F_{DB} + 600\text{ N} = 0 \quad F_{DB} = -250\text{ N}$$



(d)

The negative sign indicates that $\mathbf{F}_{DB}$ acts in the *opposite sense* to that shown in Fig. 6–10*d*.* Hence,

$$F_{DB} = 250\text{ N (T)} \qquad \textit{Ans.}$$

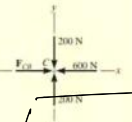
To determine $\mathbf{F}_{DC}$, we can either correct the sense of $\mathbf{F}_{DB}$ on the free-body diagram, and then apply $\Sigma F_y = 0$, or apply this equation and retain the negative sign for F_{DB}, i.e.,

$$+\uparrow \Sigma F_y = 0; \quad -F_{DC} - \tfrac{4}{5}(-250\text{ N}) = 0 \quad F_{DC} = 200\text{ N (C)} \qquad \textit{Ans.}$$

Joint C. (Fig. 6–10*e*).

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{CB} - 600\text{ N} = 0 \quad F_{CB} = 600\text{ N (C)} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 200\text{ N} - 200\text{ N} = 0 \quad \text{(check)}$$



(e)

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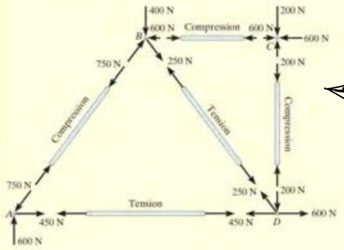
NOTE: The analysis is summarized in Fig. 6–10*f*, which shows the free-body diagram for each joint and member.



(f)

Member	Force/Magnitude	Nature
AB →	750	Compression
BC →		
BD →		
CD →		
AD →	450N	Tension

*The proper sense could have been determined by inspection, prior to applying $\Sigma F_y = 0$.

6.3 Zero-Force Members

Truss analysis using the method of joints is greatly simplified if we can first identify those members which support *no loading*. These *zero-force members* are used to increase the stability of the truss during construction and to provide added support if the loading is changed.

The zero-force members of a truss can generally be found *by inspection* of each of the joints. For example, consider the truss shown in Fig. 6–11*a*. If a free-body diagram of the pin at joint A is drawn, Fig. 6–11*b*, it is seen that members AB and AF are zero-force members. (We could not have come to this conclusion if we had considered the free-body diagrams of joints F or B simply because there are five unknowns at each of these joints.) In a similar manner, consider the free-body diagram of joint D, Fig. 6–11*c*. Here again it is seen that DC and DE are zero-force members. From these observations, we can conclude that *if only two members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members.* The load on the truss in Fig. 6–11*a* is therefore supported by only five members as shown in Fig. 6–11*d*.

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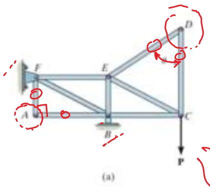
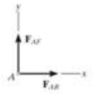


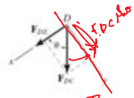
(a)



$\xrightarrow{+} \Sigma F_x = 0;\ F_{AB} = 0$
$+\uparrow \Sigma F_y = 0;\ F_{AF} = 0$

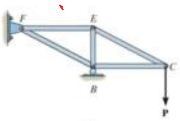
(b)

FDE
FDC
Σfx = 0
FAB = 0
Σfy = 0
fAF = 0



$+\searrow \Sigma F_y = 0;\ F_{DC} \sin\theta = 0;\ F_{DC} = 0$ since $\sin\theta \neq 0$
$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\ F_{DE} = 0$

(c)



(d)

Fig. 6–11

Σfx=0
fDE + fDC cos 90 = 0
fDE=0

Now consider the truss shown in Fig. 6–12*a*. The free-body diagram of the pin at joint D is shown in Fig. 6–12*b*. By orienting the y axis along members DC and DE and the x axis along member DA, it is seen that DA is a zero-force member. Note that this is also the case for member CA, Fig. 6–12*c*. In general then, *if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint.* The truss shown in Fig. 6–12*d* is therefore suitable for supporting the load **P**.

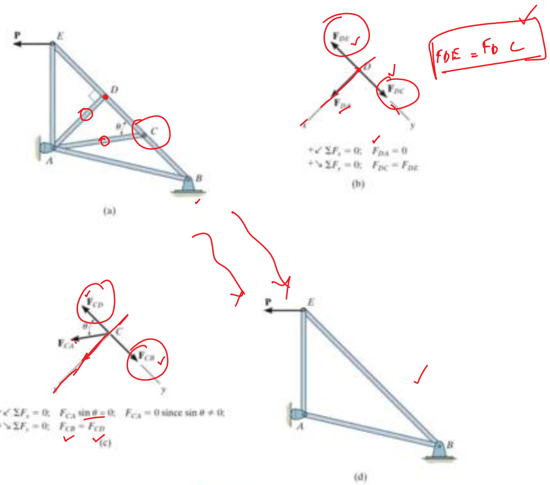
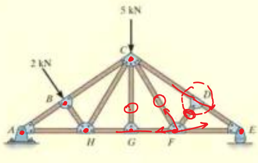


Fig. 6–12

EXAMPLE 6.4

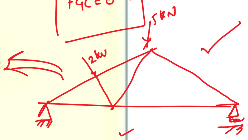
Using the method of joints, determine all the zero-force members of the *Fink roof truss* shown in Fig. 6–13*a*. Assume all joints are pin connected.

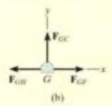


(a)

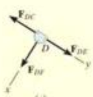
Fig. 6–13

$F_{FD} = 0$ ✓
$F_{CF} = 0$ ✓
$F_{GC} = 0$ ✓

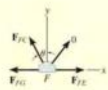




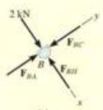
(b)



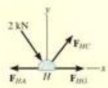
(c)



(d)



(e)



(f)

SOLUTION

Look for joint geometries that have three members for which two are collinear. We have

Joint G. (Fig. 6–13*b*).

$+\uparrow \Sigma F_y = 0;$ $\quad F_{GC} = 0$ *Ans.*

Realize that we could not conclude that *GC* is a zero-force member by considering joint *C*, where there are five unknowns. The fact that *GC* is a zero-force member means that the 5-kN load at *C* must be supported by members *CB*, *CH*, *CF*, and *CD*.

Joint D. (Fig. 6–13*c*).

$+\swarrow \Sigma F_x = 0;$ $\quad F_{DF} = 0$ *Ans.*

Joint F. (Fig. 6–13*d*).

$+\uparrow \Sigma F_y = 0;$ $\quad F_{FC} \cos\theta = 0 \quad$ Since $\theta \neq 90°$, $\quad F_{FC} = 0$ *Ans.*

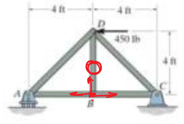
NOTE: If joint *B* is analyzed, Fig. 6–13*e*,

$+\searrow \Sigma F_x = 0;$ $\quad 2\text{ kN} - F_{BH} = 0 \quad F_{BH} = 2\text{ kN} \quad$ (C)

Also, F_{HC} must satisfy $\Sigma F_y = 0$, Fig. 6–13*f*, and therefore *HC* is *not* a zero-force member.

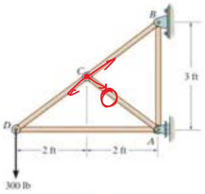
FUNDAMENTAL PROBLEMS

F6–1. Determine the force in each member of the truss. State if the members are in tension or compression.



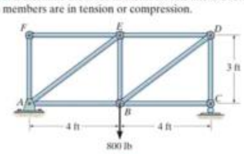
F6–1

F6–2. Determine the force in each member of the truss. State if the members are in tension or compression.



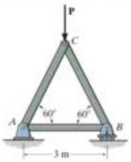
F6–2

F6–3. Determine the force in members AE and DC. State if the members are in tension or compression.



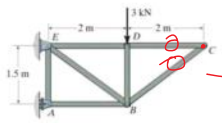
F6–3

F6–4. Determine the greatest load P that can be applied to the truss so that none of the members are subjected to a force exceeding either 2 kN in tension or 1.5 kN in compression.



F6–4

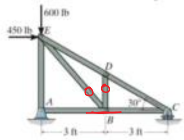
F6–5. Identify the zero-force members in the truss.



F_{DC}
F_{DB}

F6–5

F6–6. Determine the force in each member of the truss. State if the members are in tension or compression.



F6–6