28/04/2023 UNIT-3

PROBABILITY AND RANDOM VARIABLES:

→ Introduction to probability
→ Random experiment
→ Random variables → Discrete R.V / → Continuous R.V
- Definitions, Problems
→ Probability distribution ~~table~~ function of discrete random variables. (PDF)
— Problems

$\boxed{\text{expectatio-mean}}$

→ Probability density function of continuous random variables — Problems. (PDF)

→ Properties.
→ Expectations (Mean) - Defn - Problems - Properties.
→ Variance - Defn - Properties - ~~Properties~~ Problems
→ Normal Distribution (Tables)

*Probability distribution tables:

x no. of times a coin is tossed.

$= 2^2 = 4$

$S = \{HH, HT, TH, TT\}$

RE(X). Random Experiment.

S — Samples Space: HH, HT, TH, TT

R_x — Range space (or) Random space: 0, 1, 2

x	0	1	2
P(x)	$\frac{1}{4}$	$\frac{2}{4}$	1/4

$E(x) = \sum x P(x)$

$= 0 \times \frac{1}{4} + 1 \times \frac{2}{4} + 2 \times \frac{1}{4}$

$= 1$

expectation

$\boxed{\text{Variance} = E(x^2) - [E(x)]^2}$

→ All the answers in Range space are real numbers.

* Probability:

The probability is the branch of mathematics which studies the influence of chance. It is a numerical measure certainity (or) uncertainity of a random experiment.

*→ Introduction:

In everyday life; we come across statements such as

(i) Most probably it will rain today.
There is a chance rain will come (or) not.

(ii) I doubt that he will win the race.

The words 'most probably', 'chances' 'doubt' etc show uncertainity (or) probability of occurrence of an event.

* Terms related to probability:

→ Experiment: An operation can produce some well-defined outcomes, is called an experiment. Each outcome is called an event.

→ Random Experiment: An experiment in which all possible outcomes are known and the exact outcome cannot be predicted in advance is called a random experiment, i.e. outcome is not unique.

→ Trial: A single performance of an experiment is briefly called a trial.

→ Event: All outcomes are known as events.

→ Sample Space: The set of all possible outcomes in an experiment is called sample space of exhaustive events

→ Sample point: Each element in sample space is called sample point.

Eg: Tossing a coin means there are 2 chances to get head (or) tail.

Sample space $S = \{H, T\} = 2^1$

→ Tossing 2 coins.

Sample Space $= S = \{HH, HT, TH, TT\} = 2^2$

→ Throwing a dice:

$S = \{1, 2, 3, 4, 5, 6\} = 6^1$

* Random Variables:

A random variable 'x' is a function defined from a sample space 'S' to a set of all real numbers R such that the pre-image of every element of R is an ~~exacpting~~ event of S.

There are 2 types of random variables:

→ If the range set of x is finite; then 'x' is called discrete random variable.

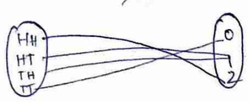
→ If the range set of x is infinite then 'x' is called continuous random variable.

(Range set is infinite means it is in intervals (or) the union of sub-intervals)

Ex: In an experiment of tossing 2 coins we have the sample space

$S = \{HH, HT, TH, TT\} = 2^2$

S X R



X = No. of times head occurs.

$P(X=0) = \frac{1}{4}$

$P(X=1) = \frac{2}{4}$

$P(X=2) = \frac{1}{4}$

$\Sigma(P(x)) = 1.$

pdf

X	0	1	2
P(x)	1/4	2/4	1/4

1

→ pdf → distribution is only applied for discrete values. (Σ)

Q) A coin tossed 3 times until a tail appears. (no. of times tails occurs)

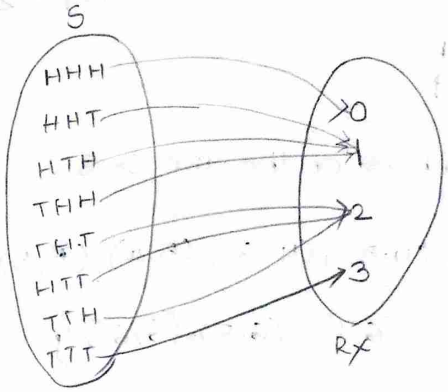
$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$
$= 2^3$

$P(X=0) =$

$X \to$ no. of times tail occured.

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

$\Sigma P(x) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{8}{8} = 1.$



Q) If a dice are rolled. The

$S = \{(1,1), (1,2), (1,3), \ldots, (6,6)\} = 6^2 = 36$

{(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)}

(i) Find the probability of that both the dice shows the same number.

$S = \{(1,1), (2,2), (3,3), (4,4), (5,5)\} = 6$

$= 6/36 = \frac{1}{6}$

Q) Find the probability that total no of dice is 9.

$X = \text{sum} = 9$.

$S = \{(3,6), (4,5), (5,4), (6,3)\} = 4$

$\sum P(x) = \frac{4}{36} = \frac{1}{9}$

Q) Total numbers on the dice > 8.

$S = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6),$
$(6,3), (6,4), (6,5), (6,6)\}$

$P(X) = \frac{10}{36}$

Q) X = Total No on the dice is 13.

The maximum possibility of sum is 12 = 6+6

so, $S = \{\phi\}$.

∴ It is an impossible event.

∴ required ~~possib~~ probability = 0 = P(X)

Q) X = Total sum is 8.

$S = \{(6,2), (5,3), (4,4), (2,6), (3,5)\}$

$= \frac{5}{36}$

Q) X = The favourable cases that 1st dice shows 3.

$S = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$

$= \frac{6}{36} = \frac{1}{6}$

Q) If 2 dice are rolled. Define $X : S \rightarrow R$

$\Rightarrow X(a,b) = 2a+b$.

$X(1,1) = 2(1)+1 = 3$
$X(1,2) = 2(1)+2 = 4$
$\vdots$
$X(6,6) = 2(6)+6 = 18$

$S = \{3, 4, \dots, 18\}$ → also known as spectrum.

Find the probability that sum is odd.

$\Rightarrow P(X) = \frac{8}{36} = \frac{2}{9}$

$S = \{(1,1), (1,3), (1,5)$
$(2,1), (2,3), (2,5)$
$(3,1), (3,3), (3,5)$
$(4,1), (4,3), (4,5)$
$(5,1), (5,3), (5,5)$
$(6,1), (6,3), (6,5)\}$

$2(1)+1 = 3$.
$\vdots$
$6(2)+5 = 17$

$P(X) = \frac{18}{36} = \frac{1}{2}$

Q) If 2 dice are rolled ~~twice~~ define $X: S \to R \ni X(a,b) = max(a,b)$. find PDF

$S = \{1, 2, \ldots, 6\}$

$max(1,2) = 2$

X	1	2	3	4	5	6
P(X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Probability Distribution: The probability Distribution of random variable X is a description of the set of possible values of x along with the probability associated with each of the possible values of X.

X	x_1	x_2	......	x_n
$P(X=x)$	P_1	P_2	......	P_n

Probability function: The probability function is a real function which assigns a value (called probability) to each value of the range of a random variable X.

→ These are of 2 types:

1. Probability mass function (or) Discrete probability function

→ Denoted by $p(x)$. For any random variable X, the function $P(x)$ satisfies the following conditions

(i) $P(x_i) \geq 0$ for each x.

(ii) $\sum_{\forall x} P(x_i) = 1$ then $P(x_i)$ is called probability mass function.

2. Probability Density function (or) Continuous probability function:

It is denoted by f(x) which is defined on continuous random variable satisfy the following conditions.

~~(i) $P(x_i) \geq 0$ for each x.~~

~~(ii) $\sum_{\forall x} P(x_i) = 1$ then $P(x_i)$ is called probability mass function.~~

(i) $f(x) \geq 0 \ \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ then $f(x)$ is called probability density function.

Probability Distribution function (or) Cumulative distribution function.

For a random variable X the probability distribution is denoted by $F(x)$ and is defined by $F(x) = P(X \leq x)$ where x is any real number.

→ If X is a discrete random variable: $F(x) = P(X \leq x) = \sum_{i=0}^{x} P(x)$

If X is continuous Random variable $F(x) =$

$P(X \le x) = \int_{-\infty}^{x} f(x)dx$

[If $a < b$

$P(a \le x \le b) = F(b) - F(a)$ & x is a discrete random variable $(-\infty, b] = (-\infty, a] \cup (a, b]$

where $(-\infty, a], (a, b]$ are disjoint.

$$P[(-\infty, b]] = P\{(-\infty, a]\} + P\{(a, b]\}$$

i.e $P[x \le b] = P(x \le a) + P(a < x \le b)$

$$P(a < x \le b) = P(x \le b) - P(x \le a)$$
$$= F(b) - F(a)$$

(2) $P(a \le x \le b) = P(x = a) + F(b) - F(a)$

(3) $P(a \le x < b) = P(x = a) + F(b) - F(a) - P(x = b)$

(4) $P(a < x < b) = F(b) - F(a) - P(x = a)$

If x is a continuous random variable $P(x = a) =$ the area of the curve $f(x)$ from a to $a = \int_{-a}^{a} f(x)dx = 0$.

$\therefore P(a < x \le b) = P(a \le x \le b) = P(a \le x < b] = P(a < x < b) = F(b) = F(a)]$

The cummulative Distribution: The cummulative distribution of a random variable X is a description of the set of all possible values of X along with the cummulative probability.

x	x_1	x_2	......	x_n
$f(x)$	$F(x_1)$	$F(x_2)$	..	$F(x_n)$

$$F(x_1) = P(x_1)$$
$$F(x_2) = P(x \le x_2) = P(x_1) + P(x_2)$$
$$\vdots$$
$$F(x_n) = P(X \le x_n) = P(x_1) + P(x_2) + \cdots P(x_n)$$

Note: Probability distribution and cummulative distribution i.e table is possible in case of "discrete random variable".

→ probability mass function (Discrete r.v)
probability distribution funⁿ (continous r.v)

Discrete R.V: Range Space is finite. → $\sum P(x_i) = 1$

Continous R.V: Range space is not finite. $\int_{-\infty}^{\infty} f(x_i) = 1$

RangeSpace :

Ex: $f(x) = \begin{cases} kx^2 & x \ge 0 \\ 0 & x < 0 \end{cases}$

Expectation (Mean): Given pdf,
$E(X)$ μ.

$$E(X) = \sum_{i=1}^{n} x_i P(x_i)$$

Mathematical Expectation $(E(x))$:— (or) Mean:- or μ (μ)

If 'X' is a Random Variable, the Expectation of X^r is defined as $E(X^r) = \begin{cases} \sum_i x_i^r P(x_i), & \text{if } X \text{ is D.R.V} \\ \int_{-\infty}^{\infty} x_i^r f(x_i), & \text{If } X \text{ is C.r.V} \end{cases}$

Properties of Expectation:-

1. If X is a random variable then prove that:

(i) $E(x+a) = E(X)+a$ where a = constant

By the defination of Expectation,

$$E(X) = \sum_{i=1}^{n} x_i P(x_i)$$

L.H.S = $E(X+a)$

$$= \sum_{i=1}^{n} (x_i+a) P(x_i)$$

$$= \sum_{i=1}^{n} x_i p(x_i) + \sum_{i=1}^{n} a\, p(x_i)$$

$$= \sum_{i=1}^{n} x_i P(x_i) + a \sum_{i=1}^{n} P(x_i)$$ (∴ By the def of Expectation)

R.H.S = $E(X) + a$

(ii) $E(aX) = aE(X)$, a = constant

By the def^n of Expect^n

$$E(X) = \sum_{i=1}^{n} x_i P(x_i) \quad \text{—— ①}$$

L.H.S $E(ax) = \sum_{i=1}^{n} a x_i P(x_i)$

$$= a \sum_{i=1}^{n} x_i P(x_i) \quad (\therefore \text{By ①})$$

R.H.S = $aE(X)$

(iii) $E(ax+b) = aE(x) + b$, where a, b are constants

L.H.S = $E(ax+b) = \sum_{i=1}^{n} (ax_i + b) P(x_i)$

$$= \sum_{i=1}^{n} a x_i P(x_i) + \sum_{i=1}^{n} b P(x_i)$$

$$= a \underbrace{\sum_{i=1}^{n} x_i P(x_i)}_{E(X)} + b \underbrace{\sum_{i=1}^{n} P(x_i)}_{1}$$

R.H.S = $aE(X) + b$.

(iv) $E(X+Y) = E(X) + E(Y)$

(v) $E(XY) = E(X) \cdot E(Y)$

$$E(XY) = \sum_{i=1}^{n} x_i y_i P(x_i y_i)$$

$=$

Variance: If X is D.r.v then $V(X) = E(X-\mu)^2$

$\sigma^2 = E(X-\mu)^2 = \sum (x_i-\mu)^2 p(x_i)$

$= \sum x_i^2 p(x_i) + \sum \mu^2 p(x_i) - 2\sum x_i \mu p(x_i)$

$= E(x^2) + \mu^2 - 2\mu E(x_i)$

$= E(x^2) + \mu^2 - 2\mu^2$

$= E(x^2) - \mu^2$

Properties of varience :-

property 1 :- $V(K) = 0$

$V(X+K) = V(X)$

$V(aX) = a^2 V(X)$

$V(aX+b) = a^2 V(X)$

$V(aX+bY) = a^2 V(X) + b^2 V(Y)$

Property:

If X is a random variable prove that

(i) $E(ax+by) = a E(X) + b E(Y)$

(ii) $V(ax+by) = a^2 V(X) + b^2 (V(Y))$.

Proof: case1 :- when X & Y are a discrete random variable

$E(X) = \sum x_i P(x_i)$

$E(ax+by) = \sum (ax_i + by_i) P(x_i+y_i)$

$= a\sum x_i P(x_i) + a\sum \cancel{x_i} P(y_i) + b\sum y_i \cancel{P}(x_i)$

$+ b\sum y_i P(y_i)$

$= a E(X) + b(E(Y))$.

$E(ax+by) = a E(X) + b E(Y)$.

case 2: When X and Y are continous random variables.

$$\boxed{E(X) = \int_{-\infty}^{\infty} x F(x)\, dx}$$

$$E(ax+by) = \int_{-\infty}^{\infty} (ax+by) F(X+Y)\, dx dy$$

$$= a \int_{-\infty}^{\infty} x F(x) dx + b \int_{-\infty}^{\infty} y F(y) dy$$

$= a E(X) + b E(Y)$

case(i) When X and Y are Discrete random variables

$V(aX+bY) = a^2 V(X) + b^2 V(Y)$

$V(X) = E(X^2) - (E(X))^2$ — ①

L.H.S $= V(aX+bY)$

$E((ax+by)^2) - [E(ax+by)]^2$

$= E(a^2x^2 + b^2y^2 + 2abxy) - (a E(X) + b E(Y))^2$

$= E(a^2x^2 + b^2y^2 + 2abxy) - (a^2(E(x))^2 + b^2(E(y))^2 + 2abE(x)E(y))$

$= a^2 E(x^2) + b^2 E(y^2) + 2abE(X)E(y) - (a^2(E(x))^2 + b^2(E(y))^2 + 2abE(x)E(y))$

$$= a^2\left(E(X^2)-(E(X))^2\right)+b^2\left(E(Y^2)-(E(Y))^2\right)$$

$$V(aX+bY) = a^2V(X)+b^2V(Y)$$

Case 2: When X and Y are continuous random variables $\left(E(X) = \int_{-\infty}^{\infty} x\,f(x)\,dx\right)$.

$$V(ax+by) = E\left((ax+by)^2\right) - \left[E(ax+by)\right]^2$$

$$= \int_{-\infty}^{\infty}(ax+by)^2 f(x+y)\,dy\,dx - \left[\int_{-\infty}^{\infty}(ax+by)f(x+y)\,dx\,dy\right]^2$$

$$= \int_{-\infty}^{\infty}(a^2x^2+b^2y^2+2abxy)f(x+y)\,dx\,dy - \left[\int_{-\infty}^{\infty}(ax+by)\,f(x+y)\,dx\,dy\right]^2$$

$$\Rightarrow a^2\int_{-\infty}^{\infty}x^2 f(x)\,dx + b^2\int_{-\infty}^{\infty}y^2 f(y)\,dy + 2ab\int_{-\infty}^{\infty}x\,f(x)\,dx\int_{-\infty}^{\infty}y\,f(y)\,dy - \left[a\int_{-\infty}^{\infty}x\cdot f(x)\,dx + b\int_{-\infty}^{\infty}y\cdot f(y)\,dy\right]^2$$

$$\Rightarrow a^2\int_{-\infty}^{\infty}x^2 f(x)\,dx + b^2\int_{-\infty}^{\infty}y^2 f(y)\,dy + 2ab\int_{-\infty}^{\infty}x\cdot f(x)\,dx\int_{-\infty}^{\infty}y\,f(y)\,dy$$

$$-a^2\left[\int_{-\infty}^{\infty}x\,f(x)\,dx\right]^2 - b^2\left[\int_{-\infty}^{\infty}y\cdot f(y)\,dy\right]^2 - 2ab\int_{-\infty}^{\infty}x\cdot f(x)\,dx\int_{-\infty}^{\infty}y\cdot f(y)\,dy$$

$$= a^2\left[\int_{-\infty}^{\infty}x^2 f(x)\,dx - \left[\int_{-\infty}^{\infty}x\,f(x)\,dx\right]^2\right] + b^2\left[\int_{-\infty}^{\infty}y^2 f(y)\,dy - \left[\int_{-\infty}^{\infty}y\cdot F(y)\,dy\right]^2\right]$$

$$= V(ax+bY) = a^2V(x)+b^2V(Y)$$

(5) Find the following using given pdf

X	-4	-3	-2	-1	0	1	2	3	4
P(X)	0.07	0.05	0.1	0.3	0	0.3	0.15	0.1	0.03

Find (i) $E(X)$.

(ii) $E(2X\pm3)$.

(iii) $E(3X+5)$

(iv) $V(X)$

(v) $V(2X\pm3)$

(i) $E(X) = \sum_{i=-4}^{4} x_i P(x_i)$

$= (-4\times0.07) + (-3\times0.05) + (-2\times0.1) + (-1\times0.3) + (0\times0) + 1(1\times0.3) + (2\times0.15) + 3(0.1) + 4\times0.03$

$E(X) = 0.09$

(ii) $E(2X\pm3) = 2E(X)+3 = 3.18$

$= 2E(X) - 3 = -2.82.$

(iii) $E(3X+5) = 3E(X)+5 = 5.27$

(iv) $V(X) = E(X^2) - (E(X))^2$

$E(X^2) = (16\times0.07) + (9\times0.05) + (4\times0.1) + (1\times0.3) + (1\times0.3) + 4(0.15) + (9\times0.1) + (6\times0.03)$

$E(X^2) = 4.55.$

(v) $V(2X\pm3) = 4V(X)$

$= 4(3.74)$

$= 14.96$

Note: If the fun^n is said to be density function then the total probability is $f(x) = 1$.

Q

$X=x_i$	1	2	3	4	5	6
$P(X=x_i)$	K	3K	5K	7K	9K	11K.

(i) find K.

$$\sum_{i=1}^{6} P(X)=1$$

$$36K=1$$

$$\boxed{K=\frac{1}{36}}$$

(ii) Mean or E(X)

$$E(X)=\sum_{i=1}^{6} x_i P(x_i)$$

$$=1\times\frac{1}{36}+2\times\frac{3}{36}+3\times\frac{5}{36}+4\times\frac{7}{36}+$$

$$5\times\frac{9}{36}+6\times\frac{11}{36}$$

$$=\frac{161}{36}=4.47$$

(iii) Variance $(\sigma^2)=E(X^2)-(E(X))^2$

$$E(X^2)=1^2\times\frac{1}{36}+2^2\times\frac{3}{36}+3^2\times\frac{5}{36}+4^2\times\frac{7}{36}+$$

$$5^2\times\frac{9}{36}+6^2\times\frac{11}{36}$$

$$E(X^2)=21.97.$$

$$[E(X)]^2=19.98$$

$$(\sigma^2)=21.97-19.98$$

$$=1.98.$$

(iv) $SD(\sigma)=\sqrt{\text{Variance}}=\sqrt{1.98}=1.41$

(v) $E(3X+4)=3E(X)+4$

$$=3(4.47)+4$$

$$=$$

(vi) $V(3X+4)=9V(X)$ $\quad (V(kX)=k^2V(X)$, $V(k)=0)$

$$=9(1.98)$$

(vii) $P(X\leq 3)$

$$=P(X=1)+P(X=2)+P(X=3)$$

$$=9K=\frac{9}{36}$$

(viii) $P(X<3)=4K=4/36$

(ix) $P(3<X<6)$

$$\Rightarrow 7K+9K=\frac{16}{36}.$$

Q) If a dice rolled twiced. Define X from $S \to R$. Such that $X(a,b) = a+b$.

$$S = \begin{Bmatrix} (1,1) & (2,1) & (3,1) & (4,1) & (5,1) & (6,1) \\ (1,2) & (2,2) & (3,2) & (4,2) & (5,2) & (6,2) \\ (1,3) & (2,3) & (3,3) & (4,3) & (5,3) & (6,3) \\ (1,4) & (2,4) & (3,4) & (4,4) & (5,4) & (6,4) \\ (1,5) & (2,5) & (3,5) & (4,5) & (5,5) & (6,5) \\ (1,6) & (2,6) & (3,6) & (4,6) & (5,6) & (6,6) \end{Bmatrix}$$

$X(1,1) = 1+1 = 2,$

$X(1,2) = X(2,1) = 3$

$X(1,3) = X(2,2) = X(3,1) = 4.$

$\vdots$

$X(6,6) = 12.$ $\qquad X = \{2,3,4,5,6,7,8,9,10,11,12\}$

Range Space or Spectrum of X.

Find probability mass function.

$X=x_i$	2	3	4	5	6	7	8	9	10	11	12
$P(X=x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

E($\sum P(X) = 1.$

Mean: $2\times\frac{1}{36} + 3\times\frac{2}{36} + 4\times\frac{3}{36} + 5\times\frac{4}{36} + 6\times\frac{5}{36} + 7\times\frac{6}{36} + 8\times\frac{5}{36} + 9\times\frac{4}{36} + 10\times\frac{3}{36} + 11\times\frac{2}{36} + 12\times\frac{1}{36}$

1.944

$E(X) = 7$

Variance :- $V(X) = E(X^2) - E(X)^2$

$E(X^2) = 2^2\times\frac{1}{36} + 3^2\times\frac{2}{26} + 4^2\times\frac{2}{36} + 5^2\times\frac{4}{36} + 6^2\times\frac{5}{36} + 7^2\times\frac{6}{36} + 8^2\times\frac{5}{36} + 9^2\times\frac{4}{36} + 10^2\times\frac{3}{36} + 11^2\times\frac{2}{36} + 12^2\times\frac{1}{36}.$

$= 54.83$

$(E(X))^2 = (7)^2 = 49$

$(\sigma^2) = 54.83 - 49 = 5.83$

when $P(X \le x) > \frac{1}{2}$. Find the min value of x.

$P(X \le 3) = P(X=2) + P(X=3) =$

$= \frac{1}{36} + \frac{2}{36} = \frac{3}{16} \not> \frac{1}{2}$

$P(X \le 4) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{6}{36} \not> \frac{1}{2}$

$P(X \le 5) = \frac{10}{36} \not> \frac{1}{2}$

$P(X \le 6) = \frac{15}{36} \not> \frac{1}{2}$

$P(X \le 7) = \frac{21}{36} > \frac{1}{2}$

So, minimum value of X is 7.

Median: If x is median of a random variable x then $P(X<x) \leq \frac{1}{2}$. Then $P(X>x) \leq \frac{1}{2}$

In particular :- If x is a continous random variable then a point "$x = M$" is said to be median of 'X'.

If $$\int_{-\infty}^{M} f(x)dx = \int_{M}^{\infty} f(x)dx = \frac{1}{2}$$

Here $f(x)$ is the p.d.f of 'x'.

Mode:- A point 'x' is said to be mode of random variable x.

→ If $f(x)$ attains maximum at x. Here $f(x)$ is the p.d.f of X.

$(\because f'(x)=0$ & $f''(x)<0)$
max.

→ p.d.f of a random variable 'X' is given by
$$f(x) = \begin{cases} \frac{1}{2}\sin x & 0 \leq x \leq \pi \\ 0 & \text{else where} \end{cases}$$

Find the mean, mode, median for x & also find $P(0<x<\pi/2)$

$$\text{mean } (\mu) = \int_{-\infty}^{\infty} x \cdot f(x)dx \quad [\because \text{limits } 0, \pi]$$

$$= \int_{0}^{\pi} x \cdot \frac{1}{2}\sin x dx$$

$$= \frac{1}{2}\Big[x(-\cos x) - (1)(-\sin x)\Big]_0^{\pi}$$

$$= \frac{1}{2}\Big[-\pi\cos\pi + 0 + \sin\pi - \sin 0\Big]$$

$$= -\frac{\pi}{2}(-1)$$

$$\text{mean} = \frac{\pi}{2}$$

Mode: $f(x) = \frac{1}{2}\sin x$

$f'(x) = \frac{1}{2}\cos x = 0$

$\cos x = 0 = \cos(2n \pm 1)\frac{\pi}{2}$

$\boxed{x = \frac{\pi}{2}}$ is a critical value.

$f''(x) = \frac{1}{2}(-\sin x)$

At $x = \pi/2$, $f''(x) = \frac{-1}{2} < 0$

$\therefore f(x)$ has max at $x = \pi/2$

Median:-

Case 1: $\int_{-\infty}^{M} f(x)dx = \frac{1}{2}$

$\Rightarrow \int_{0}^{M} \frac{1}{2}\sin x\, dx = \frac{1}{2}$

$(-\cos x)_0^M = 1$

$-\cos M + \cos 0 = 1$

$-\cos M + 1 = 1$

$\cos M = 0 = \cos \pi/2$

$\boxed{M = \pi/2}$

Case 2: $\int_{M}^{\infty} f(x)dx = \frac{1}{2}$

$\int_{M}^{\pi} \frac{1}{2}\sin x\, dx = \frac{1}{2}$

$(-\cos x)_M^{\pi} = 1$

$-\cos\pi + \cos M = 1$

$\cos M = 0$

$M = \pi/2$

Q) $E(X) = 1$, $E(X^2) = 4$. Find the mean and variance of Y, where $Y = 2X - 3$.

$E(Y)$, $V(Y)$

$E(Y) = E(2X-3) = 2E(X) - 3$

$= 2(1) - 3 = -1$

$V(Y) = V(2X-3) = 4V(X) - 0$

$= 4(3)$

$\boxed{V(Y) = 12}$

$[V(X) = E(X^2) - (E(x))^2]$

$V(X) = 4 - 1$

$= 3$

Q) Find the mean and variance of uniform probability distribution given by

$f(x) = \frac{1}{n}$, $x = 0, 1, 2, \ldots, n$.

Given pdf is discrete random variable.

$X = x_i$	0	1	2		n
$f(x)$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	..	$\frac{1}{n}$

$\text{mean}(\mu) = \sum x_i f(x)$

$= 0\times\frac{1}{n} + 1\times\frac{1}{n} + 2\times\frac{1}{n} + \cdots n\times\frac{1}{n}$

$= \frac{1}{n}[1+2+\cdots+n]$

$\mu = \frac{1}{\cancel{n}}\left[\frac{\cancel{n}(n+1)}{2}\right] = \frac{n+1}{2}$.

$V(X) = E(X^2) - (E(X))^2$

$E(X^2) = 0^2 \times \frac{1}{n} + 1^2 \times \frac{1}{n} + 2^2 \times \frac{1}{n} + \cdots n^2 \times \frac{1}{n}$

$= \frac{1}{n}(1^2 + 2^2 + \cdots + n^2) = \frac{1}{\cancel{n}} \frac{\cancel{n}(n+1)(2n+1)}{6}$

$= \frac{(n+1)(2n+1)}{6}$

$V(X) = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$

$= \frac{(n+1)}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$

$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$

$= \frac{n+1}{2}\left[\frac{n-1}{6}\right] \Rightarrow \frac{n^2-1}{12}$.

Q) 2 Dice are thrown. If X is the max of the variables ~~sum of the variables on the faces~~

(i) Find mean, variance.

X = xi	2	~~3~~	4	5	6	~~7~~ 8	9	10	~~11~~	12
P(X=xi)										

X	1	2	3	4	5	6
P(X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

mean $(\mu) = 1 \times \frac{1}{36} + 2 \times \frac{3}{36} + 3 \times \frac{5}{36} + 4 \times \frac{7}{36}$

$+ 5 \times \frac{9}{36} + 6 \times \frac{11}{36}$

$= 4.47$.

$E(X^2)$

~~Variance (σ^2)~~ $= 1^2 \times \frac{1}{36} + 2^2 \times \frac{3}{36} + 3^2 \times \frac{5}{36}$

$+ 4^2 \times \frac{7}{36} + 5^2 \times \frac{9}{36} + 6^2 \times \frac{11}{36}$

$=$ 21.97

16
× 7
112

36
× 11
36
36
396

$V(X) = E(X^2) - (E(X))^2$

$V(X) = 21.97 - (4.47)^2$

$= 1.989$.

Q) $f(x) = \frac{k}{1+x^2}$ for $-\infty < x < \infty$, find k.

$\int_{-\infty}^{\infty} \frac{k}{1+x^2} dx = 1$

$k \cdot \int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = 1 \Rightarrow k(\tan^{-1}(x))_{-\infty}^{\infty} = 1$

$= k[\tan^{-1}(\infty) - \tan^{-1}(-\infty)] = 1$

$= k\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 1$

$K\left(\frac{4\pi}{4}\right)=1$

$K(\pi)=1$

$k=\frac{1}{\pi}$

Q) Find k so that the probability density function of the random variable $(X=f(x))$

$$f(x)=\begin{cases}0 & \text{if } x\leq 0\\ Kxe^{-4x^2} & \text{if } x>0\end{cases}$$

$\Rightarrow$ The probability is 1

$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\cancel{\int_{-\infty}^{0} f(x)\,dx} + \int_{0}^{\infty} f(x)\,dx = 1$

$= \int_{0}^{\infty} K\cdot x\cdot e^{-4x^2}\,dx = 1$ $\qquad 4x^2 = t$

$8x\,dx = dt$

$= K\int_{0}^{\infty} e^{-t}\,\frac{dt}{8} = 1$ $\qquad x\,dx = \frac{dt}{8}$

$= \frac{K}{8}\int_{0}^{\infty} e^{-t}dt = 1$ $\qquad = \frac{-K}{8}\left[e^{-\infty}-e^{0}\right]$

$= \frac{K}{8}=1 \Rightarrow k=8$

Q) The length of time that the certain lady speaks to be found in the random phenomenon. If the probability function is given by $f(x)=\begin{cases}ke^{-\frac{x}{5}} .. & x\geq 0\\ 0 & \text{elsewhere.}\end{cases}$ (i) Determine the value of K.

(ii) a) what is the probability that the no. of minutes she will talk over the phone is

1) more than 10 mins $\rightarrow 0.3678$

2) <5 mins $\rightarrow 0.6321$

3) $5<x<10 \rightarrow 0.2355$

(i) $\Rightarrow \int_{0}^{\infty} K\cdot e^{-x/5}\,dx = 1$ $\qquad \frac{x}{5}=t$

$K\cdot\int_{0}^{\infty} e^{-x/5}dx = 1$ $\qquad x=5t$

$dx = 5dt$

$K\cdot\int_{0}^{\infty} e^{-t}\cdot 5\,dt = 1$

$5K\cdot\int_{0}^{\infty} e^{-t}dt = 1$

$5K\Gamma 1 = K = 1/5$

$$\frac{1}{5}\int_{10}^{\infty} e^{-x/5}\,dx = 1$$

$$\left[e\right]_{10}^{\infty}\ 5$$

→ $f(x)$ → probability density function

$F(X)$ = cumulative density function

$$f(x) = \frac{d}{dx}\left[F(x)\right]$$

$$F(X) = \int_{-\infty}^{x} f(x)\,dx.$$

Q) A c.r.v x has a distribution funⁿ

$$F(x) = \begin{cases} 0, & x \le 1 \\ K(x-1)^4, & 1 < x \le 3 \\ 1, & x > 3 \end{cases}$$

Find (i) $f(x)$ (ii) K

$$\cancel{F(x) = \int_{-\infty}}$$

$$f(x) = \frac{d}{dx}F(x) \qquad f(x) = \frac{d}{dx}(1) = 0$$

$$= \frac{d}{dx}K(x-1)^4$$

$$= K4(x-1)^3$$

$$= 4K(x-1)^3$$

$$f(x) = \begin{cases} 0 & x \le 1 \\ 4K(x-1)^3 & 1 < x \le 3 \\ 0 & x > 3 \end{cases}$$

$$F(x) = \int_1^3 4K(x-1)^3\,dx$$

$$= \cancel{4}K\left[\frac{(x-1)^4}{4}\right]_1^3 = K\left[(x-1)^4\right]_1^3 = 1$$

$$\cancel{\frac{K}{16}\left[(3-1)^4 - 0\right]} \qquad = K\left[(3-1)^4 - 0\right] = 1$$

$$\cancel{\frac{K}{16}[2]^4 \neq K = 1} \qquad = K(16) = 1$$

$$K = 1/16$$

Probability Density Function :- F(x) (or) Cumulative Density function of a random variable X is given by

$$f(x) = \begin{cases} 2kxe^{-x^2} & \text{for } x > 0 \\ 0 & x \leq 0 \end{cases}$$

Determine (i) K (ii) the distribution function for X. F(x) = ?

we have total probability of c.r.v is 1.

i.e $\int_{-\infty}^{\infty} f(x)dx = 1$

$= \int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx = 1$

$= 2K \int_{0}^{\infty} xe^{-x^2}dx = 1$ $x^2 = t$, $2xdx = dt$

$= K \int_{0}^{\infty} e^{-t}dt = 1$

$= K \left[-e^{-t}\right]_{0}^{\infty}$

$= K[0-(-1)] = \boxed{K=1}$

$\therefore f(x) = \begin{cases} 2xe^{-x^2} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$

$F(x) = \int_{-\infty}^{x} f(x)dx = \int_{0}^{x} 2xe^{-x^2}dx$

$= \int_{0}^{x} e^{-t}dt$

$= \left[-e^{-t}\right]_{0}^{x} = \left[-e^{-x^2}\right]_{0}^{x} = -e^{-x^2} + 1$

$= 1 - e^{-x^2}$

Q) A continous random variable X is defined by

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2 & \text{if } -3 \leq x < -1 \\ \frac{1}{16}(6-2x^2), & \text{if } -1 \leq x < 1 \\ \frac{1}{16}(3-x)^2, & \text{if } 1 \leq x \leq 3 \\ 0, & \text{if elsewhere} \end{cases}$$

(1) verify that f(x) is a density function and also find mean of f(x)

Now, we have to prove that $\int_{-\infty}^{\infty} f(x)dx = 1$.

$= \frac{1}{16}\left[\cancel{\int_{-\infty}^{-3} f(x)dx} + \int_{-3}^{-1} f(x)dx + \int_{-1}^{1} f(x)dx + \int_{1}^{3} f(x)dx + \cancel{\int_{3}^{\infty} f(x)dx}\right]$

$$\frac{1}{16}\left[\int_{-3}^{-1}(3+x)^2dx + \int_{-1}^{1}(6-2x^2)dx + \int_{1}^{3}(3-x)^2dx\right]$$

$$\rightarrow \frac{1}{16}\left[\left[\frac{(3+x)^3}{3}\right]_{-3}^{-1} + \left[6x-\frac{2x^3}{3}\right]_{-1}^{1} + \left[\frac{(3-x)^3}{3}\right]_1^3\right]$$

$$\rightarrow \frac{1}{16}\left\{\frac{8}{3}-0 + \left[6-\frac{2}{3}+6-\frac{2}{3}\right] + \left[+\frac{8}{3}\right]\right\}$$

$$\frac{1}{16}\left[\cancel{16}12-\frac{4}{3}\right] \Rightarrow \frac{1}{16}\left[\frac{36+\cancel{20}}{3}\right] \Rightarrow \cancel{\frac{32}{3}}\times\cancel{\frac{1}{16}} = \cancel{\frac{2}{3}}$$

$$\frac{1}{16}\times\frac{16}{3}+\frac{1}{16}\times\frac{32}{3} = \frac{3}{3} = 1$$

$$= \frac{1}{16}\left\{\frac{16}{3}+12-\frac{4}{3}\right\} = \frac{1}{16}\left\{\frac{12}{3}+12\right] = \frac{16}{16} = 1.$$

NORMAL DISTRIBUTION

Q)

$$f(x) = \begin{cases} Kx^2 & 0<x<3 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find cumulative Distribution function $F(x)$

(ii) Find $P(1<x\leq 2)$

$$\Rightarrow \int_{-\infty}^{\infty} f(x)dx = 1.$$

$$\Rightarrow \int_0^3 Kx^2 = 1 \Rightarrow K\left[\frac{x^3}{3}\right]_0^3 = K[9] = 1$$

$$K = \frac{1}{9}$$

(i) $$F(x) = \int_{-\infty}^{x} f(x)dx$$

$$= \int_0^x \frac{x^2}{9}dx = \frac{1}{9}\left[\frac{x^3}{3}\right]_0^x$$

$$= \frac{1}{27}\left[x^3\right] = \frac{x^3}{27}$$

(ii) $$\int_1^2 f(x)dx = \int_1^2 \frac{x^2}{9} \Rightarrow \frac{1}{9}\left[\frac{x^3}{3}\right]_1^2$$

$$\Rightarrow \frac{1}{27}[8-1] = \frac{7}{27}$$

Mean of problem :

$3+x=t$

$x=t-3 \quad dx=dt$

$$\Rightarrow \frac{1}{16}\left[\int_{-3}^{-1} x(3+x)^2dx + \int_{-1}^{1}x(6-2x^2)dx + \int_1^3 x(3-x)^2dx\right]$$

$$= \frac{1}{16}\left[\int_{-3}^{-1} x(9+x^2+6x) + \int_{-1}^{1}(6x-2x^3)dx + \int_1^3 x(9+x^2-6x)dx\right]$$

$$= \frac{1}{16}\left[\int_{-3}^{-1} 9x+x^3+6x^2 + \int_{-1}^{1} 6x-2x^3 dx\right.$$

$$\left. + \int_{1}^{3}(9x+x^3-6x^2)\,dx\right]$$

$$= \frac{1}{16}\left[\left[\frac{9x^2}{2}+\frac{x^4}{4}+\frac{6x^3}{3}\right]_{-3}^{-1} + \left[\frac{6x^2}{2}-\frac{2x^4}{4}\right]_{-1}^{1}\right.$$

$$\left. + \left[\frac{9x^2}{2}+\frac{x^4}{4}-\frac{6x^3}{3}\right]_{1}^{3}\right]$$

$$\frac{1}{16}\left\{\left[\cancel{\left(\frac{9}{2}+\frac{1}{4}-2\right)-\left(\frac{81}{2}+\frac{81}{4}-54\right)}\right]\right.$$

$$\left. + \left[\cancel{3-\frac{1}{2}-\left(3-\frac{1}{2}\right)}\right] + \left[\cancel{\left(\frac{81}{2}+\frac{81}{4}-54\right) - \left(\frac{9}{2}+\frac{1}{4}-2\right)}\right]\right\}$$

$$\frac{1}{16}[0] = 0$$

*NORMAL DISTRIBUTION:

→ A random variable X is said to be normal distribution, if its density function or probability distribution function is given by

$$\boxed{f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

Normal distribution is continuous random variable.

If $-\infty < x < \infty$ & $-\infty < \mu < x$ $\sigma > 0$

where $\sigma \to$ standard deviation

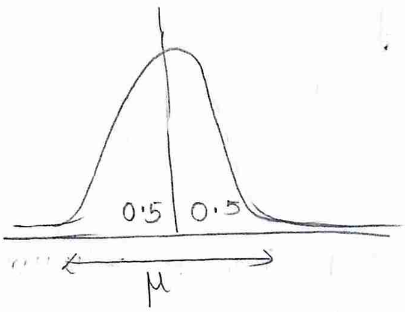
$\mu \to$ mean

*NOTE:

1) Random variable X with μ & variance σ^2 is expressed in normal form is denoted by $X \sim N(\mu, \sigma^2)$

2) A random variable X is then said to be "Normal random variable" (or) "Normal variate"

3) The curve representing normal distribution is called "Normal curve" and total area bounded by curve & x-axis is 1.



i.e. $P(a<x<b)$ = area under normal curve b/w $x=a$ & $x=b$

which is $\int_a^b f(x)\,dx$.

4) Normal distribution is limiting form of binomial distribution.

5) Normal distribution is applicable on following cases:

- Life of items subjected to wear, tear such as lights, bulbs etc.
- Length & diameter of certain products like pipes & screws.
- Height & weight of baby at birth.
- Aggregate marks obtained by student in Sem.
- Weekly sales of items in store.

* Characteristics Of Normal distribution:

1) Graph of normal distribution $y=f(x)$ in XY plane is known as normal curve.

2) The curve is a bell-shaped ~~curve~~ & symmetrical wrt μ(mean) i.e. about line $x=\mu$ & and tails on right & left sides of mean extends to ∞.
The top of bell is directly above μ.

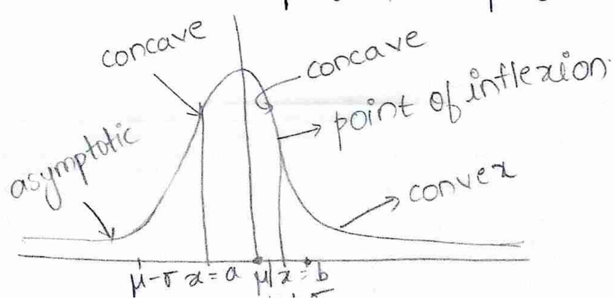
3) Area under normal curve represents total population.

4) Mean, median & mode of distribution coincide at $x=\mu$ as distribution is symmetrical; So normal curve is unimodal

5) x-axis is an asymptotic to curve.

6) Linear combination of independent normal variables is also a normal variate

7) The points of inflexion of curve are at $x=\mu\pm\sigma$ & curve changes from concave to convex at $x=\mu+\sigma$ to $x=\mu-\sigma$.



*Uses of Normal distribution:

1) The normal distribution can be used to approximate poisson distribution.

2) It has extensive use in sampling theory

3) It helps estimate parameter from statistic & confidence limits of parameter

4) It has wide use in testing statistical hypothesis & test of significance in shot always assumed that population from which sample has been drawn should be normal distribution

$$\boxed{f(x, \mu, \sigma) = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

$-\infty < \mu < \infty$ $\quad$ $-\infty < x < \infty$

$\sigma > 0$ $\quad$ $\mu, \sigma \rightarrow$ parameters.

* If x is a normal variate with mean 8 and standard deviation 4 resp. Find probability of $P(5 \le x \le 10)$

Given that $\quad$ mean $= 8 = \mu$

sd $= 4 = \sigma$

We have standard normal variate 'z'

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

$\rightarrow x = 5 \quad z = \frac{5-8}{4} = -\frac{3}{4} = -0.75 = z_1$

$\rightarrow x = 10 \quad z = \frac{10-8}{4} = \frac{2}{4} = 0.5. = z_2$

$P(5 \le x \le 10) = P(-0.75 \le z \le 0.5)$

$= |A(z_2) + A(z_1)|$ $\quad$ A → area

$\boxed{A(z) = A(-z)}$

$A(0.5) = 0.1915$ $\quad$ $A(0.75) = A(-0.75)$

$= 0.2734$

$= |0.1915 + 0.2734|$

$= \underline{\underline{0.4649}}$

$\rightarrow P(10 \le x \le 15)$. $\mu = 8 ; \sigma = 4$

$z = \frac{x-\mu}{\sigma}$

$z_1 = \frac{10-8}{4} = 0.5$ $\quad z_2 = \frac{15-8}{4} = 1.75.$

$P(0.5 \le x \le 1.75)$

$A(0.5) = 0.1915$

$A(1.75) = 0.4599$

$P(10 \le x \le 15) = P(0.5 \le x \le 1.75)$

$= \cancel{0.6514}$

0.2684

* In a sample of 1000 cases; the mean of certain test is 14 and S.D is 2.5. Assuming the distribution to be normal; Find (i) How many students score b/w 12 & 15?

(ii) how many score above 18?

(iii) how many score below 18.

(i) $P(12 \le x \le 15) = ?$

Standard Normal variate (S.N.V)

$z = \frac{x-\mu}{\sigma}$

when $x = 12$ $z = \frac{12-14}{2.5} = \frac{-2}{2.5} = -0.8 = z_1$.

$x = 15 \Rightarrow z = \frac{15-14}{2.5} = \frac{1}{2.5} = 0.4 = z_2$

$P(12 \le x \le 15) = P(-0.8 \le z \le 0.4)$

$= P(z_1) + P(z_2)$

$= 0.2881 + 0.1554$

$= 0.4435 \times 1000 = 443.5$

$= 443$

(ii) $P(x \ge 18)$

Standard Normal variate (S.N.V)

$z = \frac{x-\mu}{\sigma}$

$P(x \le 18)$ $x = 18$; $z = \frac{18-14}{2.5} = \frac{4}{2.5} = 1.6$.

$P(z \ge 1.6) = 0.4452$

$= 0.5 - 0.4452 = 0.0548 \times 1000$

$= 54.8 = 55$

~~(iii) $P(x \le 18)$~~

~~$z = \frac{x-\mu}{\sigma} = \frac{4}{2.5} = 1.6$~~

~~$P(z \le 1.6) = 0.4452 \times 1000 = 445$~~

(iii) $P(x<18) = 1 - P(x>18)$

$= 1 - 0.0548$

$= 0.9452$

0.9452×1000

$= 945$

otherway

$P(x<18) = \frac{18-14}{2.5} = 1.6$

$P(x<18) = 0.5 + A(1.6)$

$= 0.5 + 0.4452$

$= 0.9452 \times 1000 = 945.2$

* Weights of 800 students are normally distributed with $\mu = 140$ pounds and S.D = 10 pounds.

(i) Find no. of students whose weights are:

a) $138 \leq x \leq 140$

b) >152 pounds

c) ≤ 152 pounds

(i) $P(138 \leq x \leq 148)$

$x = 138$ $\quad z = \frac{x-\mu}{\sigma} = \frac{138-140}{10} = \frac{-2}{10} = -0.2$

$x = 148$ $\quad z = \frac{x-\mu}{\sigma} = \frac{148-140}{10} = \frac{8}{10} = 0.8$

$P(138 \leq x \leq 140) = P(-0.2 \leq z \leq 0.8)$

$= P(z_1) + P(z_2)$

$= 0.0793 + 0.2881$

$= 0.3674 \times 800$

$= 294$

(ii) $P(x>152)$

$P(x<152)$

$z = \frac{x-\mu}{\sigma} = \frac{152-140}{10} = 1.2$

$P(x>152) = 0.5 - 0.3849$

$= 0.1151 \times 800 = 92$

(iii) $P(x \leq 152) = 0.5 + P(z > 152)$

$= 1 - 0.1151 = \cancel{0.1151}$

$= 0.8849 \times 800$

$= 707.9 = 708$

* In a normal distribution 35% of the items are under 45 and 8% are over 64. Find the mean & standard deviation:

$P(x<45) = 35\% = 0.35 = 0.5 - 0.35 = 0.15$

$P(x>64) = 8\% = 0.08 = 0.5 - 0.08 = 0.42$

$\cancel{\frac{45-\mu}{\sigma} = 0.35}$

$P(x<45) = 0.15 \rightarrow z = 0.39 = -0.39$ (under)

$P(x>64) = 0.42 \rightarrow z = 1.41 = 1.41$ (above)

$\frac{45-\mu}{\sigma} = -0.39$ $\qquad$ $\frac{64-\mu}{\sigma} = 1.41$

$45-\mu = -0.39\sigma$ $\qquad$ $64-\mu = 1.41\sigma$

$\mu - 0.39\sigma = 45$

$\mu + 1.41\sigma = 64$

$-1.80\sigma = -19$

$\sigma = \frac{19}{1.8}$

$\sigma = 10.5$

$\mu = 49.195$

* $P(x<35) = 10\%$ $\qquad$ $\mu = ?$

$P(x>90) = 5\%$ $\qquad$ $\sigma = ?$

$P(x<35) = 10\% = 0.10 = 0.5 - 0.1 = 0.4$

$P(x>90) = 5\% = 0.05 = 0.5 - 0.05 = 0.45.$

$P(x<35) = 0.4 \rightarrow z = 1.29 = -1.29$ (below)

$P(x>90) = 0.45 \rightarrow z = 1.64$ (above)

$\frac{35-\mu}{\sigma} = -1.29$ $\qquad$ $\frac{90-\mu}{\sigma} = 1.64$

$35-\mu = -1.29\sigma$ $\qquad$ $90-\mu = 1.64\sigma$

$\mu - 1.29\sigma = 35.$

$\mu + 1.64\sigma = 90$

$+2.93\sigma = +55$

$\sigma = \frac{55}{2.93} = 18.7$

$\sigma = 18.7$

$\mu = 59.332$

* If X is normally distributed with mean mean 2 and variance 0.01 then find $P(|X-2| \geq 0.01)$?

Sol: $\mu = 2 \quad \sigma = 0.1$ $\left(\because SD = \sqrt{\text{variance}} = \sqrt{0.01} = 0.1\right)$

$x = 1.99 \qquad z = \frac{x-\mu}{\sigma} = \frac{1.99-2}{0.1} = -0.1$

$x = 2.01 \qquad z = \frac{x-\mu}{\sigma} = \frac{2.01-2}{0.1} = 0.1$

$P(|X-2| < 0.01) = P(1.99 < X < 2.01) =$

$P(-0.1 < z < 0.1) = 2P(0 < z < 0.1) =$

$2 \times 0.0398 = 0.0796$

$P(|X-2| \geq 0.01) = 1 - P(|X-2| < 0.01)$

$= 1 - 0.0796 = 0.9204$