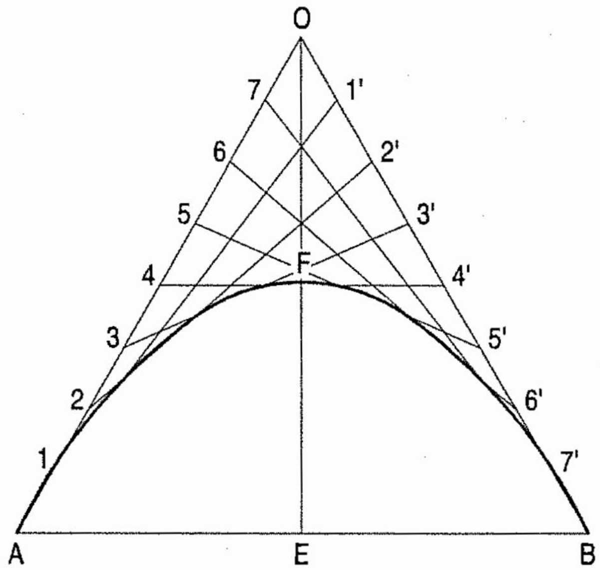
(iii) Join O with A and B. Divide lines OA and OB into the same number of equal parts, say 8.

(iv) Mark the division-points as shown in the figure.

(v) Draw lines joining 1 with 1', 2 with 2' etc. Draw a curve starting from A and tangent to lines 1-1', 2-2' etc. This curve is the required parabola.



Tangent method

FIG. 6-19

6-1-3. HYPERBOLA

Use of hyperbolical curves is made in cooling towers, water channels etc.

Rectangular hyperbola: It is a curve traced out by a point moving in such a way that the product of its distances from two fixed lines at right angles to each other is a constant. The fixed lines are called *asymptotes*.

This curve graphically represents the Boyle's Law, viz. $P \times V = a$, P = pressure, V = volume and a is constant. It is also useful in design of water channels.

General method of construction of a hyperbola:

Mathematically, we can describe a hyperbola by

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \text{ (Fig. 6-20 and fig. 6-21.)}$$

Problem 6-12. (fig. 6-20): *Construct a hyperbola, when the distance of the focus from the directrix is 65 mm and eccentricity is* $\frac{3}{2}$.

(i) Draw the directrix AB and the axis CD.

(ii) Mark the focus F on CD and 65 mm from C.

(iii) Divide CF into 5 equal divisions and mark V the vertex, on *the second division from C.*

Thus, eccentricity $= \dfrac{VF}{VC} = \dfrac{3}{2}$.

To construct the scale for the ratio $\frac{3}{2}$ draw a line VE perpendicular to CD such that $VE = VF$. Join C with E.

Thus, in triangle CVE, $\frac{VE}{VC} = \frac{VF}{VC} = \frac{3}{2}$.

(iv) Mark any point 1 on the axis and through it, draw a perpendicular to meet CE-produced at 1'.

(v) With centre F and radius equal to 1-1', draw arcs intersecting the perpendicular through 1 at P_1 and P'_1.

(vi) Similarly, mark a number of points 2, 3 etc. and obtain points P_2 and P'_2, P_3 and P'_3 etc.

(vii) Draw the hyperbola through these points.

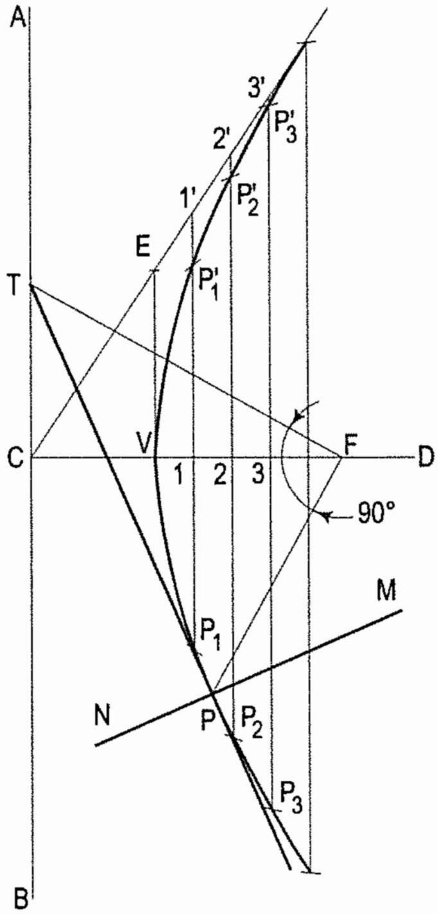


FIG. 6-20

Problem 6-13. (fig. 6-21): *To draw a hyperbola when its foci and vertices are given, and to locate its asymptotes.*

(i) Draw a horizontal line as axis and on it, mark the given foci F and F_1, and vertices V and V_1.

(ii) Mark any number of points 1, 2, 3 etc. to the right of F_1.

(iii) With F and F_1 as centres and radius, say $V2$, draw four arcs.

(iv) With the same centres and radius V_12, draw four more arcs intersecting the first four arcs at points P_2. Then these points lie on the hyperbola.

(v) Repeat the process with the same centres and radii $V1$ and V_11, $V3$ and V_13 etc. Draw the required hyperbola through the points thus obtained.

(vi) With FF_1 as diameter, draw a circle.

(vii) Through the vertices V and V_1, draw lines perpendicular to the axis, cutting the circle at four points A. From O, the centre of the circle, draw lines passing through points A. These lines are the required asymptotes.

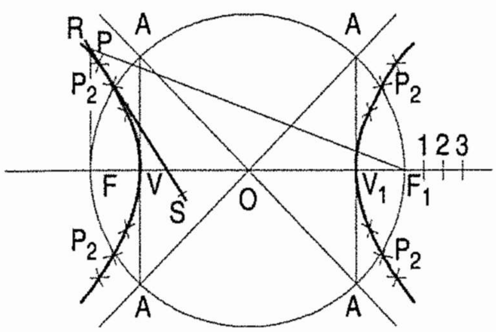


FIG. 6-21

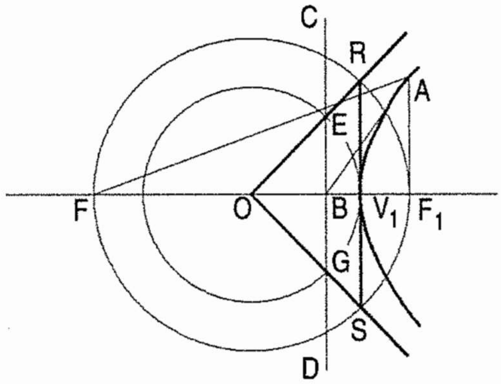


FIG. 6-22

Problem 6-14. (fig. 6-22): *To locate the directrix and asymptotes of a hyperbola when its axis and foci are given.*

From the focus F_1, draw a perpendicular to the axis intersecting the hyperbola at a point A.