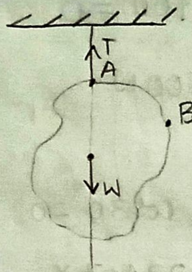
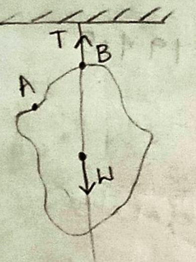
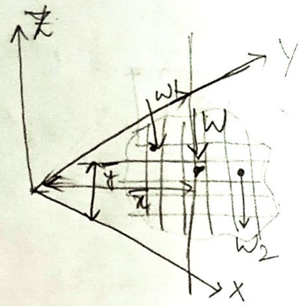
17/02/2022

CENTROIDS



* Centroid is a point where total weight of the body is concentrated.





$$W\times\bar{x} = W_1x_1+W_2x_2+W_3x_3+\dots$$

$$W\times\bar{y} = W_1y_1+W_2 y_2+W_3y_3+\dots$$

$$W=(A\times t)\,\rho.$$

$$(A\times t)\times\rho\times\bar{x} = (A_1\times t)\rho\times\bar{x}_1 + (A_2\times t)\rho\times\bar{x}_2$$

$$A\bar{x} = A_1\bar{x}_1+A_2x_2+\dots$$

$$\boxed{\bar{x} = \frac{A_1\bar{x}_1+A_2\bar{x}_2+\dots}{A}}$$

$$\boxed{\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i}}$$

III^{ly} $\boxed{\cancel{A}\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i}}$

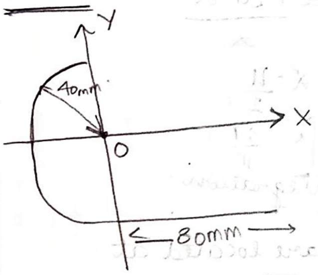
For lines: $W = (a\times L)\times\gamma$.

$$(a\times L)\times\gamma\times\bar{x} = (a\times L_1\times\rho\times x_1) + (a\times L_2\times\rho\times x_2)+\dots$$

$$\boxed{L\bar{x} = L_1\bar{x}_1+L_2\bar{x}_2+\dots}$$

$$\boxed{\bar{x} = \frac{\Sigma L_i x_i}{\Sigma L_i}} \qquad \boxed{\bar{y} = \frac{\Sigma L_i y_i}{\Sigma L_i}}$$

Q:
6-6.1

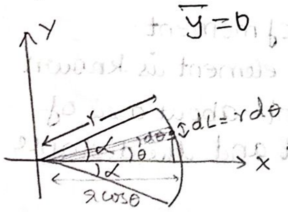


$$\bar{x} = \frac{\Sigma L_i x_i}{\Sigma L_i}$$

=

Element	L_i	x_i
40	$\pi r = 40\pi$	
80	80	

$\bar{y} = 0$



* $L\bar{x} = \int x\,dL$

$$L = \int_{-\alpha}^{\alpha} r\,d\theta = r(2\alpha)$$

$$\boxed{L = 2\alpha r}$$

$\rightarrow$ $L\bar{x} = \int x\,dL$

$$2\alpha r\bar{x} = \int_{-\alpha}^{\alpha} r\cos\theta\; r\,d\theta$$

$$\frac{2\alpha\bar{x}}{r} = \sin\alpha+\sin\alpha$$

$$\boxed{\bar{x} = \frac{r\sin\alpha}{\alpha}}$$

if $r = r$

$\alpha = \frac{\pi}{2}$

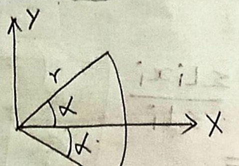
$$\boxed{\bar{x} = \frac{2r}{\pi}}$$

CENTROIDS BY INTEGRATION:

1) $A\bar{x} = \int x \, dA$ 2) $A\bar{y} = \int y \, dA$

3) $L\bar{x} = \int x \, dL$ 4) $L\bar{y} = \int y \, dL$

* CIRCULAR ARC



$\bar{y} = 0$

$\bar{x} = \frac{r \sin \alpha}{\alpha}$

if $\alpha = \frac{\pi}{2}$

$\boxed{\bar{x} = \frac{2r}{\pi}}$

Rules of selecting a strip for integration:

→ All the points of the element are located at same distance from axis of moment.

→ The position of centroid of the element is known so that moment of the element about axis of moment is product of element and the distance of its centroid from the axis.

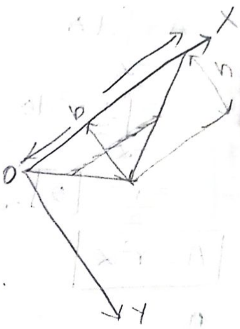
→ Centroid lies on axis of symmetry. If a element has 2 and more lines of symmetry then point of intersection of them will be the centroid.

* Triangle:

for similar triangles

$\frac{h}{b} = \frac{h-y}{x}$

$\boxed{x = \frac{b}{h}(h-y)}$



* $dA = x \, dy$

$\bar{y} A = \int y \, dA$

$\bar{y} \times \frac{1}{2} \times \cancel{b} \times h = \int y \cdot \frac{\cancel{b}}{h}(h-y) \, dy$

$\frac{\bar{y} h}{2} = \frac{1}{h} \int_0^h y(h-y) \, dy$

$= \frac{1}{h} \left[\frac{h y^2}{2} - \frac{y^3}{3} \right]_0^h$

$= \frac{1}{h} \left[\frac{h^3}{2} - \frac{h^3}{3} \right]$

$= h^2 \left[\frac{1}{6} \right] = \frac{h^2}{6}$

$\bar{y} = \frac{h}{6} \times 2$

$\boxed{\bar{y} = \frac{h}{3}}$

The centroid of the triangle wrt base is at a distance $\frac{h}{3}$ from base.

* centroid of a triangle lies at the intersection of medians.

⇒ CENTROID OF CIRCULAR AREA:

$\bar{y} = 0$

$A\bar{x} = \int x \, dA$

$dA = \frac{1}{2}(r\,d\theta)(r)$

$dA = \frac{1}{2} r^2 d\theta$

$A = \int_{-\alpha}^{\alpha} dA$

$= \int_{-\alpha}^{\alpha} \frac{1}{2} r^2 d\theta$

$= \frac{r^2}{2} [\theta]_{-\alpha}^{\alpha}$

$\boxed{A = r^2\alpha}$

$\boxed{A\bar{x} = \int x\, dA}$

$r^2\alpha\,\bar{x} = \int_{-\alpha}^{\alpha} \frac{2r}{3} \cos\theta \cdot \frac{r^2}{2} d\theta$

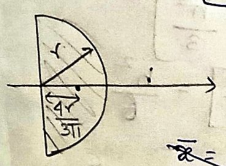
$\bar{x} = \frac{2}{3r\alpha} \cdot \frac{r^2}{2} (+\sin\theta)_{-\alpha}^{\alpha}$

$= \frac{r}{3\alpha} (2\sin\alpha)$

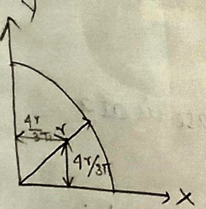
$\boxed{\bar{x} = \frac{2r\sin\alpha}{3\alpha}}$

if $\alpha = \frac{\pi}{2}$

$\boxed{\bar{x} = \frac{4r}{3\pi}}$



$\cancel{\bar{x} = \frac{r}{3\alpha} (\sin\theta)_0^{\pi/2}}$



$A = \frac{r^2}{2} \cdot \frac{\pi}{2} = \frac{\pi r^2}{4}$

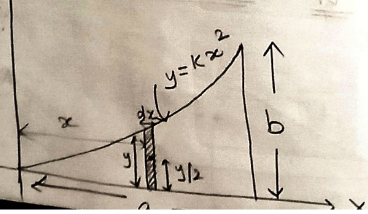
$\bar{x} = \frac{r^3 4}{3\pi r^2} (\sin\theta)_0^{\pi/2}$

$\boxed{\bar{x} = \frac{4r}{3\pi}}$

SEGMENT OF

*. ~~SPANDREL~~:

SECOND DEGREE PARABOLA:



$A\bar{x} = \int x\, dA$

$dA = dx \cdot y$

$dA = Kx^2 dx$

$A = \int_0^a Kx^2 dx$

$\boxed{A = \frac{Ka^3}{3}}$

$\bar{x} = \frac{3}{a^3K} \int_0^a K \cdot x^3 dx$

$= \frac{3K}{Ka^3} \frac{a^4}{4}$

$\boxed{\bar{x} = \frac{3a}{4}}$

By Rule : 2.

$\frac{Ka^3}{3} \bar{y} = \int_0^a y \cdot (y\,dx)$

$y = Kx^2$

$y^2 = K^2x^4$

$= \int_0^a K^2 x^4 dx$

$\bar{y} = \frac{K^2 a^5}{5} \times \frac{3}{Ka^3}$

$\boxed{\bar{y} = \frac{3Ka^2}{5}}$ $\boxed{\bar{y} = \frac{3}{5} Ka^2}$

$\frac{Ka^3}{3} \bar{y} = \int_0^a y \cdot dx \cdot \frac{y}{2}$

$b - \bar{x}$

$a - \frac{3a}{4} = \frac{a}{4}.$

$= \frac{1}{2} \int_0^a K^2 x^4 dx$

$x = a$

$y = Ka^2$

$\frac{Ka^3}{3} \bar{y} = \frac{K^2}{2} \cdot \frac{a^5}{5}$

$\boxed{\bar{y} = \frac{3Ka^2}{10}}$

$\bar{x} = \frac{b}{n+2} = \frac{a}{2+2} = \frac{a}{4}.$

$\bar{y} = \frac{3}{4} b$

$\boxed{\bar{y} = \frac{3}{10} b}$

19/02/2022

*CENTROIDS OF COMPOSITE AREA:

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} \qquad \bar{x} = \frac{\Sigma L_i x_i}{\Sigma L_i}$$

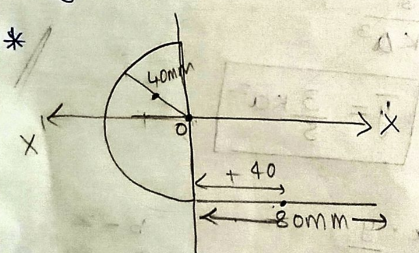
$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} \qquad \bar{y} = \frac{\Sigma L_i y_i}{\Sigma L_i}$$

⇒ Conventional signs:

1) When areas are added, the sign is positive.

2) When areas are subtracted, the sign is negative.

*



Element	L_i	x_i	y_i
	40π	$-\frac{80}{\pi}$	0
———	80	-40	[illegible]

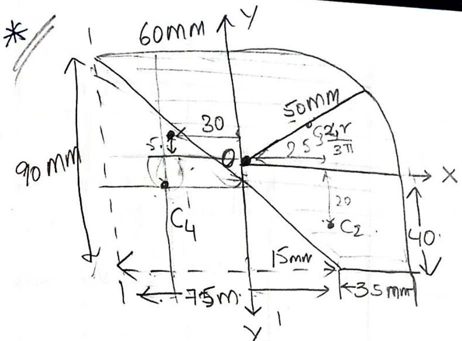
$$\frac{\Sigma L_i x_i}{\Sigma L_i} = \frac{-3200 + 3200}{40\pi + 80} = 0$$

$$\frac{\Sigma L_i y_i}{\Sigma L_i} = \frac{0 - \cancel{3200}\,80}{\cancel{40}(\pi+2)} = \frac{-80}{\pi+2}$$

$\boxed{\bar{X} = 0}$

$\boxed{\bar{Y} = -\frac{80}{\pi+2}}$ = -15.56mm

*



$\frac{4}{3} \times \frac{50}{3.14}$

rectangle = 60 × 90

rectangle = 50 × 40

$\frac{1}{4}$ circle = 50mm

$\triangle$le = b = 75m

h = 90m

A_i	x_i	y_i
5400	-30	5
2000	25	-20
$\frac{50^2 \times \pi}{4}$ = 1962.5	21.23	+21.23
-3375	-35	-10

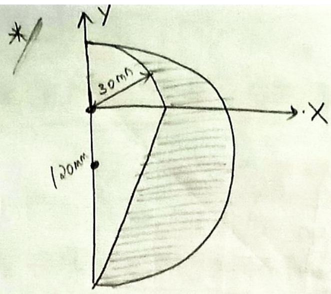
$$\frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{-162000 + 50000 + 41663.875 + 118125}{12737.5}$$

~~$\bar{X} = 16.4701$~~ $\bar{X} = 3.75$

$$\frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{27000 - 40000 + 41663.875 + 33750}{12737.5}$$

$\bar{Y} = 4.9$

*



Element	A_i	
D		

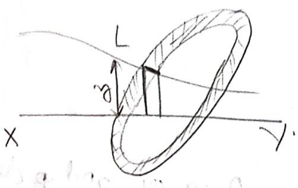
21/02/2022

THEOREM OF PAPPU'S:

Theorem 1: The surface area is the product of length of generating curve multiplied by the distance travelled by its centroid.

Theorem 2: The volume is the product of the area of the figure multiplied by length of path described by the centroid of the area.

Theorem 1



Surface area.

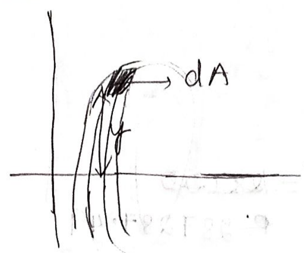
$= \int (2\pi \times y)\, dL$

$= 2\pi \int y\, dL$

$= 2\pi L \bar{y}$

$\boxed{SA = 2\pi \Sigma L_i y_i}$

Theorem 2



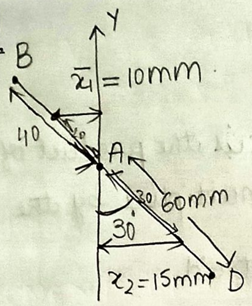
Volume $= \int 2\pi y\, dA$

$= 2\pi \int y\, dA$

$= 2\pi A \bar{y}$

$\boxed{\text{Volume} = 2\pi \Sigma A_i y_i}$

6.7.1



L_i	x_i	$L_i x_i$
40	+10	+400
60	15	+900
		1300

$SA = 2600 \times \pi$

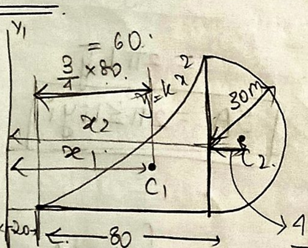
$= 8168.1 mm^2$

***SPANDREL:**

$\bar{x} = \frac{b}{n+2}$ $\bar{y} = \left(\frac{n+1}{n+2}\right)h$

$\boxed{area = \frac{bh}{n+1}}$

*
6-7.2



Area of 2nd degree parabola $= \frac{1}{3}$ of enclosing rectangle

$\frac{4r}{3\pi} = \frac{4 \times \cancel{30}^{10}}{\cancel{3}\pi} = 12.73$

	A_i	x_i
◺	$\frac{1}{3} \times 60 \times 80$ $= 1600$	60+20 =80
D	$\frac{\pi r^2}{2} = 1413$	$+112.73$

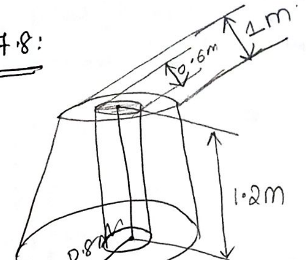
$\Sigma A_i x_i = 287287.49$

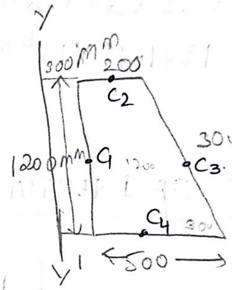
$= 287.87 \times 10^3$

$2\pi \Sigma A_i x_i = 1807.823 \times 10^3$

$V = 1.8 \times 10^6 mm^3$

6-7.8:





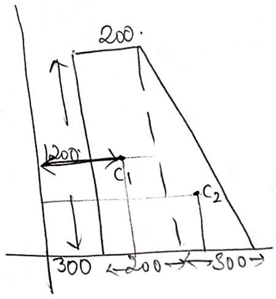
	L_i	x_i
C_1	1200	300
C_2	200	400
C_3	1236.9	300+200+150 = 650
C_4	500	550

$L_i x_i$
360000
80000
803985
275000
1518985

$= 2\pi \Sigma L_i x_i$

$= 9539225.8$

$\boxed{SA = 9.54 \times 10^6 mm^2}$



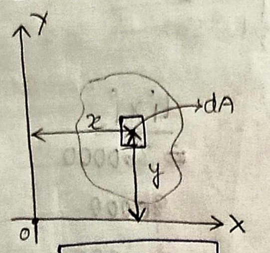
	A_i	x_i
▭	240000	400
◺	$\frac{1}{2} \times 300 \times 1200$ =180000	300+200 +100 =600

$\Sigma A_i x_i = 96 \times 10^6 + 108 \times 10^6$
$= 204 \times 10^6$

$$V = 2\pi \Sigma A_i x_i$$
$$= 6.28 \times 204 \times 10^6$$
$$= 1281.12 \times 10^6 \, mm^3.$$

24/02/2022

* MOMENT OF INERTIA:



$$\boxed{I_{xx} = \int y^2 dA, \quad I_{yy} = \int x^2 dA}$$

$$\boxed{I = \int P^2 dA}$$

P = ⊥ar distance.

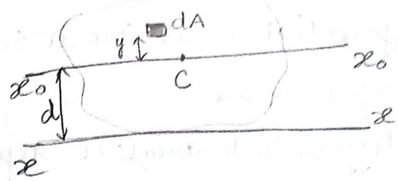
PERPENDICULAR AXIS THEOREM:

The moment of inertia of the point about an axis which is perpendicular to the plane of the point is Polar moment of Inertia.

$$I_{zz} = \int r^2 dA$$
$$= \int (x^2+y^2) dA$$
$$= \int x^2 dA + \int y^2 dA$$
$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

PARALLEL AXIS THEOREM:

If an axis passes through the centroid of the area, it is called centroidal axis.



$$I_{x_0x_0} = \int y^2 dA$$
$$I_{xx} = \int (d+y)^2 dA.$$
$$= \int d^2 dA + \int y^2 dA + \int 2dy dA$$

$$\int = A\bar{y} = A(0) = 0$$

$$\boxed{I_{xx} = I_{x_0x_0} + Ad^2}$$

I_{xx} = Moment of inertia about an axis ∥el to centroidal axis.

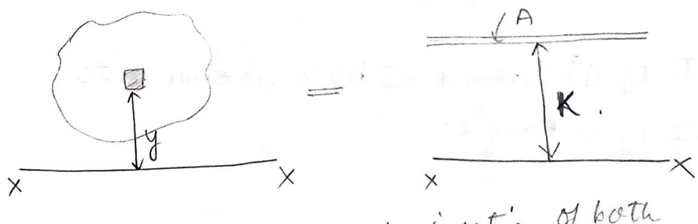
$I_{x_0x_0}$ = centroidal axis

A = area.

d = distance b/w I_{xx} & $I_{x_0x_0}$.

RADIUS OF GYRATION (K):

$$I = \int y^2 dA = \int K^2 dA = K^2 A \Rightarrow \boxed{I = K^2 A}$$



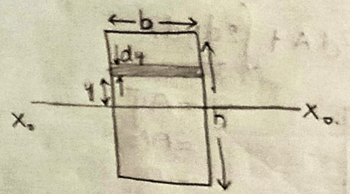
Moment of inertia of both cases are equal.

$$K = \sqrt{\frac{I}{A}}$$

MOMENT OF INERTIA BY INTEGRATION:

→ All parts of differential area are at same distance from reference axis.

→ The MOI of differential area w.r.t reference axis is known. The MOI of the area is then the summation of MOI of its elements.



About centroidal axis X_oX_o:

$$I_{x_ox_o} = \int_{-h/2}^{h/2} y^2\, b\, dy$$

$$= \frac{b}{3}\left(\frac{y^3}{3}\right)_{-h/2}^{h/2}$$

$$= \frac{b}{3}\,\frac{h^3 + h^3}{8}$$

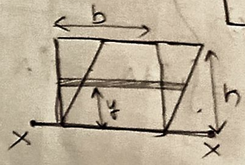
$$\boxed{I = \frac{bh^3}{12}}$$

MOI of rectangle about base:

$$= \int_0^h y^2 (b\,dy)$$

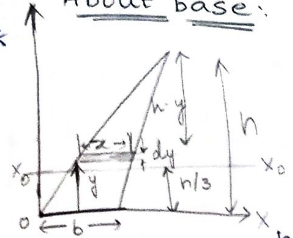
$$= \frac{b}{3}\,\frac{h^3}{1}$$

$$\boxed{I = \frac{bh^3}{3}}$$



MOI of ||gm about its base is equal to MOI of rectangle.

* About base:



$$\frac{b}{h} = \frac{x}{h-y}$$

$$\boxed{x = \frac{b}{h}(h-y)}$$

$$\int_0^h y^2\, dA = \int_0^h y^2 (x\,dy)$$

$$= \frac{b}{h}\int_0^h y^2 (h-y)\, dy$$

$$= \frac{b}{h}\left[\frac{hy^3}{3} - \frac{y^4}{4}\right]_0^h$$

$$= \frac{b}{h}\left[\frac{4h^4 - 3h^4}{12}\right]$$

$$\boxed{I = \frac{bh^3}{12}}$$

About centroidal axis:

$$I_{xx} = I_{x_ox_o} + Ad^2$$

$$\frac{bh^3}{12} = I_{xx_o} + A\frac{h^2}{9}$$

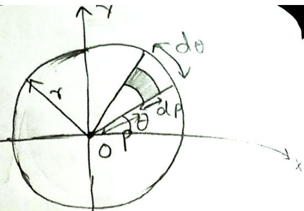
$$\frac{bh^3}{12} = I_{x_ox_o} + \frac{1}{2}\,\frac{b \times h^3}{9}$$

$$I_{x_ox_o} = bh^3\left[\frac{18-12}{18 \times 12}\right]$$

$$= \frac{bh^3 \times \cancel{6}}{18 \times \cancel{12}_{2}}$$

$$\boxed{I_{x_ox_o} = \frac{bh^3}{36}}$$

*CIRCULAR AREA:



$dA = (p\,d\theta) \times dp$

$$= \int_0^{2\pi}\int_0^{r} (p\,d\theta)\,dp\,(p\sin\theta)^2$$

$$= \int_0^{2\pi} \sin^2\theta\, d\theta \left[\frac{p^4}{4}\right]_0^r$$

$$= \frac{r^4}{4}\int_0^{2\pi} \frac{1-\cos 2\theta}{2}\, d\theta$$

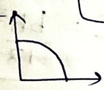
$$= \frac{r^3}{4}\left(\frac{2\pi}{2} - \frac{1}{4}(0)\right)$$

$\boxed{I = \frac{\pi r^4}{4}} \Rightarrow \cancel{I = \frac{\pi r^2}{2}}$

About any diameter.

*For semi circle; about diameter

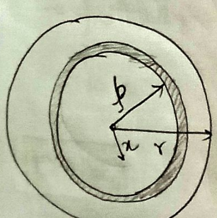
$\boxed{I = \frac{\pi r^4}{8}}$



$\boxed{I = \frac{\pi r^4}{16}}$

$\boxed{I = \frac{\pi r^4}{4}}$

$\boxed{I_{xx} = I_{yy} = \frac{\pi r^4}{4}}$ About diameter



Polar MOI of circular area

$$= \int_0^r p^2 (2\pi p \times dp)$$

$$= \left[\frac{p^4}{4}(2\pi)\right]_0^r$$

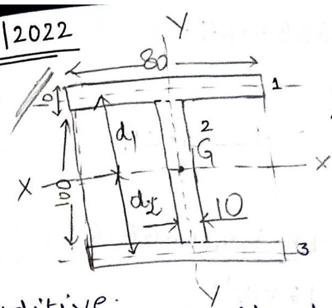
$I_{xx} = I_{yy} = \frac{\pi r^4}{4}$ $\boxed{I = \frac{\pi r^4}{2}}$

$I_{zz} = I_{xx} + I_{yy}$

$\frac{I_{zz}}{2} = I_{xx}$

$\boxed{I_{xx} = \frac{\pi r^4}{4}}$

26/02/2022



* Determine moment of inertia about its centroidal axes XX and YY.

G(40, 60)

*Additive

	Element	Area
1)	80×10	800
2)	10×100	1000
3)	80×10	800

About XX axis

① & ③ $I_{xx} = I_{x_0x_0} + Ad^2$

$I_{x_0x_0} = \frac{bh^3}{12} = \frac{80 \times 1000}{12} = 6666.67$

$d_1 = 55$ units $= d_2$

$(I_{xx})_1 = (I_{xx})_3 = 6666.67 + 800 \times (55)^2 = 2426666.67$

$(I_{xx})_2 = \frac{10 \times 10^6}{12} = 833333.34$

$I_{xx} = 5686666.673$

$= 569 \times 10^4$ units

about YY axis:

	element	area
1)	10×80	800
2)	100×10	1000
3)	10×80	800

① & ③ $I_{xx} = I_{x_0x_0} + Ad^2$

$I_{xx_0} = \frac{10 \times 512 \times 10^3}{12} = 426666.67$

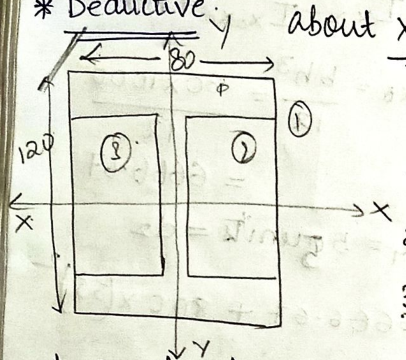
$(I_{xx_1}) = (I_{xx_3}) = 426666.67.$

$I_{xx_2} = \frac{100 \times 1000}{12} = 8333.34.$

$I_{xx} = 434999.94 + 426666.67.$

$= 86.16 \times 10^4$

* Deductive: about X-axis



Element	I_{GG}	I_{GG} (mm⁴)
1) 80×120	$\frac{80 \times (120)^3}{12}$	1152(0000)
2) 35×100	$\frac{35 \times 10^6}{12}$	2916666.17
3) 35×100	$\frac{35 \times 10^6}{12}$	2916666.67

$d_1 = 0 = d_2 = d_3.$

~~$(I_{xx})_2 + (I_{xx})_3 = 2 \times 2916.6$~~

$(I_{xx})_1 = 11520000$

$(I_{xx})_2 + (I_{xx})_3 = 2 \times 2916666.67$

$= 5833333.33.$

$I_{xx} = 5686666.67$

$= 5.6 \times 10^6 \text{ mm}^4$

about Y-axis:

Element	Area	I_{GG}.
1) 120×80	9600	512×10^4
2) 100×35	3500	357291.6
3) 100×35	3500	357291.6

$(I_{yy})_1 = 5120000$

$(I_{yy})_2 + (I_{yy})_3 =$

$= 2\left[357291.6 + 82250\right]$

$= 879083.2.$

$d_2 = d_3 =$

$= 40 - \frac{35}{2}$

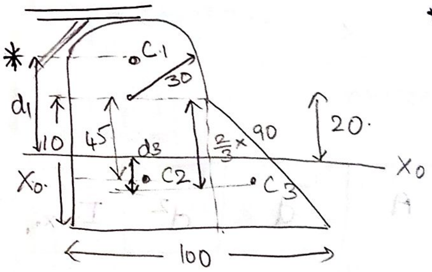
$= 40 - 17.5$

$= 23.5$

$I_{yy} = 59.99 \times 10^5$

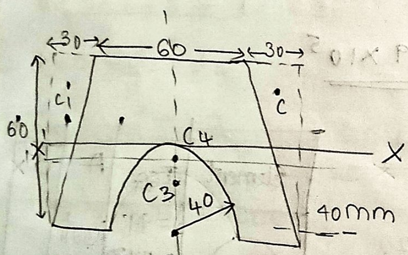
28/02/2022

*



Element	I_{GG}	A	d^2
semicircle	$= 99$ 0.11×30	$\frac{(30)^2 \pi}{2}$	
60, 90	$\frac{60 \times (90)^3}{12}$	60×90	
90, 40	$\frac{40 \times (90)^3}{36}$	$\frac{1}{2} \times 40 \times 90$	

* 7·6-10:



Element	I_{GG}	A	d	d^2	$I_{x_0x_0}$
30, 60	$\frac{30\times(60)^3}{36}$ 18×10^4	$\frac{1}{2}\times30\times60$ 900	0	0	
30, 60	18×10^4	$\frac{1}{2}\times30\times60 = 900$	0	0	
40	$0{\cdot}11\times(40)^4 = 281600$	$\frac{\pi r^2}{2} = 2512$	$d_3 = 60-20 - \frac{4\times40}{3\pi} = 23{\cdot}03$	530·38	
120, 60	$\frac{120\times(60)^3}{12} = 216\times10^4$	7200	$30-20 = 10$	100	

$$① = (I_{x_0x_0})_1 = 180000 + 0$$
$$= 180000 = (I_{x_0x_0})_2$$

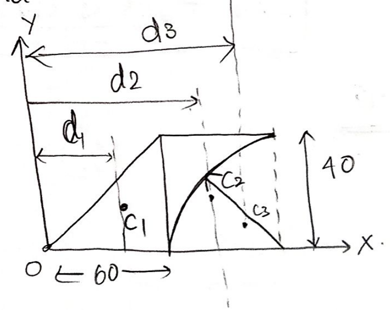
$$③\ I_{x_0x_0} = 281600 + 1332314{\cdot}56$$
$$= 161{\cdot}39\times10^4\,mm^4$$

$$④\ I_{x_0x_0} = 2160000 + 720000$$
$$= 288\times10^4\,mm^4$$

$$I_{xx} = (288 - 197{\cdot}39)\times10^4$$
$$= 90{\cdot}61\times10^4\,mm^4$$

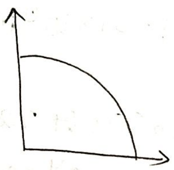
* 7-6·9

Find MOI about Y-axis



Element	I_{GG}	A	d	d^2	$I_{x_0x_0}$
40, 60	$\frac{40\times(60)^3}{36} = 24\times10^4$	1200	$\frac{2}{3}\times 60 = 40$ (20)	1600	
40, 40	$\frac{40\times(40)^3}{12} = 21.3\times10^4$	1600	80	6400	
40	$0.055\times(40)^4 = 140800$	1256	$100-16.9$ 83.014	6891.3	

$I_{xx} = \frac{\pi r^4}{16}$



$\frac{\pi r^4}{16} = I_{GG} + \frac{\pi r^2}{4}\times\left(\frac{4r}{3\pi}\right)^2$

$I_{GG} = \frac{1.57}{8} - \frac{4}{9\pi} r^4$

$= 0.055 r^4$

$(I_{xx_0})_1 =$

$240000 + 1[illegible]000$

$= 216\times10^4$

$(I_{x_0x_0})_2$

$=(21.3 + 1024)\times10^4$

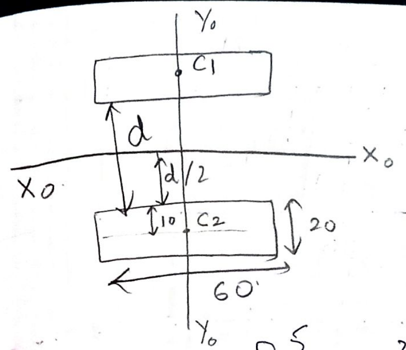
$= 1045.3\times10^4$

$(I_{x_0x_0})_3$

$= 8796272.8$

$\boxed{I_{xx} = 381.6\times10^4 \text{ mm}^4}$

* $I_{x_0} = I_{y_0}$. 7-6-15



$I_{x_0x_0} = 2\left[\frac{\cancel{60}^{5}\times(20)^3}{\cancel{12}} + 60\times20\times\left(\frac{d}{2}+10\right)^2\right]$

$I_{y_0y_0} = 2\left[\frac{\cancel{20}^{10 \; 5}\times(60)^3}{\cancel{12}_{6 \; 3}}\right]$

$I_{x_0x_0} = I_{y_0y_0}$.

$\cancel{2}\left[6\times8\times10^3 + 1200\times\left(\frac{d}{2}+10\right)^2\right]$

$= \cancel{2}\left[\frac{5}{3}\times216\times10^3\right]$

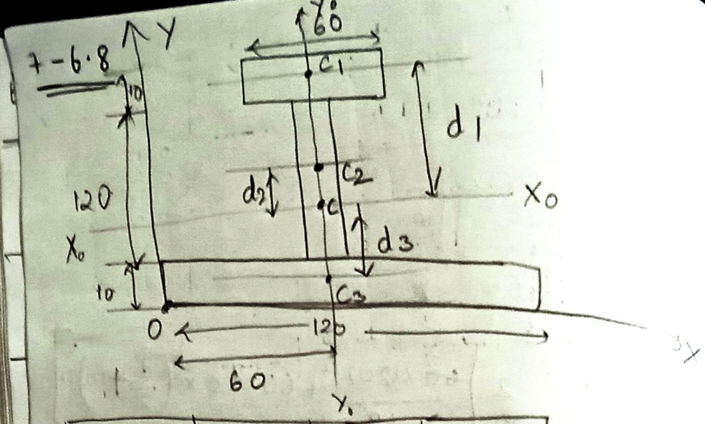
$10^{\cancel{3}\,1}(360 - 48) = 1200\times\left(\frac{d}{2}+10\right)^2$

$\frac{10\,(312)}{12} = 12$

$(260)^{1/2} = \frac{d}{2}+10$

$d = 12.24$ m.

7-6-8



Element	A_i	y_i	$A_i y_i$
60 × 10	600	135	81000
10 × 120	1200	70	84000.
120 × 10	1200	5	60000
	3000		1.71×10^5

$y_1 = 10 + 120 + \frac{10}{2} = 135$

$y_2 = 10 + \frac{120}{2} = 70$

$y_3 = \frac{10}{2} = 5.$

$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{171000}{3000} = 57$

Element	I_{GG}	A	d	d^2	$I_{x_0 x_0}$
60 × 10	$\frac{60 \times 10^3}{12} = 5000$	600	$140 - 5 - 57 = 78$	6084.	365.54×10^4
10 × 120	$\frac{10 \times (120)^3}{12} = 144 \times 10^4$	1200	$\left(10 + \frac{120}{2}\right) - 57 = 13$	169	164.2×10^4
120 × 10	$\frac{120 \times (10)^3}{12} = 10,000$	1200	$57 - \frac{10}{2} = 52.$	2704.	325.4×10^4

$(I_{x_0 x_0})_1 = 5000 + 3650400$
$= 365.54 \times 10^4.$

$(I_{x x_0})_2 = 144 \times 10^4 + 20.2 \times 10^4$
$= 164.2 \times 10^4.$

$(I_{x x_0})_3 = 10000 + 324.4 \times 10^4$
$= 325.4 \times 10^4$

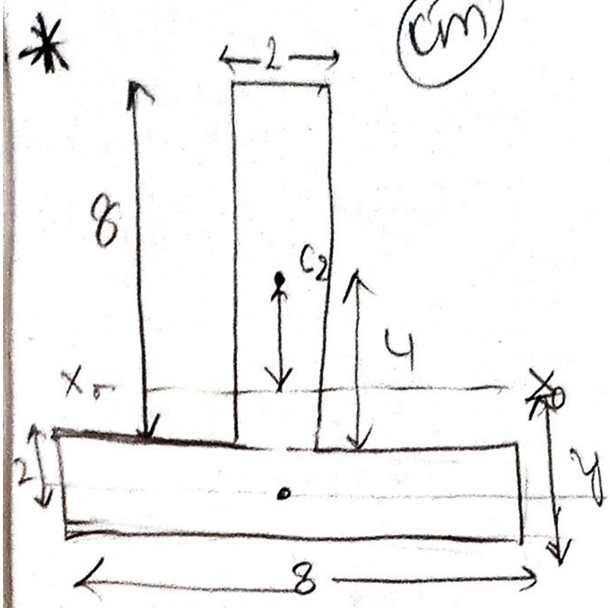
$\boxed{I_{xx} = 8.55 \times 10^6 \ mm^4}$

$I_{Y_0 Y_0}$
18×10^4
10^4
144×10^4

$\Sigma I_{YY} = 163 \times 10^4$
$= 1.63 \times 10^6 mm^4$

∵ all the centroids are on the moment axis YY_0.

*



Element	I_{GG}	A	d
20 × 80	$\frac{20\times(80)^3}{12}$ 853333.33	1600	2.5
20 × 80	$\frac{80\times(20)^3}{12}$ = 53333.3	1600	2.5

Element	A_i	y_i	$A_i y_i$
20 × 80	1600	6	9600
20 × 80	1600	1	1600
	3200		

$$\bar{y} = \frac{112\cancel{00}}{32\cancel{00}}$$

$= 3.5\,cm$

$= 350\,mm$

$d_1^2 = d_2^2 = 6.25cm$

$= 6.25\times10$

$= 62500$

$$(I_{x_0x_0})_1 = (853333.33 + 10^4\times10^2$$

$$= 1853333.3 = 185.33\times10^4$$

$$(I_{x_0x_0})_2 = 53333.3 + 10^6$$

$$= 105.3\times10^4$$

$$I_{xx} = 290.68\times10^4\ mm^4$$