A Model of Image Degradation/Restoration Process

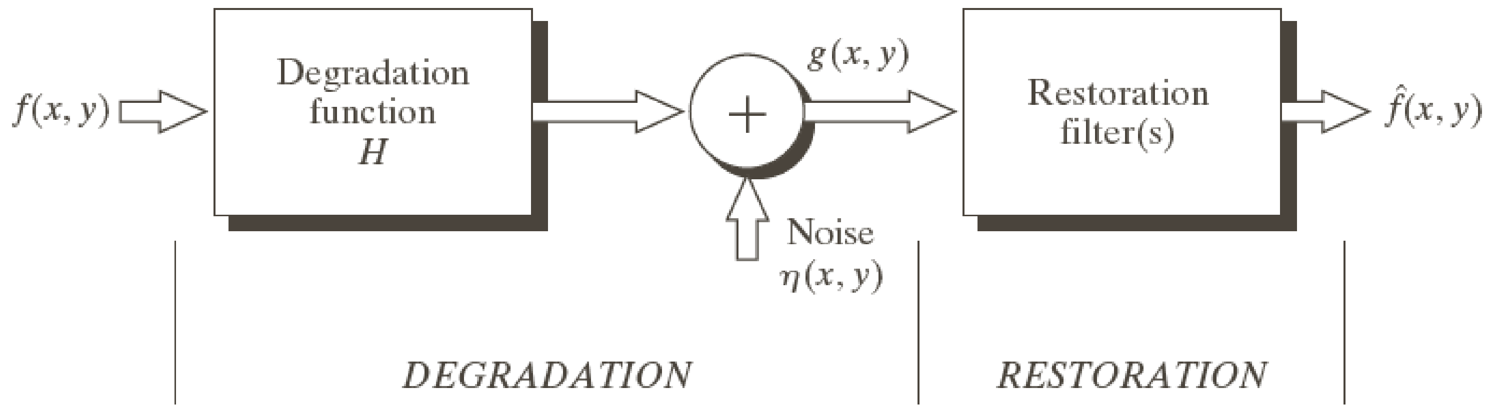


FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- Assuming that H is the identity operator, now we deal only with image degradation due to noise.
- Later we consider a number of important image degradation functions and look at methods for image restoration in the presence of both H and η

Noise Models

- The principle source of noise arise during image acquisition and/or transmission.
- Some important noise PDFs are

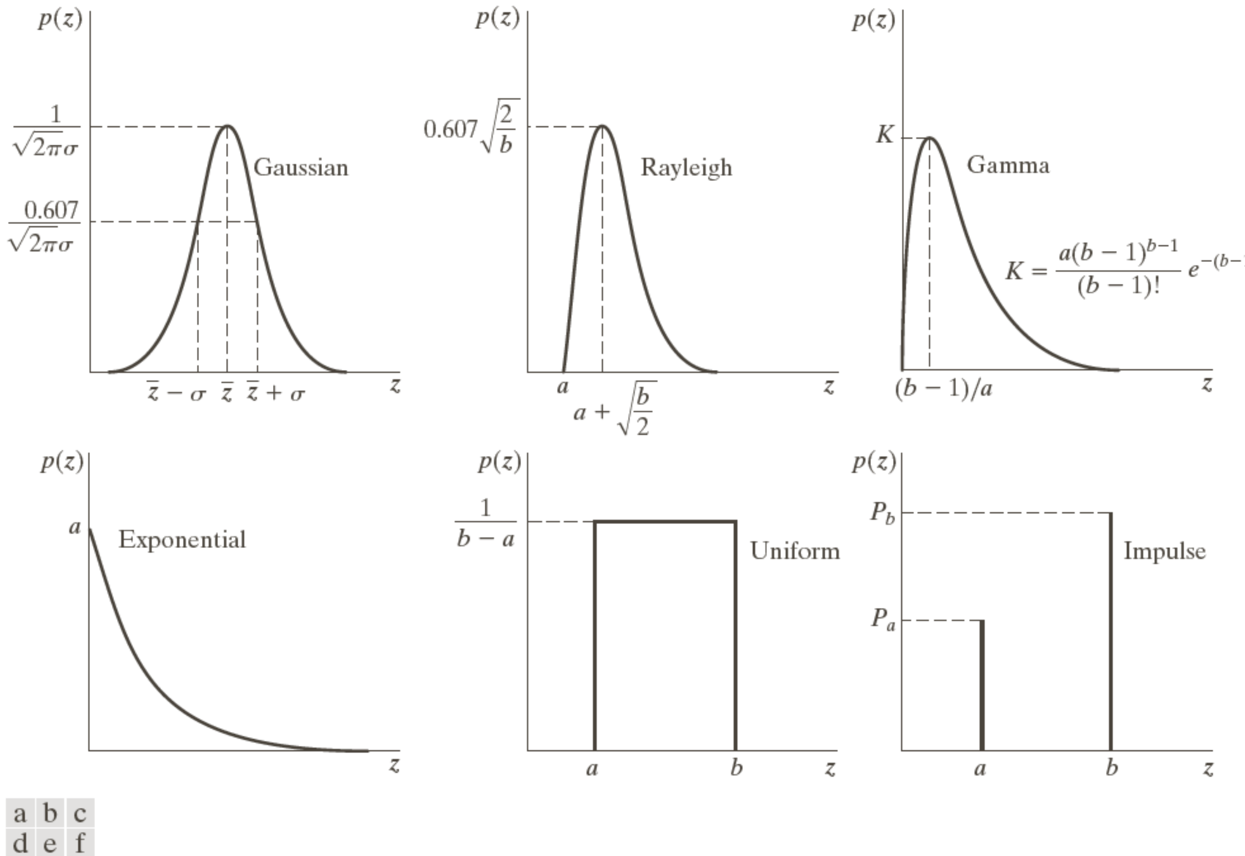


FIGURE 5.2 Some important probability density functions.

Gaussian noise

Because of its mathematical tractability in both the spatial and frequency domains, Gaussian (also called *normal*) noise models are used frequently in practice. In fact, this tractability is so convenient that it often results in Gaussian models being used in situations in which they are marginally applicable at best.

The PDF of a Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2} \tag{5.2-1}$$

where z represents intensity, $\bar{z}$ is the mean† (average) value of z, and σ is its standard deviation. The standard deviation squared, σ^2, is called the *variance* of z. A plot of this function is shown in Fig. 5.2(a). When z is described by Eq. (5.2-1), approximately 70% of its values will be in the range $[(\bar{z} - \sigma), (\bar{z} + \sigma)]$, and about 95% will be in the range $[(\bar{z} - 2\sigma), (\bar{z} + 2\sigma)]$.

Rayleigh noise

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = a + \sqrt{\pi b/4}$$

and

$$\sigma^2 = \frac{b(4-\pi)}{4} \tag{5.2-4}$$

Figure 5.2(b) shows a plot of the Rayleigh density. Note the displacement from the origin and the fact that the basic shape of this density is skewed to the right. The Rayleigh density can be quite useful for approximating skewed histograms.

Erlang (gamma) noise

The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \tag{5.2-5}$$

where the parameters are such that $a > 0$, b is a positive integer, and "!" indicates factorial. The mean and variance of this density are given by

$$\bar{z} = \frac{b}{a} \tag{5.2-6}$$

and

$$\sigma^2 = \frac{b}{a^2} \tag{5.2-7}$$

Exponential noise

The PDF of *exponential* noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \tag{5.2-8}$$

where $a > 0$. The mean and variance of this density function are

$$\bar{z} = \frac{1}{a} \tag{5.2-9}$$

and

$$\sigma^2 = \frac{1}{a^2} \tag{5.2-10}$$

Note that this PDF is a special case of the Erlang PDF, with $b = 1$. Figure 5.2(d) shows a plot of this density function.

Uniform noise

The PDF of *uniform* noise is given by

$$p(z) = \begin{cases} \dfrac{1}{b - a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases} \tag{5.2-11}$$

The mean of this density function is given by

$$\bar{z} = \frac{a + b}{2} \tag{5.2-12}$$

and its variance by

$$\sigma^2 = \frac{(b - a)^2}{12} \tag{5.2-13}$$

Figure 5.2(e) shows a plot of the uniform density.

Impulse (salt-and-pepper) noise

The PDF of (*bipolar*) *impulse* noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \qquad (5.2\text{-}14)$$

If $b > a$, intensity b will appear as a light dot in the image. Conversely, level a will appear like a dark dot. If either P_a or P_b is zero, the impulse noise is called *unipolar*. If neither probability is zero, and especially if they are approximately equal, impulse noise values will resemble salt-and-pepper granules randomly distributed over the image. For this reason, bipolar impulse noise also is called *salt-and-pepper* noise. *Data-drop-out* and *spike* noise also are terms used to refer to this type of noise. We use the terms *impulse* or *salt-and-pepper* noise interchangeably.

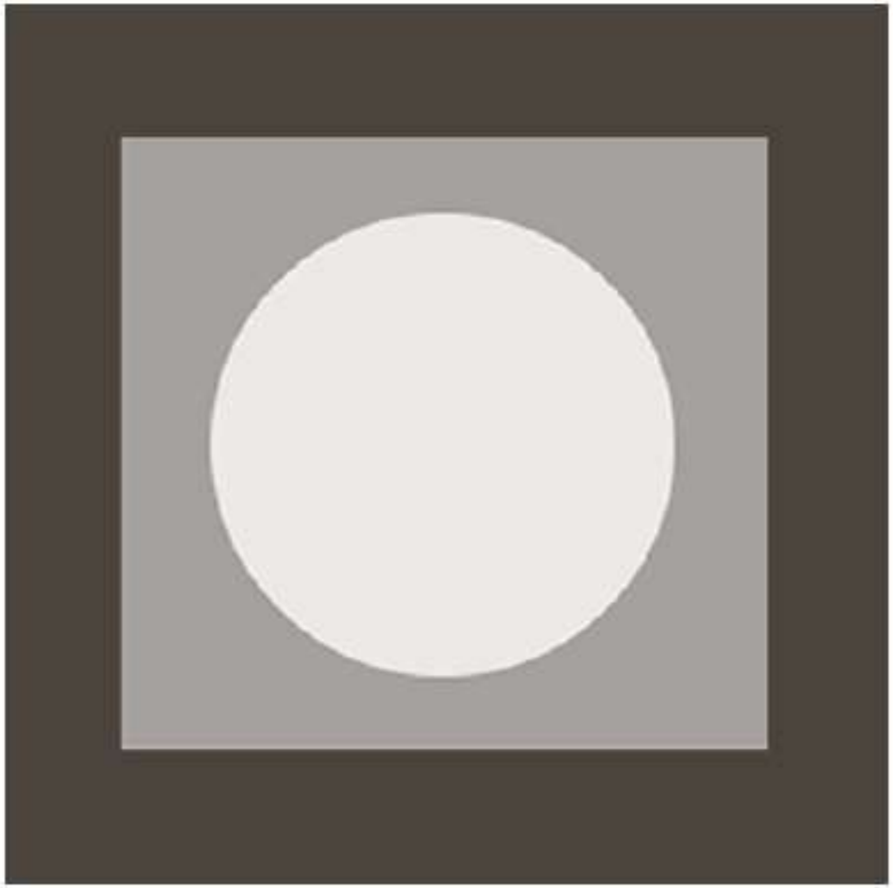


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

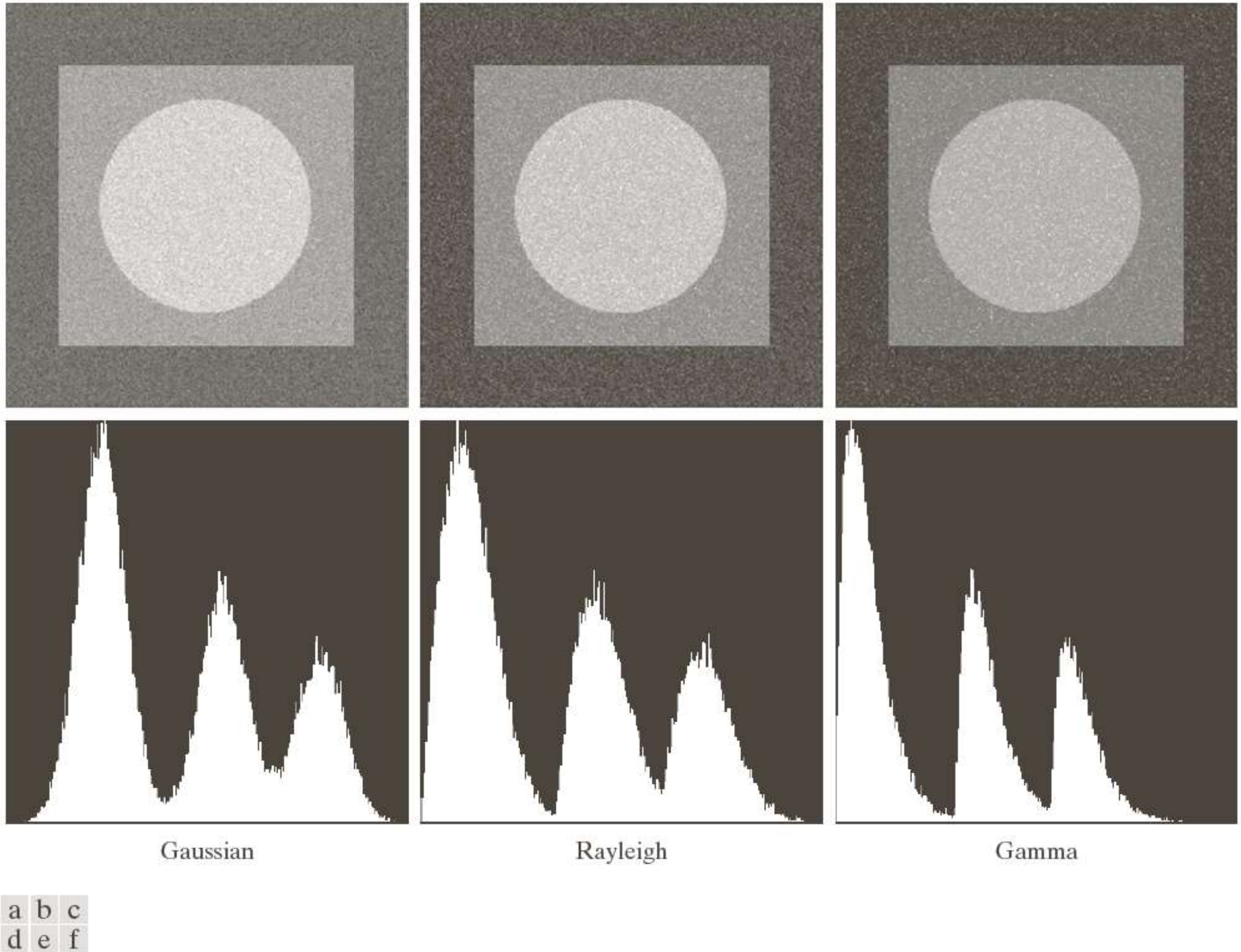


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

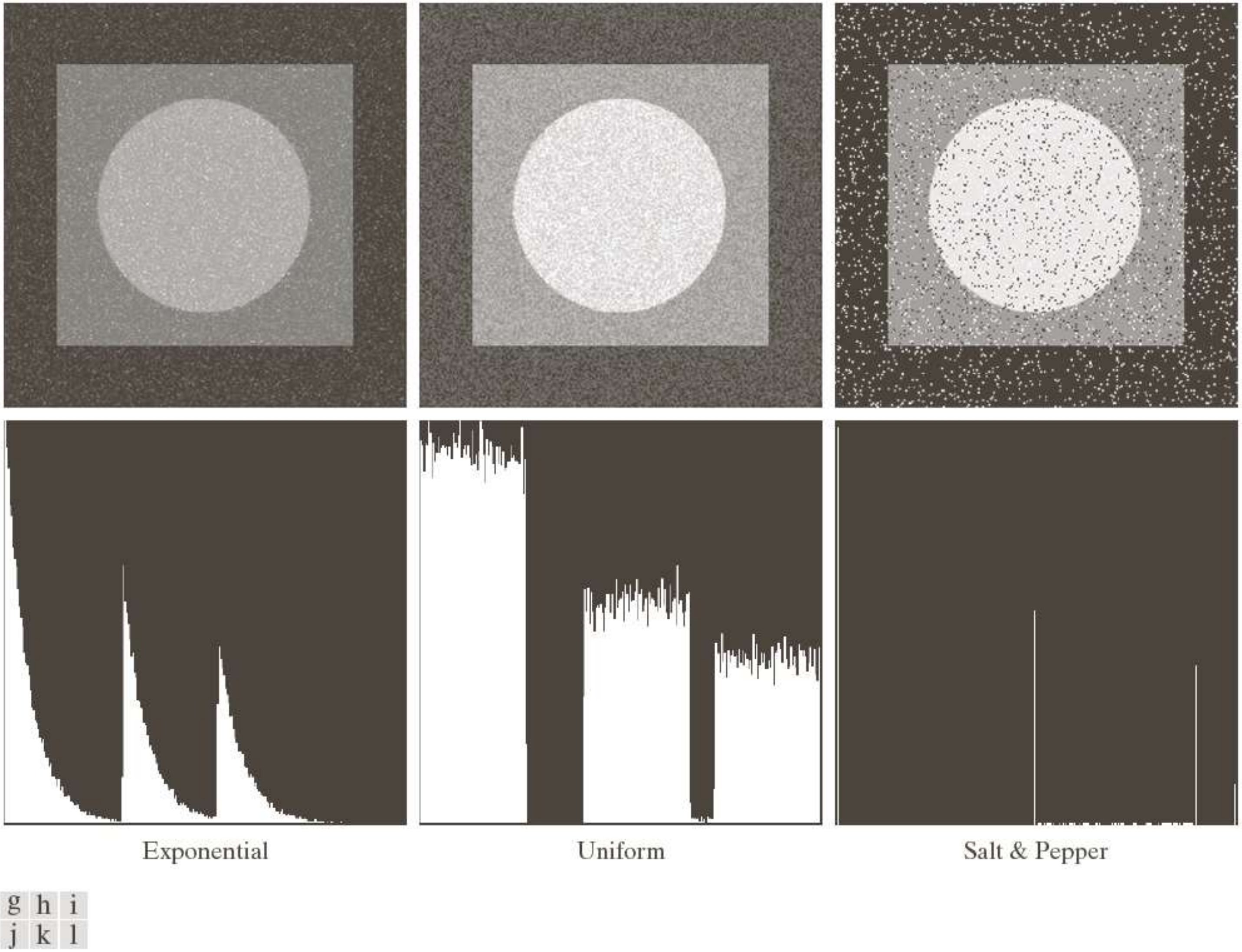
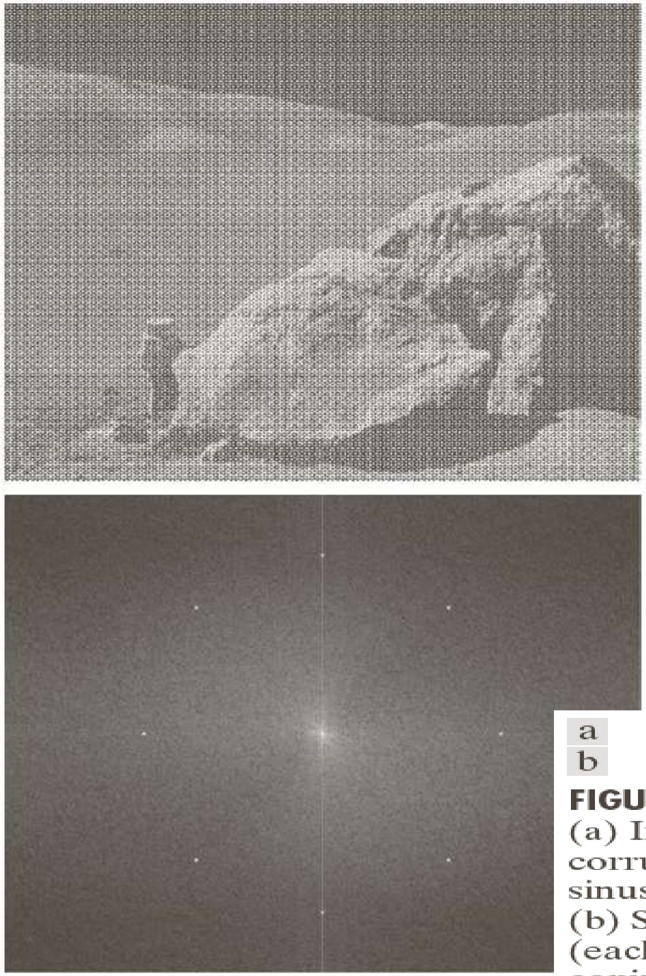


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Periodic Noise

- Periodic noise arises from electrical or electromechanical interference during acquisition.
- It is spatially dependent.



a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Estimation of noise parameters

- when imaging system is available
  - capture a set of images of flat environments(a solid gray board that is illuminated uniformly).
  - The resulting images typically are good indicators of system noise.
- When only images are available
  - Estimate the parameters of the PDF from small patch of reasonably constant background intensity.
  - Histogram is calculated using image data from the small patch.
  - The shape of the histogram identifies the closest PDF match.
  - Calculate the mean and variance of intensity levels from the small patch.

a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Restoration in the presence of Noise Only-Spatial Filtering

- Spatial filtering is the method of choice in situations when only additive random noise is present.
- When the only degradation present in an image is noise, then

$$g(x,y) = f(x,y) + \eta(x,y)$$

and

$$G(u,v) = F(u,v) + N(u,v)$$

- The noise terms are unknown, so subtracting them from g(x,y) or G(u,v) is not a realistic option.

Filters for image restoration in the presence of Noise only

- Mean filters
  - Arithmetic mean filter
  - Geometric mean filter
  - Harmonic mean filter
  - Contraharmonic mean filter
- Order statistic filters
  - Median filter
  - Max filter
  - Min filter
  - Midpoint filter
  - Alpha-trimmed mean filter

Arithmetic mean filter

This is the simplest of the mean filters. Let S_{xy} represent the set of coordinates in a rectangular subimage window (neighborhood) of size $m \times n$, centered at point (x, y). The arithmetic mean filter computes the average value of the corrupted image $g(x, y)$ in the area defined by S_{xy}. The value of the restored image $\hat{f}$ at point (x, y) is simply the arithmetic mean computed using the pixels in the region defined by S_{xy}. In other words,

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t) \qquad (5.3\text{-}3)$$

This operation can be implemented using a spatial filter of size $m \times n$ in which all coefficients have value $1/mn$. A mean filter smooths local variations in an image, and noise is reduced as a result of blurring.

Given a noisy image $g(x, y) = \begin{bmatrix} 5 & 2 & 2 \\ 5 & 0 & 1 \\ 6 & 4 & 2 \end{bmatrix}$.

Obtain the restored image, $\hat{f}(x, y)$, by removing the random noise from the image using the arithmetic mean filter.

Geometric mean filter

An image restored using a *geometric mean* filter is given by the expression

$$\hat{f}(x, y) = \left[\prod_{(s,t)\in S_{xy}} g(s, t) \right]^{\frac{1}{mn}} \tag{5.3-4}$$

Here, each restored pixel is given by the product of the pixels in the subimage window, raised to the power $1/mn$. As shown in Example 5.2, a geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process.

Harmonic mean filter

The *harmonic mean* filtering operation is given by the expression

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s, t)}} \tag{5.3-5}$$

The harmonic mean filter works well for salt noise, but fails for pepper noise. It does well also with other types of noise like Gaussian noise.

Contraharmonic mean filter

The *contraharmonic* mean filter yields a restored image based on the expression

$$\hat{f}(x, y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}} \tag{5.3-6}$$

where Q is called the *order* of the filter. This filter is well suited for reducing or virtually eliminating the effects of salt-and-pepper noise. For positive values of Q, the filter eliminates pepper noise. For negative values of Q it eliminates salt noise. It cannot do both simultaneously. Note that the contraharmonic filter reduces to the arithmetic mean filter if $Q = 0$, and to the harmonic mean filter if $Q = -1$.

Median filter

The best-known order-statistic filter is the *median filter*, which, as its name implies, replaces the value of a pixel by the median of the intensity levels in the neighborhood of that pixel:

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{\operatorname{median}}\{g(s, t)\} \tag{5.3-7}$$

The value of the pixel at (x, y) is included in the computation of the median. Median filters are quite popular because, for certain types of random noise, they provide excellent noise-reduction capabilities, with considerably less blurring than linear smoothing filters of similar size. Median filters are particularly effective in the presence of both bipolar and unipolar impulse noise. In fact, as Example 5.3 below shows, the median filter yields excellent results for images corrupted by this type of noise. Computation of the median and implementation of this filter are discussed in Section 3.5.2.

Max and min filters

Although the median filter is by far the order-statistic filter most used in image processing, it is by no means the only one. The median represents the 50th percentile of a ranked set of numbers, but you will recall from basic statistics that ranking lends itself to many other possibilities. For example, using the 100th percentile results in the so-called *max filter*, given by

$$\hat{f}(x, y) = \max_{(s,t)\in S_{xy}} \{g(s, t)\} \qquad (5.3\text{-}8)$$

This filter is useful for finding the brightest points in an image. Also, because pepper noise has very low values, it is reduced by this filter as a result of the max selection process in the subimage area S_{xy}.

The 0th percentile filter is the *min filter*:

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}} \{g(s, t)\} \qquad (5.3\text{-}9)$$

This filter is useful for finding the darkest points in an image. Also, it reduces salt noise as a result of the min operation.

Midpoint filter

The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by the filter:

$$\hat{f}(x, y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s, t)\} + \min_{(s,t)\in S_{xy}} \{g(s, t)\}\right] \tag{5.3-10}$$

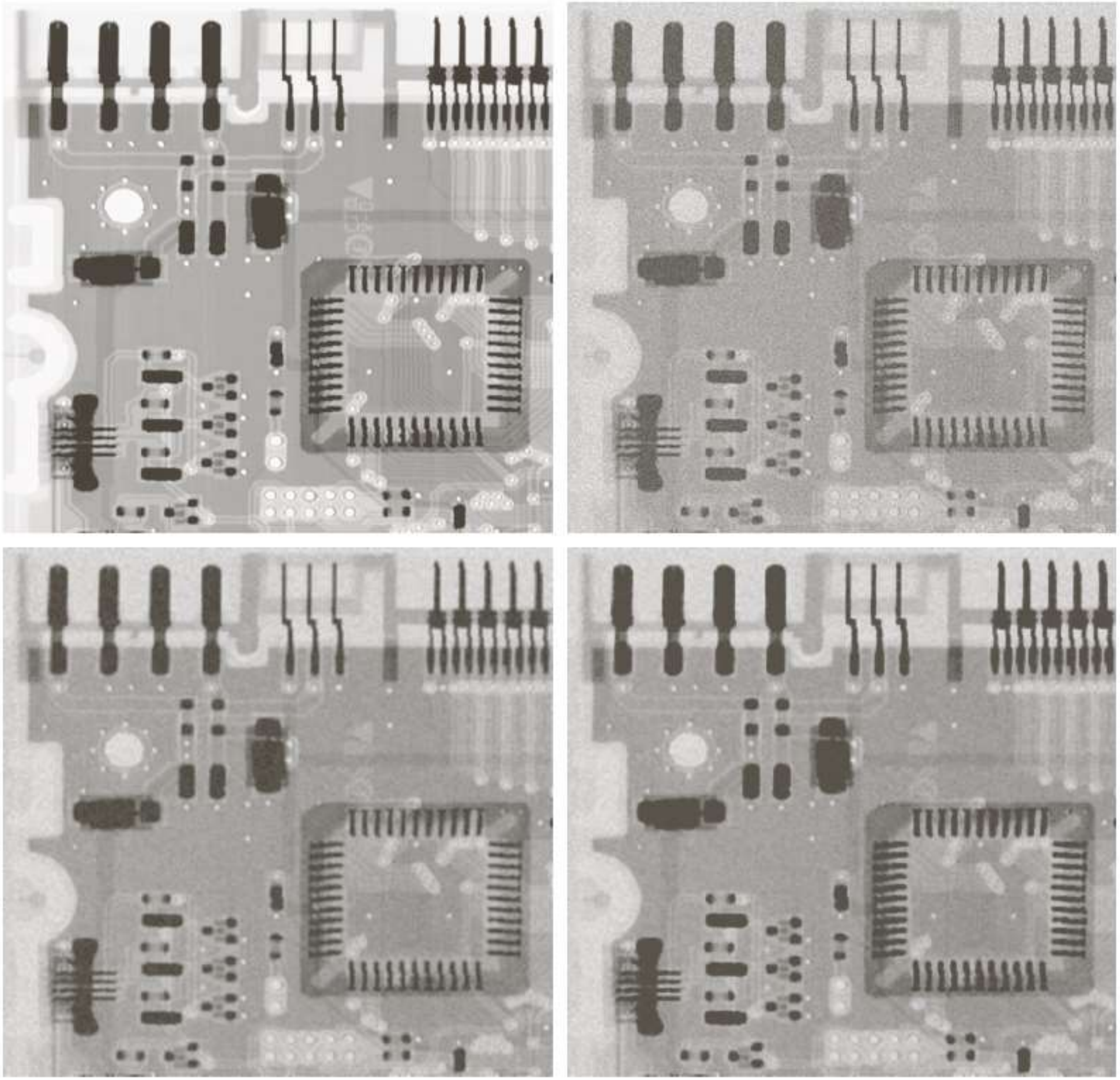
Note that this filter combines order statistics and averaging. It works best for randomly distributed noise, like Gaussian or uniform noise.

Alpha-trimmed mean filter

Suppose that we delete the $d/2$ lowest and the $d/2$ highest intensity values of $g(s, t)$ in the neighborhood S_{xy}. Let $g_r(s, t)$ represent the remaining $mn - d$ pixels. A filter formed by averaging these remaining pixels is called an *alpha-trimmed mean* filter:

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s, t) \in S_{xy}} g_r(s, t) \tag{5.3-11}$$

where the value of d can range from 0 to $mn - 1$. When $d = 0$, the alpha-trimmed filter reduces to the arithmetic mean filter discussed in the previous section. If we choose $d = mn - 1$, the filter becomes a median filter. For other values of d, the alpha-trimmed filter is useful in situations involving multiple types of noise, such as a combination of salt-and-pepper and Gaussian noise.



a b
c d

FIGURE 5.7
(a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

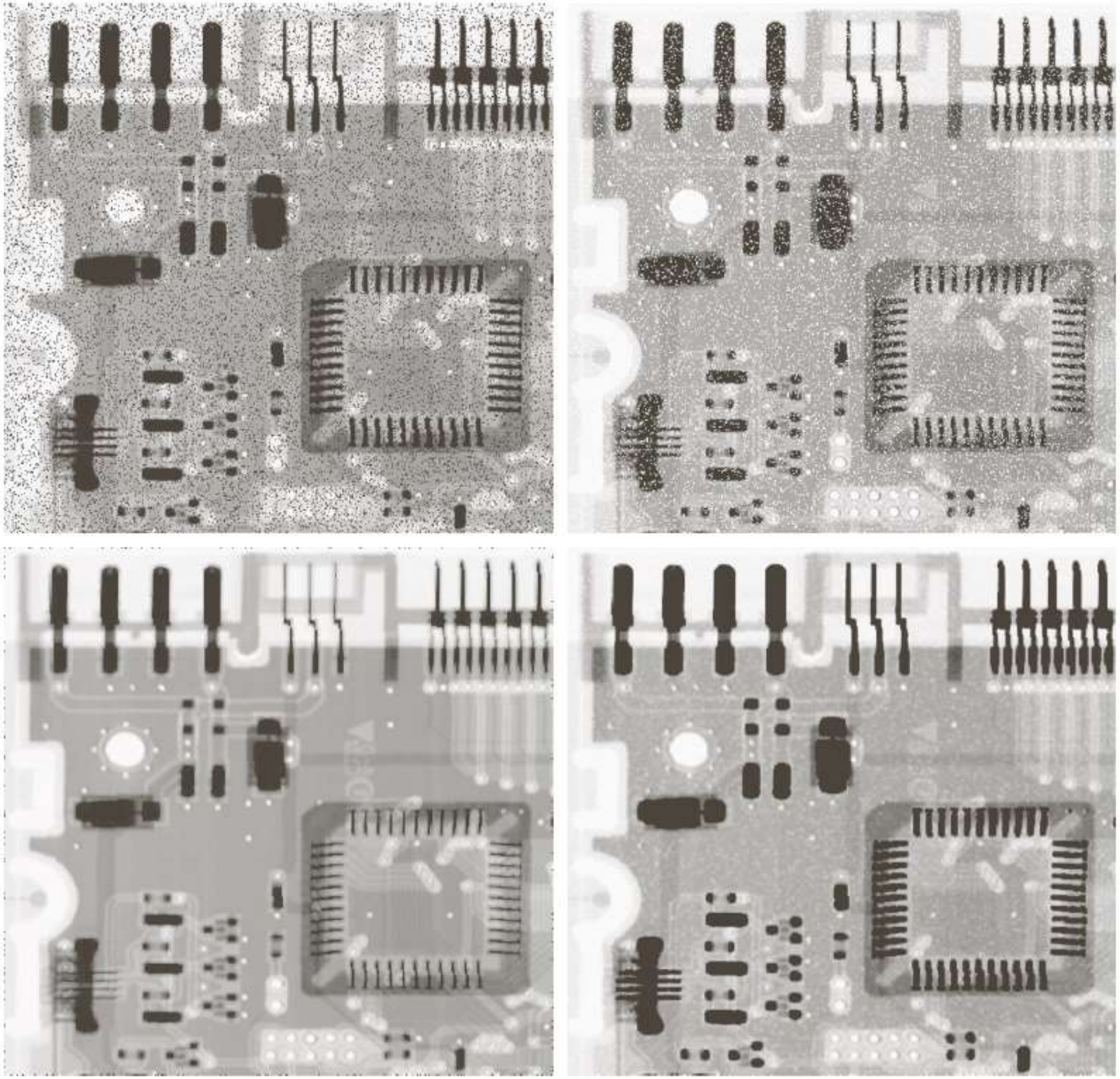
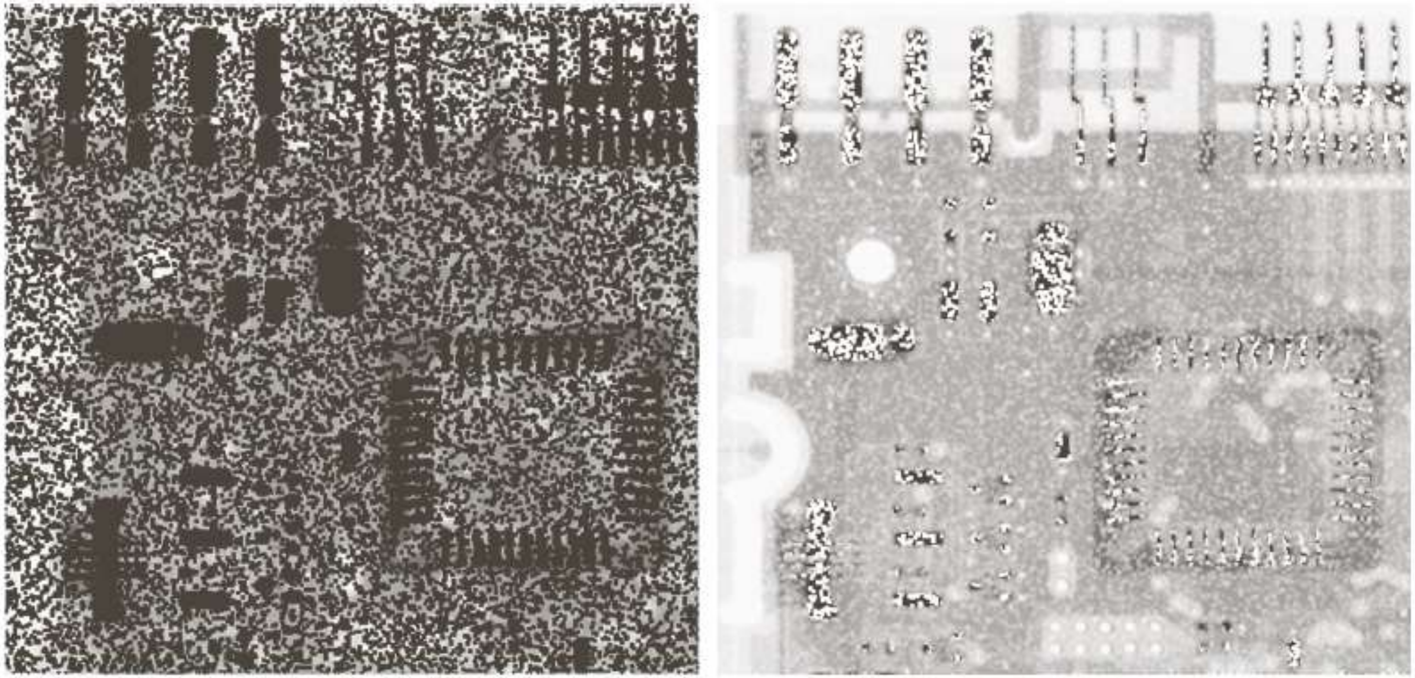
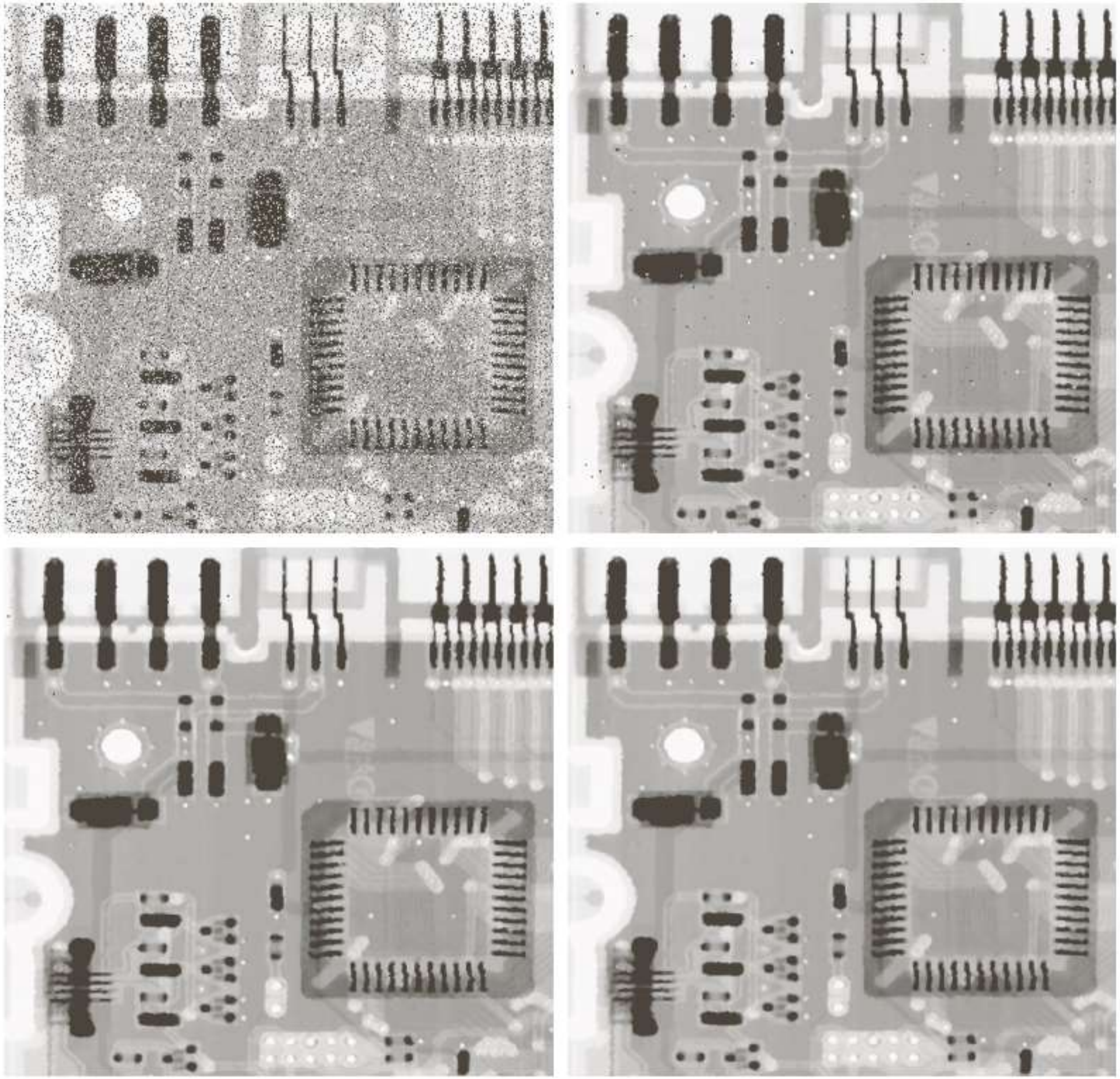


FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.



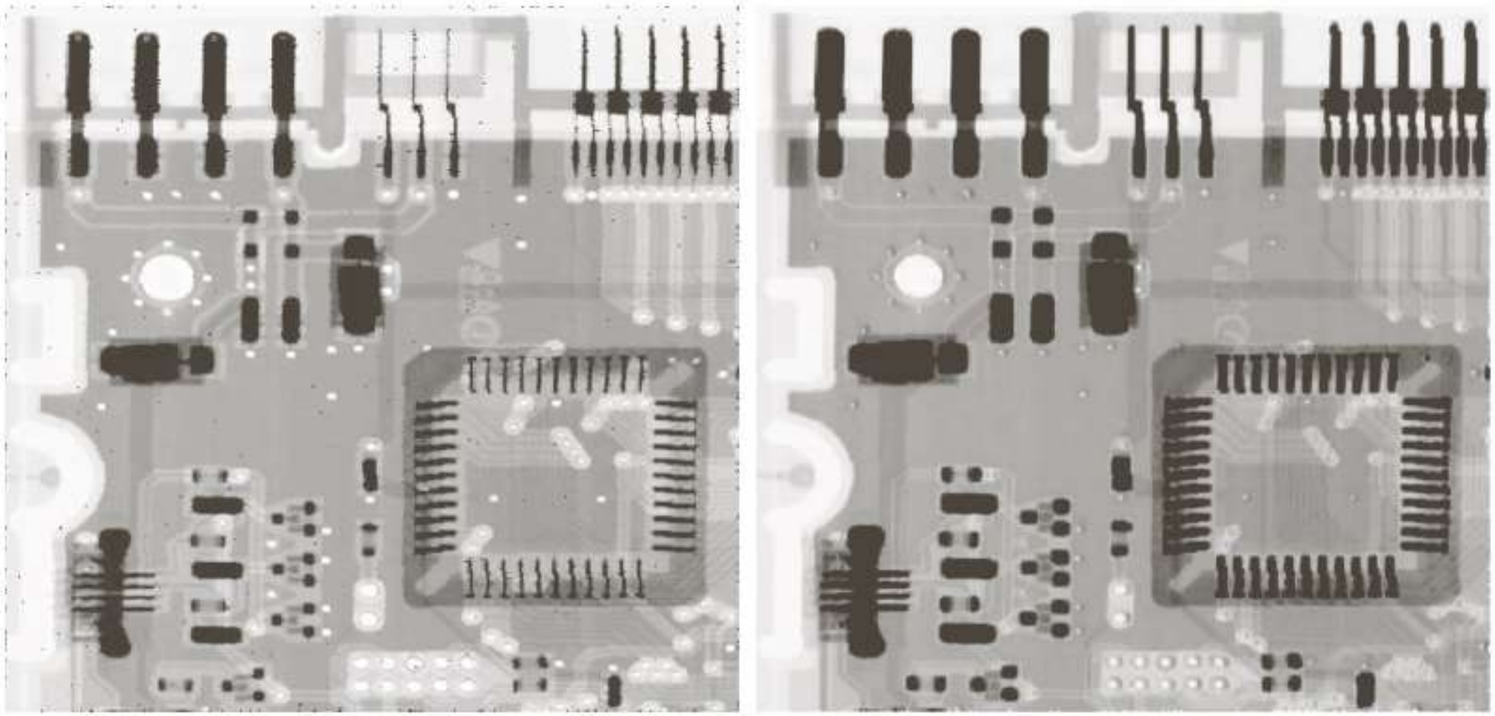
a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.



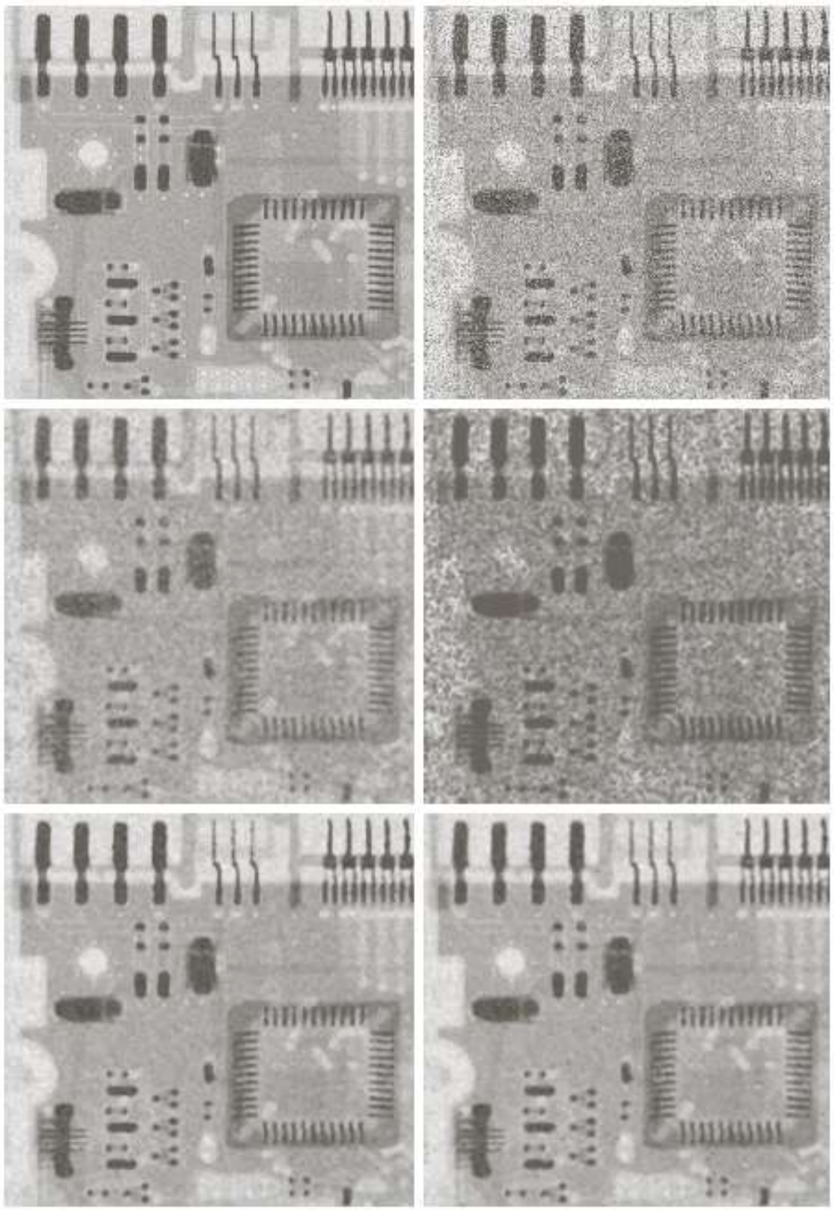
a b
c d

FIGURE 5.10
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size 3×3.
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.



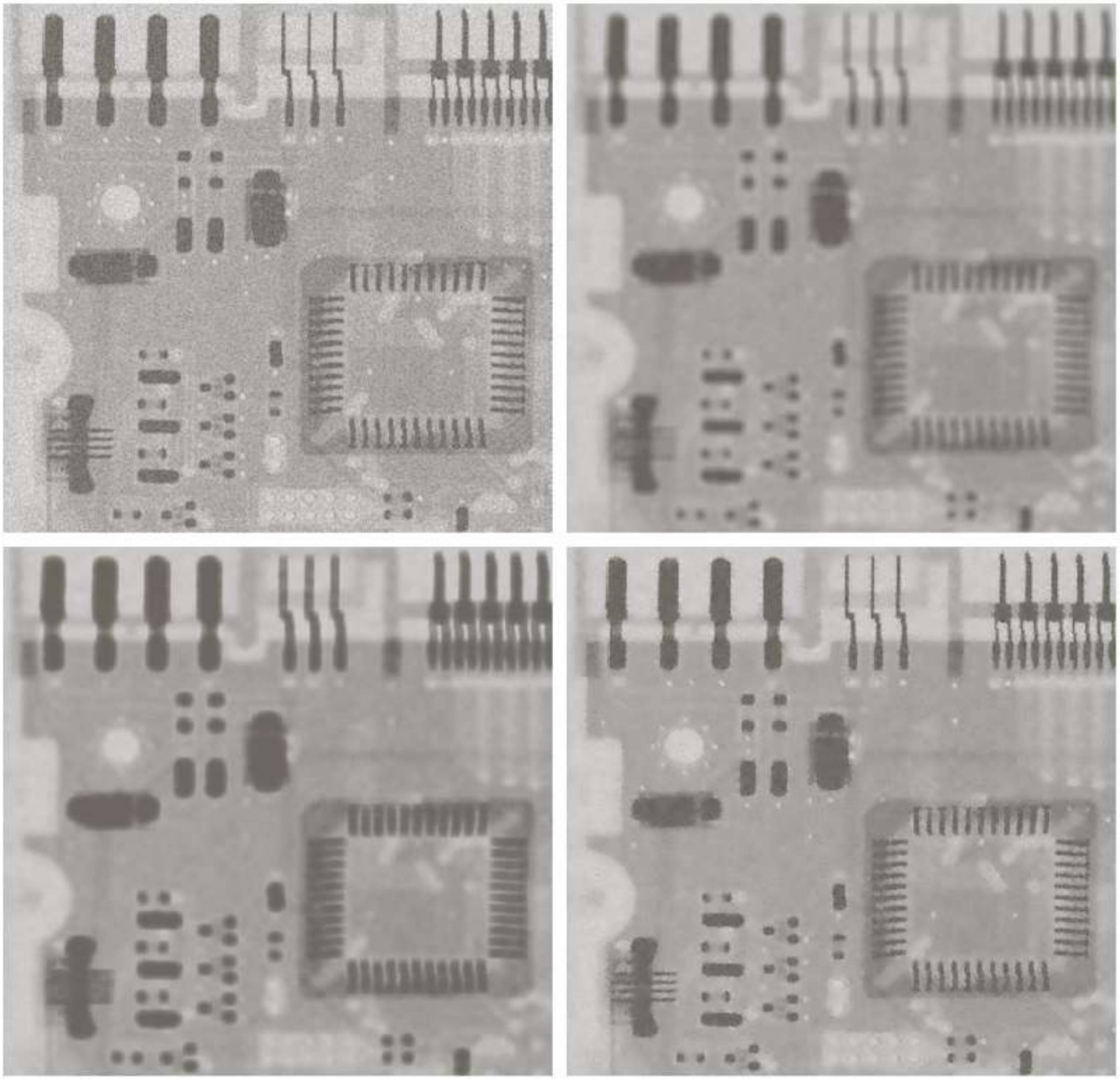
a b

FIGURE 5.11
(a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.



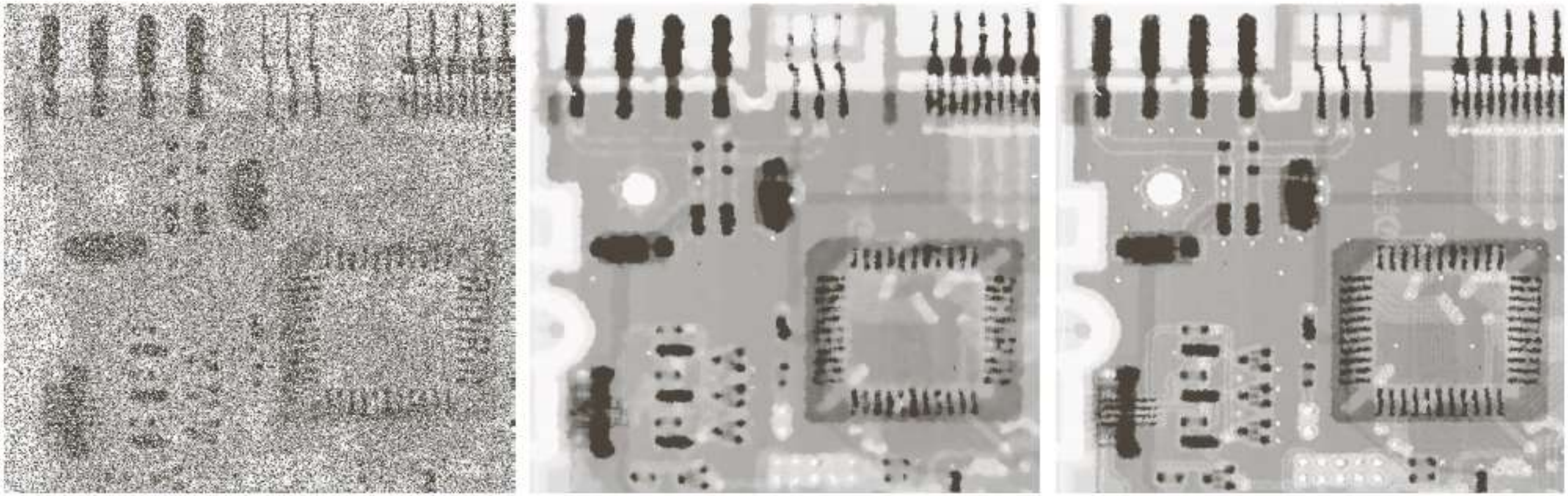
a	b
c	d
e	f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.



a b
c d

FIGURE 5.13
(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000.
(b) Result of arithmetic mean filtering.
(c) Result of geometric mean filtering.
(d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



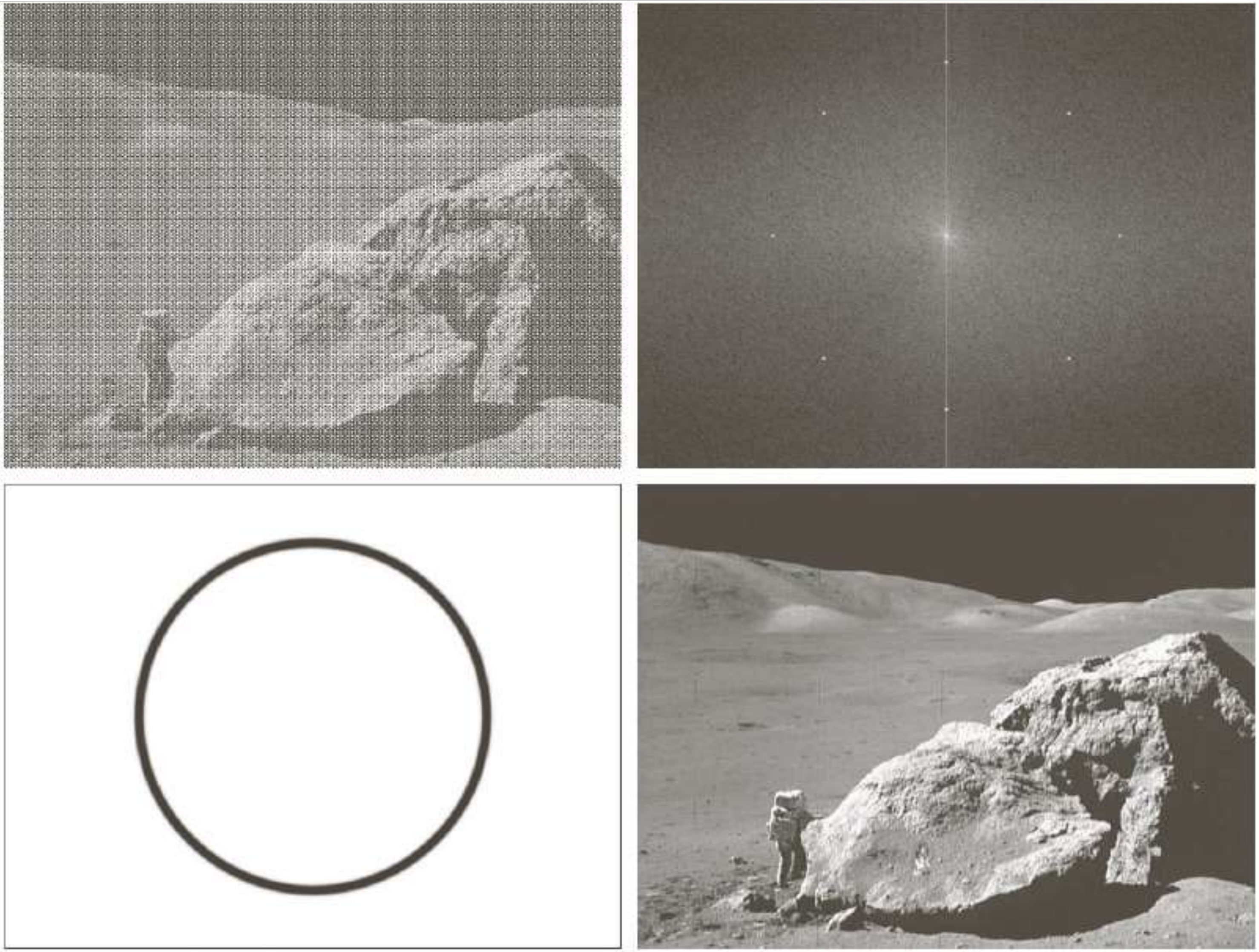
a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

Filters for removing Sinusoidal noise

a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

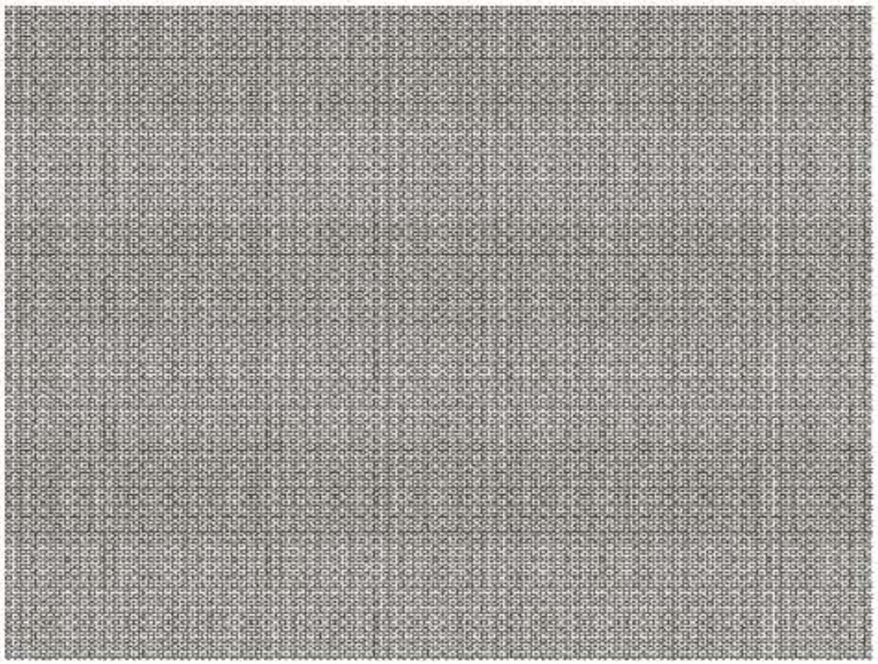
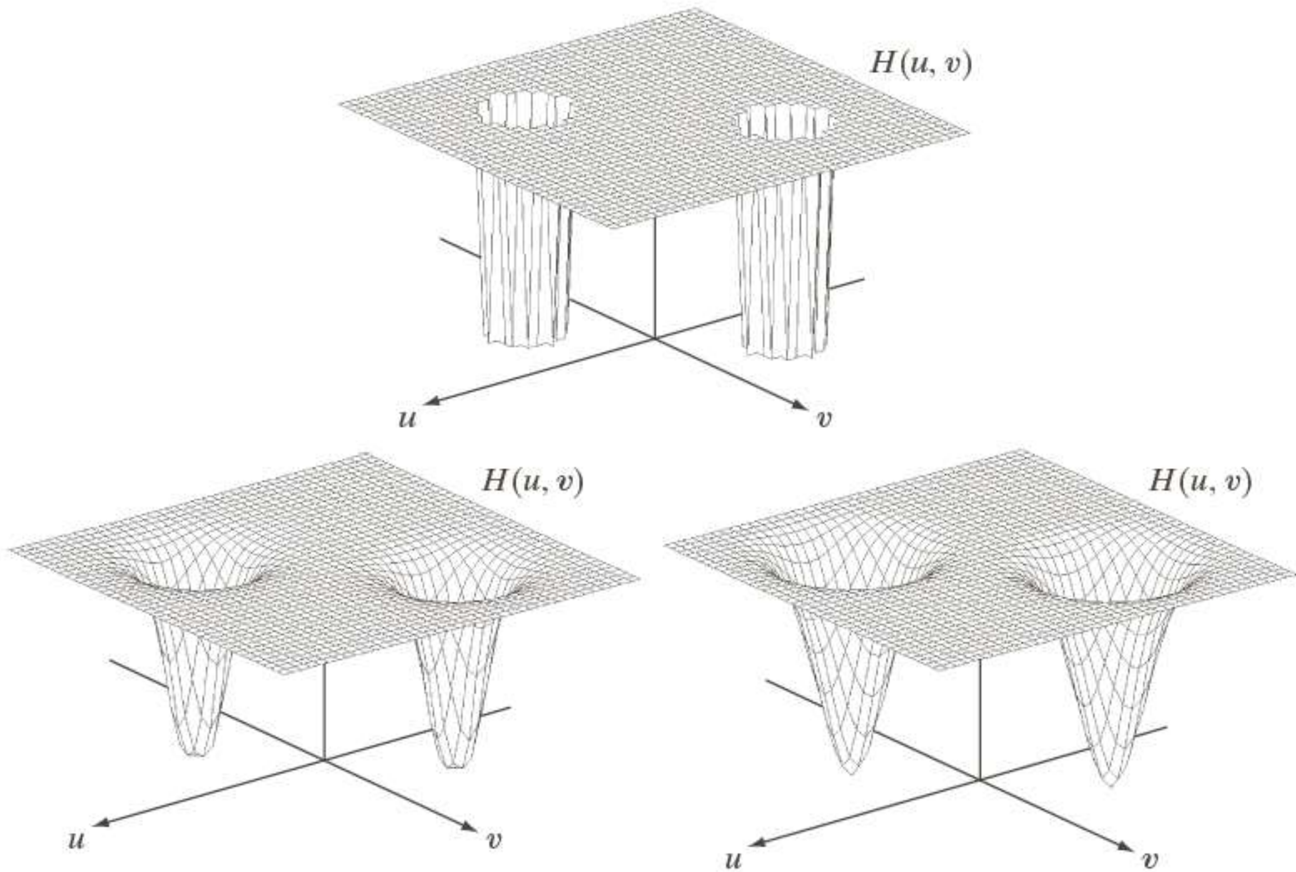
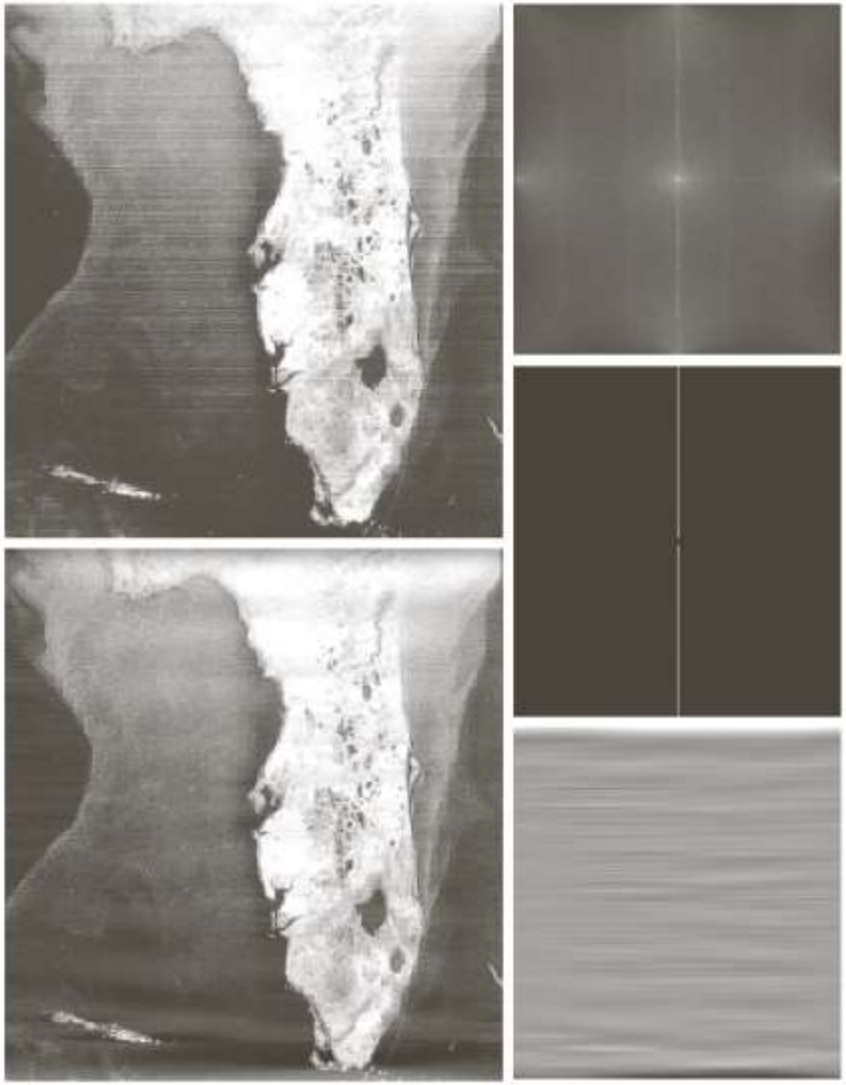


FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



a
b c

FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

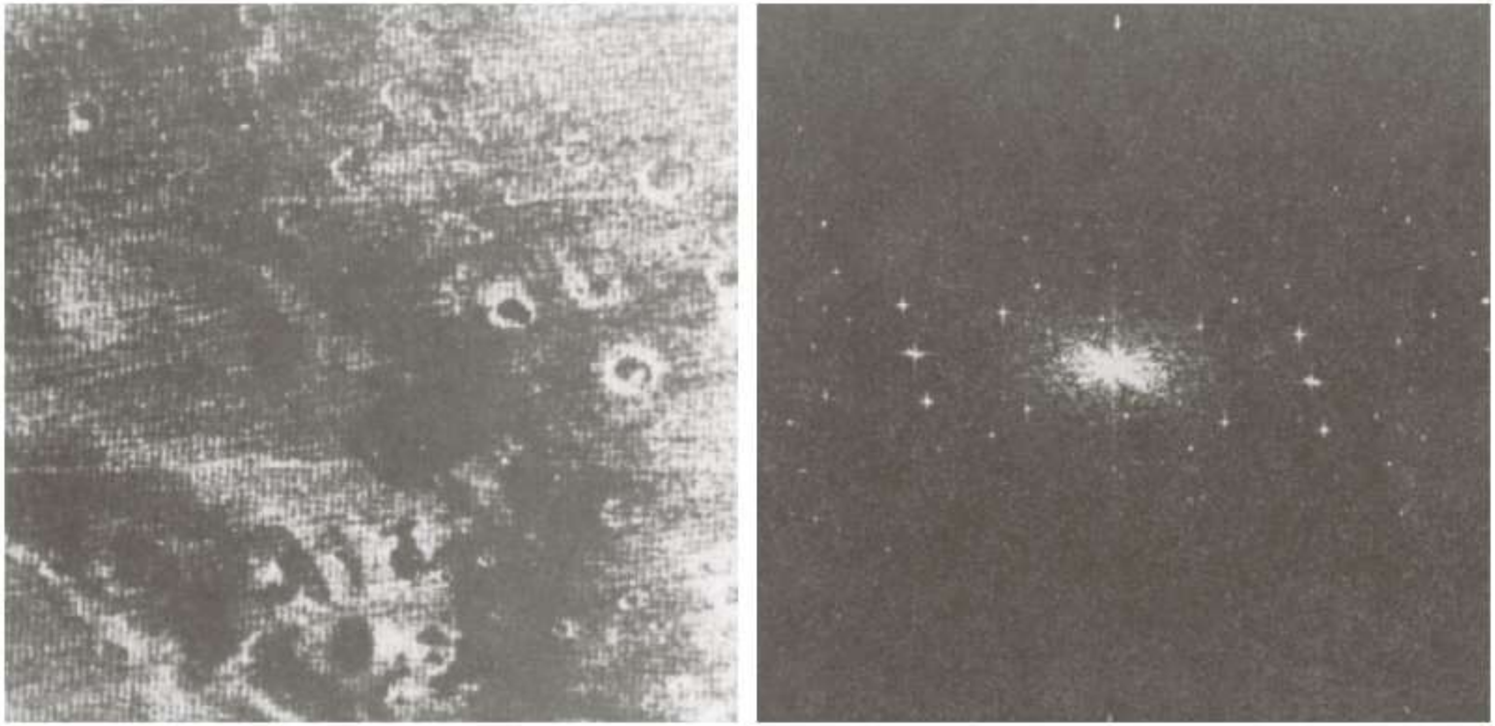


a b
c
e d

FIGURE 5.19
(a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

a b

FIGURE 5.20 (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)



Linear, position-invariant Degradation

- Linear, spatially invariant degradation system with additive noise can be modeled in the spatial domain as the convolution of the degradation function with an image , followed by addition of noise.

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

- Based on the convolution theorem, the same process can be expressed in the frequency domain as the product of the transforms of the image and degradation, followed by the addition of the transform of the noise.

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- Nonlinear and position-dependent techniques, although more general, introduce difficulties that often have no known solution or are very difficult to solve computationally.

Estimating the Degradation function

- Estimation by Image Observation
- Estimation by Experimentation
- Estimation by Modeling

Estimation by Image Observation

- From the given degraded image select a subimage, $g_s(x,y)$, such that the signal content is strong(eg, an area of high contrast)
- Process the subimage to arrive at a result that is as unblurred as possible. Let the processed subimage be denoted by $\hat{f}_s(x,y)$.
- Since the effect of noise is negligible in the subimage then the degradation is given as

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

- From the characteristics of this function, we then deduce the complete degradation function *H(u,v) based on our assumption of position invariance.*
- *For example, suppose* that a radial plot of *Hs(u,v) has the approximate shape of a Gaussian curve.*
- *We* can use that information to construct a function *H(u,v) on a larger scale, but having* the same basic shape.
- This is a laborious process used only in very specific circumstances, such as restoring an old photograph of historical value.

Estimation by Experimentation

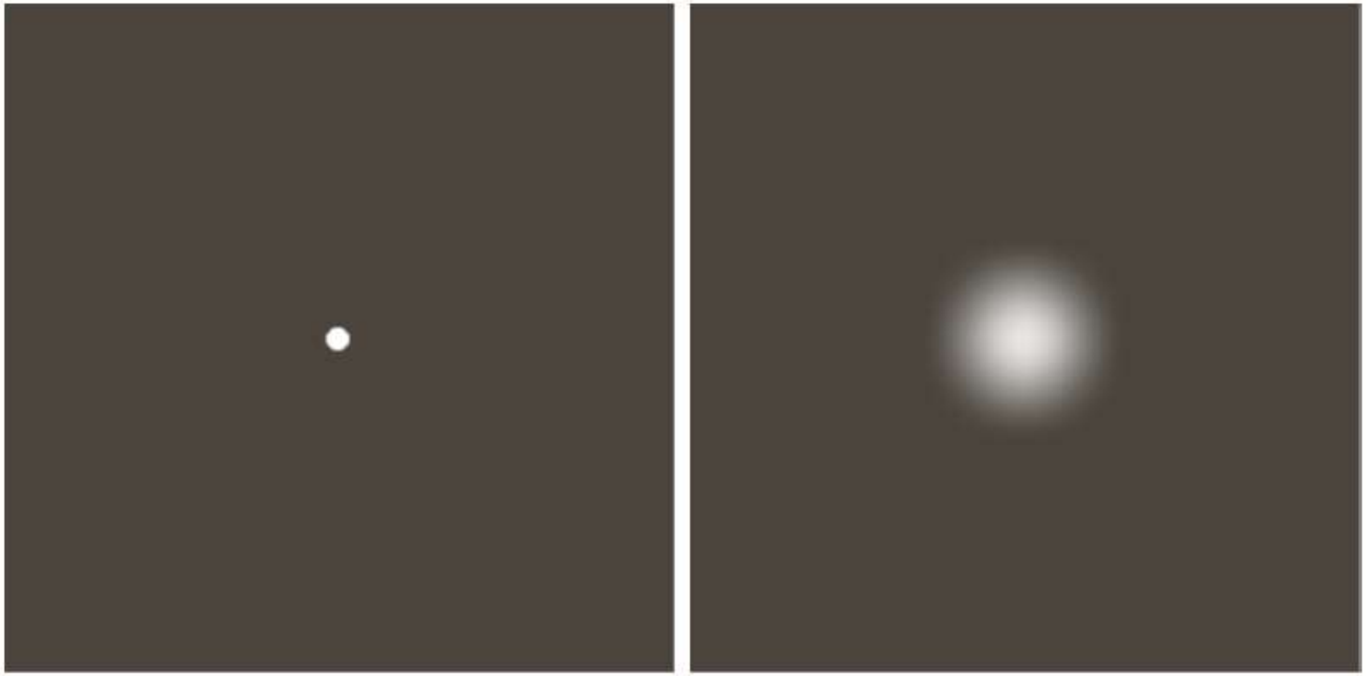
- If equipment similar to the equipment used to acquire the degraded image is available, then it is possible to obtain accurate estimate of the degradation function.
- Images similar to the degraded image can be acquired with various system settings until they are degraded as closely as possible to the image we wish to restore.
- Obtain the impulse response of the degradation by imaging an impulse (small dot of light) using the same system settings.
- We know that the Fourier transform of an impulse is a constant.
- Then the degradation function is given as

$$H(u,v) = \frac{G(u,v)}{A}$$

Where G(u,v) is the Fourier transform of observed image and A is a constant describing the strength of the impulse.

a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



Estimation by Modeling

- Degradation modeling affords insight into the image restoration problem.
- A degradation model proposed by Hufnagel and Stanley is based on the physical characteristics of atmospheric turbulence.
- This model has a familiar form:

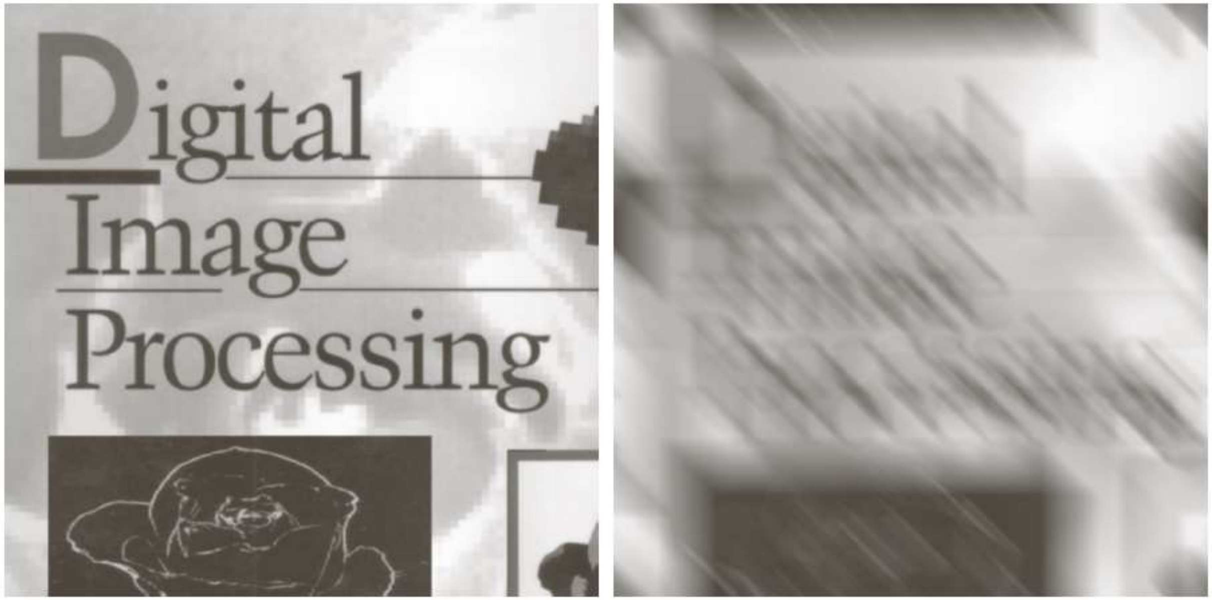
$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

where k is a constant that depends on the nature of turbulence.

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Motion blurring



$$H(u,v) = \frac{T}{\pi(ua + vb)} \sin\left[\pi(ua + vb)\right] e^{-j\pi(ua + vb)} \qquad (5\text{-}77)$$

To generate a discrete filter transfer function of size $M \times N$, we sample this equation for $u = 0, 1, 2, \ldots, M-1$ and $v = 0, 1, 2, \ldots, N-1$.

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5-77) with $a = b = 0.1$ and $T = 1$.

Inverse Filtering

- Estimate the degradation function H(u,v) using one of the three approaches as discussed earlier.
- Compute an estimate of the transform of the original image simply by dividing the transform of the degraded image, G(u,v), by the degradation function H(u,v):

$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)}$$

- Substituting the formula $G(u, v) = H(u, v)F(u, v) + N(u, v)$ in the above equation yields

$$\hat{F}(u, v) = F(u, v) + \frac{N(u, v)}{H(u, v)}$$

- The above equation tells that we cannot recover the undegraded image exactly because N(u,v) is not known.
- Another problem is, if the degradation function, H(u,v), has zero or very small values, then the ratio N(u,v)/H(u,v) will be high, which dominates the estimate $\hat{F}(u, v)$.
- One approach to get around the zero or small-value problem is to limit the filter frequencies to values near the origin, since we know that H(0,0) is usually the highest value of H(u,v) in the frequency domain.

- The image (a) was inverse filtered using the exact inverse of the degraded function (atmospheric turbulence)

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

Constrained Least squares Filtering

- We can express the equation

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

in vector-matrix form as

$$\mathbf{g} = \mathbf{Hf} + \boldsymbol{\eta}$$

For example, suppose that $g(x, y)$ is of size $M \times N$. Then we can form the first N elements of the vector $\mathbf{g}$ by using the image elements in first row of $g(x, y)$, the next N elements from the second row, and so on. The resulting vector will have dimensions $MN \times 1$. These are also the dimensions of $\mathbf{f}$ and $\boldsymbol{\eta}$, as these vectors are formed in the same manner. The matrix $\mathbf{H}$ then has dimensions $MN \times MN$.

- Matrix H is sensitive to noise.
- One way to solve noise senstivity problem is to base 0ptimality of restoration on a measure of smoothness, such as second order derivative of an image.
- Thus we desired to find the minimum of a criterion function, C, defined as

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\nabla^2 f(x, y)]^2 \tag{5.9-2}$$

subject to the constraint

$$\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\|^2 = \|\boldsymbol{\eta}\|^2 \tag{5.9-3}$$

where $\|\mathbf{w}\|^2 \triangleq \mathbf{w}^T\mathbf{w}$ is the Euclidean vector norm,† and $\hat{\mathbf{f}}$ is the estimate of the undegraded image. The Laplacian operator ∇^2 is defined in Eq. (3.6-3).

The frequency domain solution to this optimization problem is given by the expression

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right]G(u,v) \tag{5.9-4}$$

where γ is a parameter that must be adjusted so that the constraint in Eq. (5.9-3) is satisfied, and $P(u,v)$ is the Fourier transform of the function

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \tag{5.9-5}$$

We recognize this function as the Laplacian operator introduced in Section 3.6.2. As noted earlier, it is important to keep in mind that $p(x,y)$, as well as all other relevant spatial domain functions, must be properly padded with zeros prior to computing their Fourier transforms for use in Eq. (5.9-4), as discussed in Section 4.6.6. Note that Eq. (5.9-4) reduces to inverse filtering if γ is zero.

a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.