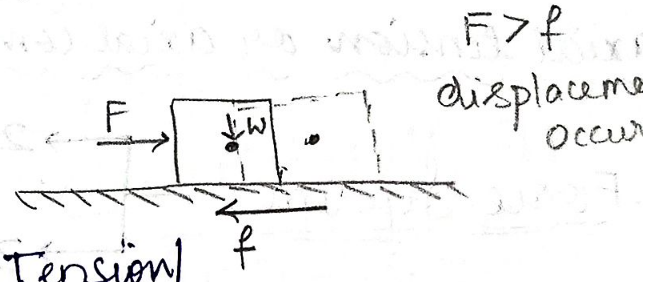
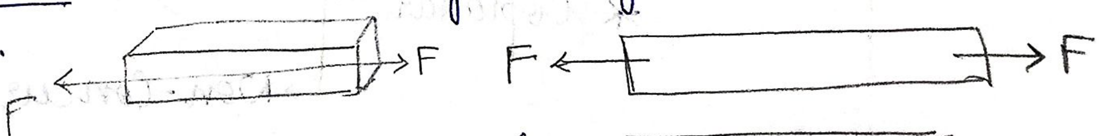
09/12/2021 # BASIC ENGINEERING MECHANICS

* **Prisamatic**: Shape and dimension are identical along the complete axis.
* **Rigid**: The objects which do not change its dimensions on application of external force.
* **Force**: External effort acting on rigid body which changes its displacements.

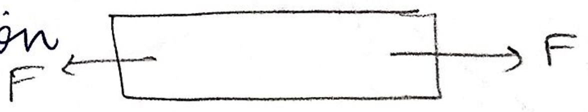
FORCE:
- → Magnitude
- → Direction
- → point of application
- → Sense [pull/push; Tension/compression]



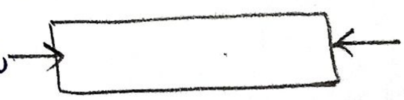
* **axial force**: Force acting along the axis of the body.



axial force:
- → axial tension



- → axial compression



* **Note**: The main component in Engineering Mechanics is force (F) which is defined as external effort acting on rigid body.

The basic engineering mechanics deals with only rigid bodies.

* Rigid body is defined as no change in

dimensions even on application of force.

* Characteristics of a force:

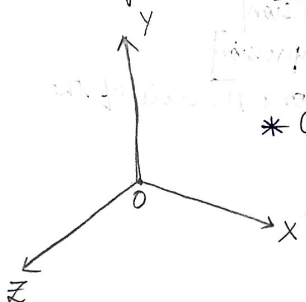
1) Magnitude 2) Direction 3) ~~&~~ point of application

4) Sense (Tension/~~Applico~~ Compression)

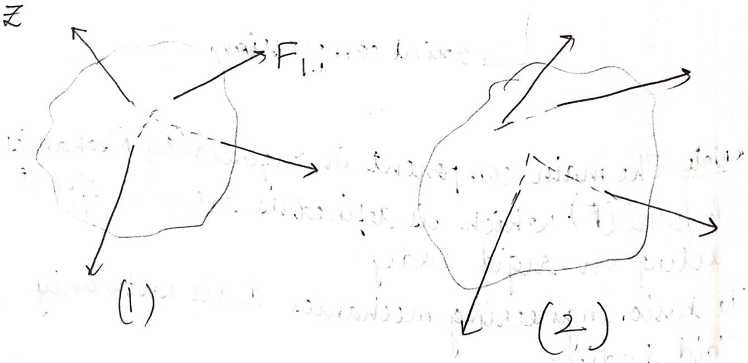
* Axial force: Force is acting along the axis of the rigid body is said to be axial force; depending upon the direction of the force it may be axial tension or axial compression.

* Force System (F.S):
- → 2D (Co-planar)
- → 3D (Spatial)

Planes
Horizontal
Vertical
Profile



* Coplanar:
- → Concurrent Coplanar F.S (1)
- → Non-Concurrent Coplanar F.S (2)



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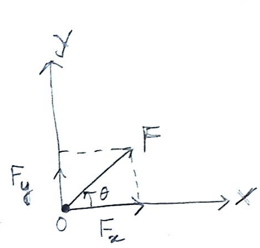
* 2D Force System: [Coplanar]

* Components of force:

The horizontal axis is called datum. $\boxed{F_x = F\cos\theta \quad F_y = F\sin\theta}$



θ → angle made by the line of action of force with datum.

* If the angle b/w the components is 90°; are called rectangular components: $\boxed{F = \sqrt{F_x^2 + F_y^2}}$

* If the angle b/w the components is not 90°; are called oblique components $\boxed{F = \sqrt{F_x^2 + F_y^2 + 2F_xF_y\cos\theta}}$

⇒ If θ = 0°: $F_x = F$, $F_y = 0$

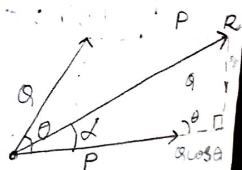
θ = 90°: $F_x = 0$, $F_y = F$

θ = 45°: $F_x = F_y = \frac{F}{\sqrt{2}}$

→ Note: ~~If the~~

COMPONENTS OF FORCE:
- → Rectangular Components (90°)
- → oblique Components (≠ 90°) (Parallelogram Law)

• OBLIQUE COMPONENTS: (θ ≠ 90°)



$$R^2 = (P + Q\cos\theta)^2 + Q^2\sin^2\theta$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Angle: $\boxed{\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}} \Rightarrow \boxed{\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)}$

Q: 2 forces 50 & 30 KN acting on the rigid body; their line of action of forces is 120°. Evaluate the resultant — magnitude & direction.

A: $R = \sqrt{2500+900+2(1500)\left(\frac{-1}{2}\right)} = \sqrt{(1000+900)}$

$= 100\sqrt{190} = 100\sqrt{190}$ KN

$= 10\sqrt{19}$ KN.

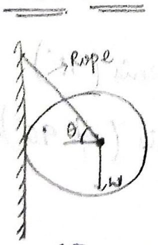
$\alpha = \tan^{-1}\left(\frac{30K\left(\frac{\sqrt{3}}{2}\right)}{\left(50+30\left(\frac{-1}{2}\right)\right)K}\right)$

$= \tan^{-1}\left(\frac{15\sqrt{3}}{35}\right) = \tan^{-1}\left(\frac{3\sqrt{3}}{7}\right)$

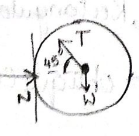
$= \tan^{-1}(0.74)$

$= 36.5°$

* FREE BODY DIAGRAM: Isolated View of component which should be analysed.



FBD



$\Sigma F_x = N\cos 0° - T\cos 45° + W\cos 90$

$\Sigma F_x = N - \frac{T}{\sqrt{2}} = 0$

$\boxed{\sqrt{2}N = T}$

$\Sigma F_y = N\sin 0° + T\sin 45° - W$

$= \frac{T}{\sqrt{2}} - W = 0$ $\quad \boxed{N = W}$

$\boxed{\sqrt{2}W = T}$

* Action of seperated elements are shown.

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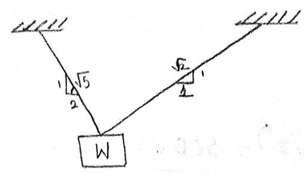
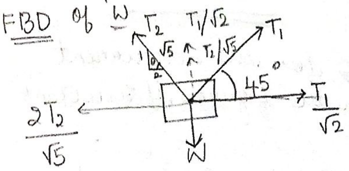


if W = 500N

FBD of 'W'



$\cos\theta = \frac{2}{\sqrt{5}}$

$\sin\theta = \frac{1}{\sqrt{5}}$

$\Sigma F_x = \frac{T_1}{\sqrt{2}} - \frac{2T_2}{\sqrt{5}} = 0$

$\boxed{\sqrt{5}T_1 = 2\sqrt{2}T_2}$

$\Sigma F_y = \frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{5}} - W = 0$

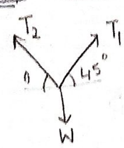
$\Rightarrow \frac{2\sqrt{2}T_2}{\sqrt{5}\cdot\sqrt{2}} + \frac{T_2}{\sqrt{5}} = 500$

$\Rightarrow \frac{3T_2}{\sqrt{5}} = 500$

$$\boxed{\begin{array}{c} T_2 = \frac{500\sqrt{5}}{3} \\ \hline T_1 = \frac{1000\sqrt{2}}{3} \end{array}}$$

⇒ Lami's theorem is applicable only when 3 concurrent co-planar forces acting on a rigid body.

Lami's theorem:

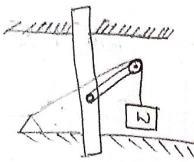


$\frac{T_1}{\sin(90+\theta)} = \frac{T_2}{\sin(90+45°)} = \frac{W}{\sin(180-(45+\theta))}$

$\frac{T_1}{\cos\theta} = \frac{W}{\sin(45+\theta)}$

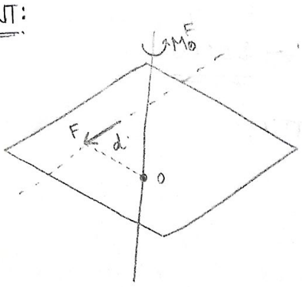
$T_1 = \frac{500\times 2/\sqrt{5}}{\frac{1}{\sqrt{2}}\cdot\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{5}}} = \frac{\frac{200\sqrt{5}\times\sqrt{10}}{3}}{\frac{200\times 5}{3}\sqrt{2}}$

FBD:



2 forces can be in equilibrium only when they are in equal magnitude; opposite in direction and collinear on line of action of force.

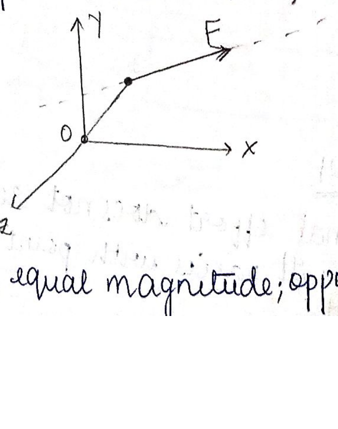
*MOMENT:



$$\boxed{M_o^F = F \cdot d}$$

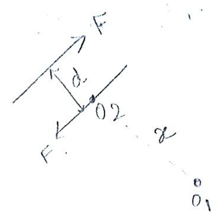
- A tendency to rotate along the lever arm(or) ⊥ar distance 'd' is called moment.
- Moment axis is the axis ⊥'ar to the plane created by the applied force and lever arm passing through moment centre.
- Line joining the reference point to the line of action of force is called position vector.

$$\boxed{M_o^F = r \times F}$$



*COUPLE: If 2 forces of equal magnitude; opposite in direction are separated by a ⊥ar distance; Such system of forces is called couple.

External effect of a couple is rotation not translation.



$$M_{O_1}^{couple} = F(d+x) - F(x)$$
$$= F \cdot d$$
$$M_{O_2}^{couple} = F \times d + 0$$
$$= Fd$$

- If a line of action of force passes through moment centre; the ⊥ar distance will be zero; hence that force does not contribute to moment of couple.

⇒ Features:

*Moment of couple does not change even when it is moved to a different place in the same plane.

* To replace a couple with a new one, the magnitude and sense of the couple should remain same ⇒ $F_1 d_1 = F_2 d_2$.

* When the couple is rotated by an angle 'θ', the sense & magnitude of moment does not change.

* When the couple is shifted to a parallel plane the sense and magnitude does not change.

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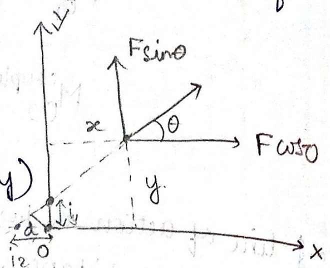
* Principle of Moments: ~~VARIGON PRI~~
[VARIGNON THEOREM]:

It states moment of force = moment sum of its components.



$M_o^F = F \times d$

$= -(F\sin\theta \times x) + (F\cos\theta \times y)$

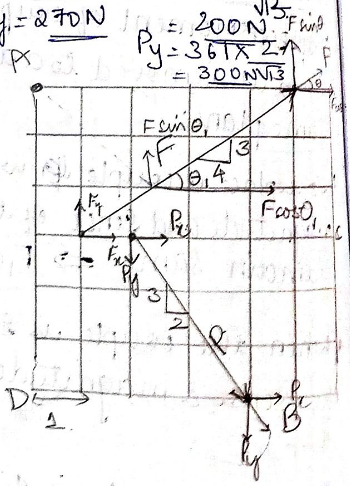
$= F\cos\theta \times i_y$

$= F\sin\theta \times i_x$

* Compute moment of force F = ~~500N~~ 450N & force P = 361N about points A, B, C & D.

$F_x = 360N$ $P_x = 361 \times \frac{3}{\sqrt{13}} = 200N$

$F_y = 270N$ $P_y = 361 \times \frac{2}{\sqrt{13}} = 300N$



$M_A^F = -F\sin\theta \times i_x = -(360 \times 3)$

$= -450 \times \frac{3}{5} \times 5$ -270×1

$= -1350$ $= -1080$

$= -1350$ -270

$= -1350$

$M_B^F = F\cos\theta \times i_y$

$= 360 \times 6$

$= 2160$

$= 360 \times 3 + 270 \times 4$

$= 1080 + 1080$

$= 2160$

$M_C^F = 270 \times 1 + 360 \times 3$

$= 270 + 1080 = 1350 Nm$

$M_D^F = 360 \times 3 - 270 \times 1$

$= 1080 - 270$

$= 810$

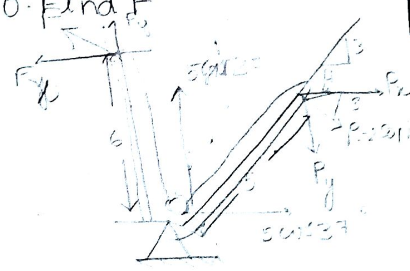
$M_A^P = 200 \times 3 + 300 \times 2 = 0$

$M_B^P = +300 \times 1 = +300 Nm$

$M_C^P = 300 \times 4 = 1200 Nm$

$M_D^P = 300 \times 4 = -1200 Nm$

* Rocker Arm shown moment of force F about O balances that of P about O. Find F.



$P_x = 250 \times \frac{4}{5} = 200N$

$P_y = 250 \times \frac{3}{5} = 150N$

$M_O^P = 200 \times 5 \times \frac{3}{5}$

$+ 150 \times 5 \times \frac{4}{5}$

$= 600 + 600 = 1200 Nm$

$6 \times F \times \frac{2}{\sqrt{5}} = 1200$

$F = \frac{600\sqrt{5}}{6} = \frac{600 \times 2.2}{6}$

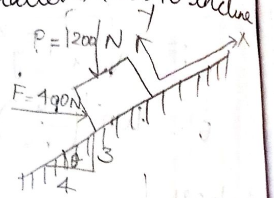
$= 2.2 \times 100 = 223N$

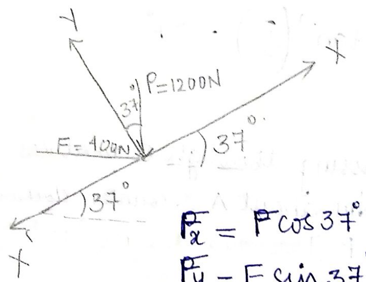
Moment about O

$M_o^F = -(1)(60) + 120\times3$

$= 360-60 = 300\,Nm$

* The body on incline is subjected to force as shown. Find the components of each force along X & Y-axes oriented parallel & ⊥ar to incline.





$F_x = F\cos37^\circ = \overset{80}{\cancel{400}}\times\frac{4}{\cancel{5}} = 320N$

$F_y = F\sin37^\circ = 400\times\frac{3}{5} = 240N$

$P_x = -P\sin37^\circ = -\overset{240}{\cancel{1200}}\times\frac{3}{\cancel{5}} = -720N$

$P_y = -P\cos37^\circ = -1200\times\frac{4}{5} = -960N$

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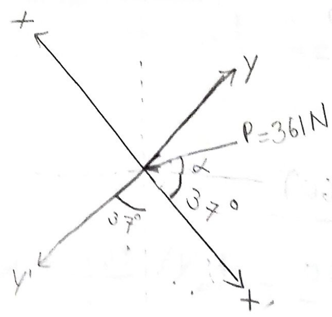
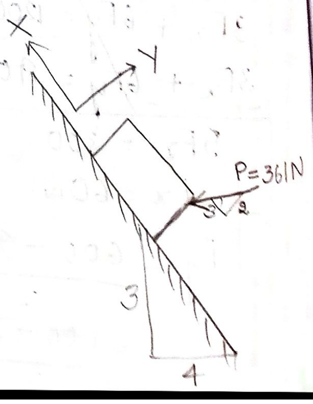




$\cancel{\Sigma F_x = 0}$

$P_x = P\cos(\alpha+\theta)$

$= 361\cos(33.824+37)$

$= 361\cos(70.824)$

$= 361\times0.32$

$\simeq 120N$

$\tan\alpha = \frac{2}{3}$

$\alpha = 33.824$

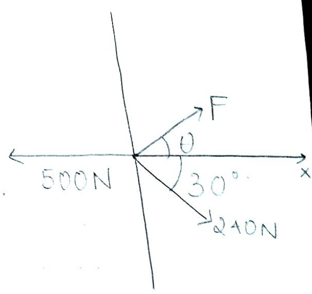
$P_y = P\sin(\alpha+\theta)$

$= 361\sin(70.824)$

$= 361\times0.944$

$= \underline{340.7N}$

* The force system shown has a resultant of 200N pointing up ~~of~~ on Y-axis. Compute the values of θ & F to give this resultant.



$\Sigma F_x = 0$

$F\cos\theta - 500 + 240\cos30^\circ = 0$

$F\cos\theta = 500 - 240\times\frac{\sqrt3}{2}$

$= 500 - 120\times\sqrt3$

$= 292.16N$

$\Sigma F_y = 0$

$F\sin\theta - 240\sin30^\circ = 200$

$F\sin\theta = 120N + 200 = 320N$

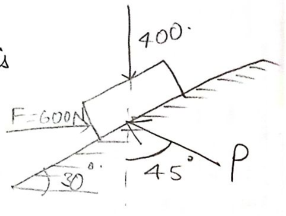
$\tan\theta = \frac{320}{292.16} = 1.09$

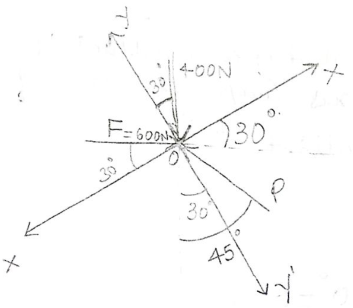
$\theta = 47.64^\circ$

$F = \frac{320}{0.73} = \underline{438.3N}$

prototype

* Block shown is acted on by its weight W= 400 N and horizontal force is 600N and the pressure exerted by the plane 'P'. The resultant(R) of these forces is ||el to incline. Determine P & R. Does the block move up or down?





$\Sigma F_y = -400 + P\cos 15° + 600\sin 30° \cos 30° = 0$

$\cancel{6400}$

$P\cos 15° = \frac{600}{2} + 200\sqrt{3}$

$P = \cancel{\frac{700}{\cos 15}} \quad \frac{300+346}{\cos 15°}$

$= \cancel{670N} \quad = \frac{646}{0.96}$

$= 672.9$

ΣF_x

$R = -400\sin 30° - P\sin 15° + 600\cos 30°$

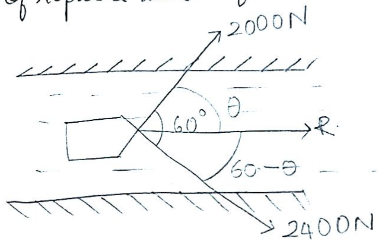
$= -200 - 670 \times 0.25 + 300\sqrt{3}$

$= -200 - 167.5 + 519$

$= -367.5 + 519$

$= 151.5N$

* Two locomotives on opp. banks of canal pull a vessel moving parallel to banks by means of 2 horizontal ropes. Tensions in these ropes are 2000N & 2400N while angle b/w them is 60°. Find the resultant pull on the vessel and angle b/w each of ropes & sides of canal.



$\Sigma F_y = 0$

$\cancel{2000}_{10} \sin\theta - \cancel{2400}^{12} \sin(60-\theta) = 0$

$5\sin\theta = 6\left[\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right]$

$5\sin\theta = 3\sqrt{3}\cos\theta - 3\sin\theta$

$8\sin\theta = 3\sqrt{3}\cos\theta$

$\tan\theta = \frac{3\sqrt{3}}{8} = 0.64$

$\theta = 32.9°$

$\Sigma F_x = R$

$R = 2000\cos\theta + 2400\cos(60-\theta) = R$

$R = 0.83 \times 2000 + 2400 \times 0.89$

$= 2136 + 1660 = \underline{3796N}$

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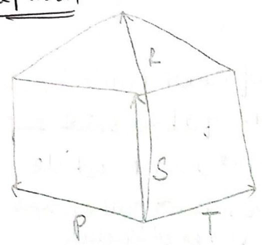

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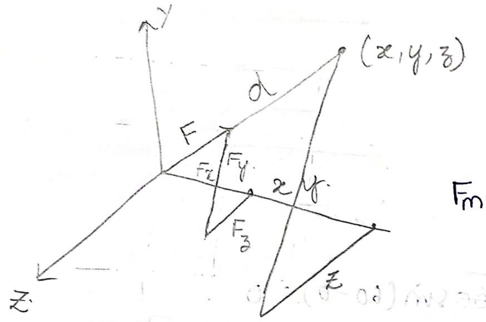

Fm ⇒ Force multiplier

$$\frac{F_x}{x} = \frac{F_y}{y} = \frac{F_z}{z} = \frac{F}{d} = F_m =$$

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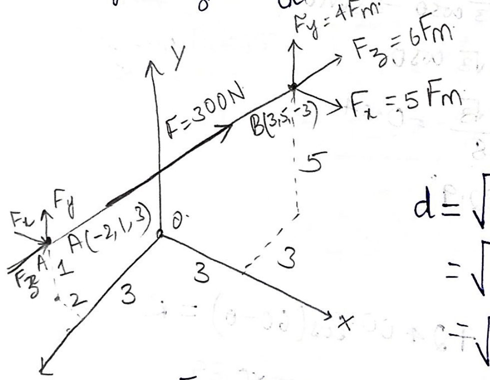


$d = \sqrt{25+16+36}$

$= \sqrt{25+52}$

$= \sqrt{77}$

$= 8.77$ units

$F_m = \frac{300}{8.77}$

$= 34.2$ N/units

$F_x = 5 \times 34.2 = 171.03$ N

$F_y = 136.8$ N

$F_z = -205.2$ N.

~~For~~

* Find the resultant of component force system which consists of forces :- directed from D towards A, B, C respectively.

Force	x	y	z	d	Fm :	Xcomp	Ycomp	Zcomp.
T=300N(DA)	5	−10	0.	11.12	26.8	134	−268	0
P=200N(DB)	−5	−10	−3	11.57	17.28	−86	−172.8	−51.24
F=500N(DC)	−5	−8	4.	10.24	48.8	~~[illegible]~~ −244	−390.4	195.2
						−196.5	−831.3	143.96.

D (5, 10, 0) A (10, 0, 0)

D (5, 10, 0) B (0, 0, −3)

D (5, 10, 0) C (0, 2, 4).

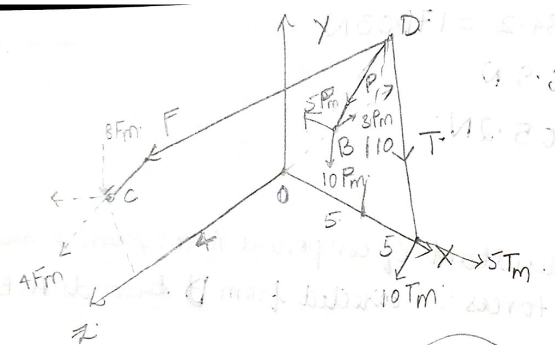
Figure

$R = \sqrt{F_x^2 + F_y^2 + F_z^2}.$

$= \sqrt{38612.25 + 691059.69 + 20724.48}$

$= 866.22$ N.

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

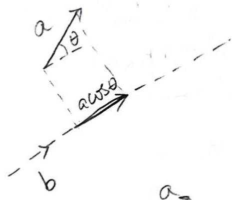


$* \; F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

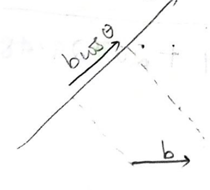
$$\frac{F_x}{x} = \frac{F_y}{y} = \frac{F_z}{z} = F_m = \frac{F}{d}$$

$$F = \frac{F}{d}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\boxed{F = F_m(x\hat{i} + y\hat{j} + z\hat{k})}$$



$$\frac{ab\cos\theta}{|b|} = \frac{\bar{a}\cdot\bar{b}}{|b|} = a \cdot \hat{n}_b$$



$$\frac{\bar{a}\cdot\bar{b}}{|a|} = b \cdot \hat{n}_a$$

Component of force along any defined direction is obtained by taking dot product b/w force vector and the unit vector along desired line of action.

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1) In the tripod shown; the P & F act as shown along legs DC & DB in terms of its force multiplier; determine components of force F along the direction of P and also angle b/w F & P. Note that points B & C are in same vertical & that 'c' is located 2m about the horizontal plane containing B & A.

~~D(0,5,0)~~

D(5,10,0)
C(0,2,4) } F

D(5,10,0)
B(0,0,−3) } P

$$F = F_m(-5\hat{i} - 8\hat{j} + 4\hat{k})$$

$$\bar{P} = P_m = (-5\hat{i} - 10\hat{j} - 3\hat{k})$$

$$\hat{n}_p = P_m(-5\hat{i} - 10\hat{j} - 3\hat{k}) / P_m\sqrt{134}$$

$$= \cancel{P_m\sqrt{25+100+9}}$$

$$= \cancel{P_m}\sqrt{134}$$

$$\hat{n}_p = \frac{P_m(-5\hat{i} - 10\hat{j} - 3\hat{k})}{P_m(11.5)} = \frac{-5\hat{i} - 10\hat{j} - 3\hat{k}}{11.5}$$

Component of 'F' in direction of P

$= \hat{n}_p \cdot F$

$= \dfrac{F_m(25+80-12)}{11.5} = \dfrac{93F_m}{11.5}$

$= 8.04F_m$

$\cos\theta = \dfrac{93}{11.5 \times \sqrt{64+25+16}}$

$= \dfrac{93}{11.5\times\sqrt{105}} = \dfrac{93}{11.5\times 10.24}$

$= 0.789 \simeq 0.8$

$\boxed{\theta \simeq 38^\circ}$

* In figure a boom AC is supported by a ball & socket joint at C. And by the cables BE & AD. If the force multiplier of a force F acting from B to E $F_m = 10N/m$ and that of force P acting from A to D is $P_m = 20N/m$. Find the component of each force along AC. B(4,−5,0) E(0,3,6)

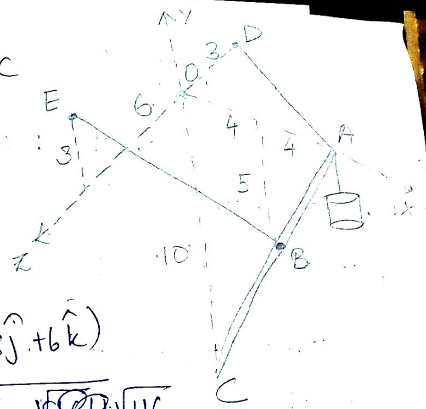
A(8,0,0) D(0,0,−3)

C = (0,−10,0)

$\bar{F}$ = B to E $= (-4\hat{i}+8\hat{j}+6\hat{k})$

$\bar{P}$ = A to D $= (-8\hat{i}-3\hat{k})$

$\overline{AC} = (-8\hat{i}-10\hat{j})$.



Component of 'F' along AC

$= \hat{n}_{AC} \cdot F =$

$\hat{n}_F = \dfrac{F_m(-4\hat{i}+8\hat{j}+6\hat{k})}{\sqrt{164}\sqrt{16+64+36}}$

$= \dfrac{10\times(-4\hat{i}+8\hat{j}+6\hat{k})}{\sqrt{164}\sqrt{116}}$

$= \dfrac{10\times(-4\hat{i}+8\hat{j}+6\hat{k})}{\sqrt{116}}$

$\hat{n}_{AC} = \dfrac{-8\hat{i}-10\hat{j}}{\sqrt{164}}$

~~$= \dfrac{10}{10.7}(32-80) = \dfrac{10\times(48)}{10.7} = -44.8N$~~

$\hat{n}_p = \dfrac{20(-8\hat{i}-3\hat{k})}{\sqrt{64+9}} = \dfrac{-20(8\hat{i}+3\hat{k})}{\sqrt{73}}$

$= \dfrac{-20(8\hat{i}+3\hat{k})}{8.5}$

F along 'AC'

$= \hat{n}_{AC}(F)$

$= \dfrac{-48\times 10}{12.8}$

$= -37.5$

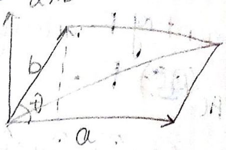
Component of 'P' along AC.

$= \hat{n}_{AC}(P)$

$= \dfrac{-20}{8.5}\dfrac{(-64)}{12.8} \simeq 100N$

*CROSS PRODUCT: (APPLICATION: 1) ⊥ar distance 2) moment of force abt centre)

$a \times b = ab\sin\theta\, \hat{n}$



1) Not commutative ⇒ $A \times B \neq B \times A$

2) $nA \times B = A \times nB \Rightarrow (n - \text{scalar})$

3) $A \times (B \times C) \neq (A \times B) \times C \rightarrow$ Not associative when multiplied with vector.

$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$

$(a \times b) \times c = (a \cdot c)b - (b \cdot c) \cdot a$

4) Distributive over addition.

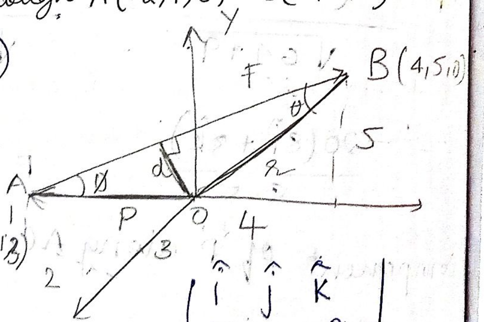
$A \times (B + C) = (A \times B) + (A \times C)$

* Find the shortest distance from origin to the line passing through A(−2,1,3) & B(4,5,0).

$\bar{AB} = 6\hat{i} + 4\hat{j} - 3\hat{k}$

$\bar{r} = (4\hat{i} + 5\hat{j})$

$\bar{P} = -2\hat{i} + \hat{j} + 3\hat{k}$



$\bar{r} \times \bar{AB}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & -3 \\ 4 & 5 & 0 \end{vmatrix} \qquad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 0 \\ 6 & 4 & -3 \end{vmatrix}$$

$= \hat{i}(+15) - \hat{j}(12) + \hat{k}(14) = \cancel{15\hat{i} - 12\hat{j} + 14\hat{k}}$

$= -15\hat{i} + 12\hat{j} - 14\hat{k}$

$\bar{p} \times \bar{AB}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 3 \\ 6 & 4 & -3 \end{vmatrix} = \hat{i}(-15) - \hat{j}(-12) + \hat{k}(-14)$$

$= -15\hat{i} + 12\hat{j} - 14\hat{k}$

$|r \times AB| = r\sin\theta \cdot (AB)$

$\sqrt{225 + 144 + 196} = d\,(AB)$

$\dfrac{23.76}{\sqrt{61}} = \dfrac{23.76}{7.8} = d = 3.04$ units

$36 + 16 + 9 = 61$

$|p \times AB| = p\sin\phi\,(AB)$

$d = \dfrac{23.76}{\sqrt{61}} = \dfrac{23.76}{7.8} = 3.04$

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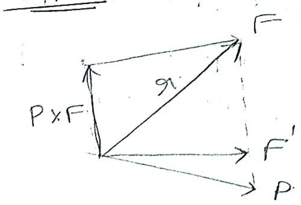
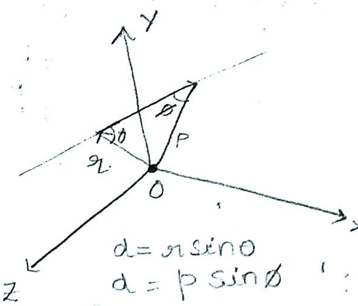




$d = r\sin\theta$

$d = p\sin\phi$

$M_O^F = F \cdot r\sin\theta = F \cdot P\sin\phi$

Note:

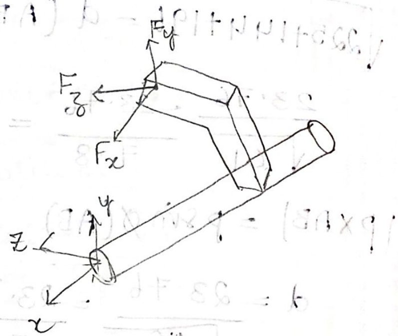
Moment of a force about a moment centre is a cross product of position vector and force vector but not vice versa.

...od :

Recall

$$r \times F = \begin{vmatrix} x & y & z \\ F_x & F_y & F_z \\ \hat{i} & \hat{j} & \hat{k} \end{vmatrix}$$

$$= \hat{i}(yF_z - zF_y) - \hat{j}(xF_z - zF_x) + \hat{k}(xF_y - yF_x)$$



* Tripod consists of 3 parts joined at B & $F_m = 10$ N/m acting along $\bar{D}$ to $\bar{A}$.
Find the moment of F about C and about line CB.
Also find the value of P acting bar $\overline{DB}$ from B to B that will cause moment of P about axis AC to be 2000 N/m

M_C^F

M_{CB}^F

$M_{AC}^P = 2000$ Nm

D(4,10,0) C(8,0,0)

A(0,3,4) B(0,0,−3)

$$\bar{F} = 10(-4\hat{i} - 7\hat{j} + 4\hat{k})$$

$$\overline{CB} = -4\hat{i} + 10\hat{j} + 0\hat{k}$$

$$M_C^F = \overline{CD} \times F$$

$$= \begin{vmatrix} -4 & 10 & 0 \\ -4 & -7 & 4 \\ \hat{i} & \hat{j} & \hat{k} \end{vmatrix} 10 = \hat{i}(40) - \hat{j}(-16) + \hat{k}(68)$$

$$= [40\hat{i} + 16\hat{j} + 68\hat{k}]\,10$$

∵ Moment is a vector its component about arbitrary axis through moment centre can be found as dot product of moment with unit vector $\hat{n}$ which specifies the direction of its arbitrary axis.

C(8,0,0)

B(0,0,−3)

$$\overline{CB} = -8\hat{i} - 0\hat{j} - 3\hat{k}$$

$$\hat{n}_{CB} = \frac{-8\hat{i} + 0\hat{j} - 3\hat{k}}{\sqrt{64+9}} = \frac{-8\hat{i} - 3\hat{k}}{\sqrt{73}}$$

$$M_{CB}^F = \hat{n}_{CB} \cdot M_C^F$$

$$= \left(\frac{-8\hat{i} - 0\hat{j} - 3\hat{k}}{\sqrt{73}}\right) \cdot (400\hat{i} + 160\hat{j} + 680\hat{k})$$

$$= \frac{-3200 - 0 - 2040}{\sqrt{73}}$$

$$= \frac{-5240}{\sqrt{73}} = -613.3 \text{ Nm}$$

D(4,10,0) B(0,0,−3)

$$P = P_m(-4\hat{i} - 10\hat{j} - 3\hat{k})$$

$M_C^P = (-4\hat{i} + 10\hat{j} + 0\hat{k}) \times (P_m(-4\hat{i} - 10\hat{j} - 3\hat{k}))$

$A(0,3,4)$ $C(8,0,0) \Rightarrow 8\hat{i} - 3\hat{j} - 4\hat{k}$

$M_{AC}^P = \hat{n}_{AC} \cdot M_C^P$

$$2000 = \left[\frac{8\hat{i} - 3\hat{j} - 4\hat{k}}{\sqrt{89}}\right] \cdot P_m \begin{vmatrix} -4 & 10 & 0 \\ -4 & -10 & -3 \\ \hat{i} & \hat{j} & \hat{k} \end{vmatrix}$$

$$2000\sqrt{89} = P_m[8\hat{i} - 3\hat{j} - 4\hat{k}][-30\hat{i} + 12\hat{j} + 80\hat{k}]$$

$$2000\sqrt{89} = (-240 - 36 - 320) P_m$$

$$P_m = \frac{2000 \times 9.43}{-596}$$

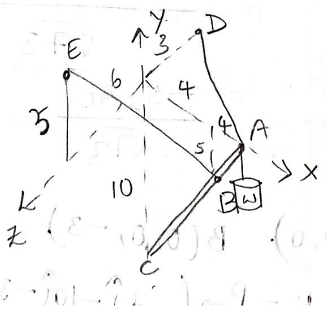
$= -31.6$

560
36

$$\bar{P} = 31.6(-4\hat{i} - 10\hat{j} - 3\hat{k})$$

Q: A boom AC is supported by a ball & socket joint at C & by the cables BE & AD. If the force multiplier of force F acting from B to E is $F_m = 10N/m$, find (a) the moment of F about point C (b) the moment of F about point D (c) moment of F about a line from C to D.

$F = F_m \lambda$



M_C^F

$B(8,-5,0)$ $E(0,3,6)$

$A(8,0,0)$ $D(0,0,-3)$

$C(0,10,0)$

$F \Rightarrow B$ to $E = F_m(-4\hat{i} + 8\hat{j} + 6\hat{k})$

$\overline{CD} = 0\hat{i} + 10\hat{j} - 3\hat{k}$

$\cancel{M_C^F = \overline{CD} \times F}$

$$\cancel{= \begin{vmatrix} 0 & 10 & -3 \\ -40 & 80 & 60 \\ \hat{i} & \hat{j} & \hat{k} \end{vmatrix} = 840\hat{i} + 120\hat{j} - 400\hat{k}}$$

$\cancel{F \Rightarrow B \text{ to } E = F_m(-4\hat{i} + 8\hat{j} + 6\hat{k})}$

$\cancel{\overline{DA} = 8\hat{i} + 0\hat{j} + 3\hat{k}}$

$M_D^F = \overline{DA} \times \bar{F}$

$$= \begin{vmatrix} 8 & 0 & 3 \\ -4 & 8 & 6 \\ i & j & k \end{vmatrix} = (-24\hat{i} - 60\hat{j} + 64\hat{k})10$$

$\overline{CB} = -4\hat{i} - 5\hat{j} - 0\hat{k}$

$$M_C^F = \begin{vmatrix} +4 & +5 & 0 \\ -4 & 8 & 6 \\ i & j & k \end{vmatrix} = (+30\hat{i} - 24\hat{j} + 52\hat{k})10$$

$= +300\hat{i} - 240\hat{j} + 520\hat{k}$

$\overline{DB} = +4\hat{i} - 5\hat{j} + 3\hat{k}$

$$M_D^F = \begin{vmatrix} +4 & -5 & +3 \\ -4 & 8 & 6 \\ i & j & k \end{vmatrix}(10) = -540\hat{i} - 360\hat{j} + 120\hat{k}$$

$\overline{CD} = 0\hat{i} + 10\hat{j} - 3\hat{k}$

$M_{CD}^F =$

$$\hat{n}_{CD} = \frac{10\hat{j} - 3\hat{k}}{\sqrt{109}}$$

$M_{CD}^F = \hat{n}_{CD} \cdot M_C^F$

$$= \frac{10\hat{j} - 3\hat{k}}{\sqrt{109}} \cdot (300\hat{i} - 240\hat{j} + 520\hat{k})$$

$= \frac{2400 - 1560}{10.44} = -3+7.31\,Nm.$

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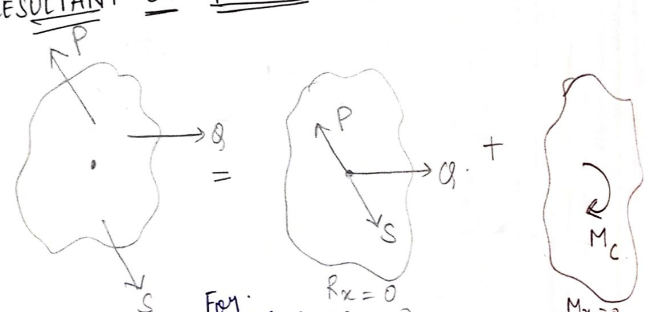
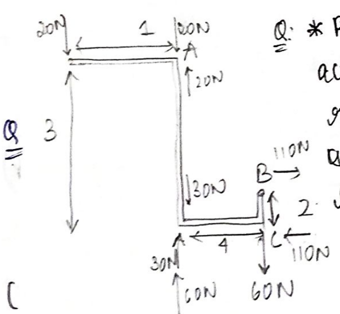
RESULTANT OF GENERAL FORCE SYSTEM:

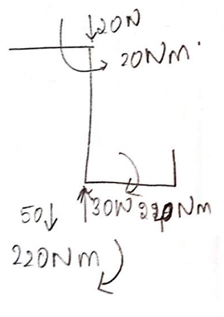


For equilibrium: $R_x = 0$, $R_y = 0$, $R_z = 0$

$M_x = 0$
$M_y = 0$
$M_z = 0$



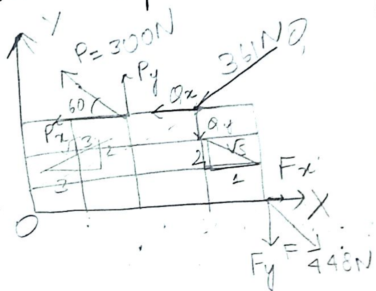
Q: * Replace the system of forces acting on a frame by a resultant force 'R' through 'A' ~~aedea~~ and a couple acting horizontally to B&C.



Resultant force $= 50N \downarrow$

Resultant moment $= 220Nm$ clockwise

* A flat plate subjected to co-planar system of forces. The inscribed grid ~~sq aacm~~ with each square of 1m. locates each slope & force. Determine resultant & its x & y intercepts.



$F_x = 448 \frac{1}{\sqrt{5}}$

$= \frac{448}{2.23} = 200.8N$

$F_y = -448 \times \frac{2}{\sqrt{5}}$

$= -401.6N$

$P_x = -300\cos 60°$

$= -150N$

$P_y = +300\sin 60°$

$= 150\sqrt{3} = 259.5N$

$Q_x = -361 \times \frac{3}{\sqrt{13}}$

$= \frac{-1083}{3.6}$

$= -300.8N$

$Q_y = -361 \times \frac{2}{\sqrt{13}}$

$= -200.5$

$(\Sigma Force_x) = R_x$

$= -250N$

$R_y = -342.6N$

$M_o = -(+150 \times 3) - (259.5 \times 2)$
$- (300.8 \times 3) + (200.5 \times 3)$
$+ (+401.6 \times 4)$

$= -450 - 519 - 902.4 + 601.5 + 1606.4$

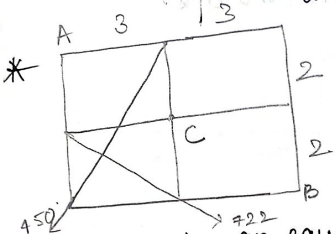
$= 2207.9 - 1871.4 = 336.5\,Nm$

M_o^R $336.5 = R_y \times i_x$

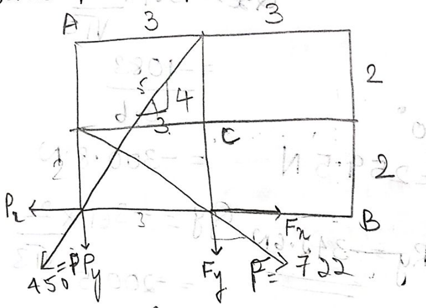
$i_x = \frac{336.5}{+342.6} = 0.98\,m$

$M_0^R = 336.5 = R_x \times l_y$

$l_y = \frac{336.5}{250} = 1.346m$



Replace loads by an equivalent force ~~through~~ 'C' and a couple acting through A & B. Solve if the forces of the couple are horizontal & vertical.



$F_x = 722 \times \frac{2}{\sqrt{13}}$

$= 401.1N$

$F_y = +\frac{722 \times 3}{\sqrt{13}}$

$= +601.6N$

$P_x = -\cancel{450}^{90} \times \frac{3}{\cancel{5}}$

$= -270N$

$P_y = -\cancel{450}^{90} \times \frac{4}{\cancel{5}}$

$= -360N$

$R_x = -270 + 601.6 = 331.6$

$R_y = -360 + 401.1 = 761.1$

$R = \sqrt{109958.56 + 579273.21}$

$= \underline{830.19N}$

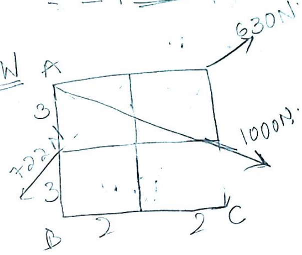
$M^C = P_x \times 2 - P_y \times 3 - Q_x \times 2$

$-540 + 1080 - 601.6 \times 2$

$= -540 - 1203.2$

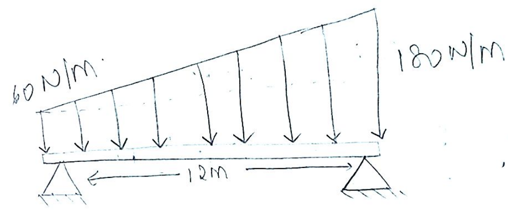
$= -\underline{1743.2 Nm}$

HW

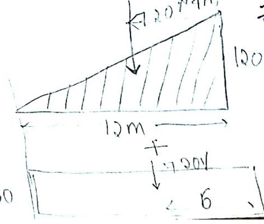


Compute resultant of 3 forces acting on plate. Locate its intersection with AB & BC.

*) Calculate resultant & its position



: When loading diagram is given; resultant is equal to area of the diagram. and the position may be identified either using principal of moments or just by identifying centroid of loading diagram.



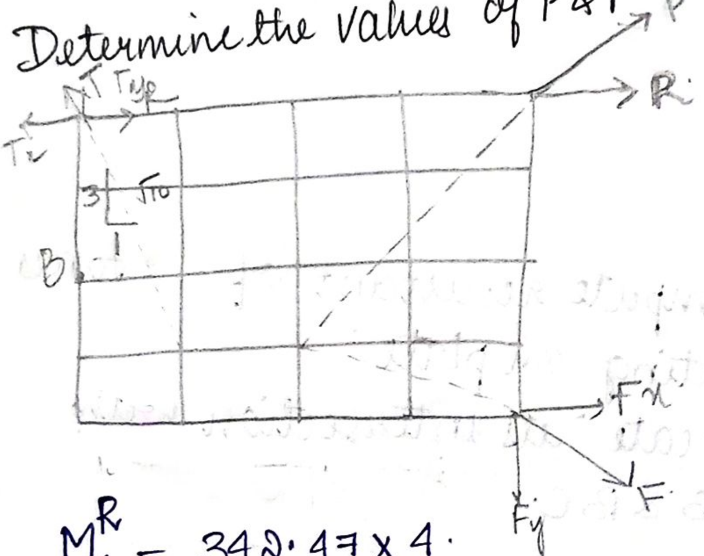
$720 \times 2\bar{x} = 720 \times 8 + 720 \times 6$

$\bar{x} = \frac{\cancel{720} \times 14}{\cancel{720} \times 2} = \underline{7m}$

$y(0) = y_0$..

* 3 forces shown in figure are required to cause a horizontal resultant acting through A. If T=3..

Determine the values of P & F.



$$T_x = 361 \times \frac{1}{\sqrt{10}}$$

$$= \frac{361}{3.16}$$

$$= 114.1N.$$

$$T_y = 361 \times \frac{3}{\sqrt{10}}$$

$$= \underline{\underline{342.47N}}$$

$$M_A^R = 342.47 \times 4.$$
$$- F_x \times 4 = 0$$

$$\underline{\underline{F_x = 342.47N}}$$

$$F_x = F \frac{2}{\sqrt{5}},$$

$$F = \frac{342.47 \times 2.23}{2} = 381.6N$$

$$F_y = \frac{F}{\sqrt{5}} = \frac{381.6}{\sqrt{5}} = \underline{\underline{171.23}} N.$$

Q:

$$342.47 + \frac{3P}{\sqrt{13}} - 171.23 = 0$$

$$P = \frac{171.24 \times \sqrt{13}}{3}$$

$$\underline{\underline{P = -205.4N}}$$

~~RAT~~

$$-P_y \times 4 \; 342.47 \times 4 + 171.23 \times 4 = 0$$

$$P_y = \frac{684.92 - 1369.88}{4}$$

$$= -171.24N$$