Definition: An *m-way search tree* is either empty or satisfies the following properties:

(1) The root has at most m subtrees and has the following structure:

$$n,\ A_0,\ (E_1, A_1),\ (E_2, A_2),\ \cdots,\ (E_n, A_n)$$

where the A_i, $0 \le i \le n < m$, are pointers to subtrees, and the E_i, $1 \le i \le n < m$, are elements. Each element E_i has a key $E_i.K$.

(2) $E_i.K < E_{i+1}.K$, $1 \le i < n$.

(3) Let $E_0.K = -\infty$ and $E_{n+1}.K = \infty$. All keys in the subtree A_i are less than $E_{i+1}.K$ and greater than $E_i.K$, $0 \le i \le n$.

(4) The subtrees A_i, $0 \le i \le n$, are also m-way search trees. □

Definition: A *B-tree of order m* is an *m*-way search tree that either is empty or satisfies the following properties:

(1) The root node has at least two children.

(2) All nodes other than the root node and external nodes have at least $\lceil m/2 \rceil$ children.

(3) All external nodes are at the same level. □

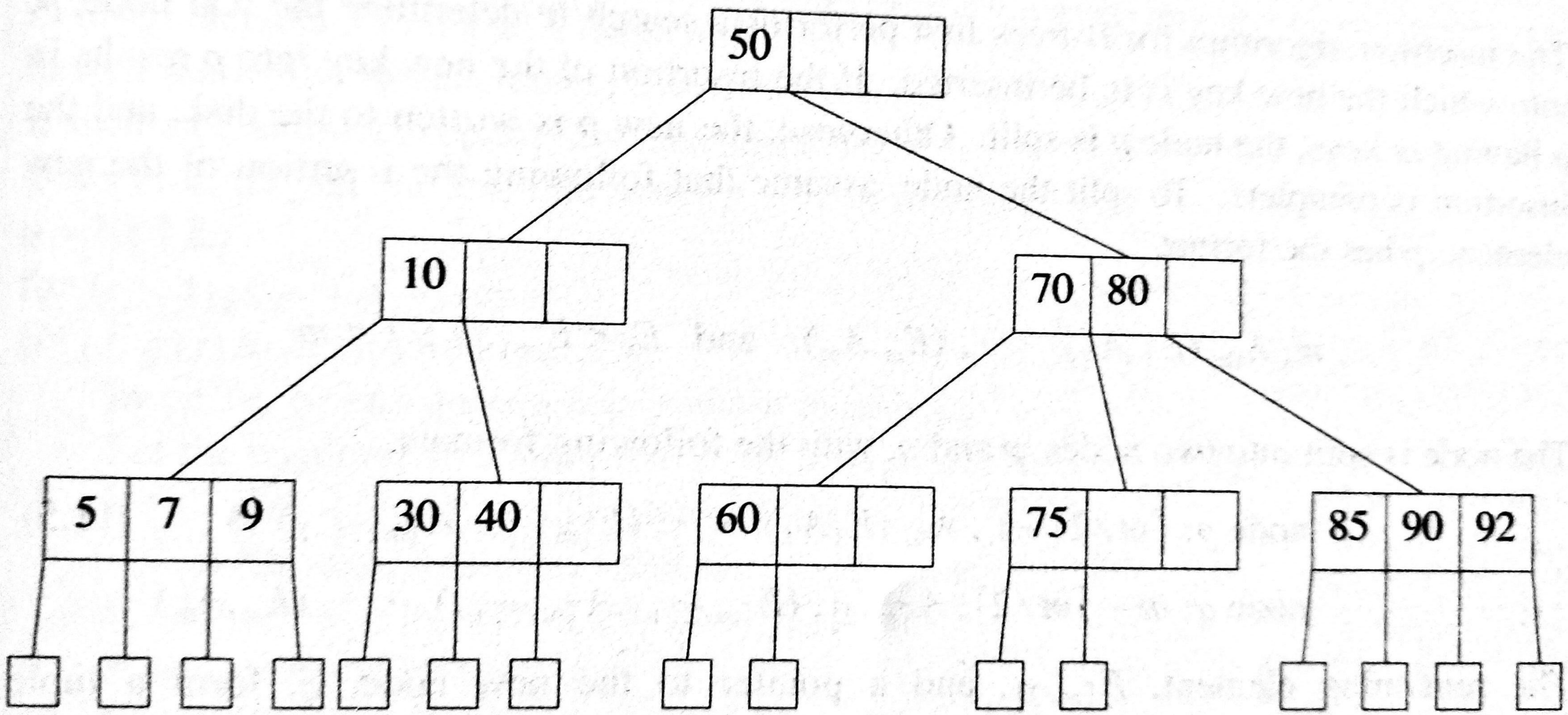


Figure 11.3: Example of a 2-3-4 tree

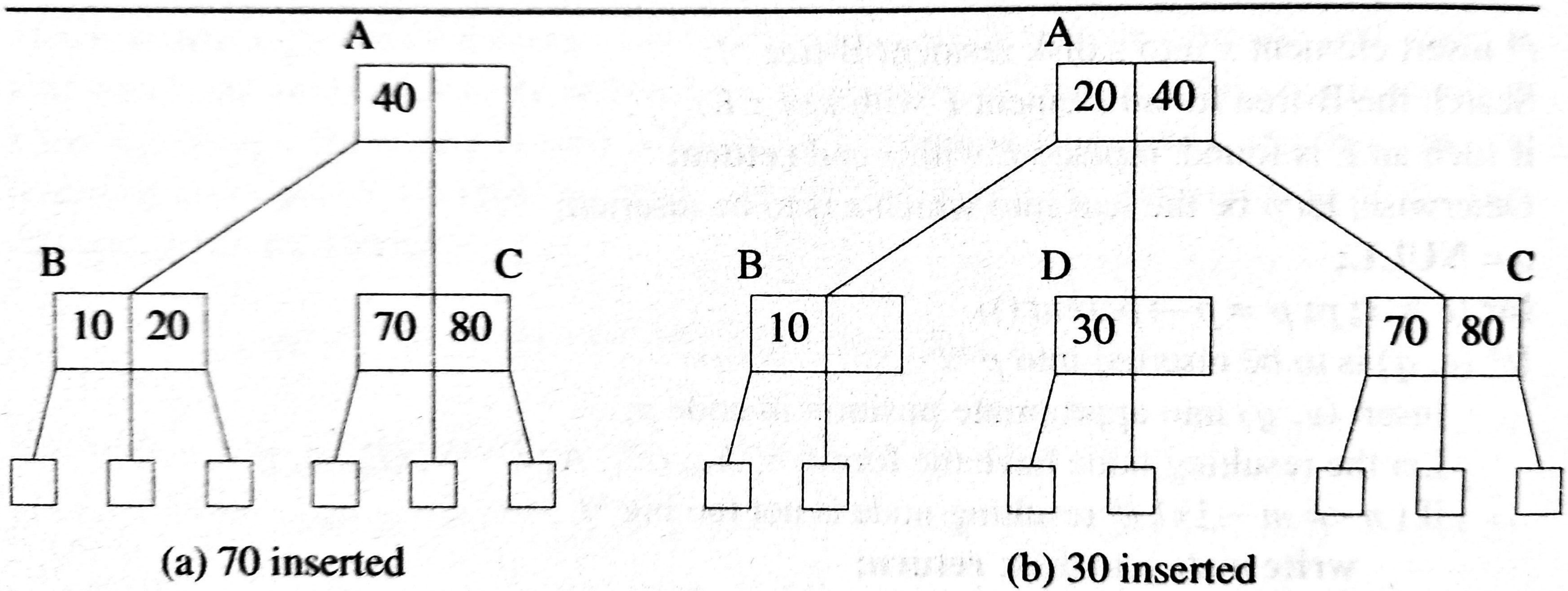
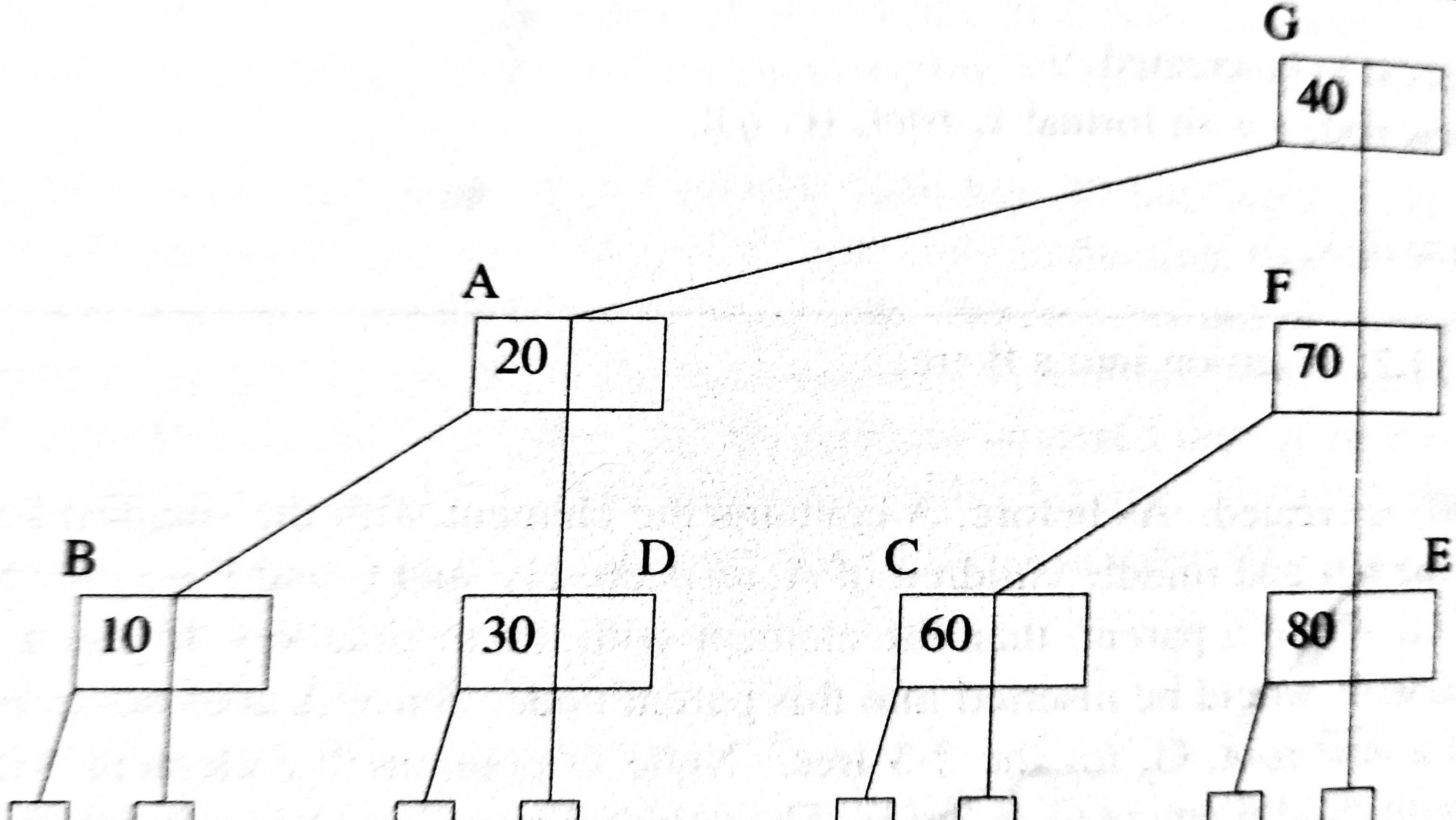
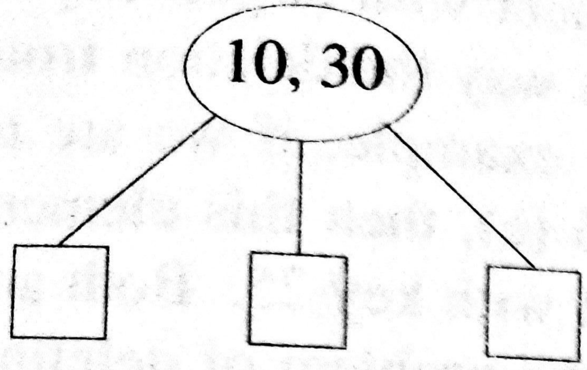


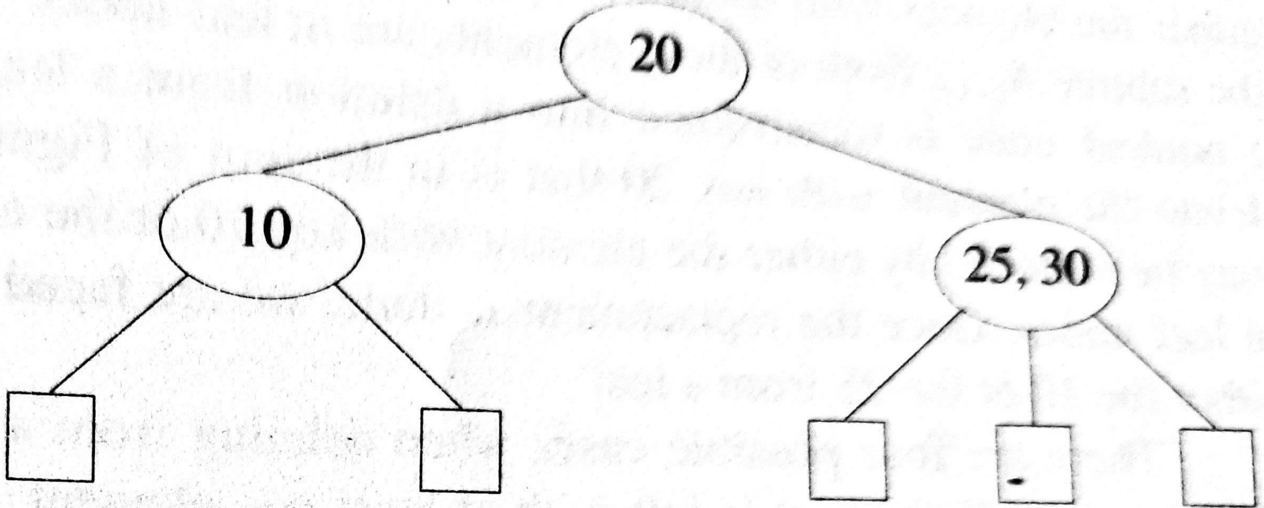
Figure 11.4: Insertion into the 2-3 tree of Figure 11.2



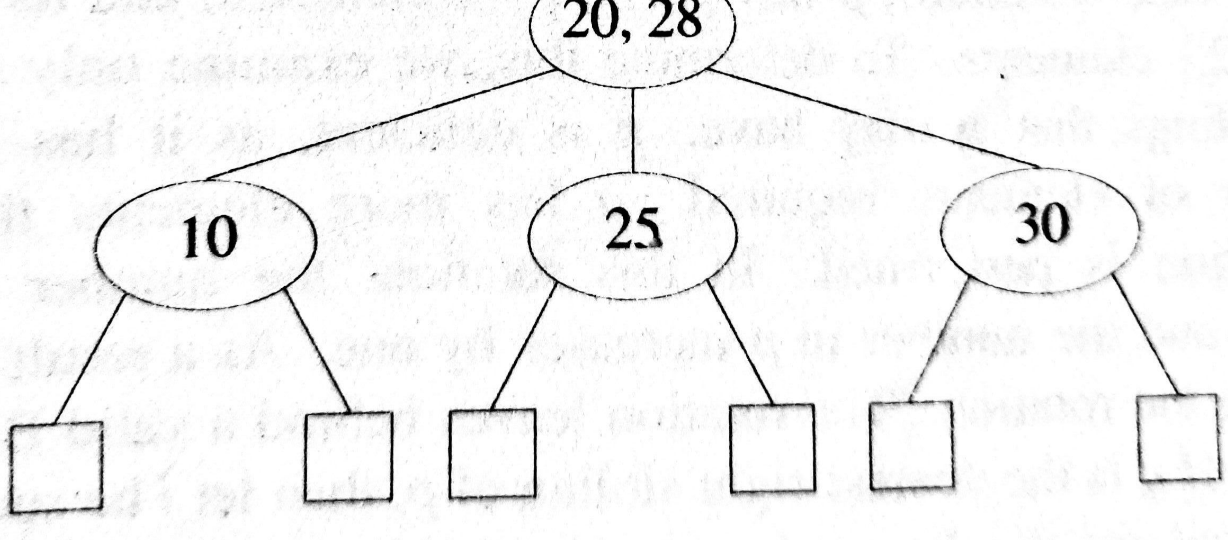
G
40
A
20
F
70
B
10
D
30
C
60
E
80



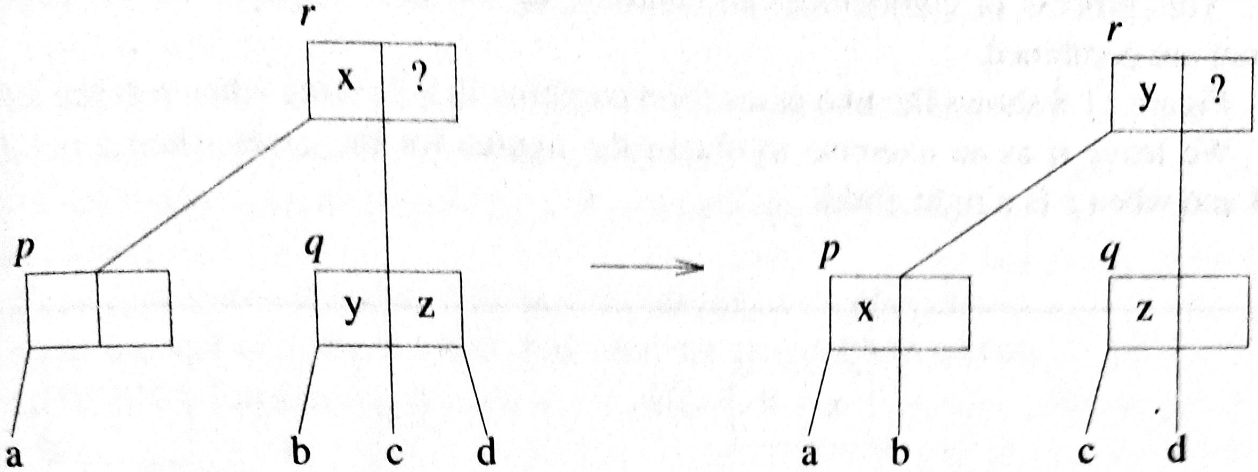
(a) $p = 1, s = 0$



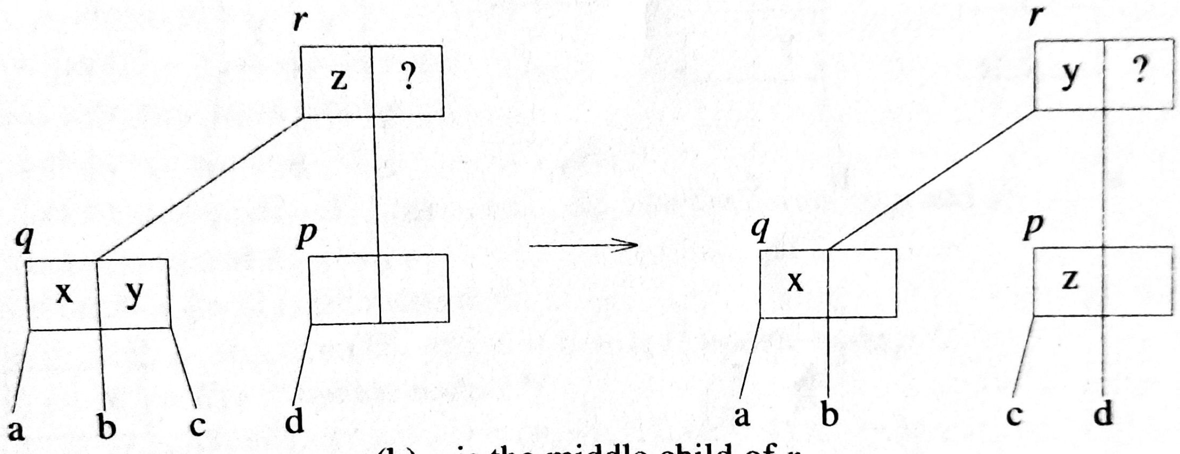
(b) $p = 3, s = 1$



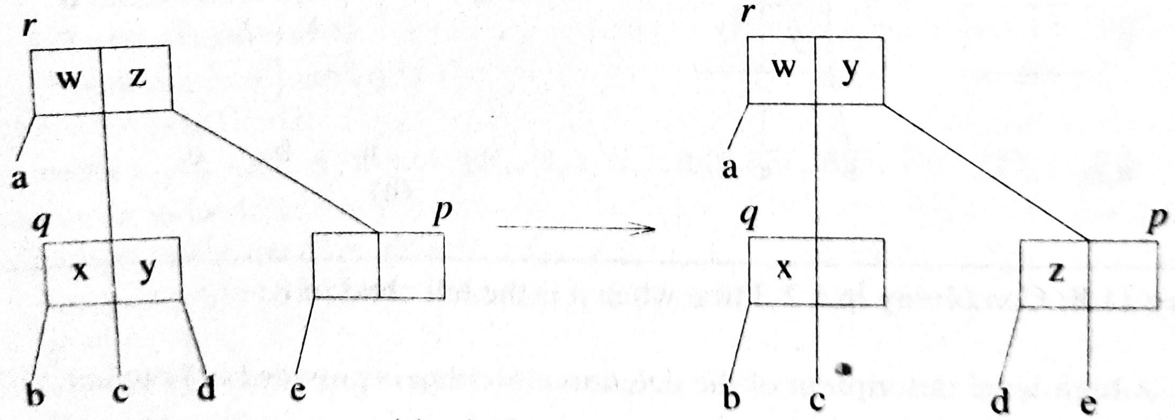
(c) $p = 4, s = 2$



(a) *p* is the left child of *r*



(b) *p* is the middle child of *r*



(c) *p* is the right child of *r*

Figure 11.7: The three cases for rotation in a 2-3 tree

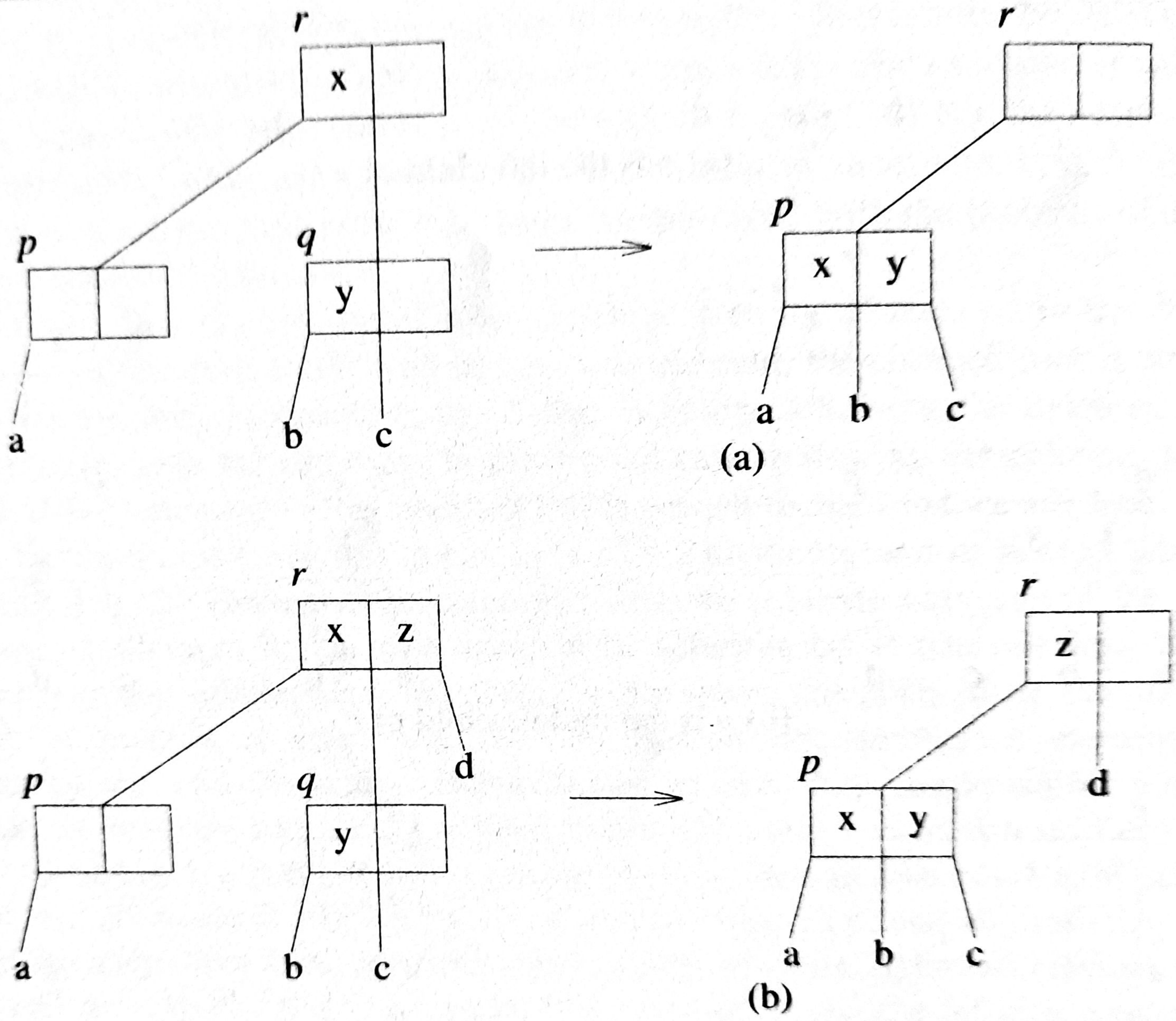


Figure 11.8: Combining in a 2-3 tree when p is the left child of r