** CALCULATION OF ELECTRON DENSITY:

Let $dn \rightarrow$ no. of e^-s whose energy lie in the energy interval E and $E+dE$ in the conduction band.

$$\boxed{dn = Z(E) \cdot f(E) \cdot dE} \quad \text{--- (1)}$$

$Z(E)\,dE \longrightarrow$ density of state in energy interval E & $E+dE$.

$f(E) \Rightarrow$ probability that a state of energy is occupied by an e^-.

$$\boxed{n = \int_{E_c}^{\infty} Z(E)\, f(E)\, dE} \quad \text{--- (2)}$$

density of state

$$Z(E)\,dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2} E^{1/2} dE \quad ; E > E_c$$

- The bottom level of CB(E_c) corresponds to PE of e^- at rest

$E - E_c$ = Kinetic energy of e^- in higher levels of CB.

$$\boxed{\frac{Z(E)}{dE} = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{1/2} dE} \quad \text{—(3)}$$

Probability of an e^-: $\boxed{f(E) = \frac{1}{1+e^{(E-E_F)/K_BT}}}$

no. of available energy states => no. of e^-.

- Fermi dirac distribution function can be approximated to Boltzmann function.

$$\boxed{f(E) = e^{-(E-E_F)/K_BT}} \quad \text{—(4)}$$

Sub (3) & (4) in (2)

$$n = \frac{4\pi}{h^3} (2m^*e)^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_F)/KT} dE$$

$$n = \frac{4\pi}{h^3} (2m^*e)^{3/2} \cdot \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_F+E_c-E_c)/KT} dE$$

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{(E_F-E_c)/KT} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-\frac{(E-E_c)}{KT}} dE \quad \text{—(5)}$$

Above integral is of standard form whose solution is given by

$$\boxed{\int_0^{\infty} x^{1/2} e^{-ax} dx = \frac{\sqrt{\pi}}{2a^{3/2}}}$$

where $a = \frac{1}{KT}$; $x = (E - E_c)$

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{(E_F - E_c)/KT}\left(\frac{\sqrt{\pi}}{4}(KT)^{3/2}\right)$$

$$\boxed{n = 2\left[\frac{2\pi m_e^* KT}{h^2}\right]^{3/2} e^{-(E_c - E_F)/KT}}$$

$$N_c = 2\left[\frac{2\pi m^*_e KT}{h^2}\right]^{3/2}$$

= Effective density of states in CB and its is temp dependent.

In Si, at 300K $N_c = 2.8 \times 10^{25}/m^3$.

$$\boxed{n = N_c e^{-(E_c - E_F)/KT}}$$

for ~~p~~ hole concentration

$$\boxed{p = N_v e^{-(E_F - E_V)/KT}}$$

$n = N_c e^{-(E_c - E_F)/KT}$; $p = N_v e^{-(E_F - E_V)/KT}$

concentration of free e^-s in CB — concentration of holes in VB.

Intrinsi carrier concentration

$$\boxed{n_i^2 = np}$$

* CONCENTRATION OF HOLES :

Let $dp \rightarrow$ no. of holes whose energy lie in the energy interval E & $E+dE$ in the valence band:

$$\boxed{dp = Z(E)\cdot\left(1-f(E)\right)\cdot dE} \quad \text{—(1)}$$

$Z(E)\,dE \longrightarrow$ density of state in energy interval E & $E+dE$

$(1-f(E)) \Rightarrow$ probability that a state of energy is occupied by a hole:

$$\boxed{p = \int_{-\infty}^{E_V} Z(E)\left(1-f(E)\right)\cdot dE} \quad \text{—(2)}$$

$$1-f(E) = 1 - \frac{1}{1+e^{(E-E_F)/K_BT}}$$

$$= \frac{e^{(E-E_F)/K_BT}}{1+e^{(E-E_F)/K_BT}}$$

$$1-f(E) = \frac{1}{e^{-(E-E_F)/K_BT}+1}$$

$$\boxed{1-f(E) = \frac{1}{1+e^{-(E-E_F)/K_BT}}} \quad \text{—(3)}$$

density of state :

$$Z(E)\,dE = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} E^{1/2}\,dE \; ; \quad E<E_V.$$

$E_V - E$ = K.E. of hole in lower energy levels of V.B.

$$\boxed{Z(E)\,dE = \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V - E)^{1/2}\,dE} \quad -④$$

Sub ③ & ④ in ②

$$p = \int_{-\infty}^{E_V} Z(E)(1 - f(E))\,dE$$

$$\boxed{1 - f(E) = e^{(E - E_F)/K_BT}} \quad -③$$

$$p = \int_{-\infty}^{E_V} \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V - E)^{1/2}\, e^{(E - E_F + E_V - E_V)/K_BT}\, dE$$

$$\cancel{p = \int_{-\infty}^{E_V} 4\pi}$$

$$p = \frac{4\pi}{h^3}(2m_h^*)^{3/2}\, e^{E_V - E_F} \int_{-\infty}^{E_V} e^{-(E_V - E)/K_BT} (E_V - E)^{1/2}\, dE$$

$$\boxed{p = \frac{4\pi}{h^3}(2m_h^*)^{3/2} e^{(E_V - E_F)/K_BT} \int_{-\infty}^{E_V} (E_V - E)^{1/2}\, e^{-\frac{(E_V - E)}{K_BT}}\, dE} \quad -⑤$$

$$\int_0^{\infty} x^{1/2} e^{-ax}\, dx = \frac{\sqrt{\pi}}{2a^{3/2}}$$

where $a = \frac{1}{KT}$ $\quad x = (E_V - E)$

$$p = \frac{4\pi}{h^3}(2m_h^*)^{3/2} e^{-(E_F - E_V)/KT} \left[\frac{\sqrt{\pi}(KT)^{3/2}}{2}\right]$$

$$p = \frac{4\pi}{h^3} \cdot 2\left[\frac{2\pi m_h^* KT}{h^2}\right]^{3/2} e^{-(E_F - E_V)/KT}$$

$$N_V = 2\left[\frac{2\pi m_h^* KT}{h^2}\right]^{3/2}$$

= Effective density of states in valence band.

$$\boxed{p = N_V e^{-(E_F - E_V)/KT}}$$

INTRINSIC CARRIER CONCENTRATION:

$$\boxed{n_i = n = p}$$

$$n_i^2 = np$$

$$= N_C N_V \cdot \exp\left[\frac{-\{(E_C - E_F) - (E_F - E_V)\}}{KT}\right]$$

$$= N_C N_V \cdot \exp\left[\frac{(-E_C + \cancel{E_F} - \cancel{E_F} + E_V)}{KT}\right]$$

$$n_i^2 = N_C N_V e^{(E_V - E_C)/K_B T}$$

$$\boxed{n_i^2 = N_C N_V e^{-E_g/K_B T}}$$

$$n_i^2 = 4\left[\frac{2\pi KT}{h^2}\right]^3 (m_e^* m_h^*)^{3/2} e^{-E_g/KT}$$

$$\boxed{n_i = 2\left[\frac{2\pi KT}{h^2}\right]^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2KT}}$$