

Vedic Maths:

Speed Maths Methods:

① Any number $\times 11$

$$\begin{array}{r} 25 \times 11 \\ \downarrow \quad \downarrow \\ 2 \quad 7 \quad 5 \end{array} = 25 \times 11 = 275$$

$$\begin{array}{r} 65 \times 11 \\ \downarrow \quad \downarrow \\ 6 \quad (1) \quad 5 \end{array} = 65 \times 11 = 715$$

② Even number $\times 5$

$$\begin{array}{r} 12 \times 5 \\ \hline 12/2 = 6 \end{array} \Rightarrow 60$$

$$\begin{array}{r} 42 \times 5 \\ \hline 42/2 = 21 \end{array} \Rightarrow 210$$

③ Odd number $\times 5$

$$47 \times 5$$

↓

$$47 - 1 = 46 \frac{1}{2} = 23$$

$$\Rightarrow 235$$

$$99 \times 5$$

$$\downarrow$$

$$99 - 1 = 98 \frac{1}{2} = 49$$

$$\Rightarrow 495$$

④ Square of any number (1-99)

$$34^2 \Rightarrow$$

$$3^2 \quad 4^2 \Rightarrow 3 \times 4 = 12$$

$$\downarrow 916$$

$$34^2 = 1156$$

$$\begin{array}{r} 12 \\ (+) \ 12 \\ \hline 1156 \end{array}$$

$$23^2 \Rightarrow$$

$$2^2 \quad 3^2$$

$$2 \times 3 = 06$$

$$23^2 = 529$$

$$\begin{array}{r} 06 \\ (+) \ 06 \\ \hline 529 \end{array}$$

$$96^2 \Rightarrow$$

$$9^2 \quad 6^2$$

$$9 \times 6 = 54$$

$$96^2 = 9216$$

$$\begin{array}{r} 8136 \\ (+) \ 58 \\ \hline 9216 \end{array}$$

⑤ Square of number ends with 5
(upto 99)

$$35^2 \Rightarrow$$

$$3+1 = 4$$

$$3 \times 4 = 12$$

$$\downarrow 5^2$$

$$35^2 = 1225$$

$$95^2 \Rightarrow$$

$$9+1 = 10$$

$$9 \times 10 = 90$$

$$\downarrow 5^2$$

$$95^2 = 9025$$

Concept of Unit Digits

→ The unit digits are categorized into 3 different categories:

① Unit of one's/base place 0, 1, 5 & 6
 ② Unit digit of base one's place 4 & 9

$$0^{\text{any}} = 0$$

$$1^{\text{any}} = 1$$

$$5^{\text{any}} = 5$$

$$6^{\text{any}} = 6$$

$$4^{\text{any}} = 4^{\text{odd}} = -4$$

$$4^{\text{even}} = -6$$

$$9^{\text{any}} = 9^{\text{odd}} = -9$$

$$9^{\text{even}} = -1$$

③ Unit digit of base of 2, 3, 7 & 8

$$2^{\text{any}} \Rightarrow 2, 4, 6, 8 \dots$$

$$3^{\text{any}} \Rightarrow 3, 9, 7, 1 \dots$$

$$7^{\text{any}} \rightarrow 7, 9, 3, 1 \dots$$

$$8^{\text{any}} \rightarrow 8, 4, 2, 6 \dots$$

As for every 4 numbers the same series repeats, hence the cyclicity is 4

① $254\underline{=69}$ ⇒ 4⁸ is the last digit of the number = 4^{even} ends with = 6

$$\textcircled{2} \quad 589\underline{=475} \Rightarrow 9^5 = 9^{\text{odd}} = 9$$

$$\textcircled{3} \quad 125\underline{=453} \Rightarrow 5^3 = 5$$

$$\textcircled{4} \quad 2356\underline{=458} \Rightarrow 6^8 = 6$$

$$\textcircled{5} \quad 369\underline{=6725} \Rightarrow 9^5 = 9$$

$$\textcircled{6} \quad 897\underline{=126} \Rightarrow 7^6 = 7 \times \frac{31}{4} \text{ rem } 2 \text{ in series} \rightarrow 9$$

$$\textcircled{7} \quad 1458 \xrightarrow{3654} \frac{3654}{4} \quad 813 \quad \text{rem } 2 \rightarrow \frac{2^{\text{nd}} \text{ no in series}}{4}$$

$$\textcircled{8} \quad 393 \xrightarrow{7438} \frac{7438}{4} \quad 1859 \quad \text{rem } 2 \rightarrow 9$$

$$\textcircled{9} \quad 12359 \xrightarrow{129} \frac{129}{4} \quad 32 \quad \text{rem } 1 \rightarrow 8$$

$$\textcircled{10} \quad 36541 \xrightarrow{658} \frac{658}{4} \quad 18 \quad \text{rem } 1 \rightarrow 1$$

$$\#(1!)^1 + (2!)^2 + (3!)^3 + (4!)^4 + (5!)^5 + (6!)^6 +$$

~~(7!)^7 + (8!)^8 + (9!)^9 + (10!)^10~~

$$= 1 + (2)^2 + (6)^3 + (24)^4 + (120)^5 + (720)^6$$

$$= 1 + 4 + 6 + 6 + 0 + 0$$

$$= 7$$

$$\#(1!)^1 + (2!)^2 + (3!)^3 + (4!)^4 + (5!)^5 + (6!)^6 + (7!)^7 + \dots + 100 \text{ terms}$$

$$= 1 + 4 + 6 + 6 + 0 + 0 + 0 + \dots = 7$$

Finding highest Powers

① Find the highest power of 2^n in $10!$

① Method

$$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ \cancel{2} \times \cancel{5} \quad \cancel{2}^3 \times \cancel{2}^3 \times \cancel{2}^2 \times \cancel{2}^1 = 8$$

② Method

$$\frac{10}{2} \rightarrow 5 \Rightarrow \frac{5}{2} \rightarrow 2 \cancel{+ 1} \quad \frac{2}{2} \rightarrow 1$$

$$\Rightarrow 5 + 2 + 1 = 8$$

② Find highest power of 2^n in the

following

$$\textcircled{11} \quad 20! \Rightarrow \frac{20}{2} \overset{\text{quo}}{\rightarrow} 10 \rightarrow \frac{10}{2} \overset{\text{quo}}{=} 5 \rightarrow \frac{5}{2} \overset{\text{quo}}{=} 2 \rightarrow \frac{2}{2} = 1$$

$$10 + 5 + 2 + 1 = 18$$

b) $134!$ ~~lowest digit problem~~

$$\frac{134}{2} = 67 \rightarrow \frac{67}{2} = 33 \rightarrow \frac{33}{2} = 19 \rightarrow \frac{19}{2} = 8$$

$$\frac{8}{2} = 4 \rightarrow \frac{4}{2} = 2 \rightarrow \frac{2}{2} = 1$$

$$\Rightarrow 67 + 33 + 19 + 8 + 4 + 2 + 1 = 131$$

c) $157!$ highest power of 3^n

$$\Rightarrow \frac{157}{3} = 52 \rightarrow \frac{52}{3} = 17 \rightarrow \frac{17}{3} = 5 \rightarrow \frac{5}{3} = 1$$

$$52 + 17 + 5 + 1 = 75$$

find no. of zeros: (Trailing zeros)

$$10 \rightarrow 5^1 \times 2^1 \rightarrow 1 \text{ zero}$$

$$100 \rightarrow 5^2 \times 2^2 \rightarrow 2 \text{ zero}$$

$$1000 \rightarrow 5^3 \times 2^3 \rightarrow 3 \text{ zero}$$

$$\therefore 5^n \times 2^n \rightarrow n \text{ zeroes}$$

① find the no. of zeroes in the following

ⓐ $36! \Rightarrow 5^n$ & 2^n highest

$$\frac{36}{5} = 7 \rightarrow \frac{7}{5} = 1 \quad \frac{36}{2} = 18 \rightarrow \frac{18}{2} = 9$$

$$7+1 \quad \frac{9}{2} = 4 \rightarrow \frac{4}{2} = 2$$

$$\frac{2}{2} = 1$$

$$18+9+4+2+1$$

No. of trailing zeroes = smallest power of any 5 (or) 2

$$= 5^8 \times 2^{34} = 8 \text{ zeroes}$$

$$\textcircled{b} \quad 115 \times 95 \times 64 \times 25 \times 20$$

2010X ~~$\frac{1}{5^5} \times 12\$$~~

$\frac{1}{5} \frac{1}{23} \frac{1}{5} \frac{1}{19} \frac{1}{2^6} \frac{1}{5^2}$ $2^2 \times 5 = 5 \text{ zeroes}$

\textcircled{c} 157!

$$5^n \text{ nlt n zeroes} \quad \frac{2^n}{2^n} \text{ nlt b/wt}$$

$$\frac{157}{5} = 31 \rightarrow \frac{31}{5} = 6$$

$$\frac{6}{5} = 1$$

$$31 + 6 + 1 = 38$$

$$\frac{157}{2} = 78 \rightarrow \frac{78}{2} = 39$$

$$\frac{39}{2} = 19 \rightarrow \frac{19}{2} = 9$$

$$\frac{9}{2} \Rightarrow 4 \rightarrow \frac{4}{2} = 2$$

$$\frac{2}{2} = 1$$

$$78 + 39 + 19 + 9 + 4 + 3$$

$$= 149$$

$$\text{No. of zeroes} = 38$$

No. of factors :

\textcircled{1} Total no. of factors

Given number 'n' can be written

$a^p \times b^q \times c^r$ i.e. a,b,c are prime then the
total no. of factor = $(p+1)(q+1)(r+1)$

a) 450

$$\begin{array}{|r|l|} \hline 2 & 450 \\ \hline 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline 1 & \\ \hline \end{array}$$

$$= 2^1 \times 3^2 \times 5^2$$

$$= (1+1)(2+1)(2+1)$$

$$= 2 \times 3 \times 3$$

$$= 18 \text{ factors}$$

b) $12^2 \times 25 \times 32$

$$12^2 \rightarrow (3 \times 2^2)^2 = 3^2 \times 2^4$$

$$25 \rightarrow 5^2$$

$$32 \rightarrow 2^5$$

$$= 2^9 \times 3^2 \times 5^2$$

$$= (9+1)(2+1)(2+1)$$

$$= 10 \times 3 \times 3$$

$$= 90 \text{ factors}$$

2. No. of odd factors?

considering only the odd bases

a) 450

Total no. of factors = 18 factors

$$\Rightarrow 2 \times 3^2 \times 5^2$$

$$(2+1)(2+1) = 9$$

3. The even factors:

a) 450

Total no. of factors = 18

Odd factors = 9

Even factors = Total - Odd

$$= 18 - 9$$

= 9 factors

b) $12^2 \times 25 \times 32$

Total no. of factors = $90 \rightarrow 2^9 \times 3^2 \times 5^2$

Odd factors = $(2+1)(2+1)$

$$= 9$$

Even factors = Total - Odd

$$= 90 - 9$$

$$= 81 \text{ factors}$$

4. Prime factors:

$$n = a^p \times b^q \times c^r$$

Total no. of prime factors = $p+q+r$

a) $450 = 2 \times 3^2 \times 5^2$

b) $12^2 \times 25 \times 32$

prime factors = $1+2+2$

$$= 2^9 \times 3^2 \times 5^2$$

$$= 5$$

prime factors

$$= 9+2+2$$

$$= 13$$

5. Different prime factors

$$n = a^p \times b^q \times c^r$$

No. of base values $\rightarrow (a, b, c)$ it means 3

$$\text{a) } 450 = 2 \times 3^2 \times 5^2 \quad \text{b) } 12^2 \times 35 \times 32 \\ = 2^9 \times 3^2 \times 5^2$$

Ans. 3 (2,3,5)

Ans. 3 (2,3,5)

- In how many ways can 450 be expressed as product of 2 factors.

- a) 9 b) 12 c) 18 d) 24

for even $\rightarrow \frac{\text{no. of factors}}{2}$

for odd $\rightarrow \frac{\text{no. of factors} + 1}{2}$

Total no. of factors of 450 = 18

$$\text{Ans} = \frac{18}{2} = 9$$

Divisibility Rules:

- For 1 : All are divisible by 1.
 - For 2 : All even numbers are divisible
 - For 3 & 9 : sum of digits divisible by 3 & 9
 - For 4 : Last two digits of the number should be divisible by 4.
 - For 5 : The numbers ending with 5 & 0 are divisible by 5
 - For 6 : The numbers that are divisible by both 2 & 3 (for more than 3 digits)
 - For 7 : what ever, the given number take last three numbers from the remaining the digits existing before them.
- a) 22050 \Rightarrow 22|050
- | | | |
|-----|----|----|
| 050 | 22 | |
| | | 28 |

\therefore 28 is divisible by 7
So, 22050 is also done

$$b) 7965 \mid 123$$

$$\begin{array}{r} 7965 \\ 123 \\ \hline 7 \end{array}$$

$\overline{84} \overset{3}{\cancel{9}}$
 -7
 $\hline 83$

$$7) 835 (119$$

$$\begin{array}{r} -7 \downarrow \\ 13 \\ -7 \downarrow \\ 65 \\ -63 \\ \hline 2 \end{array}$$

It's not divisible by 7

- For 8: last 3 digits of the number is divisible by 8
- For 10: the last digit ending with 0 is divisible by 10
- For 11: in a number, the difference between sum of odd place digits and even place digits must be either 0 or

- divisible by 11.
- For 12: the number should be divisible by both 4 & 3 \Rightarrow co-prime combination $\Rightarrow \text{H.C.F} = 1$
- For 13:
it has same divisibility rule of 7
- For 24:
the numbers which are divisible by 8 and 3.

- For 72:
the numbers which are divisible by 9 and 8
- For 25:
the last two digits are divisible by 25

- 1) 243P51 divisible by 9, then value of P
 a) 3 b) 1 c) 2 d) 4

$$2+4+3+5+1 = 15$$

$15+P$ must be divisible by 9

$$15+P = 18 \Rightarrow 15 + \frac{3}{P} = 18$$

- 2) 42573k divisible by 72, find value of k.
 a) 4 b) 5 c) 6 d) 7

$$4+2+5+7+3 = 21$$

$$21+k = 27$$

$$\Rightarrow k=6$$

- 3) find A and B in 18563145B is divisible by 88

To be divisible by 88, it should be divisible by 8 and 11

$$A+5+3+4+B = A+B+12$$

$$8+6+1+5 = 20$$

$$20 - (A+B+12) = 8 - (A+B) = 0$$

$$A+B = 8$$

- a) 2,6 b) 3,6 c) 4,6 d) 2,4

$$2+6 \Rightarrow A+B$$

The only no in between 450 to 459 is "456" is divisible by "8"

$$\text{so, } B=6$$

- 4) If 41895A8 is divisible by 44. find 'A'
 a) 2 b) 4 c) 6 d) 8

$$4 \quad 1 \quad 8 \quad 9 \quad 5 \quad A \quad 8 \Rightarrow (4+16+5)-(10+A)=10 \\ 25-10-A=10$$

$$\begin{array}{r} 12 \\ 48 \\ \hline A \end{array} \Rightarrow \text{divisible by 4}$$

$$A=18-11 \\ A=4$$

- 5) A number when divided by 342 gives a remainder 47. When the same number is divided by 19, what would be the remainder?

- a) 3 b) 6 c) 7 d) 9

Given, "an unknown number"

$$\text{Remainder} = 47$$

Again divided by a num. 19.

$$\frac{47}{19} = 9$$

- 6) A number when divided by 5 leaves a remainder 3. What is the remainder when the square of the same number is divided by 5?

- a) 9 b) 5 c) 0 d) 4

(S) $342 \sqrt[n]{n}$

$$n = \frac{342q}{19} + \frac{47}{19}$$

$$\Downarrow$$

$$0 + 9 \Rightarrow 9!$$

~~$\sqrt[n]{n}$~~ ~~$\sqrt[n]{n^2}$~~

~~$\sqrt[5]{n}$~~ ~~$\sqrt[5]{n^2}$~~

~~$n = 5q + 3$~~

~~$n^2 = 25q^2 + 30q + 9$~~

~~$(5q+3)^2 = 25q^2 + 30q + 9$~~

~~$25q^2 + 25q + 9 + 20q + 3 = 25q^2 + 48q + 12$~~

⑥ $\bullet \frac{8}{5} = 3 \quad \frac{64}{5} = 4$

$\bullet \frac{13}{5} = 3 \quad \frac{169}{5} = 4$

$$\bullet \frac{18}{5} = 3 \Rightarrow \frac{8^2}{5} = \frac{324}{5} = 4$$

$$\bullet \frac{23}{5} = 3 \Rightarrow \frac{0409}{5} = 4$$

Hence the remainder = 4

Remainder theorem

$$1. \frac{14 \times 15}{8}$$

$$8 - 14 \rightarrow -6$$

$$8 - 15 \rightarrow -7$$

$$\frac{-6 \times -7}{8} = \frac{42}{8}$$

Remainder = 2

$$2. \frac{69 \times 63 \times 64}{66}$$

$$-66 + 69 \rightarrow 6$$

$$-66 + 63 \rightarrow 3$$

$$-66 + 64 \rightarrow 2$$

$$\frac{-6 \times -3 \times -2}{66} = \frac{-24}{66} \Rightarrow 66 - 24 = 42$$

Remainder = 42

$$3. \frac{988}{8} \Rightarrow \frac{988}{8} \Rightarrow \text{Power is even}$$

$$\frac{9}{8} \Rightarrow 9 - 8 = 1$$

$$4. \frac{899}{7} \Rightarrow \frac{8 \times 118 + 1}{7} \text{ (using } 8^3 = 512)$$

$$\text{Remainder} = 8$$

$$5. \frac{899}{7} \Rightarrow \frac{8}{7} = 1$$

$$\text{Remainder} = 1$$

$$6. \frac{9^{100}}{7} \Rightarrow \frac{9}{7} = 9 - 7 = 2$$

$\frac{2^{100}}{7}$ (Numerator should be near to denominator $\frac{6}{7}$)

$$\frac{2^{99} \cdot 2}{7} = \frac{(2^3)^{33} \cdot 2}{7} = \frac{8^{33} \times 2}{7}$$

$$8 - 7 = 1 \quad 2 - 7 = -5 \text{ (or) } 2$$

$$\therefore \text{Remainder} = 2 \times 1$$

7. Find the remainder of 11^8 when it is divisible by 7.

$$\frac{11^8}{7} \Rightarrow 11 - 7 = 4 \quad \frac{4^8}{7} = \frac{2^{16}}{7} = \frac{(2^3)^5 \cdot 2}{7}$$

$$\frac{8^5 \cdot 2}{7} = \frac{8}{7} \times \frac{2}{7}$$

$$\text{Remainder} = 1 \times 2 = 2$$

8. Find the remainder when 21^{875} is divided

by 17

$$\frac{21^{875}}{17} = \frac{21}{17} = 21 - 17 = 4$$

$$\frac{4^{875}}{17} = \frac{(2)^{875 \times 2}}{17} = \frac{(2^4)^{437} \times 4}{17}$$

$$= \frac{(16)^{437} \times 4}{17} = \frac{16}{17} \times \frac{4}{7}$$

$$\Rightarrow \frac{-4}{7} \Rightarrow 17 - 4 = 13,$$

$$9. 3^{87} + 5^{87} \text{ divided by } 26$$

$$3^{87} = (3^3)^{29} = (27)^{29}$$

$$5^{87} = (5^2)^{43} \times 5 = 25^{43} \times 5$$

$$\frac{3^{87} + 5^{87}}{26} = \frac{(27)^{29} + (25)^{43} \times 5}{26}$$

$$= \frac{27}{26} + \frac{25 \times 5}{26}$$

$$= 1 + \underline{(-1) \times 5}$$

$$26 - 5 = 21 \quad \rightarrow$$

$$\text{Remainder} = 1 + 21 = 22$$

10. Find the remainder when $1! + 2! + 3! + 4! + 5! + \dots + 99! + 100!$ is divided by the product of first 7 natural numbers.

$$\Rightarrow \frac{1! + 2! + 3! + 4! + 5! + \dots + 99! + 100!}{7!}$$

$\frac{1}{7!}$

$$= \frac{1}{7!} + \frac{2!}{7!} + \frac{3!}{7!} + \frac{4!}{7!} + \frac{5!}{7!} + \frac{6!}{7!} \\ + \left[\frac{7!}{7!} + \dots + \frac{99!}{7!} + \frac{100!}{7!} \right]$$

Rem: 0

$$= \frac{1}{7} + \frac{2}{7} + \frac{6}{7} + \frac{24}{7} + \frac{120}{7} + \frac{720}{7} + 0 + 0 \\ + \dots + 0$$

$$= 1 + 2 + 6 + 24 + 120 + 720 = 873$$

$$\Rightarrow \frac{873}{7!} = \frac{873}{5040} = 873 \text{ (Remainder)}$$

$$(ii) \frac{37 \times 43 \times 57}{7}$$

$$\frac{37}{7} = ② \quad \frac{43}{7} = 1 \quad \frac{57}{7} = 1$$

$$\text{Remainder} = 2 \times 1 \times 1$$

$$= 2 \quad \square$$

* The number obtained by interchanging the two digits of a 2-digit number is more than the original number by 27. If the sum of the two digits is 13. What is original number

- a) 67 b) 76 c) 85 d) 58

~~(@)~~ 76 - 67 = 9 @ 85 - 58 = 27

~~(@)~~ 76 - 67 = 9 @ 58 - 85 = 27

larger number when

* A number being successively divided by 3, 5 and 8 leaves remainder 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed.

Let the unknown number be 'n'

~~(@)~~ 3, 5, 8 q's
 ↓ ↓ ↓
 1 4 7 r's
 ← order

$$\Rightarrow n = 8q + 7 \quad \Rightarrow \frac{120q + 118}{8}$$

$$3(5(8q+7)+4) + 1$$

$$= 3(40q + 35 + 4) + 1 \quad \Rightarrow \frac{118}{8} = 6$$

$$= 120q + 117 + 1 \quad \frac{15q + 14}{5} = 4$$

$$= 120q + 118 \quad \frac{3q + 2}{3} = 2 \text{ (R)}$$

The Remainders : 6, 4, 2

* A number being successively divided by 5, 4 and 3 leaves remainder 2, 2 and 1 respectively. Find the respective remainders if the order of divisors be reversed.

$$\begin{array}{c} 5, 4, 3 \\ \downarrow \quad \downarrow \quad \downarrow \\ 2 \quad 2 \quad 1 \end{array} \quad \begin{array}{c} 3 \sqrt{ } n \\ \hline \quad \quad \quad 1 \end{array}$$

order

$$n = 3q + 1$$

$$5(4(3q+1) + 2) + 2 = 5(12q + 6) + 2$$

$$= 60q + 32$$

$$\frac{60q+32}{3} = (20q+10)3 + \underline{\underline{2}}$$

$$\frac{20q+10}{4} = (5q+2)4 + \underline{\underline{2}}$$

$$\frac{5q+2}{5} = 2(\text{Re})$$

Remainders : 2, 2, 2

* Finding last two digits:

when base ends with '1':

$$(41)^{\underline{\underline{293}}} = 4 \times 3 = \underline{\underline{12}}$$

so last two digits : 21

\Rightarrow when a given num base ends with '1'
last digit is always '1' and last two
second digit is beside the unit place num

multiples with the power unit place and
the product's unit digit.

* Squares:

$$89^2 = (100 - 89)$$

$= 11^2$ is the last two digits (121)

$$= \underline{\underline{21}}$$

$$(81)^2 = (100 - 81)$$

$$64^2 = (64 - 50)$$

$$= 14^2 (196)$$

$$= 19^2 (361)$$

$$= \underline{\underline{61}}$$

$$\textcircled{1} \quad (19\underline{\underline{6}}1)^{\underline{\underline{3843}}} + (19\underline{\underline{6}}1)^{\underline{\underline{481}}}$$

$$\downarrow \quad \downarrow$$

$$\underline{\underline{21}} \quad + \quad \underline{\underline{61}}$$

$$\Rightarrow \underline{\underline{82}} \text{ last two digits}$$

* when the base number ends with 3, 9, 7, 1

$$\begin{aligned} 3 &\rightarrow \underline{\underline{03}} \\ 3^2 &\rightarrow \underline{\underline{09}} \\ 3^3 &\rightarrow \underline{\underline{27}} \\ 3^4 &\rightarrow \underline{\underline{81}} \end{aligned}$$

$$\begin{aligned} 9 &\rightarrow 9 \\ 9^2 &\rightarrow \underline{\underline{81}} \end{aligned}$$

$$\begin{aligned} 7 &\rightarrow 7 \\ 7^2 &\rightarrow \underline{\underline{49}} \\ 7^3 &\rightarrow \underline{\underline{343}} \\ 7^4 &\rightarrow \underline{\underline{2401}} \end{aligned}$$

$$① 3^{100}$$

$$\Rightarrow (3^4)^{25} = (\underline{\underline{81}})^{25} = \underline{\underline{01}}$$

$$② 7^{2008}$$

$$\Rightarrow (7^4)^{502} = (\underline{\underline{2401}})^{502}$$

$$③ 9^{264} \Rightarrow (9^2)^{132} = (\underline{\underline{81}})^{132} = \underline{\underline{01}}$$

$$\begin{aligned} ④ 19^{266} &\Rightarrow (19^2)^{133} = (\underline{\underline{361}})^{133} = \underline{\underline{81}} \\ ⑤ 33^{288} &\Rightarrow (33^4)^{72} = (\underline{\underline{21}})^{72} = \underline{\underline{41}} \\ ⑥ 87^{474} & (100 - 87)^{474} = (\underline{\underline{13}})^{237} \\ &= (\underline{\underline{869}})^{237} \\ &= ((\underline{\underline{69}})^2)^{118} \times \underline{\underline{1}} \\ &= (\underline{\underline{61}})^{118} \times \underline{\underline{1}} \\ &= (-81) \times \underline{\underline{1}} \\ &= \underline{\underline{89}} \end{aligned}$$

* when the base number is '2'

$$(2^{10})^1 \rightarrow 24 \text{ (odd)}$$

$$(2^{10})^2 \rightarrow 76 \text{ (even)}$$

Hint: $2^n \times 76 \rightarrow (n > 2)$ if
 ⇒ the last two digits are same as

$$\textcircled{1} \quad 8 \times 76 \quad \text{so, last 2 digits is } 08$$

$$2^3 \times 76$$

$$\textcircled{2} \quad 64 \times 76 \quad 2^6 \times 76$$

so, last 2 digits is 64

$$\textcircled{3} \quad (64)^{236} = (2^6)^{236}$$

$$= 2^{1410} \times 2^6$$

$$= (2^{10})^{141} \times 2^6$$

$$= (-76) \times 64$$

last two digits are 64

$$\textcircled{4} \quad (62)^{586}$$

$$(31 \times 2)^{586} = (31)^{586} \times (2)^{586}$$

$$= (-81) \times 76 \times 64$$

$$\begin{array}{r} 64 \\ \times 81 \\ \hline 64 \\ 512 \end{array}$$

$$= (-81) \times 64$$

$$= (-84)$$

$$\textcircled{5} \quad (54)^{380} = (2 \times 27)^{380}$$

$$= (2^{10})^{38} \times (3^3)^{380}$$

$$= 76 \times 3^{1140}$$

$$= 76 \times (81)^{285}$$

$$= 76 \times (-01)$$

Last two digits are 76
 * when the base number ends with 5

$$\text{odd} \quad (a5)^{ny} \rightarrow \text{odd} \rightarrow 75$$

$$\text{odd} \quad (a5)^{ny} \rightarrow \text{even} \rightarrow 25$$

$$\text{even} \quad (a5)^{ny} \rightarrow \text{odd} \rightarrow 25$$

$$\text{even} \quad (a5)^{ny} \rightarrow \text{even} \rightarrow 75$$

$$\text{even} \quad (a5)^{ny} \rightarrow \text{odd} \rightarrow 25$$

① $\underline{125}$ ⑥ 5 → odd
even

last 2 digits : 25 last 2 digits : 75

③ $\underline{35}$ ③ 2 → even
odd
last 2 digits : 25 last 2 digits : 75

LCM & HCF :

* L.C.M of $\frac{x}{a,b}$ × H.C.F of $\frac{y}{a,b}$ = $a^* b$

* L.C.M of a fraction = $\frac{\text{LCM of Num}}{\text{HCF of Den}}$

* H.C.F of a fraction = $\frac{\text{HCF of Num}}{\text{LCM of Den}}$

LCM = highest power of common factors (product)

H.C.F = lowest power of common factors (product)

② $\underline{75}$ ③ 5 → odd
odd

① find the LCM & HCF of 144, 180, 192

$$144 \rightarrow 12 \times 12 \quad 180 \rightarrow 2^3 \times 3 \times 5 \\ = 2^4 \times 3^2$$

$$192 \rightarrow 32 \times 6 \\ = 2^6 \times 3$$

$$\text{H.C.F} = 2^2 \times 3 = 12$$

$$\text{L.C.M} = 2^6 \times 3^2 \times 5 = 2880$$

② what is the least number by which 2800 should be multiplied so that product may be perfect square

* Perfect sq. number = (prime) even

* Perfect cu. num = (prime) multiple of 3

* Perfect sq. of cu number = (prime) 7 must be

$2800 = 7 \times 5^2 \times 2^4$ multiplied to get

③ The HCF of two numbers is 4 and their LCM is 576. If one of the numbers is 64, find the other number?

$$4 \times 576 = 64 \times x$$

$$\Rightarrow x = \frac{4 \times 576}{64}$$

$$x = 36$$

④ If HCF and LCM of 2 numbers are 13 and 455. If one of the numbers is 75 and 125. Then find the other number?

$$\text{HCF} = 13 \quad \text{LCM} = 455$$

$$455 = 13 \times 35$$

$$13 \times 5$$

$$5 \quad 7$$

$$13 \times 7$$

$$(65) \quad (91)$$

Unit : 2

- ① Ratio ② Ages ③ Average
- ④ Partnership ⑤ Alligation or mixtures

Ratio :

1. $A:B = 3:4$ and $B:C = 5:6$ then
 $A:B:C$ is ?

Method - 1

$$\begin{array}{ll} A:B & B:C \\ 3:4 & 5:6 \\ (\times 5) & (\times 4) \end{array}$$

$$15 \quad 20 \quad 24$$

$$A:B:C = 15:20:24$$

Method - 2

$$A:B:C$$

$$3:4:6$$

$$15:20:24$$

$$= A:B:C$$

2. $A:B = 2:3$ and, $B:C = 4:5$ and
 $C:D = 6:7$ then $A:B:C:D$ is ?

$$A:B:C:D$$

S: times

$$2:3$$

$$\rightarrow 4:5$$

$$\rightarrow 6:7$$

$$8:12:15$$

$$\rightarrow 6:7$$

$$48:72:90:105$$

3. The sum of 3 numbers is 98. If the ratio of the first to second is 2:3 and that of the second to the third 5:8, then the second number is?

$$A+B+C = 98$$

$$10n + 15n + 24n$$

$$= (10+15+24)n = 98$$

$$49n = 98$$

$$n = 2$$

$$\therefore \text{Second number} = 15n$$

$$= 30$$

$$A:B:C$$

$$2:3 \rightarrow 5:8$$

$$10:15:24$$

4. If the ratio of the ages of two friends A and B is the ratio 3:5 and that of B and C is 3:5 and the sum of the ages is 147, then how old is B?

$$A:B = 3:5 = B:C$$

$$A:B:C$$

$$9:15:25 \Rightarrow 9+15+25 = 49$$

$$(49) \times 3 = 147$$

$$\text{Given, } A+B+C = 147$$

The age of B is $= 15 \times 3 = 45$ yrs
5. The ratio of money with Ram and Gopal is 7:17 and that with Gopal and Krishna is 7:17, if Ram has Rs 490, Krishna has?

$$R:G = 7:17 = G:K$$

$$49:119:289$$

$$R:G:K$$

$$49n = 490 \Rightarrow n = 10$$

Krishna has

$$\text{Rs } 2890$$

6. It was intended that Rs. 117 be divided among Chinky, Pinky and Rinky in the ratio 4:3:2 but by mistake the distribution was made in the proportion $\frac{1}{4} : \frac{1}{3} : \frac{1}{2}$. How much does Rinky gain by error?

Pinky $\rightarrow P$ Rinky $\rightarrow R$ Chinky $\rightarrow C$

$$\text{Given, } C+P+R = 117$$

$$C:P:R \\ 4:3:2 \rightarrow \frac{1}{4 \times 3}, \frac{1}{3 \times 4}, \frac{1}{2 \times 6} \\ 3:4:6$$

Actual:

$$C:P:R = 4:3:2$$

$$C+P+R = 4+3+2 = 9$$

$$\text{Rinky's part} = \frac{2}{9} \times 117$$

$$= ₹ 26$$

$$\text{Gain} = 154 - 26 = ₹ 28$$

Error:

$$C:P:R = 3:4:6$$

$$C+P+R = 13$$

$$\text{Rinky's part} = \frac{6}{13} \times 117$$

$$= ₹ 54$$

7. Rs. 10250 is divided among Arvind, Akash & Raghav in such a way that if Rs 75, Rs. 125 & Rs. 50 be deducted from their respective shares, they have them in the ratio 2:3:5. Then Akash has?

Arvind $\rightarrow A$ Akash $\rightarrow B$ Raghav $\rightarrow C$

Given,

$$A+B+C = 10250$$

$$A:B:C = 2:3:5 \Rightarrow 2+3+5 = 10$$

$$\text{Deducted money} = 125 + 75 + 50$$

$$= 250$$

$$\text{Actual money} = 10250 - 250 = 10000$$

$$\text{Akash's share} = \frac{3}{10} \times 10,000 = 3000$$

$$\text{Actual Akash's share} = 3000 + 125$$

$$= ₹ 3125$$

8. Two numbers are in the ratio 3:5 if 9 is subtracted from each, the new numbers are in the ratio of 12:23. The small number is?

a) 28

b) 33

c) 49

d) 55

Method - 1: $\frac{3x-9}{5x-9} = \frac{12}{23} \Rightarrow 69x - 207 = 60x - 108$

$$9x = 207 - 108$$

$$9x = 99 \Rightarrow x = 11$$

Method - 2

I (smallest)

$$\underbrace{3 : 2}_{\times 11} \quad 5$$

II (largest)

$$\rightarrow 33 : 55$$

$$\underbrace{12 : 23}_{\times 2} \rightarrow 24 : 46$$

The smallest number is 33

9. The ratio of two natural numbers is 8:11 if a number is added to both the numbers the ratio become 13:16. If the larger number exceeds the number by 36. Find the number added?

First $\left[\begin{array}{l} 8 : 11 \\ \times 2 \\ \hline 16 : 22 \end{array} \right] \times 5 \quad \frac{36}{3} = 12$

Second $\left[\begin{array}{l} 13 : 16 \\ \times 3 \\ \hline 39 : 48 \end{array} \right] \times 5 \quad 12 \times 5 = 60$

Larger number exceeds by smallest number by 36

\therefore 60 is the number added
10. Two numbers are in the ratio 5:11 if 8 is added to both the numbers they will be in the ratio 1:2. What is the sum of two numbers?

Method - 1

$$\frac{5x+8}{11x+8} = \frac{1}{2}$$

$$10x + 16 = 11x + 8$$

$$x = 8$$

$$5(8) + 11(8) = 16(8)$$

$$= 128$$

Method - 2

$$\left[\begin{array}{l} 5 : 11 \\ \times 6 \\ \hline 30 : 66 \end{array} \right] \times 8 \quad \left[\begin{array}{l} 1 : 2 \\ \times 6 \\ \hline 6 : 12 \end{array} \right] \times 8$$

$$5 + 11 = 16 \times 8$$

$$= 128$$

Path Analysis:

When a same number is added to both numbers then the difference between the ratio is same, then we're going to multiply it with sum when it's added.

→ If same number is not added or subtracted to both sides of ratio then this method isn't applicable.

11. Two numbers are in the ratio 4:9, if 4 is added to both the numbers then the new ratio becomes 21:46. What is the difference between A & B?

$$\frac{4x+4}{9x+4} = \frac{21}{46}$$

$$184x + 184 = 189x + 84$$

$$184 - 84 = 5x$$

$$5x = 100 \Rightarrow (x=20)$$

$$\begin{array}{rcl} 4:9 & \xrightarrow{\times 4} & 20:45 \\ 5x5 & & 5x5 \\ 21:46 & \xrightarrow{\downarrow 25} & 21:46 \end{array}$$

exceeds by '4'

$$4x = 80, 9x = 180$$

$$\text{Difference} = 180 - 80 \\ = 100$$

$$20 \times 4 = 80$$

$$45 \times 4 = 180$$

$$\text{diff} = 180 - 80$$

$$= 100$$

12. A mixture contains milk and water in the ratio 8:3 on adding 3 liters of water then the ratio becomes 2:1. The quantity of milk in the mixture is?

a. 24

b. 22

c. 26

d. 20

Method - 1

milk = 8 parts

Ans must be divisible

by '8'

Ans: 24

Method - 2

$$\frac{8x}{3x+3} = \frac{2}{1}$$

$$8x = 6x + 6$$

$$2x = 6$$

$$x = 3$$

$$\text{Ans} = 8 \times 3 = 24$$

"24 liters of milk
in the mixture"

13. In a box the ratio b/w the number of cashews and the no. of grapes is 7:13. If 8 cashews and 11 grapes are eaten the ratio between no. of cashews and the no. of grapes become 1:2. Find the original number of cashews?

- a) 65 b) 35 c) 40 d) 30

Method - 1

$$\frac{7x-8}{13x-11} = \frac{1}{2}$$

$$14x - 16 = 13x - 11$$

$$x = 16 - 11 = 5$$

$$7(5) = 35 \text{ (Ans)}$$

14. Seats for Mathematics, Physics and Biology in a school are in ratio 5:7:8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of seats after increased?

- a. 2:3:4 b. 6:7:8 c. 6:8:9 d. none of these

Maths - M. Physics - P. Biology - B

Given M:P:B = 5:7:8

500 700 800 (let)

$$\begin{array}{cccc}
 40\% & \downarrow & 50\% & \downarrow \\
 +40 \times \frac{500}{100} = 200 & & +50 \times \frac{700}{100} = 350 & \\
 700 & 1050 & 1400 & \\
 \end{array}
 \Rightarrow \frac{300}{14} : \frac{1050}{21} : \frac{1400}{28} = 2 : 3 : 4$$

15. A Bag contains Rs.600 in the form of 1 rupee, 50 paise, and 25 paise coins in the ratio 3:4:12. The no. of 25 paise coins are?

- a) 899 b) 905 c) 895 d) 900

Rs : 50 paise : 25 paise
25 paise = 12

Answer should be divisible by 12

Ans. 900

$$\begin{array}{ccc}
 3 & : & 4 \\
 \downarrow & & \downarrow \\
 3RS & : & 2RS \\
 3+3+2 = 8RS & &
 \end{array}$$

Total rupees = 600

$$\text{No. of 25 paise coins} = \frac{600}{8} \times 12$$

$$= 75 \times 12 = 900$$

Problems on Ages:

↑ ago/hence

Present age

↓ after/later

Remainder: always calculate from present age. (option elimination)

1. Ram's age is 3 times of Anil. In 12 yrs
Ram's age will be double the age of Anil
Ram's present age is
- a. 27 b. 32 c. 36 d. 40

82ⁿ method:

Ram Anil
Present: $3x$ x

after 12 yrs: $3x+12$ $x+12$

$$\Rightarrow 2y \quad y$$

$$3x+12 = 2y$$

$$x+12 = y$$

$$2y - 3x = 12$$

$$2y - 2x = 24$$

$$-x = -12$$

$$x = 12$$

Ram's present age = $3x = 36$ yrs

2. Ravi's age is 6 times of Arun. After 18 years Ravi's age will be three times that the age of Arun. Ravi's present age is
- a. 69 b. 72 c. 84 d. 62

Ravi	Arun
present	$6x$
after 18 yrs	$6x+18$
	$x+18$
	$3y$
	y

$$6x+18 = 3y$$

$$x+18 = y$$

$$3y - 6x = 18$$

$$3y - 3x = 54$$

$$+3x = 36$$

$$x = 12$$

Ravi's age = $12(6)$
 $= 72$

3. Present age of Sahul and Amar are in the ratio of 3:2 respectively after 3 yrs. Hence the ratio of their ages will be 9:7. What will be Sahul's present age.

Path analysis:

Sahul : Amar \rightarrow Sahul's age is 6 yrs

$$\text{present} \quad 3 : 2 = ⑥ : 4 \quad | \cdot 3$$

$$\text{After 3 yrs} \quad 9 : 7 = 9 : 7$$

$$\text{equation method: } 2 \rightarrow 3(2) = \text{Sahul's age} \quad (3x+3)7 = (2x+3)9$$

$$\begin{array}{l} \text{present} \quad ③x \quad 2x \\ \text{After 3 yrs} \quad 3x+3 \quad 2x+3 \\ \qquad\qquad\qquad 9 \quad 7 \end{array} \quad 21x + 21 = 18x + 27$$

$$3x = 6 \quad | \div 3$$

$$x = 2$$

4. Present age of Sateesh and Ajay are in the ratio of 2:5 respectively. Six years hence, the ratio of their ages will become 7:13. what is Sateesh's present age?

- a) 8 b) 16 c) 9 d) 15

Eq^n method: Sateesh Ajay
 present: $2 : 5$ $④ : ⑩$ $| \cdot 3 \times 2$
 After 6 yrs: $7 : 13$ $7 : 13$

$$\frac{2x+6}{5x+6} = \frac{7}{13} \Rightarrow 13(2x+6) = 7(5x+6)$$

$$\Rightarrow 26x + 78 = 35x + 42 \Rightarrow 9x = 36$$

$$x = 4$$

Sateesh age = 2x
 $= 2(4) = 8$
 $x = 8$

5. Present age of Sarathi and Anand are in the ratio of 7:8 respectively. Eight years ago, the ratio of their ages became 5:6. What is Anand's present age?

- a) 32 b) 46 c) 12 d) 24

$$\begin{array}{l} ⑤ \quad 1 \quad ① \\ | \cdot 8 \quad | \cdot 4 \\ 7x : 8] \cdot 4 \quad 1 \end{array}$$

$$7x : 8 \quad | \cdot 2 \times 4$$

$$5 : 6$$

$$\text{Eq^n method:}$$

$$\frac{7x+8}{8x+8} = \frac{5}{6}$$

$$(7x+8)6 = 5(8x+8)$$

$$42x - 40 = 40x - 40$$

$$2x = 8 \Rightarrow x = 4$$

Anand's age:
 $8(x) = 8 \times 4 = 32$ yrs

6. Sum of ages of father and daughter is 50 yrs. Before 5 yrs, father was 7 times as old than present age of father and daughter, then present age of father and daughter is

- a. 35, 15 b. 38, 12 c. 40, 10 d. 42, 8

$$f + D = 50 \quad 7D = F \\ (\text{at present}) \quad (5 \text{ yrs ago})$$

$$40 - 5 = 35 = F \quad 7(D) = f$$

$$10 - 5 = 5 = D \quad 7(5) = 35$$

\Rightarrow Hence ages as per option is 40, 10

7. Sum of ages of mother and son is 86 yrs. 5 years ago mother was 3 times as son, then present age of mother is

- a. 35 b. 38 c. 40 d. 62

Given that,

$$\text{sum of mother \& son ages} = 86 \\ 86 - 62 = 24 \checkmark \quad 86 - 40 = 46$$

$$62 - 5 = 57$$

$$24 - 5 = 19$$

$$\text{The } 3(18) = 54$$

$$3(19) = 57$$

Hence, verified.

8. The ages of two persons differ by 10 yrs. If 6 yrs ago the elder one was twice as old as younger one. Find their present age of younger one

Given that, Elder younger

$$\text{present : } x - y = 10 \\ 6 \text{ yrs ago : } x - 6 \quad y - 6$$

$$x - 6 = 2(y - 6)$$

$$x - 2y = -6 \Rightarrow 2y - x = 6 \\ -y + x = 10$$

\therefore the age of younger = 16 yrs $y = 16$

9. ~~The present age of~~ Before 10 yrs her mother's age is 4 times older than her daughter. After 10 yrs the mother will be twice older than her daughter. Present age of A is

- A. 5 B. 10 C. 20 D. 30

Before 10 yrs $x-10$ $y-10$ $(x-10) = 4(y-10)$

Present x y

After 10 yrs $x+10$ $y+10$ $(x+10) = 2(y+10)$

$$\Rightarrow x-10 = 4y - 40 \Rightarrow 4y - x = 30$$

$$x+10 = 2y + 20 \quad \cancel{x-2y = 10}$$

$$2y = 40$$

$$y = 40/2 = 20 \text{ yrs}$$

Present age of A (daughter) = 20 yrs

10. After 5 years the age of father will be thrice the age of his son, whereas 5 years ago he was 7 times as old as his son. Father present age is.

- A. 35 B. 40 C. 45 D. 50

5 yrs ago : $x-5$ $y-5$ $\Rightarrow x-5 = 7(y-5)$

present : x y

5 yrs after : $x+5$ $y+5$ $x+5 = 3(y+5)$

~~$x-7y = 30$~~

$$x-7y = 30$$

~~$x-3y = 10$~~

$$4y = 40$$

$$y = 10 \text{ yrs}$$

$$x-5 = 7(5)$$

$$x = 35 + 5$$

$$x = 40 \text{ yrs}$$

Father's present age is 40 yrs

11. The total age of A and B is 16 yrs more than the total age of B and C. C is how many years younger than A

$$A+B' = B+C+16$$

$$A-C = 16$$

\Rightarrow 16 yrs younger than A

12. The average age of 3 pupil is 18 years
And the ages are in proportion of 1:2:3. The ratio of ages of pupil after 2 yrs is?
- a. 15:20:3 b. 11:20:29 c. 18:14:78
d. 25:23:24

Given Rat,

$$A:B:C = 1:2:3$$

$$\frac{A+B+C}{3} = 18$$

$$A+B+C = 54$$

$$(1+2+3)x = 54$$

$$(x=9)$$

$$A's \text{ age} = x = 9$$

$$B's \text{ age} = 2x = 18$$

$$C's \text{ age} = 3x = 27$$

Present ages : 9: 18: 27

After 2 yrs : 9+2: 18+2: 27+2
11: 20: 29

13. Anand is younger than Roopa by 5 years. If their ages are in ratio of 3:2, how old is Roopa?

Given that,

Difference of their ages = 5 yrs

Ratio of their ages = 3:2

Roopa $\downarrow \times 5$ Anand $\downarrow \times 5$

15 : 10

Roopa's age = 15 years

Anand's age = 10 years

Partnerships

Simple Partnership:

Ratio = Invest of A : Invest of B :
Invest of C

$$6300 : 4200 : 10500 \\ 3 : 2 : 5$$

$$\text{share of A in profit} = \frac{3}{10} \times 12100 \\ = \text{Rs } 3630$$

Compound Partnership:

Ratio = Invest of A * Time : Ratio :

* A, B and C invested Rs 6300, Rs 4200 and Rs 10500 respectively, in a partnership business. Find the share of A in profit of Rs. 12100 after a year?

- A. Rs. 3630 B. Rs 2840 C. Rs 3200
 D. Rs. 5600

* A, B and C invested Rs. 38000, Rs 40000 and Rs 42000 respectively, in a partnership business. find the share of A in profit of Rs 54000 after a year

$$38000 : 40000 : 42000$$

$$19 : 20 : 21$$

$$\frac{19}{80} \times \frac{900}{54000} = 17100$$

* A, B and C started a business in which A invested Rs 10000/- for 1 year, B invested Rs. 20000/- for 2 years, C invested Rs 30000/- for 3 years. At the end the profit received by them is Rs. Rs. 5600/- what is C's share

$$10,000 : 40,000 : 90,000$$

$$1 : 4 : 9$$

$$\frac{9}{14} \times 5400 = 3600$$

* Raj invested Rs 76000 in a business. After few months Mani joined him and invests Rs. 57000. At the end of year both of them share the profit at the ratio of 2:1. After how many months Mani joined with Raj?

$$\frac{76000 \times 12^6}{57000 \times x} = \frac{2}{1}$$

$$12 - 8 = 4 \text{ months}$$

$$\frac{76 \times 6}{57} = x$$

$$x = 8$$

* A and B entered into a partnership investing Rs. 28000 and Rs. 30000 respectively. After 4 months C also joined the business with an investment of Rs. 35000. What is the share of C in

an annual profit of Rs 47000?
 A. Rs 18000 B. Rs 18000 C. Rs 17000
 D. Rs 14000

$$\begin{array}{rcl} 25000 & : & 30000 : 35000 \\ \times 12 & & \times 12 & \times 8 \\ 15 & : & 18 & : 14 \end{array}$$

$$\frac{14}{47} \times \frac{10,000}{47000} = 14,000$$

*A and B entered into a partnership investing Rs 16000 and Rs 12000 respectively. After 3 months A withdraw Rs 5000 while B invests Rs 5000 more, after 3 more months C joined in the business with an investment of Rs 21000. What is

the share of C in an annual profit of Rs 52800?
 $(16000 \times 3) + (11000 \times 9)$
 $: (12000 \times 3) + (17000 \times 9)$
 $: 21000 \times 6$

$$48000 + 99000 : 36000 + 153000$$

$$\begin{array}{rcl} 1 & & 126000 \\ \frac{48}{99} & & \\ \hline 147 & & \end{array} \quad \begin{array}{rcl} 147 & : & 189 : 126 \\ 49 & 93 & 42 \\ \hline 189 & & \end{array}$$

The share of C

$$= \frac{42}{184} \times 52800$$

$$= 14400$$

* A, B and C enter into a partnership & their shares are in the ratio $1/2 : 1/3 : 1/4$. After 2 months, A withdraws half of his capital and after 10 months, a profit of ~~Rs. 378~~³⁷⁸ is divided among them. Find

B's share?

$$6 : 4 : 3$$

$$\begin{matrix} 6 \times 2 \\ + \\ 3 \times 10 \end{matrix} \quad 4 \times 12 : 3 \times 12$$

$$46 : 48 : 36$$

$$7 : 8 : 6$$

$$\frac{8}{21} \times \frac{18}{378} = \text{Rs } 144\frac{1}{3}$$

$$\frac{168}{2} \quad \frac{21}{8}$$

* A, B and C invested capitals in the ratio 7 : 3 : 2. At the end of business term, they received the profits in ratio 2 : 3 : 7. Find the ratio of time for which they contributed their capital.

A	B	C
7	3	2
$7x$	$3y$	$2z$

$$\frac{7x}{3y} = \frac{2}{3} \quad \frac{3y}{2z} = \frac{3}{7}$$

$$\begin{matrix} x : 4 \\ 2 : 7 \end{matrix} \quad \begin{matrix} y : 7 \\ 2 : 7 \end{matrix}$$

$$4 : 14 : 49$$

A and B invest in a business in the ratio 3:2. If 5% of the total profit goes to charity and A's share is Rs. 855, the total profit is:

A : B	profit	charity	Total profit
(57) 3 : 2 (38)	95	5	100
<u>5</u> [<u>95</u>] <u>95</u>	$\Rightarrow 57 \times 15 = 855$	$\sqrt{15}$	1800

Averages:

* A family consists of 2 grandparents, two parents and three grandchildren. The average age of the grandparents is 67 yrs, that of the parents is 35 yrs and that of grandchildren is 8 yrs. What is the

$$\begin{array}{ll}
 \text{avg age of the family} & \text{sum} \\
 a = 67 & 67 \times 2 = 134 \\
 & 35 \\
 & 8 \\
 & 8 \times 3 = \frac{24}{228} \\
 & \Rightarrow \frac{228}{7} = 32.5
 \end{array}$$

* The average salary of all the workers in a workshop is Rs. 8000. The avg salary of 7 technicians is Rs 12000 and the avg salary of the rest is Rs. 6000. The total no. of workers in the workshop is?

$$\begin{array}{l}
 a = 8000 \quad (\text{Total} = n) \Rightarrow 800n \\
 7 \text{ tech} = 12000 \quad \Rightarrow 84000 \\
 600n - 42000
 \end{array}$$

$$8000\gamma = 84000 + 6000\gamma - 42000$$

$$2000 \times 2 = 4200$$

$$\Rightarrow n = 21$$

* 9 people went to a hotel for combined dinner. 8 people out of 9 spend Rs. 30 each on dinner. 9th person spend Rs. 20 more than the average of all 9 persons. Find the total expenditure on dinner.

$$\text{avg} = a$$

$$8 \times 30 = 240$$

$$\frac{240 + [a+20]}{9} = a$$

$$260 = 80$$

$$a = \frac{32.5}{g}$$

$$a = 32.5$$

~~Total exp = 292 est~~

$$\text{Total exp} = 240 + 525$$

$$= 292.5$$

* In an exam, the avg marks for 80 students of class V is 35. The avg of marks in section A of class is 55 while the avg of marks in section B is 30. Find the no. of students in VB

$$\begin{array}{c}
 80 \text{ students} \rightarrow \begin{array}{l} a \\ 35 \end{array} \quad \begin{array}{l} \text{option} \\ \text{eli:} \\ \hline 30+55 \\ \hline 2 \end{array} \\
 \swarrow \quad \searrow \\
 \begin{array}{l} \text{Sec A} \\ \text{Sec B} \end{array} \rightarrow \begin{array}{l} 30 \\ 55 \end{array} = \frac{85}{2}
 \end{array}$$

$$\frac{d}{80} = 35 \quad n = 2800$$

$$\begin{array}{r}
 80 \\
 \downarrow \quad \downarrow \\
 16 \quad 64 \\
 \Rightarrow \quad \begin{array}{r}
 30 \\
 \times 64 \\
 \hline
 1920 \\
 + 880 \\
 \hline
 2800
 \end{array}
 \end{array}$$

* The avg age of 12 students in a group is 14 yrs. When teacher's age is included to it, the avg rises by 2.
What's the teacher's age

$$\frac{x}{12} = 14 \quad x = 168$$

$$\frac{168+x}{13} = 16$$

$$168+x = 16 \times 13$$

$$x = -168 + 208$$

$$\begin{array}{r}
 16 \\
 \times 13 \\
 \hline
 208 \\
 -168 \\
 \hline
 40
 \end{array}$$

$$x = 40 \text{ yrs (teacher's age)}$$

* The avg weight of 15 students in a grp is 20 kgs. When Sameera's weight is included to it, the avg increase by 5. What is sameera's weight in kgs.

$$\begin{aligned}
 x &= 20 \times 15 \\
 &= 300
 \end{aligned}$$

$$\begin{array}{r}
 300 + x = 25 \\
 \hline
 16
 \end{array}$$

$$300 + x = 400$$

$$\boxed{x = 100 \text{ kgs}}$$

* The avg age of 4 brothers and their family is 11 yrs. When father's age is

excluded. The avg decreases by 8.
What is father's age?

avg = a

$$5 \times 11 = 55 \text{ yrs}$$

$$4 \times 6 = 24$$

$$\begin{array}{r} 55 \\ - 24 \\ \hline 31 \text{ yrs} \end{array}$$

* In a class, there are 20 Boys whose avg is decreased by 2 months, when one boy aged 18 yrs is replaced by a new boy. Find the age of new boy

$$\text{avg} = a$$

$$\frac{20a - 18 + x}{20} = a - \frac{2}{12} \text{ yrs}$$

$$20a - 18 + x = 20a - \frac{4}{12} \times 20 \text{ yrs}$$

$$x = 14 \text{ yrs } 3 \text{ months}$$

* The avg weight of 8 persons increases by 1.5 kg. If a person weighing 65 kg is replaced by a new person, what could be the weight of new persons?

$$\frac{\text{sum}}{8} = a + 1.5$$

$$\frac{\text{sum} - 65 + x}{8} = a + \frac{65 + x}{8}$$

$$(a + 1.5) 8 - 65 + x$$

$$\text{avg} = a$$

$$\frac{8a - 65 + x}{8} = a + 1.5$$

$$8a - 68 + x = 8a + 12$$

$$\boxed{x = 77}$$

* The avg age of 8 persons in a committee is increased by 2 yrs when two men aged 35 yrs and 45 yrs are substituted by two women. Find the avg age of these two women.

$$\text{avg} = a$$

$$\frac{8a + x - 35 - 45}{8} = a + 2$$

$$8a + x - 80 = 8a + 16$$

$$x = 96$$

$$\text{avg age} = \frac{96}{2} = 48$$

* A mathematics teacher tabulated the marks secured by 35 students of 8th class. The avg of their marks was 72. If the marks secured by Reema was written as 36 instead of 86 then find correct avg marks upto two decimal places

- a) 73.41
- b) 74.31
- c) 72.43
- d) 73.43

$$[72 \times 35] - 36 + 86$$

$$\frac{[72 \times 35] \times 50}{35} = 72 + \frac{50}{35}$$
$$= 72 + \frac{10}{7}$$
$$= 73.43$$